

# hw6

**1**

**a**

$$\begin{aligned} E[\hat{\tau}_{wdim}] &= E\left[\frac{1}{n_1} \sum_{D_i=1} w_i Y_i\right] - E\left[\frac{1}{n_0} \sum_{D_i=0} w_i Y_i\right] \\ &= E\left[w(X)Y(1) \mid D=1\right] - E\left[w(X)Y(0) \mid D=0\right] \\ &= E\left[E[w(X)Y(0) \mid X, D=1] \mid D=1\right] - E\left[E[w(X)Y(1) \mid X, D=0] \mid D=0\right] \quad \text{iterated expectation} \\ &= E\left[w(X)E[Y(0) \mid X] \mid D=1\right] - E\left[w(X)E[Y(1) \mid X] \mid D=0\right] \quad \text{CI} \\ &= E[w(X)f_1(X) \mid D=1] - E[w(X)f_0(X) \mid D=0] \\ &= E[f_1(X)] - E[f_0(X)] \quad \text{WC-ATE} \\ &= E[E[Y(1) \mid X]] - E[E[Y(0) \mid X]] \\ &= E[Y(1)] - E[Y(0)] \\ &= \text{ATE} \end{aligned}$$

**b**

$$\begin{aligned} E[\hat{\tau}_{wdim}] &= E\left[\frac{1}{n_1} \sum_{D_i=1} w_i Y_i\right] - E\left[\frac{1}{n_0} \sum_{D_i=0} w_i Y_i\right] \\ &= E[w(X)f_1(X) \mid D=1] - E[w(X)f_0(X) \mid D=0] && \text{as shown above} \\ &= E\left[\frac{1}{n_1} \sum_{D_i=1} w_i f_1(X_i)\right] - E\left[\frac{1}{n_0} \sum_{D_i=0} w_i f_0(X_i)\right] \\ &= E\left[\frac{1}{n} \sum_{D_i=1} f_1(X_i)\right] - E\left[\frac{1}{n} \sum_{D_i=0} f_0(X_i)\right] && \text{EWC-ATE} \\ &= E[E[Y(1) \mid X]] - E[E[Y(0) \mid X]] \\ &= E[Y(1)] - E[Y(0)] \\ &= \text{ATE} \end{aligned}$$

**c**

**d**

**2**

```
set.seed(394)
library(tidyverse)
library(gt)

# Sample size
n <- 1500 #X
x <- rnorm(n)

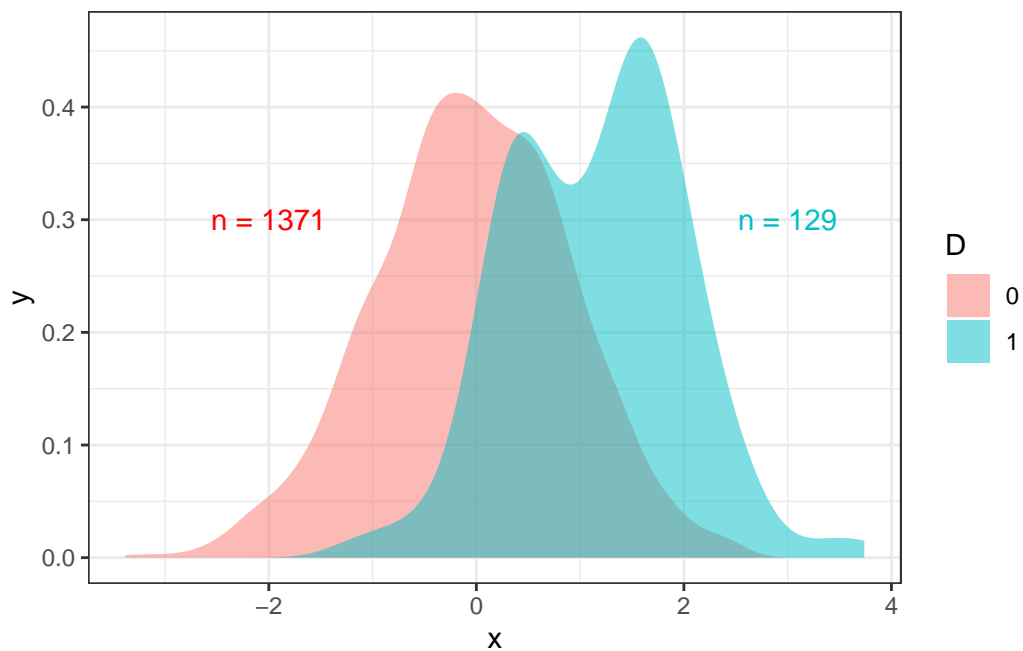
#D
prob <- exp(1.5*x-3.25) / (1 + exp(1.5*x-3.25))
d <- 1*(runif(n)<prob)

#Y
y <- x + rnorm(n, sd=0.75)

data <- tibble(
  x = x,
  d = d,
  y = y
)
```

a

```
data %>%  
  ggplot(aes(x = x, fill = factor(d))) +  
  geom_density(  
    alpha = 0.5,  
    color = NA  
  ) +  
  annotate("text", x = -2, y = 0.3, label = "n = 1371", color = "red") +  
  annotate("text", x = 3, y = 0.3, label = "n = 129", color = "#00BFC4") +  
  labs(  
    fill = "D"  
  ) +  
  theme_bw()
```



```
data %>%  
  count(d)
```

```
# A tibble: 2 x 2  
  d     n  
<dbl> <int>  
1     0 1371
```

2 1 129