SYDE 556/750

Simulating Neurobiological Systems
Lecture 5: Feed-Forward Transformation

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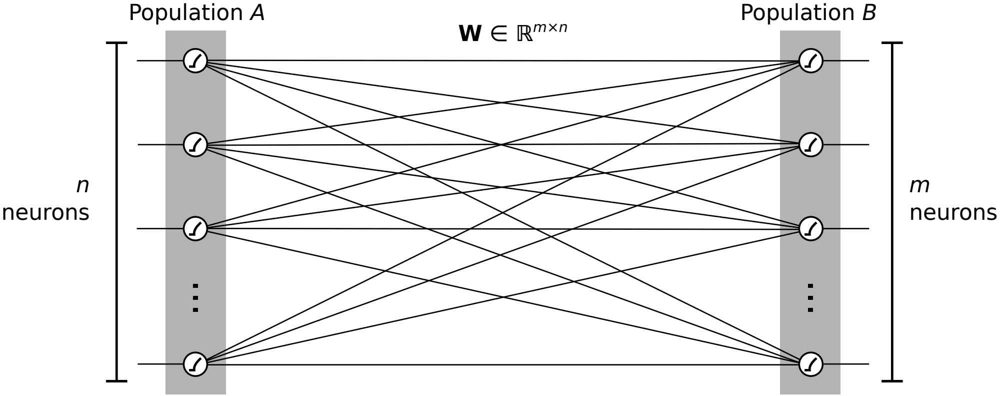
Introduction

- We've only talked about representation til now
  - What about computation?
- We start by focusing on the state of a network after learning and development
- A kind of hypothesis testing and generation

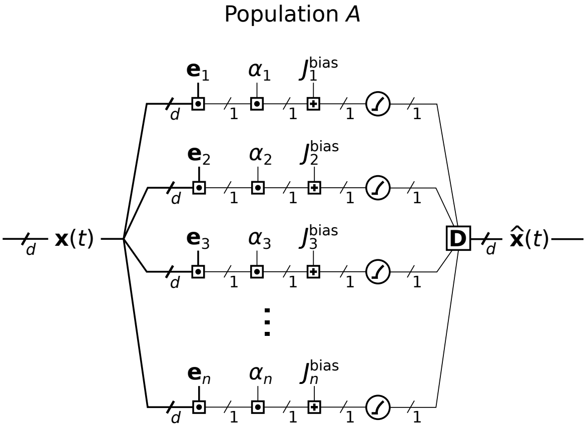
DALL-E AI Generated Art, 2022

NEF Principle 2: Transformation



NEF Principle 2 – Transformation
Connections between populations describe *transformations* of neural representations. Transformations are functions of the variables represented by neural populations.

A Tale of Two Populations (I)



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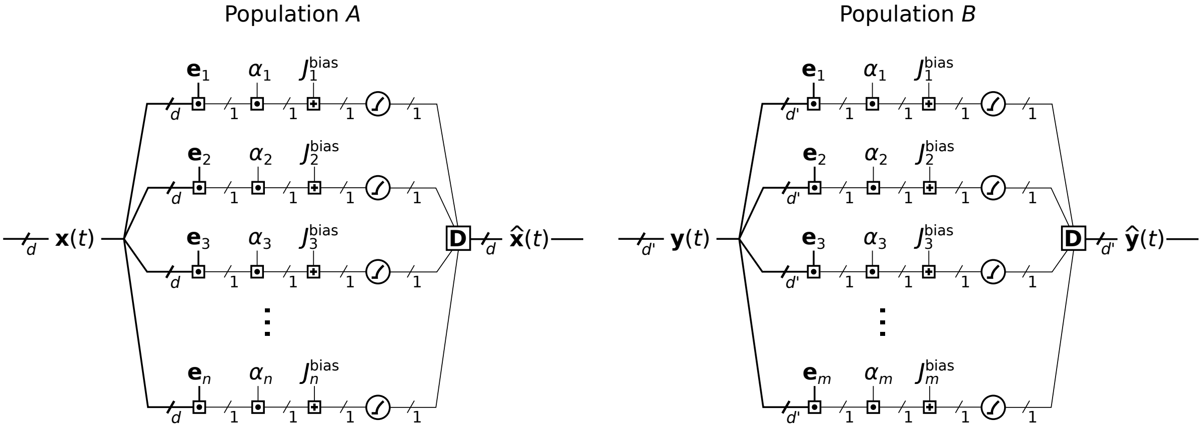
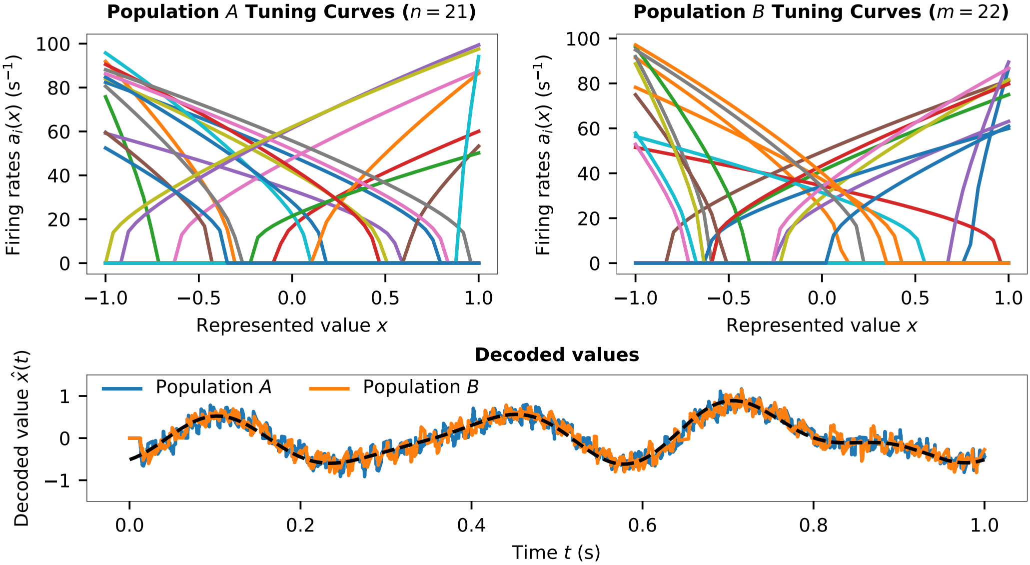
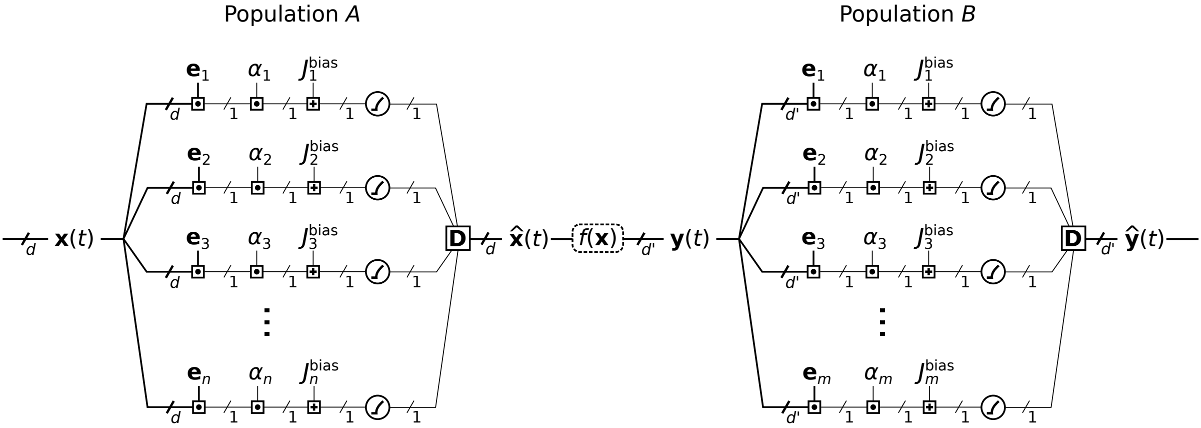


Communication Channel Experiment: Same input signal



A Tale of Two Populations (II)



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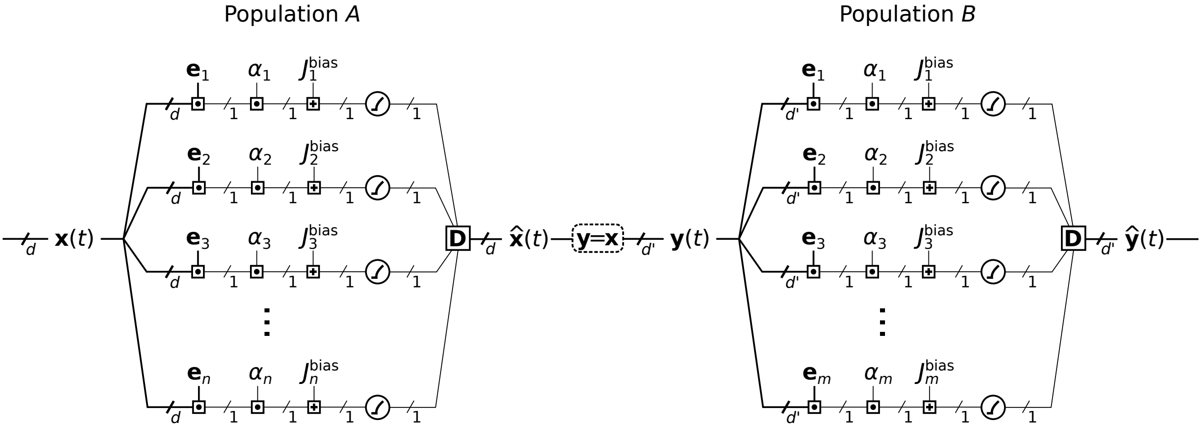
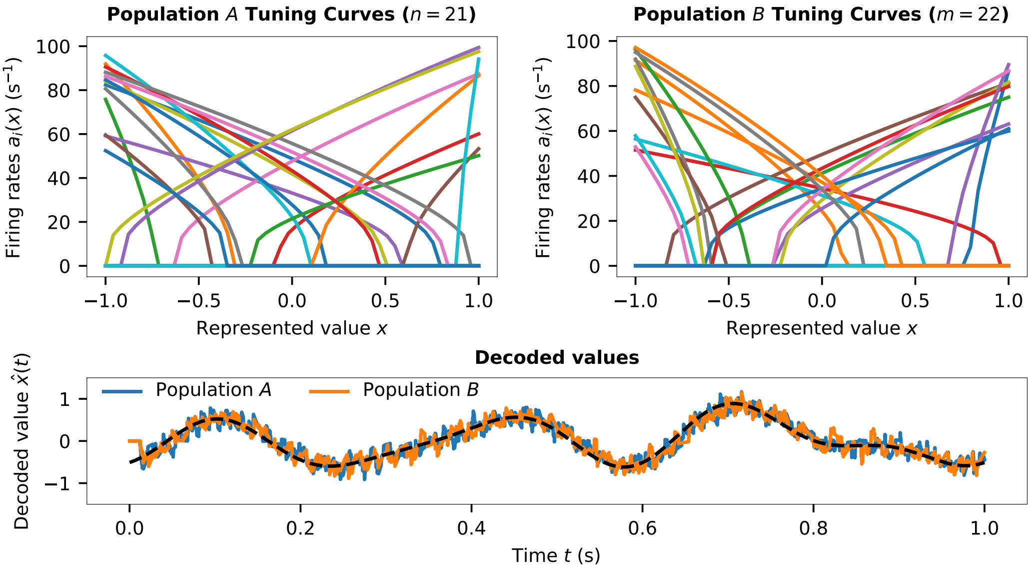


Communication Channel Experiment: Populations in series



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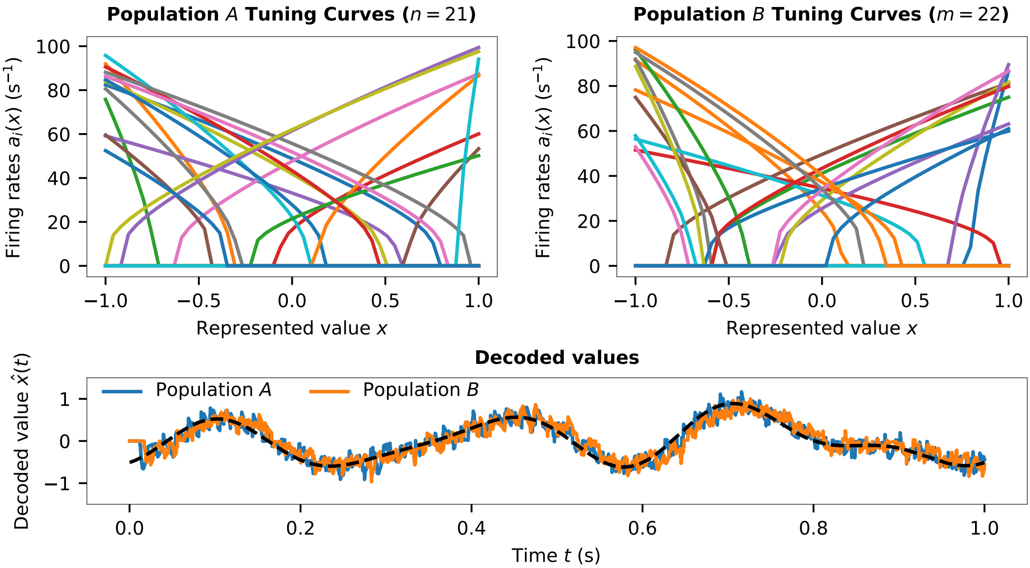
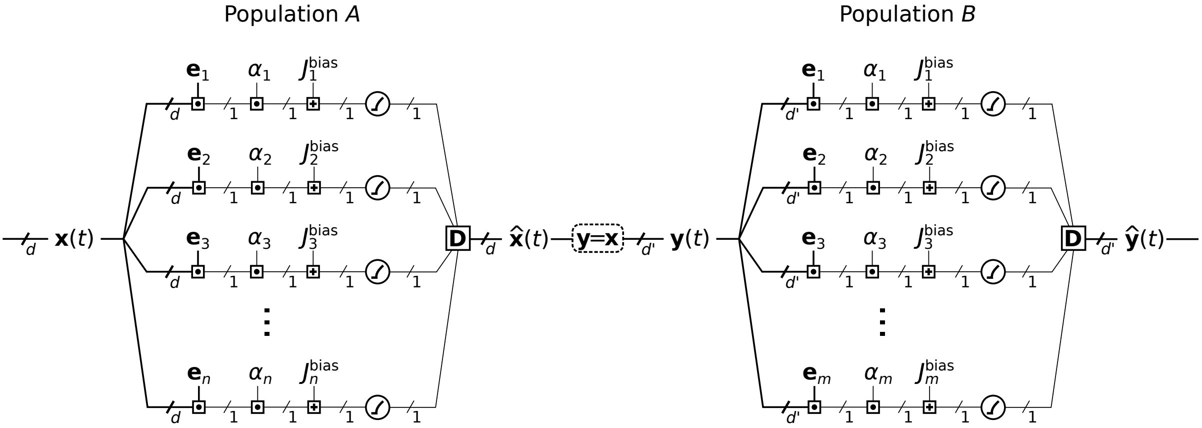


Computing Synaptic Weights: Step 1 – Encoding Matrix



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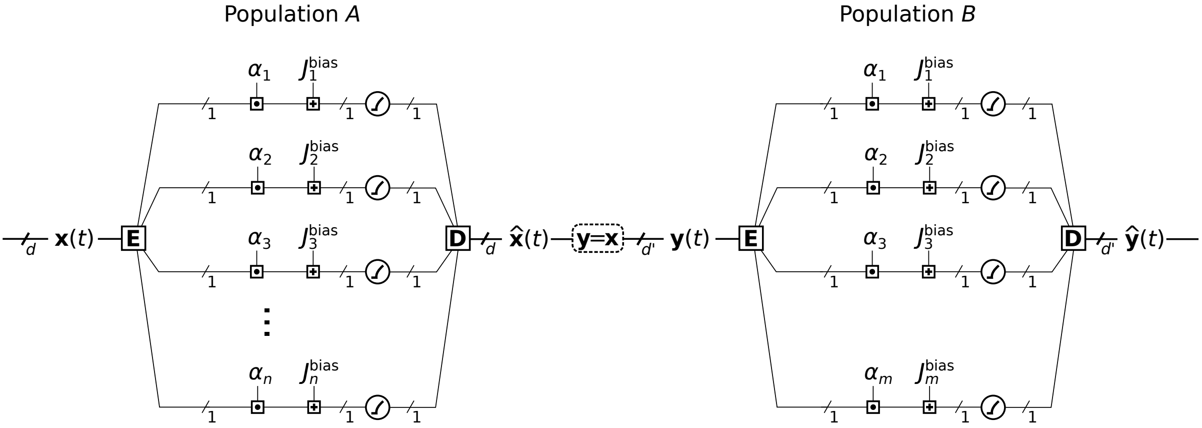
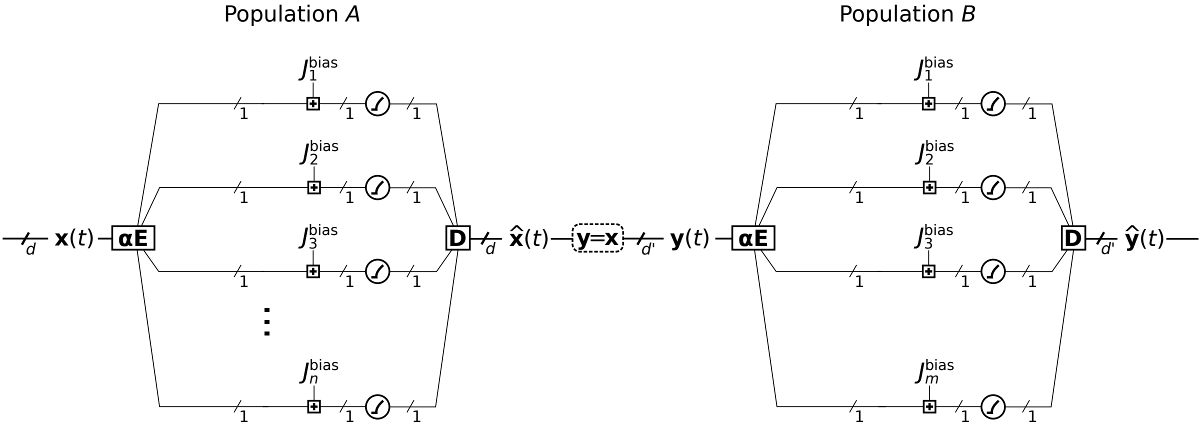
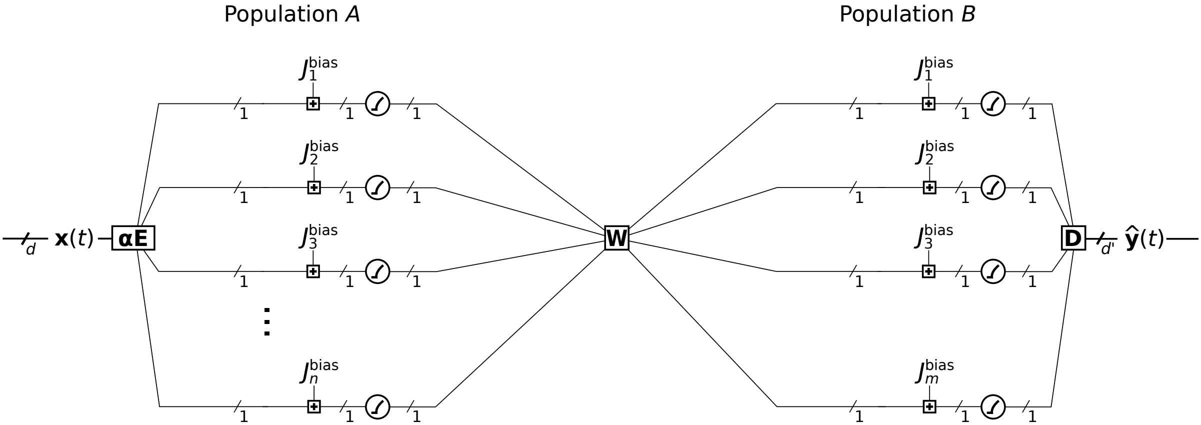


Computing Synaptic Weights: Step 2 – Scaled Encoding Matrix



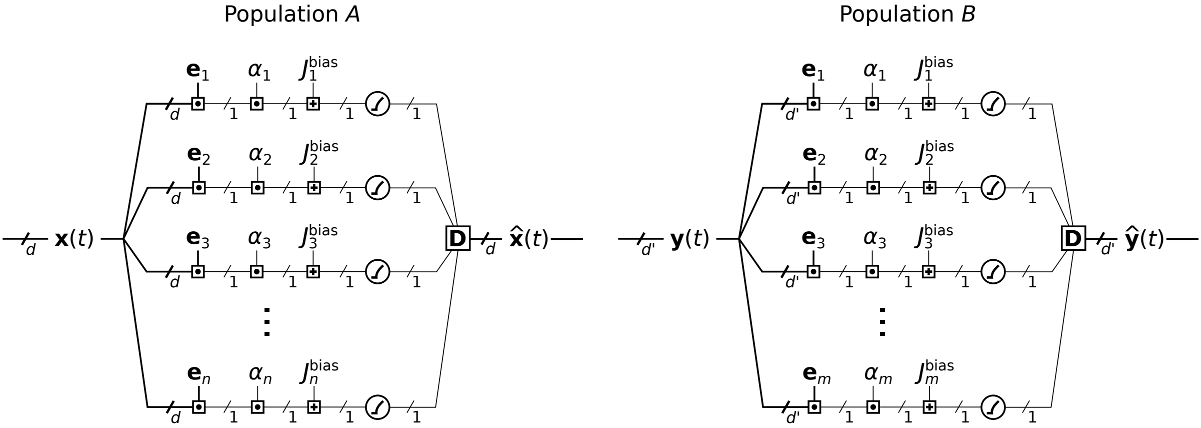
Computing Synaptic Weights: Step 3 – $\mathbf{W} = \mathbf{E}\mathbf{D}$



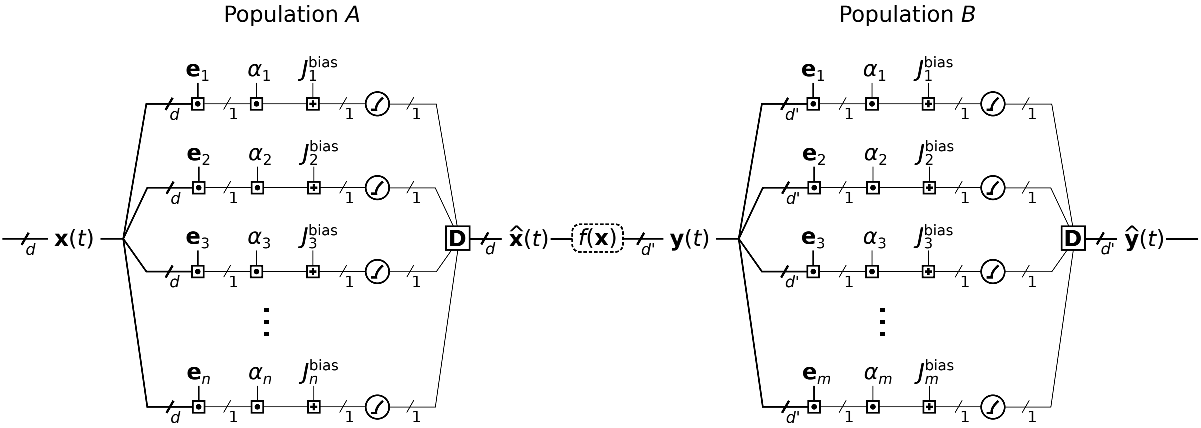
Computational Complexity

- Weights - multiplying $\mathbf{a} \in \mathbb{R}^n$ with $\mathbf{W} \in \mathbb{R}^{m \times n}$ is $\mathcal{O}(nm)$ i.e., $\approx \mathcal{O}(n^2)$
- Decoding - $\hat{\mathbf{x}} = \mathbf{D}\mathbf{a}$ is $\mathcal{O}(dn)$
- Encoding - $\mathbf{J} = \mathbf{E}\hat{\mathbf{x}} + \mathbf{J}_{\text{bias}}$ is $\mathcal{O}(dm)$
- Encoding/Decoding - $\mathcal{O}(d(n+m))$ or $\approx \mathcal{O}(dn)$ for $n = m$
- So if d is small we get a linear complexity $\mathcal{O}(n)$
- Therefore, sequential decoding and re-encoding saves a lot of time compared to using actual synaptic weights
- One reason why Nengo is so fast compared to other SNN simulators

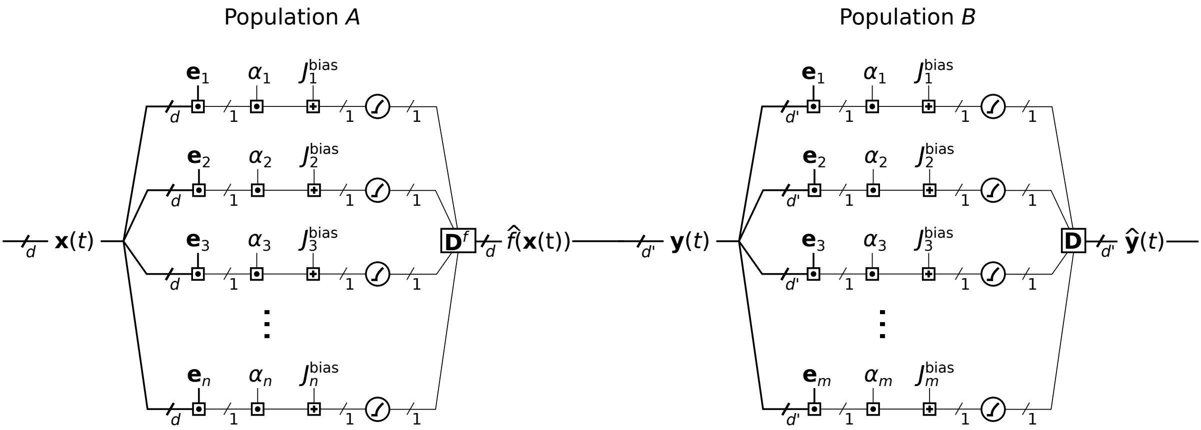
Computing Functions



Computing Functions

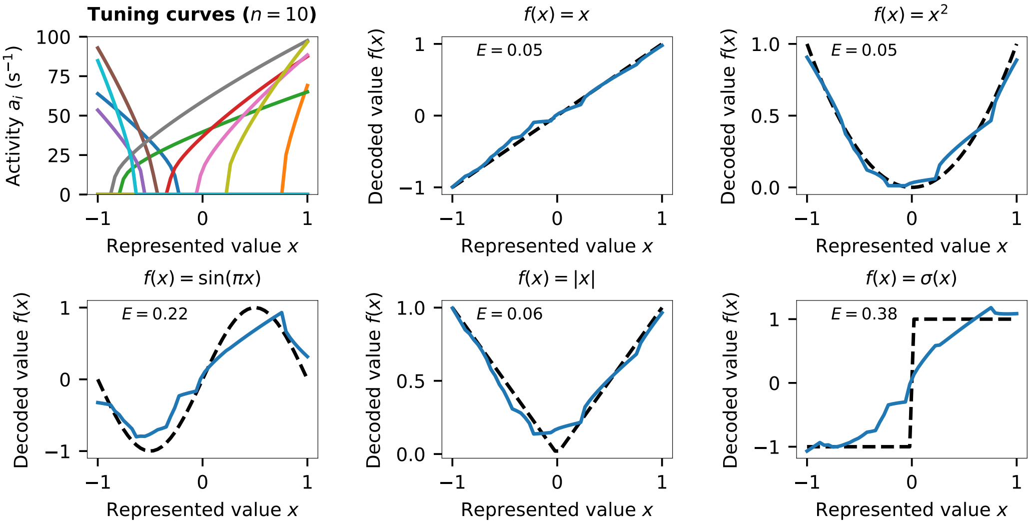


Computing Functions

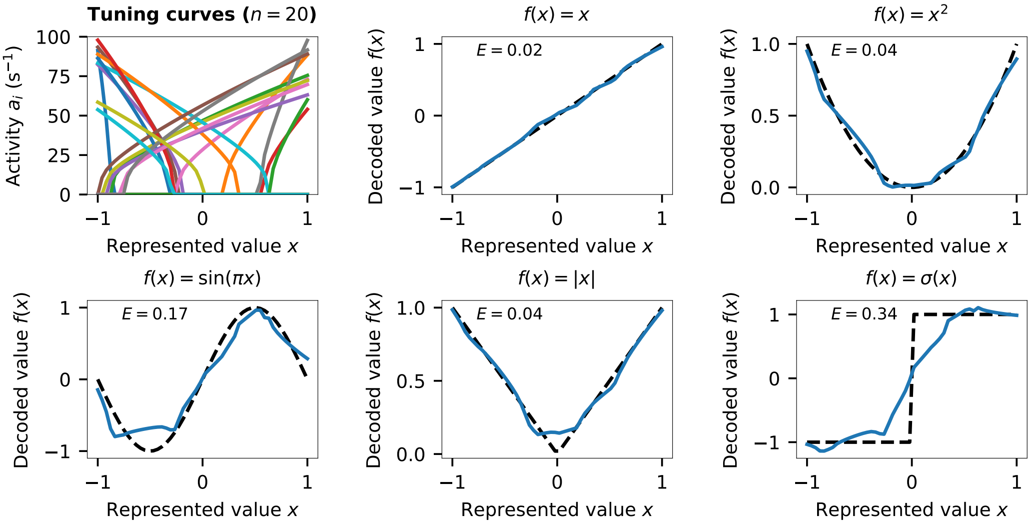


Function Decoder $\mathbf{D}^f = \left((\mathbf{A}\mathbf{A}^\mathsf{T} + N\sigma^2\mathbf{I})^{-1}\mathbf{A}\mathbf{Y}^\mathsf{T}\right)^\mathsf{T}$, where $\left(\mathbf{Y}\right)_{ik} = \left(f(\mathbf{x}_k)\right)_i$

Decoding Functions – Using a Few Neurons



Decoding Functions – Using More Neurons



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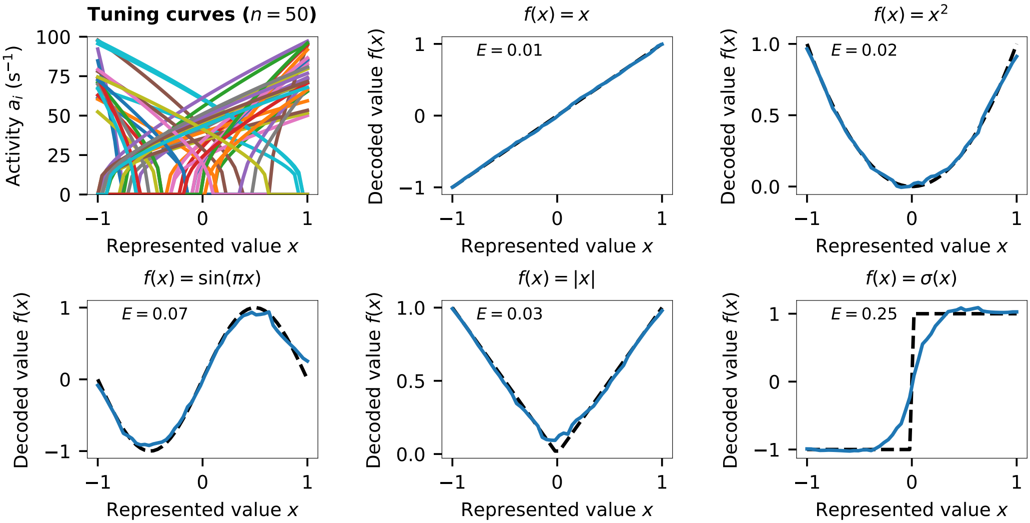

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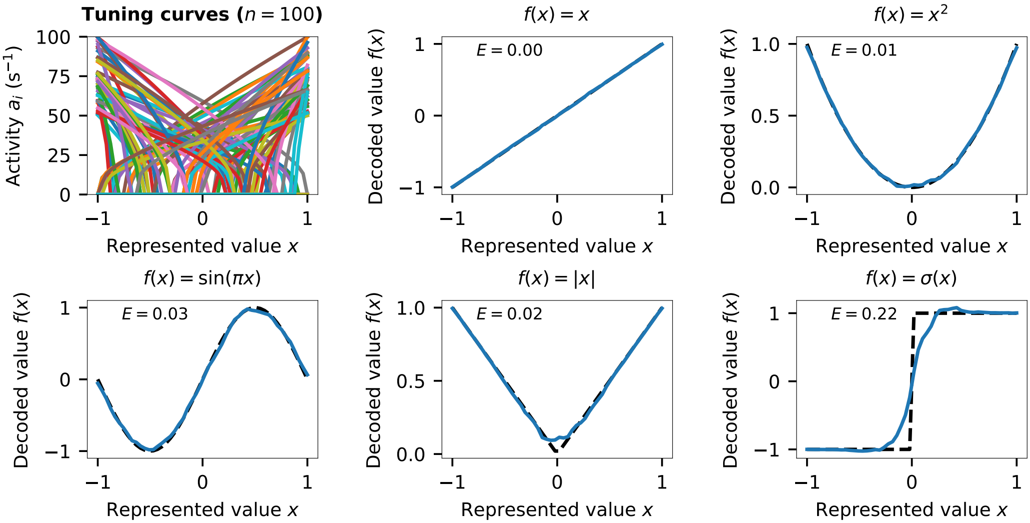
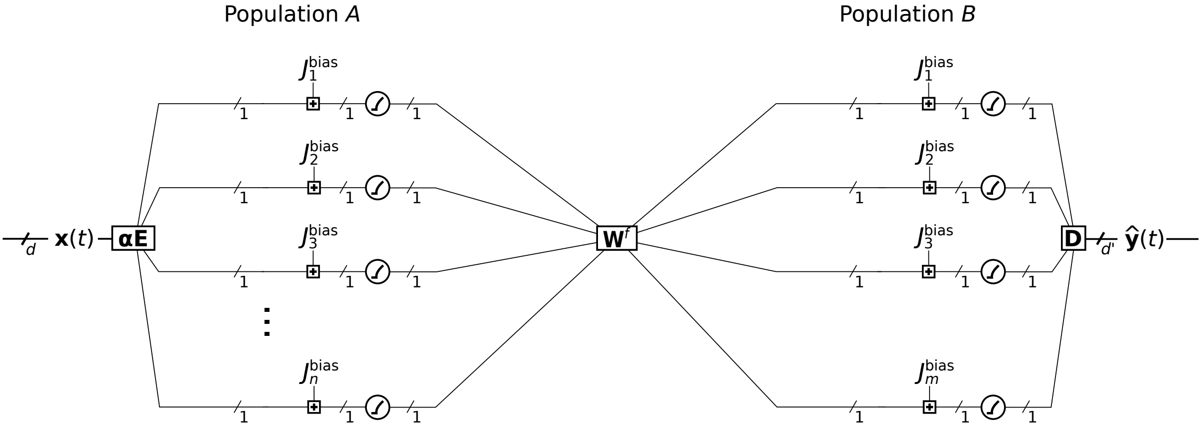


Computing Functions – Weight Matrix



$$\mathbf{W}^f = \mathbf{E}\mathbf{D}^f$$

Recipe for any feedforward transformation

1. Define encoding for two populations (input/output)
2. Write the transformation with the represented input variables
3. Solve for the $\mathbf{D}^f$ for that transformation for the input population
4. Sub 3. into the encoding for the output variable

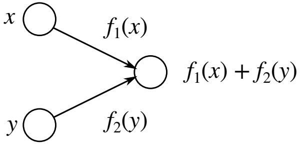
Computing Multivariate Functions

Homogenous population Heterogenous population

Linear connection Inh. connection Exc. connection

Linear Superposition

$$W^{f_1}\mathbf{a}_1(\mathbf{x}) + W^{f_2}\mathbf{a}_2(\mathbf{y})$$



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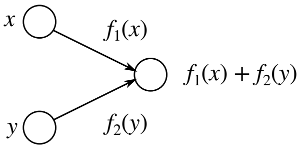
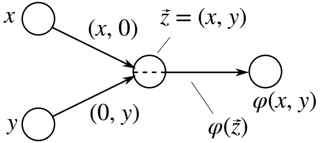


Nonlinear Functions

Multi-dimensional $\mathbf{z}$



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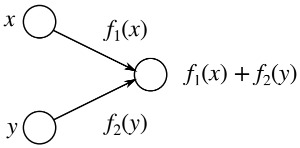

Nonlinear Functions

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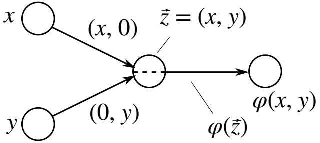
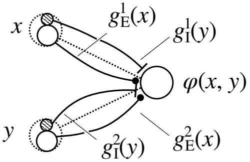


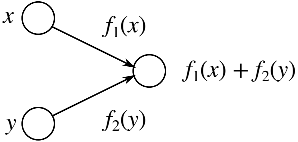
(Dendritic Computation)

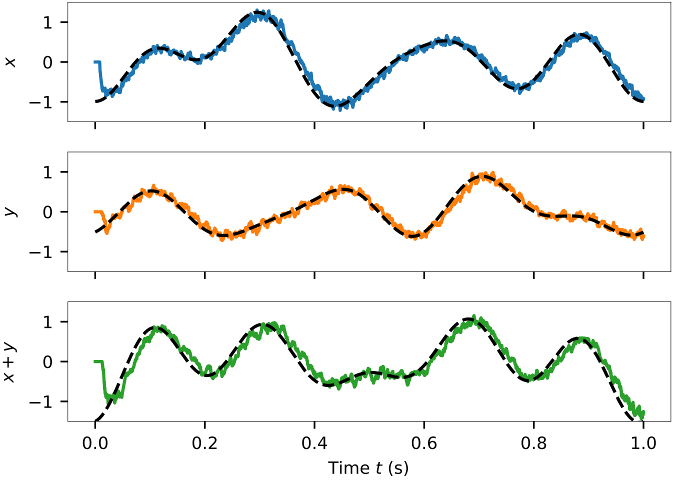
Exploit dendritic nonlinearity



Computing Multivariate Functions – Linear Superposition

Linear Superposition

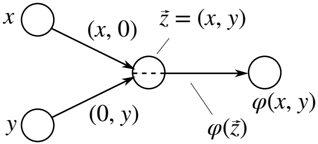


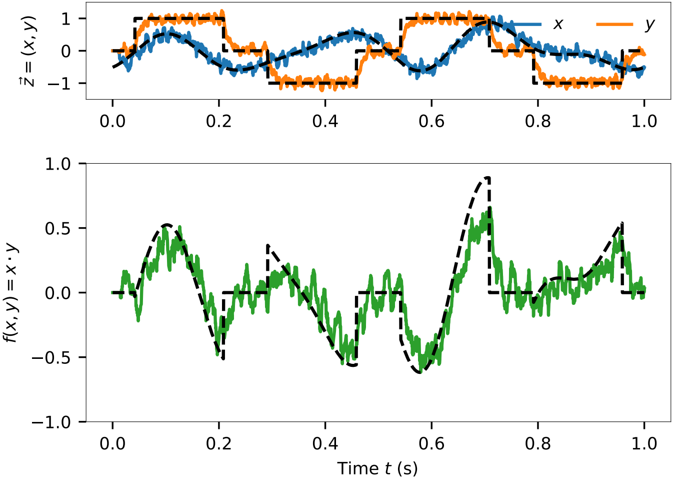


Computing Multivariate Functions – Multiplication

Nonlinear Functions

Multi-dimensional $\mathbf{z}$





Computing Multivariate Functions – Multiplication

Nonlinear Functions

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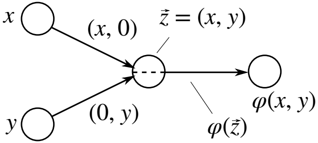

Multiplication is useful…

- Gating of signals
- Attention effects
- Binding
- Statistical inference

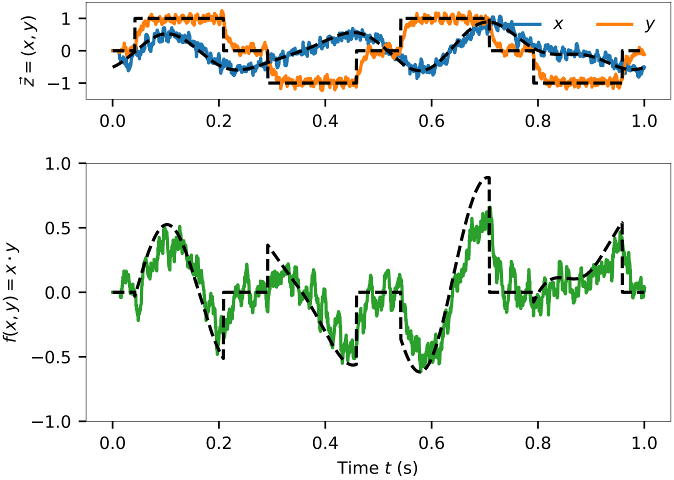

Image sources

Title slide
"Yellow Butterfly"
Author: Albert Bierstadt, circa 1890.
From Wikimedia.