# Informed Search

## Use “heuristics” to guide the search

- What is a heuristic?
  - Guidelines/Intuitions that are based on information outside the problem formulation
- “Best” first
  - Greedy Search
  - A*
- Many variations of A* are used
  - Hill-climbing / Simulated Annealing

## Best-first search

- Idea:

  use an evaluation function for each node; estimate of ***"desirability"***

$\Rightarrow$ expand most desirable unexpanded node.

- Implementation:

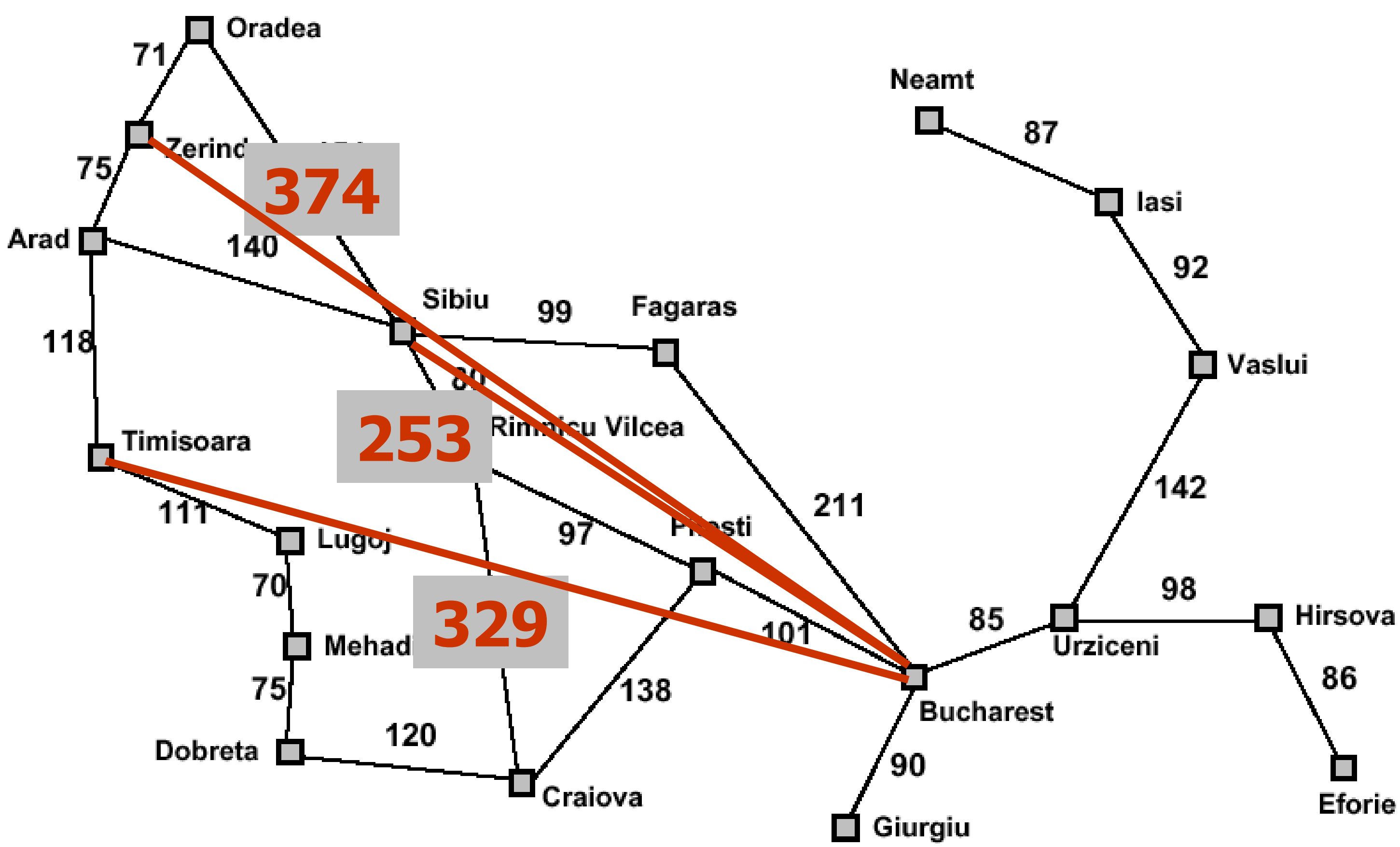
  **QueueingFn** = insert successors in decreasing order of **"desirability"**

- Examples of Best First search:

  Greedy search

  A* search

# Romania with step costs in km




Straight–line distance to Bucharest

| | |
|---|---|
| **Arad** | 366 |
| **Bucharest** | 0 |
| **Craiova** | 160 |
| **Dobreta** | 242 |
| **Eforie** | 161 |
| **Fagaras** | 178 |
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| **Zerind** | 374 |

## Greedy search

- Estimation function:

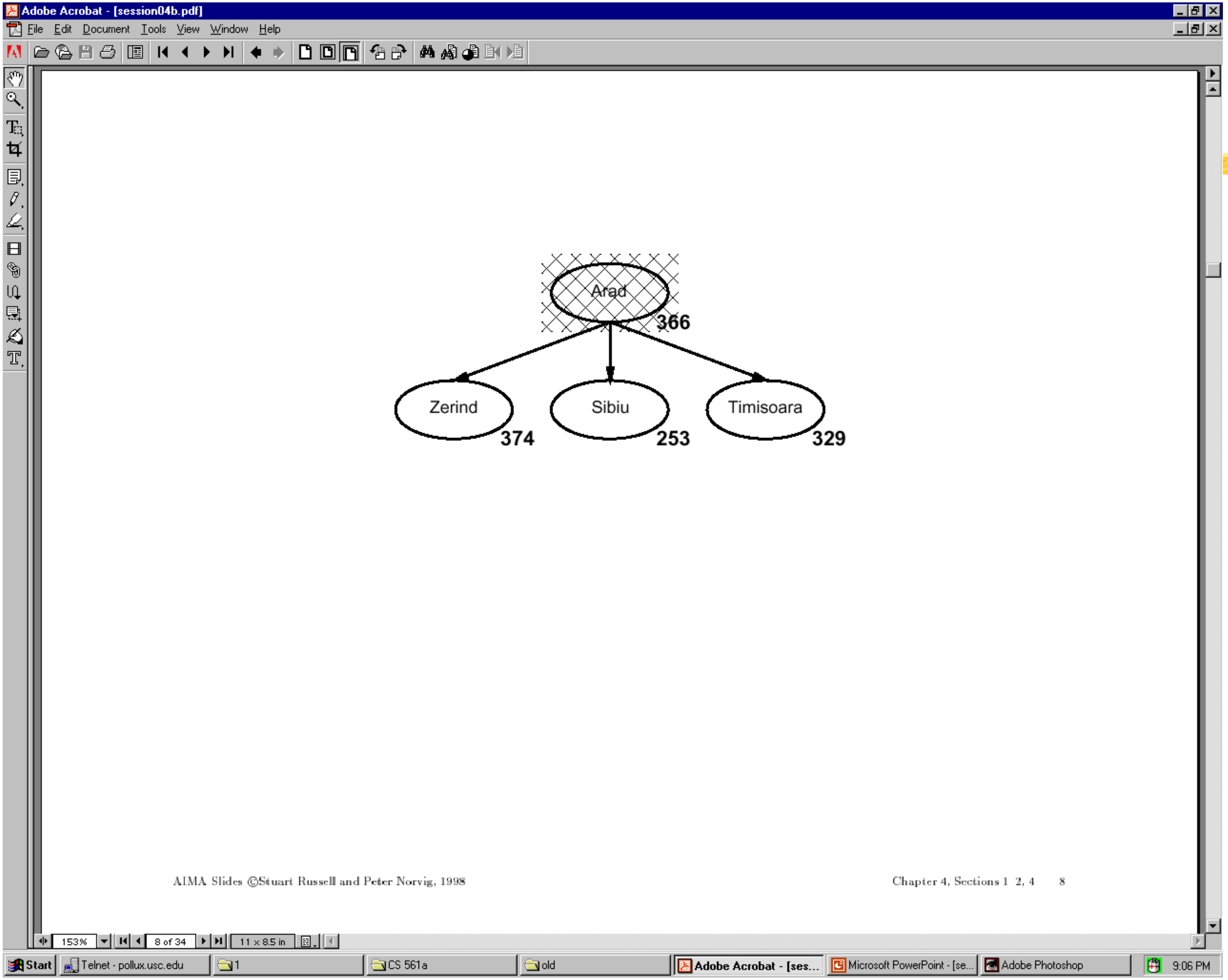
  $h(n)$ = estimate of cost from $n$ to goal (h ➔ heuristic)

- For example:

  $h_{SLD}(n)$ = **straight-line distance** from $n$ to Bucharest

- Greedy search expands first the node that **appears promising** to be closest to the goal, according to $h(n)$. I.e., node expanded will be the one with the best $h(n)$

# Greedy search example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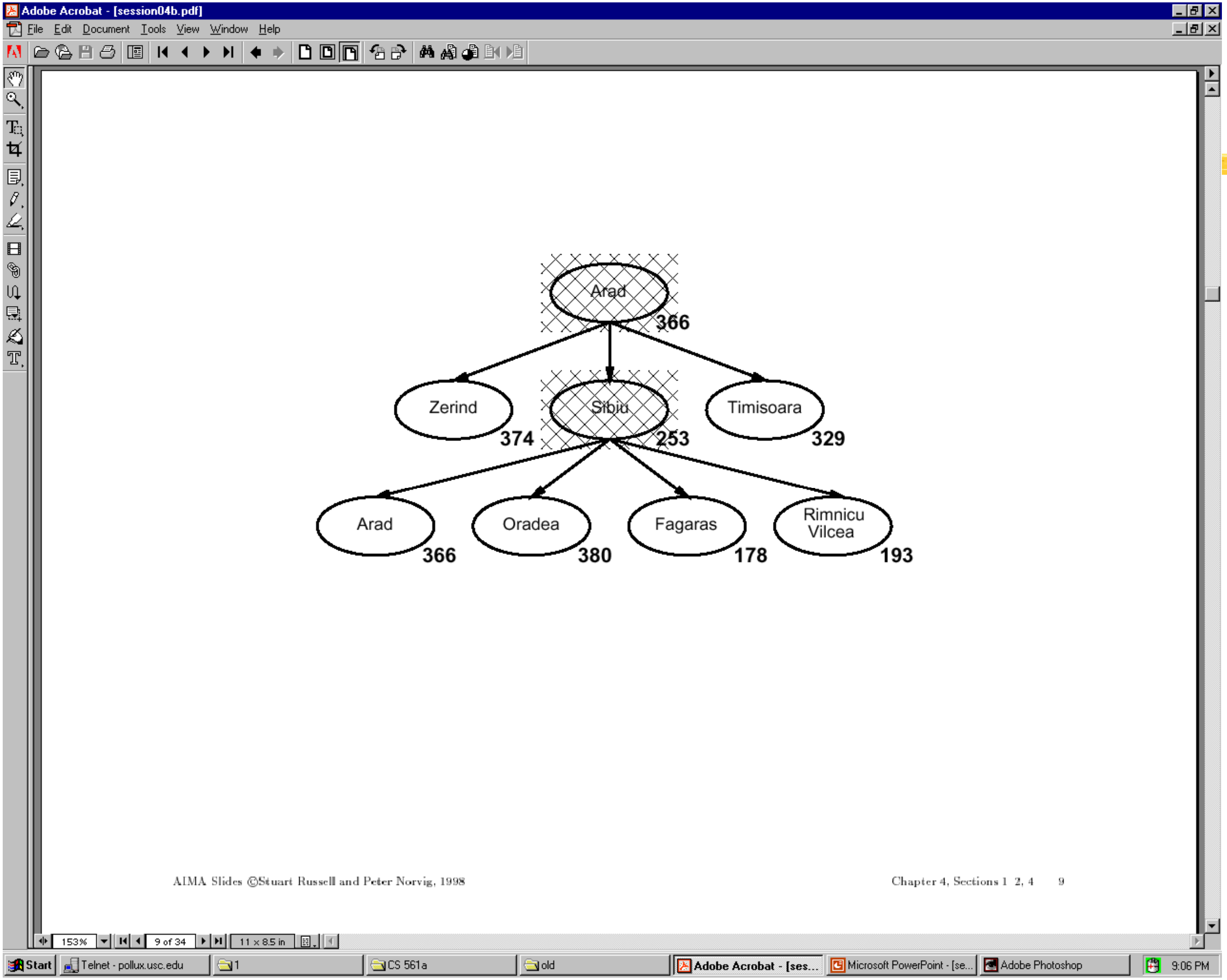

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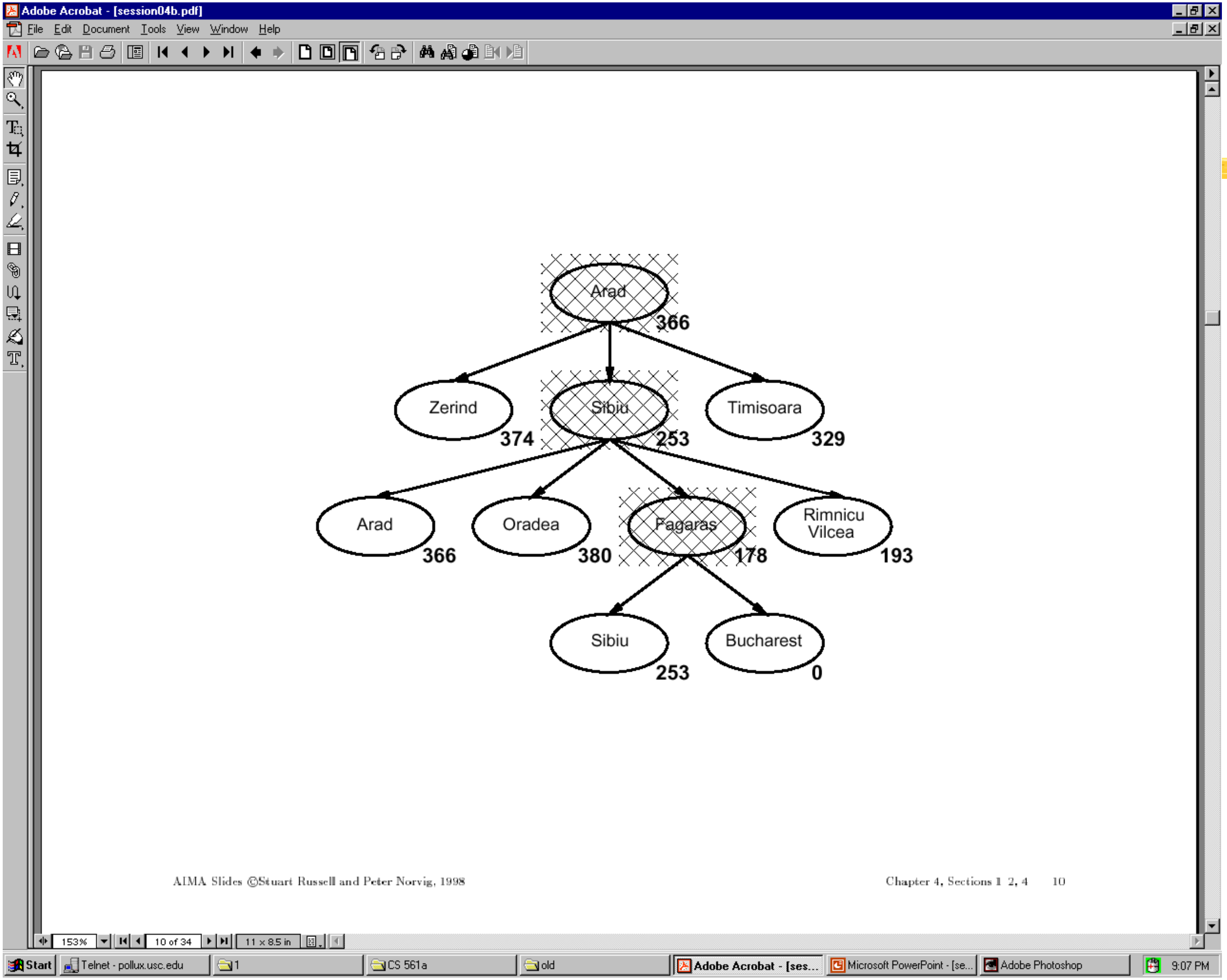
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# Properties of Greedy Search

- Complete?

- Time?

- Space?

- Optimal?

# Properties of Greedy Search

- Complete? No – can get stuck in loops
  e.g., Iasi > Neamt > Iasi > Neamt > …
  Complete in finite space with repeated-state checking.
- Time? O(b^m) but a "good" heuristic can provide dramatic improvement
- Space? O(b^m) – keeps all nodes in memory
- Optimal? No.

## A* search

- Basic Idea: avoid expanding paths that are **already** expensive

  evaluation function: $f(n) = g(n) + h(n)$ with:

  $g(n)$ – cost so far to reach $n$

  $h(n)$ – estimated cost to goal from $n$

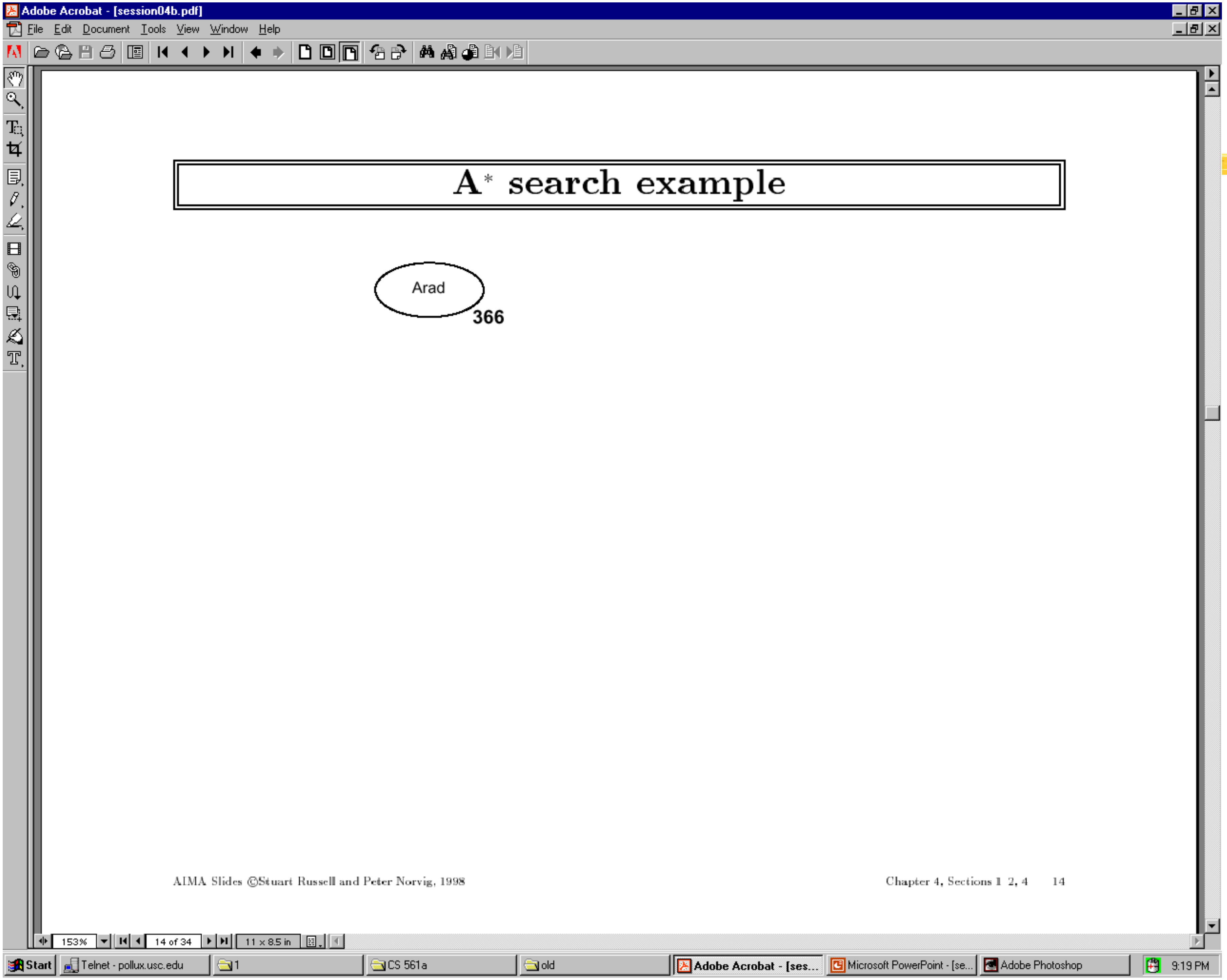
  $f(n)$ – estimated total cost of path through $n$ to goal

- A* search uses an **admissible** heuristic, that is,

  $h(n) \le h^*(n)$ where $h^*(n)$ is the **true or actual** cost from $n$.

  For example: $h_{SLD}(n)$ never overestimates actual road distance from n to the destination.

- Theorem: A* search is optimal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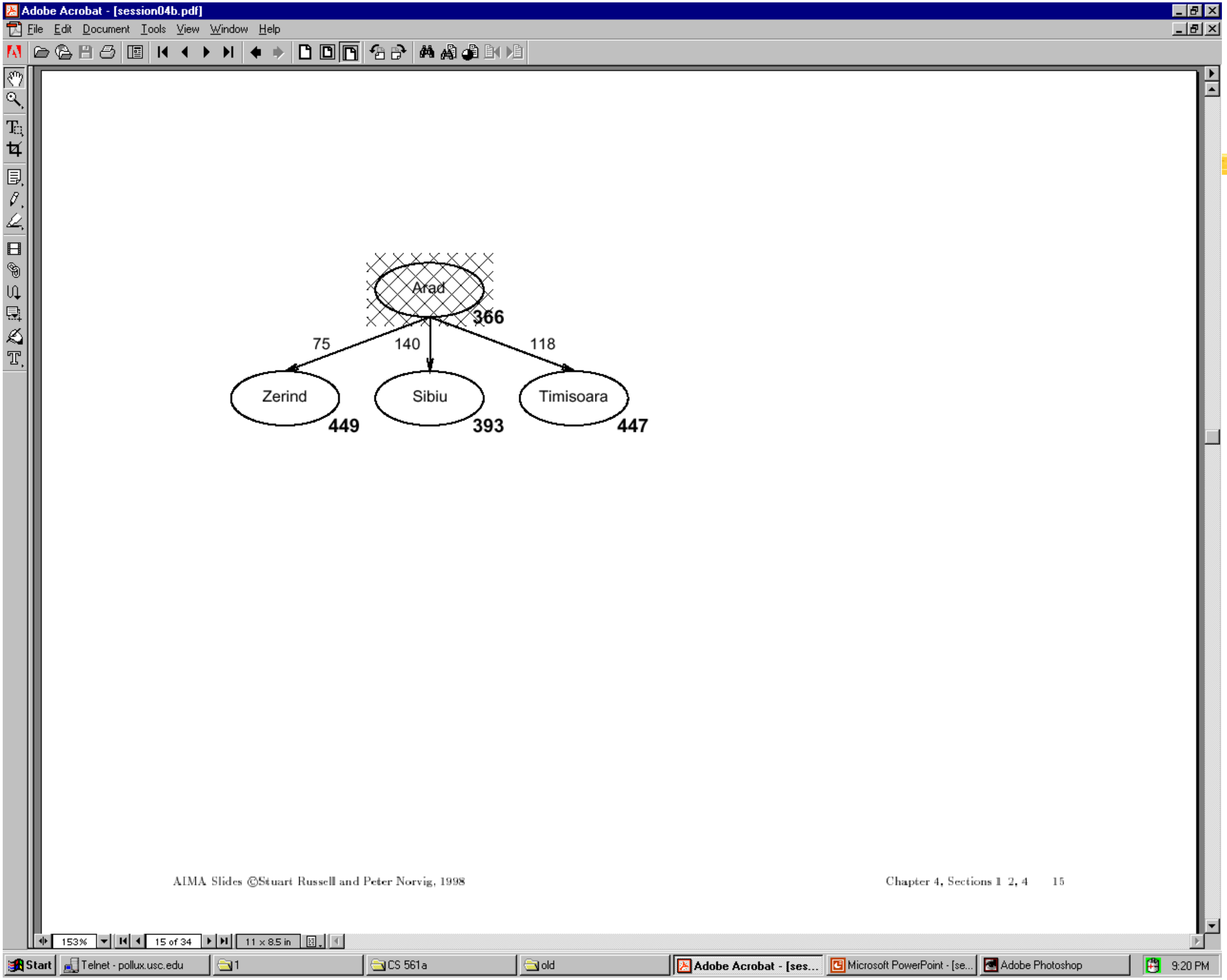
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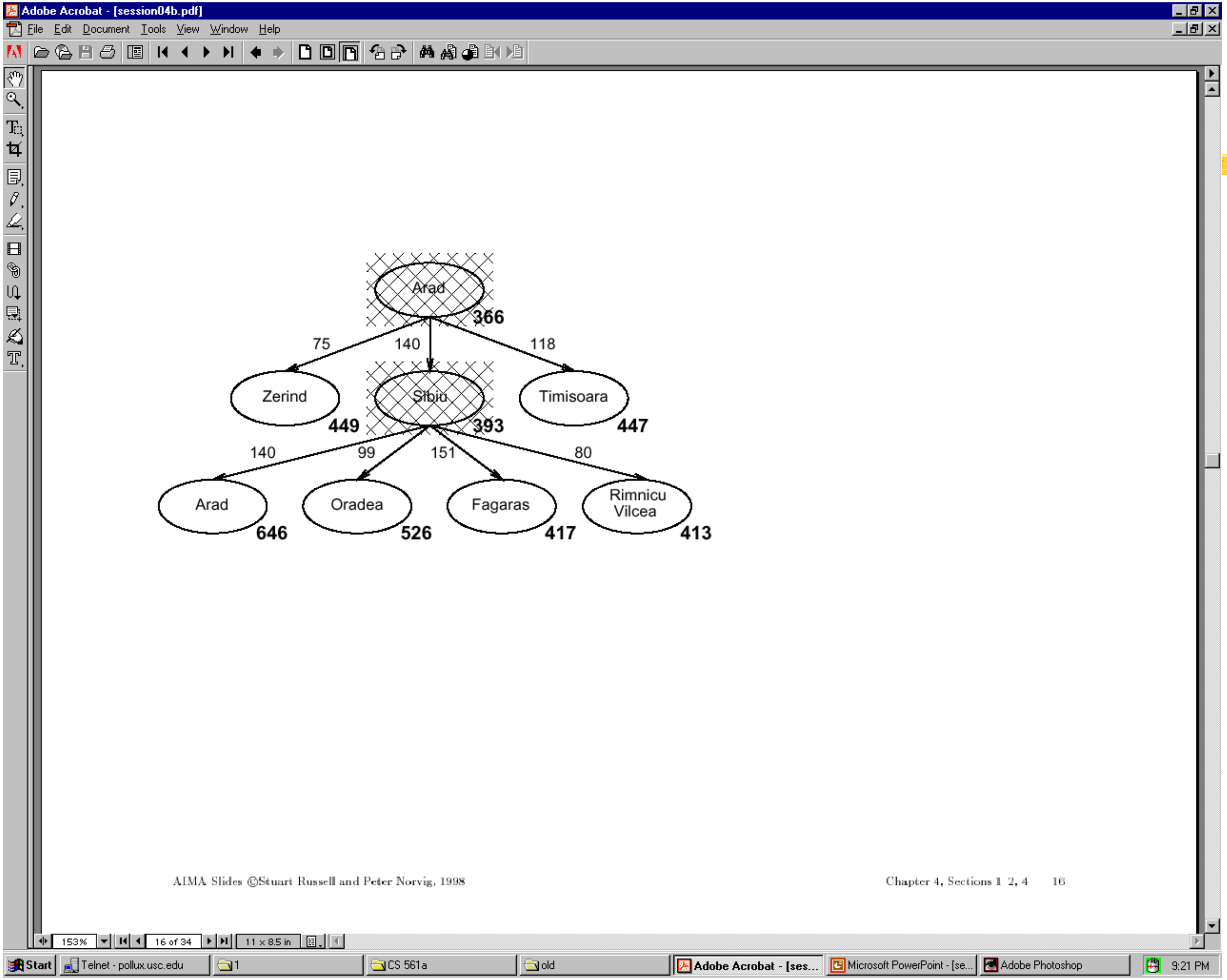
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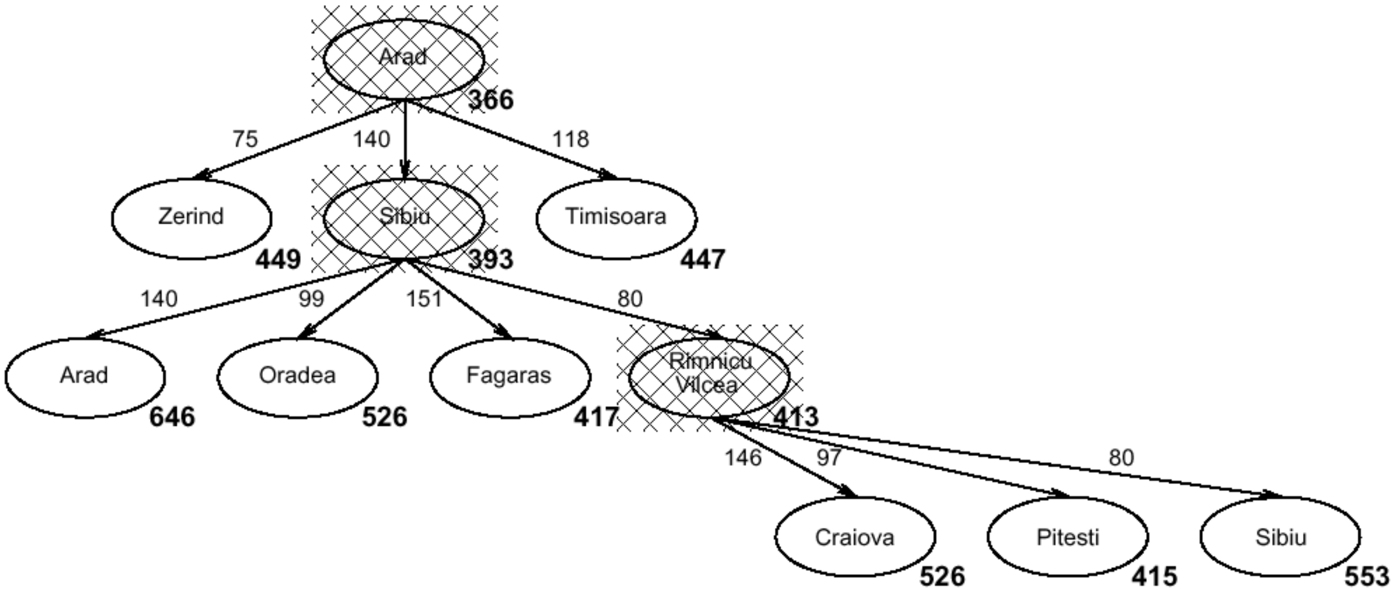

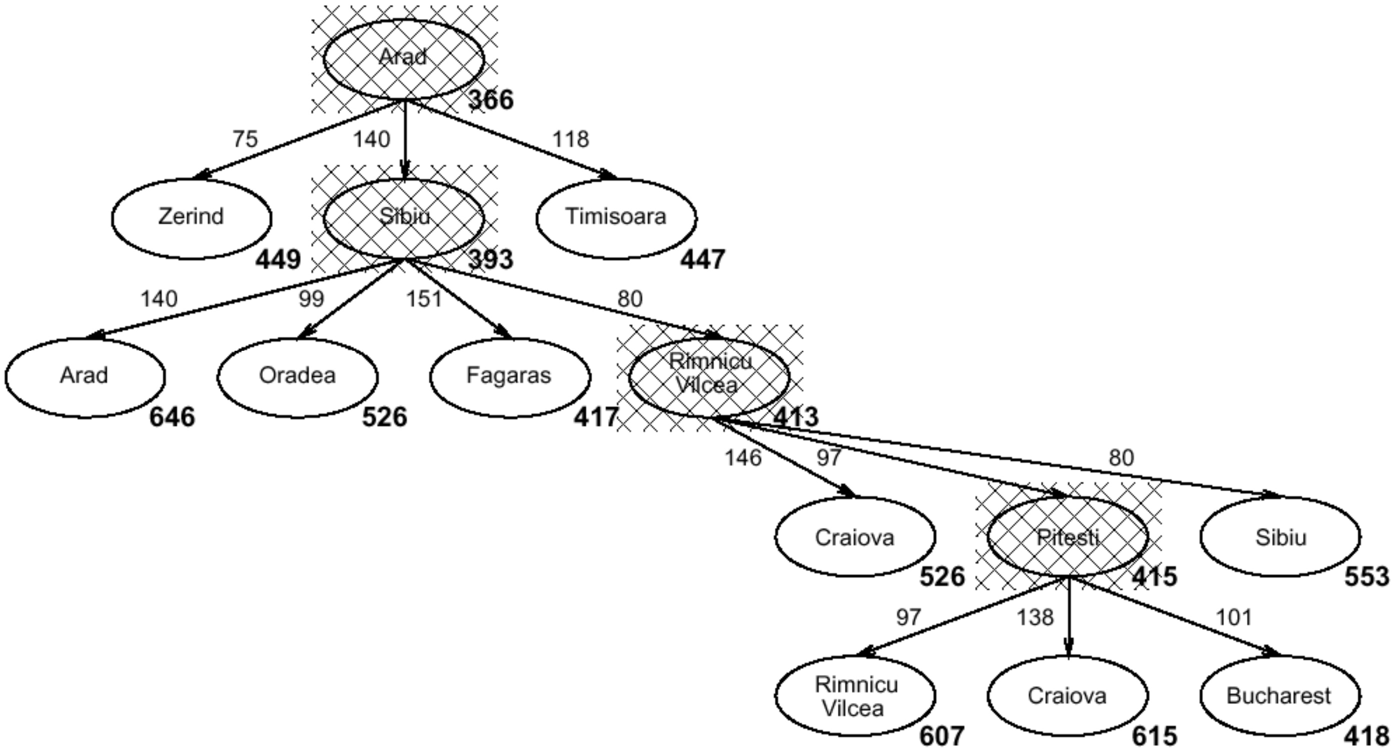
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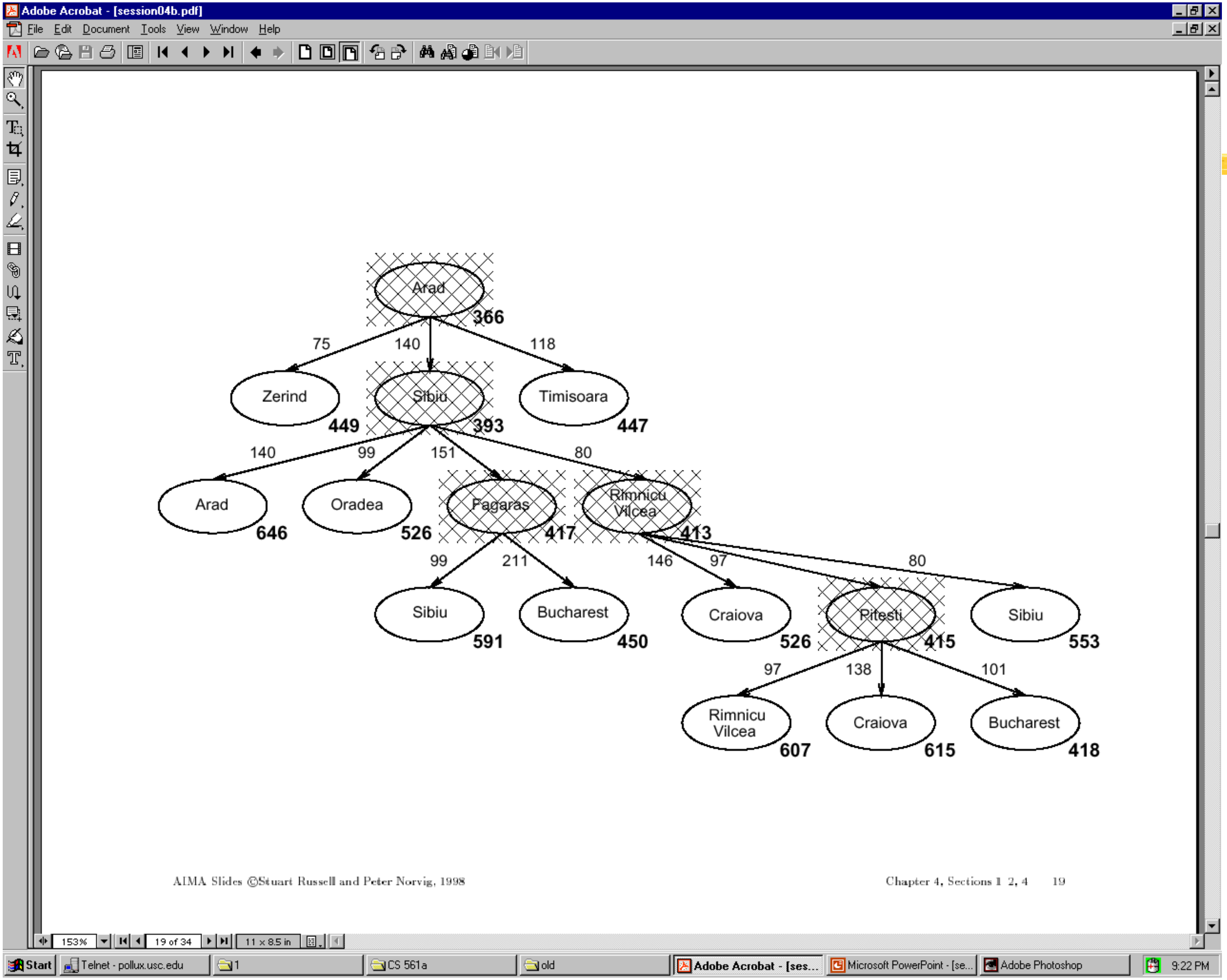
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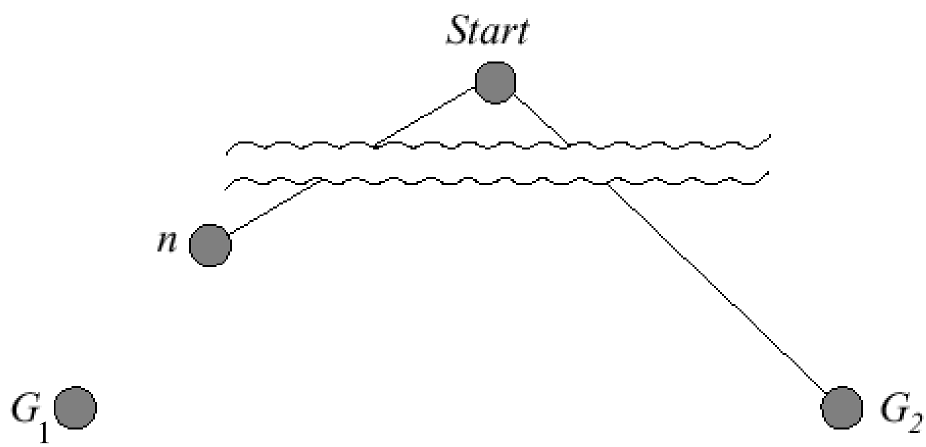
# Optimality of A* (an outline of standard proof)

Suppose some suboptimal goal $G_2$ has been generated and is in the queue. Let $n$ be an unexpanded node on a shortest path to an optimal goal $G_1$.



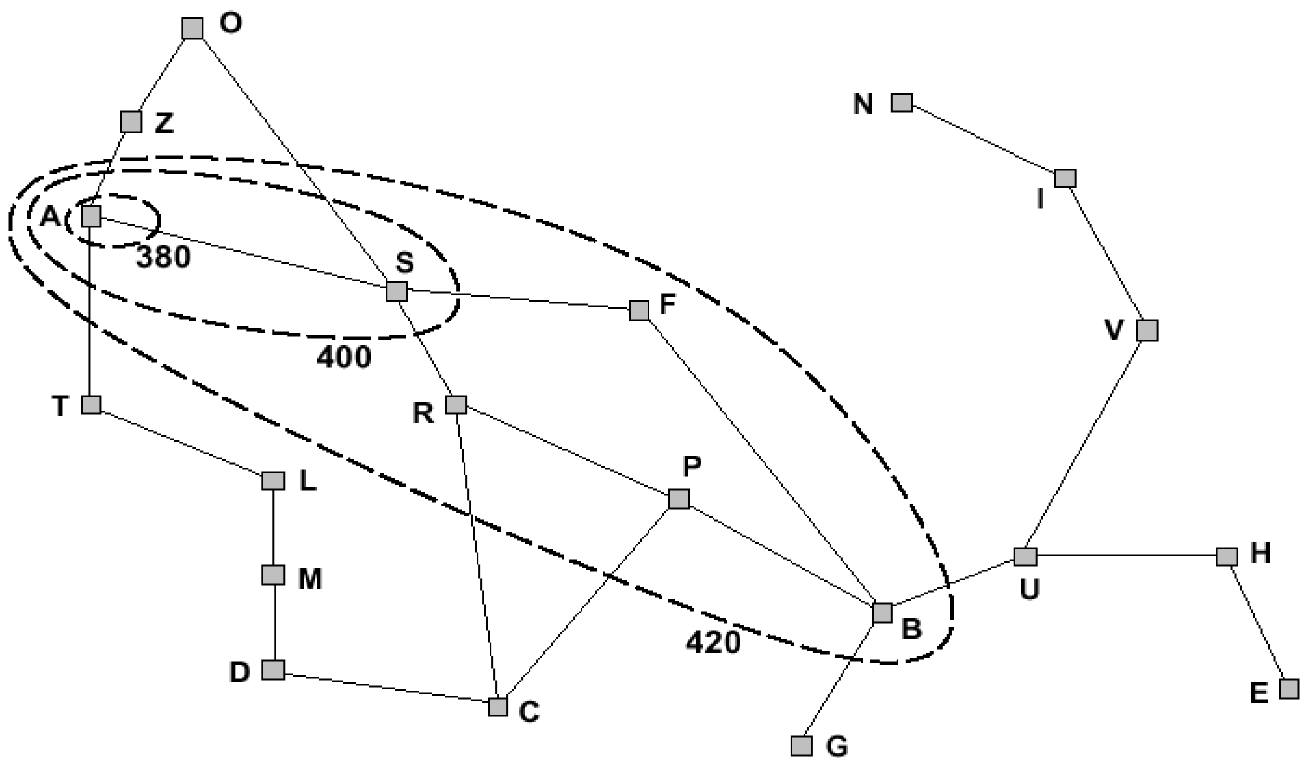

$$\begin{aligned} f(G_2) &= g(G_2) && \text{since } h(G_2) = 0 \\ &> g(G_1) && \text{since } G_2 \text{ is suboptimal} \\ &\geq f(n) && \text{since } h \text{ is admissible} \end{aligned}$$

Since $f(G_2) > f(n)$, A* will never select $G_2$ for expansion

# Optimality of A* (Another, perhaps more useful proof)

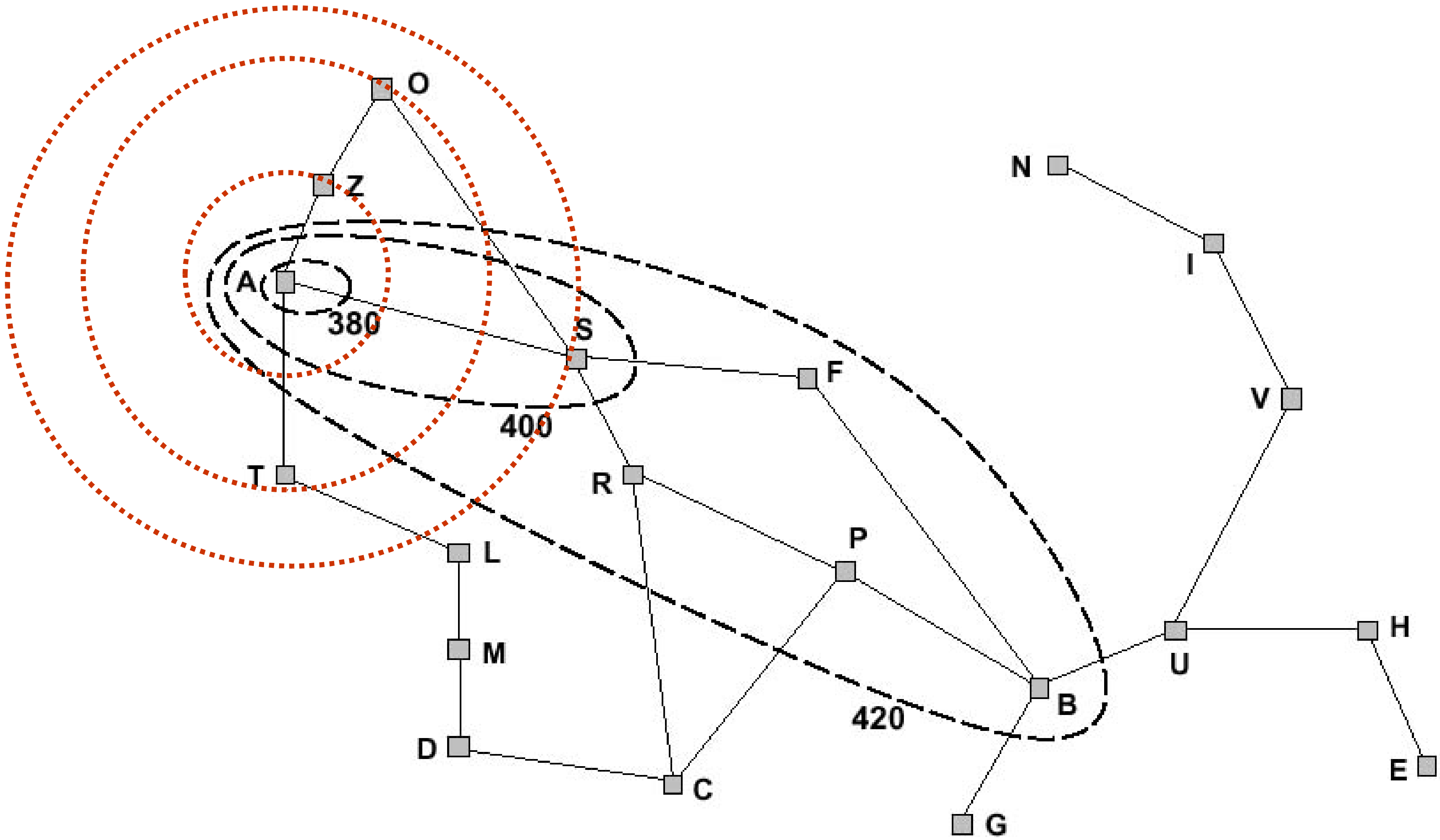
Lemma: A* expands nodes in order of increasing $f$ value

Gradually adds "$f$-contours" of nodes (cf. breadth-first adds layers)
Contour $i$ has all nodes with $f = f_i$, where $f_i < f_{i+1}$



# f-contours

How do the contours look like when h(n) =0?



## Properties of A*

- Complete?
- Time?
- Space?
- Optimal?

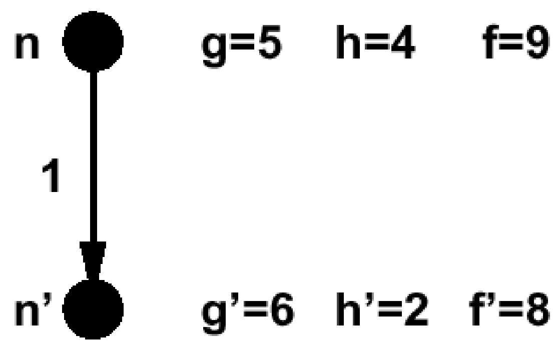
# Properties of A*

- Complete? Yes, unless infinitely many nodes with $f \leq f(G)$
- Time? Exponential in [(relative error in h) x (length of solution)]
- Space? Keeps all nodes in memory
- Optimal? Yes – cannot expand $f_{i+1}$ until $f_i$ is finished

## Proof of lemma: pathmax

For some admissible heuristics, $f$ may *decrease* along a path

E.g., suppose $n'$ is a successor of $n$

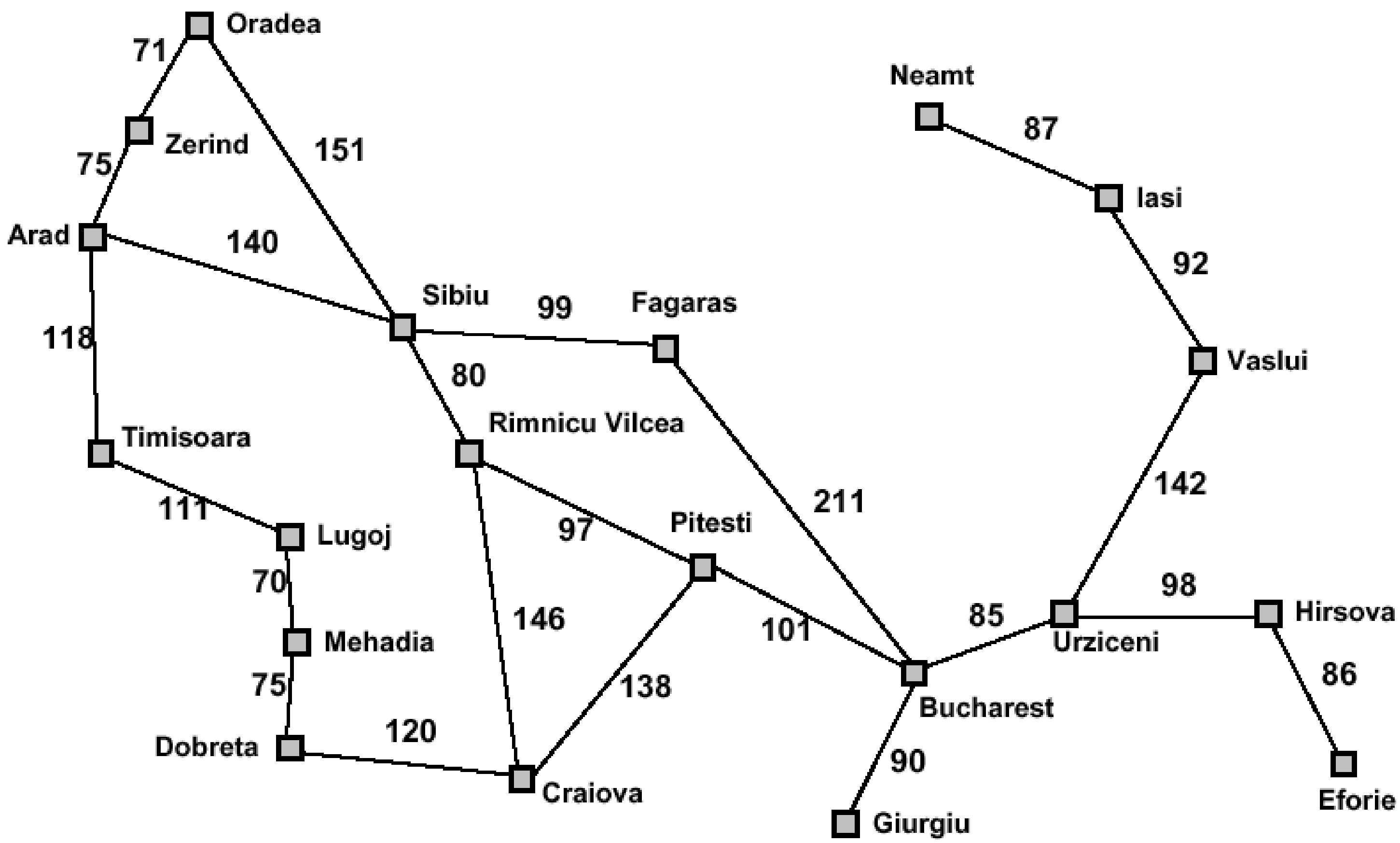



But this throws away information!
$f(n) = 9 \Rightarrow$ true cost of a path through $n$ is $\geq 9$
Hence true cost of a path through $n'$ is $\geq 9$ also

Pathmax modification to A$^*$:
Instead of $f(n') = g(n') + h(n')$, use $f(n') = max(g(n') + h(n'), f(n))$

With pathmax, $f$ is always nondecreasing along any path

# Romania with step costs in km – SLD heuristic

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# Exercise: Search Algorithms

The following figure shows a portion of a partially expanded search tree. Each arc between nodes is labeled with the cost of the corresponding operator, and the leaves are labeled with the value of the heuristic function, *h*.

Which node (use the node's letter) will be expanded next by each of the following search algorithms?

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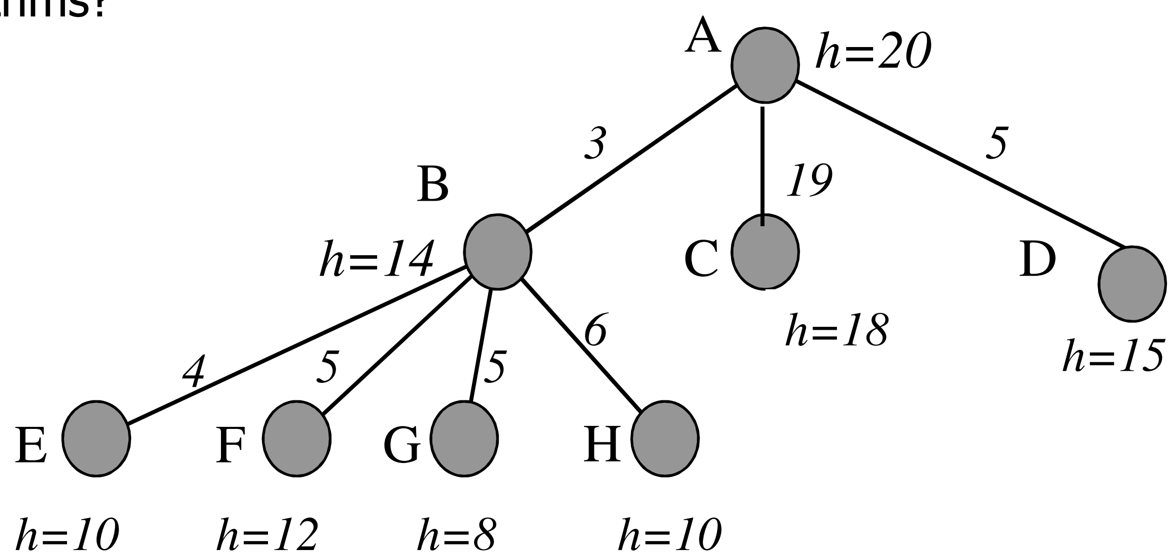

# Depth-first search

Node queue: initialization

| # | state | depth | path cost | parent # |
|---|---|---|---|---|
| 1 | A | 0 | 0 | -- |

# Depth-first search

Node queue: add successors to queue front; empty queue from top

| # | state | depth | path cost | parent # |
|---|---|---|---|---|
| 2 | B | 1 | 3 | 1 |
| 3 | C | 1 | 19 | 1 |
| 4 | D | 1 | 5 | 1 |
| 1 | A | 0 | 0 | -- |

## Depth-first search

Node queue: add successors to queue front; empty queue from top

| # | state | depth | path cost | parent # |
|---|---|---|---|---|
| 5 | E | 2 | 7 | 2 |
| 6 | F | 2 | 8 | 2 |
| 7 | G | 2 | 8 | 2 |
| 8 | H | 2 | 9 | 2 |
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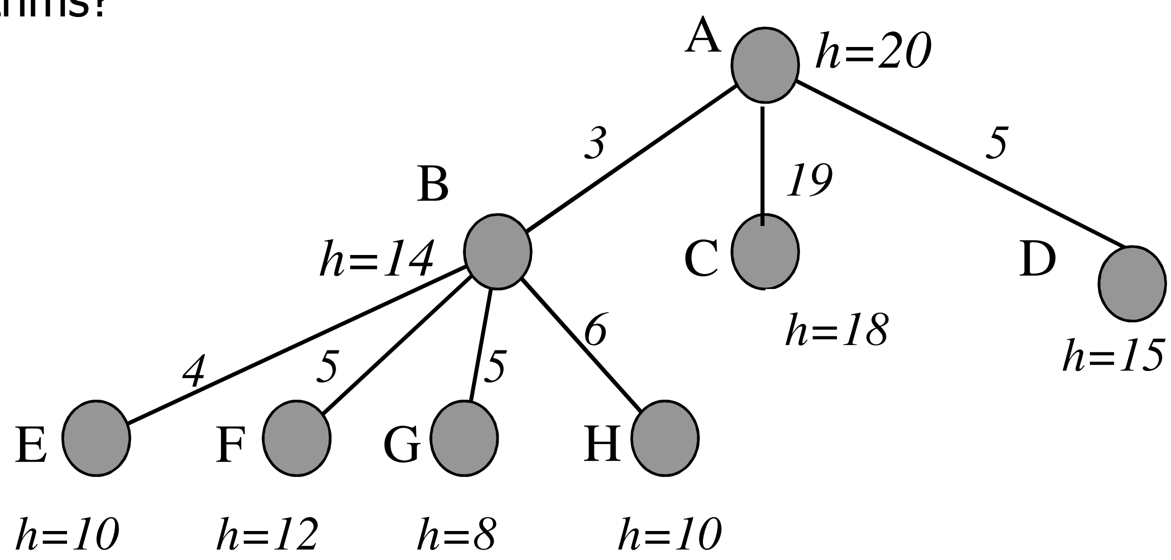

# Breadth-first search

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| # | state | depth | path cost | parent # |
|---|---|---|---|---|
| 1 | A | 0 | 0 | -- |

# Breadth-first search

Node queue: add successors to queue end; empty queue from top

| # | state | depth | path cost | parent # |
|---|---|---|---|---|
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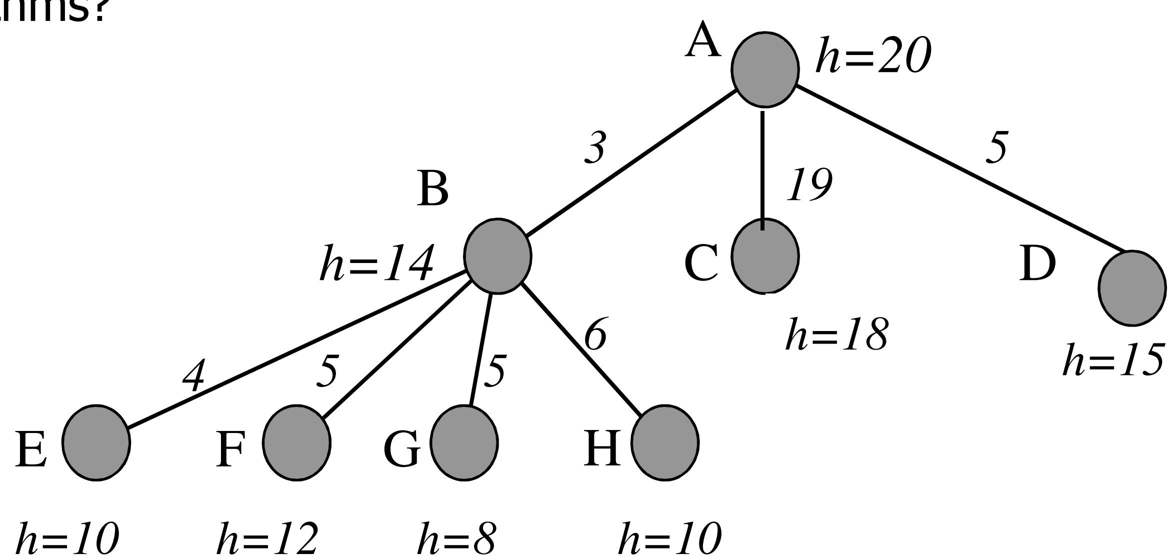

## Uniform-cost search

Node queue: initialization

| # | state | depth | path cost | parent # |
|---|---|---|---|---|
| 1 | A | 0 | 0 | -- |

# Uniform-cost search

Node queue: add successors to queue so that entire queue is sorted by path cost so far; empty queue from top

| # | state | depth | path cost | parent # |
|---|---|---|---|---|
| 1 | A | 0 | 0 | -- |
| 2 | B | 1 | 3 | 1 |
| 3 | D | 1 | 5 | 1 |
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# Uniform-cost search

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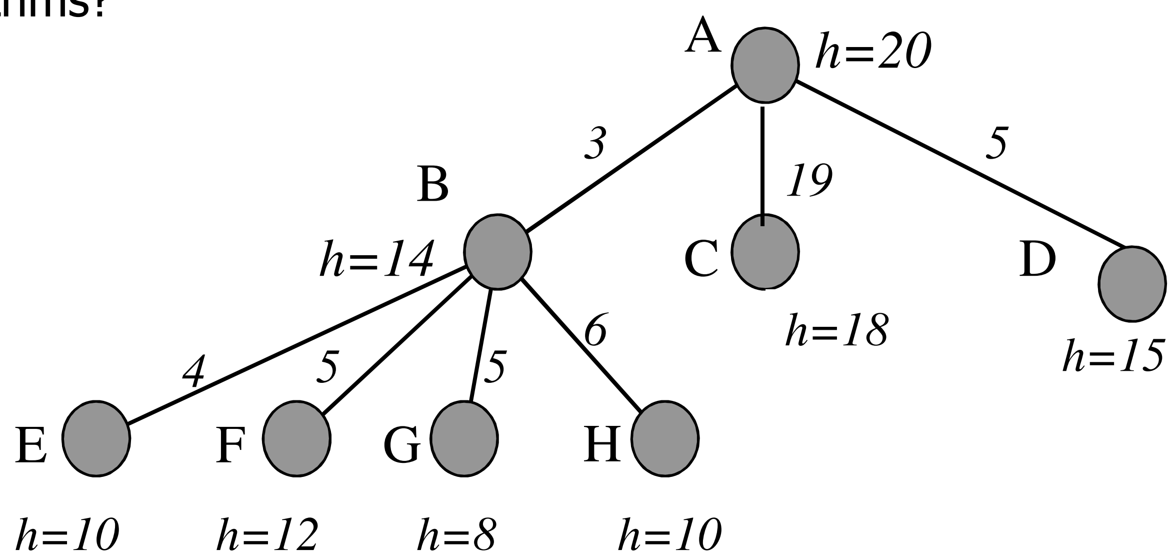

# Greedy search

Node queue: initialization

| # | state | depth | path cost | cost to goal | total cost | parent # |
|---|---|---|---|---|---|---|
| 1 | A | 0 | 0 | 20 | 20 | -- |

# Greedy search

Node queue: Add successors to queue, sorted by cost to goal.

| # | state | depth | path cost | cost to goal | total cost | parent # |
|---|---|---|---|---|---|---|
| 1 | A | 0 | 0 | 20 | 20 | -- |
| 2 | B | 1 | 3 | 14 | 17 | 1 |
| 3 | D | 1 | 5 | 15 | 20 | 1 |
| 4 | C | 1 | 19 | 18 | 37 | 1 |

Sort key

# Greedy search

Node queue: Add successors to queue, sorted by cost to goal.

| # | state | depth | path cost | cost to goal | total cost | parent # |
|---|---|---|---|---|---|---|
| 1 | A | 0 | 0 | 20 | 20 | -- |
| 2 | B | 1 | 3 | 14 | 17 | 1 |
| 5 | G | 2 | 8 | 8 | 16 | 2 |
| 7 | E | 2 | 7 | 10 | 17 | 2 |
| 6 | H | 2 | 9 | 10 | 19 | 2 |
| 8 | F | 2 | 8 | 12 | 20 | 2 |
| 3 | D | 1 | 5 | 15 | 20 | 1 |
| 4 | C | 1 | 19 | 18 | 37 | 1 |

## Greedy search

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| 6 | H | 2 | 9 | 10 | 19 | 2 |
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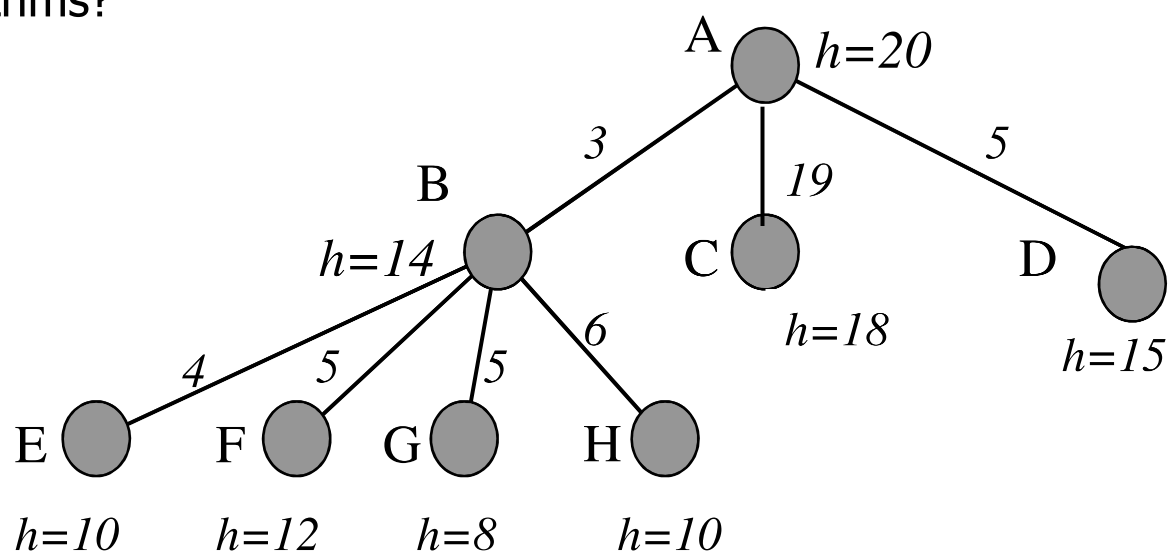

# A* search

Node queue: initialization

| # | state | depth | path cost | cost to goal | total cost | parent # |
|---|---|---|---|---|---|---|
| 1 | A | 0 | 0 | 20 | 20 | -- |

# A* search

Node queue: Add successors to queue, sorted by total cost.

| # | state | depth | path cost | cost to goal | total cost | parent # |
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| 1 | A | 0 | 0 | 20 | 20 | -- |
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| 4 | C | 1 | 19 | 18 | 37 | 1 |

Sort key

# A* search

Node queue: Add successors to queue front, sorted by total cost.

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|---|---|---|---|---|---|---|
| 1 | A | 0 | 0 | 20 | 20 | -- |
| 2 | B | 1 | 3 | 14 | 17 | 1 |
| 5 | G | 2 | 8 | 8 | 16 | 2 |
| 6 | E | 2 | 7 | 10 | 17 | 2 |
| 7 | H | 2 | 9 | 10 | 19 | 2 |
| 3 | D | 1 | 5 | 15 | 20 | 1 |
| 8 | F | 2 | 8 | 12 | 20 | 2 |
| 4 | C | 1 | 19 | 18 | 37 | 1 |

# A* search

Node queue: Add successors to queue front, sorted by total cost.

| # | state | depth | path cost | cost to goal | total cost | parent # |
|---|---|---|---|---|---|---|
| 1 | A | 0 | 0 | 20 | 20 | -- |
| 2 | B | 1 | 3 | 14 | 17 | 1 |
| 5 | G | 2 | 8 | 8 | 16 | 2 |
| 6 | E | 2 | 7 | 10 | 17 | 2 |
| 7 | H | 2 | 9 | 10 | 19 | 2 |
| 3 | D | 1 | 5 | 15 | 20 | 1 |
| 8 | F | 2 | 8 | 12 | 20 | 2 |
| 4 | C | 1 | 19 | 18 | 37 | 1 |

## Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles
$h_2(n)$ = total Manhattan distance
(i.e., no. of squares from desired location of each tile)

| | | |
|---|---|---|
| 5 | 4 | |
| 6 | 1 | 8 |
| 7 | 3 | 2 |

**Start State**

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 8 | | 4 |
| 7 | 6 | 5 |

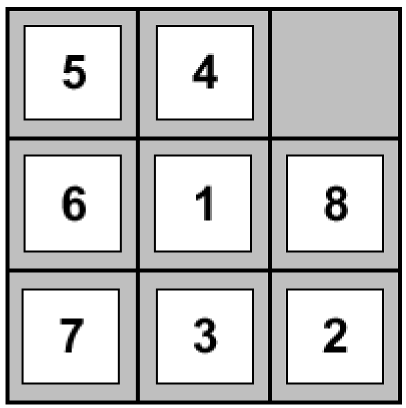
**Goal State**

$h_1(S)$ =??
$h_2(S)$ =??

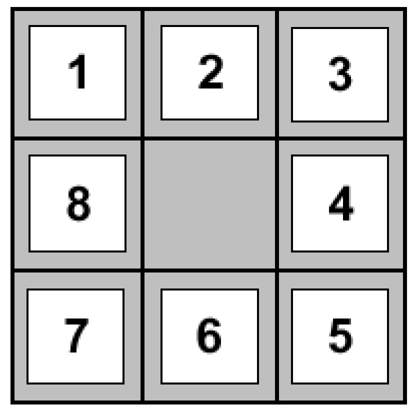
## Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles
$h_2(n)$ = total Manhattan distance
(i.e., no. of squares from desired location of each tile)




Start State




Goal State

$h_1(S) = ??$ 7
$h_2(S) = ??$ 2+3+3+2+4+2+0+2 = 18

## Admissibility By Defining a “Relaxed” Problem

- Admissible heuristics can be derived from the exact solution cost of a relaxed version of the problem.

- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution.

- If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution.