SYDE 556/750

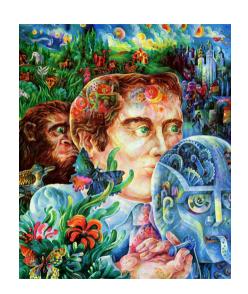
Simulating Neurobiological Systems Lecture 10: Symbols and Symbol-like Representations

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- Content: Terry Stewart, Andreas Stöckel, Chris Eliasmith





Classical Representation of Knowledge

▶ "The number eight comes after the number nine":

► "All dogs chase cats":

$$\forall x \forall y (\mathbf{isDog}(x) \land \mathbf{isCat}(y)) \rightarrow \mathbf{doesChase}(x, y).$$

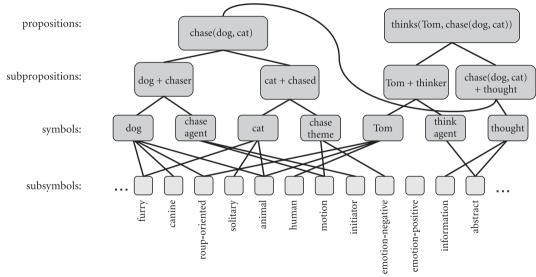
▶ "Anne knows that Bill thinks that Charlie likes Dave":

$$knows(ANNE, "thinks(BILL, 'likes(CHARLIE, DAVE)')")$$
.

Jackendoff's Challenges

- ► The Binding Problem
- ► The Problem of Two
- ► The Problem of Variables
- Working Memory versus Long-Term Memory

Solution Attempt 1: Neural Synchrony (I)

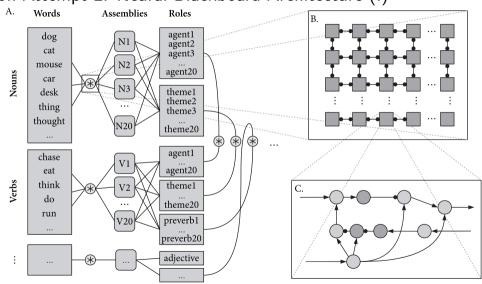


Solution Attempt 1: Neural Synchrony (II)

- Solves the binding problem
- Localist representation
- Unclear how to solve problems 2 to 4

- Unclear how these oscillations are generated and controlled
- Unclear how the representations are processed
- Exponential explosion of neurons required to represent concepts

Solution Attempt 2: Neural Blackboard Architecture (I)



Solution Attempt 2: Neural Blackboard Architecture (II)

- Fewer resources than LISA
- Solves all four of Jackendoffs challenges (according to the authors)
- Explains limitations of human sentence representation
- (At least partially) localist representation

- Particular structure; does not match biology
- Only considers representation, no control structures

Solution Attempt 3: Vector Operators

Idea: High-dimensional vectors $\mathbf{x} \in \mathbb{R}^d$ represent symbols; bind using tensor product

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_1b_1 & a_1b_2 & a_1b_3 \\ a_2b_1 & a_2b_2 & a_2b_3 \\ a_3b_1 & a_3b_2 & a_3b_3 \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \otimes \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} & a_{12} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \\ a_{21} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} & a_{22} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} a_{11}b_{11} & a_{11}b_{12} & a_{12}b_{11} & a_{12}b_{12} \\ a_{21}b_{11} & a_{21}b_{12} & a_{22}b_{11} & a_{22}b_{12} \\ a_{21}b_{21} & a_{21}b_{22} & a_{22}b_{21} & a_{22}b_{22} \end{pmatrix}$$

$$= \begin{pmatrix} a_{11}b_{11} & a_{11}b_{22} & a_{12}b_{21} & a_{12}b_{22} \\ a_{21}b_{11} & a_{21}b_{12} & a_{22}b_{11} & a_{22}b_{12} \\ a_{21}b_{21} & a_{21}b_{22} & a_{22}b_{21} & a_{22}b_{22} \end{pmatrix}$$

$$(\text{Outer product})$$

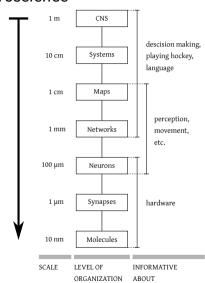
igoplus Scales extremely poorly d^n for n binding operations

A Deeper Problem: Cognitive Science vs. Neuroscience

► Trying very hard to map purely symbolic architectures onto neurons.

- Neural aspects are treated as mere implementation details.
- ► Instance of top-down modelling: High-level cognitive architectures are mapped onto biology.
- ► Hope of many cognitive scientists:

 If successful, neurons do not matter.



Binding Operator Properties

i. Preservation of Dimensionality

$$\circledast: \mathbb{R}^d \times \mathbb{R}^d \longrightarrow \mathbb{R}^d$$

ii. Approximately Reversible

$$\mathbf{x} \approx (\mathbf{x} \circledast \mathbf{y}) \circledast \mathbf{y}^{-1}$$

iii. Dissimilar to Inputs

$$0 \approx \langle \mathbf{x} \circledast \mathbf{y}, \mathbf{x} \rangle, 0 \approx \langle \mathbf{x} \circledast \mathbf{y}, \mathbf{y} \rangle$$

Sentence Encoding Revisited

► "The number eight comes after the number nine":

NUMBER
$$\circledast$$
 EIGHT + SUCC \circledast NINE.

► "The dog chases the cat":

$$DOG \circledast SUBJ + CAT \circledast OBJ + CHASE \circledast VERB$$
.

"Anne knows that Bill thinks that Charlie likes Dave":

$$\begin{split} \text{SUBJ} \circledast \text{ANNE} + \text{ACT} \circledast \text{KNOWS} + \text{OBJ} \circledast \\ \left(\text{SUBJ} \circledast \text{BILL} + \text{ACT} \circledast \text{THINKS} + \text{OBJ} \circledast \right. \\ \left(\text{SUBJ} \circledast \text{CHARLIE} + \text{ACT} \circledast \text{LIKES} + \text{OBJ} \circledast \text{DAVE} \right) . \end{split}$$

Compression of information; graceful degradation; depends on *d*

Using the Reversibility Property to Answer Questions

► "A blue square and a red circle:"

$$\mathbf{x} = \mathtt{BLUE} \circledast \mathtt{SQUARE} + \mathtt{RED} \circledast \mathtt{CIRCLE}$$
.

► "Which object is blue?"

$$\mathbf{y} = (\mathtt{BLUE} \circledast \mathtt{SQUARE} + \mathtt{RED} \circledast \mathtt{CIRCLE}) \circledast \mathtt{BLUE}^{-1}$$

$$= (\mathtt{BLUE} \circledast \mathtt{SQUARE}) \circledast \mathtt{BLUE}^{-1} + (\mathtt{RED} \circledast \mathtt{CIRCLE}) \circledast \mathtt{BLUE}^{-1}$$

$$\approx \mathtt{SQUARE} + \underbrace{\mathtt{RED} \circledast \mathtt{CIRCLE} \circledast \mathtt{BLUE}^{-1}}_{\text{"noise"}}$$

$$\approx \mathtt{SQUARE}.$$

△ Supposes that there is a set of valid symbols ⇒ "Cleanup Memory"

VSAs: Potential Binding Operators (I)

$$\begin{pmatrix}
1 \\ 0 \\ 1 \\ 0
\end{pmatrix} \oplus \begin{pmatrix}
1 \\ 1 \\ 0 \\ 0
\end{pmatrix} = \begin{pmatrix}
0 \\ 1 \\ 1 \\ 0
\end{pmatrix}$$
(XOR)
$$\begin{pmatrix}
A \\ B \\ C \\ D
\end{pmatrix} \odot \begin{pmatrix}
E \\ F \\ G \\ H
\end{pmatrix} = \begin{pmatrix}
AE \\ BF \\ CG \\ DH
\end{pmatrix}$$
(Hadamard Product)

VSAs: Potential Binding Operators (II)

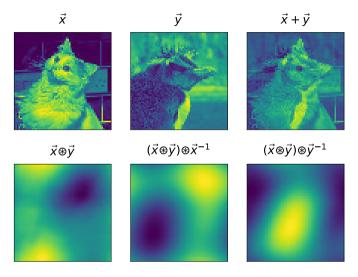
$$\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} \circledast \begin{pmatrix} E \\ F \\ G \\ H \end{pmatrix} = \begin{pmatrix} AE + BH + CG + DF \\ AF + BE + CH + DG \\ AG + BF + CE + DH \\ AH + BG + CF + DE \end{pmatrix}$$
 (Circular Convolution)

Circular Convolution is a "compressed" outer product:

$$\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} \otimes \begin{pmatrix} E \\ F \\ G \\ H \end{pmatrix} = \begin{pmatrix} AE & AF & AG & AH \\ BE & BF & BG & BH \\ CE & CF & CG & CH \\ DE & DF & DG & DH \end{pmatrix}$$

(Outer Product)

Circular Convolution: Dissimilarity and Reversibility



Circular Convolution: Encoding Numbers

- ▶ Spaun uses an interesting encoding to capture number relations.
- ► Start with a random vector ONE, then

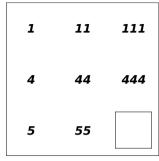
$$\label{eq:two_state} \begin{split} \mathsf{TWO} &= \mathsf{ONE} \circledast \mathsf{ONE} \,, \\ \mathsf{THREE} &= \mathsf{ONE} \circledast \mathsf{TWO} = \mathsf{ONE} \circledast \mathsf{ONE} \circledast \mathsf{ONE} \,, \\ \mathsf{NUMBER-}k &= \underbrace{\mathsf{ONE} \circledast \mathsf{ONE} \circledast \ldots \circledast \mathsf{ONE}}_{k\text{-times}} \\ &= \mathsf{ONE}^k \,. \end{split}$$

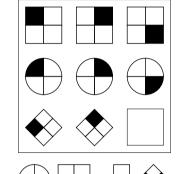
▶ Which is a slight abuse of notation. Also note,

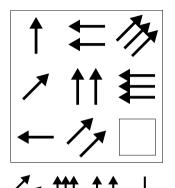
$$\mathsf{ONE}^k = \mathcal{DFT}^{-1} \big(\mathcal{DFT} (\mathsf{ONE})^k \big) ,$$

Where inside the bracket is a Hadamard exponent.

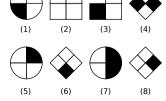
Raven's Progressive Matrices (I)

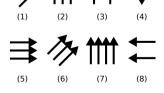


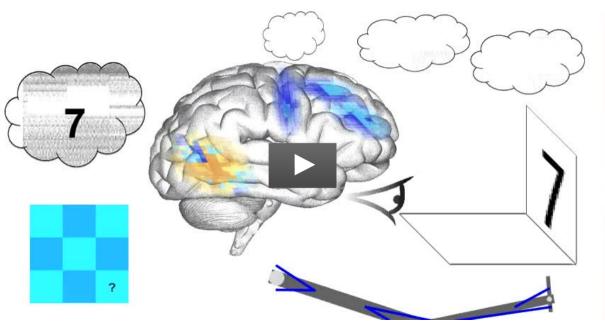




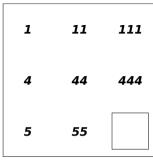








Raven's Progressive Matrices (II)

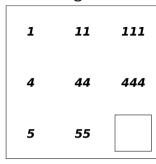


3	<i>55</i>	111	44
(1)	(2)	(3)	(4)
444	555	999	33
(5)	(6)	(7)	(8)

Representing cells:

C1 = ONE
$$\circledast$$
 P1,
C2 = ONE \circledast P1 + ONE \circledast P2,
C3 = ONE \circledast P1 + ONE \circledast P2 + ONE \circledast P3,
C4 = ONE \circledast P1 + ONE \circledast P2 + ONE \circledast P3,
C5 = FOUR \circledast P1 + FOUR \circledast P2,
C6 = FOUR \circledast P1 + FOUR \circledast P2 + FOUR \circledast P3,
C7 = FIVE \circledast P1,
C8 = FIVE \circledast P1 + FIVE \circledast P2.

Raven's Progressive Matrices (III)



3	<i>55</i>	111	44
(1)	(2)	(3)	(4)
444	555	999	33
(5)	(6)	(7)	(8)

Extracting the horizontal rule:

$$\begin{split} &\text{T1} = \text{C2} \circledast \text{C1}^{-1} \,, & &\text{T4} = \text{C6} \circledast \text{C5}^{-1} \,, \\ &\text{T2} = \text{C3} \circledast \text{C2}^{-1} \,, & &\text{T5} = \text{C8} \circledast \text{C7}^{-1} \,, \\ &\text{T3} = \text{C5} \circledast \text{C4}^{-1} \,. & &\text{} \end{split}$$

$${\rm T} = \frac{{\rm T1} + {\rm T2} + {\rm T3} + {\rm T4} + {\rm T5}}{5} \, .$$

Making a prediction:

$$\label{eq:c9} \begin{split} \texttt{C9} &= \texttt{C8} \circledast \texttt{T} \\ &\approx \texttt{FIVE} \circledast \texttt{P1} + \texttt{FIVE} \circledast \texttt{P2} + \texttt{FIVE} \circledast \texttt{P3} \,. \end{split}$$

Image sources

Title slide

Wikimedia.

Bell telephone magazine, 1922, American Telephone and Telegraph Company