Phys 112 : Lecture 23

Notes for the Fall 2017 Physics 112 Course taught by Professor Holzapfel
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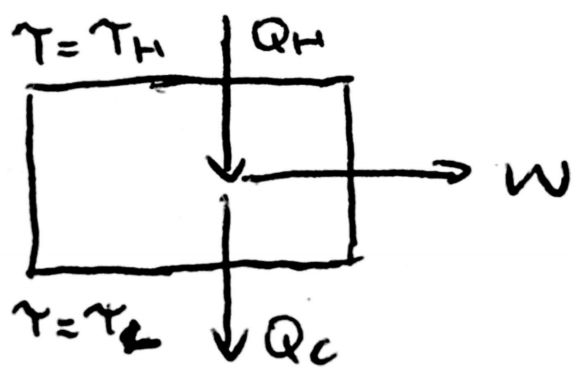
1 Heat engines

For a practical application of thermodynamics, we might imagine creating machines to convert work to heat, and vice versa. The first example of a machine we might make is a heat engine, which converts heat into mechanical work.



In this case, we have two heat reservoirs, one at a high temperature τ_H, one at a low temperature τ_L; and heat flows through the machine and is converted into work W (through some mechanical process, imagine heating a piston so the piston expands and lifts a small object). Now, we know that we have:

$$dQ = \tau d\sigma$$

So by this equation, we know that the entropy change for the two heat reservoirs are given by:

$$\Delta\sigma_H = \frac{Q_H}{\tau_H}; \qquad \Delta\sigma_L = \frac{Q_L}{\tau_L}$$

Now, we know by the second law of thermodynamics tells us the overall entropy must increase (though perhaps not strictly increase). If that's the case, then our best-case scenario (most efficiency) occurs when there is no overall entropy change, i.e. we have:

$$\Delta\sigma_H = \Delta\sigma_L; \qquad \frac{Q_H}{\tau_H} = \frac{Q_L}{\tau_L}; \qquad Q_L = \left(\frac{\tau_L}{\tau_H}\right) Q_H$$

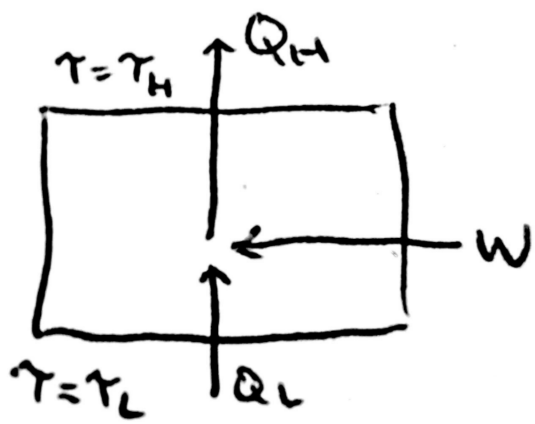
Now by conservation of energy, we have that

$$W = Q_H - Q_L = \frac{\tau_H - \tau_L}{\tau_H} Q_H$$

So now we can define the efficiency as:

$$n_c = \frac{W}{Q_H} = \frac{\tau_H - \tau_L}{\tau_H}$$

where n_c is the carnot efficiency, the highest possible efficiency for a physical heat engine. We can interpret this efficiency as the percentage of the input heat that is converted into work. If we wanted to, we could check that the efficiency for real engines is always bounded above by the carnot efficiency, by enforcing the inequality $\Delta\sigma_L \geq \Delta\sigma_H$, by the second law of thermodynamics.



Now, we can use the exact same maths to describe many other machines that convert between heat and work. For example, we might imagine a refregirator, which uses mechanical energy to pump heat out of a system of interest. So now, we have that:

$$W = Q_H - Q_L = \frac{\tau_H - \tau_L}{\tau_H} Q_H = \frac{\tau_H - \tau_L}{\tau_L} Q_L$$

so we can define the efficiency of the fridge to be:

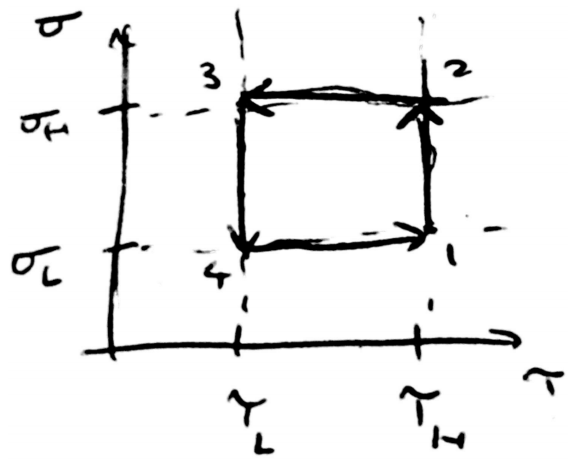
$$\gamma_c = \frac{Q_L}{W} = \frac{\tau_L}{\tau_H - \tau_L}$$

which can be interpreted as the fraction of the work that gets extracted out of the system of interest as heat energy (Q_L). Finally, we can consider a heat pump, which takes in work to pump heat into the system of interest. The diagram for this system is the same as the diagram for the fridge, except that the system of interest is now the hotter heat reservoir, and we define our efficiency to be:

$$\frac{Q_H}{W} = \frac{\tau_H}{\tau_H - \tau_L}$$

2 Carnot Cycle

So far, we haven't actually expounded upon how the different engines we have been considering actually work internally. To do this, we usually cast the question in terms of a cycle in a particular phase-space; for now we will be working in the "ST" phase-space (entropy plotted against temperature).



The diagram above shows the 'carnot cycle' of an engine, which starts at point 1, then goes to point 2, point 3, point 4, then back to point 1. The system above is connected to two heat reservoirs of temperature τ_H and τ_L, and we can find the change in entropy:

$$\Delta\sigma_L = \Delta\sigma_H = \sigma_H - \sigma_L$$

Now, we know that

$$dU = \tau d\sigma - pdV$$

so we have that:

$$\int dU = \int \tau d\sigma - pdV = 0$$

when the integral is taken over the entire loop. This implies that:

$$W = \int pdV = \int \tau d\sigma$$

$$W = \tau_H(\sigma_H - \sigma_L) - \tau_L(\sigma_H - \sigma_L) = (\tau_H - \tau_L)(\sigma_H - \sigma_L)$$

Now, looking at the $1 \to 2$ transition in the cycle, we can find Q_H to be:

$$\tau_H \Delta\sigma = \tau_H(\sigma_H - \sigma_L)$$

so we have:

$$\frac{W}{Q_H} = \frac{\tau_H - \tau_L}{\tau_H} = n_c$$

i.e. the process shown above reaches the maximum efficiency, the carnot efficiency.