CS 547: Sensing and Planning in Robotics

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(Slides adapted from Thrun, Burgard, Fox book slides)

Bayes Filters: Framework

• Given:

Stream of observations z and action data u:

$$d_t = \{u_1, z_1, \dots, u_t, z_t\}$$

- Sensor model P(z|x).
- Action model P(x|u,x').
- Prior probability of the system state P(x).

• Wanted:

- Estimate of the state X of a dynamical system.
- The posterior of the state is also called Belief:

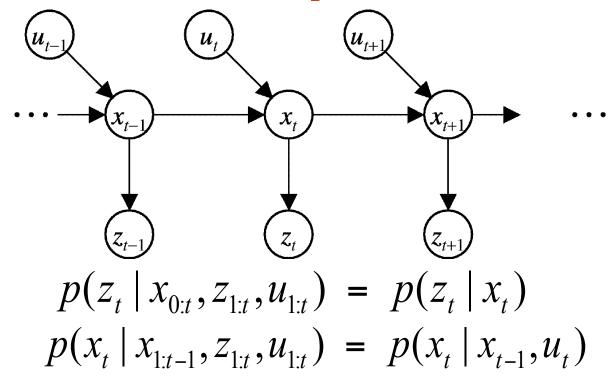
$$Bel(x_t) = P(x_t | u_1, z_1, ..., u_t, z_t)$$

Recursive Bayesian Updating

Markov assumption: z_n is independent of $d_1, ..., d_{n-1}$ if we know x_n

$$\begin{split} Bel_{new}(x) &= P(x \mid d_1, ..., d_{n-1}, z_n) \\ &= \frac{P(z_n \mid x, d_1, ..., d_{n-1}) P(x \mid d_1, ..., d_{n-1})}{P(z_n \mid d_1, ..., d_{n-1})} \\ &= \frac{P(z_n \mid x) P(x \mid d_1, ..., d_{n-1})}{P(z_n \mid d_1, ..., d_{n-1})} \\ &= \eta P(z_n \mid x) P(x \mid d_1, ..., d_{n-1}) \\ &= \eta P(z_n \mid x) Bel_{old}(x) \end{split}$$

Markov Assumption



Underlying Assumptions

- Static world
- Independent noise
- Perfect model, no approximation errors

Recursive Bayesian Updating

Action Update

$$\begin{split} Bel_{new}(x) &= P(x \mid d_1, ..., d_{n-1}, u_n) \\ &= \int P(x \mid d_1, ..., d_{n-1}, u_n, x_{n-1}) P(x_{n-1} \mid d_1, ..., d_{n-1}, u_t) dx_{t-1} \\ &= \int P(x \mid u_n, x_{n-1}) P(x_{n-1} \mid d_1, ..., d_{n-1}) dx_{t-1} \\ &= \int P(x \mid u_n, x_{n-1}) Bel_{old}(x_{n-1}) dx_{t-1} \end{split}$$

$Bel(x_t) = \eta \ P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \ Bel(x_{t-1}) \ dx_{t-1}$

```
Algorithm Bayes_filter( Bel(x),d ):
2.
     \eta=0
3.
      If d is a perceptual data item z then
4.
         For all x do
             Bel'(x) = P(z \mid x)Bel(x)
5.
             \eta = \eta + Bel'(x)
6.
7.
   For all x do
             Bel'(x) = \eta^{-1}Bel'(x)
8.
9.
      Else if d is an action data item u then
10.
         For all x do
             Bel'(x) = \int P(x \mid u, x') Bel(x') dx'
11.
      Return Bel'(x)
12.
```

Discrete Bayes Filter Algorithm

```
Algorithm Discrete_Bayes_filter( Bel(x),d ):
2.
     \eta=0
3. If d is a perceptual data item z then
4.
        For all x do
            Bel'(x) = P(z \mid x)Bel(x)
5.
            \eta = \eta + Bel'(x)
6.
7. For all x do
            Bel'(x) = \eta^{-1}Bel'(x)
8.
9.
     Else if d is an action data item u then
10.
        For all x do
             Bel'(x) = \sum P(x \mid u, x') Bel(x')
11.
12.
     Return Bel'(x)
```

z = observation

u = action

x = state

Bayes Filters

$$\begin{array}{ll} \boxed{\textit{Bel}(x_t)} = P(x_t \mid u_1, z_1 \, ..., u_t, z_t) \\ \text{Bayes} &= \eta \; P(z_t \mid x_t, u_1, z_1, ..., u_t) \; P(x_t \mid u_1, z_1, ..., u_t) \\ \text{Markov} &= \eta \; P(z_t \mid x_t) \; P(x_t \mid u_1, z_1, ..., u_t) \\ \text{Total prob.} &= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_1, z_1, ..., u_t, x_{t-1}) \\ & \qquad \qquad P(x_{t-1} \mid u_1, z_1, ..., u_t) \; dx_{t-1} \\ \text{Markov} &= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; P(x_{t-1} \mid u_1, z_1, ..., u_t) \; dx_{t-1} \\ &= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; P(x_{t-1} \mid u_1, z_1, ..., z_{t-1}) \; dx_{t-1} \\ &= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; Bel(x_{t-1}) \; dx_{t-1} \end{array}$$

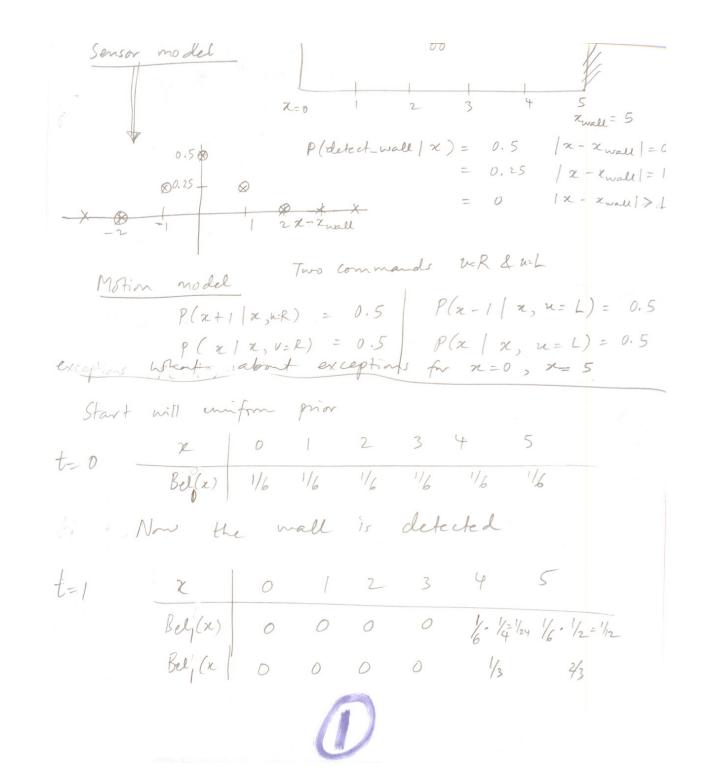
Bayes Filters are Familiar!

$$Bel(x_t) = \eta \ P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \ Bel(x_{t-1}) \ dx_{t-1}$$

- Kalman filters
- Particle filters
- Hidden Markov models
- Dynamic Bayesian networks
- Partially Observable Markov Decision Processes (POMDPs)

Summary

- Bayes rule allows us to compute probabilities that are hard to assess otherwise.
- Under the Markov assumption, recursive Bayesian updating can be used to efficiently combine evidence.
- Bayes filters are a probabilistic tool for estimating the state of dynamic systems.



Now a command W = L is given $P(o \mid x=0, w=L) = | P(1 \mid x=0, w=R) = |$ $P(5 \mid x=5, w=R) = | P(4 \mid x=5, v=L) = |$ $P(4 \mid x=5, v=L$

X 0 1 2 3 4 5

Bel3(2), 0 0 0 0 14.5/4 0

Bel3(2) 0 0 0 0 1 0

What if we redo this for the case where there is no special action model for the 1=0 x 0 1 2 3 4 5 B (x) 1/6 1/6 1/6 1/6 1/6 Now wall is detected B,(x) 0 0 0 0 1/3 2/3 Now command u = L is given B_L(z) 0 0 0 1/6 1/2(1/3) 1/2(1/3) B_L(z) 0 0 0 1/6 1/2(1/3) 82(2) 0 0 1/6 1/2 1/3 Now the wall is detected (say) B₃(x) 0 0 0 0 1/8 1/6

What if you started with a non-uniform $\begin{cases} t = 0 & 2 & 3 & 4 & 5 \\ B_0(z) & 1 & 0 & 0 & 0 & 0 & 0 \end{cases}$ DEPLETION EXAMPLE)
Now wall is detected te 1 2 0 1 2 3 4 5 11 B,(2) 0 0 0 0 0 0 0 0 What if you started with a non-mitom prior but not zero everywhere t=0 2 0 1 2 3 4 5
B(x) 19/20 1/100 1/100 1/100 1/100 1/100 Now wall is detected $t=1 \times 0 1 2 3 4 5$ $B_1(x) 0 0 0 0 1/400 1/200$ B₁(z) 0 0 0 0 1/3 2/3

