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THE EFFECTS OF BINARITY ON PLANET OCCURRENCE RATES MEASURED BY TRANSIT SURVEYS

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ABSTRACT

This derives the equations of the paper.

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1. PRELIMINARIES

1.1. *Searchable Distance*

We can detect a signal if

$$\frac{\text{signal}}{\text{noise}} \sim \frac{\delta_{\text{obs}}}{(L/d^2)^{-1/2}}, \quad \delta_{\text{obs}} : \text{observed depth} \tag{1}$$

is above some threshold (it would probably make more sense to include the duration information, but that would anyway be a trivial extension and so we omit it here for brevity). Thus, the maximum searchable distance scales as

$$d(\delta_{\text{obs}}, L_{\text{sys}}) \propto \delta_{\text{obs}} \cdot L_{\text{sys}}^{1/2}. \tag{2}$$

We assume that the signal is detected if and only if a given star is searchable.

Using this distance, the number of searchable stars N_{s} is given by

$$N_{\text{s}}(\delta_{\text{obs}}, L_{\text{sys}}) = n_0 \delta_{\text{obs}}^3 L_{\text{sys}}^{3/2}, \tag{3}$$

where n_0 is a normalization constant proportional to the volume density of the systems with a certain type of interest (e.g. single star, binary). We neglect the dependence of the normalization n_0 on the stellar type.

1.2. *Relation between Apparent and Actual Stellar Properties*

We assume that the apparent property of an unresolved binary is the same as that of the primary:

$$M_{\text{a}} = M_1, \quad R_{\text{a}} = R_1, ... \tag{4}$$

We also assume the stellar radius and luminosity is uniquely related to the stellar mass.

Given these assumptions, the total luminosity of the system is

$$L_{\text{sys}} = L_1 + L_2 = L(M_{\text{a}}) + L(qM_{\text{a}}), \tag{5}$$

where $q = M_2/M_1$. Note that L_{sys} is the true value (because M_{a} is the true primary mass), while the system luminosity estimated by an observer would be $L(M_{\text{a}})$, which based on the apparent stellar parameters (unless the observer has a priori knowledge on the distance to a given system).

1.3. *Apparent Number of Searchable Stars*

Given the apparent signal δ_{obs} and stellar mass M_{a}, the maximum searchable distance for singles and binaries are proportional to $\delta_{\text{obs}} \cdot L(M_{\text{a}})^{1/2}$ and $\delta_{\text{obs}} \cdot [L(M_{\text{a}}) + L(qM_{\text{a}})]^{1/2}$. Thus, the apparent number of searchable stars (i.e. points in the sky), which will be selected by an ignorant observer, is

$$\begin{aligned} N_{\text{s,a}}(\delta_{\text{obs}}, M_{\text{a}}) &= n_{\text{s}} \delta_{\text{obs}}^3 L(M_{\text{a}})^{3/2} \left\{ 1 + \int \mathrm{d}q\, f(q) \frac{\text{BF}}{1-\text{BF}} \left[1 + \frac{L(qM_{\text{a}})}{L(M_{\text{a}})} \right]^{3/2} \right\} \\ &\equiv N_{\text{s}}^0(\delta_{\text{obs}}, L(M_{\text{a}})) \left[1 + \mu(\text{BF}, M_{\text{a}}) \right], \end{aligned} \tag{6}$$

where N_s^0 is the number of searchable singles (this agrees with the actual value), $f(q)$ is the binary mass ratio distribution, and $\mathrm{BF} = n_\mathrm{b}/(n_\mathrm{s} + n_\mathrm{b})$ is the binary fraction in a volume-limited sample.

2. APPARENT OCCURRENCE RATE — GENERAL FORMULA

A group of astronomers wants to measure the mean number of planets of a certain type per star of a certain type. They simply observe a set points on the sky selected in a magnitude-limited way (or whatever, actually?) and detect n planets of a desired class. Then they compute the occurrence as

$$\Lambda(\mathcal{P}, \mathcal{S}) = \frac{n(\mathcal{P}, \mathcal{S})}{N_\mathrm{s}(\mathcal{P}, \mathcal{S})} \times \frac{1}{p_\mathrm{tra}(\mathcal{P}, \mathcal{S})}. \tag{7}$$

Here N_s is the number of stars (among those initially selected) around which the planets of interest are searchable; p_tra is the transit probability.

Let's see how the binary modifies this. The occurrence is computed based on the apparent planetary/stellar parameters $\mathcal{P}_\mathrm{a}$, $\mathcal{S}_\mathrm{a}$:

$$\Lambda_\mathrm{a}(\mathcal{P}_\mathrm{a}, \mathcal{S}_\mathrm{a}) = \frac{n(\mathcal{P}_\mathrm{a}, \mathcal{S}_\mathrm{a})}{N_\mathrm{s,a}(\mathcal{P}_\mathrm{a}, \mathcal{S}_\mathrm{a})} \times \frac{1}{p_\mathrm{tra}(\mathcal{P}_\mathrm{a}, \mathcal{S}_\mathrm{a})}. \tag{8}$$

Here, N_s and p_tra are computed for the apparent parameter $\mathcal{P}_\mathrm{a}$ and $\mathcal{S}_\mathrm{a}$. In the presence of dilution, planets with $(\mathcal{P}_\mathrm{a}, \mathcal{S}_\mathrm{a})$ are associated with systems of many different planetary and stellar properties, so n_a is given by the convolution of the true occurrence, $\Lambda(\mathcal{P}, \mathcal{S})$, and number of searchable stars that give $(\mathcal{P}_\mathrm{a}, \mathcal{S}_\mathrm{a})$ when the true system actually has $(\mathcal{P}, \mathcal{S})$, $\mathcal{N}(\mathcal{P}_\mathrm{a}, \mathcal{S}_\mathrm{a}; \mathcal{P}, \mathcal{S})$:

$$n(\mathcal{P}_\mathrm{a}, \mathcal{S}_\mathrm{a}) = \sum_i n^i(\mathcal{P}_\mathrm{a}, \mathcal{S}_\mathrm{a}) = \sum_i \int \mathrm{d}\mathcal{P}\mathrm{d}\mathcal{S}\, \mathcal{N}_\mathrm{s}^i(\mathcal{P}_\mathrm{a}, \mathcal{S}_\mathrm{a}; \mathcal{P}, \mathcal{S}) \cdot \Lambda^i(\mathcal{P}, \mathcal{S}) \cdot p_\mathrm{tra}(\mathcal{P}, \mathcal{S}), \tag{9}$$

where i specifies the type of true host stars (0: single, 1: primary, 2: secondary).

So the problem essentially reduces to the evaluation of

$$\mathcal{N}_\mathrm{s}(\mathcal{P}_\mathrm{a}, \mathcal{S}_\mathrm{a}; \mathcal{P}, \mathcal{S}) \tag{10}$$

for planets around single stars, primaries in binaries, and secondaries in binaries.

3. EVALUATION OF $\mathcal{N}$

Let us explicitly write $\mathcal{P} = r$ and $\mathcal{S} = M$; R and L are uniquely determined from the assumed mass–radius–luminosity relation. We neglect the P dependence.

3.1. *Single Stars*

For $i = 0$,

$$\mathcal{N}_\mathrm{s}^0(\mathcal{P}_\mathrm{a}, \mathcal{S}_\mathrm{a}; \mathcal{P}, \mathcal{S}) = \delta(\mathcal{P}_\mathrm{a} - \mathcal{P})\delta(\mathcal{S}_\mathrm{a} - \mathcal{S}) N_\mathrm{s}^0(\mathcal{P}, \mathcal{S}), \tag{11}$$

so

$$n^0(\mathcal{P}_\mathrm{a}, \mathcal{S}_\mathrm{a}) = N_\mathrm{s}^0(\mathcal{P}_\mathrm{a}, \mathcal{S}_\mathrm{a}) \cdot \Lambda^0(\mathcal{P}_\mathrm{a}, \mathcal{S}_\mathrm{a}) \cdot p_\mathrm{tra}(\mathcal{P}_\mathrm{a}, \mathcal{S}_\mathrm{a}). \tag{12}$$

If all the stars are singles, this yields

$$\Lambda_\mathrm{a}(\mathcal{P}_\mathrm{a}, \mathcal{S}_\mathrm{a}) = \Lambda^0(\mathcal{P}_\mathrm{a}, \mathcal{S}_\mathrm{a}), \tag{13}$$

as expected (now $\mu = 0$ and $N_\mathrm{s,a} = N_\mathrm{s}^0$) — the true occurrence is recovered.

3.2. *Primaries in Binaries*

Since we assume $\mathcal{S}_\mathrm{a} = \mathcal{S}_1$,

$$\mathcal{N}_\mathrm{s}^1(r', M'; r, M) \propto \delta(M' - M). \tag{14}$$

In this case, $\mathcal{N}_\mathrm{s}^1$ is non-zero only at $r' = R'\sqrt{D'}$, where

$$D' = \left[\frac{r}{R(M')}\right]^2 \times \frac{L(M')}{L_\mathrm{sys}(M', q)} \tag{15}$$

and the normalization is given by the number of searchable binaries (for signal D'):

$$n_\mathrm{b} D'^3 L_\mathrm{sys}^{3/2} = N_\mathrm{s}^0(D', L(M')) \cdot \mu(\mathrm{BF}, M'). \tag{16}$$

Thus,

$$\mathcal{N}_\mathrm{s}^1(r', M'; r, M) = \int \mathrm{d}q\, f(q) \mathcal{N}_{\mathrm{s},q}^1(r', M'; r, M; q), \tag{17}$$

where $f(q)$ is the binary mass ratio distribution and

$$\begin{aligned}\mathcal{N}_{\mathrm{s},q}^1(r', M'; r, M; q) = {} & N_\mathrm{s}^0(D', L(M')) \cdot \mu(\mathrm{BF}, M') \\ & \times \delta\left(r' - r\sqrt{\frac{L(M')}{L_\mathrm{sys}(M', q)}}\right) \delta(M' - M).\end{aligned} \tag{18}$$

3.3. *Secondaries in Binaries*

In this case, $M = qM_1 = qM'$, so

$$\mathcal{N}_\mathrm{s}^2(r', M'; r, M) \propto \delta\left(M' - \frac{M}{q}\right). \tag{19}$$

Again $\mathcal{N}_\mathrm{s}^2$ is non-zero only at $r' = R'\sqrt{D'}$, but this time

$$D' = \left[\frac{r}{R(qM')}\right]^2 \times \frac{L(qM')}{L_\mathrm{sys}(M', q)}. \tag{20}$$

The normalization remains the same as the previous case (we are counting the searchable stars at a given observed depth D', total luminosity of the binary is the same). Thus,

$$\mathcal{N}_\mathrm{s}^2(r', M'; r, M) = \int \mathrm{d}q\, f(q) \mathcal{N}_{\mathrm{s},q}^2(r', M'; r, M; q), \tag{21}$$

where

$$\mathcal{N}_{\rm s}^2(r', M'; r, M; q) = N_{\rm s}^0(D', L(M')) \cdot \mu({\rm BF}, M') \times \delta\left(r' - r\sqrt{\left[\frac{R(M')}{R(qM')}\right]^2 \frac{L(qM')}{L_{\rm sys}(M', q)}}\right) \delta\left(M' - \frac{M}{q}\right). \quad (22)$$

4. RESULT

4.1. *Marginalization over the True Properties*

Let's integrate out $\mathcal{P}$ and $\mathcal{S}$.

$$n^0(r_{\rm a}, M_{\rm a}) = \int {\rm d}r{\rm d}M\, \mathcal{N}_{\rm s}^0(r_{\rm a}, M_{\rm a}; r, M) \cdot \Lambda^0(r, M) \cdot p_{\rm tra}(M) \quad (23)$$

$$= \int {\rm d}r{\rm d}M\, N_{\rm s}^0(\delta_{\rm obs}, L(M_{\rm a}))\delta(r_{\rm a} - r)\delta(M_{\rm a} - M) \cdot \Lambda^0(r, M) \cdot p_{\rm tra}(M) \quad (24)$$

$$= N_{\rm s}^0(\delta_{\rm obs}, L(M_{\rm a})) \cdot \Lambda^0(r_{\rm a}, M_{\rm a}) \cdot p_{\rm tra}(M_{\rm a}). \quad (25)$$

$$n^1(r_{\rm a}, M_{\rm a}) = \int {\rm d}r{\rm d}M\, \mathcal{N}_{\rm s}^1(r_{\rm a}, M_{\rm a}; r, M) \cdot \Lambda^1(r, M) \cdot p_{\rm tra}(M) \quad (26)$$

$$= \int {\rm d}q\, f(q) \int {\rm d}r{\rm d}M\, \mathcal{N}_{{\rm s},q}^1(r_{\rm a}, M_{\rm a}; r, M; q) \cdot \Lambda^1(r, M) \cdot p_{\rm tra}(M) \quad (27)$$

$$= N_{\rm s}^0(\delta_{\rm obs}, L(M_{\rm a})) \cdot p_{\rm tra}(M_{\rm a}) \cdot \mu({\rm BF}, M_{\rm a}) \int \frac{{\rm d}q}{\alpha}\, f(q)\, \Lambda^1(r^*, M_{\rm a}), \quad (28)$$

where $r^* = r_{\rm a}/\alpha$ and

$$\alpha(q, M_{\rm a}) = \sqrt{\frac{L(M_{\rm a})}{L_{\rm sys}(M_{\rm a}, q)}}. \quad (29)$$

Finally,

$$n^2(r_{\rm a}, M_{\rm a}) = \int {\rm d}r{\rm d}M\, \mathcal{N}_{\rm s}^2(r_{\rm a}, M_{\rm a}; r, M) \cdot \Lambda^2(r, M) \cdot p_{\rm tra}(M) \quad (30)$$

$$= \int {\rm d}q\, f(q) \int {\rm d}r{\rm d}M\, \mathcal{N}_{{\rm s},q}^2(r_{\rm a}, M_{\rm a}; r, M; q) \cdot \Lambda^2(r, M) \cdot p_{\rm tra}(M) \quad (31)$$

$$= N_{\rm s}^0(\delta_{\rm obs}, L(M_{\rm a})) \cdot \mu({\rm BF}, M_{\rm a}) \int \frac{q{\rm d}q}{\beta}\, f(q)\, \Lambda^2(r^{**}, qM_{\rm a})\, p_{\rm tra}(qM_{\rm a}), \quad (32)$$

where $r^{**} = r_{\rm a}/\beta$ and

$$\beta(q, M_{\rm a}) = \frac{R(M_{\rm a})}{R(qM_{\rm a})}\sqrt{\frac{L(qM_{\rm a})}{L_{\rm sys}(M_{\rm a}, q)}}. \quad (33)$$

4.2. *Final Formula*

Note again that the denominators of $\Lambda_{\rm a}$ are the same for singles, primaries, and secondaries: $N_{\rm s,a}(\delta_{\rm obs}, M_{\rm a})$ and $p_{\rm tra}(M_{\rm a})$. This is because we are calculating the occurrence at the same apparent planet/star properties, and the observer can never distinguish binaries from singles (and adopt the primary properties for binaries). Using the results above, the apparent occurrence is thus given by

$$\Lambda_{\rm a}(r_{\rm a}, M_{\rm a}) = \frac{1}{1+\mu({\rm BF}, M_{\rm a})} \times \tag{34}$$

$$\left\{\Lambda^0(r_{\rm a}, M_{\rm a}) + \mu({\rm BF}, M_{\rm a}) \left[\int \frac{{\rm d}q}{A}\, f(q)\, \Lambda^1\left(\frac{r_{\rm a}}{A}, M_{\rm a}\right) + \int \frac{q{\rm d}q}{B}\, f(q)\, \Lambda^2\left(\frac{r_{\rm a}}{B}, qM_{\rm a}\right) \frac{R(qM_{\rm a})}{R(M_{\rm a})} q^{-1/3}\right]\right\}, \tag{35}$$

where

$$A = \left[1 + \frac{L(qM_{\rm a})}{L(M_{\rm a})}\right]^{-1/2}, \quad B = \frac{R(M_{\rm a})}{R(qM_{\rm a})}\left[1 + \frac{L(M_{\rm a})}{L(qM_{\rm a})}\right]^{-1/2}. \tag{36}$$

5. EXAMPLES

5.1. *Twin Binary*

We have $f(q) = \delta(q-1)$. Thus

$$\Lambda_{\rm a}(r_{\rm a}, M_{\rm a}) = \frac{1}{1+\mu({\rm BF}, M_{\rm a})}\left\{\Lambda^0(r_{\rm a}, M_{\rm a}) + \mu({\rm BF}, M_{\rm a}) \cdot \sqrt{2}\left[\Lambda^1(\sqrt{2}\, r_{\rm a}, M_{\rm a}) + \Lambda^2(\sqrt{2}\, r_{\rm a}, M_{\rm a})\right]\right\}, \tag{37}$$

where

$$\mu({\rm BF}, M_{\rm a}) = \int {\rm d}q\, f(q) \frac{\rm BF}{1-{\rm BF}}\left[1 + \frac{L(qM_{\rm a})}{L(M_{\rm a})}\right]^{3/2} = 2^{3/2} \cdot \frac{\rm BF}{1-{\rm BF}}. \tag{38}$$

5.1.1. *Same Planets*

If $\Lambda^i(r, M) = Z^i \cdot \delta(r - r_0)$ (all the planets have the same radius),

$$\Lambda_{\rm a}(r_{\rm a}, M_{\rm a}) = \frac{1}{1+\mu({\rm BF})}\left[Z^0 \cdot \delta(r_{\rm a} - r_0) + (Z^1 + Z^2) \cdot \mu({\rm BF}) \cdot \delta\left(r_{\rm a} - \frac{r_0}{\sqrt{2}}\right)\right]. \tag{39}$$

This reproduces Eq.(9) of the draft, after correcting for the smaller number of apparently searchable stars for the diluted planets (factor of $(1/2)^3$). In the limit of ${\rm BF} \to 1$ ($\mu \to \infty$), the above formula yields $\Lambda_{\rm a} = (Z^1 + Z^2)\, \delta(r_{\rm a} - r_0/\sqrt{2})$.

5.2. *Power Law World*

If we assume

$$L(M) \sim M^\alpha \sim R^\alpha, \tag{40}$$

we find

$$A = (1 + q^\alpha)^{-1/2}, \quad B = q^{-1}(1 + q^{-\alpha})^{-1/2}, \tag{41}$$

so

$$\Lambda_{\mathrm{a}}(r_{\mathrm{a}}, M_{\mathrm{a}}) = ... \tag{42}$$

We may further assume

$$f(q) \sim q^{\beta}, \quad \Lambda(r) \sim r^{\gamma} \tag{43}$$

and keep calculating...