

**SYDE 556/750**

**Simulating Neurobiological Systems**  
**Lecture 7: Temporal Basis Functions**

Chris Eliasmith

October 21, 2022

- ▶ Slide design: Andreas Stöckel
- ▶ Content: Terry Stewart, Andreas Stöckel, Chris Eliasmith

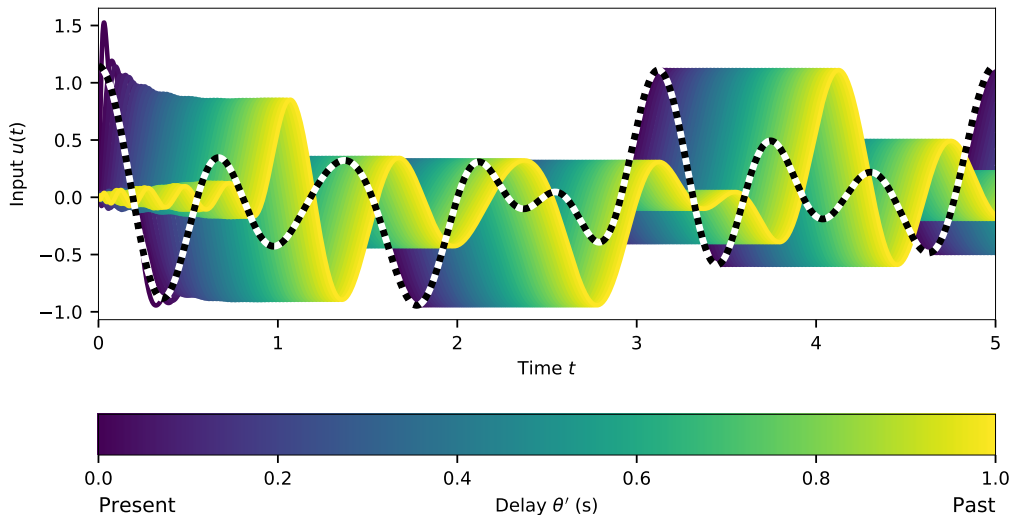


UNIVERSITY OF  
**WATERLOO**

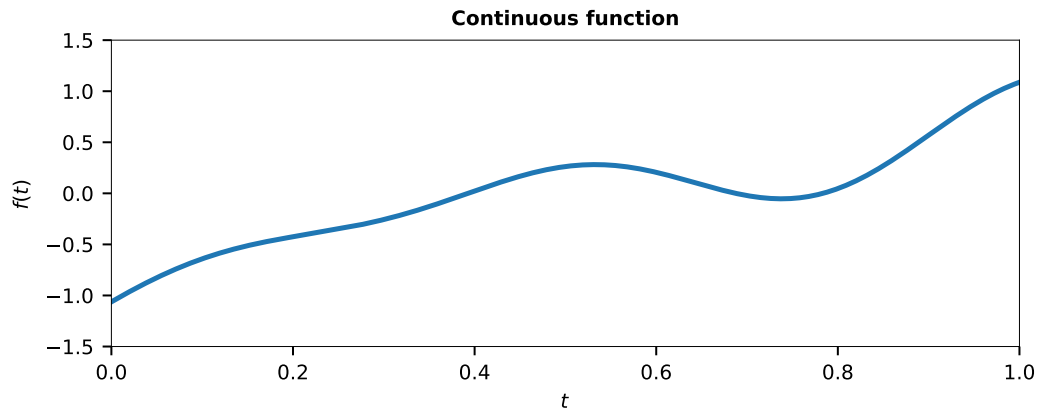
FACULTY OF  
ENGINEERING



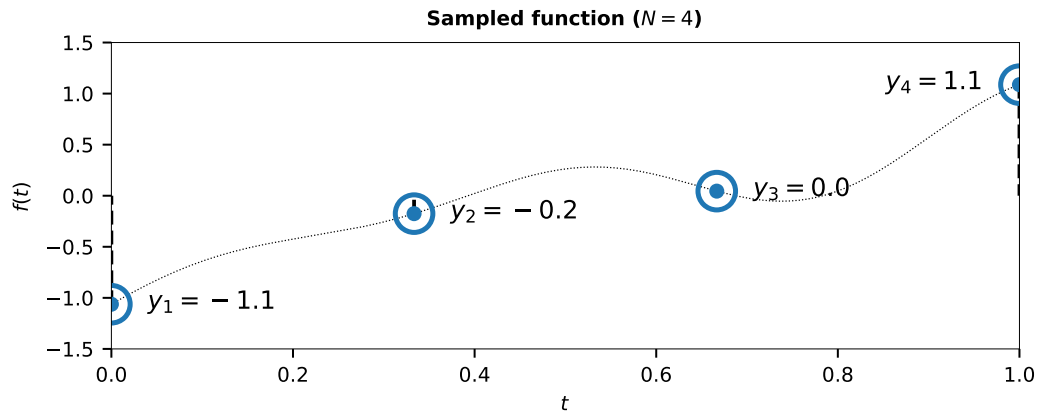
# Representing Stimulus Histories



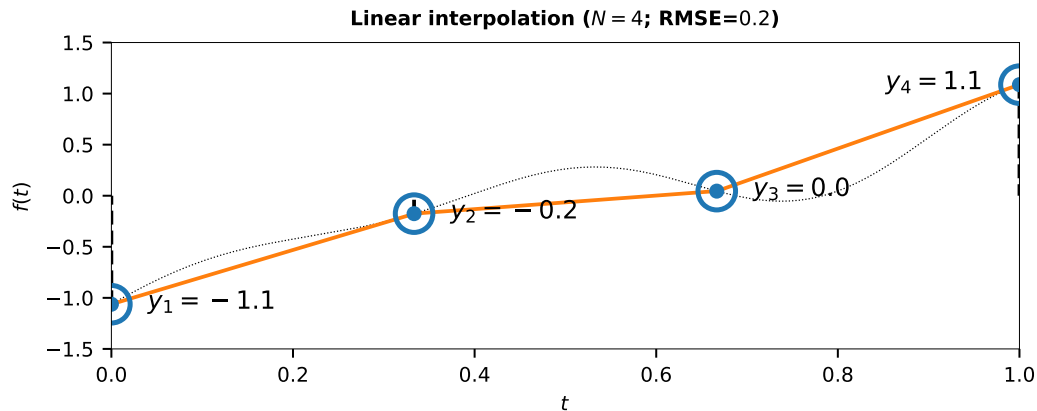
# Representing Functions: Sampling



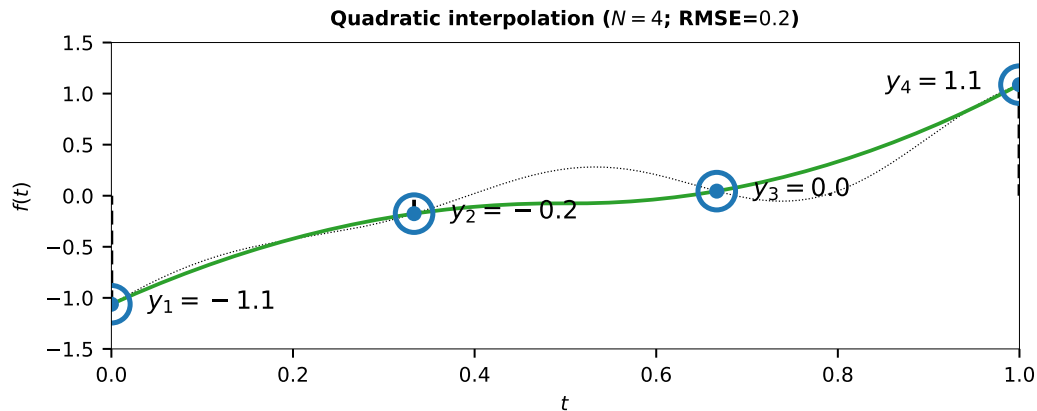
# Representing Functions: Sampling



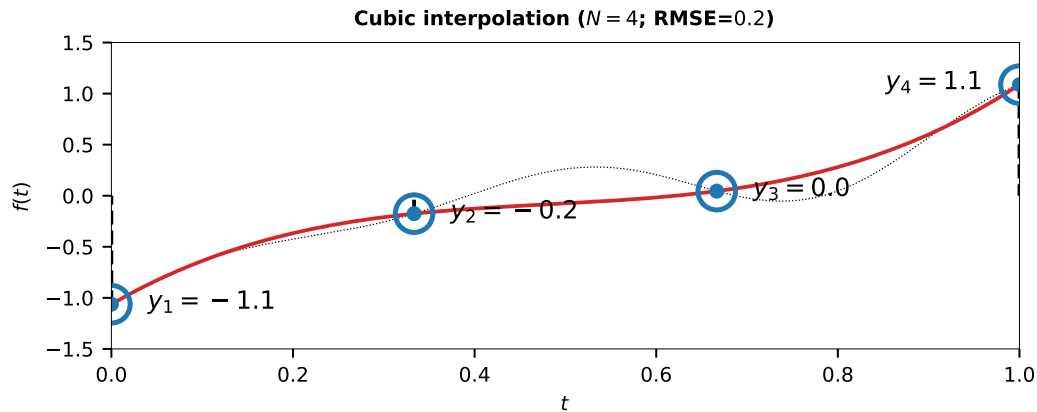
# Representing Functions: Sampling



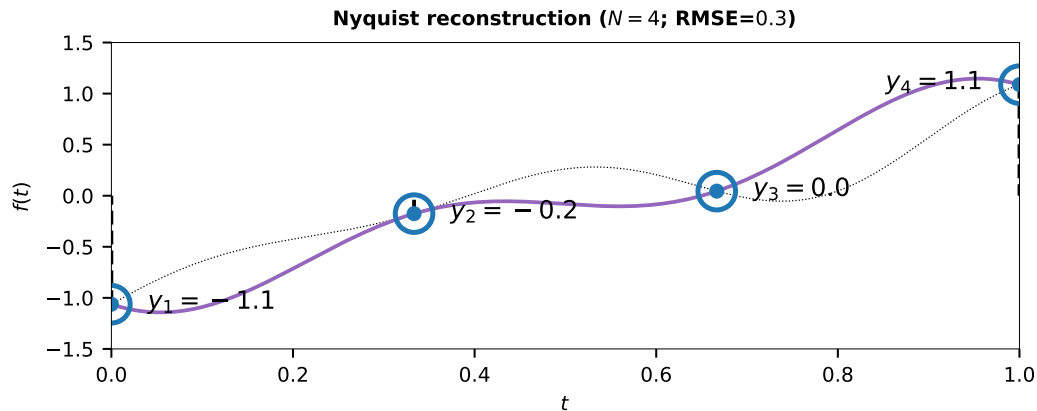
# Representing Functions: Sampling



# Representing Functions: Sampling

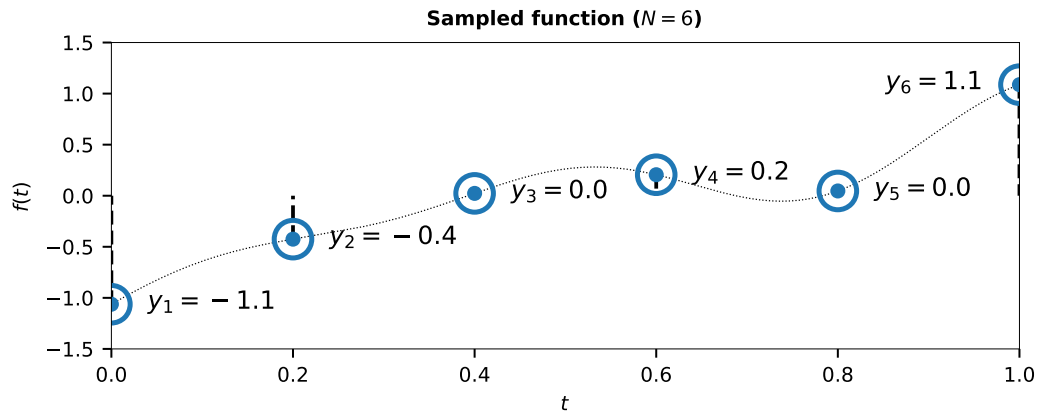


# Representing Functions: Sampling

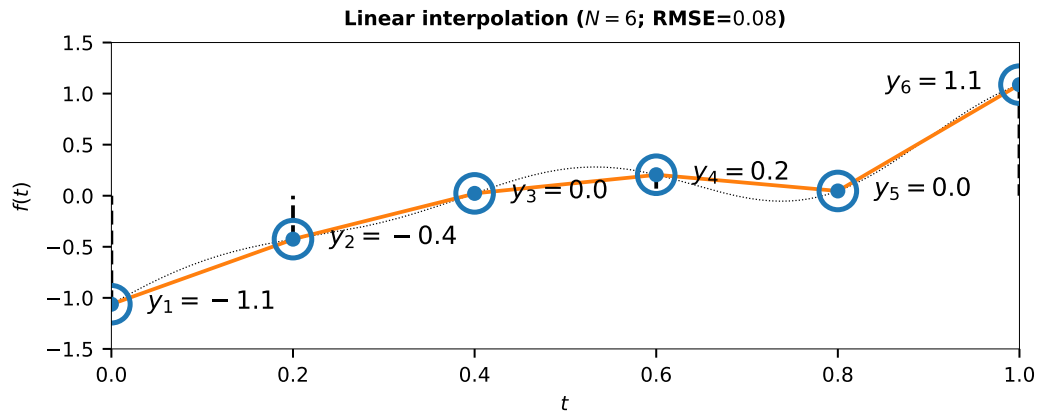




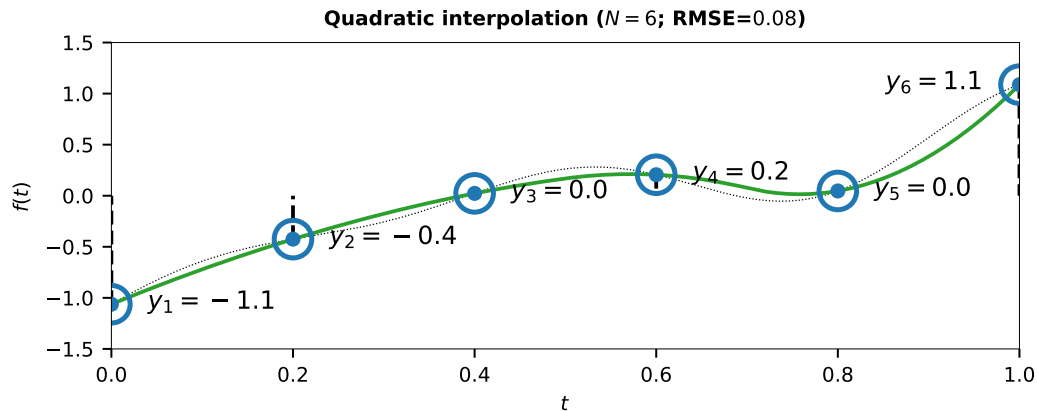
# Representing Functions: Sampling



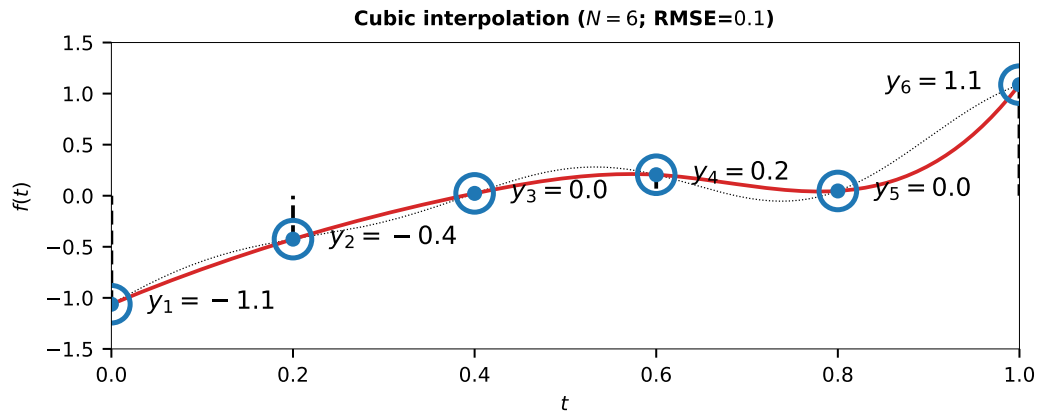
# Representing Functions: Sampling



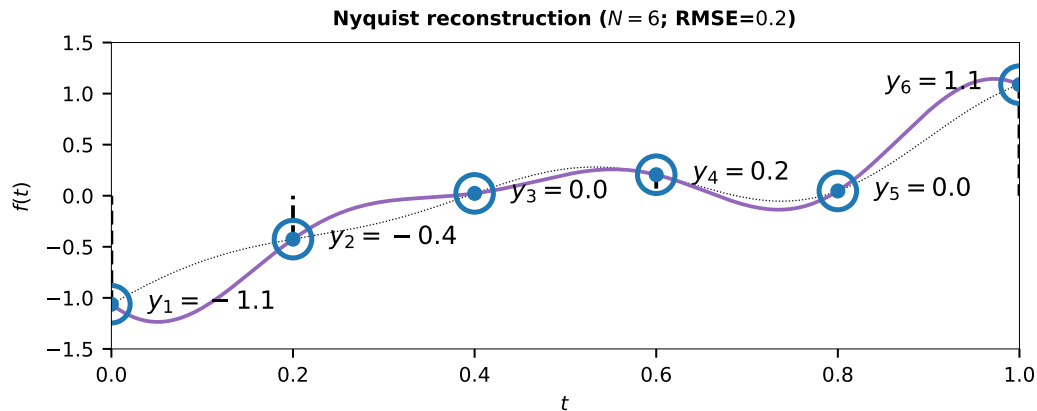
# Representing Functions: Sampling



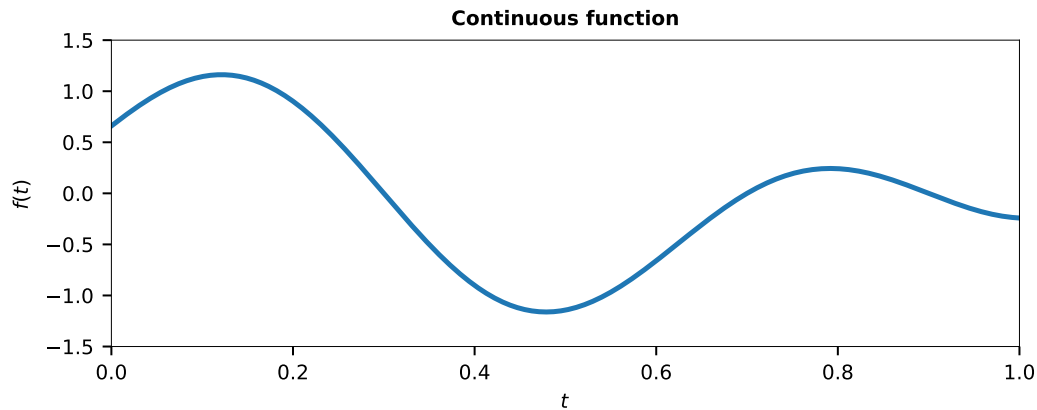
# Representing Functions: Sampling



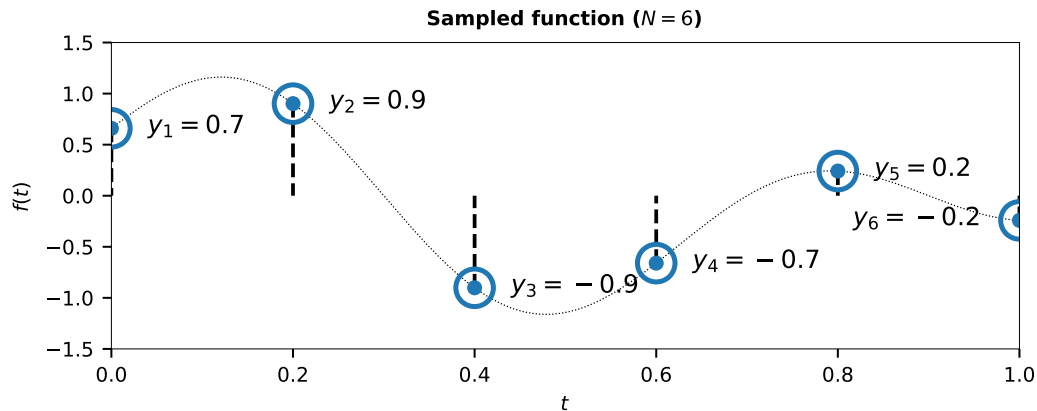
# Representing Functions: Sampling



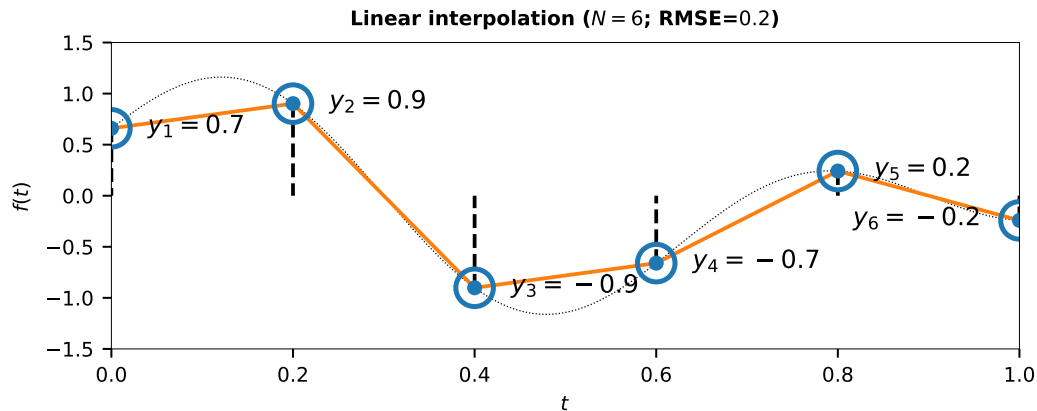
# Representing Functions: Sampling



# Representing Functions: Sampling

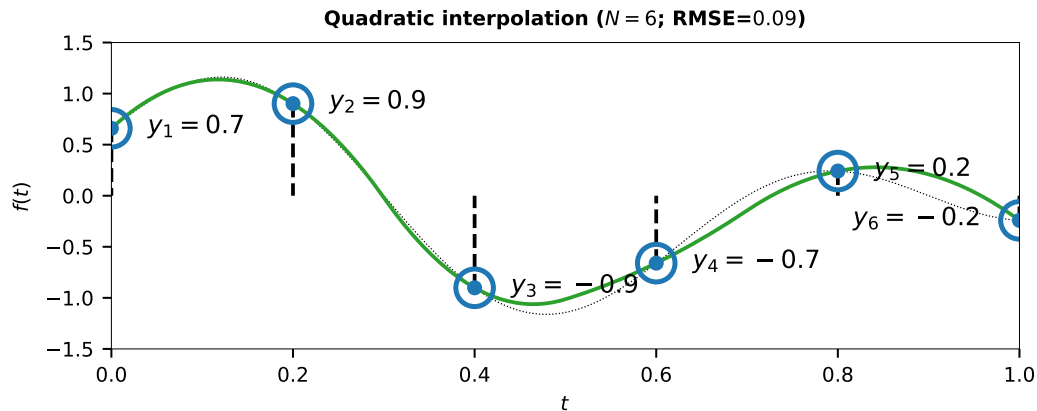


# Representing Functions: Sampling

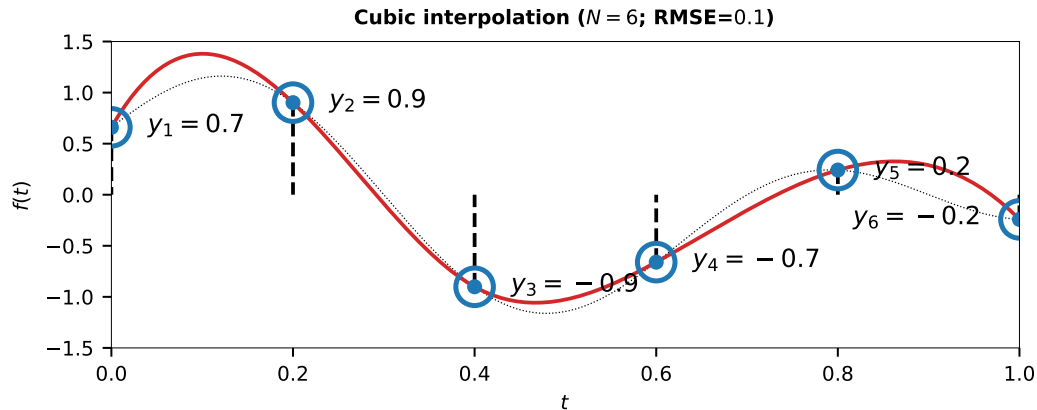




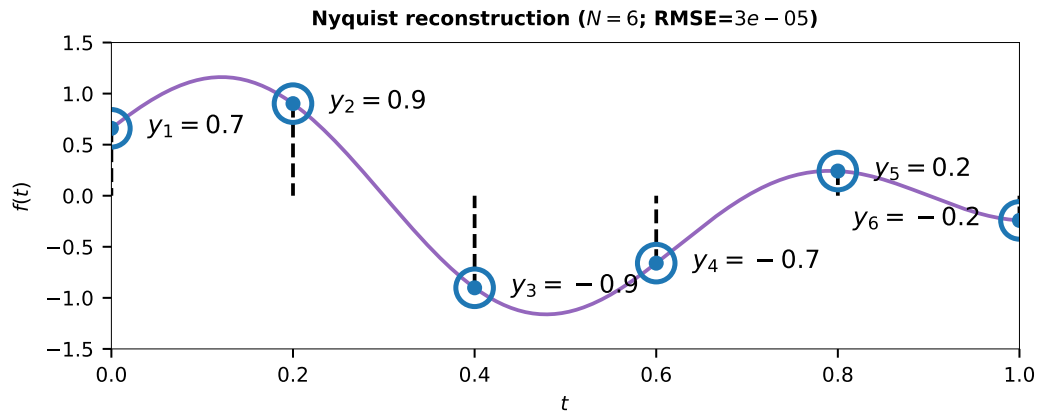
# Representing Functions: Sampling



# Representing Functions: Sampling

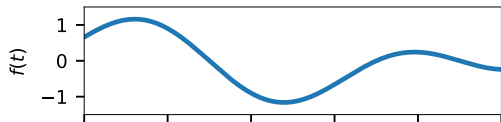


# Representing Functions: Sampling

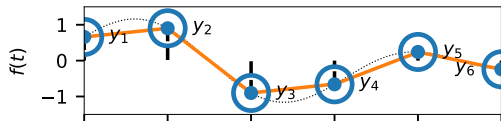


# Representing Functions: Sampling

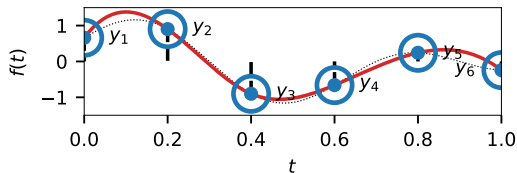
**Continuous function**



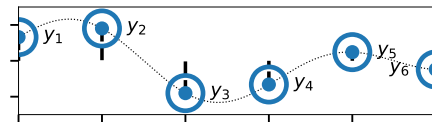
**Linear interpolation ( $N = 6$ ; RMSE=0.2)**



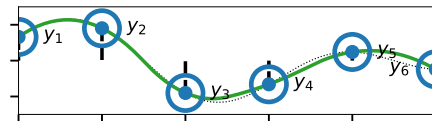
**Cubic interpolation ( $N = 6$ ; RMSE=0.1)**



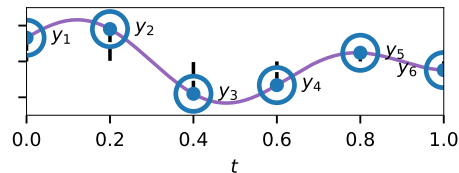
**Sampled function ( $N = 6$ )**



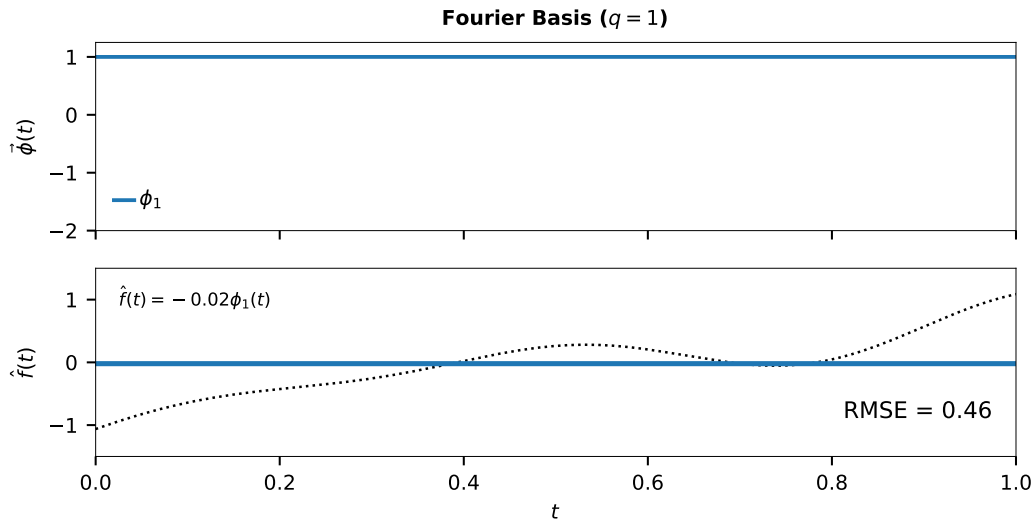
**Quadratic interpolation ( $N = 6$ ; RMSE=0.09)**



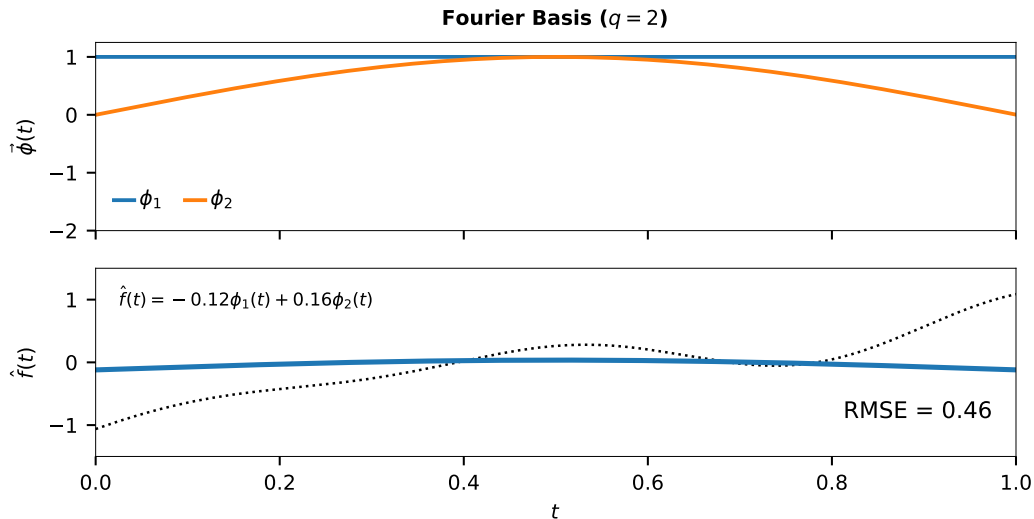
**Nyquist reconstruction ( $N = 6$ ; RMSE=3e-05)**



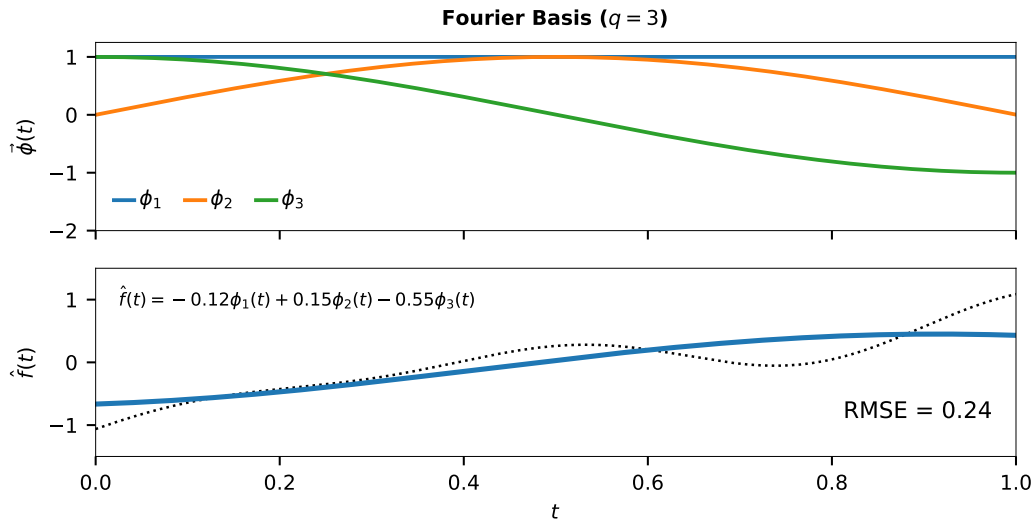
# Representing Functions: Fourier Basis



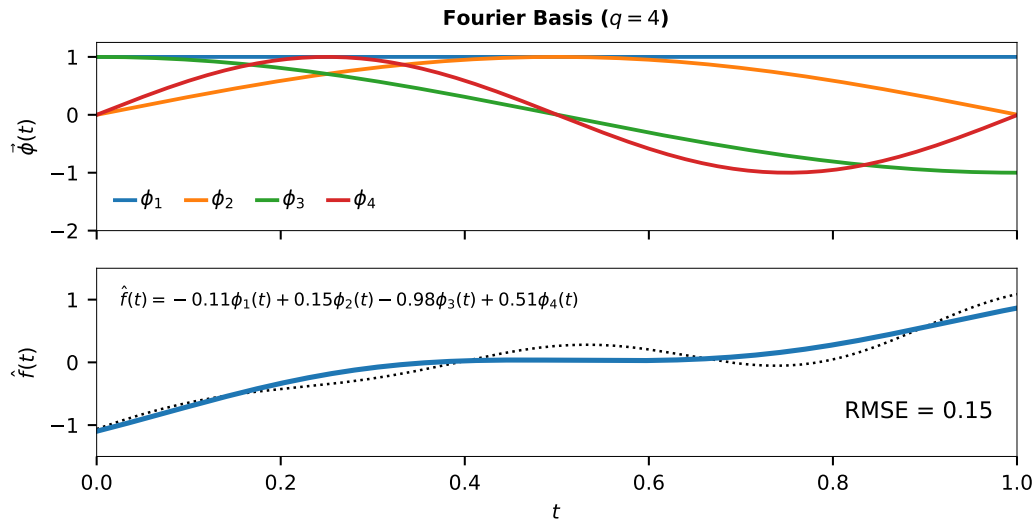
# Representing Functions: Fourier Basis



# Representing Functions: Fourier Basis

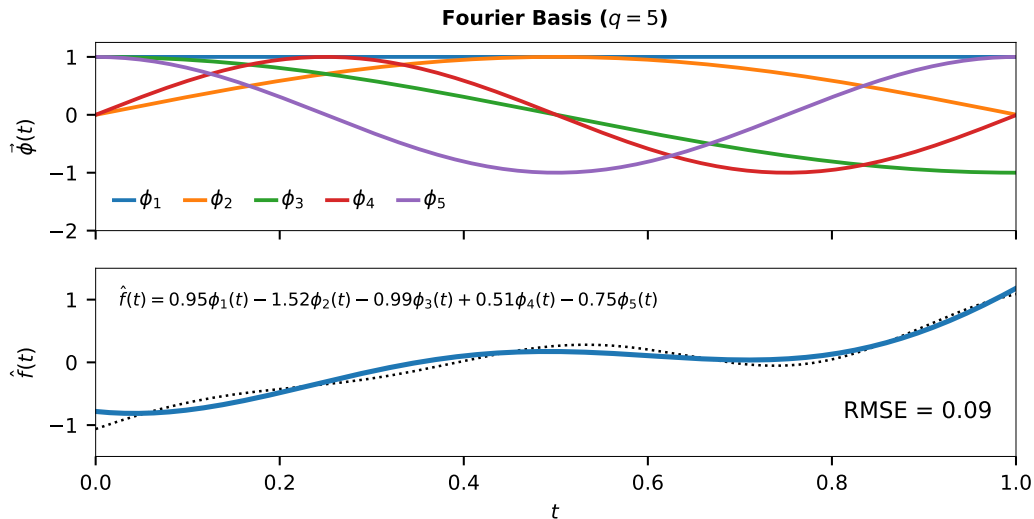


# Representing Functions: Fourier Basis

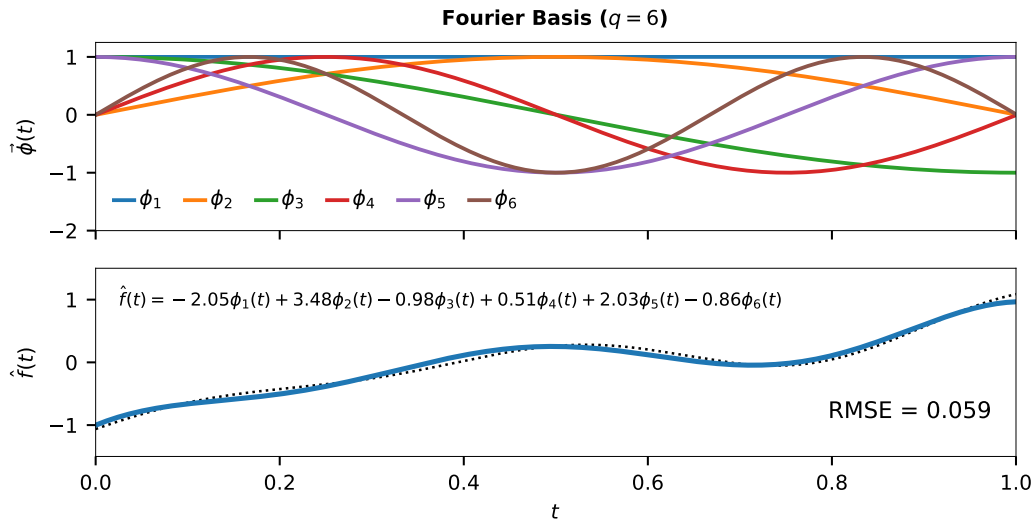




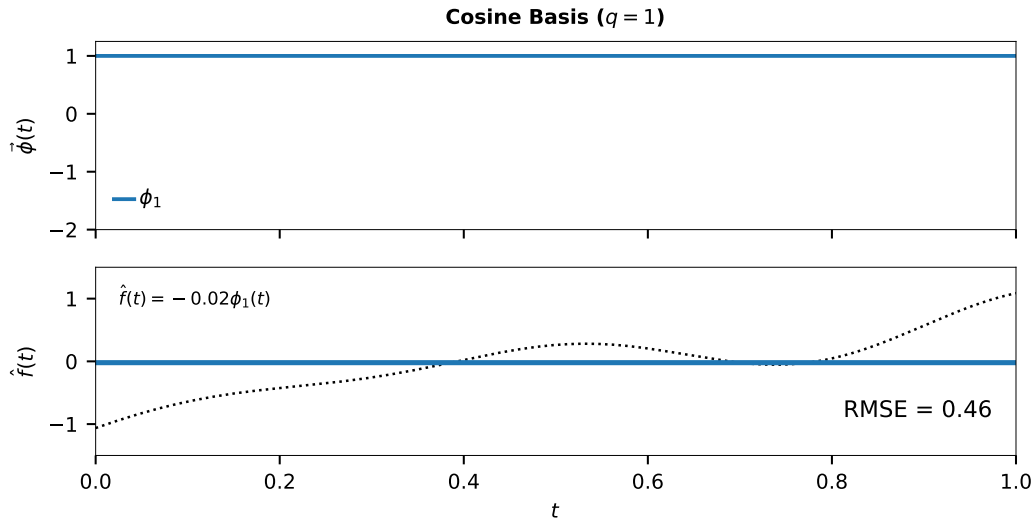
# Representing Functions: Fourier Basis



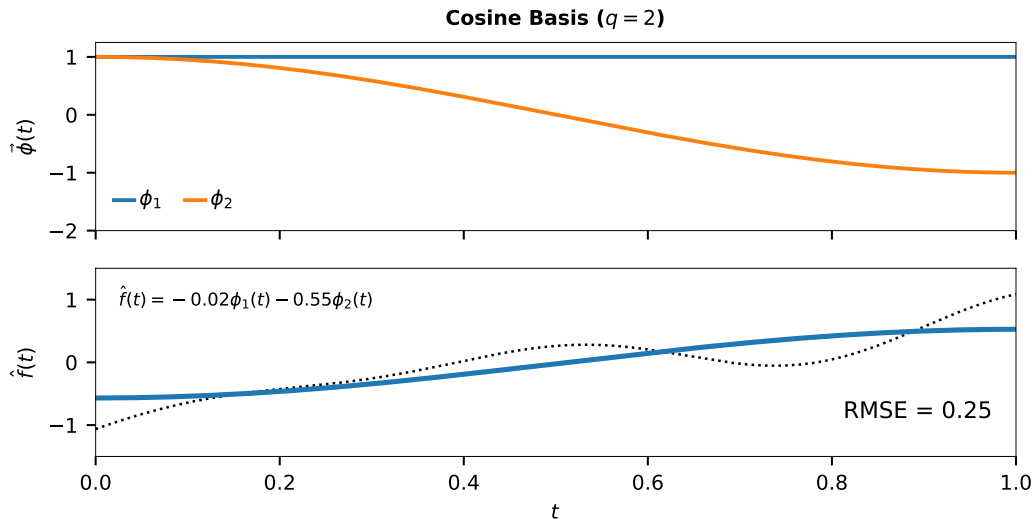
# Representing Functions: Fourier Basis



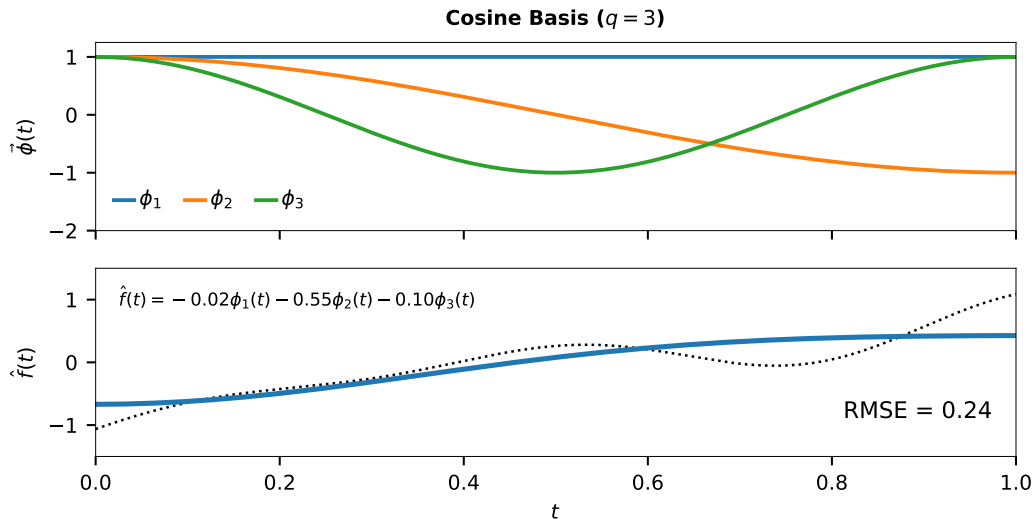
# Representing Functions: Cosine Basis



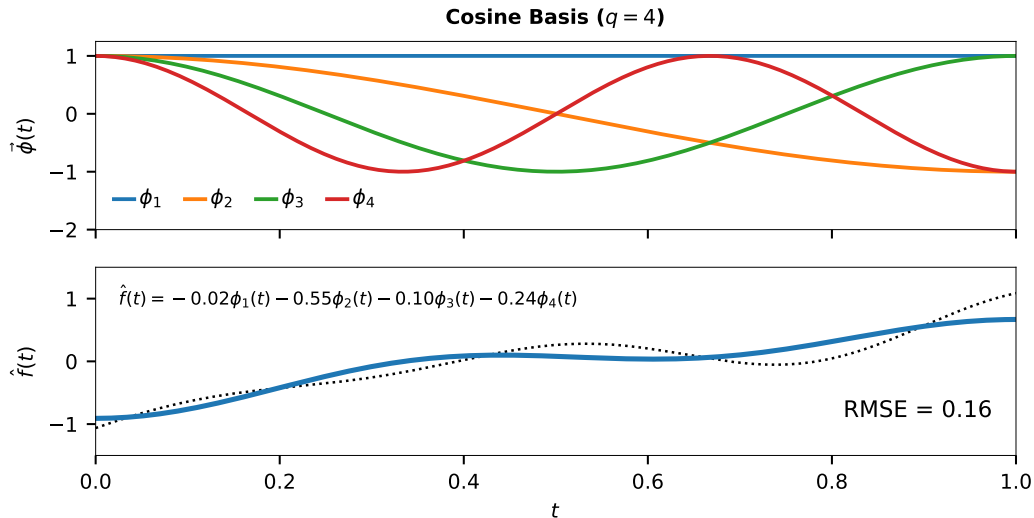
# Representing Functions: Cosine Basis



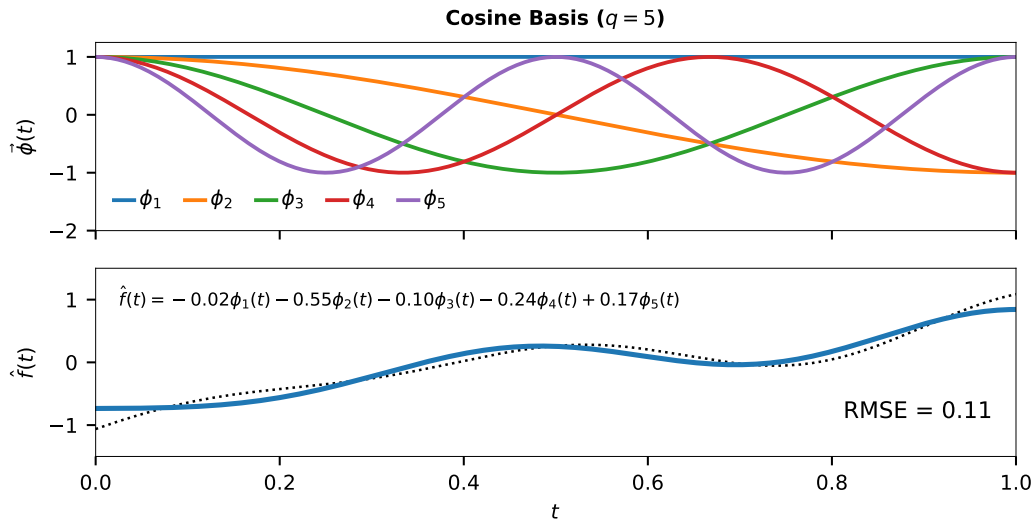
# Representing Functions: Cosine Basis



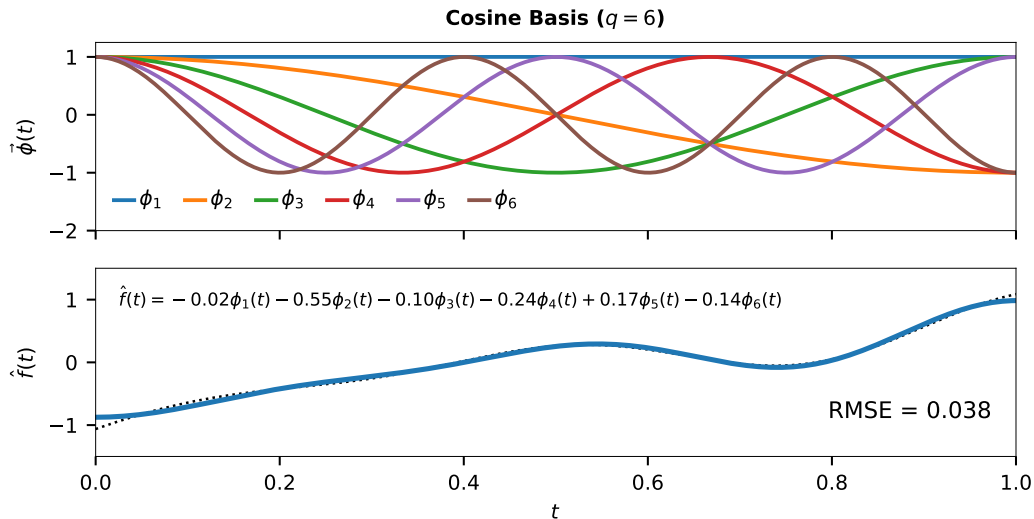
# Representing Functions: Cosine Basis



# Representing Functions: Cosine Basis

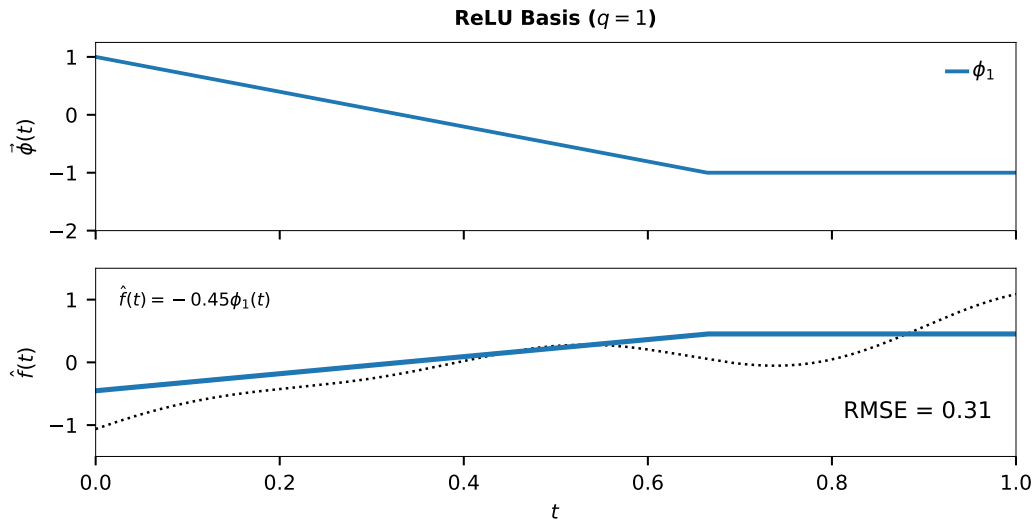


# Representing Functions: Cosine Basis

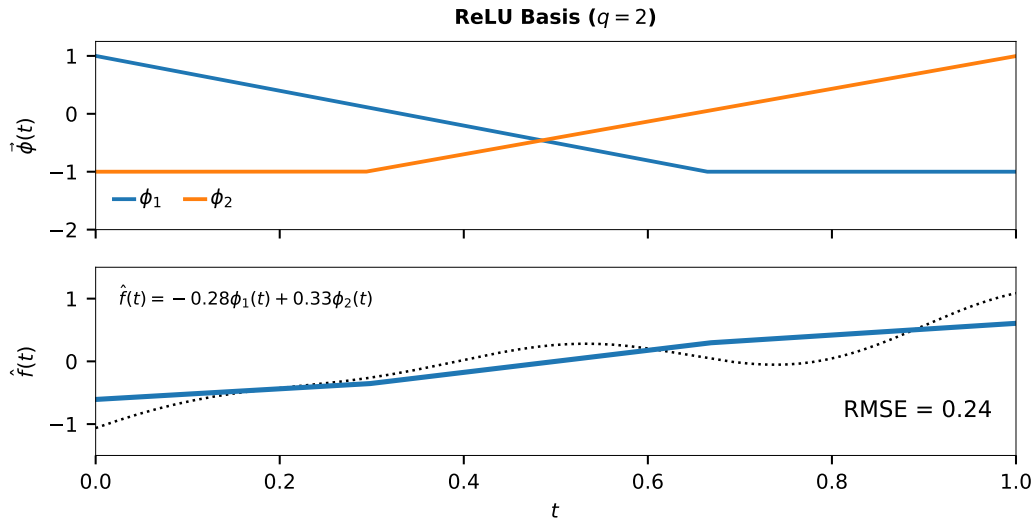




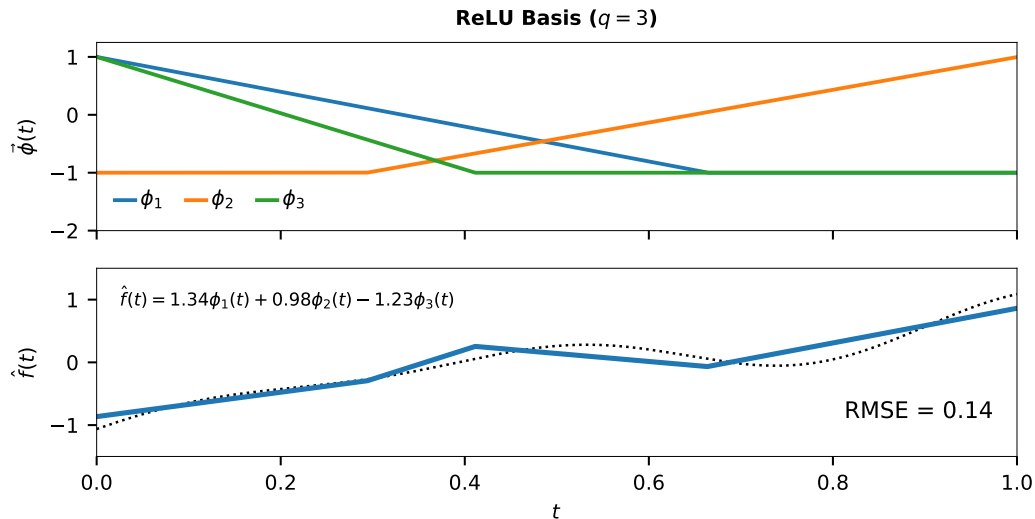
# Representing Functions: ReLU Basis



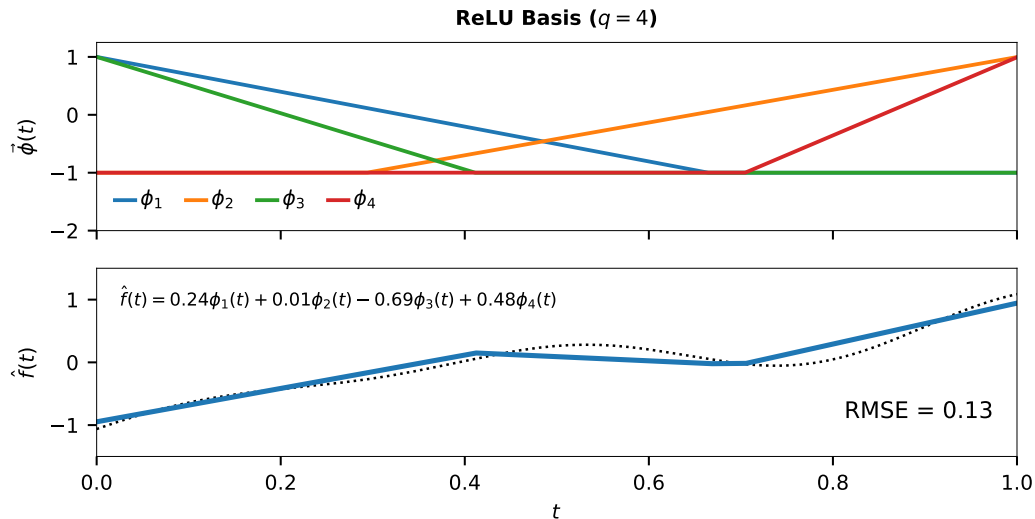
# Representing Functions: ReLU Basis



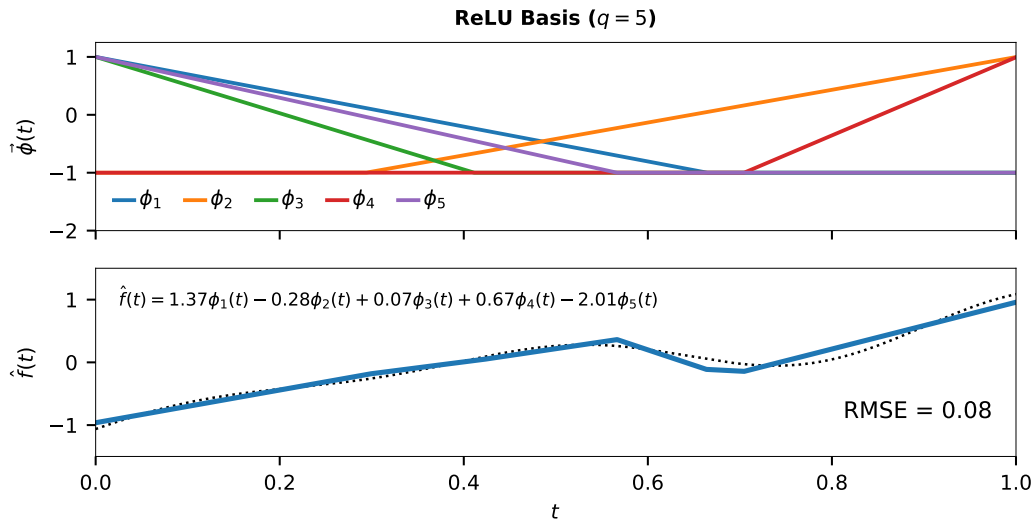
# Representing Functions: ReLU Basis



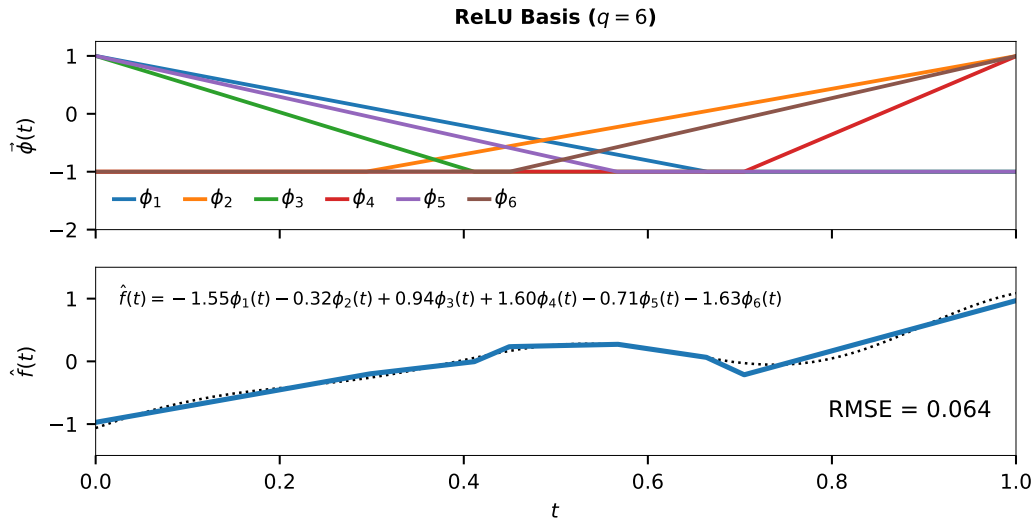
# Representing Functions: ReLU Basis



# Representing Functions: ReLU Basis

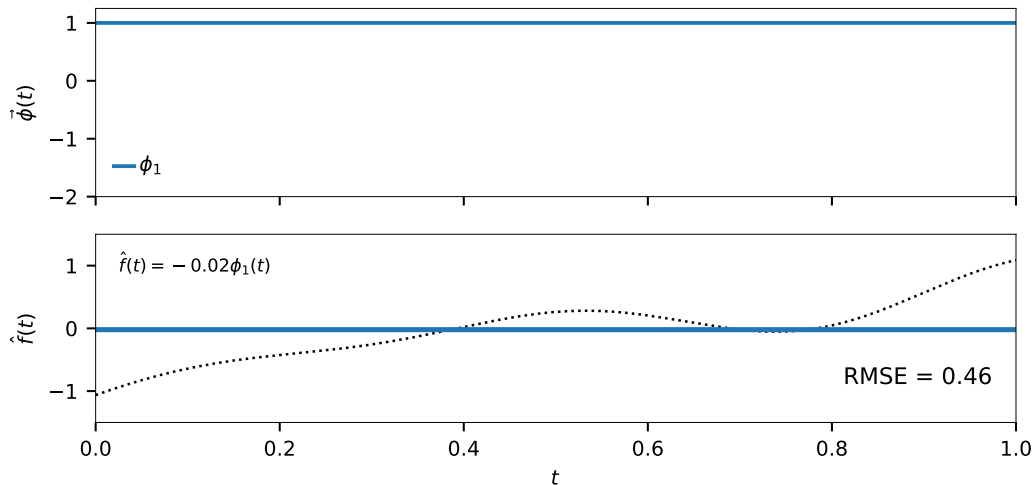


# Representing Functions: ReLU Basis



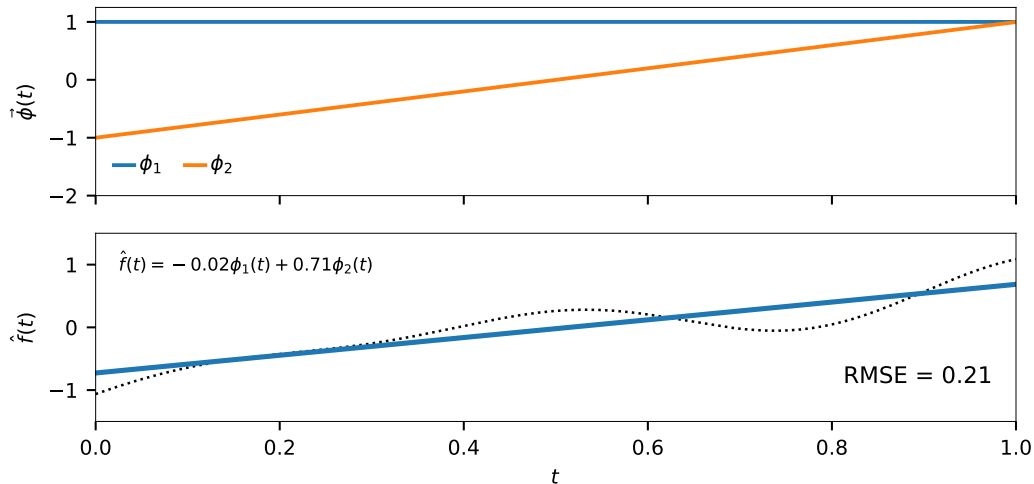
# Representing Functions: Legendre Basis

**Legendre Polynomials ( $q = 1$ )**



# Representing Functions: Legendre Basis

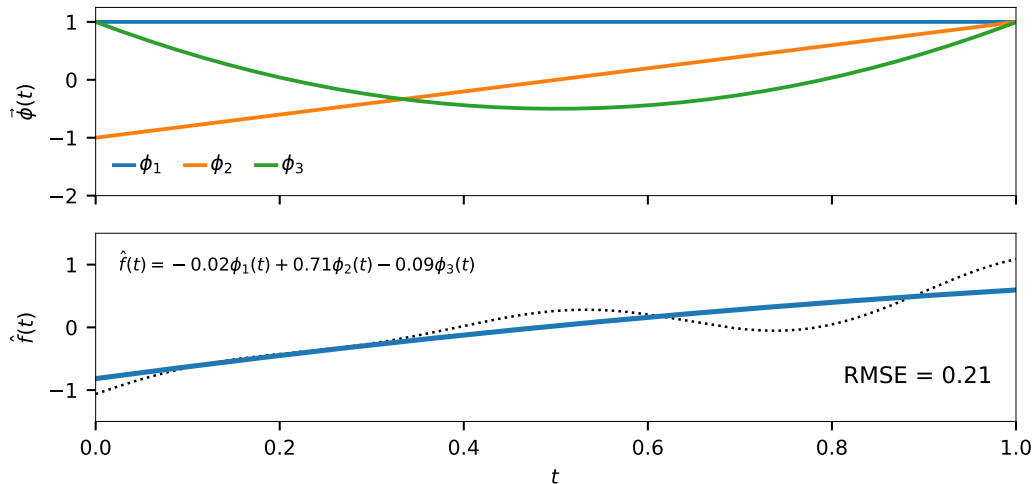
**Legendre Polynomials ( $q = 2$ )**





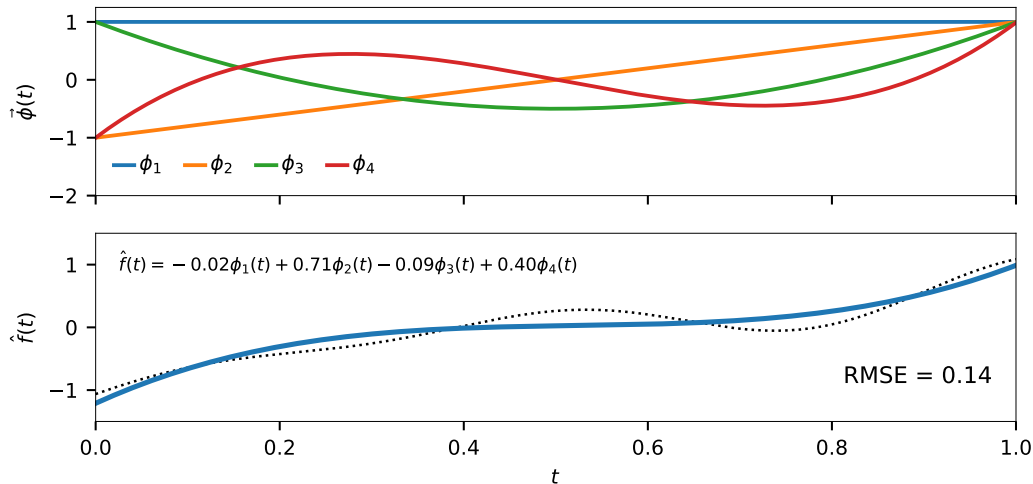
# Representing Functions: Legendre Basis

**Legendre Polynomials ( $q = 3$ )**



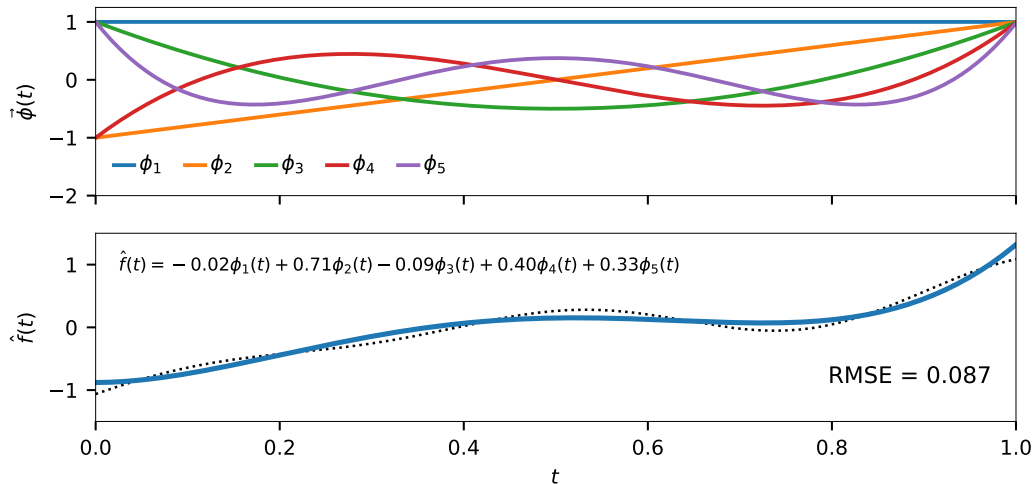
# Representing Functions: Legendre Basis

**Legendre Polynomials ( $q = 4$ )**



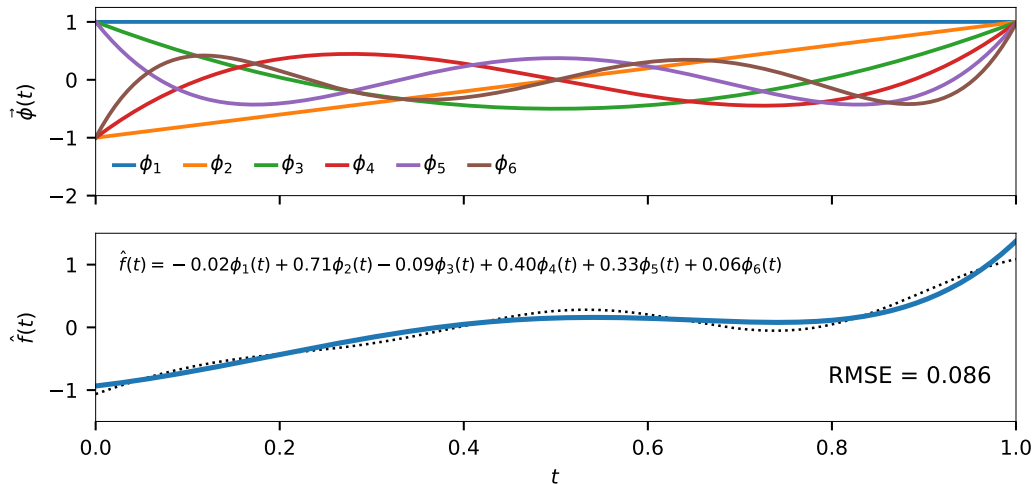
# Representing Functions: Legendre Basis

**Legendre Polynomials ( $q = 5$ )**

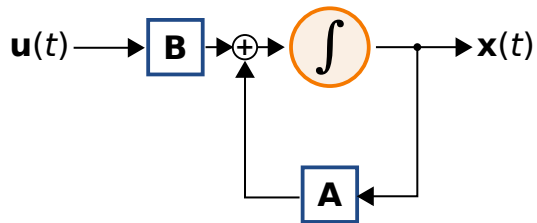


# Representing Functions: Legendre Basis

**Legendre Polynomials ( $q = 6$ )**



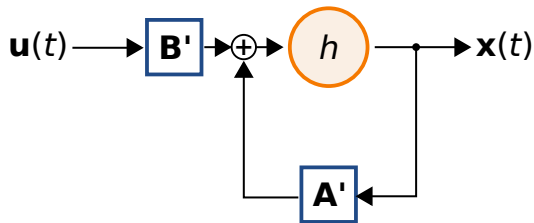
## Implementing the Delay Network



$$\theta \mathbf{A} = a_{ij} \in \mathbb{R}^{q \times q},$$

$$\theta \mathbf{B} = b_i \in \mathbb{R}^q,$$

$$\mathbf{A}' = \tau \mathbf{A} + \mathbf{I}$$

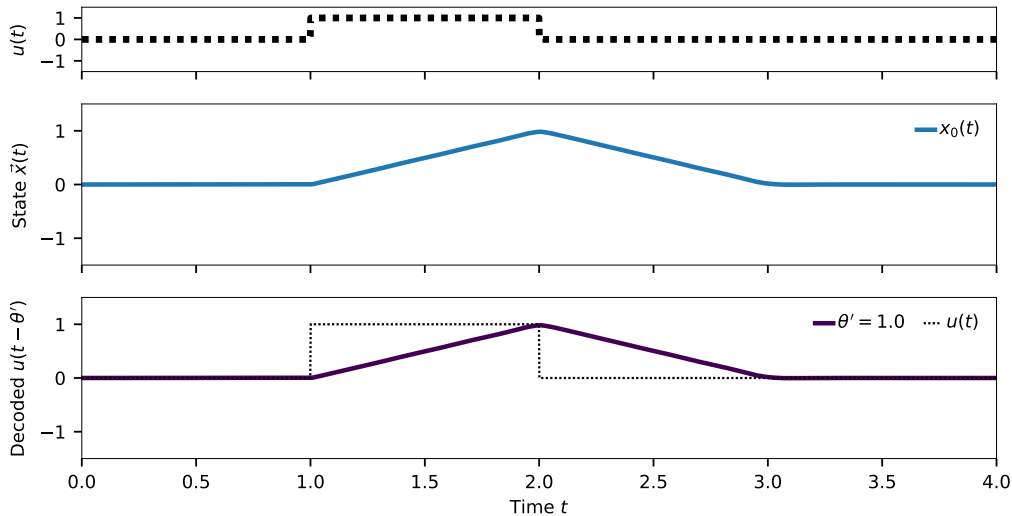


$$a_{ij} = \begin{cases} (2i+1)(-1) & i < j, \\ (2i+1)(-1)^{i-j+1} & i \geq j \end{cases}$$

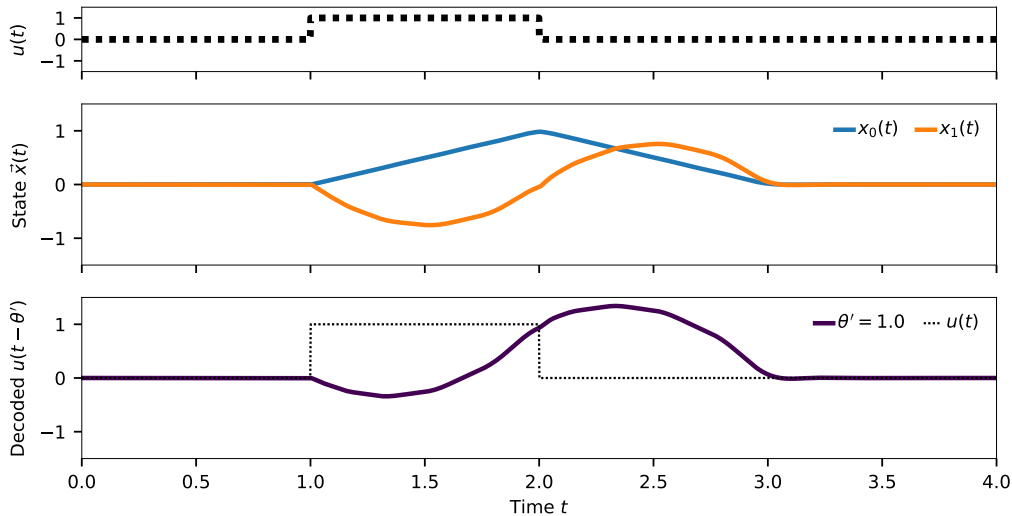
$$b_i = (2i+1)(-1)^i$$

$$\mathbf{B}' = \tau \mathbf{B}$$

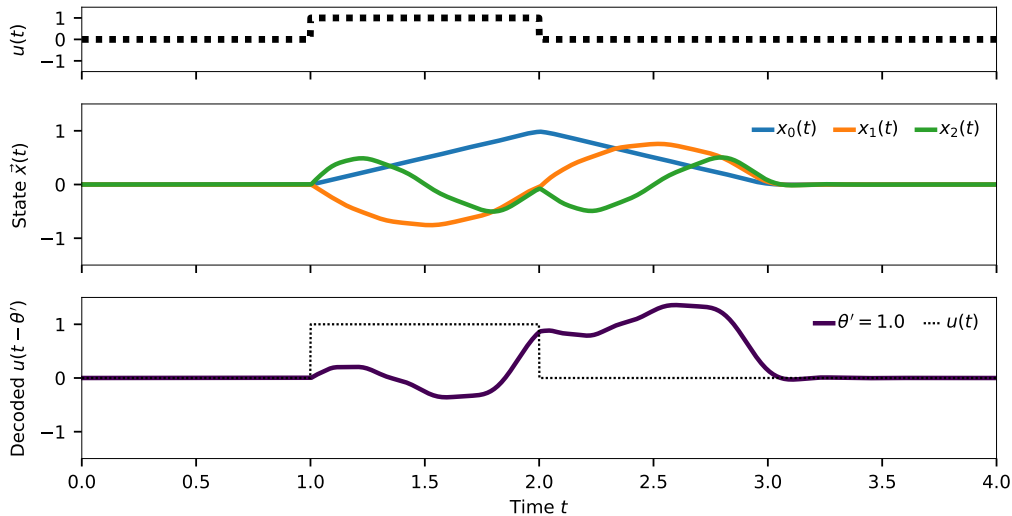
## Delay Network: Step Function



## Delay Network: Step Function

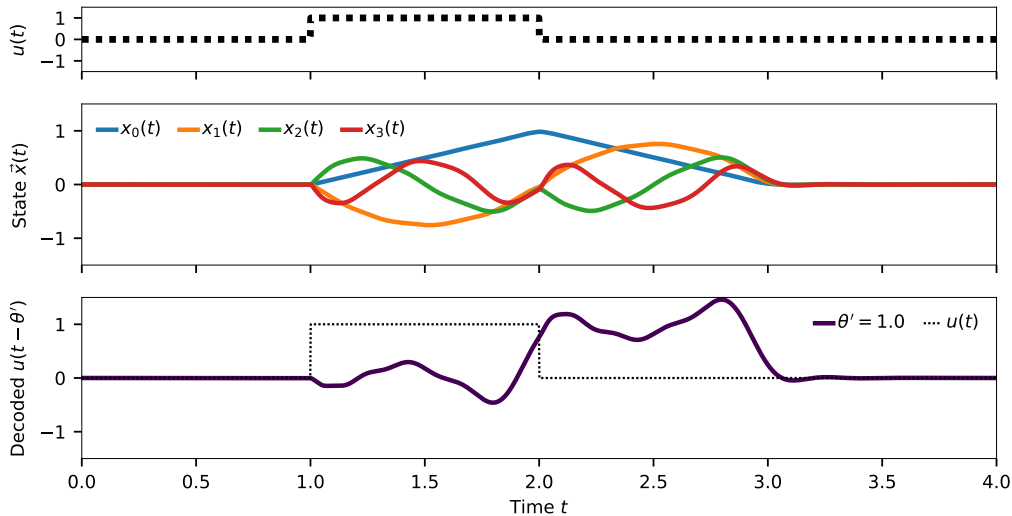


## Delay Network: Step Function

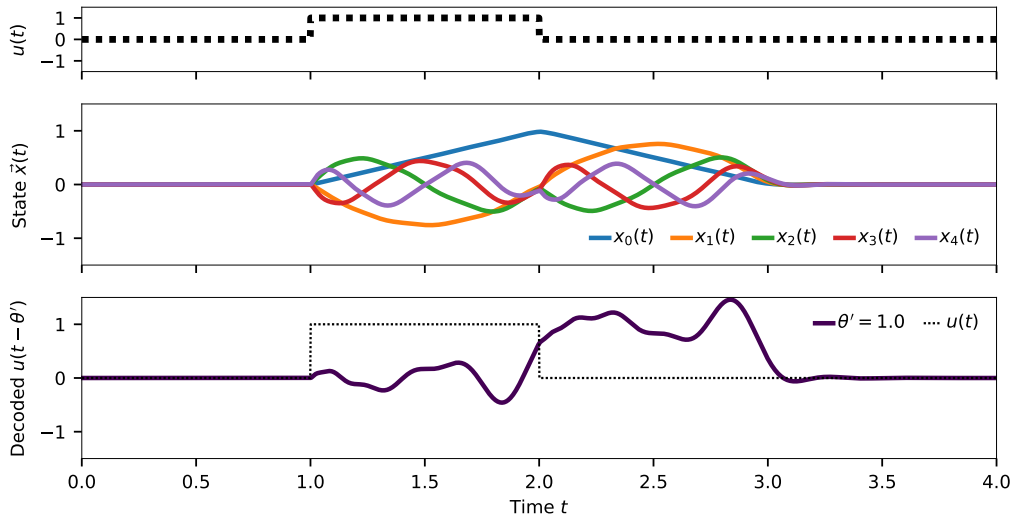




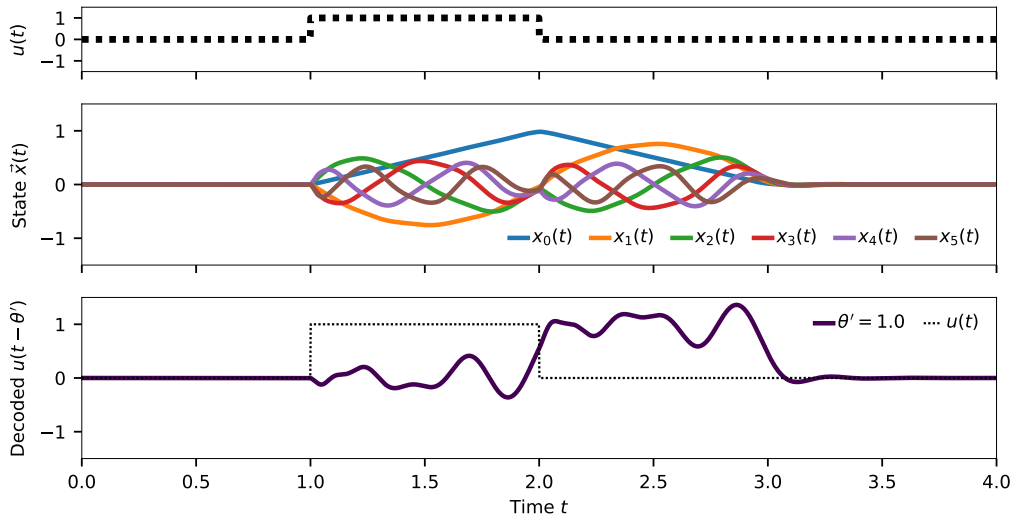
## Delay Network: Step Function



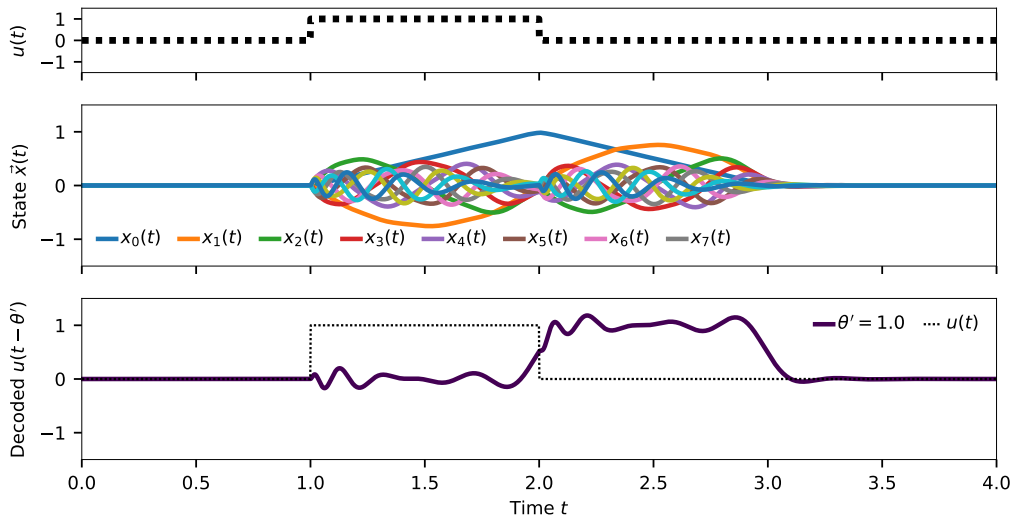
# Delay Network: Step Function



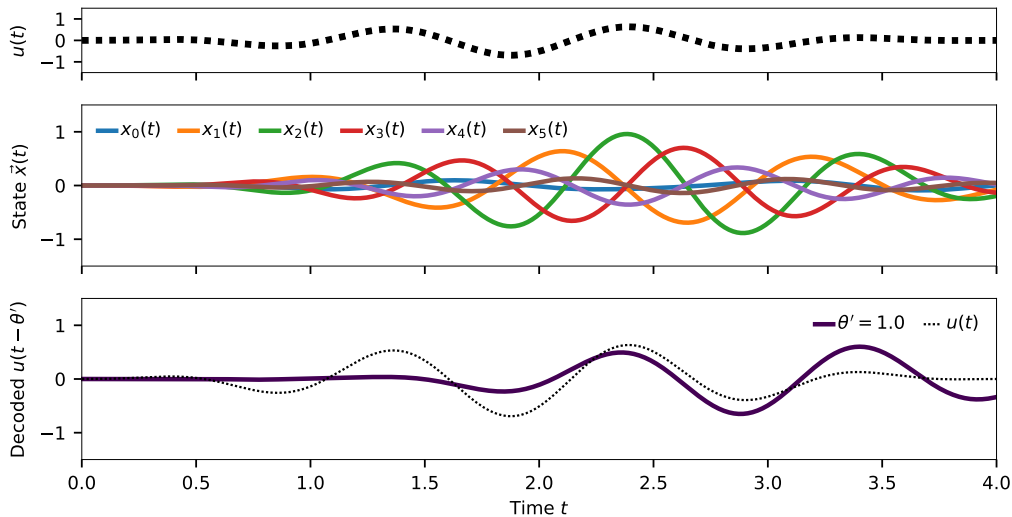
# Delay Network: Step Function



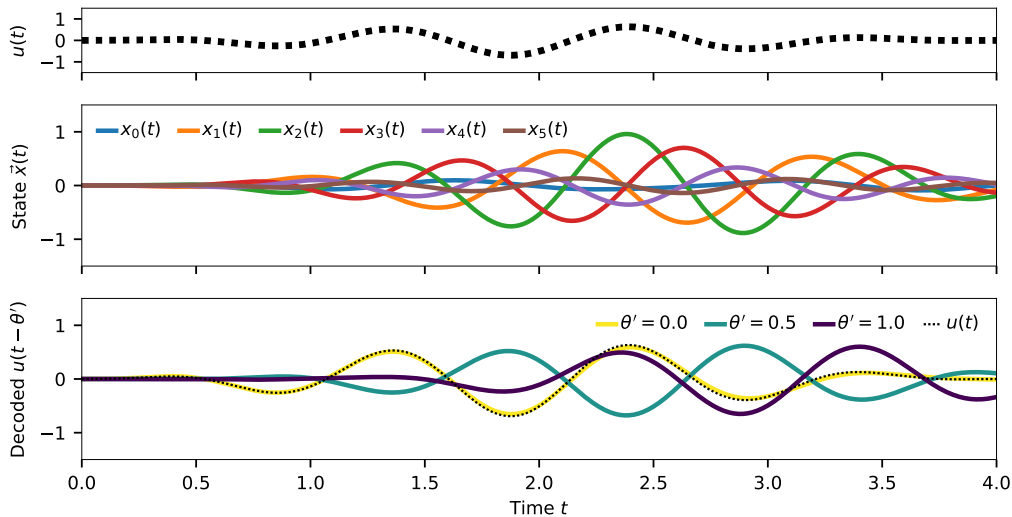
# Delay Network: Step Function



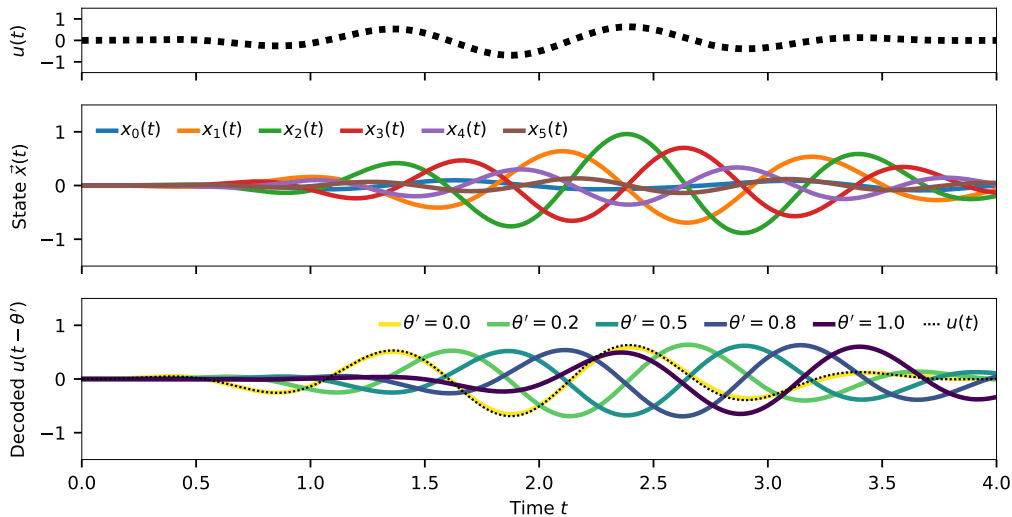
# Delay Network: Windowed Sine Function



# Delay Network: Windowed Sine Function



# Delay Network: Windowed Sine Function



# Image sources

## **Title slide**

Infrared Photograph of a Sundial Near the Einstein Tower in Potsdam, Germany

Author: DrNRNowaczyk, 2007.

From Wikimedia.