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1. DID WE NORMALIZE THE NUMBER OF SEARCHABLE STARS CORRECTLY?

Recall how we evaluated

$$N_{\mathrm{det}}(r_{\mathrm{a}}, M_{\mathrm{a}}) = \sum_i N^i_{\mathrm{det}}(r_{\mathrm{a}}, M_{\mathrm{a}}) \tag{1}$$

$$N_{\mathrm{det}}(r_{\mathrm{a}}, M_{\mathrm{a}}) = \sum_i \int \mathrm{d}r\mathrm{d}M\, \mathcal{N}^i_{\mathrm{s}}(r_{\mathrm{a}}, M_{\mathrm{a}}; r, M) \cdot \Gamma^i(r, M) \cdot p_{\mathrm{tra}}(r, M). \tag{2}$$

The fancy-looking $\mathcal{N}^i_{\mathrm{s}}(r_{\mathrm{a}}, M_{\mathrm{a}}; r, M)$ term is supposed to mean "the number of searchable stars of type i (per unit $r_{\mathrm{a}}, M_{\mathrm{a}}$) that give $(r_{\mathrm{a}}, M_{\mathrm{a}})$ when the true system actually has (r, M)".

For $i = 0$, we wrote

$$\mathcal{N}^0_{\mathrm{s}}(r_{\mathrm{a}}, M_{\mathrm{a}}; r, M) = \delta(r_{\mathrm{a}} - r)\delta(M_{\mathrm{a}} - M)N^0_{\mathrm{s}}(r, M), \tag{3}$$

where $N^0_{\mathrm{s}}(r, M)$ is the number of searchable singles with true parameters (r, M). Note that this $N^0_{\mathrm{s}}(r, M)$ should indeed be written a function of the *true* parameters, and only once integrated will it give a function of apparent parameters.

We then get

$$\begin{aligned} N^0_{\mathrm{det}}(r_{\mathrm{a}}, M_{\mathrm{a}}) &= \int \mathrm{d}r\mathrm{d}M \mathcal{N}^0_{\mathrm{s}}(r_{\mathrm{a}}, M_{\mathrm{a}}; r, M)\Gamma^0(r, M)p_{\mathrm{tra}}(r, M) \\ &= \int \mathrm{d}r\mathrm{d}M \delta(r_{\mathrm{a}} - r)\delta(M_{\mathrm{a}} - M)N^0_{\mathrm{s}}(r, M)\Gamma^0(r, M)p_{\mathrm{tra}}(r, M) \\ &= N^0_{\mathrm{s}}(r_{\mathrm{a}}, M_{\mathrm{a}})\Gamma^0(r_{\mathrm{a}}, M_{\mathrm{a}})p_{\mathrm{tra}}(M_{\mathrm{a}}). \end{aligned} \tag{4}$$

For $i = 1$, by definition,

$$N^1_{\mathrm{det}}(r_{\mathrm{a}}, M_{\mathrm{a}}) = \int \mathrm{d}r\mathrm{d}M\, \mathcal{N}^1_{\mathrm{s}}(r_{\mathrm{a}}, M_{\mathrm{a}}; r, M) \cdot \Gamma^i(r, M) \cdot p_{\mathrm{tra}}(r, M). \tag{5}$$

Just as before, we must marginalize out the binary distribution:

$$\mathcal{N}^1_{\mathrm{s}}(r_a, M_a; r, M) = \int \mathrm{d}q\, f(q)\mathcal{N}^1_{\mathrm{s},q}(r_a, M_a; r, M; q). \tag{6}$$

We can thus write (order of integration does not matter)

$$N^1_{\mathrm{det}}(r_{\mathrm{a}}, M_{\mathrm{a}}) = \int \mathrm{d}r\mathrm{d}M\mathrm{d}q\, f(q)\mathcal{N}^1_{\mathrm{s}}(r_{\mathrm{a}}, M_{\mathrm{a}}; r, M; q) \cdot \Gamma^i(r, M) \cdot p_{\mathrm{tra}}(r, M). \tag{7}$$

Now I'm pretty sure that we need to write

$$\mathcal{N}^1_{\mathrm{s}}(r_{\mathrm{a}}, M_{\mathrm{a}}; r, M; q) = \delta(r_{\mathrm{a}} - r\mathcal{A}(q))\delta(M_{\mathrm{a}} - M)N^1_{\mathrm{s}}(r, M; q), \tag{8}$$

where $\mathcal{A}(q)$ is the necessary correction for dilution, defined identically as before, but now $N^1_{\mathrm{s}}(r, M; q)$ is the number of primaries that are searchable for planets when the system has true parameters (r, M, q).

This is immediately different from what we've done previously. We previously wrote

$$N_{\mathrm{s}}^{i}(\delta_{\mathrm{obs}}, L_{\mathrm{sys}}) \propto n_i \delta_{\mathrm{obs}}^3 L_{\mathrm{sys}}^{3/2}, \tag{9}$$

where $N_{\mathrm{s}}^{i}(\delta_{\mathrm{obs}}, L_{\mathrm{sys}})$ is the number of searchable stars of type i, given *apparent* parameters.

Now, instead, when I write $N_{\mathrm{s}}^{1}(r, M; q)$, I'm saying: *given true parameters*, how many searchable primaries are there? I'm pretty sure this latter question is the correct one to ask for purposes of determining $\mathcal{N}_{\mathrm{s}}^{1}(r_{\mathrm{a}}, M_{\mathrm{a}}; r, M; q)$. **If you disagree, then don't bother reading the rest**. Assuming we're asking the right question, let's find out the effect on the $\mathcal{N}_{\mathrm{s}}^{1}$ quantity. First, note

$$d_{\mathrm{det}}^{1}(r, R, M, q) \propto \left(\frac{L(M_{\mathrm{a}})}{L(M_{\mathrm{a}}) + L(qM_{\mathrm{a}})} \right) \cdot \left(\frac{r}{R} \right)^2 \cdot \left(L(M_{\mathrm{a}}) + L(qM_{\mathrm{a}}) \right)^{1/2}, \tag{10}$$

where I ignored the T_{dur} dependence, and wrote the terms for dilution, geometric transit depth, and system luminosity. Taking the cube,

$$N_{\mathrm{s}}^{1}(r, R, M, q) \propto n_b \frac{L(M_{\mathrm{a}})^3}{(L(M_{\mathrm{a}}) + L(qM_{\mathrm{a}}))^{3/2}} \left(\frac{r}{R} \right)^6. \tag{11}$$

This number of searchable stars, given the true system parameters, will be what goes into determining the number of detections (given apparent parameters).

I think this also means $f(q)$ in Eq. 7 should be $f_{\mathrm{vl}}(q)$ — the volume limited binary distribution. Before, we (I) said it would be magnitude limited. The reasoning for why it must be volume-limited will become clear in Eq. 16, where it will be evident that doing otherwise would be erroneously accounting for the Malmquist bias twice.

Let's compute the contribution of primaries to the overall rate density,

$$\Gamma_{\mathrm{a}}^{1}(r_{\mathrm{a}}, M_{\mathrm{a}}) = \frac{N_{\mathrm{det}}^{1}(r_{\mathrm{a}}, M_{\mathrm{a}})}{N_{\mathrm{s,a}}(r_{\mathrm{a}}, M_{\mathrm{a}})} \frac{1}{p_{\mathrm{tra}}(r_{\mathrm{a}}, M_{\mathrm{a}})}. \tag{12}$$

As before, we have

$$N_{\mathrm{s,a}}(r_{\mathrm{a}}, M_{\mathrm{a}}) = N_{\mathrm{s}}^{0}(r_{\mathrm{a}}, M_{\mathrm{a}})(1 + \mu), \tag{13}$$

where $\mu \equiv N_d/N_s^0$ is, as we've agreed before, a number that is determined by the binary mass ratio distribution, the binary fraction, and nothing else.

From Eq. 7 and Eq. 8, we now get

$$N_{\mathrm{det}}^{1}(r_{\mathrm{a}}, M_{\mathrm{a}}) = p_{\mathrm{tra}}(M_{\mathrm{a}}) \int \frac{\mathrm{d}q\, f_{\mathrm{vl}}(q)}{\mathcal{A}(q)} N_{\mathrm{s}}^{1} \left(r = \frac{r_{\mathrm{a}}}{\mathcal{A}(q)}, M = M_{\mathrm{a}}, q = q \right) \Gamma^1 \left(\frac{r_{\mathrm{a}}}{\mathcal{A}(q)}, M_{\mathrm{a}} \right), \tag{14}$$

where I'm keeping the explicit $N_{\mathrm{s}}^{1}(r, M, q)$ dependence to indicate I am referring to Eq. 11, the number of searchable primaries given real parameters. Plugging into

Eq. 12,

$$\Gamma^1_{\rm a}(r_{\rm a}, M_{\rm a}) = \frac{1}{1+\mu}\int \frac{{\rm d}q\, f_{\rm vl}(q)}{\mathcal{A}(q)} \frac{N^1_{\rm s}\left(r = \frac{r_{\rm a}}{\mathcal{A}(q)}, M = M_{\rm a}, q = q\right)}{N^0_{\rm s}(r_{\rm a}, M_{\rm a})} \Gamma^1\left(\frac{r_{\rm a}}{\mathcal{A}(q)}, M_{\rm a}\right) \tag{15}$$

$$= \frac{(n_b/n_s)}{1+\mu}\int \frac{{\rm d}q\, f_{\rm vl}(q)}{\mathcal{A}(q)} \left(\frac{L(M_{\rm a})}{L(M_{\rm a}) + L(qM_{\rm a})}\right)^{3/2} \mathcal{A}(q)^{-6}\Gamma^1\left(\frac{r_{\rm a}}{\mathcal{A}(q)}, M_{\rm a}\right) \tag{16}$$

$$= \frac{(n_b/n_s)}{1+\mu}\int {\rm d}q\, f_{\rm vl}(q)\mathcal{A}(q)^{-4}\Gamma^1\left(\frac{r_{\rm a}}{\mathcal{A}(q)}, M_{\rm a}\right). \tag{17}$$

If you compute out the various limiting cases, it turns out for $f_{\rm vl}(q) = \delta(q-1)$, the above is exactly the same as what we had before. However, if $f_{\rm vl}(q) = q^\beta/\mathcal{N}_q$, it is slightly different.