

A multilayer network approach to polarity-driven cell-fate patterning in mammary organoids

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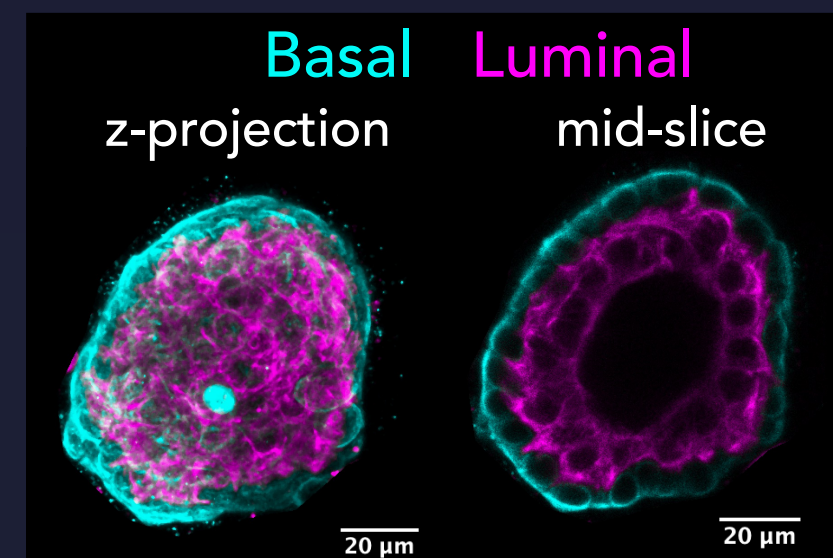
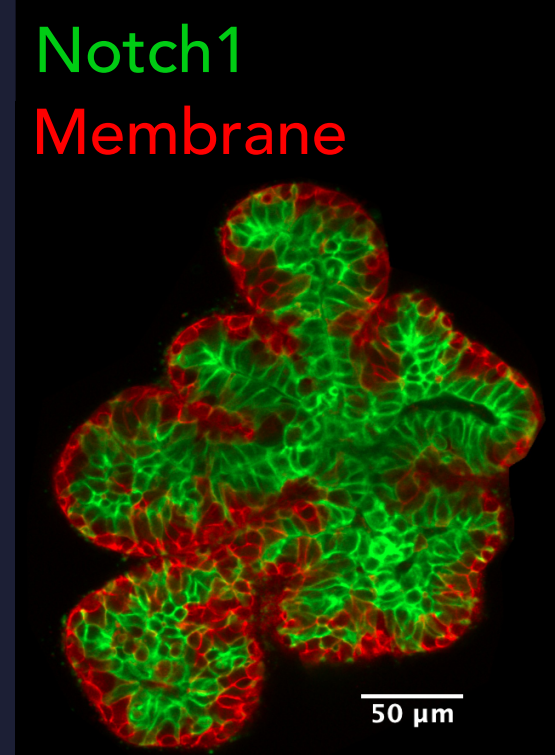
1

Mammary organoids and cell-fate patterning

Mammary organoids are 3D *in vitro* biological models derived from mammary glands that preserve the **bilayer tissue structure** and **cellular functions** [1].

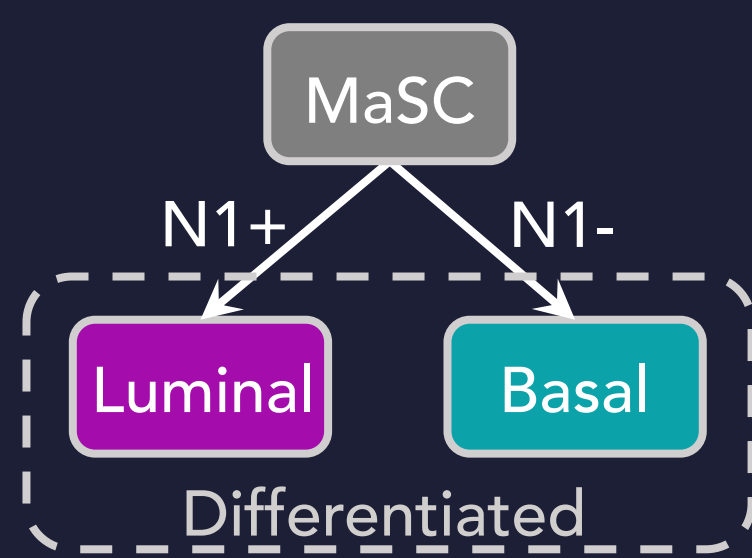
Coordinated control of cell-fate decisions, essential for healthy development, is highly dependent on the cellular microenvironment in mammary organoids [2].

Among many other factors, cell-fate decisions are regulated by Notch1 [3].

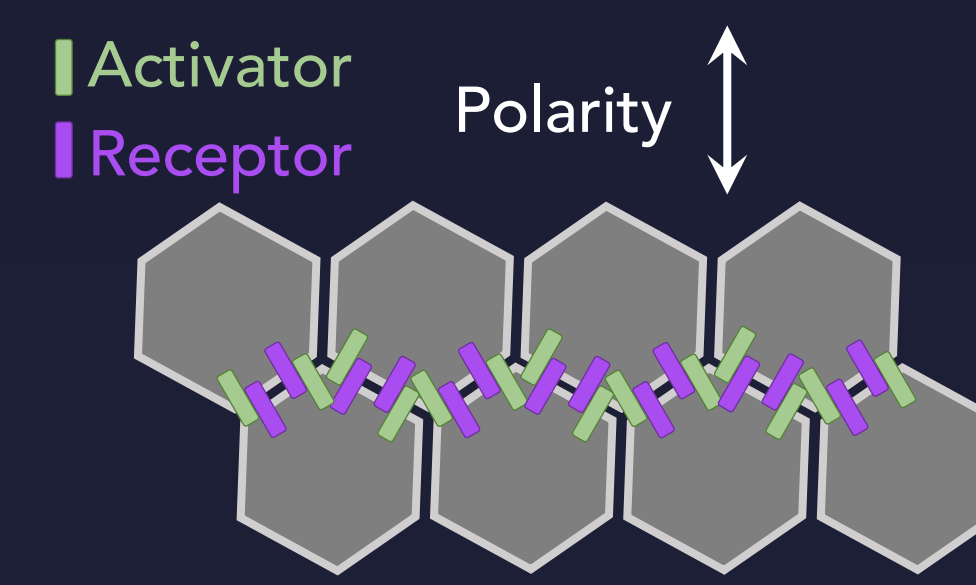


This generates **laminar patterns** of Notch1 in the bilayer cellular architecture.

However, the mechanisms that robustly generate these patterns are yet to be identified.



Modelling cell signalling polarity with weighted networks



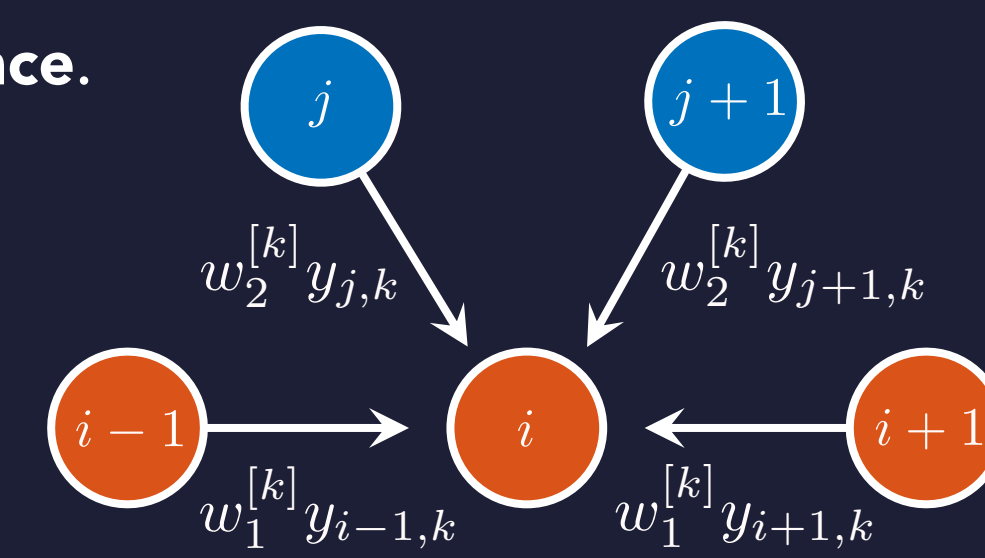
Cell signalling polarity is the **asymmetric distribution** of pathway signal activators and receptors on the surface of the membrane [4].

Many cell-fate determinants including Notch1 have been observed to exhibit polarity characteristics in the developing tissues [5].

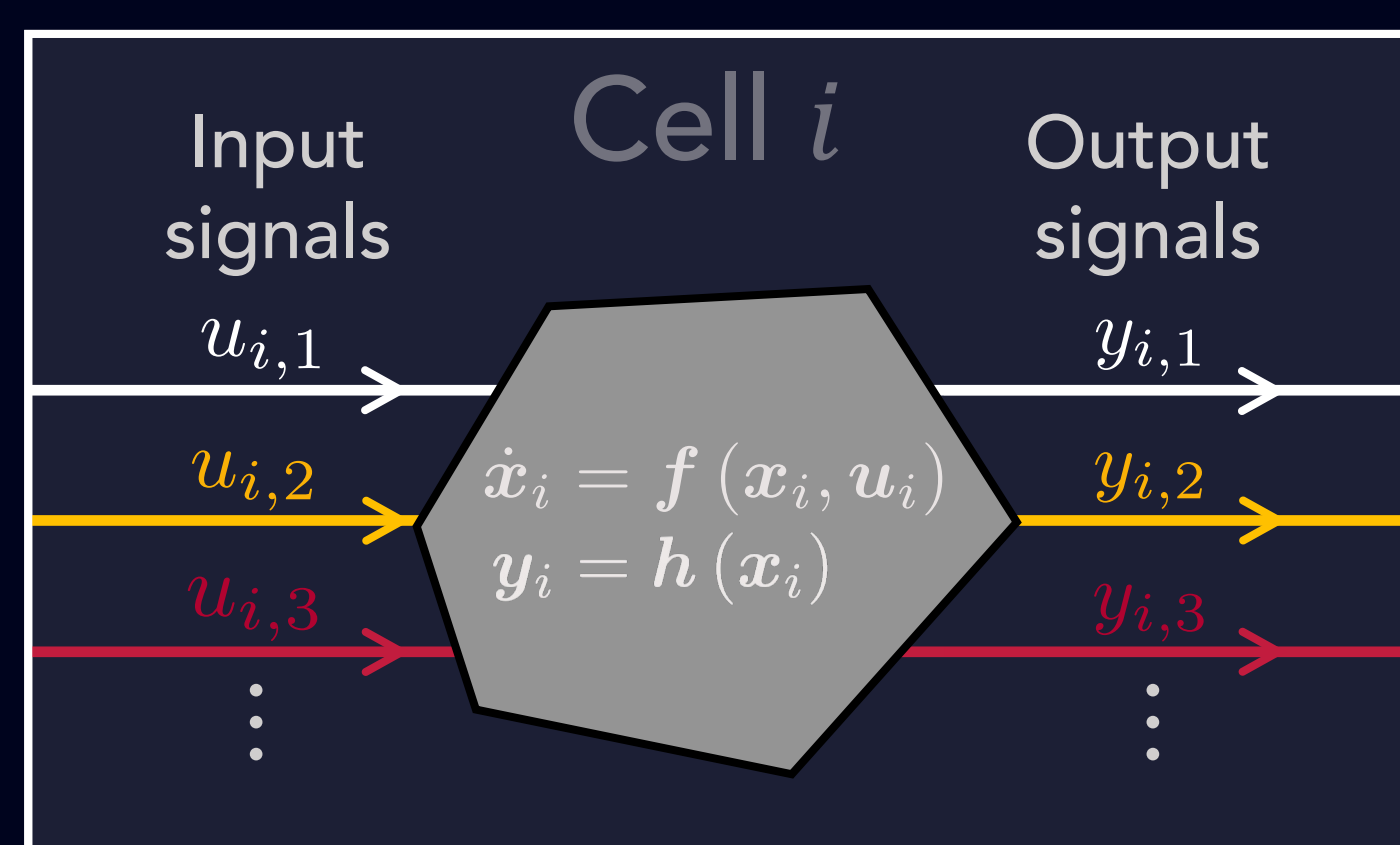
A topological (network) approach to cell-cell connectivity allows for **spatial dimension independence**.

Signal polarity edge weights for \mathcal{G}_k
 $w_1^{[k]}$ - same layer $w_2^{[k]}$ - different layer

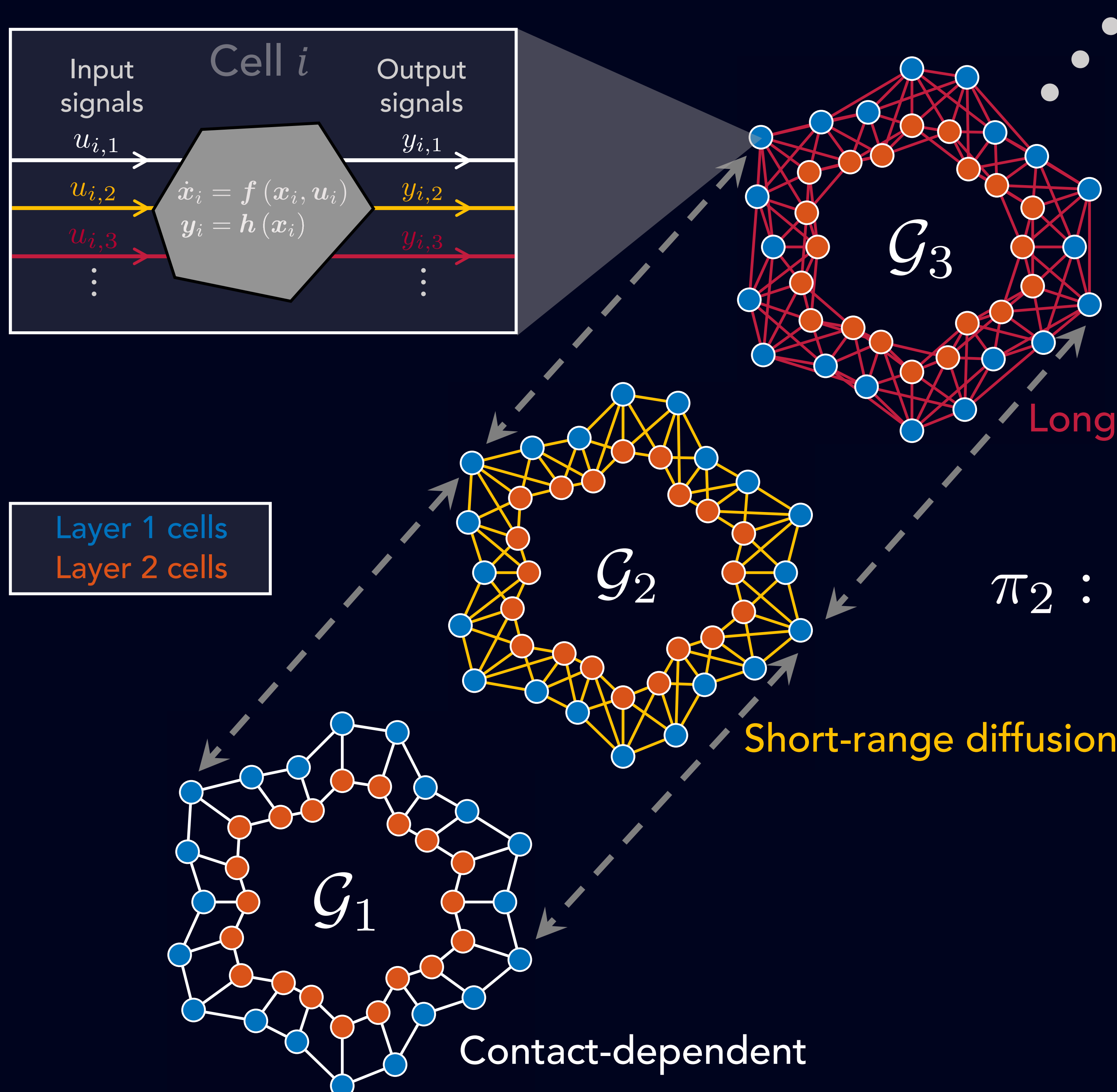
Aim: quantify the relative amount of polarity required to induce laminar patterns of cell-fate determinants.



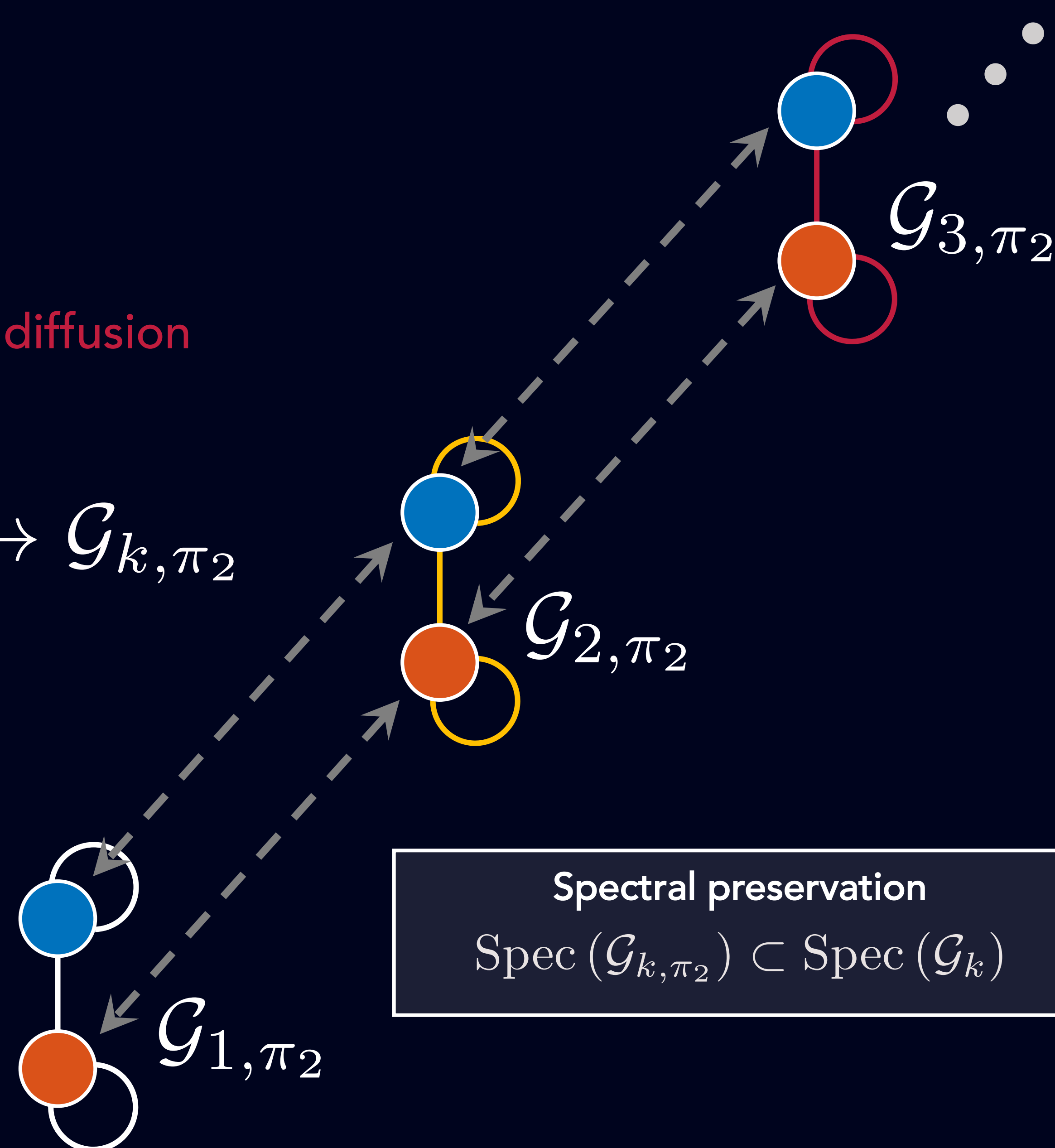
Interconnected dynamical systems for intracellular interactions



Multilayer networks for multiple channels of intercellular signaling



Quotient networks for laminar pattern templating



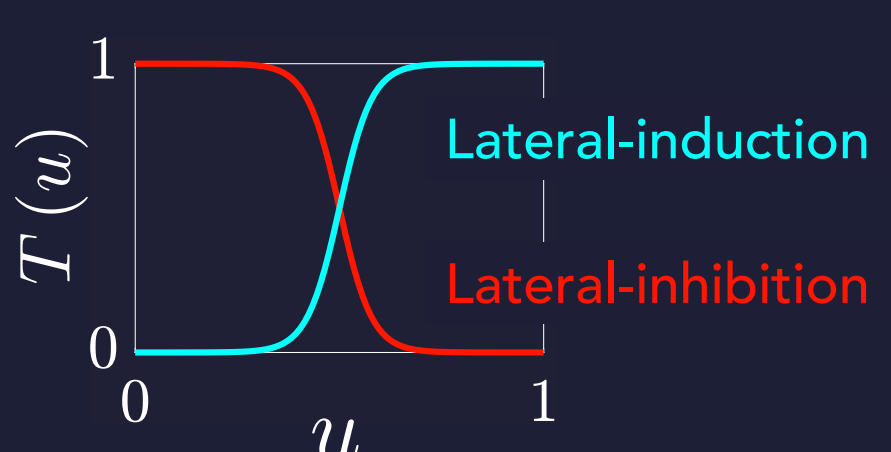
Spectral preservation
 $\text{Spec}(\mathcal{G}_{k,\pi_2}) \subset \text{Spec}(\mathcal{G}_k)$

3

Monotone IO kinetics generate predictable patterns

A signal input/output (IO) interconnected dynamical system is said to be monotone if the signal transfer function $T: U \rightarrow Y$ is monotone.

Bounded and monotonic intercellular signals produce **convergent** and **non-oscillatory** intracellular dynamics that are highly dependent on cell-cell connectivity.



Laminar pattern existence

Spatially-driven homogenous steady state instability in the quotient system:

$$\prod_{i=1}^r (1 - \mu_i) < 0$$

$$\mu_i \in \text{Spec}(\Lambda_{\min} \partial_u T(u^*))$$

$$\Lambda_{\min} = \text{diag}(\min(\text{Spec}(\mathcal{G}_{1,\pi_2})), \dots, \min(\text{Spec}(\mathcal{G}_{r,\pi_2})))$$

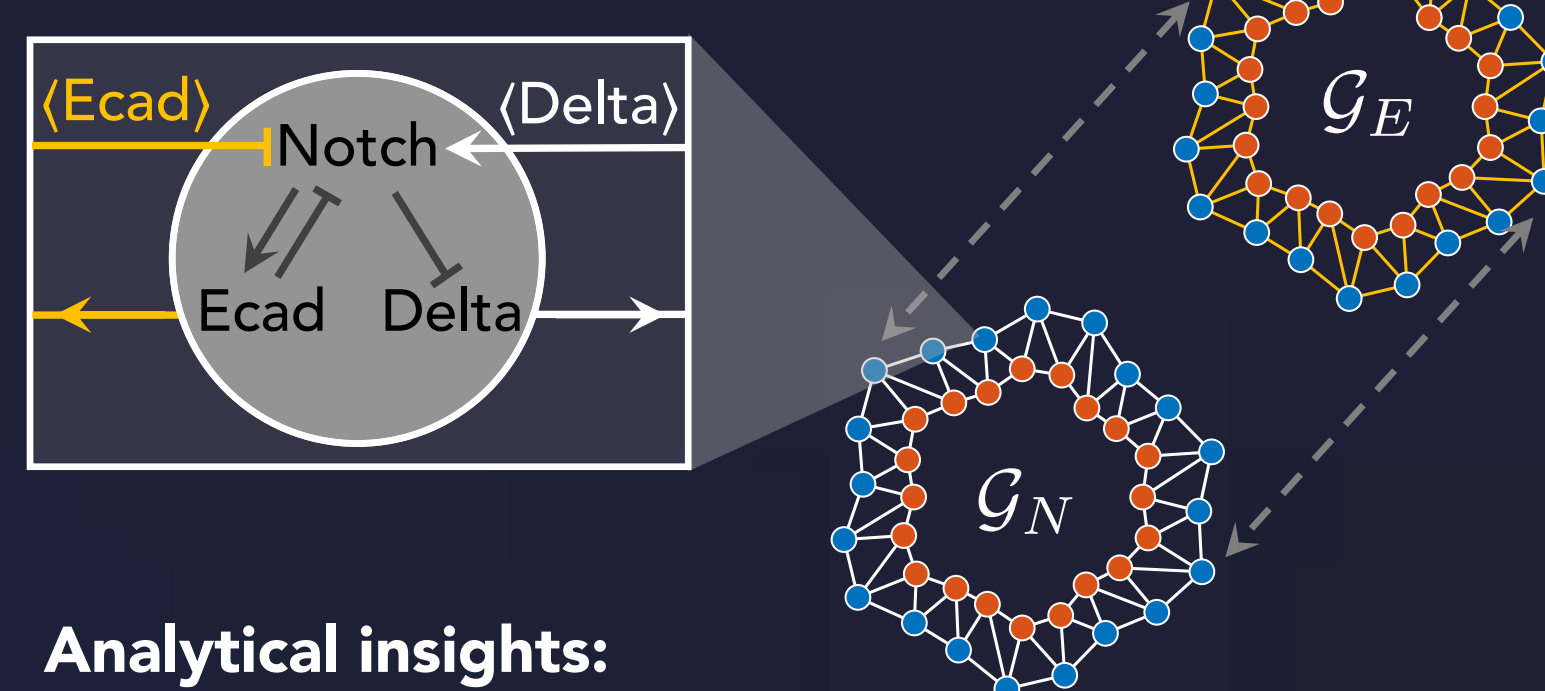
Global laminar pattern convergence

A sufficient condition for quotient patterns to be preserved in the large-scale system:

$$\min(\text{Spec}(\mathcal{G}_{k,\pi_2})) = \min(\text{Spec}(\mathcal{G}_k))$$

Notch-Ecad crosstalk laminar pattern analysis in bilayers

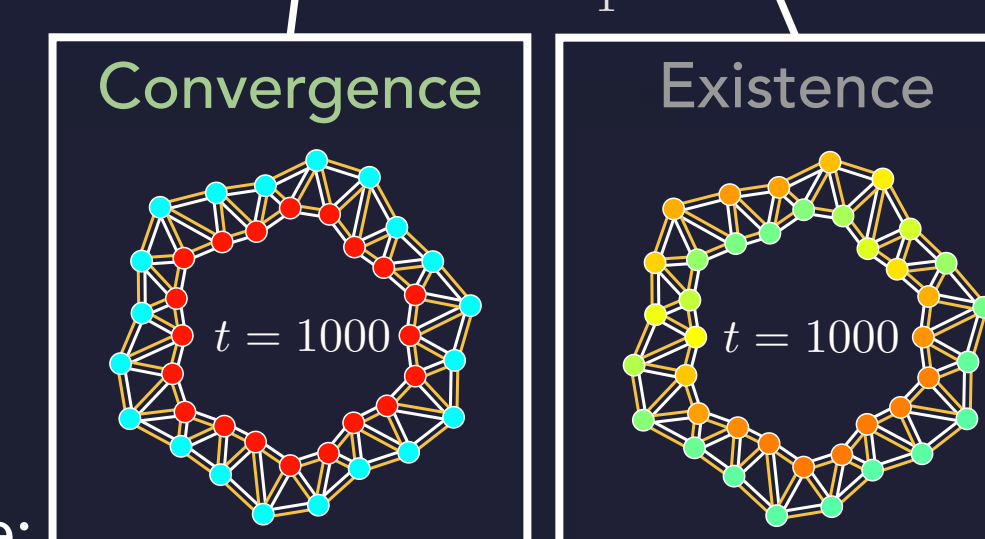
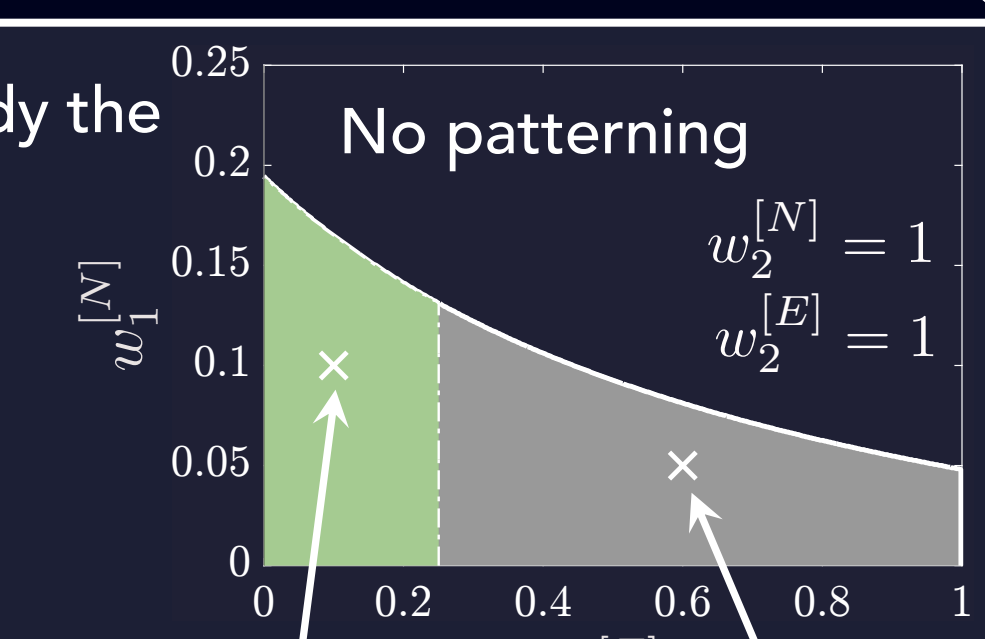
Using our generic pattern analysis framework, we study the polarity required for Notch laminar patterns with the Notch-Ecad-Delta kinetics [6]:



Analytical insights:

- Delta-Notch localisation on the basal-luminal interface initiates pattern existence;
- Ecad localisation is required for pattern convergence:

$$w_1^{[k]} < 4w_2^{[k]} \quad (k \in \{N, E\})$$



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References

