Numerical Simulation of Axion Dynamics in the Early Universe

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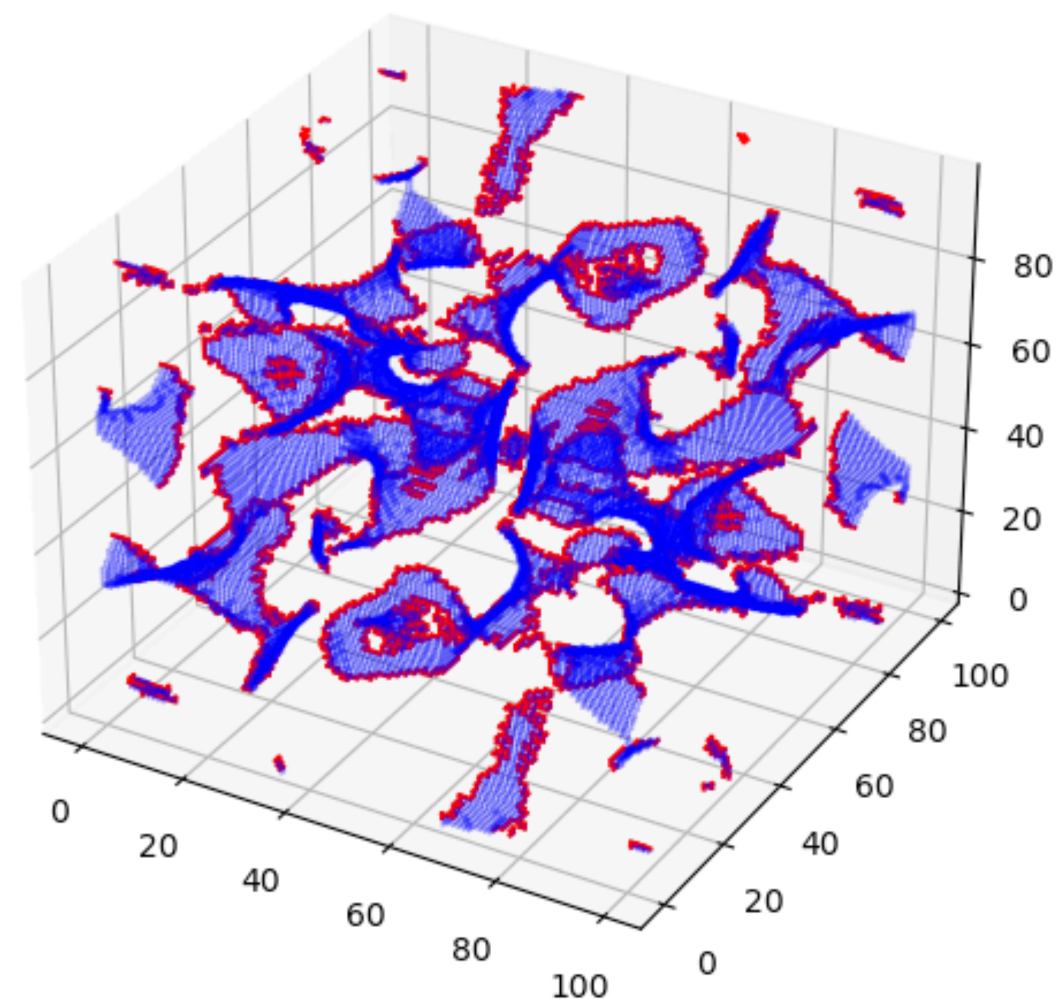
Final Project for Cosmology 212

Main Reference:
M. Buschmann, J.W. Foster, B.R. Safdi
arxiv.org/abs/1906.00967



25 April 2023

Axions in Cosmology

- Attractive dark matter candidate that also solves the strong CP problem

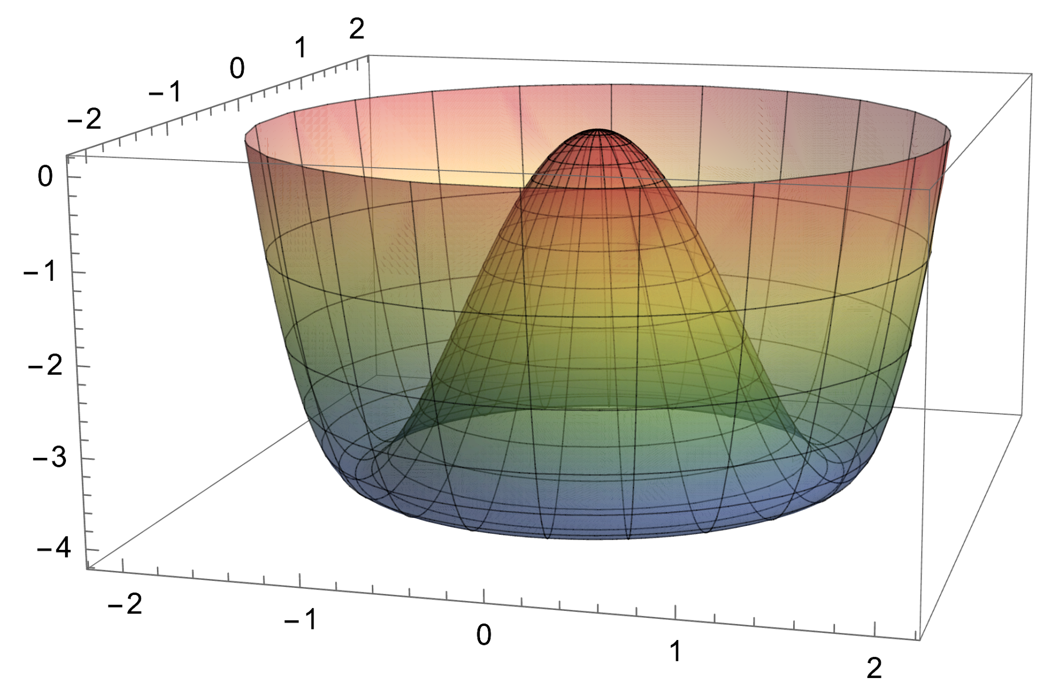
$$\mathcal{L} \supset \left(\theta + \frac{a}{f_a}\right) \frac{g^2}{32\pi^2} \tilde{F}_{\mu\nu} F^{\mu\nu}$$

- Introduce complex scalar ϕ that has global U(1) Peccei-Quinn symmetry, that gets spontaneously broken so $\phi \sim f_a e^{i\frac{a}{f_a}}$

- Two cosmological scenarios:

  - Pre-inflationary PQ breaking (uniform field, no clumps)

  - Post-inflationary PQ breaking (axion clumps, and potential experimental detection! We simulate this scenario)



Axion potential after Peccei-Quinn breaking, before coupling to QCD

Cosmic History

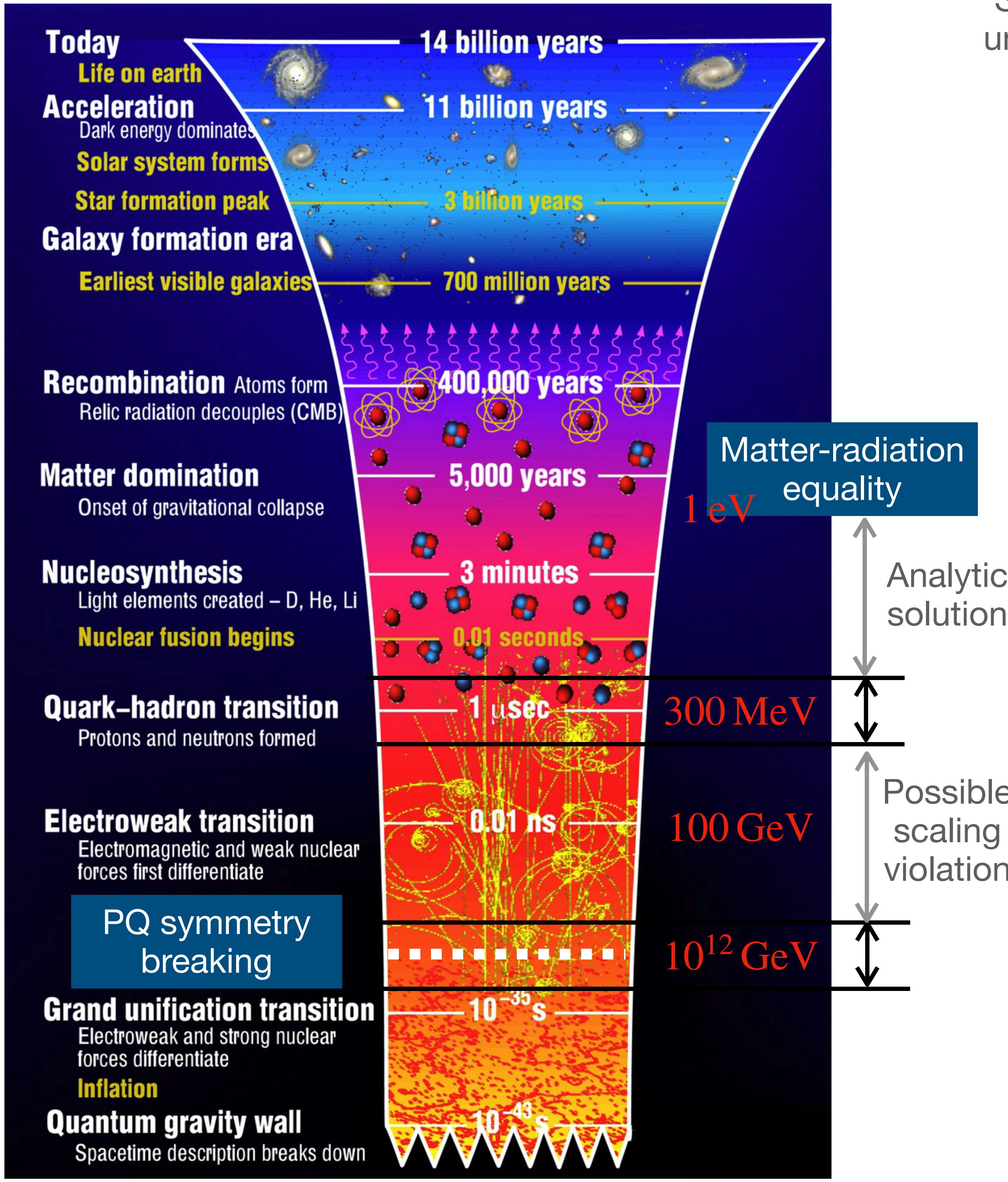


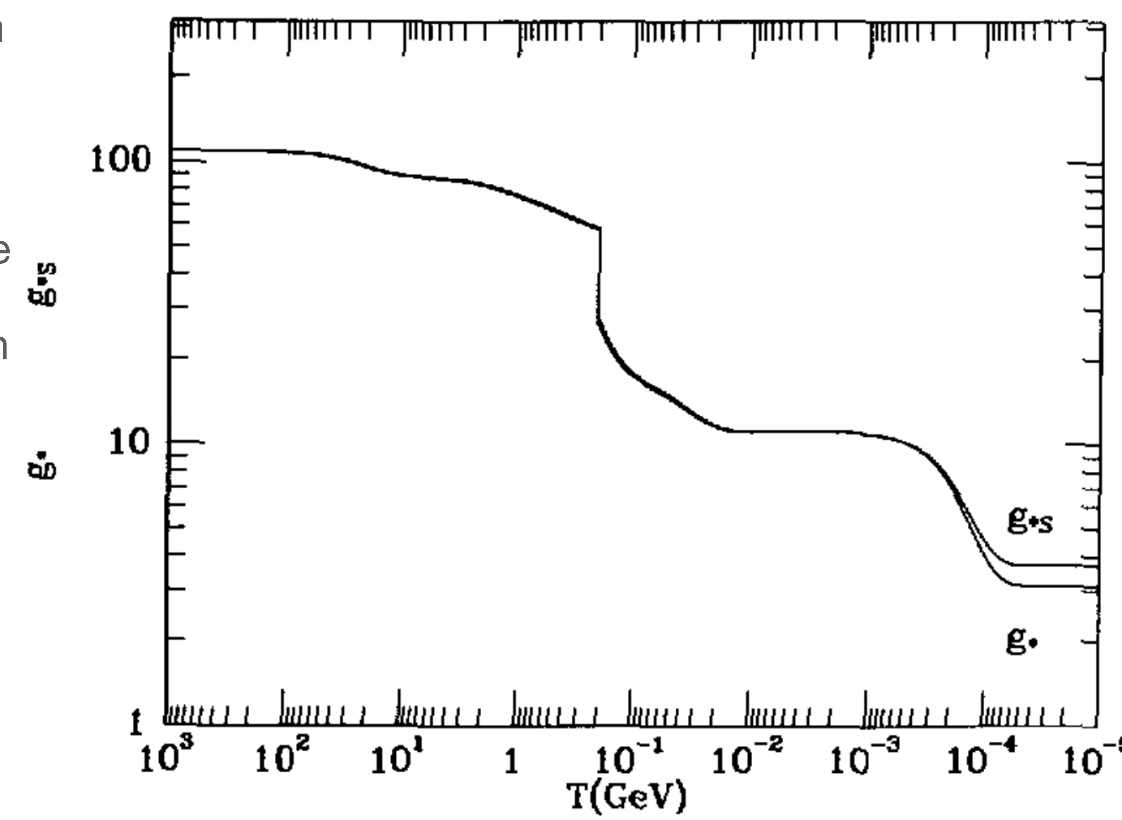
Image credit:
https://www.ctc.cam.ac.uk/outreach/origins/big_bang_three.php

Since we work in post-inflationary, but still radiation-dominated universe, cosmological equations are fairly simple to write down:

$$H = \sqrt{\frac{4\pi^3}{45}} g_*^{\frac{1}{2}} T^2 m_{\text{Planck}}^{-1} \approx 1.660 g_*^{\frac{1}{2}} T^2 m_{\text{Planck}}^{-1}$$

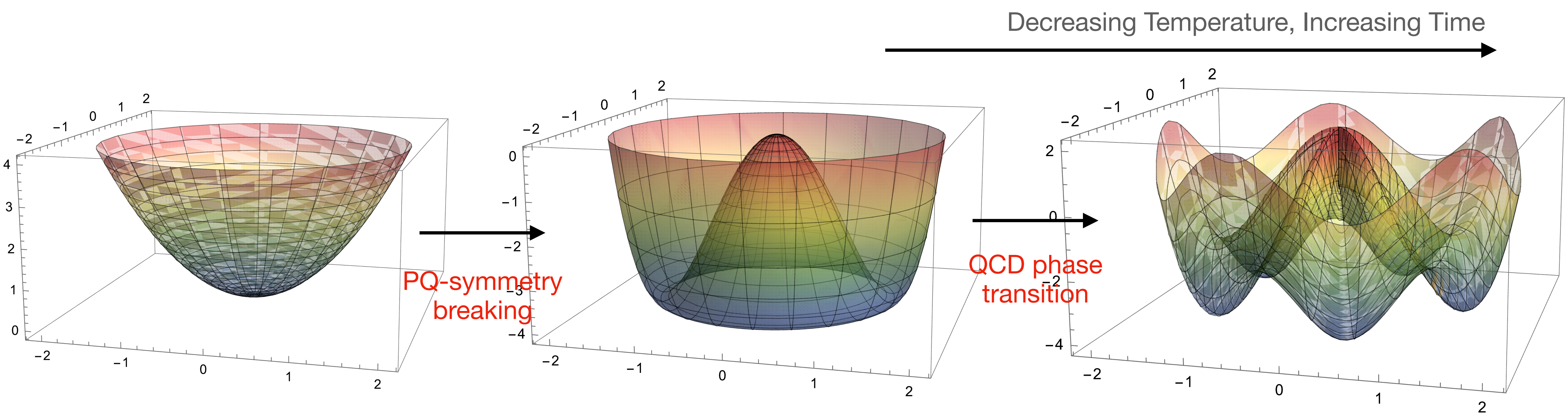
$$\eta^{PQ} := \frac{R}{R_1^{PQ}} = \left(\frac{45}{4\pi^3 g_*}\right)^{\frac{1}{4}} \sqrt{f_a m_{\text{Pl}}}\, T^{-1}$$

$$g_* = N_{\text{bosons}} + \frac{7}{8} N_{\text{fermions}}$$



The Early Universe by Kolb & Turner

Physical Expectations



$N = 5$

- Lagrangian that governs axion dynamics

$$\mathcal{L}_a = \frac{1}{2}|\partial_\mu \Phi|^2 - \frac{\lambda}{4}(|\Phi|^2 - f_a^2)^2 - \frac{\lambda T^2}{6}|\Phi|^2 - \frac{m_a(T)^2 f_a^2}{N^2}[1 - \cos(N \mathrm{Arg}(\Phi))]$$

Nonperturbative model

- Topological defects (stable if $N > 1$, otherwise decay):
    - Strings (1 dim.): the field winds around the strings ($a \to a + 2\pi$)
    - Domain walls (2 dim.): separate different vacua
- Oscillons (3 dim.): large overdensities cause by nonlinearity

Algorithm Details

- Initial condition: thermal distribution

$$\langle \phi_i(\vec{k})\phi_j(\vec{k}')\rangle = \frac{(2\pi)^3 n_k}{\omega_k}\delta(\vec{k}+\vec{k}')\delta_{ij}$$

$$\langle \dot{\phi}_i(\vec{k})\dot{\phi}_j(\vec{k}')\rangle = (2\pi)^3 n_k \omega_k \delta(\vec{k}+\vec{k}')\delta_{ij}$$

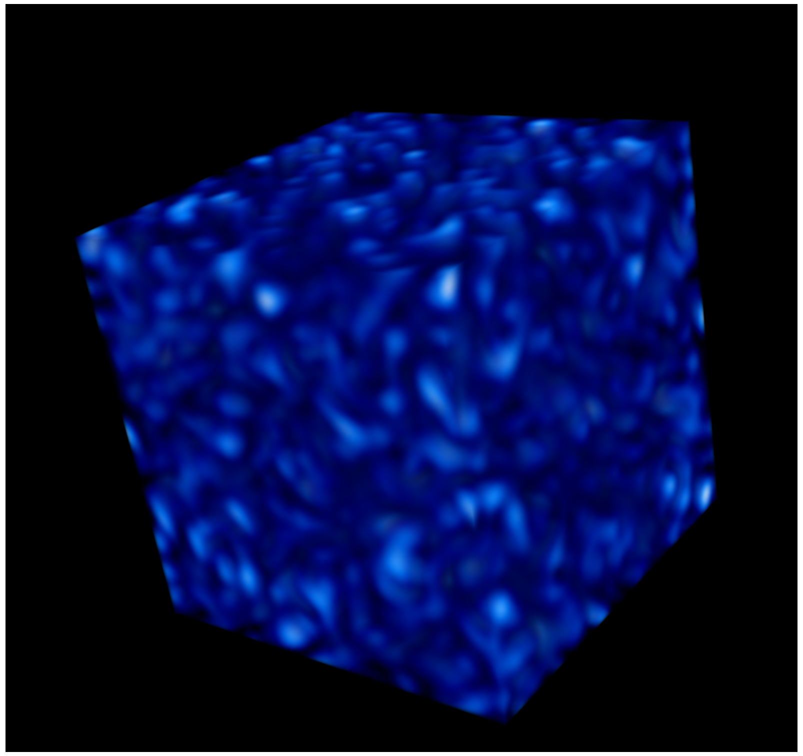
$$n_k = \frac{1}{e^{\omega_k/T}-1}, \quad \omega_k = \sqrt{|\vec{k}|^2 + m_{\text{eff}}^2},$$

- Equation of motion simulated via RKN:

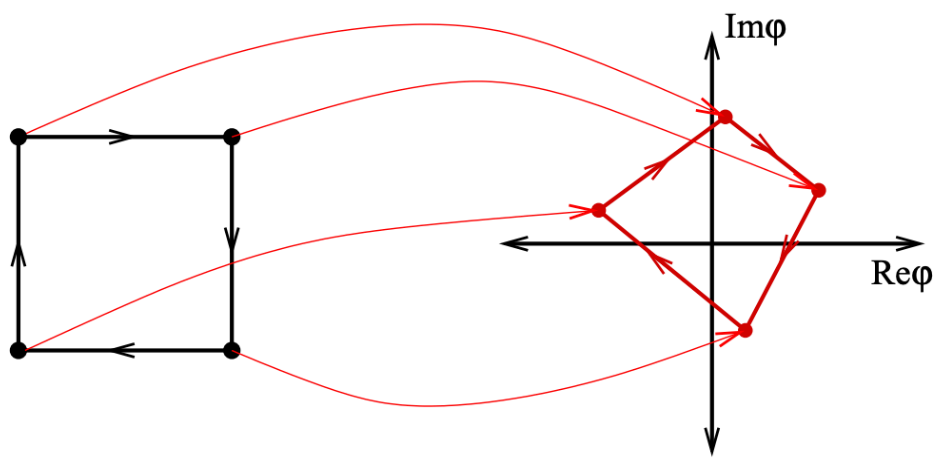
$$\left[\frac{\partial^2}{\partial t^2} + 3H(t)\frac{\partial}{\partial t} - \frac{1}{R(t)^2}\nabla^2\right]\Phi(\vec{x},t) = \frac{\partial V_{\text{eff}}}{\partial \Phi^*}$$

- 100^3 comoving grid, 25 Hubble length physical size (at $H = f_a$)

- String and Domain Wall detection for $\mathrm{N} = 1$ using "naive" algorithm shown on right

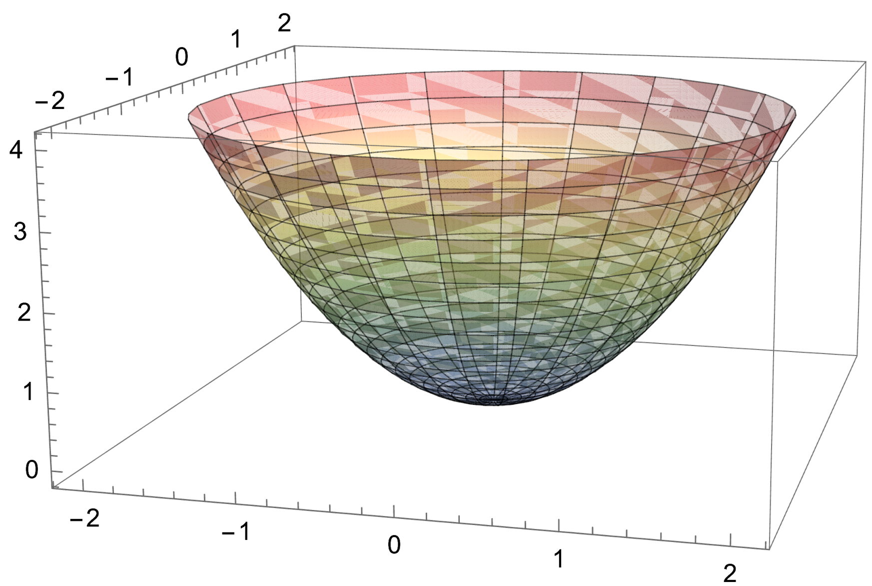


Example of thermal configuration

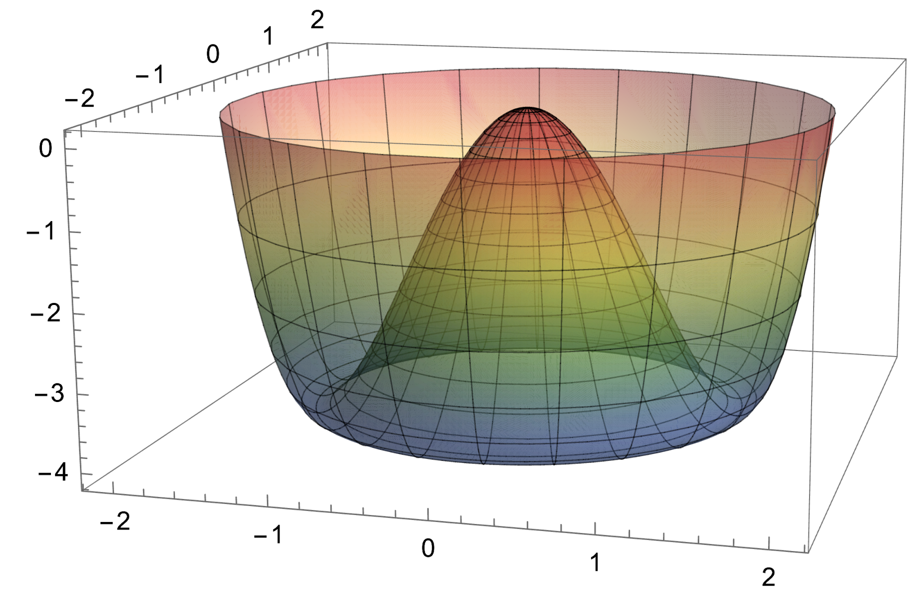


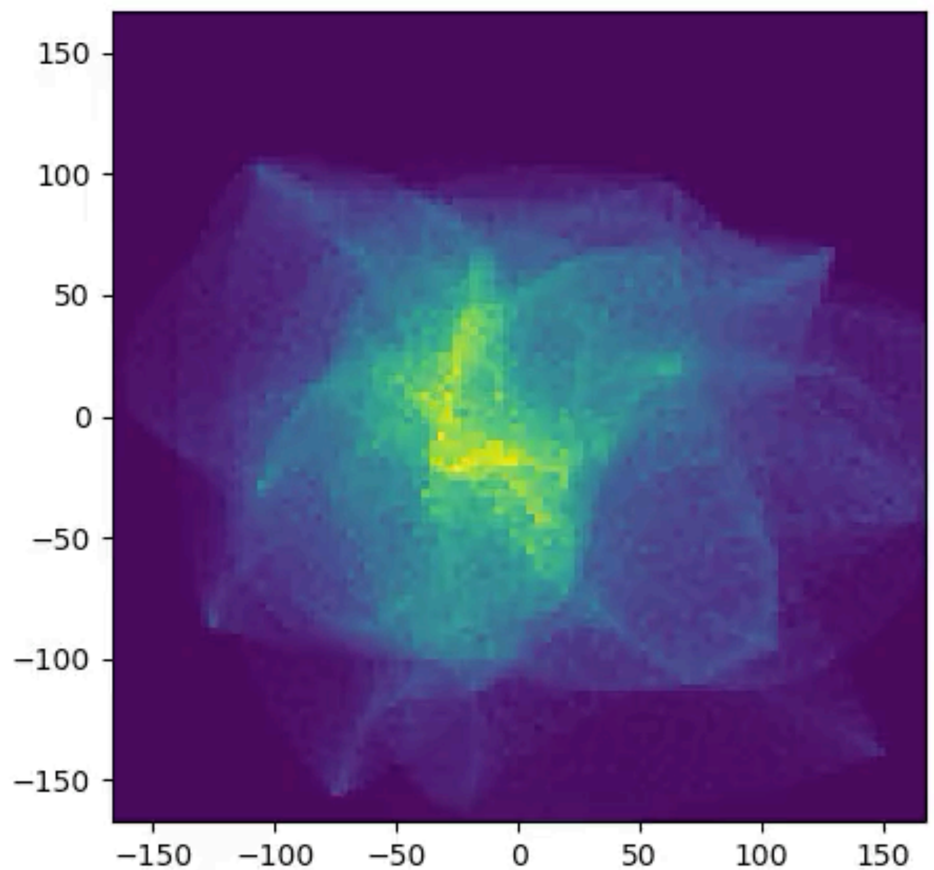
If the phase of the field circles around the origin when you go around a plaquette, then we tag the plaquette as being pierced by an axion string. The edge that passes the negative real axis is said to contain a domain wall (assuming post-QCD-transition vev points along positive real axis)
https://arxiv.org/pdf/1509.00026.pdf

Peccei-Quinn breaking

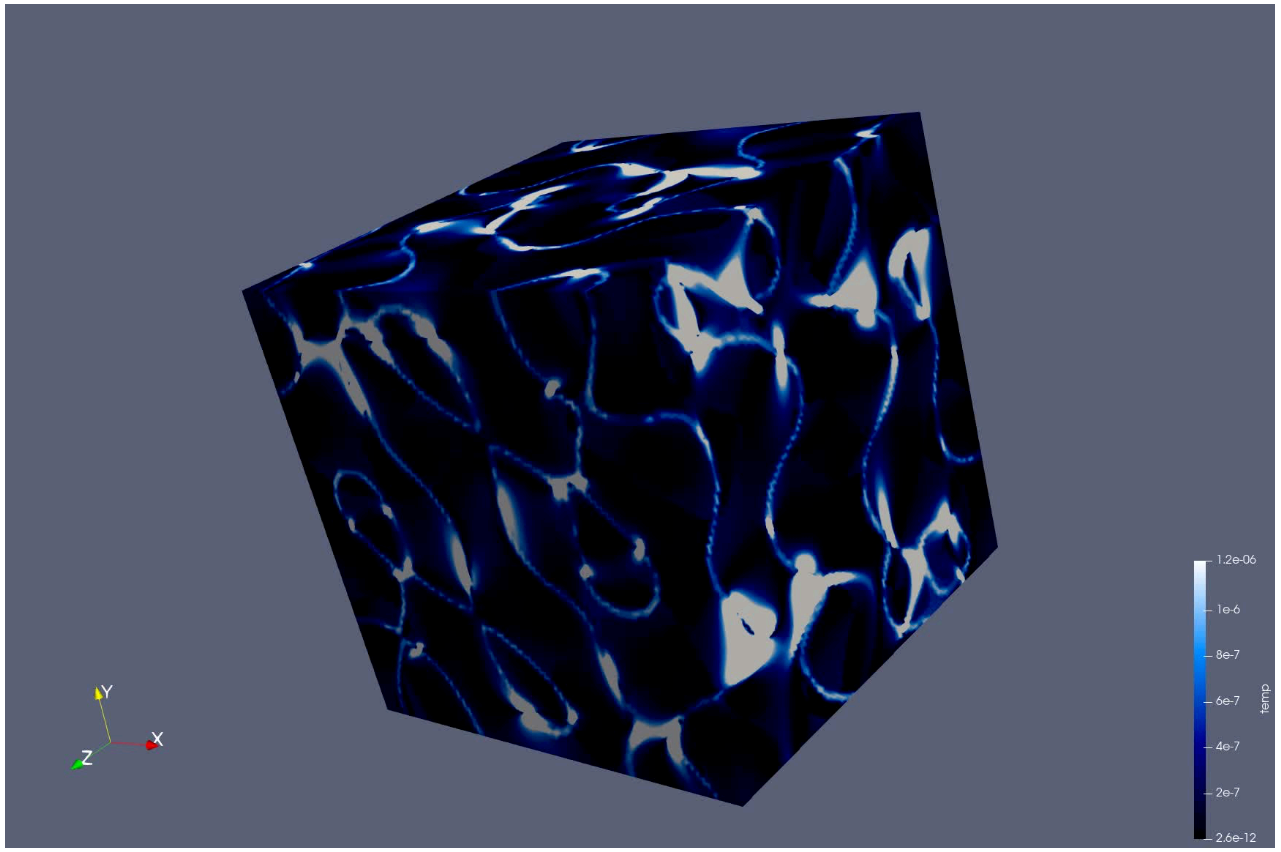


PQ-symmetry breaking



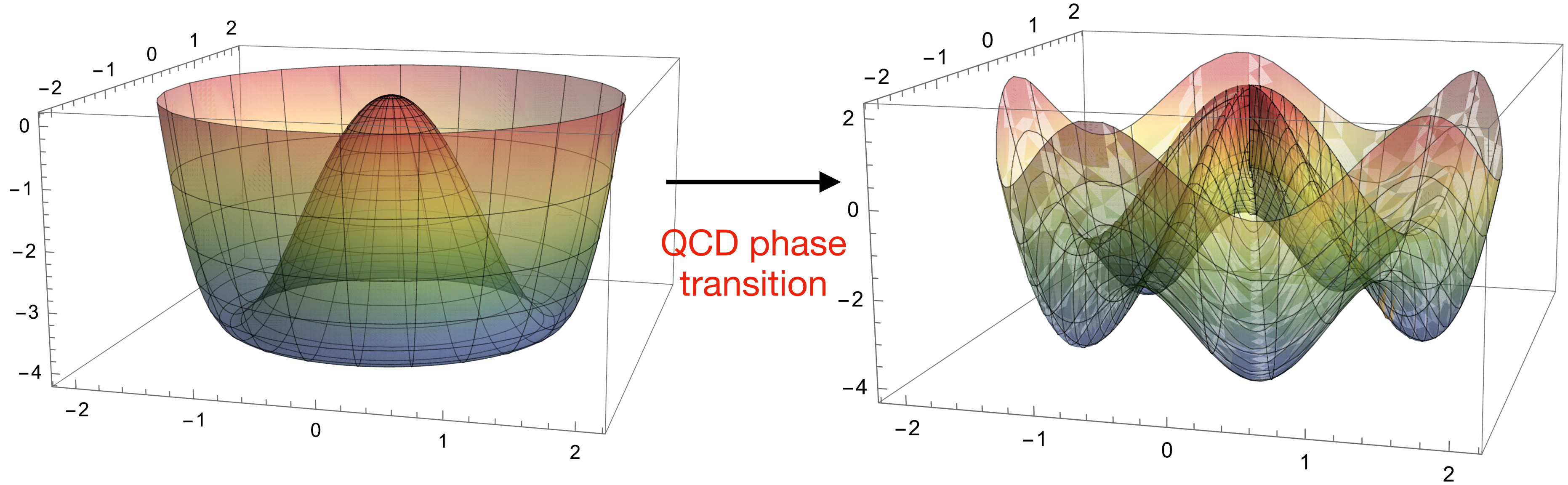


Movie showing the field undergoing dynamic symmetry breaking
(Refer to /presentation/figs/PQmovie.mp4)

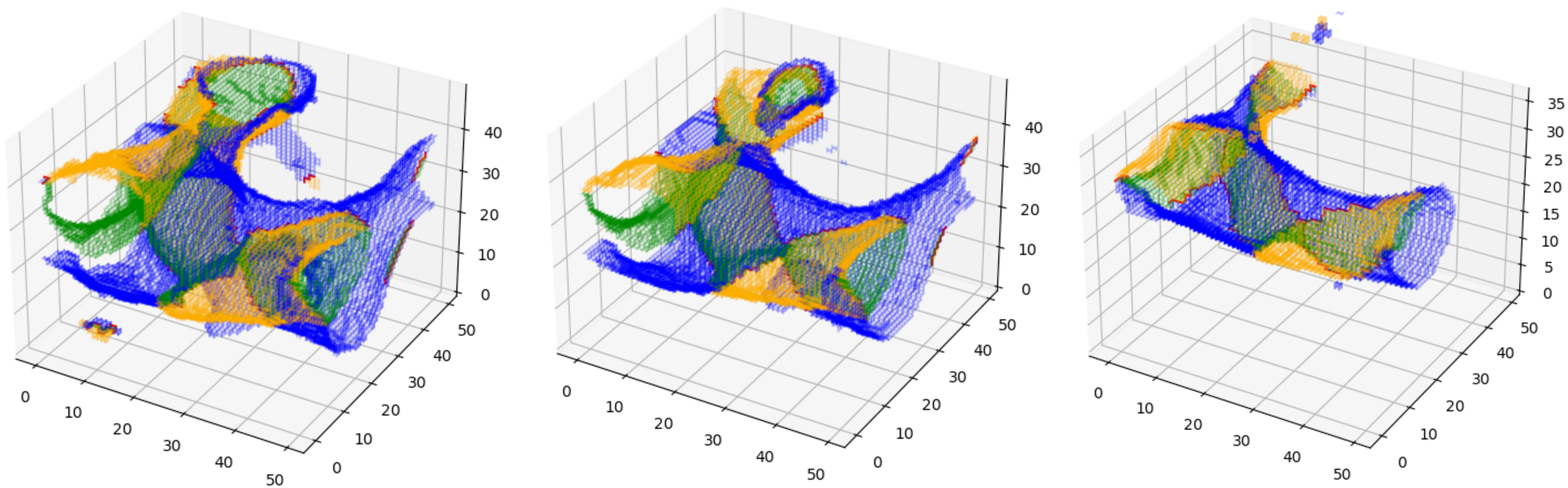


Movie showing typical domain-wall dynamics on surface of the cubic lattice
(Refer to /presentation/figs/PQ_surface_density.avi)

QCD phase transition $N = 3$

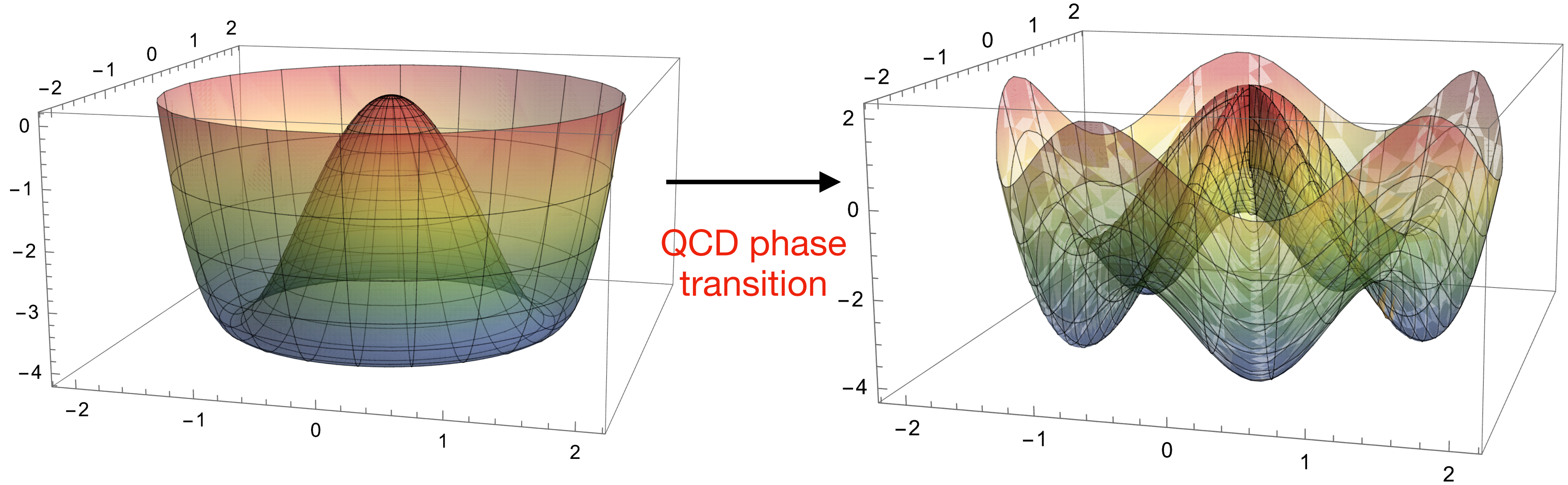


Axion potential is modified by coupling to instantons, and breaks the U(1) symmetry to a discrete subgroup



Stable large-scale string+domain wall networks, ruled out experimentally

QCD phase transition N = 1



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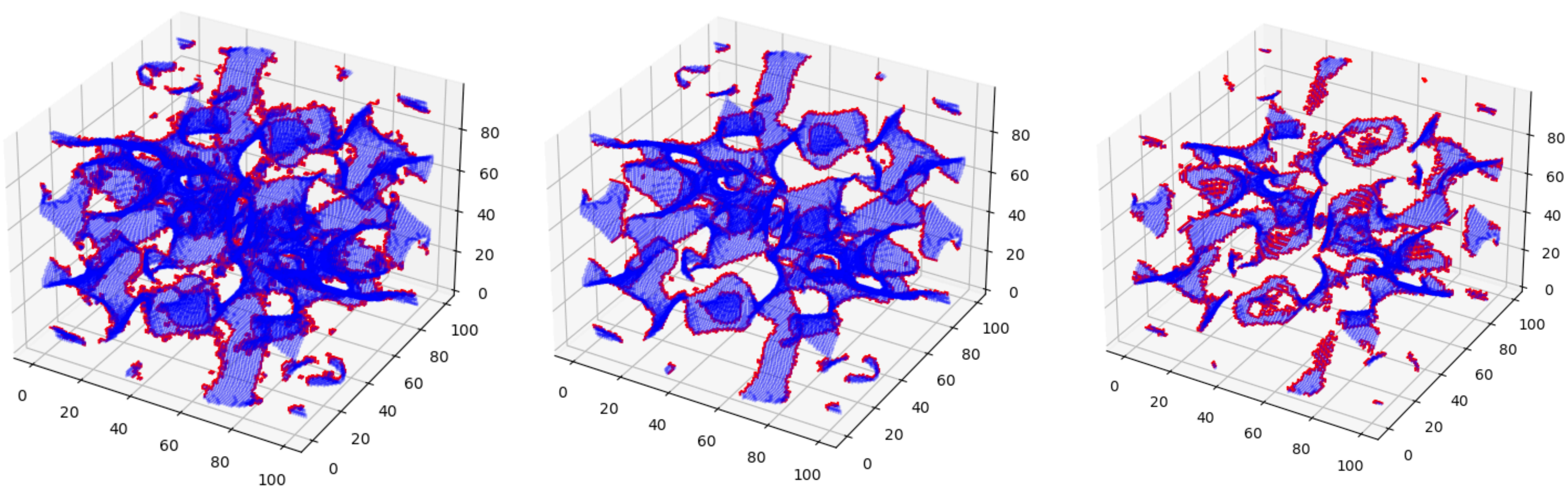

String+domain walls evaporate, and leave clusters of energy that can potentially be detected

Thanks!

Diagrams of potentials made in Mathematica
3D-plots of energy density made in Paraview
All other plots/code used standard python libraries (numpy/scipy/matplotlib)