hw4

1

a

W, Y

b

X, W, Y

c

Y, Z

d

Y,Z,T

е

- $\bullet \quad X-Y-T \\ \bullet \quad X-Y-Z-T$
- X-Y-W-Z-T
- X W Y T
- X-W-Y-Z-T
- X W Z T
- X-W-Z-Y-T

f

•
$$X - Y - T$$

•
$$X-Y-Z-T$$

$$X - Y - Z - T$$

$$X - W - Y - T$$

$$\bullet \ X-W-Y-Z-T \\ \bullet \ X-W-Z-T$$

•
$$X-W-Z-T$$

2

a

$$\begin{array}{c} U_X \\ U_Y \stackrel{\downarrow}{\downarrow} U_Z \\ \stackrel{\downarrow}{\downarrow} \stackrel{\swarrow}{\downarrow} \stackrel{\downarrow}{\downarrow} \end{array}$$

b

$$E[Z|Y=3] = \sum_{z} z P(Z=z|Y=3) = 0 \ + \ \dots \ + \ 0 + \frac{3}{16} \cdot 1 + 0 + \ \dots \ + 0 = \frac{3}{16}$$

C

$$E[Z|X=3] = \sum_{z} z P(Z=z|X=3) = \frac{1}{16}$$

d

We apply the Markov condition here:

$$E[Z|X=1,Y=3] = \sum_{z} zP(Z=z|X=1,Y=3) = \sum_{z} zP(Z=z|Y=3) = \frac{3}{16}$$

3

a

- $\{X,Z_2\}$ are d-separated by $\{Z_3,Z_1\}$
- $\{X,Y\}$ are d-separated by $\{W,Z_2,Z_3\}$
- $\{Z_1, W\}$ are d-separated by $\{X\}$
- $\{Z_1,Z_2\}$ are d-separated by $\{\}$
- $\{Z_1,Y\}$ are d-separated by $\{W,Z_3,Z_2\}$
- $\{Z_3, W\}$ are d-separated by $\{X\}$
- $\{Z_2, W\}$ are d-separated by $\{X\}$

b

- $\{W, Z_3\}$ are d-separated by $\{X\}$
- $\{W, Z_1\}$ are d-separated by $\{X\}$

C

Yes.

4

a

Needs Z and then any combination of A, B, C, D

- $\{Z,D\},\{Z,C\},\{Z,B\},\{Z,A\}$
- $\{Z, D, C\}, \{Z, D, B\}, \{Z, D, A\}, \{Z, C, A\}, \{Z, B, A\}, \{Z, B, C\}$ $\{X, D, C, B\}, \{Z, D, C, A\}, \{Z, D, B, A\}, \{Z, B, C, A\}$
- $\{Z, D, C, B, A\}$

The minimal ones are

• $\{Z, D\}, \{Z, C\}, \{Z, B\}, \{Z, A\}$

b

must include z or c or both and then any combination of B, A, X, W