hw4

1

a

W, Y

b

X, W, Y

c

Y, Z

d

Y, Z, T

e

- $X - Y - T$
- $X - Y - Z - T$
- $X - Y - W - Z - T$
- $X - W - Y - T$
- $X - W - Y - Z - T$
- $X - W - Z - T$
- $X - W - Z - Y - T$

f

- $X - Y - T$
- $X - Y - Z - T$
- $X - W - Y - T$
- $X - W - Y - Z - T$
- $X - W - Z - T$

2

a

$$\begin{tikzcd}
 & U_X \arrow[d, dashed] & \\
U_Y \arrow[d, dashed] & X \arrow[dl] & U_Z \arrow[d, dashed] \\
Y \arrow[rr] & & Z
\end{tikzcd}$$

b

$$E[Z|Y=3] = \sum_z zP(Z=z|Y=3) = 0 \ + \ ... \ + \ 0 + \frac{3}{16} \cdot 1 + 0 + \ ... \ + 0 = \frac{3}{16}$$

c

$$E[Z|X=3] = \sum_z zP(Z=z|X=3) = \frac{1}{16}$$

d

We apply the Markov condition here:

$$E[Z|X=1, Y=3] = \sum_z zP(Z=z|X=1, Y=3) = \sum_z zP(Z=z|Y=3) = \frac{3}{16}$$

3

a

- $\{X, Z_2\}$ are d-separated by $\{Z_3, Z_1\}$
- $\{X, Y\}$ are d-separated by $\{W, Z_2, Z_3\}$
- $\{Z_1, W\}$ are d-separated by $\{X\}$
- $\{Z_1, Z_2\}$ are d-separated by $\{\}$
- $\{Z_1, Y\}$ are d-separated by $\{W, Z_3, Z_2\}$
- $\{Z_3, W\}$ are d-separated by $\{X\}$
- $\{Z_2, W\}$ are d-separated by $\{X\}$

b

- $\{W, Z_3\}$ are d-separated by $\{X\}$
- $\{W, Z_1\}$ are d-separated by $\{X\}$

c

Yes.

4

a

Needs Z and then any combination of A, B, C, D

- $\{Z, D\}, \{Z, C\}, \{Z, B\}, \{Z, A\}$
- $\{Z, D, C\}, \{Z, D, B\}, \{Z, D, A\}, \{Z, C, A\}, \{Z, B, A\}, \{Z, B, C\}$
- $\{X, D, C, B\}, \{Z, D, C, A\}, \{Z, D, B, A\}, \{Z, B, C, A\}$
- $\{Z, D, C, B, A\}$

The minimal ones are

- $\{Z, D\}, \{Z, C\}, \{Z, B\}, \{Z, A\}$

b

must include Z or C or both and then any combination of B, A, X, W, can also just include C only.

- $\{Z, C\}, \{C\}$

5

a

$$P(X, Y, Z) = P(X|Z)P(Y|X, Z)P(Z)$$

b

$$\begin{tikzcd} & Z \arrow[dr] & \\ X = x \arrow[rr] & & Y \end{tikzcd}$$

c

$$\begin{aligned} P(Y, Z \mid \mathrm{do}(X = x)) &= P_m(Z)P_m(X = x|Z)P_m(Y|X = x, Z) \\ &= P(Z)P(Y|X = x, Z) \end{aligned}$$

$$\begin{aligned} P(Y \mid \mathrm{do}(X = x)) &= \sum_z P(Y, Z = z \mid \mathrm{do}(X = x)) \\ &= \sum_z P(Z = z)P(Y|X = x, Z = z) \end{aligned}$$

d

$$P(X, Y, Z) = P(X)P(Y|X)P(Z|X, Y)$$

e

$$\begin{tikzcd} & Z & \\ X = x \arrow[ur] \arrow[rr] & & Y \arrow[ul] \end{tikzcd}$$

f

$$
\begin{aligned}
P(Y, Z \mid \mathrm{do}(X = x)) &= P_m(X = x)P_m(Y|X = x)P_m(Z|X = x, Y) \\
&= P(Y|X = x)P(Z|X = x, Y)
\end{aligned}
$$

$$
\begin{aligned}
P(Y \mid \mathrm{do}(X = x)) &= \sum_z P(Y, Z \mid \mathrm{do}(X = x)) \\
&= \sum_z P(Y|X = x)P(Z = z|X = x, Y) \\
&= P(Y|X = x)\sum_z P(Z = z|X = x, Y) \\
&= P(Y|X = x) \cdot 1
\end{aligned}
$$

g

No (as we'd expect)

h

For common cause $\{Z\}$ satisfies the backdoor criterion and so we use tha adjustment formula to get

$$
P(Y \mid \mathrm{do}(X = x)) = \sum_z P(Y|X = x, Z = z)P(Z = z)
$$

which is exactly what we got in part (c)

For collider there are no backdoor paths into X. I suppose $\{\}$ sort of vacuously satisfies the backdoor criterion which indeed gives us

$$
P(Y \mid \mathrm{do}(X = x)) = \sum_{\{\}} P(Y|X = x) = P(Y|X = x)
$$

which again is what we got previously.