

hw5

1

a

Identify the ATE under conditional ignorability:

$$\begin{aligned} E[Y(1) - Y(0)] &= E\left[E[Y(1) \mid X]\right] - E\left[E[Y(0) \mid X]\right] && \text{iterated expectation} \\ &= E\left[E[Y(1) \mid X, D = 1]\right] - E\left[E[Y(0) \mid X, D = 0]\right] && \text{CI} \end{aligned}$$

b

We can write that

$$\begin{aligned}
\hat{\tau}_{Lin} &= \underset{\tau}{\operatorname{argmin}} \sum_{i=1}^n \left[Y_i - \left(\tau D_i + (X_i - \bar{X}_1)^T \hat{\beta}_{Lin} + D_i (X_i - \bar{X}_1)^T \hat{\gamma}_{Lin} \right) \right]^2 \\
&= \underset{\tau}{\operatorname{argmin}} \sum_{i=1}^n \left[\left(Y_i - (X_i - \bar{X}_1)^T \hat{\beta}_{Lin} - D_i (X_i - \bar{X}_1)^T \hat{\gamma}_{Lin} \right) - \tau D_i \right]^2 \\
&= \frac{1}{n_1} \sum_{D_i=1} \left(Y_i - (X_i - \bar{X}_1)^T \hat{\beta}_{Lin} - D_i (X_i - \bar{X}_1)^T \hat{\gamma}_{Lin} \right) - \frac{1}{n_0} \sum_{D_i=0} \left(Y_i - (X_i - \bar{X}_1)^T \hat{\beta}_{Lin} \right) \\
&= \underbrace{\left[\frac{1}{n_1} \sum_{D_i=1} Y_i - \frac{1}{n_0} \sum_{D_i=0} Y_i \right]}_{\hat{\tau}_{dim}} - \frac{1}{n_1} \sum_{D_i=1} (X_i - \bar{X}_1)^T \hat{\beta}_{Lin} - \frac{1}{n_1} \sum_{D_i=1} (X_i - \bar{X}_1)^T \hat{\gamma}_{Lin} + \\
&\quad \frac{1}{n_0} \sum_{D_i=0} (X_i - \bar{X}_1)^T \hat{\beta}_{Lin} \\
&= \hat{\tau}_{dim} - \frac{1}{n_1} \sum_{D_i=1} X_i^T \hat{\beta}_{Lin} + \bar{X}_1^T \hat{\beta}_{Lin} - \frac{1}{n_1} \sum_{D_i=1} X_i^T \hat{\gamma}_{Lin} + \bar{X}_1^T \hat{\gamma}_{Lin} + \frac{1}{n_0} \sum_{D_i=0} X_i^T \hat{\beta}_{Lin} - \bar{X}_1^T \hat{\beta}_{Lin} \\
&= \hat{\tau}_{dim} - \frac{1}{n_0} \sum_{D_i=0} X_i^T \hat{\beta}_{Lin} - \bar{X}_1^T \hat{\beta}_{Lin} \\
&= \frac{1}{n_1} \sum_{D_i=1} Y_i - \frac{1}{n_0} \sum_{D_i=0} Y_i - \frac{1}{n_0} \sum_{D_i=0} X_i^T \hat{\beta}_{Lin} - \frac{1}{n_1} \sum_{D_i=1} X_i^T \hat{\beta}_{Lin}
\end{aligned}$$

but now,

$$\frac{1}{n_0} \sum_{D_i=0} Y_i - \frac{1}{n_0} \sum_{D_i=0} X_i^T \hat{\beta}_{Lin} = \frac{1}{n_0} \sum_{D_i=0} \left(Y_i - X_i^T \hat{\beta}_{Lin} \right) = 0$$

Because the residuals must sum to zero for this portion of the model. And so we are left with

$$\frac{1}{n_1} \sum_{D_i=1} Y_i - \frac{1}{n_1} \sum_{D_i=1} X_i^T \hat{\beta}_{Lin}$$

as desired.

2

a

```
library(tidyverse)
dgp_p2 <- function(n) {
  tibble(
    x1 = rnorm(n, 0, 1),
    x2 = rchisq(n, 1),
    prob_D = exp(0.5*x1 + 0.5*x2 - 0.5)/(1 + exp(0.5*x1 + 0.5*x2 - 0.5)),
    D = rbinom(n, 1, prob_D),
    prob_Y = exp(0.6*x1 + 0.2*x2 + 0.5*x1*x2)/(1 + exp(0.6*x1 + 0.2*x2 + 0.5*x1*x2)),
    Y = rbinom(n, 1, prob_Y)
  ) %>%
  select(-prob_D, - prob_Y)
}

att_est <- function(dat) {
  mod_y <- glm(Y ~ x1 + x2 + x1*x2, dat[dat$D == 0, ], family = "binomial")
  p1 <- mean(dat[dat$D == 1, "Y", drop = T])
  p2 <- mean(predict(mod_y, newdata = dat[dat$D==1, ], type = "response"))
  est_att <- p1 - p2

  return(est_att)
}

bootstrap <- function(dat){
  ests <- rep(0, 500)
  for(i in 1:500) {
    dat_boot <- dat %>%
      slice_sample(n = 500, replace = T)

    ests[i] <- att_est(dat_boot)
  }
  return(ests)
}

sim <- function(df) {
  dim <- lm(Y ~ D, df)$coefficients[2]
  est_att <- att_est(df)
```

```

boots <- bootstrap(df)
ci <- quantile(boots, c(0.025, 0.975))

tibble(
  dim = dim,
  att_hat = est_att,
  lower = ci[1],
  upper = ci[2]
)
}

data_list <- list()
for(i in 1:500){
  data_list[[i]] <- dgp_p2(500)
}

res <- data_list %>%
  map_dfr(sim)

```

Looks unbiased

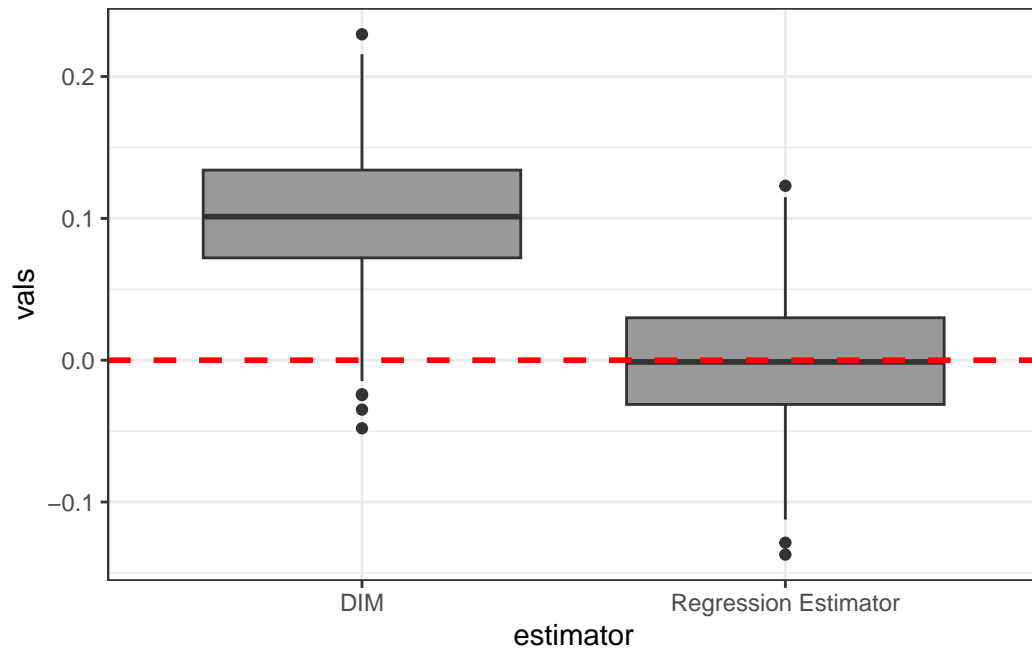
```

library(here)

res <- read_csv(here("data", "hw5_res.csv"))

res %>%
  pivot_longer(cols = c(dim, att_hat), names_to = "estimator", values_to = "vals") %>%
  mutate(estimator = case_when(
    estimator == "att_hat" ~ "Regression Estimator",
    estimator == "dim" ~ "DIM"
  )) %>%
  ggplot(aes(x = estimator, y = vals)) +
  geom_boxplot(fill = "grey60") +
  geom_hline(yintercept = 0, linetype = "dashed", color = "red", linewidth = 1) +
  theme_bw()

```



Correct coverage

```
res %>%
  rowwise() %>%
  mutate(covers = between(0, lower, upper)) %>%
  ungroup() %>%
  summarise(coverage = mean(covers))
```

```
# A tibble: 1 x 1
  coverage
  <dbl>
1    0.944
```

3