# Physics 112 : Lecture 11

Notes for the Fall 2017 Physics 112 Course taught by Professor Holzapfel
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## 1 Flux

Let

$$dE = U_{\nu\sigma} dV d\Omega d\nu$$

be the energy density per volume, per solid angle, per frequency interval. What we have right now is the energy density of photons inside the cavity, and we want to change it to an energy flux; where we care about the *direction* the photons are travelling. To do this, imagine placing a small cylinder, with base area $dA$, and length $cdt$, inside our cavity, so that we have:

$$dE = U_{\nu\sigma}(dAcdt)d\Omega d\nu$$

Recalling our definition for the intensity from last lecture; we have equality:

$$I_\nu dAdtd\Omega d\nu = U_{\nu\Omega}(dAcdt)d\Omega d\nu$$

so that we have:

$$I_\nu = cU_{\nu\Omega}$$

From last lecture, we have $U_\nu$, the energy density per specific frequency. To relate $U_\nu$ to $U_{\nu\Omega}$, we have the relation: $U_\nu = \int U_{\nu\Omega} d\Omega$, and in the case that the radiation is homogoneous in every direction, then we simply have $U_\nu = 4\pi U_{\nu\Omega}$, so we have:

$$I_\omega = \frac{c}{4\pi}\left(\frac{\hbar}{c^2\pi^2}\frac{\omega^3}{e^{\hbar\omega/\tau}-1}\right)$$

where the $4\pi$ comes from integrating over the sphere uniformly, and the other terms come from the expression for $U_\omega$ that we derived last lecture. We can also now write down $I_\nu$, by dividing by a factor of $d\omega/d\nu = 2\pi$. Now, recall our definition for the specific flux:

$$F_\nu = \int I_\nu \cos\theta d\Omega = I_\nu \int_0^{\pi/2}\int_0^{2\pi} \cos\theta(\sin\theta d\theta d\phi) = \pi I_v$$

where we calculate the specific flux for the case where we have isotropic radiation, and we are considering a black body absorber placed inside the cavity, and only looking at the flux through one of the surfaces. If we integrated $\theta$ from 0 to $pi$ instead, we would have gotten 0 as our answer, as the flux through the front cancels the flux through the back. Now, we can calculate the total flux:

$$F = \int F_\nu d\nu = \int \left( \frac{c}{4} U_\omega \right) d\omega = \frac{c}{4} \int U_\omega d\omega = \frac{c}{4} U$$

[1] From last lecture, we know what $U$ is, so we have:

$$F = \sigma_B T^4; \qquad \sigma_B = \frac{\pi^2 k_B^4}{60 \hbar^3 c^2} \approx 5.67 \cdot 10^{-8} \frac{W}{m^2 K^4}$$

where $\sigma_B$ is known as the stefan boltzmann constant. So, for some toy examples, we can calculate the power lost by a radiating body:

$$P = \int F dA = FA = A \sigma_B T^4 = 1 \cdot \sigma_B \cdot 300^4 \approx 460W$$

for a body of 1 metre squared, at a normal temeperature of 300 kelvin, emits 460 Watts of power. That's not a lot; but its not nothing either. If we now want to find the $\nu$ that contributes the most to the blackbody radiation, we can differentiate our expression for $I_\nu$ with respect to $\nu$ and set this equal to zero, and solving this we find that:

$$h\nu_{\mathrm{max}} = 2.82kT$$

note that this is NOT the same as finding the wavelength that contributes the most to the radiation. The reason is that secretly we are finding the maximum of $I_\nu d\nu$, and for lambda $I_\lambda d\lambda$, which are different things, because $d\nu \neq d\lambda$. You have to remember if you are considering per frequency interval, or per wavelength interval, because it is important.

Now, there are two different 'parts' to our spectrum $I_\nu$, when $h\nu << kT$ then we are in the classical regime, and approximating our formula for $I_\nu$ by replacing the exponential with a taylor series to first order, we find that:

$$I_\nu = 2kT \frac{\nu^2}{c^2}$$

In the 'wien' portion of the spectrum, we instead have that $h\nu >> kT$, and approximating $I_\nu$ in the usual way (by dropping the 1), and we can find that:

$$I_\nu = \frac{2h\nu}{\lambda^2} e^{-h\nu/kT}$$

## 2 Emmisivity and Absorbtivity

Sadly, real black bodies don't exist! We define a quantity $\epsilon$ known as the emissivity, which describes the proportion of photons that a body actually emits/absorbs compared to an

[1] note that all our capital $U$'s here are actually small $u$'s by our naming convention from last lecture; i.e. they are all energy densities. Implicitly, they are all densities over volume, but additional subscripts mean additional degrees of freedom that they are densities over, and that you need to integrate over

ideal black body radiator. (Not that the emissivity is the *same* as the 'absorbtivity', i.e. the proportion of energy a body emits is the same as the proportion it absorbs. We can see this by a thought experiment, place an object with emissivity $\epsilon$ in a box with an ideal black body radiator, then they should converge to the same temperature, which only happens when the emissivity is the same as the absorbtivity). So, for an imperfect body we have:

$$F = \epsilon(\nu) F_\nu; \qquad F = \int \epsilon(\nu) F_\nu d\nu$$

where $\epsilon(\nu)$ is the emissivity dependent on the frequency. Fortunately, for many practical applications, we can take emissivity to be a constant, which makes these integrals a lot easier.

As a final note, we can consider the momentum flux through a surce from photons impinging on the surface, so we have:

$$p = \int \frac{dF}{c} \cos\theta d\Omega = \frac{1}{c} \int I \cos^2\theta$$

where we pick up an extra factor of $\cos\theta$ on top of the factor that we picked up for calculating the flux, in order to accomidate for the fact that we only care about momentum in the perpendicular direction. Also, note that to find the actual pressure, the screen has to actually reflect the photon back; so we need to double the momentum flux that we found above. All in all, we find that:

$$P = \frac{1}{3} U$$

where $P$ is the pressure.