Population and Harvesting Analysis of Ben Tre Inc. Clams - Project Report

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October 2024

1 The Problem and Given Data

We have been asked to give assistance to the Vietnamese Ministry of Agriculture and Rural Development (MARD) on two questions pertaining Ben Tre Inc. Ben Tre Inc. is a small commercial clam farm located on a sand flat in the Mekong Delta in Vietnam that harvests hard saltwater clams (*Meretrix Lyrata*). The MARD estimates that in the absence of harvesting, the Ben Tre sand flat can support 780,000 clams. Currently, the MARD has licensed Ben Tre to harvest 1300 clams per day and MARD scientists have observed that at this rate, the clam population will decline and stabilize at 500,000.

Additionally, on January 13, 2022, a local steel plant illegally dumped a large volume of wastewater containing high concentrations of cyanide and phenol (toxic chemicals) into the coastal water near the sand. The event destabilized the ecosystem, instantly killing 50% of the clams. Local authorities declared a state of emergency, forcing Ben Tre to immediately suspend all harvesting operations.

Our task is to advise on two questions:

1. Before the catastrophe, Ben Tre Inc. has petitioned an increase in their clam harvesting license from 1300 to 1500 clams per day. If the clam population return to pre-catastrophe level (i.e. 500,000), how would this increase in their harvesting rate affect the clam population in the long run?
2. How many days should Ben Tre wait before they can resume harvesting at the rate of 1300 clams per day? How much would this suspension cost them given that their average sell price is \$0.35 a piece (MARD will include this amount in their fine imposed to the steel plant)? Finally, after Ben Tre resumes harvesting at 1300 clams per day, how much longer will it take for the clam population to be back within 10% of its pre-castrophe level?

2 Model

2.1 Description of Model

We will use a harvesting model as the following:

$$\frac{dP}{dt} = R\left(1 - \frac{P}{K}\right)P - H \tag{1}$$

We choose to use this differential equation as the population growth rate of the clams is self-limited by the clam farm's carrying capacity K and the harvesting rate H is constant as regulated by the MARD.

2.2 Variables

The variables in the model are the dependent variable P, which is the population of clams, and the independent variable, t which is time.

2.3 Parameters and Initial Conditions

The parameters are

- H, which is the harvesting rate. It may take different values depending on the specific case in question. We will let
  - H_1 be the original constant harvesting rate of 1300 clams per day
  - H_2 be the proposed constant harvesting rate of 1500 clams per day
- R, which is the reproduction rate when the population is close to 0.
- K, which is the carrying capacity that stays constant at 780,000.
- P_1^* and P_2^*, which are the two steady states of Equation 1 in 2.1. The former is the unstable steady state of $280,000$ and the latter is the stable steady state of $500,000$.

P_2^* is given in the problem. We show how we derive R in 3.1 and P_1^* in 3.3.1.

2.4 Modeling Assumptions

The model makes the following modeling assumptions:

1. We assume that the carrying capacity and harvesting rate are constant and not subject to significant alteration.
2. We assume that the growth mechanism is bounded in such a way where the growth rate decreases linearly with the population.
3. We ignore the possibility of further extrinsic effects on the clam farm (i.e. another chemical assault, significant release of predators, etc.)

3 Solving the Model

3.1 Finding the Maximum Reproduction Rate R

We begin by solving by setting $\frac{dP}{dt}$ from Equation 1 equal to 0 since the environment is initially at equilibrium and R is not given. Let $H = H_1$ and $P = P_1^*$.

$$\begin{aligned}
\frac{dP}{dt} &= R\left(1 - \frac{P}{K}\right)P - H \\
0 &= R\left(1 - \frac{P_1^*}{K}\right)P_1^* - H_1 \\
R &= \frac{H_1}{\left(1 - \frac{P_1^*}{K}\right)P_1^*} \\
R &= 0.00724
\end{aligned}$$

3.2 Long Term Behavior Analysis when the Harvesting Rate is 1500 Clams Per Day

To observe the long-term behavior of the clam population when the harvesting rate is 1500 clams per day, we conduct a phase line analysis shown in Figure 1. Notice that Equation 1 when $H = H_2$ has no zeros and is always negative. This implies that the population is always declining, and thus the clams will be exterminated if Ben Tre Inc. harvests 1500 clams per day. Accordingly, we advise against increasing Ben Tre Inc.'s harvesting license from 1300 to 1500 clams per day.

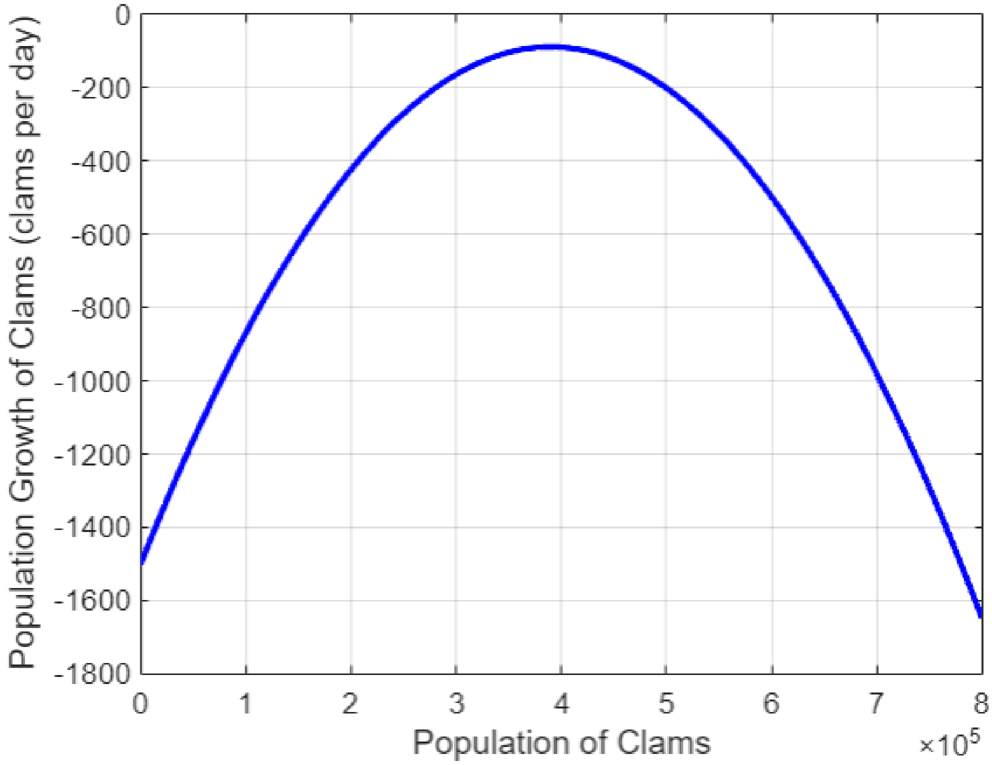

Figure 1: Phase line diagram for the model when the harvesting rate is 1500 clams per day

3.3 Analyzing Effect of Chemical Catastrophe

3.3.1 Calculating Loss due to Catastrophe

We let $H = 0$ and solve for P in Equation 1:

$$\frac{dP}{dt} = R\left(1 - \frac{P}{K}\right)P$$

We separate our variables and integrate.

$$\int \frac{K}{P(K-P)}\,du = \int R\,dt$$

Which implies that

$$\ln\left|\frac{P}{K-P}\right| = Rt + C.$$

We let $P_c = \frac{P_2^*}{2} = 250,000$ and substitute it to P. Then we solve for C analytically.

$$C = \ln\left|\frac{P_c}{K-P_c}\right|$$

Using C to solve for t in terms of P we find that

$$t = \frac{\ln\left(\frac{P}{K-P}\right) + \ln\left|\frac{P_c}{K-P_c}\right|}{R}. \tag{2}$$

In order to find how many days t Ben Tre must wait before they can begin harvesting, we must find the population P beyond which the population growth rate $\frac{dP}{du}$ is no longer negative even in the presence of harvesting. Let that P be P_1^*. Setting Equation 2 equal to 0 and solving for P_1^*, we find that once the population surpasses $280,000$, which is the unstable steady state, the population will grow to the stable steady state of $500,000$ even in the presence of harvesting at a rate of $1,300$ clams per day. This means that we set $P = P_1^* = 280,000$ in Equation 5 and solve for t.

$$t = \frac{\ln\left(\frac{P_1^*}{K-P}\right) + \ln\left|\frac{P_c}{K-P_c}\right|}{R} = 23.7 \text{ days}$$

Due to measurement error in the parameters of the model, we suggest waiting 25 days, at which point the population will be about 281,703 and comfortably above the steady state of 280,000. This means that harvesting can resume on February 8, 2022. Suspending all harvesting for 25 days, at a sale price of \$0.35/clam and a harvesting rate of 1,300 clams/day will cost the company \$11,375.00.

3.3.2 Calculating Long Term Effect on Clam Population

We begin with Equation 2 which describes the population growth of clams after they Ben Tre resumes harvesting. Since the harvesting constant is in the model, we cannot solve the model analytically with separation of variables like we did in section 3.3.1. Rather, we will use ODE45, an adaptive time-step numerical method for solving ODEs built into MatLab. Using ODE45 with initial condition $P(0) = 281,703$, we get the solution curve found in Figure 2.

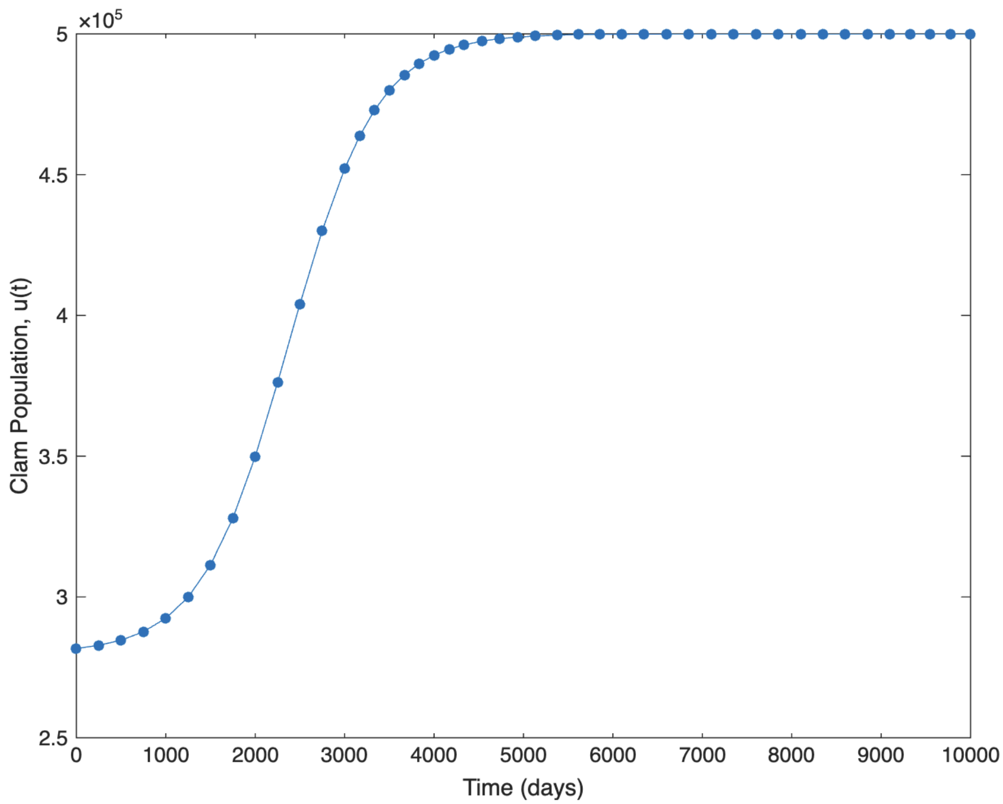

Figure 2: Time course graph of the solution of the model with harvesting at 1,300 clams per day and initial population of 281,703 solved using ODE45 in Matlab

Using this solution curve, we see that it takes approximately 2,977 days after harvesting resumes for the population of clams to reach 450,000 which is 90% of the pre-catastrophe population. Therefore, if Ben Tre follows our advice to wait 25 days before resuming harvesting at a rate of 1,300 clams per day, it will take

3002 days from the day of the catastrophe for the population of clams to reach 90% of its initial population. This will occur on April 3, 2030.

4 Summary of Results

The requested answers are provided hereafter:

- Ben Tre Inc. should *not* be allowed to increase their harvesting license to 1500 clams per day as it would cause the clam population to go extinct even at pre-catastrophe levels.
- We suggest that Ben Tre Inc. wait 25 days before they can resume harvesting, which would be on February 8, 2022. This suspension would cost Ben Tre Inc. $11,375.00.
- We project that it would take about 2,977 days for the population of clams to reach 450,000 (which is 90% of the pre-catastrophe population) after resuming to harvesting as before (i.e. 1300 clams per day).