

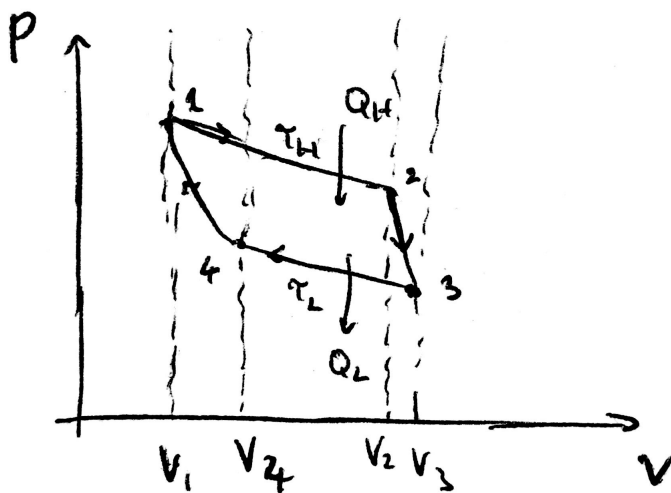
# Phys 112 : Lecture 24

Notes for the Fall 2017 Physics 112 Course taught by Professor Holzapfel  
prepared by Joshua Lin (email: joshua.z.lin@gmail.com)

November 16, 2017

## 1 PV Diagrams

Remember from last lecture, we were expressing physical engines that exchange between heat and work as closed curves in some phase space - before we were working in the Temperature Entropy space, but we can also work in the Pressure Volume case:



Now, we can calculate all our quantities of interest for this particular cycle. For the  $1 \rightarrow 2$  transition, the heat input  $Q_H$  has to equal the work done by the system  $W = \int p dV$  since the internal energy doesn't change (set by the temperature). So we have:

$$Q_H = \int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} \frac{N\tau}{V} dV = N\tau_H \ln\left(\frac{V_2}{V_1}\right)$$

Now, from point  $2 \rightarrow$  point 3 on the phase diagram, we don't have any heat flow, so the work done by the system is equal to negative of the change of internal entropy, so we have that:

$$W = -\Delta U = -C_V(\tau_3 - \tau_2) = \frac{3}{2}N(\tau_H - \tau_L)$$

Recall that for adiabatic processes, we derived in a past lecture that:

$$PV^\gamma = \text{Constant}; \quad TV^{\gamma-1} = \text{Constant}; \quad \gamma = \frac{C_P}{C_V}$$

and we have that  $\gamma = 5/3$  for monatomic gases, and  $7/5$  for the diatomic gases. Now it turns out that the particular value of  $\gamma$  doesn't matter; our calculations always hold since the gammas vanish. But for the sake of simplicity, we will only treat the case where we have a monatomic gas. So we have that:

$$\tau_L V_3^{2/3} = \tau_H V_2^{2/3}; \quad \frac{V_3}{V_2} = \left( \frac{\tau_H}{\tau_L} \right)^{3/2}$$

Now, from point 3  $\rightarrow$  point 4 on the phase diagram, we have the same situation as in 1  $\rightarrow$  2, but in reverse in some sense. So we have to do some work on the system, so we have:

$$W = N\tau_L \ln \left( \frac{V_3}{V_4} \right) = Q_L$$

Finally, the last step, from point 4  $\rightarrow$  point 1, we have the same situation as point 2  $\rightarrow$  point 3:

$$W = C_V(\tau_1 - \tau_4) = \frac{3}{2}N(\tau_H - \tau_L)$$

so we have:

$$\frac{V_4}{V_1} = \left( \frac{\tau_H}{\tau_L} \right)^{3/2}$$

so using our equation from before, we have:

$$\frac{V_4}{V_1} = \frac{V_3}{V_2}; \quad \frac{V_3}{V_4} = \frac{V_2}{V_1}$$

Now, the total work is going to be:

$$W = W_{12} + W_{23} - W_{34} - W_{41}$$

where  $W_{ij}$  is the absolute value of the work done going from  $i$  to  $j$  (as described above in equations, since the Professor wants everything to be positive). Notice that  $W_{23} = W_{41}$  so those two terms cancel. So we have:

$$W = N \left[ \tau_H \ln \left( \frac{V_2}{V_1} \right) - \tau_L \ln \left( \frac{V_3}{V_4} \right) \right] = N(\tau_H - \tau_L) \ln \left( \frac{V_2}{V_1} \right)$$

Now, we also have that:

$$Q_H = N\tau_H \ln \left( \frac{V_2}{V_1} \right)$$

so the efficiency of the system shown; we have:

$$\eta = \frac{W}{Q_H} = \frac{\tau_H - \tau_L}{\tau_H} = \eta_c$$

the carnot efficiency!!

## 2 Reversible and Irreversible Processes

<sup>1</sup> If we are thinking about a system in Energy Entropy phase space, when we go from state  $U_1, \sigma_1$  to  $U_2, \sigma_2$ , then there must be a reversible process that does this process for me, which costs  $dW_{\text{rev}}$  amount of work, and  $dQ_{\text{rev}}$  amount of heat. For any irreversible proces, we have that  $dW_{\text{irrev}} > dW_{\text{rev}}$  (we need to do more work), but since we have:

$$dU = dW_{\text{irrev}} + dQ_{\text{irrev}} = dW_{\text{rev}} + dQ_{\text{rev}}$$

so we have that in this case,  $dQ_{\text{irrev}} < dQ_{\text{rev}}$ , i.e. there is *less* heat flow in the irreversible case.

As a model problem, we can firs consider the case of an isothermal expansion:

$$dW_{\text{rev}} = - \int_{V_1}^{V_2} p dV = -N\tau \ln \left( \frac{V_2}{V_1} \right) < 0$$

is the work done on the system, and we have:

$$\begin{aligned} dU &= 0; & dQ_{\text{rev}} &= -dW_{\text{rev}} > 0 \\ d\sigma &= \sigma_2 - \sigma_1 = \frac{dQ_{\text{rev}}}{\tau} = N \ln \left( \frac{V_2}{V_1} \right) \end{aligned}$$

We could start with the same initial state and end with the same final state if we just did a Free expansion, in which case:

$$dU = 0; \quad dW_{\text{irrev}} = dQ_{\text{irrev}} = 0$$

So we have verified that:

$$dW_{\text{irrev}} > dW_{\text{rev}}; \quad dW_{\text{irrev}} < dQ_{\text{rev}}$$

As another model problem, we can consider two systems of temperature  $\tau_1, \tau_2$ , placed in thermal contact. We have that:

$$\begin{aligned} dU_1 &= dQ_1 = \tau_1 d\sigma_1; & dU_2 &= dQ_2 = \tau_2 d\sigma_2 \\ dQ_1 + dQ_2 &= 0 \\ d\sigma_{\text{irrev}} &= \frac{dQ_1}{\tau_1} + \frac{dQ_2}{\tau_2} = \frac{dQ_1(\tau_2 - \tau_1)}{\tau_2 \tau_1} \end{aligned}$$

As another model problem, we can consider isothermal work. In this case, we have that  $d\tau = 0$ . So our equations are given by:

$$\begin{aligned} dW_{\text{rev}} &= \Delta F; & F &= U - \tau\sigma \\ dQ &= \tau d\sigma = d(\tau\sigma) \\ dW &= dU - dQ = dU - d(\tau\sigma) = dF \end{aligned}$$

Now, since an ideal gas internal energy only depends on temperautre, we have that:

$$dU = 0; \quad dW = -dW = dF$$

---

<sup>1</sup>From now on, the convention is that work refers to work done on the system, and heat refers to heat flowing into the system