Versatile Mathematics COMMON MATHEMATICAL APPLICATIONS

Instructor Solution Manual

NOT FOR USE BY STUDENTS

Josiah Hartley Frederick Community College

Val Lochman
Frederick Community College

Erum Marfani Frederick Community College

2nd Edition

2020

This solution manual was created to accompany $Versatile\ Mathematics$, an OER textbook developed at Frederick Community College in Maryland.

The solutions are shown here solely for the benefit of instructors, and this manual should not be shared with students.

Contents

1	Financial Mathematics 1.2 Applied Percentage Problems 1.3 Simple and Compound Interest 1.4 Saving for Retirement 1.5 Mortgages and Credit Cards 1.6 Income Tax 2.6
2	Growth Models 2 2.1 Linear Models 2 2.2 Quadratic Models 3 2.3 Exponential Models 3 2.4 Logistic Models 3
3	Statistics 4 3.1 Gathering Data 4 3.2 Visualizing Data 4 3.3 Describing Data with Statistics 5 3.4 Linear Regression 5 3.5 The Normal Distribution 5
4	Probability64.1 Basic Concepts of Probability64.2 The Addition Rule and the Rule of Complements64.3 The Multiplication Rule and Conditional Probability74.4 Counting Methods7
5	Linear Programming75.1 Linear Functions and Their Graphs75.2 Systems of Linear Equations85.3 Systems of Linear Inequalities85.4 Linear Programming8
6	Logic 9 6.1 Statements and Logical Operations 9 6.2 Conditionals and Equivalence 9 6.3 Logic Rules 10 6.4 Arguments 10
7	Set Theory 10 7.1 Basic Concepts 10 7.2 Set Operations 11

	7.3 Properties of Set Operations	
8	Graph Theory	117
	8.1 Introduction to Graphs	118
	8.2 Euler and Hamilton Paths	
	8.3 Shortest Paths	128
	8.4 Trees	131

1

Financial Mathematics

SECTION 1.2 Applied Percentage Problems

For problems 1-8, convert each number to a percent.

 $\bigcirc 1 \quad \frac{1}{2}$

Answer to 1: 50%

 $\frac{1}{2} \times 100\% = 0.5 \times 100\% = 50\%$

(2) 0.04

Answer to 2: $\boxed{4\%}$

 $0.04 \times 100\% = 4\%$

 $3 \frac{3}{5}$

Answer to 3: 60%

 $\frac{3}{5} \times 100\% = 0.6 \times 100\% = 60\%$

(4) 0.79

Answer to 4: 79%

 $0.79 \times 100\% = 79\%$

(5) 1.35

Answer to 5: 135%

 $1.35 \times 100\% = 135\%$

6 $\frac{10}{4}$

Answer to 6: 250%

 $\frac{10}{4} \times 100\% = 2.5 \times 100\% = 250\%$

(7) 12.5

Answer to 7: 1250%

 $12.5\times 100\% = 1250\%$

8) 0.0378

Answer to 8: 3.78%

 $0.0378 \times 100\% = 3.78\%$

For problems 9-16, convert each percent to a decimal.

9 33%

Answer to 9: 0.33

 $33\% = \frac{33}{100} = 0.33$

(10) 2.6%

Answer to 10: 0.026

 $2.6\% = \frac{2.6}{100} = 0.026$

$$124\% = \frac{124}{100} = 1.24$$

(12) 1240.5%

$$1240.5\% = \frac{1240.5}{100} = 12.405$$

13) 42%

$$42\% = \frac{42}{100} = 0.42$$

14) 4.5%

$$4.5\% = \frac{4.5}{100} = 0.045$$

(15) 0.003%

$$0.003\% = \frac{0.003}{100} = 0.00003$$

 $(16) \frac{1}{4}\%$

$$\frac{1}{4}\% = 0.25\% = \frac{0.25}{100} = 0.0025$$

(17) What is 12% of 72?

$$x = (12\%)(72) = (0.12)(72) = 8.64$$

(18) Ninety is what percentage of 200?

$$90 = (x)(200) \longrightarrow x = \frac{90}{200} = 0.45 = 45\%$$

(19) Twenty is 15% of what?

$$20 = (15\%)(x) \longrightarrow x = \frac{20}{15\%} = \frac{20}{0.15} = 133.33$$

(20) Six is what percentage of 40?

$$6 = (x)(40) \longrightarrow x = \frac{6}{40} = 0.15 = 15\%$$

(21) Forty is 91% of what?

Answer to 21: 43.96

$$40 = (91\%)(x) \longrightarrow x = \frac{40}{91\%} = \frac{40}{0.91} = 43.96$$

(22) What is 65% of 65?

Answer to 22: 42.25

$$x = (65\%)(65) = (0.65)(65) = 42.25$$

23 In the fall of 2009 FCC enrolled 6,233 students. Of those enrolled, 2,810 are in the 18-21 age group. What percent of FCC students does this represent?

Answer to 23: 45.1%

$$\frac{2810}{6233} = 0.451 = 45.1\%$$

24) Patrick left an \$8 tip on a \$50 restaurant bill. What percent tip is that?

Answer to 24: 16%

$$\frac{8}{50} = 0.16 = 16\%$$

25) Ireland has a 23% VAT (value-added tax, similar to a sales tax). How much will the VAT be on a purchase of a €250 item?

Answer to 25: €57.50

$$(23\%)(250) = (0.23)(250) = 57.5$$

Employees in 2012 paid 4.2% of their gross wages towards social security (FICA tax). How much would someone earning \$45,000 a year pay toward social security?

Answer to 26: \$1890

$$(4.2\%)(45,000) = (0.042)(45,000) = 1890$$

27 A project on Kickstarter was aiming to raise \$15,000 for a precision coffee press. They ended up with 714 supporters, raising 557% of their goal. How much did they raise?

Answer to 27: \$83,550

$$(557\%)(15,000) = (5.57)(15,000) = 83,550$$

28 Another Kickstarter project for an iPad stylus raised 1,253% of their goal, finishing with a total of \$313,250 from 7,511 supporters. What was their original goal?

Answer to 28: \$25,000

$$(1253\%)(x) = 313,250 \longrightarrow x = \frac{313,250}{1253\%} = \frac{313,250}{12.53} = 25,000$$

(29) One year ago the median price for a home was \$275,000. Now the current median price for a home is \$235,000. What was the percent decrease in the median price of a home over the last year?

Answer to 29: | 14.55%

$$275,000 - 235,000 = 40,000 \longrightarrow \frac{40,000}{275,000} = 0.1455 = 14.55\%$$

(30) There were 943 tornadoes reported in the U.S. in 2013, and 897 tornadoes were reported in 2014. What percent decrease was there from 2013 to 2014?

Answer to 30: 4.88%

$$943 - 897 = 46 \longrightarrow \frac{46}{943} = 0.0488 = 4.88\%$$

(31) The population of a town increased from 3,250 in 2008 to 4,300 in 2010. Find the absolute and percent increase.

Answer to 31: absolute: 1050; percent: 32.3%

Absolute:
$$4300 - 3250 = 1050$$

Percent: $\frac{1050}{3250} = 0.323 = 32.3\%$

(32) The number of CDs sold in 2010 was 114 million, down from 147 million the previous year. Find the absolute and percent decrease.

Answer to 32: absolute: 33 million; percent: 22.45%

Absolute:
$$147 - 114 = 33$$

Percent: $\frac{33}{147} = 0.2245 = 22.45\%$

- (33) A company wants to decrease their energy use by 15%.
 - (a) If their electric bill is currently \$2,200 a month, what will their bill be if they're successful?

Answer to 33(a): \$1870

$$(2200)(85\%) = (2200)(0.85) = 1870$$

(b) If their next bill is \$1,700 a month, were they successful? What percent decrease was there from the current bill?

Answer to 33(b): Yes; 22.73%

$$2200 - 1700 = 500 \longrightarrow \frac{500}{2200} = 0.2273 = 22.73\%$$

- (34) A store is hoping an advertising campaign will increase their number of customers by 30%. They currently have about 80 customers per day.
 - (a) How many customers will they have if their campaign is successful?

Answer to 34(a): 104

$$(80)(130\%) = (80)(1.3) = 104$$

(b) If they increase to 120 customers a day, were they successful? What percent increase is this from the current level?

Answer to 34(b): Yes; 50%

$$120 - 80 = 40 \longrightarrow \frac{40}{80} = 0.5 = 50\%$$

(35) An article reports that "attendance dropped 6% this year, to 300." What was the attendance before the drop?

Answer to 35: | 319

$$(x)(94\%) = 300 \longrightarrow x = \frac{300}{94\%} = \frac{300}{0.94} \approx 319$$

(36) An article reports that "sales have grown by 30% this year, to \$200 million." What were sales before the growth?

Answer to 36: 153.85 million

$$(x)(130\%) = 200 \longrightarrow x = \frac{200}{130\%} = \frac{200}{1.3} = 153.85$$

(37) The U.S. federal debt at the end of 2001 was \$5.77 trillion, and grew to \$6.20 trillion by the end of 2002. At the end of 2005 it was \$7.91 trillion, and grew to \$8.45 trillion by the end of 2006. Calculate the absolute and relative increase for 2001-2002 and 2005-2006. Which year saw a larger relative increase in federal debt?

Answer to 37: 0.43 trillion; 7.45%; 0.54 trillion; 6.83%; 2001-2002

2001-2002 (absolute):
$$6.20-5.77=0.43$$
 trillion 2001-2002 (relative): $\frac{0.43}{5.77}=0.0745=7.45\%$ 2005-2006 (absolute): $8.45-7.91=0.54$ trillion 2005-2006 (relative): $\frac{0.54}{7.91}=0.0683=6.83\%$

(38) A TV originally priced at \$799 is on sale for 30% off. There is then a 9.2% sales tax. Find the price after including the discount and sales tax.

Answer to 38: \$610.76

$$(799)(0.7) = 559.30 \longrightarrow (559.30)(1.092) = 610.76$$

- (39) The Walden University had 47,456 students in 2010, while Kaplan University had 77,966 students. Complete the following statements.
 - (a) Kaplan's enrollment was _____% larger than Walden's.

Answer to 39(a): 64.29%

$$77,966 - 47,456 = 30,510 \longrightarrow \frac{30,510}{47,456} = 0.6429 = 64.29\%$$

(b) Walden's enrollment was $___\%$ smaller than Kaplan's.

Answer to 39(b): 39.13%

$$77,966 - 47,456 = 30,510 \longrightarrow \frac{30,510}{77,966} = 0.3913 = 39.13\%$$

(c) Walden's enrollment was _____% of Kaplan's.

Answer to 39(c): | 60.87%

$$\frac{47,456}{77,966} = 0.6087 = 60.87\%$$

- (40) In the 2012 Olympics, Usain Bolt ran the 100 m dash in 9.63 seconds. Jim Hines won the 1968 gold with a time of 9.95 seconds.
 - (a) Bolt's time was _____% faster than Hines'.

Answer to 40(a): | 3.22%

$$9.95 - 9.63 = 0.32 \longrightarrow \frac{0.32}{9.95} = 0.0322 = 3.22\%$$

(b) Hines' time was _____% slower than Bolt's.

Answer to 40(b): 3.32%

$$9.95 - 9.63 = 0.32 \longrightarrow \frac{0.32}{9.63} = 0.0332 = 3.32\%$$

(c) Hines' time was _____% of Bolt's.

Answer to 40(c): 103.32%

$$\frac{9.95}{9.63} = 1.0332 = 103.32\%$$

A store has clearance items that have been marked down by 60%. They are having a sale, advertising an additional 30% off clearance items. What percent of the original price do you end up paying?

Answer to 41: 28%

$$(0.4)(0.7) = 0.28 = 28\%$$

A publisher marks up a textbook by 65%, and a bookstore further marks up the textbook by 35%. What percentage of the original cost do you pay?

Answer to 42: | 222.75%

(1.65)(1.35) = 2.2275 = 222.75%

SECTION 1.3 Simple and Compound Interest

In problems 1-3, a principal amount is borrowed at the given interest rate for the given period of time. Find the loan's future value F, or the amount due at the end of the time, if the loan uses simple interest.

Principal: \$3000 Interest rate: 7%Time: 2 years

Answer to 1: \$3420

F = P(1 + rt) = 3000(1 + (0.07)(2)) = 3420

Principal: \$2700 4%Interest rate: Time: 3 years

Answer to 2: \$3024

F = P(1 + rt) = 2700(1 + (0.04)(3)) = 3024

\$7500 Principal: (3)Interest rate: 3.5%Time: 18 months

Answer to 3: \$7893.75

$$F = P(1+rt) = 7500(1+(0.035)(1.5)) = 7893.75$$

In problems 4-6, a principal amount is borrowed at the given interest rate for the given period of time. If the future value is given, find the principal (present value) if the loan uses simple interest.

\$9000 Future value: Interest rate: 5.5%Time: 1 year

Answer to 4: \$8530.81

$$F = P(1+rt) \longrightarrow P = \frac{F}{1+rt} = \frac{9000}{1+(0.055)(1)} = 8530.81$$

Future value: \$7700 Interest rate: 6%Time: 4 years

Answer to 5: \$6209.68

$$F = P(1+rt) \longrightarrow P = \frac{F}{1+rt} = \frac{7700}{1+(0.06)(4)} = 6209.68$$

Answer to 6: \$783.83

$$F = P(1+rt) \longrightarrow P = \frac{F}{1+rt} = \frac{800}{1+(0.0275)(0.75)} = 783.83$$

In problems 7–9, a principal amount is borrowed at the given interest rate for the given period of time, and interest is compounded as stated. Find the loan's future value F, or the amount due at the end of the time.

 $\begin{array}{c} {\rm Principal:} & \$1200 \\ {\rm Interest\ rate:} & 5\% \\ {\rm Compounding:} & {\rm Annually} \\ {\rm Time:} & 3\ {\rm years} \end{array}$

Answer to 7: \$1389.15

$$F = P\left(1 + \frac{r}{n}\right)^{nt} = 1200\left(1 + \frac{0.05}{1}\right)^{(1)(3)} = 1389.15$$

Principal: \$5700
Interest rate: 3.5%
Compounding: Monthly
Time: 24 months

Answer to 8: \$6112.67

$$F = P\left(1 + \frac{r}{n}\right)^{nt} = 5700\left(1 + \frac{0.035}{2}\right)^{(12)(2)} = 6112.67$$

Principal: \$3000
Interest rate: 5.32%
Compounding: Continuously
Time: 48 months

Answer to 9: \$3711.41

$$F = Pe^{rt} = 3000e^{(0.0532)(4)} = 3711.41$$

In problems 10–12, a principal amount is borrowed at the given interest rate for the given period of time, and interest is compounded as stated. If the future value is given, find the principal.

Answer to 10: \$13,814.66

$$F = P\left(1 + \frac{r}{n}\right)^{nt} \longrightarrow P = \frac{F}{\left(1 + \frac{r}{n}\right)^{nt}} = \frac{17,500}{\left(1 + \frac{0.03}{1}\right)^{(1)(8)}} = 13,814.66$$

Future value: \$18,000
Interest rate: 5.6%
Compounding: Daily
Time: 18 months

Answer to 11: \$16,549.87

$$F = P\left(1 + \frac{r}{n}\right)^{nt} \longrightarrow P = \frac{F}{\left(1 + \frac{r}{n}\right)^{nt}} = \frac{18,000}{\left(1 + \frac{0.056}{365}\right)^{(365)(1.5)}} = 16,549.87$$

Future value: \$9000 Interest rate: 7.48%

Compounding: Continuously
Time: 60 months

Answer to 12: \$6191.79

$$F = Pe^{rt} \longrightarrow P = \frac{F}{e^{rt}} = \frac{9000}{e^{(0.0748)(5)}} = 6191.79$$

(13) A friend lends you \$200 for a week, which you agree to repay with 5% one-time interest. How much will you have to repay?

Answer to 13: \$10

$$(200)(0.05) = 10$$

(14) Suppose you obtain a \$3,000 T-note with a 3% annual rate, paid quarterly, with maturity in 5 years. How much interest will you earn?

Answer to 14: \$483.55

$$F = P\left(1 + \frac{r}{n}\right)^{nt} = 3000\left(1 + \frac{0.03}{4}\right)^{(4)(5)} = 3483.44 \longrightarrow 3483.44 - 3000 = 483.44$$

- (15) A student took out a simple interest loan for \$2,400 for two years at an annual rate of 7%.
 - (a) What is the interest on the loan?

Answer to 15(a): \$336

$$I = Prt = (2400)(0.07)(2) = 336$$

(b) How much will the student have to repay at the end of two years?

Answer to 15(b): \$2736

$$P + I = 2400 + 336 = 2736$$

- (16) A loan of \$2,040 has been made at a 5.7% annual simple interest rate for four months.
 - (a) What is the interest on the loan?

Answer to 16(a): \$38.76

$$I = Prt = (2040)(0.057)\left(\frac{4}{12}\right) = 38.76$$

(b) Find the future value of the loan.

Answer to 16(b): \$2078.76

$$P + I = 2040 + 38.76 = 2078.76$$

- (17) You deposit \$2000 in an account earning 3% interest compounded monthly.
 - (a) How much will you have in the account in 20 years?

Answer to 17(a): \$3641.51

$$F = P\left(1 + \frac{r}{n}\right)^{nt} = 2000\left(1 + \frac{0.03}{12}\right)^{(12)(20)} = 3641.51$$

(b) How much interest will you earn?

Answer to 17(b): \$1641.51

$$I = F - P = 3641.51 - 2000 = 1641.51$$

- (18) You deposit \$10,000 in an account earning 4% interest compounded weekly.
 - (a) How much will you have in the account in 25 years?

Answer to 18(a): \$27,172.37

$$F = P\left(1 + \frac{r}{n}\right)^{nt} = 10,000\left(1 + \frac{0.04}{52}\right)^{(52)(25)} = 27,172.37$$

(b) How much interest will you earn?

Answer to 18(b): \$17,172.37

$$I = F - P = 27,172.37.37 - 10,000 = 17,172.37$$

How much would you need to deposit in an account earning 6% compounded monthly in order to have \$6,000 in the account in 8 years?

Answer to 19: \$3717.14

$$F = P\left(1 + \frac{r}{n}\right)^{nt} \longrightarrow P = \frac{F}{\left(1 + \frac{r}{n}\right)^{nt}} = \frac{6000}{\left(1 + \frac{0.06}{12}\right)^{(12)(8)}} = 3717.14$$

(20) How much would you need to deposit in an account earning 5% compounded quarterly in order to have \$20,000 in the account in 4 years?

Answer to 20: \$16,394.93

$$F = P\left(1 + \frac{r}{n}\right)^{nt} \longrightarrow P = \frac{F}{\left(1 + \frac{r}{n}\right)^{nt}} = \frac{20,000}{\left(1 + \frac{0.05}{4}\right)^{(4)(4)}} = 16,394.93$$

21) If you deposit \$5400 in an account earning 4.35% interest compounded continuously, how much will the account hold in 18 months?

Answer to 21: \$5764.10

$$F = Pe^{rt} = 5400e^{(0.0435)(1.5)} = 5764.10$$

22 If you take out a loan for \$7700 at 6.7% interest compounded continuously, how much will you have to pay back in 5 years?

Answer to 22: \$10,764.14

$$F = Pe^{rt} = 7700e^{(0.067)(5)} = 10.764.14$$

How much do you need to deposit today at 4% interest compounded continuously in order to have \$4000 in 2 years?

Answer to 23: \$3692.47

$$F = Pe^{rt} \longrightarrow P = \frac{F}{e^{rt}} = \frac{4000}{e^{(0.04)(2)}} = 3692.47$$

(24) If you find a CD offering 5.8% interest compounded continuously, how much should you deposit if you are saving up to refinish your kitchen in 3 years and you estimate that will take \$15,000?

Answer to 24: \$12,604.45

$$F = Pe^{rt} \longrightarrow P = \frac{F}{e^{rt}} = \frac{15,000}{e^{(0.058)(3)}} = 12,604.45$$

- 25) You have \$12,000 to invest for 3 years. Find how much you'll have at the end of the 3 years if you earn 4% interest compounded
 - (a) annually

$$F = P\left(1 + \frac{r}{n}\right)^{nt} = 12,000\left(1 + \frac{0.04}{1}\right)^{(1)(3)} = 13,498.37$$

(b) monthly

$$F = P\left(1 + \frac{r}{r}\right)^{nt} = 12,000\left(1 + \frac{0.04}{12}\right)^{(12)(3)} = 13,527.26$$

(c) daily

$$F = P\left(1 + \frac{r}{n}\right)^{nt} = 12,000\left(1 + \frac{0.04}{365}\right)^{(365)(3)} = 13,529.87$$

(d) continuously

$$F = Pe^{rt} = 12,000^{(0.04)(3)} = 13,529.96$$

- (26) You would like to have \$8000 saved in 3 years. Find how much you'll have to invest now to reach that goal if you earn 6% interest compounded
 - (a) annually

$$F = P\left(1 + \frac{r}{n}\right)^{nt} \longrightarrow P = \frac{F}{\left(1 + \frac{r}{n}\right)^{nt}} = \frac{8000}{\left(1 + \frac{0.06}{1}\right)^{(1)(3)}} = 6716.95$$

(b) monthly

$$F = P\left(1 + \frac{r}{n}\right)^{nt} \longrightarrow P = \frac{F}{\left(1 + \frac{r}{n}\right)^{nt}} = \frac{8000}{\left(1 + \frac{0.06}{1}\right)^{(1)(3)}} = 6685.16$$

(c) daily

$$F = P\left(1 + \frac{r}{n}\right)^{nt} \longrightarrow P = \frac{F}{\left(1 + \frac{r}{n}\right)^{nt}} = \frac{8000}{\left(1 + \frac{0.06}{1}\right)^{(1)(3)}} = 6682.26$$

(d) continuously

$$F = Pe^{rt} \longrightarrow P = \frac{F}{e^{rt}} = \frac{8000}{e^{(0.06)(3)}} = 6682.16$$

(27) How long will it take to double an investment at 7% compounded continuously?

$$2P = Pe^{0.07t} \longrightarrow 2 = e^{0.07t} \longrightarrow \ln 2 = 0.07t \longrightarrow t = \frac{\ln 2}{0.07} = 9.9$$

(28) How long will it take to double an investment at 4.6% compounded continuously?

$$2P = Pe^{0.046t} \longrightarrow 2 = e^{0.046t} \longrightarrow \ln 2 = 0.046t \longrightarrow t = \frac{\ln 2}{0.046} = 15.07$$

SECTION 1.4 Saving for Retirement

In problems 1-3, a periodic deposit is made into an annuity with the given terms. Find how much the annuity will hold at the end of the specified amount of time.

 $\begin{array}{c|cccc} & \operatorname{Regular\ deposit} & \$250 \\ & \operatorname{Interest\ rate} & 4\% \\ & \operatorname{Frequency} & \operatorname{Monthly} \\ & \operatorname{Time} & 15 \ \operatorname{years} \\ & \operatorname{Future\ value} & ? \end{array}$

Answer to 1: \$61,522.62

$$F = \frac{PMT\left[\left(1 + \frac{r}{n}\right)^{nt} - 1\right]}{\left(\frac{r}{n}\right)} = \frac{250\left[\left(1 + \frac{0.04}{12}\right)^{(12)(15)} - 1\right]}{\left(\frac{0.04}{12}\right)} = 61,522.62$$

Answer to 2: \$60,009.21

$$F = \frac{PMT\left[\left(1 + \frac{r}{n}\right)^{nt} - 1\right]}{\left(\frac{r}{n}\right)} = \frac{10\left[\left(1 + \frac{0.05}{365}\right)^{(365)(12)} - 1\right]}{\left(\frac{0.05}{365}\right)} = 60,009.21$$

Regular deposit \$2000
Interest rate 3%
Frequency Yearly
Time 22 years
Future value ?

Answer to 3: \$61,073.56

$$F = \frac{PMT\left[\left(1 + \frac{r}{n}\right)^{nt} - 1\right]}{\left(\frac{r}{n}\right)} = \frac{2000\left[\left(1 + \frac{0.03}{1}\right)^{(1)(22)} - 1\right]}{\left(\frac{0.03}{1}\right)} = 61,073.56$$

In problems 4–6, find how much should be regularly deposited into an annuity with the given terms in order to have the specified final amount in the account.

Regular deposit ?
Interest rate 5%
Frequency Monthly
Time 18 years
Future value \$50,000

Answer to 4: \$143.18

$$PMT = \frac{F\left(\frac{r}{n}\right)}{\left[\left(1 + \frac{r}{n}\right)^{nt} - 1\right]} = \frac{50,000\left(\frac{0.05}{12}\right)}{\left[\left(1 + \frac{0.05}{12}\right)^{(12)(18)} - 1\right]} = 143.18$$

 $\begin{array}{ccc} Frequency & Weekly \\ Time & 10 \ years \\ Future \ value & \$27,000 \end{array}$

Answer to 5: \$37.92

$$PMT = \frac{F\left(\frac{r}{n}\right)}{\left[\left(1 + \frac{r}{n}\right)^{nt} - 1\right]} = \frac{27,000\left(\frac{0.06}{52}\right)}{\left[\left(1 + \frac{0.06}{52}\right)^{(52)(10)} - 1\right]} = 37.92$$

Regular deposit ?

(6)

(7)

(8)

(9)

Interest rate 3.5% Frequency Yearly Time 35 years

Future value \$200,000

Answer to 6: \$2999.67

$$PMT = \frac{F\left(\frac{r}{n}\right)}{\left\lceil \left(1 + \frac{r}{n}\right)^{nt} - 1 \right\rceil} = \frac{200,000\left(\frac{0.035}{1}\right)}{\left\lceil \left(1 + \frac{0.035}{1}\right)^{(1)(35)} - 1 \right\rceil} = 2999.67$$

In problems 7–9, you want to be able to withdraw the specified amount periodically from a payout annuity with the given terms. Find how much the account needs to hold to make this possible.

 $\begin{array}{cc} \text{Regular withdrawal} & \$1000 \\ \text{Interest rate} & 5\% \end{array}$

Frequency Monthly
Time 20 years

Account balance ?

Answer to 7: \$151,525.31

$$P = \frac{PMT\left[1 - \left(1 + \frac{r}{n}\right)^{-nt}\right]}{\left(\frac{r}{n}\right)} = \frac{1000\left[1 - \left(1 + \frac{0.05}{12}\right)^{-(12)(20)}\right]}{\left(\frac{0.05}{12}\right)} = 151,525.31$$

Regular withdrawal \$200

Interest rate 3%

Frequency Weekly
Time 15 years

Account balance ?

Answer to 8: \$125,593.56

$$P = \frac{PMT\left[1 - \left(1 + \frac{r}{n}\right)^{-nt}\right]}{\left(\frac{r}{n}\right)} = \frac{200\left[1 - \left(1 + \frac{0.03}{52}\right)^{-(52)(15)}\right]}{\left(\frac{0.03}{52}\right)} = 125,593.56$$

Regular withdrawal \$20,000

Interest rate 5.5%

Frequency Yearly

Time 25 years

Account balance ?

Answer to 9: \$268,278.65

$$P = \frac{PMT\left[1 - \left(1 + \frac{r}{n}\right)^{-nt}\right]}{\left(\frac{r}{n}\right)} = \frac{20,000\left[1 - \left(1 + \frac{0.055}{1}\right)^{-(1)(25)}\right]}{\left(\frac{0.055}{1}\right)} = 268,278.65$$

In problems 10–12, you expect to have the given amount in an account with the given terms. Find how much you can withdraw periodically in order to make the account last the specified amount of time.

Answer to 10: \$1950.59

$$PMT = \frac{P\left(\frac{r}{n}\right)}{1 - \left(1 + \frac{r}{n}\right)^{-nt}} = \frac{300,000\left(\frac{0.04}{12}\right)}{1 - \left(1 + \frac{0.04}{12}\right)^{-(12)(18)}} = 1950.59$$

Answer to 11: \$380.29

$$PMT = \frac{P\left(\frac{r}{n}\right)}{1 - \left(1 + \frac{r}{n}\right)^{-nt}} = \frac{250,000\left(\frac{0.05}{52}\right)}{1 - \left(1 + \frac{0.05}{52}\right)^{-(52)(20)}} = 380.29$$

Answer to 12: \$4135.57

$$PMT = \frac{P\left(\frac{r}{n}\right)}{1 - \left(1 + \frac{r}{n}\right)^{-nt}} = \frac{1,000,000\left(\frac{0.0285}{12}\right)}{1 - \left(1 + \frac{0.0285}{12}\right)^{-(12)(30)}} = 4135.57$$

- (13) You deposit \$200 each month into an account earning 3% interest compounded monthly.
 - (a) How much will you have in the account in 30 years?

Answer to 13(a): \$116,547.38

$$F = \frac{PMT\left[\left(1 + \frac{r}{n}\right)^{nt} - 1\right]}{\left(\frac{r}{n}\right)} = \frac{200\left[\left(1 + \frac{0.03}{12}\right)^{(12)(30)} - 1\right]}{\left(\frac{0.03}{12}\right)} = 116,547.38$$

(b) How much total money will you put into the account?

Answer to 13(b): \$72,000

$$(200)(12)(30) = 72,000$$

(c) How much total interest will you earn?

Answer to 13(c): \$44,547.38

$$116,547.38 - 72,000 = 44,547.38$$

- (14) You deposit \$1000 each year into an account earning 8% interest compounded annually.
 - (a) How much will you have in the account in 10 years?

$$F = \frac{PMT\left[\left(1 + \frac{r}{n}\right)^{nt} - 1\right]}{\left(\frac{r}{n}\right)} = \frac{1000\left[\left(1 + \frac{0.08}{1}\right)^{(1)(10)} - 1\right]}{\left(\frac{0.08}{1}\right)} = 14,486.56$$

(b) How much total money will you put into the account?

$$(1000)(1)(10) = 10,000$$

(c) How much total interest will you earn?

$$14,486.56 - 10,000 = 4,486.56$$

(15) Evelyn has \$500,000 saved for retirement in an account earning 6% interest, compounded monthly. How much will she be able to withdraw each month if she wants to take withdrawals for 20 years?

$$PMT = \frac{P\left(\frac{r}{n}\right)}{1 - \left(1 + \frac{r}{n}\right)^{-nt}} = \frac{500,000\left(\frac{0.06}{12}\right)}{1 - \left(1 + \frac{0.06}{12}\right)^{-(12)(20)}} = 3582.16$$

Luke already knows that he will have \$750,000 when he retires. If he sets up a payout annuity for 30 years in an account paying 7% interest, how much could the annuity provide each month?

$$PMT = \frac{P\left(\frac{r}{n}\right)}{1 - \left(1 + \frac{r}{n}\right)^{-nt}} = \frac{750,000\left(\frac{0.07}{12}\right)}{1 - \left(1 + \frac{0.07}{12}\right)^{-(12)(30)}} = 4989.77$$

- (17) Michael is planning for retirement, and he estimates that he'll want to be able to withdraw \$2500 each month for 30 years once he retires. He opens a Roth IRA and finds investments that he expects to return 5% interest compounded monthly.
 - (a) How much will he need to have in the account when he retires in order to meet his goal?

$$P = \frac{PMT \left[1 - \left(1 + \frac{r}{n} \right)^{-nt} \right]}{\left(\frac{r}{n} \right)} = \frac{2500 \left[1 - \left(1 + \frac{0.05}{12} \right)^{-(12)(30)} \right]}{\left(\frac{0.05}{12} \right)} = 465,704.04$$

(b) How much will he have to deposit each month for the next 40 years in order to get this balance at retirement?

$$PMT = \frac{P\left(\frac{r}{n}\right)}{1 - \left(1 + \frac{r}{n}\right)^{-nt}} = \frac{465,704.04\left(\frac{0.07}{12}\right)}{1 - \left(1 + \frac{0.07}{12}\right)^{-(12)(40)}} = 305.18$$

(c) How much interest will his deposits earn?

$$465,704.04 - (305.18)(12)(40) = 319,217.64$$

- (18) Rachel is planning for retirement, and she estimates that she'll want to be able to withdraw \$1800 each month for 25 years once she retires. She opens a Roth IRA and finds investments that she expects to return 3.75% interest compounded monthly.
 - (a) How much will she need to have in the account when she retires in order to meet her goal?

$$P = \frac{PMT\left[1 - \left(1 + \frac{r}{n}\right)^{-nt}\right]}{\left(\frac{r}{n}\right)} = \frac{1800\left[1 - \left(1 + \frac{0.0375}{12}\right)^{-(12)(25)}\right]}{\left(\frac{0.0375}{12}\right)} = 350, 105.19$$

(b) How much will she have to deposit each month for the next 40 years in order to get this balance at retirement?

$$PMT = \frac{P\left(\frac{r}{n}\right)}{1 - \left(1 + \frac{r}{n}\right)^{-nt}} = \frac{350,105.19\left(\frac{0.0375}{12}\right)}{1 - \left(1 + \frac{0.0375}{12}\right)^{-(12)(40)}} = 315.19$$

(c) How much interest will her deposits earn?

$$350, 105.19 - (315.19)(12)(40) = 198,816.05$$

19 Faith is 27 years old, she plans to retire at age 65, and she expects to live to age 92. She expects that her investments can earn an average return of 8% until retirement, and after retirement, she plans to earn 4%. If she wants to be able to withdraw \$2500 per month after retirement, how much should she start saving each month?

$$\begin{aligned} \text{Balance at retirement: } P &= \frac{2500 \left[1 - \left(1 + \frac{0.04}{12} \right)^{-(12)(27)} \right]}{\left(\frac{0.04}{12} \right)} = 494,845.51 \\ \text{Deposits before retiring: } PMT &= \frac{494,845.51 \left(\frac{0.08}{12} \right)}{1 - \left(1 + \frac{0.08}{12} \right)^{-(12)(38)}} = 167.50 \end{aligned}$$

20 Caleb is 30 years old, he plans to retire at age 68, and he expects to live to age 90. He expects that his investments can earn an average return of 6% until retirement, and after retirement, he plans to earn 5%. If he wants to be able to withdraw \$2000 per month after retirement, how much should he start saving each month?

$$\begin{aligned} \text{Balance at retirement: } P &= \frac{2000 \left[1 - \left(1 + \frac{0.05}{12} \right)^{-(12)(22)} \right]}{\left(\frac{0.05}{12} \right)} = 319,856.32 \\ \text{Deposits before retiring: } PMT &= \frac{319,856.32 \left(\frac{0.6}{12} \right)}{1 - \left(1 + \frac{0.06}{12} \right)^{-(12)(38)}} = 183.38 \end{aligned}$$

SECTION 1.5 Mortgages and Credit Cards

1 If you take out an auto loan of \$8500 at 5% interest for 48 months, what will your monthly payment be?

Answer to 1: \$195.75

$$PMT = \frac{P\left(\frac{r}{n}\right)}{1 - \left(1 + \frac{r}{n}\right)^{-nt}} = \frac{8500\left(\frac{0.05}{12}\right)}{1 - \left(1 + \frac{0.05}{12}\right)^{-(12)(4)}} = 195.75$$

(2) If you borrow \$13,000 to buy a boat, and the bank charges 7% interest for 72 months, how much will you have to pay each month?

Answer to 2: \$221.64

$$PMT = \frac{P\left(\frac{r}{n}\right)}{1 - \left(1 + \frac{r}{n}\right)^{-nt}} = \frac{8500\left(\frac{0.05}{12}\right)}{1 - \left(1 + \frac{0.05}{12}\right)^{-(12)(4)}} = 221.64$$

3 Janine bought \$3000 of new furniture on credit. Because her credit score isn't very good, the store is charging her a fairly high interest rate on the loan: 16%. If she agreed to pay off the furniture over two years, how much will she have to pay each month?

Answer to 3: \$146.89

$$PMT = \frac{P\left(\frac{r}{n}\right)}{1 - \left(1 + \frac{r}{n}\right)^{-nt}} = \frac{8500\left(\frac{0.05}{12}\right)}{1 - \left(1 + \frac{0.05}{12}\right)^{-(12)(4)}} = 146.89$$

(4) Carly financed a new \$1200 television at 12% for 48 months. How much will she have to pay every month to pay this off?

Answer to 4: \$31.60

$$PMT = \frac{P\left(\frac{r}{n}\right)}{1 - \left(1 + \frac{r}{n}\right)^{-nt}} = \frac{8500\left(\frac{0.05}{12}\right)}{1 - \left(1 + \frac{0.05}{12}\right)^{-(12)(4)}} = 31.60$$

(5) If you want to buy a car, and you can afford a monthly payment of \$175, how large of a loan can you get at 4.8% interest over 60 months?

Answer to 5: \$9318.55

$$P = \frac{PMT\left[1 - \left(1 + \frac{r}{n}\right)^{-nt}\right]}{\left(\frac{r}{n}\right)} = \frac{175\left[1 - \left(1 + \frac{0.048}{12}\right)^{-(12)(5)}\right]}{\left(\frac{0.048}{12}\right)} = 9318.55$$

(6) Mary is going to finance new office equipment at a 2% rate over a 4 year term. If she can afford monthly payments of \$100, how much can she pay for the new office equipment?

Answer to 6: \$4609.33

$$P = \frac{PMT\left[1 - \left(1 + \frac{r}{n}\right)^{-nt}\right]}{\left(\frac{r}{n}\right)} = \frac{175\left[1 - \left(1 + \frac{0.048}{12}\right)^{-(12)(5)}\right]}{\left(\frac{0.048}{12}\right)} = 4609.33$$

(7) If you buy a \$33,000 car for \$1000 down and monthly payments of \$685 for 60 months, how much will you pay in total for the car?

Answer to 7: \$42,100

$$1000 + (685)(60) = 42,100$$

(8) A car costs \$27,000, and you're offered a loan that requires \$800 down and a monthly payment of \$575 for 60 months, how much will you pay in interest?

Answer to 8: \$8300

$$800 + (575)(60) = 35,300 \longrightarrow 35,300 - 27,000 = 8300$$

- $\fbox{ \ \, 9)}$ A car dealership offers a loan with 3.5% interest for 60 months, and you plan to purchase a car for \$18,000.
 - (a) What will your monthly payment be?

Answer to 9(a): \$327.45

$$PMT = \frac{P\left(\frac{r}{n}\right)}{1 - \left(1 + \frac{r}{n}\right)^{-nt}} = \frac{18,000\left(\frac{0.035}{12}\right)}{1 - \left(1 + \frac{0.035}{12}\right)^{-(12)(5)}} = 327.45$$

(b) How much will you pay in total for the car?

Answer to 9(b): \$19,647

$$(327.45)(60) = 19,647$$

(c) How much will you pay in interest over the life of the loan?

Answer to 9(c): \$1647

$$19,647 - 18,000 = 1647$$

- You plan to purchase a \$21,000 car, and your bank offers you a loan at 4.5% interest for 48 months.
 - (a) What will your monthly payment be?

Answer to 10(a): \$478.87

$$PMT = \frac{P\left(\frac{r}{n}\right)}{1 - \left(1 + \frac{r}{n}\right)^{-nt}} = \frac{21,000\left(\frac{0.045}{12}\right)}{1 - \left(1 + \frac{0.045}{12}\right)^{-(12)(4)}} = 478.87$$

(b) How much will you pay in total for the car?

Answer to 10(b): \$22,985.76

$$(478.87)(48) = 22,985.76$$

(c) How much will you pay in interest over the life of the loan?

Answer to 10(c): \$1985.76

$$22,985.76 - 21,000 = 1985.76$$

- (11) You want to buy a \$200,000 home, and you have \$40,000 saved up. The bank offers a 30-year mortgage at 3.8% interest.
 - (a) If you expect to pay \$6000 in closing costs, what percentage down payment can you afford?

Answer to 11(a): 17%

$$40,000 - 6000 = 34,000 \longrightarrow \frac{34,000}{200,000} = 0.17 = 17\%$$

(b) If you put less than 20% down, you'll need to pay mortgage insurance. Will you require mortgage insurance?

$$17\% < 20\% \longrightarrow \text{Yes}$$

(c) What will the principal be on the loan?

$$200,000 - 34,000 = 166,000$$

(d) What will your monthly P&I payment be?

$$PMT = \frac{P\left(\frac{r}{n}\right)}{1 - \left(1 + \frac{r}{n}\right)^{-nt}} = \frac{166,000\left(\frac{0.038}{12}\right)}{1 - \left(1 + \frac{0.038}{12}\right)^{-(12)(30)}} = 773.49$$

(e) In addition to principal and interest, the property taxes will be 1.5% of the home value per year, homeowners insurance will be \$750 per year, and the mortgage insurance (if needed, according to part (b)) will be \$25 per month. What will your total monthly payment amount be?

Property taxes:
$$(0.015)(200,000)=3000$$
 per year, or $\frac{3000}{12}=250$ per month Homeowners insurance: $\frac{750}{12}=62.50$ Mortgage insurance: 25

Total:
$$773.49 + 250 + 62.50 + 25 = 1110.99$$

(f) How much will you pay in total over 30 years in principal and interest?

$$(773.49)(12)(30) = 278,456.40$$

(g) How much interest will you pay in total?

$$278,456.40 - 200,000 = 78,456.40$$

- (12) You want to buy a \$375,000 home, and you have \$84,000 saved up. The bank offers a 20-year mortgage at 3.2% interest.
 - (a) If you expect to pay \$8000 in closing costs, what percentage down payment can you afford?

$$84,000 - 8000 = 76,000 \longrightarrow \frac{76,000}{375,000} = 0.203 = 20\%$$

(b) If you put less than 20% down, you'll need to pay mortgage insurance. Will you require mortgage insurance?

$$20\% \ge 20\% \longrightarrow \text{No}$$

(c) What will the principal be on the loan?

$$375,000 - 76,000 = 299,000$$

(d) What will your monthly P&I payment be?

Answer to 12(d): \$1688.34

$$PMT = \frac{P\left(\frac{r}{n}\right)}{1 - \left(1 + \frac{r}{n}\right)^{-nt}} = \frac{299,000\left(\frac{0.032}{12}\right)}{1 - \left(1 + \frac{0.032}{12}\right)^{-(12)(20)}} = 1688.34$$

(e) In addition to principal and interest, the property taxes will be 1.5% of the home value per year, homeowners insurance will be \$825 per year, and the mortgage insurance (if needed, according to part (b)) will be \$30 per month. What will your total monthly payment amount be?

Answer to 12(e): \$2225.84

Property taxes: (0.015)(375,000) = 5625 per year, or $\frac{5625}{12} = 468.75$ per month Homeowners insurance: $\frac{825}{12} = 68.75$ Mortgage insurance: 0

Total: 1688.34 + 468.75 + 68.75 + 0 = 2225.84

(f) How much will you pay in total over 20 years in principal and interest?

Answer to 12(f): \$405,201.60

(1688.34)(12)(20) = 405, 201.60

(g) How much interest will you pay in total?

Answer to 12(g): \$30,201.60

$$405,201.60 - 375,000 = 30,201.60$$

- (13) You can afford a \$900 per month mortgage payment. You've found a 30-year loan at 4% interest.
 - (a) How big of a loan can you afford?

$$P = \frac{PMT\left[1 - \left(1 + \frac{r}{n}\right)^{-nt}\right]}{\left(\frac{r}{n}\right)} = \frac{900\left[1 - \left(1 + \frac{0.04}{12}\right)^{-(12)(30)}\right]}{\left(\frac{0.04}{12}\right)} = 188,515.12$$

(b) How much total money will you pay the bank?

Answer to 13(b): \$324,000

$$(900)(12)(30) = 324,000$$

(c) How much of that money is interest?

Answer to 13(c): \$135,484.88

$$324,000 - 188,515.12 = 135,484.88$$

- You can afford a \$1790 per month mortgage payment. You've found a 15-year loan at 3.25% interest.
 - (a) How big of a loan can you afford?

$$P = \frac{PMT\left[1 - \left(1 + \frac{r}{n}\right)^{-nt}\right]}{\left(\frac{r}{n}\right)} = \frac{1790\left[1 - \left(1 + \frac{0.0325}{12}\right)^{-(12)(15)}\right]}{\left(\frac{0.0325}{12}\right)} = 254,743.07$$

$$(1790)(12)(15) = 322,200$$

(c) How much of that money is interest?

$$322,200 - 254,743.07 = 67,456.93$$

If the interest rate on a 30-year mortgage for \$175,000 were changed from 3.8% to 3.1%, how much would you save over the life of the loan?

Original loan payment:
$$PMT = \frac{175,000 \left(\frac{0.038}{12}\right)}{1 - \left(1 + \frac{0.038}{12}\right)^{-(12)(30)}} = 815.43$$
Total cost of original loan: $(815.43)(12)(30) = 293,554.80$
Second loan payment: $PMT = \frac{175,000 \left(\frac{0.031}{12}\right)}{1 - \left(1 + \frac{0.031}{12}\right)^{-(12)(30)}} = 747.28$
Total cost of second loan: $(747.28)(12)(30) = 269,020.80$

Second loan payment:
$$PMT = \frac{175,000 \left(\frac{0.031}{12}\right)}{1 - \left(1 + \frac{0.031}{12}\right)^{-(12)(30)}} = 747.28$$

Total cost of second loan: (747.28)(12)(30) = 269,020.80

Difference:
$$293,554.80 - 269,020.80 = 24,543$$

16) How much would you save (over the life of the loan) on a 20-year mortgage at 4.5% if you reduced the amount you borrowed from \$300,000 to \$260,000?

Original loan payment:
$$PMT = \frac{300,000 \left(\frac{0.045}{12}\right)}{1 - \left(1 + \frac{0.045}{12}\right)^{-(12)(20)}} = 1897.95$$
Total cost of original loan: $(1897.95)(12)(20) = 455.508$

Total cost of original loan: (1897.95)(12)(20) = 455,508Interest cost of original loan: 455,508 - 300,000 = 155,508

Interest cost of original loan:
$$455,508 - 300,000 = 155,508$$

Second loan payment: $PMT = \frac{260,000\left(\frac{0.045}{12}\right)}{1 - \left(1 + \frac{0.045}{12}\right)^{-(12)(20)}} = 1644.89$
Total cost of second loan: $(1644.89)(12)(20) = 394,773.60$
Interest cost of second loan: $304.773.60 - 260.000 = 134.773.60$

Interest cost of second loan: 394,773.60 - 260,000 = 134,773.60

Difference:
$$155,508 - 134,773.60 = 20,734.40$$

(17) If you borrow \$250,000, how much could you save over the life of the loan if you took out a 15-year mortgage at 4.5% instead of a 30-year mortgage at 4%?

Original loan payment:
$$PMT = \frac{250,000 \left(\frac{0.04}{12}\right)}{1 - \left(1 + \frac{0.04}{12}\right)^{-(12)(30)}} = 1193.54$$

Total cost of original loan: $(1193.54)(12)(30) = 429,674.40$

Total cost of original loan:
$$(1193.54)(12)(30) = 429,674.40$$

Second loan payment: $PMT = \frac{250,000\left(\frac{0.045}{12}\right)}{1-\left(1+\frac{0.045}{12}\right)^{-(12)(15)}} = 1912.48$

Total cost of second loan: (1912.48)(12)(15) = 344,246.40

Difference:
$$429,674.40 - 344,246.40 = 85,428$$

- (18) Suppose you take out a \$315,000 mortgage for 30 years at 4.5% interest.
 - (a) Find the monthly payment on this mortgage.

Answer to 18(a): \$1596.06

$$PMT = \frac{315,000 \left(\frac{0.045}{12}\right)}{1 - \left(1 + \frac{0.045}{12}\right)^{-(12)(30)}} = 1596.06$$

(b) Fill out the first two rows of the amortization schedule below.

Payment	Interest Payment	Principal Payment	Balance of
Number			Loan
	Prt	PMT - I	Balance -
			Prin. PMT
1	= (315,000)(0.045/12)	= 1596.06 - 1181.25	= 315,000 -
			414.81
	= \$1181.25	= \$414.81	=
			\$314, 585.19
		150000 115000	
2	=	= 1596.06 - 1179.69	=
	(314, 585.19)(0.045/12)		314,585.19 -
			416.37
	= \$1179.69	= \$416.37	=
			\$314, 168.82

- (19) Suppose you take out a \$180,000 mortgage for 15 years at 3.7% interest.
 - (a) Find the monthly payment on this mortgage.

Answer to 19(a): \$1304.54

$$PMT = \frac{180,000 \left(\frac{0.037}{12}\right)}{1 - \left(1 + \frac{0.037}{12}\right)^{-(12)(15)}} = 1304.54$$

(b) Fill out the first two rows of the amortization schedule below.

Payment	Interest Payment	Principal Payment	Balance of
Number			Loan
	Prt	PMT - I	Balance -
			Prin. PMT
1	= (180,000)(0.037/12)	= 1304.54 - 555	= 180,000 -
			749.54
	= \$555	$= \boxed{\$749.54}$	=
		[*** - ***	\$179, 250.46
2	<u>-</u>	= 1304.54 - 552.69	=
	(179, 250.46)(0.037/12)		179,250.46 -
			751.85
	= \$552.69	= \$751.85	=
			\$178,498.61

20 Suppose your VISA card calculates interest using the average daily balance method, and the monthly interest rate is 1.4%. The itemized billing for the month of April is shown below.

Detail	Date	Amount
Unpaid balance	April 1	\$1100
Payment received	April 3	\$500
New computer	April 11	\$750
Books	April 15	\$65
Mattress	April 28	\$600
Last day of billing period	April 30	
Payment Due Date	May 7	

(a) Find the average daily balance.

$$\frac{(1100)(2) + (600)(8) + (1350)(4) + (1415)(13) + (2015)(3)}{30} = \frac{36,840}{30} = 1228$$

(b) Find the interest due for this month.

$$(1228)(0.014)(1) = 17.19$$

(c) Find the total balance owed on the last day of the billing period.

$$2015 + 17.19 = 2032.19$$

(d) This credit card requires a \$20 minimum payment or 1/36 of the amount due, whichever is higher. What is the minimum monthly payment due for this month?

$$\frac{2032.19}{36} = 56.45$$

(21) Suppose your MasterCard calculates interest using the average daily balance method, and the monthly interest rate is 2.1%. The itemized billing for the month of August is shown below.

Detail	Date	Amount
Unpaid balance	August 1	\$300
Payment received	August 9	\$100
Tuition	August 10	\$4500
Textbooks	August 18	\$350
Groceries	August 25	\$180
Last day of billing period	August 31	
Payment Due Date	September 7	

(a) Find the average daily balance.

$$\frac{(300)(8) + (200)(1) + (4700)(8) + (5050)(7) + (5230)(7)}{31} = \frac{112,160}{31} = 3618.06$$

(b) Find the interest due for this month.

$$(3618.06)(0.021)(1) = 75.98$$

(c) Find the total balance owed on the last day of the billing period.

$$5230 + 75.98 = 5305.98$$

(d) This credit card requires a \$15 minimum payment or 1/24 of the amount due, whichever is higher. What is the minimum monthly payment due for this month?

$$\frac{5305.98}{24} = 221.08$$

SECTION 1.6 Income Tax

(1) A ______ is a tax at a consistent rate.

Answer to 1: flat tax

 $oxed{2}$ A _____ is a tax for which the rate increases for higher taxed amounts

Answer to 2: progressive tax

(3) A ______ is a tax for which the rate decreases for higher taxed amounts

Answer to 3: regressive tax

4 If the property taxes are \$1800 on a home valued at \$140,000, what is the effective tax rate?

Answer to 4: 1.29%

$$\frac{1800}{140,000} = 0.0129 = 1.29\%$$

(5) In state A, the gas tax is 28 cents per gallon, where the average pre-tax cost of gas is \$2.58 per gallon. In state B, the gas tax is 25 cents per gallon, where the average pre-tax cost of gas is \$2.50. Which state has a lower gas tax rate?

Answer to 5: State A

State A:
$$\frac{0.28}{2.58} = 0.1085 = 10.85\%$$

State B: $\frac{0.25}{2.50} = 0.1 = 10\%$

6 If the sales tax is \$16.05 on a purchase of \$214, what is the sales tax rate?

Answer to 6: | 7.5%

$$\frac{16.05}{214} = 0.075 = 7.5\%$$

(7) Using the 2020 tax table given in the textbook, how much would a single taxpayer owe on a taxable income of \$55,000?

Answer to 7: \$7890

$$(9875)(0.10) + (40, 125 - 9875)(0.12) + (55,000 - 40,125)(0.22) = 7890$$

(8) Using the 2020 tax table given in the textbook, how much would a married couple filing jointly owe on a taxable income of \$92,000?

Answer to 8: \$11,820

$$(19,750)(0.10) + (80,250 - 19,750)(0.12) + (92,000 - 80,250)(0.22) = 11,820$$

For problems 9-14, use the 2020 tax table given in the textbook to calculate the tax owed by each taxpayer.

(9)

Taxpayer: Single Gross income: \$75,000

Deductions: \$18,000: mortgage interest

> \$2500: property taxes \$2000: charitable donations \$300: cost of tax preparation

Tax credit:

Answer to 9: | \$6870

Deductions: 18,000 + 2500 + 200 + 300 = 21,000 (greater than standard) Taxable income: 75,000 - 21,000 = 54,000

Initial tax: (9875)(0.10) + (40,125 - 9875)(0.12) + (54,000 - 40,125)(0.22) = 7670Apply credit: 7670 - 800 = 6870

10

Taxpayer: Single Gross income: \$40,000

Deductions: 10,000: mortgage interest

\$2000: property taxes \$300: charitable donations

Tax credit: \$1300

Answer to 10: \$1814.50

Deductions: 10,000 + 2000 + 300 = 12,300 (lower than standard; use standard) Taxable income: 40,000 - 12,400 = 27,600Initial tax: (9875)(0.10) + (27,600 - 9875)(0.12) = 3114.50

Apply credit: 3114.50 - 1300 = 1814.50

(11)

Taxpayer: Married, filing jointly

Gross income: \$85,500

Deductions: \$5000: charitable donations

\$3750: state taxes

Tax credit: \$750

Answer to 11: | \$6139

Deductions: 5000 + 3750 = 8750 (lower than standard; use standard) Taxable income: 85,500 - 24,800 = 60,700Initial tax: (19,750)(0.10) + (60,700 - 19,750)(0.12) = 6889Apply credit: 6889 - 750 = 6139

Taxpayer: Married, filing jointly

Gross income: \$52,000

Deductions: \$9000: mortgage interest \$4500: charitable donations

\$1500: theft loss \$1800: state taxes

Tax credit: \$1400

Answer to 12: \$1469

Deductions: 9000 + 4500 + 1500 + 1800 = 16,800 (lower than standard; use standard) Taxable income: 52,000 - 24,800 = 27,200Initial tax: (19,750)(0.10) + (27,200 - 19,750)(0.12) = 2869Apply credit: 2869 - 1400 = 1469

Head of Household Taxpayer:

Gross income: \$104,000

Deductions: \$18,000: mortgage interest

\$5300: property taxes \$4800: state taxes

Tax credit: none

Answer to 13: | \$14,032.50

Deductions: 18,000+5300+4800=28,100 (greater than standard) Taxable income: 104,000-28,100=75,900

Initial tax:

(14,100)(0.10) + (53,700 - 14,100)(0.12) + (75,900 - 40,125)(0.22) = 14,032.50

Apply credit: N/A

(14)

Head of Household Taxpayer:

Gross income: \$43,000

Deductions: \$3700: property taxes

\$3650: state taxes

Tax credit: none

Answer to 14: \$2640

Deductions: 3700 + 3650 = 7350 (lower than standard; use standard)

Taxable income: 43,000 - 18,650 = 24,350

Initial tax: (14,100)(0.10) + (24,350 - 14,100)(0.12) = 2640

Apply credit: N/A

2

Growth Models

SECTION 2.1 Linear Models

- (1) Marko currently has 20 tulips in his yard. Each year he plants 5 more.
 - (a) Find a linear model of the form $P_t = P_0 + dt$ to describe the number of tulips Marko has at a given point in time.

Answer to 1(a): $P_t = 20 + 5t$

(b) How many tulips will Marko have in 7 years?

Answer to 1(b): 55 tulips

 $P_t = 20 + 5(7) = 55$

(c) When will Marko have 65 tulips?

Answer to 1(c): In 9 years

 $65 = 20 + 5t \longrightarrow 45 = 5t \longrightarrow t = 9$

- 2 Pam is a DJ. Every week she buys 3 new albums to add to her collection. She currently owns 450 albums.
 - (a) Find a linear model of the form $P_t = P_0 + dt$ to describe the number of albums Pam has at a given point in time.

Answer to 2(a): $P_t = 450 + 3t$

(b) How many albums will Pam have in 11 weeks?

Answer to 2(b): 483 albums

 $P_t = 450 + 3(11) = 483$

(c) When will Pam have 489 albums?

Answer to 2(c): In 13 weeks

 $489 = 450 + 3t \longrightarrow 39 = 3t \longrightarrow t = 13$

- (3) A store did \$40,000 in sales in 2016, and \$62,000 in 2018.
 - (a) Assuming the store's sales are growing linearly, find the growth rate d.

Answer to 3(a): d = 11,000

 $62,000 = 40,000 + d(2) \longrightarrow 22,000 = 2d \longrightarrow d = 11,000$

(b) Write a linear model to describe this store's sales from 2016 onward.

Answer to 3(b): $P_t = 40,000 + 11,000t$

(c) Predict the store's sales in 2025.

Answer to 3(c): \$139,000

 $P_t = 40,000 + 11,000(9) = 139,000$

(d) When do you expect the store's sales to exceed \$100,000?

Answer to 3(d): t = 5.45; about halfway through 2021

 $100,000 = 40,000 + 11,000t \longrightarrow 60,000 = 11,000t \longrightarrow t = 5.45$

- (4) A small town had 340 homes in 2010, and by 2020, this had grown to 375.
 - (a) Assuming the number of homes is growing linearly, find the growth rate d.

Answer to 4(a): d = 3.5

 $375 = 340 + d(10) \longrightarrow 35 = 10d \longrightarrow d = 3.5$

(b) Write a linear model to describe the number of homes in this town from 2010 onward.

Answer to 4(b): $P_t = 340 + 3.5t$

(c) Predict how many homes there will be in this town in 2034.

Answer to 4(c): 424 homes

$$P_t = 340 + 3.5(24) = 424$$

(d) When do you expect the number of homes to exceed 500?

Answer to 4(d): t = 45.7; mid to late 2055

$$500 = 340 + 3.5t \longrightarrow 160 = 3.5t \longrightarrow t = 45.7$$

- (5) A population of beetles is growing according to a linear growth model. Initially, there were 13 beetles, and 8 weeks later, there were 42 beetles.
 - (a) Write a linear model to describe the number of beetles over time, using weeks as the unit of time.

Answer to 5(a): $P_t = 13 + 3.625t$

$$42 = 13 + 8d \longrightarrow d = 3.625$$

(b) How many beetles are there expected to be 14 weeks after the initial point?

Answer to 5(b): approximately 64 beetles

$$P_t = 13 + 3.625(14) = 63.75$$

(c) When do you expect the number of beetles to exceed 70?

Answer to 5(c): t = 15.7

$$70 = 13 + 3.625t \longrightarrow t = 15.7$$

- 6 The number of streetlights in a town is growing linearly. Four months ago there were 130 lights. Now there are 146 lights.
 - (a) Write a linear model to describe the number of streetlights in the town over time, using months as the unit of time.

Answer to 6(a): $P_t = 130 + 4t$

$$146 = 130 + 4d \longrightarrow d = 4$$

(b) How many streetlights are expected a year from now (not a year from the beginning)?

Answer to 6(b): 194 streetlights

$$P_t = 130 + 4(16) = 194$$

(c) When do you expect the number of streetlights to exceed 200?

Answer to 6(c): |t| = 17.5

$$200 = 130 + 4t \longrightarrow t = 17.5$$

- $\overbrace{7}$ In 1990, there were 112 nuclear power plants in the U.S. By 2019, this number had fallen to 96
 - (a) Write a linear model to describe the number of nuclear power plants from 1990 onward.

Answer to 7(a): $P_t = 112 - 0.55t$

$$96=112+29d \longrightarrow d=-0.55$$

(b) Using this linear model, predict the number of nuclear power plants in the U.S. in 2030.

Answer to 7(b): 90 power plants

$$P_t = 112 - 0.55(40) = 90$$

(c) When do you expect the number of nuclear power plants to reach 80?

Answer to 7(c): t = 58.2; early 2048

$$80 = 112 - 0.55t \longrightarrow t = 58.2$$

- (8) In 1990, approximately 1,820,000 violent crimes were reported in the U.S. By 2019, this number had fallen to approximately 1,200,000.
 - (a) Write a linear model to describe the number of violent crimes in the U.S. from 1990 onward.

Answer to 8(a):
$$P_t = 1,820,000 - 21,379t$$

$$1,200,000 = 1,820,000 + 29d \longrightarrow d = -21,379$$

(b) Using this linear model, predict the number of violent crimes in 2040.

$$P_t = 1,820,000 - 21,379(50) = 751,050$$

(c) When do you expect the number of violent crimes to reach 1,000,000?

Answer to 8(c):
$$t = 38.4$$
; about halfway through 2028

$$1,000,000 = 1,820,000 - 21,379t \longrightarrow t = 38.4$$

9 The table below shows the average annual cost of health insurance for a single individual, from 1999 to 2019, according to the Kaiser Family Foundation.

Year	\mathbf{Cost}
1999	\$2,196
2000	\$2,471
2001	\$2,689
2002	\$3,083
2003	\$3,383
2004	\$3,695
2005	\$4,024
2006	\$4,242
2007	\$4,479
2008	\$4,704
2009	\$4,824
2010	\$5,049
2011	\$5,429
2012	\$5,615
2013	\$5,884
2014	\$6,025
2015	\$6,251
2016	\$6,435
2017	\$6,690
2018	\$6,896
2019	\$7,186

(a) Using only the data from the first and last years, build a linear model to describe the cost of individual health insurance from 1999 onward.

Answer to 9(a):
$$P_t = 2196 + 249.5t$$

$$7186 = 2196 + 20d \longrightarrow d = 249.5$$

(b) Using this linear model, predict the cost of insurance in 2030.

$$P_t = 2196 + 249.5(31) = 9930.5$$

(c) According to this model, when do you expect the cost of individual insurance to reach \$12,000?

Answer to 9(c):
$$t = 39.3$$
; early 2038

$$12,000 = 2196 + 249.5t \longrightarrow t = 39.3$$

(d) Using a calculator or spreadsheet program, build a linear regression model to describe the cost of individual insurance from 1999 onward.

Answer to 9(d):
$$P_t = 2381.7 + 243.97t$$

(e) Using the regression model, predict the cost of insurance in 2030.

$$P_t = 2381.7 + 243.97(31) = 9944.77$$

(f) According to the regression model, when do you expect the cost of individual insurance to reach \$12,000?

Answer to 9(f):
$$t = 39.4$$
; early to mid 2038

$$12,000 = 2381.7 + 243.97t \longrightarrow t = 39.4$$

(10) The table below shows the average annual cost of health insurance for a family, from 1999 to 2019, according to the Kaiser Family Foundation.

Year	\mathbf{Cost}	
1999	\$5,791	
2000	\$6,438	
2001	\$7,061	
2002	\$8,003	
2003	\$9,068	
2004	\$9,950	
2005	\$10,880	
2006	\$11,480	
2007	\$12,106	
2008	\$12,680	
2009	\$13,375	
2010	\$13,770	
2011	\$15,073	
2012	\$15,745	
2013	\$16,351	
2014	\$16,834	
2015	\$17,545	
2016	\$18,142	
2017	\$18,764	
2018	\$19,616	
2019	\$20,576	

(a) Using only the data from the first and last years, build a linear model to describe the cost of family health insurance from 1999 onward.

Answer to 10(a):
$$P_t = 5{,}791 + 739.25t$$

$$20,576 = 5,791 + 20d \longrightarrow d = 739.25$$

(b) Using this linear model, predict the cost of insurance in 2025.

$$P_t = 5,791 + 739.25(26) = 25,011.5$$

(c) According to this model, when do you expect the cost of family insurance to reach \$30,000?

Answer to 10(c):
$$t = 32.7$$
; mid to late 2031

$$30,000 = 5,791 + 739.25t \longrightarrow t = 32.7$$

(d) Using a calculator or spreadsheet program, build a linear regression model to describe the cost of family insurance from 1999 onward.

Answer to 10(d):
$$P_t = 6050.3 + 724.72t$$

(e) Using the regression model, predict the cost of insurance in 2025.

$$P_t = 6050.3 + 724.72(26) = 24,893.02$$

(f) According to the regression model, when do you expect the cost of family insurance to reach \$30,000?

Answer to 10(f):
$$t = 33.0$$
; beginning of 2032

$$30,000 = 6050.3 + 724.72t \longrightarrow t = 33.0$$

SECTION 2.2 Quadratic Models

1 The table below shows the distance that a baseball travels after being hit at various angles.

Angle (degrees)	Distance (feet)
10	115.6
15	157.2
20	189.2
24	220.8
30	253.8
34	269.2
40	284.8
45	285.0
48	277.4
50	269.2
58	244.2
60	231.4
64	180.4
0.1	100.1

(a) Use a graphing calculator or spreadsheet program to find a quadratic model that best fits this data, using angle as x and distance as y.

Answer to 1(a):
$$y = -0.174x^2 + 14.521x - 21.898$$

(b) Based on this model, what distance is expected for a ball hit at 55°?

$$y = -0.174(55)^2 + 14.521(55) - 21.898 = 250.41$$

(c) What distance is expected for a ball hit at 75°?

$$y = -0.174(75)^2 + 14.521(75) - 21.898 = 88.43$$

(d) Which of the two previous predictions is likely to be more reliable?

(e) What angle above 45° would you expect to yield a distance of 200 feet?

Answer to
$$1(e)$$
: 63.3°

Used calculator intersect function

(2) A ball is dropped from a height of a little over 5 feet, and the height is measured at small intervals. The table below shows the results.

Time (seconds)	Height (feet)
0.00	5.235
0.04	5.160
0.08	5.027
0.12	4.851
0.16	4.631
0.20	4.357
0.24	4.030
0.28	3.655
0.32	3.234
0.36	2.769
0.40	2.258
0.44	1.635

(a) Use a graphing calculator or spreadsheet program to find a quadratic model that best fits this data, using time as x and height as y.

Answer to 2(a):
$$y = -15.64x^2 - 1.24x + 5.23$$

(b) Based on this model, what height is expected after 0.30 seconds?

Answer to 2(b): 3.450 feet

$$y = -15.64(0.3)^2 - 1.24(0.3) + 5.23 = 3.450$$

(c) What height is expected after 0.52 seconds?

Answer to 2(c): 0.356 feet

$$y = -15.64(0.52)^2 - 1.24(0.52) + 5.23 = 0.356$$

(d) Which of the two previous predictions is likely to be more reliable?

Answer to 2(d):

The one for 0.3 seconds; interpolation instead of extrapolation

(e) When do you expect the height of the ball to be 1 foot?

Answer to 2(e): 0.48 seconds

Used calculator intersect function

(3) The table below shows the amount spent on movie theater tickets in the U.S. from 1997 to 2003.

Year	Spending (billions of dollars)
1997	6.3
1998	6.9
1999	7.9
2000	8.6
2001	9.0
2002	9.6
2003	9.9

(a) Use a graphing calculator or spreadsheet program to find a quadratic model that best fits the data. Let t represent the year, with t = 0 in 1997.

Answer to 3(a): $P_t = -0.049t^2 + 0.911t + 6.217$

(b) Based on this model, how much would you expect to be spent on movie theater tickets in 2008?

Answer to 3(b): \$10.3 billion

$$P_t = -0.049(11)^2 + 0.911(11) + 6.217 = 10.3$$

(c) When would you expect movie theater ticket expenditure to fall to \$5 billion?

Answer to 3(c): t = 19.8; late 2016

Used calculator intersect function

(4) The table below shows the number of FM radio stations in the U.S. from 1997 to 2003.

Year	Stations
1997	5542
1998	5662
1999	5766
2000	5892
2001	6051
2002	6161
2003	6207

(a) Use a graphing calculator or spreadsheet program to find a quadratic model that best fits the data. Let t represent the year, with t=0 in 1997.

Answer to 4(a): $P_t = -3.26t^2 + 136.64t + 5529.76$

(b) Based on this model, how many FM stations would you expect there to be in 2010?

Answer to 4(b): 6755 stations

$$P_t = -3.26(13)^2 + 136.64(13) + 5529.76 = 6755.1$$

(c) When would you expect the number of stations to first reach 6500?

Answer to 4(c):
$$t = 9.1$$
; early 2006

Used calculator intersect function

(5) The table below shows college textbook sales in the U.S. from 2000 to 2005.

Year	Textbook Sales (millions of dollars)
2000	4265
2000	4203 4571
2002	4899
2003	5086
2004	5479
2005	5703

(a) Use a graphing calculator or spreadsheet program to find a quadratic model that best fits the data. Let t represent the year, with t = 0 in 2000.

Answer to 5(a):
$$P_t = -2.68t^2 + 301.99t + 4270.07$$

(b) Based on this model, how much would you expect to be spent on college textbooks in 2015?

Answer to 5(b):
$$$8197$$$
 million, or just over $$8$$ billion

$$P_t = -2.68(15)^2 + 301.99(15) + 4270.07 = 8196.9$$

(c) When would you expect textbook sales to first reach \$7 billion (\$7000 million)?

Answer to 5(c):
$$t = 9.9$$
; late 2009

Used calculator intersect function

(6) The table below shows the average amount of time spent per person on entertainment per year from 2000 to 2005.

Year	Hours
2000	3492
2001	3540
2002	3606
2003	3663
2004	3757
2005	3809

(a) Use a graphing calculator or spreadsheet program to find a quadratic model that best fits the data. Let t represent the year, with t=0 in 2000.

Answer to 6(a):
$$P_t = 2.36t^2 + 53.73t + 3488.57$$

(b) Based on this model, how many hours would you expect the average person to spend on entertainment in 2012?

$$P_t = 2.36(12)^2 + 53.73(12) + 3488.57 = 4473.2$$

(c) When would you expect the average amount of time to reach 4000?

Answer to 6(c):
$$t = 7.2$$
; early 2007

Used calculator intersect function

SECTION 2.3 Exponential Models

In problems 1-4, use a calculator to solve for the unknown variable, x or t.

 $(1) 10^x = 5$

Answer to 1: x = 0.699

$$x = \frac{\ln 5}{\ln 10} = 0.699$$

 $(2) 7^t = 100$

Answer to 2: t = 2.367

$$t = \frac{\ln 100}{\ln 7} = 2.367$$

(3) $4(10^x) = 9$

Answer to 3: x = 0.352

$$10^x = \frac{9}{4} = 2.25 \longrightarrow x = \frac{\ln 2.25}{\ln 10} = 0.352$$

 $\boxed{\mathbf{4}} \ 22(1.065)^{0.05t} = 37$

Answer to 4: t = 7.751

$$(1.065)^{0.05t} = \frac{37}{22} \longrightarrow 0.05t = \frac{\ln(37/22)}{\ln 1.065} = 8.2553 \longrightarrow t = \frac{8.2553}{1.065} = 7.751$$

- (5) The population of the District of Columbia was approximately 572,000 in 2000, and has been growing at a rate of about 1.15%.
 - (a) Write an exponential model of the form $P_t = P_0(1+r)^t$ to describe the population of DC from 2000 onward.

Answer to 5(a):
$$P_t = 572,000(1+0.0115)^t$$

(b) If this trend continues, what will DC's population be in 2025?

$$P_t = 572,000(1.0115)^{25} = 761,278$$

(c) When does this model predict that DC's population will reach 800,000?

Answer to 5(c):
$$t = 29.3$$
; early 2029

$$800,000 = 572,000(1.0115)^t \longrightarrow 1.3986 = 1.0115^t \longrightarrow t = 29.3$$

- (6) Baltimore's population in 2010 was approximately 620,000, and has been decreasing at a rate of about 0.5% per year.
 - (a) Write an exponential model of the form $P_t = P_0(1+r)^t$ to describe the population of Baltimore from 2010 onward.

Answer to 6(a):
$$P_t = 620,000(1-0.005)^t$$

(b) If this trend continues, what will Baltimore's population be in 2030?

$$P_t = 620,000(0.995)^{20} = 560,858$$

(c) When does this model predict that Baltimore's population will reach 500,000?

Answer to 6(c):
$$t = 42.9$$
; end of 2052

$$500,000 = 620,000(0.995)^t \longrightarrow 0.8065 = 0.995^t \longrightarrow t = 42.9$$

7 Diseases tend to spread exponentially. In the early days of AIDS, the growth rate was around 190%. In 1983, about 1700 people in the US died of AIDS. If the trend had continued unchecked, how many people would have died from AIDS in 1990?

Answer to 7: 2,932,479

$$P_t = 1700(1+1.9)^7 = 2,932,479$$

(8) The population of the world in 1987 was 5 billion and the annual growth rate was estimated at 2 percent. If the world population followed an exponential growth model, find the projected world population in 2015.

Answer to 8: 8,705,121,031

$$P_t = 5,000,000,000(1+0.02)^{28} = 8,705,121,031$$

- (9) The population of Maryland was 5.17 million in 1999, and it grew to 6.05 million in 2019.
 - (a) Assuming that the population is growing exponentially, find the growth rate r for Maryland's population.

Answer to 9(a): r = 0.8%

$$6,050,000 = 5,170,000(1+r)^{20} \longrightarrow 1.17 = (1+r)^{20} \longrightarrow r = 0.0079$$

(b) Write an exponential model to describe the population of Maryland from 1999 onward.

Answer to 9(b): $P_t = 5,170,000(1+0.008)^t$

(c) What is Maryland's population expected to be in 2030?

Answer to 9(c): 6,618,614

$$P_t = 5,170,000(1.008)^{31} = 6,618,614$$

(d) When do you expect that Maryland's population will reach 8 million?

Answer to 9(d): t = 54.8; late 2053

$$8,000,000 = 5,170,000(1.008)^t \longrightarrow 1.547 = 1.008^t \longrightarrow t = 54.8$$

- (10) The population of Virginia was 6.87 million in 1999, and it grew to 8.54 million in 2019.
 - (a) Assuming that the population is growing exponentially, find the growth rate r for Virginia's population.

Answer to 10(a): r = 1.1%

$$8,540,000 = 6,870,000(1+r)^{20} \longrightarrow 1.243 = (1+r)^{20} \longrightarrow r = 0.011$$

(b) Write an exponential model to describe the population of Virginia from 1999 onward.

Answer to 10(b): $P_t = 6,870,000(1+0.011)^t$

(c) What is Virginia's population expected to be in 2022?

Answer to 10(c): 8,835,544

$$P_t = 6,870,000(1.011)^{23} = 8,835,544$$

(d) When do you expect that Virginia's population will reach 10 million?

Answer to 10(d): t = 34.3; early 2033

$$10,000,000 = 6,870,000(1.011)^t \longrightarrow 1.4556 = 1.011^t \longrightarrow t = 34.3$$

- (11) A bacteria culture is started with 300 bacteria. After 4 hours, the population has grown to 500 bacteria. If the population grows exponentially according to the formula $P_t = P_0(1+r)^t$,
 - (a) Find the growth rate r and write the full formula.

Answer to 11(a):
$$r = 13.6\%; P_t = 300(1.136)^t$$

$$500 = 300(1+r)^4 \longrightarrow 1.667 = (1+r)^4 \longrightarrow r = 0.136$$

(b) If this trend continues, how many bacteria will there be in one day?

Answer to 11(b): 6400 bacteria

$$P_t = 300(1.136)^{24} = 6400$$

(c) How long will it take for this culture to triple in size?

Answer to 11(c): 8.6 hours

$$900 = 300(1.136)^t \longrightarrow 3 = 1.136^t \longrightarrow t = 8.6$$

- A native wolf species has been reintroduced into a national forest. Originally 200 wolves were transplanted, and after 3 years, the population had grown to 270 wolves. If the population grows exponentially according to the formula $P_t = P_0(1+r)^t$,
 - (a) Find the growth rate r and write the full formula.

Answer to 12(a):
$$r = 10.5\%; P_t = 200(1.105)^t$$

$$270 = 200(1+r)^3 \longrightarrow 1.35 = (1+r)^3 \longrightarrow r = 0.105$$

(b) If this trend continues, how many wolves will there be in ten years?

Answer to 12(b): 543 wolves

$$P_t = 200(1.105)^{10} = 543$$

(c) If this trend continues, how long will it take the wolf population to double?

Answer to 12(c): | 6.9 years

$$400 = 200(1.105)^t \longrightarrow 2 = 1.105^t \longrightarrow t = 6.9$$

(13) In 2009, the average compensation for CEOs in the U.S. was approximately \$10,800,000, and by 2016, this had risen to about \$12,800,000. By comparison, the average compensation for workers was \$54,700 in 2009 and \$55,800 in 2016. Assume that both values are growing according to an exponential model. Find the growth rate for both salaries; which is higher?

Answer to 13: The CEO growth rate is higher, at 2.5%, versus 0.3% for workers

CEOs:

$$12,800,000 = 10,800,000(1+r)^7 \longrightarrow 1.185 = (1+r)^7 \longrightarrow r = 0.025$$

Workers:

$$55,800 = 54,700(1+r)^7 \longrightarrow 1.02 = (1+r)^7 \longrightarrow r = 0.003$$

(14) In 2008, approximately 131 million people voted in the U.S. general election, compared to about 139 million people in 2016. The total population of the U.S. was 304 million in 2008 and 323 million in 2016. Assume that both levels are growing exponentially. Find the growth rate for both populations; which is higher?

Answer to 14: Population growth rate is higher, at 0.76%, versus 0.74% for voters

Voters:

$$139,000,000 = 131,000,000(1+r)^8 \longrightarrow 1.061 = (1+r)^8 \longrightarrow r = 0.0074$$

Population:

$$323,000,000 = 304,000,000(1+r)^8 \longrightarrow 1.0625 = (1+r)^8 \longrightarrow r = 0.0076$$

(15) The table below shows the population of Canada from 2010 to 2019.

Year	Population (millions)
2010	34.0
2011	33.5
2012	34.7
2013	35.1
2014	35.4
2015	35.7
2016	35.1
2017	36.5
2018	37.1
2019	37.6

(a) Use a graphing calculator or spreadsheet program to build an exponential regression model, letting t=0 in 2010.

Answer to 15(a):
$$P_t = 33.675(1.011)^t$$

(b) What does this model predict that the population of Canada will be in 2035?

$$P_t = 33.675(1.011)^{25} = 44.3$$

(c) When does this model predict that Canada's population will reach 40 million?

Answer to 15(c):
$$t = 15.7$$
; mid to late 2025

$$40 = 33.675(1.011)^t \longrightarrow 1.188 = 1.011^t \longrightarrow t = 15.7$$

(16) The table below shows the population of Mexico from 2010 to 2019.

Year	Population (millions)
2010	114.1
2011	115.7
2012	117.3
2013	118.8
2014	120.4
2015	121.9
2016	123.3
2017	124.8
2018	126.2
2019	127.6

(a) Use a graphing calculator or spreadsheet program to build an exponential regression model, letting t=0 in 2010.

Answer to 16(a):
$$P_t = 114.363(1.012)^t$$

(b) What does this model predict that the population of Mexico will be in 2040?

$$P_t = 114.363(1.012)^{30} = 163.6$$

(c) When does this model predict that Mexico's population will reach 145 million?

Answer to 16(c):
$$t = 19.9$$
; end of 2029

$$145 = 114.363(1.012)^t \longrightarrow 1.268 = 1.012^t \longrightarrow t = 19.9$$

SECTION 2.4 Logistic Models

(1) One hundred trout are seeded into a lake. Absent constraint, their population will grow by 70% a year. If the lake can sustain a maximum of 2000 trout, use a logistic growth model to estimate the number of trout after 2 years.

Answer to 1: 352 trout

$$P_t = \frac{2000}{1 + \left(\frac{2000}{100} - 1\right)e^{-(0.7)(2)}} = 352$$

(2) Ten blackberry plants started growing in a yard. Absent constraint, blackberries will spread by 200% a month. If the yard can only sustain 50 plants, use a logistic growth model to estimate the number of plants after 2 months.

Answer to 2: 47 plants

$$P_t = \frac{50}{1 + \left(\frac{50}{10} - 1\right)e^{-(2)(2)}} = 47$$

(3) A certain community consists of 1000 people, and one individual has a particularly contagious strain of influenza. Assuming the community has not had vaccination shots and are all susceptible, the spread of the disease in the community is modeled by

$$P_t = \frac{1000}{1 + 999e^{-0.3t}}$$

where P_t is the number of people who have contracted the flu after t days.

(a) How many people have contracted the flu after 10 days? Round your answer to the nearest whole number.

Answer to 3(a): 20 people

$$P_t = \frac{1000}{1 + 999e^{-0.3(10)}} = 20$$

(b) What is the carrying capacity for this model? Does this make sense?

Answer to 3(b): 1000 people (the size of the community)

(c) How many days will it take for 750 people to contract the flu? Round your answer to the nearest whole number.

Answer to 3(c): 27 days

$$750 = \frac{1000}{1 + 999e^{-0.3t}} \longrightarrow t = 26.68 \text{ (used calculator)}$$

(4) A herd of 20 white-tailed deer is introduced to a coastal island where there had been no deer before. Their population is predicted to increase according to

$$P_t = \frac{100}{1 + 4e^{-0.14t}}$$

where P_t is the number of deer expected in the herd after t years.

(a) How many deer will be present after 2 years? Round your answer to the nearest whole number.

Answer to 4(a): 25 deer

$$P_t = \frac{100}{1 + 4e^{-0.14(2)}} = 24.9$$

(b) What is the carrying capacity for this model?

Answer to 4(b): 100 deer

(c) How many years will it take for the herd to grow to 50 deer? Round your answer to the nearest whole number.

$$50 = \frac{100}{1 + 4e^{-0.14t}} \longrightarrow t = 9.9 \text{ (used calculator)}$$

(5) The table below shows the population of California from 2010 to 2019.

Year	Population (millions)
2010	37.3
2011	37.6
2012	38.0
2013	38.3
2014	38.6
2015	38.9
2016	39.2
2017	39.4
2018	39.5
2019	39.5

(a) Use a graphing calculator to build a logistic regression model, letting t=0 in 2010.

Answer to 5(a):
$$P_t = \frac{40.591}{1 + 0.090e^{-0.145t}}$$

(b) What does this model predict that the population of California will be in 2025?

$$P_t = \frac{40.591}{1 + 0.090e^{-0.145(15)}} = 40.2$$

(c) When does this model predict that California's population will reach 40 million?

Answer to 5(c):
$$t = 12.5$$
; midway through 2022

$$40 = \frac{40.591}{1 + 0.090e^{-0.145t}} \longrightarrow t = 12.5 \text{ (used calculator)}$$

(d) According to this model, what is the carrying capacity for California's population?

(6) The table below shows the population of Florida from 2010 to 2019.

Year	Population (millions)
2010	18.7
2011	19.1
2012	19.3
2013	19.6
2014	19.9
2015	20.2
2016	20.6
2017	21.0
2018	21.2
2019	21.5

(a) Use a graphing calculator to build a logistic regression model, letting t = 0 in 2010.

Answer to 6(a):
$$P_t = \frac{93.286}{1 + 3.983e^{-0.020t}}$$

(b) What does this model predict that the population of Florida will be in 2030?

$$P_t = \frac{93.286}{1 + 3.983e^{-0.020(20)}} = 25.4$$

(c) When does this model predict that Florida's population will reach 23 million?

Answer to 6(c):
$$t = 13.2$$
; early 2023

$$23 = \frac{93.286}{1 + 3.983e^{-0.020t}} \longrightarrow t = 13.2 \text{ (used calculator)}$$

(d) According to this model, what is the carrying capacity for Florida's population?

Answer to 6(d): about 93.3 million

Statistics

SECTION 3.1 Gathering Data

For proviems 1-2, identify the population and sample.
1 A political scientist surveys 28 of the current 106 representatives in a state's congress. Of them, 14 said they were supporting a new education bill, 12 said they were not supporting the bill, and 2 were undecided.
Answer to 1: Population: 106 representatives; sample: 28 surveyed
2 The city of Frederick has 9500 registered voters. There are two candidates for the city council in an upcoming election: Marfani and Rahman. The day before the election, a telephone poll of 350 randomly selected registered voters was conducted. Of them, 112 said they would vote for Marfani, 207 said they would vote for Rahman, and 31 were undecided. Answer to 2: Population: 9500 registered voters; sample: 350 randomly selected
For problems 3-4, decide whether the sampling method described is likely to produce a representative sample, and why or why not.
A marriage counselor is interested in the studying divorce rates, so she gives her clients a survey.
Answer to 3: No; clients of a marriage counselor are more likely to get divorced
4 A fitness center wants to see how much use their treadmills get, so they pick random times during the day and record how many treadmills are in use each time. Answer to 4: Yes; no obvious bias
For problems 5–10, determine the type of sampling used in the given scenario.
(5) A high school principal polls 50 freshmen, 50 sophomores, 50 juniors, and 50 seniors regarding policy changes for after school activities.
Answer to 5: Stratified
6 To check their accuracy, the Census Bureau draws a sample of several city blocks and recounts everyone in those blocks.
Answer to 6: Cluster
(7) A pollster walks around a busy shopping mall and asks people passing by how often they shop at the mall.
Answer to 7: Convenience
(8) Police at a DUI checkpoint stop every tenth car to check whether the driver is sober.

(10) The provost at a university wants to know how a particular policy is affecting faculty, so she randomly selects 3 members of each department to survey.

(9) A restaurant samples 100 sales from the past week by numbering all their receipts, generating 100 random numbers, and picking the receipts that correspond to those numbers.

Answer to 10: Stratified

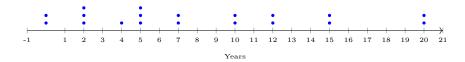
Answer to 8: Systematic

Answer to 9: | Simple random

SECTION 3.2 Visualizing Data

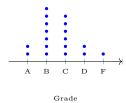
1 Nineteen immigrants to the U.S. were asked how many years, to the nearest year, they have lived in the U.S. The data are as follows:

Draw a dot plot to summarize this data.



(2) A group of students earned the following final grades:

Draw a dot plot to summarize this data.



(3) A store tracked how many iPads were sold each day for fifty days, and their data is below.

Construct a frequency table (including a relative frequency column) to describe this data.

iPads Sold	Frequency	Relative Frequency
0	5	0.10
1	8	0.16
2	14	0.28
3	13	0.26
4	6	0.12
5	4	0.08

4 Twenty students were asked how many hours they worked per day. Their responses are as follows:

6 3 3 5 2 7 4 5 2 5 3 5 6 4 4 3 3

Construct a frequency table (including a relative frequency column) to describe this data.

Hours	Frequency	Relative Frequency
2	3	0.15
3	5	$0.15 \\ 0.25$
4	3	0.15
5	6	0.30
6	2	0.10
7	1	0.05

(5) Fifty part-time students were asked how many courses they were taking this semester. The (incomplete) results are shown below. Fill in the blank cells to complete the table.

Number of Courses	Frequency	Relative Frequency
1	30	0.6
$\overline{2}$	15	0.3
3	5	0.1

(6) A group of 20 students were polled and asked what year they belonged to, whether they were freshmen (FR), sophomores (SO), juniors (JR), or seniors (SR). The results are written below.

FR $_{\rm JR}$ SOJRSRFRSOSO SOSOSRSR SR FRSRSOSRSOJRJR

Construct a frequency table (including a relative frequency column) to describe this data.

Year	Frequency	Relative Frequency
$_{ m FR}$	3	0.15
SO	7	0.35
$_{ m JR}$	4	0.20
SR	6	0.30

(7) A group of 20 registered voters were polled and asked what party they belonged to, whether they were Republicans (R), Democrats (D), Green Party members (G), or independent (I). The results are written below.

GD \mathbf{R} \mathbf{D} Ι R \mathbf{R} D Ι D Ι R \mathbf{R} D R D

Construct a frequency table (including a relative frequency column) to describe this data.

Party	Frequency	Relative Frequency
R.	Q	0.40
n D	8	0.40
$\overline{\mathrm{G}}$	1	0.05
T	3	0.15

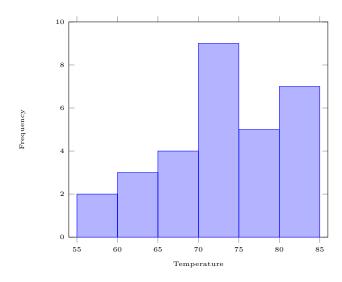
The following is the average daily temperature for Frederick, Maryland for the month of June:

> 74, 60, 58, 58, 64, 67, 64, 74, 72, 70, 78, 80, 80, 79, 80, 80, 70, 83, 76, 78,81, 78, 81, 70, 70, 71, 66, 66, 68, 74.

(a) Construct a grouped frequency and relative frequency distribution using a class width of 5, starting at 55.

Temperature	Frequency	Relative Frequency
55–59	2	0.07
60-64	$\frac{2}{3}$	0.10
65 – 69	4	0.13
70 – 74	9	0.30
75 - 79	5	0.17
80-84	7	0.23
75-79		0.17

(b) Construct a histogram from the frequency distribution.



(9) A researcher gathered data on hours of video games played by school-aged children and young adults. She collected the following data:

> 0, 0, 1, 1, 1, 2, 2, 3, 3, 3,4, 4, 4, 4, 5, 5, 5, 6, 6, 7,

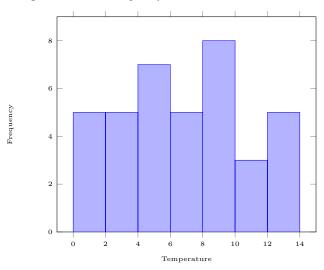
7, 7, 8, 8, 8, 8, 8, 9, 9, 9,

10, 10, 11, 12, 12, 12, 12, 13.

(a) Construct a grouped frequency and relative frequency distribution using a class width of 2, starting at 0.

Temperature	Frequency	Relative Frequency
0-1	5	0.13
2-3	5	0.13
4-5	7	0.18
6-7	5	0.13
8–9	8	0.21
10-11	3	0.08
12-13	5	0.13

(b) Construct a histogram from the frequency distribution.



For exercises 10–13, use the frequency table below, which contains the total number of deaths worldwide as a result of earthquakes for the period from 2000 to 2012.

Year	Total Number of Deaths
2000	231
2001	21,357
2002	11,685
2003	33,819
2004	228,802
2005	88,003
2006	6,605
2007	712
2008	88,011
2009	1,790
2010	320,120
2011	21,953
2012	768
Total	823,356

10 What is the frequency of deaths measured from 2006 through 2009?

$$6,605 + 712 + 88,011 + 1,790 = 97,118$$

11) What percentage of deaths occurred after 2009 (from 2010 onwards)?

Answer to 11: 41.6%

$$320, 120 + 21, 953 + 768 = 342, 841 \longrightarrow \frac{342, 841}{823, 356} = 0.416$$

(12) What is the relative frequency of deaths that occurred in 2003 or earlier?

Answer to 12: 0.081 or 8.1%

$$231 + 21,357 + 11,685 + 33,819 = 67,092 \longrightarrow \frac{67,092}{823,356} = 0.081$$

(13) What is the percentage of deaths that occurred in 2004?

Answer to 13: 27.8%

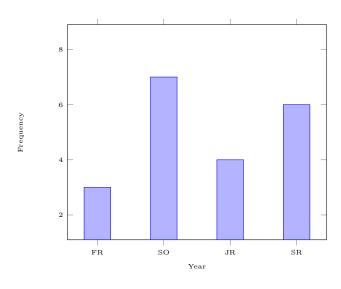
$$\frac{228,802}{823,356} = 0.278$$

Grades	Frequency
50–55	2
55–60	4
60-70	9
70-80	15
80-90	7
90 and above	4

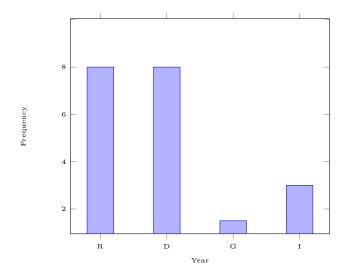
- (a) The classes do not all have the same width.
- (b) The classes overlap.
- (c) There are open-ended classes.
- (d) All of the above.

Answer to 14: (d) All of the above

(15) Draw a bar chart for the dataset in problem 6.



(16) Draw a bar chart for the dataset in problem 7.



(17) The scores for a math test are shown below, ordered from smallest to largest.

Build a stem-and-leaf plot for this data.

Stems	Leaves
4	2 9 9
5	3 5 5
6	1378899
7	2 3 4 8
8	0 3 8 8 8
9	$0\ 2\ 4\ 4\ 4\ 5\ 6$
10	0

(18) A basketball team's scores for the last 30 games are shown below, ordered from smallest to largest.

Build a stem-and-leaf plot for this data.

	Leaves
3	2 2 3 4 8
4	0 2 2 3 4 6 7 7 8 8 8 9
5	$\begin{array}{c} 2\ 2\ 3\ 4\ 8 \\ 0\ 2\ 2\ 3\ 4\ 6\ 7\ 7\ 8\ 8\ 8\ 9 \\ 0\ 0\ 1\ 2\ 2\ 2\ 3\ 4\ 6\ 7\ 7 \end{array}$
6	0 1

(19) The following stem-and-leaf plots compare the ages of 30 actors and 30 actresses at the time they won the Oscar award for Best Actor or Actress.

Actors	Stems	Actresses
	2	146667
98753221	3	00113344455778
88776543322100	4	11129
6651	5	
210	6	011
6	7	4
	8	0

(a) What is the age of the youngest actor to win an Oscar?

(b) What is the age difference between the oldest and the youngest actress to win an Oscar?

$$80 - 21 = 59$$

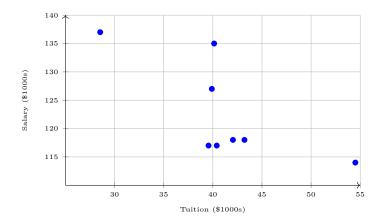
(c) What is the oldest age shared by two actors to win an Oscar?

Answer to
$$19(c)$$
: 56 years

20) The table below shows the yearly tuition of 8 universities, as well as the average midcareer salaries for graduates of each university.

University	Tuition (\$)	Salary (\$)
Princeton	28,540	137,000
Harvey Mudd	40,133	135,000
CalTech	39,900	127,000
MIT	42,050	118,000
Lehigh University	43,220	118,000
NYU-Poly	39,565	117,000
Babson College	40,400	117,000
Stanford	$54,\!506$	114,000

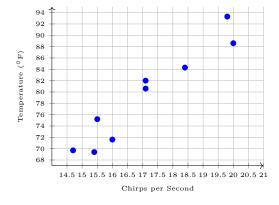
Draw a scatterplot for this data, using x to represent tuition and y to represent salary.



(21) The table below shows the frequency of chirps for the striped ground cricket compared to the ambient temperature.

Chirps per Second	Temperature $(^{\circ}F)$
20.0	88.6
16.0	71.6
19.8	93.3
18.4	84.3
17.1	80.6
15.5	75.2
14.7	69.7
17.1	82.0
15.4	69.4

Draw a scatter plot for this data, using \boldsymbol{x} to represent the chirping frequency and \boldsymbol{y} to represent temperature.



SECTION 3.3 Describing Data with Statistics

For problems 1–18, use the dataset shown in the table below. This is a sample of 15 players in Major League Baseball, chosen from the starting lineups of teams in 2019. The table shows the team, age, position, height, and salary for each player, as well as several statistics from that season. These include the number of games they played (G), their batting average (AVE) (the proportion of their at-bats for which they got a hit), and their home runs (HR).

Name	Team	\mathbf{Age}	Height	\mathbf{G}	AVE	$^{\mathrm{HR}}$	Salary
Cedric Mullins	Orioles	25	173 cm	22	.094	0	\$557,500
Tim Anderson	White Sox	26	185 cm	123	.335	18	\$1,400,000
Christin Stewart	Tigers	25	183 cm	104	.233	10	\$556,400
Alex Gordon	Royals	35	185 cm	150	.266	13	\$20,000,000
Jonathan Schoop	Twins	27	185 cm	121	.256	23	\$7,500,000
Marcus Semien	Athletics	29	183 cm	162	.285	33	\$5,900,000
Yandy Diaz	Rays	28	188 cm	79	.267	14	\$558,400
Randal Grichuk	Blue Javs	28	188 cm	151	.232	31	\$5,000,000
Josh Donaldson	Braves	33	185 cm	155	.259	37	\$23,000,000
Joey Votto	Reds	36	188 cm	142	.261	15	\$25,000,000
Cody Bellinger	Dodgers	24	193 cm	156	.305	47	\$605,000
Ryan Braun	Brewers	35	188 cm	144	.285	22	\$19,000,000
Maikel Franco	Phillies	27	185 cm	123	.234	17	\$5,200,000
Ian Kinsler	Padres	37	183 cm	87	.217	9	\$3,750,000
Marcell Ozuna	Cardinals	28	$185~\mathrm{cm}$	130	.241	29	\$12,250,000

Note: for problems 1-18, use calculator to find answer; no work shown

1) Find the mean age of the players.	
	Answer to 1: 29.53 years
2 Find the mean height of the players.	
	Answer to 2: 185.13 cm
3 Find the mean salary for the players.	
	Answer to 3: \$8,685,153
4 Find the median number of games played (G) for the pla	yers.
	Answer to 4: 130 games
5 Find the median number of home runs (HR) for the play	ers.
A	answer to 5: 21.2 home runs
6 Find the median batting average (AVE) for the players.	
	Answer to 6: 259
7 Find the mode(s) for the height of the players.	
	Answer to 7: 185 cm
8 Find the mode(s) for the ages of the players.	
	Answer to 8: 28 years
9 For the players' ages, which is greater, the mean or the n	nedian?
	Answer to 9: Mean

Answer to 10: Mean

Answer to 11: 13 years

(10) For the players' salaries, which is greater, the mean or the median?

Find the range of the ages of the players.

Find the range of the heights of the players.

Answer to 12: | 20 cm

Find the range of the number of games played by the players.

Answer to 13: | 140 games

Find the standard deviation for the players' salaries.

Answer to 14: \$8,839,865

Find the standard deviation for the number of home runs hit by the players.

Answer to 15: 12.3 home runs

Find the standard deviation for the players' batting averages.

Answer to 16: | .053

Calculate the Five Number Summary for the players' heights.

Answer to 17: 173, 183, 185, 188, 193

Calculate the Five Number Summary for the number of games played by the players.

Answer to 18: 22, 104, 130, 150, 162

Twenty-five randomly selected students were asked how many movies they had watched the previous week. The results are shown below.

Number of Movies Frequency

Find the mean and median for the number of movies watched per student.

Answer to 19: Mean: 1.68; Median: 1

$$\begin{array}{l} \text{Mean: } \frac{(0)(5)+(1)(9)+(2)(6)+(3)(4)+(4)(1)}{25}=\frac{42}{25}=1.68 \\ \text{Median: } \frac{n+1}{2}=\frac{26}{2}=13 \text{, so it is in the 13th position, where there is a 1.} \end{array}$$

Twenty students were asked how many hours they worked per day. The table below shows the results.

Number of Hours	Frequency
_	_
2	3
3	5
4	3
5	6
6	2
7	1

Find the mean and median for the number of hours worked per student.

Answer to 20: Mean: 4.1; Median: 4

Mean:
$$\frac{(2)(3) + (3)(5) + (4)(3) + (5)(6) + (6)(2) + (7)(1)}{20} = \frac{82}{20} = 4.5$$

 $\begin{array}{l} \text{Mean: } \frac{(2)(3)+(3)(5)+(4)(3)+(5)(6)+(6)(2)+(7)(1)}{20}=\frac{82}{20}=4.1\\ \text{Median: } \frac{n+1}{2}=\frac{21}{2}=10.5, \, \text{so it is between the 10th and 11th positions; average is}\\ \frac{4+4}{2}=4. \end{array}$

$$\frac{4+4}{2} = 4.$$

Find the weighted average of the student's grades listed below, using the given percentage value for each category.

Assignment	Score	Weight
Test 1	81	30%
Test 2	88	30%
Homework Quizzes	92 89	$\frac{10\%}{10\%}$
Final Exam	84	20%

Answer to 21: 85.6

$$(81)(0.3) + (88)(0.3) + (92)(0.1) + (89)(0.1) + (84)(0.2) = 85.6$$

Find the weighted average of the student's grades listed below, using the given point value for each category.

Assignment	\mathbf{Score}	Points
Tests	86	200
Projects	94	100
Homework	90	50
Final Exam	83	150

Answer to 22: | 87.1

$$\frac{(86)(200) + (94)(100) + (90)(50) + (83)(150)}{200 + 100 + 50 + 150} = 87.1$$

23) In a neighborhood donut shop, one type of donut has 530 calories, three types of donuts have 330 calories, four types of donuts have 320 calories, seven types of donuts have 410 calories, and five types of donuts have 380 calories. Find the mean and median calories of the donuts.

Answer to 23: Mean: 378.5; Median: 410

$$\begin{aligned} \text{Mean: } \frac{(1)(530) + (3)(330) + (4)(320) + (7)(410) + (5)(380)}{1 + 3 + 4 + 7 + 5} &= \frac{7570}{20} = 378.5 \\ \text{Median: } \frac{n+1}{2} &= \frac{21}{2} = 10.5, \text{ so it is between the 10th and 11th positions; average is} \\ \frac{410 + 410}{2} &= 410. \end{aligned}$$

24) In a recent issue of IEEE Spectrum, 84 engineering conferences were announced. Four conferences lasted 2 days, 36 lasted 3 days, 18 lasted 4 days, 19 lasted 5 days, 4 lasted 6 days, 1 lasted 7 days, 1 lasted 8 days, and 1 lasted 9 days. Find the mean and median length (in days) of an engineering conference.

Answer to 24: Mean: 3.94; Median: 4

$$\frac{(4)(2)+(36)(3)+(18)(4)+(19)(5)+(4)(6)+(1)(7)+(1)(8)+(1)(9)}{4+36+18+19+4+1+1+1}=\frac{331}{84}=3.94$$
 Median: $\frac{n+1}{2}=\frac{85}{2}=42.5$, so it is between the 42nd and 43rd positions; average is $\frac{4+4}{2}=4$.

$$\frac{4+4}{2} = 4$$

SECTION 3.4 Linear Regression

(1) If the value of r is 0.91 for a dataset comparing two variables, what does that tell us?

Answer to 1: There is a strong positive linear association

(2) If the value of r is -0.43 for a dataset comparing two variables, what does that tell us?

Answer to 2: There is a weak negative linear association

(3) Compute the equation of the regression line for a dataset that has the following statistics:

$$\overline{x} = 5$$
, $s_x = 2$, $\overline{y} = 1350$, $s_y = 100$, $r = 0.70$

Answer to 3: $\hat{y} = 35x + 1175$

Slope:
$$a = r \cdot \frac{s_y}{s_x} = (0.70) \left(\frac{100}{2}\right) = 35$$

Intercept: $b = \overline{y} - a\overline{x} = 1350 - (35)(5) = 1175$

(4) Compute the equation of the regression line for a dataset that has the following statistics:

$$\overline{x} = 152$$
, $s_x = 24.5$, $\overline{y} = 26$, $s_y = 2.7$, $r = -0.82$

Answer to 4: $\hat{y} = -0.09x + 39.7$

Slope:
$$a = r \cdot \frac{s_y}{s_x} = (-0.82) \left(\frac{2.7}{24.5}\right) = -0.09$$

Intercept: $b = \overline{y} - a\overline{x} = 26 - (-0.09)(152) = 39.7$

For problems 5–8, use the dataset shown in the table below. This is a sample of 15 players in Major League Baseball, chosen from the starting lineups of teams in 2019. The table shows the team, age, position, height, and salary for each player, as well as several statistics from that season. These include the number of games they played (G), their batting average (AVE) (the proportion of their at-bats for which they got a hit), and their home runs (HR).

Name	Team	\mathbf{Age}	Height	\mathbf{G}	AVE	$^{\mathrm{HR}}$	Salary
Cedric Mullins	Orioles	25	173 cm	22	.094	0	\$557,500
Tim Anderson	White Sox	26	185 cm	123	.335	18	\$1,400,000
Christin Stewart	Tigers	25	183 cm	104	.233	10	\$556,400
Alex Gordon	Royals	35	185 cm	150	.266	13	\$20,000,000
Jonathan Schoop	Twins	27	185 cm	121	.256	23	\$7,500,000
Marcus Semien	Athletics	29	183 cm	162	.285	33	\$5,900,000
Yandy Diaz	Rays	28	188 cm	79	.267	14	\$558,400
Randal Grichuk	Blue Jays	28	188 cm	151	.232	31	\$5,000,000
Josh Donaldson	Braves	33	185 cm	155	.259	37	\$23,000,000
Joey Votto	Reds	36	188 cm	142	.261	15	\$25,000,000
Cody Bellinger	Dodgers	24	193 cm	156	.305	47	\$605,000
Ryan Braun	Brewers	35	188 cm	144	.285	22	\$19,000,000
Maikel Franco	Phillies	27	185 cm	123	.234	17	\$5,200,000
Ian Kinsler	Padres	37	183 cm	87	.217	9	\$3,750,000
Marcell Ozuna	Cardinals	28	185 cm	130	.241	29	\$12,250,000

- 5 Suppose we want to try to predict a player's salary based on the number of home runs they hit (HR).
 - (a) Before doing any calculations, does it seem likely that there will be a strong association between these two variables? If so, which direction do you expect for the association?

Answer to 5(a): Yes; positive

(b) Calculate the value of the correlation coefficient, r, using a calculator.

Answer to 5(b): r = 0.137

(c) Interpret the value of r; specifically, describe the direction of the trend and the strength of the linear association.

Answer to 5(c): Positive, but negligible

(d) Find the equation of the regression line for this association. (Note: this may not be meaningful, depending on the value of r, but we can still use it for practice.)

Answer to 5(d): $\hat{y} = 98,257x + 6,602,098$

(e) Ignoring the possibility that the regression line may not be a good fit for the data, use this regression line to predict the salary of a player who hits 21 home runs. Then predict the salary of a player who hits 70 home runs. Which prediction is likely to be more accurate?

Answer to 5(e): \$8,665,495; \$13,480,088; the first one (interpolation)

21 home runs: $\hat{y} = 98,257(21) + 6,602,098 = 8,665,495$ 70 home runs: $\hat{y} = 98,257(70) + 6,602,098 = 13,480,088$

- (6) Suppose we want to try to predict a player's height based on their age.
 - (a) Before doing any calculations, does it seem likely that there will be a strong association between these two variables? If so, which direction do you expect for the association?

Answer to 6(a): No

(b) Calculate the value of the correlation coefficient, r, using a calculator.

Answer to 6(b): r = 0.121

(c) Interpret the value of r; specifically, describe the direction of the trend and the strength of the linear association.

Answer to 6(c): Positive, but negligible

(d) Find the equation of the regression line for this association. (Note: this may not be meaningful, depending on the value of r, but we can still use it for practice.)

Answer to 6(d): $\hat{y} = 0.117x + 181.7$

(e) Ignoring the possibility that the regression line may not be a good fit for the data, use this regression line to predict the height of a player who is 26 years old.

Answer to 6(e): 184.7 cm

 $\hat{y} = 0.117(26) + 181.7 = 184.7$

- (7) Suppose we want to try to predict the number of home runs that a player hits (HR) based on the number of games they play (G).
 - (a) Before doing any calculations, does it seem likely that there will be a strong association between these two variables? If so, which direction do you expect for the association?

Answer to 7(a): Yes; positive

(b) Calculate the value of the correlation coefficient, r, using a calculator.

Answer to 7(b): r = 0.76

(c) Interpret the value of r; specifically, describe the direction of the trend and the strength of the linear association.

Answer to 7(c): Positive, moderate

(d) Find the equation of the regression line for this association. (Note: this may not be meaningful, depending on the value of r, but we can still use it for practice.)

Answer to 7(d): $\hat{y} = 0.25x - 9.5$

(e) Ignoring the possibility that the regression line may not be a good fit for the data, use this regression line to predict how many home runs a player will hit if he plays 100 games.

Answer to 7(e): 15.5 home runs

$$\hat{y} = 0.25(100) - 9.5 = 15.5$$

57

- (8) Suppose we want to try to predict a player's batting average based on the number of home runs they hit.
 - (a) Before doing any calculations, does it seem likely that there will be a strong association between these two variables? If so, which direction do you expect for the association?

Answer to 8(a): Yes; positive

(b) Calculate the value of the correlation coefficient, r, using a calculator.

Answer to 8(b): r = 0.6

(c) Interpret the value of r; specifically, describe the direction of the trend and the strength of the linear association.

Answer to 8(c): Positive, moderate

(d) Find the equation of the regression line for this association. (Note: this may not be meaningful, depending on the value of r, but we can still use it for practice.)

Answer to 8(d): $\hat{y} = 0.002x + 0.2$

(e) Ignoring the possibility that the regression line may not be a good fit for the data, use this regression line to predict the batting average of a player who hits 12 home runs. Then predict the batting average of a player who hits 58 home runs. Which prediction is likely to be more accurate?

Answer to 8(e): 224; .316; the first one (interpolation)

12 home runs: $\hat{y} = 0.002(12) + 0.2 = .224$ 58 home runs: $\hat{y} = 0.002(58) + 0.2 = .316$

(9) The data set below shows the GMAT scores for five MBA students and the students' grade point averages (GPA) upon graduation.

 GMAT
 660
 580
 480
 710
 600

 GPA
 3.7
 3.0
 3.2
 4.0
 3.5

(a) Calculate r, the correlation coefficient between these two variables.

Answer to 9(a): r = 0.82

(b) Interpret the value of r; specifically, describe the direction of the trend and the strength of the linear association.

Answer to 9(b): Positive; strong

(c) Compute the regression line for predicting GPA from GMAT score.

Answer to 9(c): $\hat{y} = 0.004x + 1.21$

(d) Predict the GPA of a student who gets a score of 500 on the GMAT.

Answer to 9(d): | 3.21

 $\hat{y} = 0.004(500) + 1.21 = 3.21$

(e) Does the student with a GMAT score of 580 have a higher or lower GPA than the one predicted by the regression line?

Answer to 9(e): Lower

 $\hat{y} = 0.004(580) + 1.21 = 3.53$ (actual GPA is 3.0)

(10) The data set below shows the mileage and selling prices of eight used cars of the same model.

Mileage 21,000 34,000 41,000 43,000 65,000 72,000 76,000 84,000 \$16,000 Price \$11,000 \$13,000 \$14,000 \$10,000 \$12,000 \$7,000 \$7,000

(a) Calculate r, the correlation coefficient between these two variables.

Answer to 10(a): r = -0.84

(b) Interpret the value of r; specifically, describe the direction of the trend and the strength of the linear association.

Answer to 10(b): Negative; strong

TO COMPUTE THE REGRESSION THE TOLD RECICINE DITCE HOW HINEAS	Compute the regression line for predicting price from n	nileag
--	---	--------

Answer to 10(c): $\hat{y} = -0.118x + 17,688$

(d) Predict the price of a car with 30,000 miles.

Answer to 10(d): \$14,148

$$\hat{y} = -0.118(30,000) + 17,688 = 14,148$$

(e) Does the car with 43,000 miles on it have a higher or lower price than the one predicted by the regression line?

Answer to 10(e): Higher

 $\hat{y} = -0.118(43,000) + 17,688 = 12,614$ (actual price is 14,000)

SECTION 3.5 The Normal Distribution

- 1 The heights of American adult males are normally distributed with a mean of 177 cm and a standard deviation of 7.4 cm. Find the range of heights that contain approximately
 - (a) 68% of the data

Answer to 1(a): (169.6, 184.4)

(b) 95% of the data

Answer to 1(b): (162.2, 191.8)

(c) 99.7% of the data

Answer to 1(c): (154.8, 199.2)

- (2) Suppose that babies' weights are normally distributed with a mean of 3.23 kg and a standard deviation of 0.87 kg. Find the range of weights that contain approximately
 - (a) 68% of the data

Answer to 2(a): (2.36, 4.1)

(b) 95% of the data

Answer to 2(b): (1.49, 4.97)

(c) 99.7% of the data

Answer to 2(c): (0.62, 5.84)

- (3) Suppose that the scores on a statewide standardized test are normally distributed with a mean of 72 and a standard deviation of 4. Estimate the percentage of scores that were
 - (a) between 68 and 76.

Answer to 3(a): 68%

(b) above 76.

Answer to 3(b): 16%

(c) below 64.

Answer to 3(c): 2.5%

(d) between 68 and 84.

Answer to 3(d): 97.35%

- 4 Water usages in American showers are normally distributed, with the average shower using 17.2 gallons, and a standard deviation of 2.5 gallons. Estimate the percentage of showers that used
 - (a) more than 22.2 gallons.

Answer to 4(a): 2.5%

(b) less than 14.7 gallons.

Answer to 4(b): 16%

	(c)	between 12.2 and 22.2 gallons.	
			Answer to 4(c): 95%
	(d)	between 9.7 and 19.7 gallons.	Answer to 4(d): 83.85%
		AT scores are approximately normally distribution of 95. Estimate the percentage of scores	ated with a mean of 547 and a standard
	(a)	between 262 and 832.	
	(L)	al C40	Answer to 5(a): 99.7%
	(D)	above 642.	Answer to 5(b): 16%
	(c)	below 262.	Answer to 5(c): 0.15%
	(d)	between 262 and 452.	Answer to 5(d): 15.85%
	\$7,50	pose that wedding costs in the Caribbean are 00 and a standard deviation of \$975. Estimate cost	
	(a)	between \$6525 and \$9450.	
	(b)	above \$9450.	Answer to 6(a): 81.5%
	(6)	above \$5 100.	Answer to 6(b): 2.5%
	(c)	below \$6525.	
	(d)	between \$4575 and \$10,425.	Answer to 6(c): 16%
	(u)	between \$1575 and \$10,125.	Answer to 6(d): 99.7%
	of \$3	selling prices for homes in a certain communit 321,000 and a standard deviation of \$38,000. community with selling prices	
	(a)	between $$283,000$ and $$397,000$.	
	(h)	hotmon \$245,000 and \$425,000	Answer to 7(a): 81.5%
	(D)	between \$245,000 and \$435,000.	Answer to 7(b): 97.35%
	(c)	below \$245,000.	Answer to 7(c): 2.5%
	(d)	above \$245,000.	
_			Answer to 7(d): 97.5%
	a no	pose that the time that a new roof will last ormal distribution, with a mean of 25 years mate the percentage of roofs that last	
	(a)	longer than 30 years.	1 0/) 100/
	(b)	between 20 and 40 years.	Answer to 8(a): 16%
			Answer to 8(b): 83.85%
	(c)	less than 20 years.	Answer to 8(c): 16%
	(d)	more than 40 years.	
			Answer to 8(d): 0.15%

- (9) The widths of platinum samples manufactured at a factory are normally distributed, with a mean of 1.1 cm and a standard deviation of 0.2 cm. Find the z-scores that correspond to each of the following widths.
 - (a) 1.5 cm

Answer to 9(a):
$$z=2$$

$$z = \frac{x - \overline{x}}{s} = \frac{1.5 - 1.1}{0.2} = 2$$

(b) 0.94 cm

Answer to 9(b):
$$z = -0.8$$

$$z = \frac{x - \overline{x}}{s} = \frac{0.94 - 1.1}{0.2} = -0.8$$

- (10) The average resting heart rate of a population is 88 beats per minute, with a standard deviation of 12 bpm. Find the z-scores that correspond to each of the following heart rates.
 - (a) 120 bpm

Answer to 10(a):
$$z = 2.67$$

$$z = \frac{x - \overline{x}}{s} = \frac{120 - 88}{12} = 2.67$$

(b) 71 bpm

Answer to 10(b):
$$z = -1.42$$

$$z = \frac{x - \overline{x}}{s} = \frac{71 - 88}{12} = -1.42$$

The average height of American adult males is 177 cm, with a standard deviation of 7.4 cm. Meanwhile, the average height of Indian males is 165 cm, with a standard deviation of 6.7 cm. Which is taller relative to his nationality, a 175-cm American man or a 162-cm Indian man?

American:
$$z = \frac{x - \overline{x}}{\frac{s}{x}} = \frac{175 - 177}{7.4} = -0.27$$

Indian: $z = \frac{x - \overline{x}}{\frac{s}{x}} = \frac{162 - 165}{6.7} = -0.45$

12 Kyle and Ryan take entrance exams at two different universities. Kyle scores a 430 on an exam with a mean of 385 and a standard deviation of 70, while Ryan scores a 31 on an exam with a mean of 28 and a standard deviation of 4.5. Which do you think is more likely to be accepted at their university of choice?

Kyle:
$$z = \frac{x - \overline{x}}{s} = \frac{430 - 385}{70} = 0.64$$

Ryan: $z = \frac{x - \overline{x}}{s} = \frac{31 - 28}{4.5} = 0.67$

(13) A doctor measured serum HDL levels in her patients, and found that they were normally distributed with a mean of 63.4 and a standard deviation of 3.8. Find the serum HDL levels that correspond to the following z-scores.

(a)
$$z = -0.85$$

Answer to 13(a):
$$x = 60.17$$

$$z = \frac{x - \overline{x}}{s} \longrightarrow x = zs + \overline{x} = (-0.85)(3.8) + 63.4 = 60.17$$

(b)
$$z = 1.33$$

Answer to 13(b):
$$x = 68.45$$

$$z = \frac{x - \overline{x}}{s} \longrightarrow x = zs + \overline{x} = (1.33)(3.8) + 63.4 = 68.45$$

(14) If the distribution of weight of newborn babies is approximately normal, with a mean of 3.23 kilograms and a standard deviation of 0.87 kilograms, find the weights that correspond to the following z-scores.

(a)
$$z = 2.20$$

Answer to 14(a):
$$x = 5.14$$

$$z = \frac{x - \overline{x}}{s} \longrightarrow x = zs + \overline{x} = (2.20)(0.87) + 3.23 = 5.14$$

(b)
$$z = -1.73$$

Answer to 14(b):
$$x = 1.72$$

$$z = \frac{x - \overline{x}}{s} \longrightarrow x = zs + \overline{x} = (-1.73)(0.87) + 3.23 = 1.72$$

(15) What is the margin of error for a poll with a sample size of 2000 people?

$$\frac{1}{\sqrt{n}} = \frac{1}{\sqrt{2000}} = 0.022 = 2.2\%$$

(16) What is the margin of error for a poll with a sample size of 150 people?

$$\frac{1}{\sqrt{n}} = \frac{1}{\sqrt{150}} = 0.082 = 8.2\%$$

(17) If you want a poll to have a margin of error of 2.5%, how large will your sample have to be?

Answer to 17:
$$n = 1600$$

$$\frac{1}{\sqrt{n}} = 0.025$$

$$\frac{1}{0.025} = \sqrt{n}$$

$$\left(\frac{1}{0.025}\right)^2 = n$$

$$1600 = n$$

18 If you want a poll to have a margin of error of 1%, how large will your sample have to be?

Answer to 18:
$$n = 10,000$$

$$\frac{1}{\sqrt{n}} = 0.01$$

$$\frac{1}{0.01} = \sqrt{n}$$

$$\left(\frac{1}{0.01}\right)^2 = n$$

$$10,000 = n$$

Probability

SECTION 4.1 Basic Concepts of Probability

(1) A fair die is rolled. Find the probability of getting 4.

Answer to 1: $\left[\frac{4}{6}\right]$
(2) A fair die is rolled. Find the probability of getting less than 3.
Answer to 2: $\boxed{\frac{2}{6}}$
(3) A fair die is rolled. Find the probability of getting at least 5.
Answer to 3: $\boxed{\frac{2}{6}}$
4 You have a bag with 20 cherries, 14 sweet and 6 sour. If you pick a cherry at random what is the probability that it will be sweet?
Answer to 4: $\frac{14}{20}$
(5) A ball is drawn randomly from a jar that contains 6 red balls, 2 white balls, and 5 yellow balls. Find the probability of drawing a white ball.
Answer to 5: $\boxed{\frac{2}{13}}$
6 Suppose you write each letter of the alphabet on a different slip of paper and put th slips into a hat. What is the probability of drawing one slip of paper from the hat a random and getting a consonant? Note that a consonant is any letter that is not a vowe $(a, e, i, o, or u)$.
Answer to 6: $\boxed{\frac{21}{26}}$
(7) In a survey, 205 people indicated they prefer cats, 160 indicated they prefer dogs, and 40 indicated they don't enjoy either pet. Find the probability that if a person is chosen a random, they prefer cats.
Answer to 7: $\boxed{\frac{205}{405}}$
(8) A group of people were asked if they had run a red light in the last year. 150 responder "yes" and 185 responded "no." Find the probability that if a person is chosen at random they have run a red light in the last year.
Answer to 8: $\boxed{\frac{150}{335}}$
(9) A U.S. roulette wheel has 38 pockets: 1 through 36, 0, and 00. 18 are black, 18 are red, and 2 are green. A play has a dealer spin the wheel and a small ball in opposit directions. As the ball slows to stop, it can land with equal probability on the 38 slots Find the probability of the ball landing on green.
Answer to 9: $\frac{2}{38}$
(10) A glass jar contains 6 red, 5 green, 8 blue and 3 yellow marbles. If a single marble i

chosen at random from the jar, what is the probability of choosing

Answer to 10(a):

Answer to 10(b):

(a) a red marble?

(b) a green marble?

(c) a blue marble?

Answer to 10(c):
$$\frac{8}{22}$$

(11) Lisa has a large bag of coins. After counting the coins, she recorded the counts in the table below. She then decided to draw some coins at random, replacing each coin before the next draw.

Quarters	Nickels	Dimes	Pennies
27	18	34	21

(a) What is the probability that Lisa obtains a quarter on the first draw?

Answer to 11(a):
$$\frac{27}{100}$$

(b) What is the probability that Lisa obtains a penny or a dime on the second draw?

Answer to 11(b):
$$\frac{55}{100}$$

(c) What is the probability that Lisa obtains at most 10 cents worth of money on the third draw?

Answer to 11(c):
$$\frac{73}{100}$$

(d) What is the probability that Lisa does not get a nickel on the fourth draw?

Answer to 11(d):
$$\frac{82}{100}$$

(e) What is the probability that Lisa obtains at least 10 cents worth of money on the fifth draw?

Answer to 11(e):
$$\frac{61}{100}$$

(12) Suppose you roll a pair of six-sided dice.

(a) List all possible outcomes of this experiment.

Answer to 12(a): listed below (36 total possibilities

$$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

(b) What is the probability that the sum of the numbers on your dice is exactly 6?

Answer to 12(b):
$$\frac{5}{36}$$

(c) What is the probability that the sum of the numbers on your dice is at most 4?

Answer to 12(c):
$$\frac{6}{36}$$

(d) What is the probability that the sum of the numbers on your dice is at least 9?

Answer to 12(d):
$$\frac{10}{36}$$

(13) I asked my Facebook friends to complete a two-question survey. They answered the following questions: Which beverage do you prefer in the morning: coffee or tea? What is your gender? I summarized the results in following table:

	Coffee	Tea	Total
Female	37	24	61
\mathbf{Male}	22	31	53
Total	59	55	114

(a) What is the probability that I select a friend who prefers coffee?

Answer to 13(a): $\frac{59}{114}$

(b) What is the probability that I select a friend who is female?

Answer to 13(b): $\frac{61}{114}$

(c) What is the probability that I select a friend who is male and prefers tea?

Answer to 13(c): $\frac{31}{114}$

(14) A poll was taken of 14,056 working adults aged 40-70 to determine their level of education. The participants were classified by sex and by level of education. The results were as follows.

Education Level	Male	Female	Total
High School or Less	3141	2434	5575
Bachelor's Degree	3619	3761	7380
Master's Degree	534	472	1006
Ph.D.	52	43	95
Total	7346	6710	14,056

A person is selected at random. Compute the following probabilities:

(a) The probability that the selected person is a male

Answer to 14(a): $\frac{7346}{14,056}$

(b) The probability that the selected person does not have a Ph.D.

Answer to 14(b): $\begin{vmatrix} 13,961\\14,056 \end{vmatrix}$

(c) The probability that the selected person has a Master's degree

Answer to 14(c): $\frac{1006}{14,056}$

(d) The probability that the selected person is female and has a Master's degree

Answer to 14(d): $\frac{472}{14,056}$

SECTION 4.2 The Addition Rule and the Rule of Complements

1 A jar contains 6 red marbles numbered 1 to 6 and 8 blue marbles numbered 1 to 8. A marble is drawn at random from the jar. Find the probability the marble is red or odd-numbered.

Answer to 1:
$$\frac{10}{14}$$

$$\frac{6}{14} + \frac{7}{14} - \frac{3}{14} = \frac{10}{14}$$

(2) A jar contains 4 red marbles numbered 1 to 4 and 10 blue marbles numbered 1 to 10. A marble is drawn at random from the jar. Find the probability the marble is blue or even-numbered.

Answer to 2:
$$\boxed{\frac{12}{14}}$$

$$\frac{10}{14} + \frac{7}{14} - \frac{5}{14} = \frac{12}{14}$$

- (3) You draw one card from a standard 52-card deck.
 - a) What is the probability of selecting a King or a Queen?

Answer to 3a:
$$\frac{8}{52}$$

$$P(K \text{ or } Q) = P(K) + P(Q) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52}$$

b) What is the probability of selecting a face card or a 10?

Answer to 3b:
$$\boxed{\frac{16}{52}}$$

$$P(F \text{ or } 10) = P(F) + P(10) = \frac{12}{52} + \frac{4}{52} = \frac{16}{52}$$

c) What is the probability of selecting a spade or a heart?

Answer to 3c:
$$\boxed{\frac{26}{52}}$$

$$P(S \text{ or } H) = P(S) + P(H) = \frac{13}{52} + \frac{13}{52} = \frac{26}{52}$$

d) What is the probability of selecting a red card or a black card?

Answer to 3d:
$$\frac{52}{52}$$

$$P(R \text{ or } B) = P(R) + P(B) = \frac{26}{52} + \frac{26}{52} = \frac{52}{52}$$

- (4) You are dealt a single card from a standard 52-card deck.
 - a) Find the probability that you are not dealt a diamond.

Answer to 4a:
$$\frac{39}{52}$$

$$P(not\ D) = 1 - P(D) = 1 - \frac{13}{52} = \frac{39}{52}$$

b) Find the probability that you are not dealt a face card.

Answer to 4b:
$$\boxed{\frac{40}{52}}$$

$$P(not\ F) = 1 - P(F) = 1 - \frac{12}{52} = \frac{40}{52}$$

c) Find the probability that you are not dealt an Ace.

Answer to 4c:
$$\frac{48}{52}$$

$$P(not\ A) = 1 - P(A) = 1 - \frac{4}{52} = \frac{48}{52}$$

d) Find the probability that you are not dealt a jack or a king.

Answer to 4d:
$$\boxed{\frac{44}{52}}$$

$$P(not\ (J\ or\ K)) = 1 - (P(J) + P(K)) = 1 - \left(\frac{4}{52} + \frac{4}{52}\right) = \frac{44}{52}$$

(5) Consider the following information about a group of 130 FCC students:

Gender	Right-handed	Left-handed	Total
Female	58	13	71
Male	47	12	59
Total	105	25	130

If one person is randomly selected from the group, what is the probability this student is female or left-handed?

Answer to 5:
$$\frac{83}{130}$$

$$P(F \text{ or } L) = P(F) + P(L) - P(F \text{ and } L) = \frac{71}{130} + \frac{25}{130} - \frac{13}{130} = \frac{83}{130}$$

6 The table below shows the number of survey subjects who have received and not received a speeding ticket in the last year, and the color of their car. Find the probability that a randomly chosen person has a red car or got a speeding ticket.

	Speeding Ticket	No Speeding Ticket	Total
Red Car	15	135	150
Not Red Car	45	470	515
Total	60	605	665

Answer to 6:
$$\frac{195}{665}$$

$$P(R \text{ or } S) = P(R) + P(S) - P(R \text{ and } S) = \frac{150}{665} + \frac{60}{665} - \frac{15}{665} = \frac{195}{665}$$

- (7) Suppose you roll a blue six sided die and a red six sided die, and add their totals. Find the probability of rolling:
 - a) A 7 or 11.

Answer to 7a:
$$\boxed{\frac{8}{36}}$$

$$P(7 \text{ or } 11) = P(7) + P(11) - P(7 \text{ and } 11) = \frac{6}{36} + \frac{2}{36} - \frac{0}{36} = \frac{8}{36}$$

b) An even number or a number less than 6.

Answer to 7b:
$$\boxed{\frac{24}{36}}$$

$$P(E \ or \ < 6) = P(E) + P(< 6) - P(E \ and \ < 6) = 1 - (P(7) + P(9) + P(11))$$

= $1 - \left(\frac{6}{36} + \frac{4}{36} + \frac{2}{36}\right) = \frac{24}{36}$

c) A prime number or a number greater than 5.

Answer to 7c:
$$\frac{3}{36}$$

$$P(P \ or \ > 5) = P(4) = \frac{3}{36}$$

Answer to 8a:
$$\frac{5}{11}$$

$$P(W \text{ or } G) = P(W) + P(G) = \frac{4}{11} + \frac{1}{11} = \frac{5}{11}$$

b) A black counter or a green counter?

Answer to 8b:
$$\frac{7}{11}$$

$$P(B \text{ or } G) = P(B) + P(G) = \frac{6}{11} + \frac{1}{11} = \frac{7}{11}$$

c) Not a green counter?

Answer to 8c:
$$\frac{10}{11}$$

$$P(not\ G) = 1 - P(G) = 1 - \frac{1}{11} = \frac{10}{11}$$

(9) A poll was taken of 14,056 working adults aged 40-70 to determine their level of education. The participants were classified by sex and by level of education. The results were as follows.

Education Level	Male	Female	Total
High School or Less	3141	2434	5575
Bachelor's Degree	3619	3761	7380
Master's Degree	534	472	1006
Ph.D.	52	43	95
Total	7346	6710	14,056

A person is selected at random. Compute the following probabilities:

a) The probability that the selected person does not have a Ph.D.

Answer to 9a:
$$13,961 \over 14,056$$

$$P(not\ P) = 1 - P(P) = 1 - \frac{95}{14,056} = \frac{13,961}{14,056}$$

b) The probability that the selected person does not have a Master's degree.

Answer to 9b:
$$13,961 \over 14,056$$

$$P(not\ M) = 1 - P(M) = 1 - \frac{1006}{14,056} = \frac{13,050}{14,056}$$

c) The probability that the selected person is female or has a Master's degree.

Answer to 9c:
$$\frac{7244}{14,056}$$

$$P(F \ or \ M) = P(F) + P(M) - P(F \ and \ M) = \frac{6710}{14,056} + \frac{1006}{14,056} - \frac{472}{14,056} = \frac{7244}{14,056} + \frac{1006}{14,056} = \frac{1006}{14,056} =$$

d) The probability that the selected person is male or has a Ph.D.

Answer to 9d:
$$\boxed{\frac{7389}{14,056}}$$

$$P(M \text{ or } P) = P(M) + P(P) - P(M \text{ and } P) = \frac{7346}{14,056} + \frac{95}{14,056} - \frac{52}{14,056} = \frac{7389}{14,056} = \frac{1389}{14,056} = \frac$$

SECTION 4.3 The Multiplication Rule and Conditional Probability

1 You have a box of chocolates that contains 50 pieces, of which 30 are solid chocolate, 15 are filled with cashews and 5 are filled with cherries. All the candies look exactly alike. You select a piece, eat it, select a second piece, eat it, and finally eat one last piece. Find the probability of selecting a solid chocolate followed by two cherry-filled chocolates.

Answer to 1:
$$\frac{600}{117,600} = 0.0051$$

$$P(S, C, C) = P(S) \cdot P(C|S) \cdot P(C|S, C) = \frac{30}{50} \cdot \frac{5}{49} \cdot 448 = \frac{600}{117,600} = 0.0051$$

(2) You roll a fair six-sided die twice. Find the probability of rolling a 6 the first time and a number greater than 2 the second time.

Answer to 2:
$$\frac{4}{36} = 0.1111$$

$$P(6, > 2) = P(6) \cdot P(> 2) = \frac{1}{6} \cdot \frac{4}{6} = \frac{4}{36} = 0.1111$$

- (3) A math class consists of 25 students, 14 female and 11 male. Three students are selected at random, one at a time, to participate in a probability experiment. Compute the probability that:
 - (a) A male is selected, then two females.

Answer to 3(a):
$$\boxed{\frac{2002}{13,800} = 0.1451}$$

$$P(M, F, F) = P(M) \cdot P(F|M) \cdot P(F|M, F) = \frac{11}{25} \cdot \frac{14}{24} \cdot \frac{13}{23} = \frac{2002}{13,800} = 0.1451$$

(b) A female is selected, then two males.

Answer to 3(b):
$$\boxed{\frac{1540}{13,800} = 0.1116}$$

$$P(F,M,M) = P(F) \cdot P(M|F) \cdot P(M|F,M) = \frac{14}{25} \cdot \frac{11}{24} \cdot \frac{10}{23} = \frac{1540}{13,800} = 0.1116$$

(c) Two females are selected, then one male.

Answer to 3(c):
$$\frac{2002}{13,800} = 0.1451$$

$$P(F, F, M) = P(F) \cdot P(F|F) \cdot P(M|F, F) = \frac{14}{25} \cdot \frac{13}{24} \cdot \frac{11}{23} = \frac{2002}{13,800} = 0.1451$$

(d) Three males are selected.

Answer to 3(d):
$$\boxed{\frac{990}{13,800} = 0.0717}$$

$$P(M,M,M) = P(M) \cdot P(M|M) \cdot P(M|M,M) = \frac{11}{25} \cdot \frac{10}{24} \cdot \frac{9}{23} = \frac{990}{13,800} = 0.0717$$

(e) Three females are selected.

Answer to 3(e):
$$\boxed{\frac{2184}{13,800} = 0.1583}$$

$$P(F,F,F) = P(F) \cdot P(F|F) \cdot P(F|F,F) = \frac{14}{25} \cdot \frac{13}{24} \cdot \frac{12}{23} = \frac{2184}{13,800} = 0.1583$$

- 4 A large cooler contains the following drinks: 6 lemonade, 8 Sprite, 15 Coke, and 7 root beer. You randomly pick two cans, one at a time (without replacement).
 - (a) What is the probability that you get 2 cans of Sprite?

Answer to 4(a):
$$\frac{56}{1260} = 0.0444$$

$$P(S,S) = P(S) \cdot P(S|S) = \frac{8}{36} \cdot \frac{7}{35} = \frac{56}{1260} = 0.0444$$

(b) What is the probability that you do not get 2 cans of Coke?

Answer to 4(b):
$$\frac{1050}{1260} = 0.8333$$

$$P(not\ C,C) = 1 - P(C) \cdot P(C|C) = 1 - \frac{15}{36} \cdot \frac{14}{35} = 1 - \frac{210}{1260} = \frac{1050}{1260} = 0.8333$$

(c) What is the probability that you get either 2 root beer or 2 lemonade?

Answer to 4(c):
$$\frac{72}{1260} = 0.0571$$

$$\begin{split} P(R,R\ or\ L,L) &= P(R,R) + P(L,L) = P(R) \cdot P(R|R) + P(L) \cdot P(L|L) \\ &= \frac{7}{36} \cdot \frac{6}{35} + \frac{6}{36} \cdot \frac{5}{35} = \frac{72}{1260} = 0.0571 \end{split}$$

(d) What is the probability that you get one can of Coke and one can of Sprite?

Answer to 4(d):
$$\frac{240}{1260} = 0.1905$$

$$P(C,S) + P(S,C) = P(C) \cdot P(S|C) + P(S) \cdot P(C|S) = \frac{15}{36} \cdot \frac{8}{35} + \frac{8}{36} \cdot \frac{15}{35} = \frac{240}{1260} = 0.1905$$

(e) What is the probability that you get two drinks of the same type?

Answer to 4(e):
$$\frac{338}{1260} = 0.2683$$

$$P(L,L) + P(S,S) + P(C,C) + P(R,R) = \frac{6}{36} \cdot \frac{5}{35} + \frac{8}{36} \cdot \frac{7}{35} + \frac{15}{36} \cdot \frac{14}{35} + \frac{7}{36} \cdot \frac{6}{35} = \frac{338}{1260} = 0.2683$$

- (5) My top drawer contains different colored socks: 14 are white, 10 are black, 6 are pink, and 4 are blue. All socks in the drawer are loose. Every morning I randomly select 2 socks, one at a time. Calculate the following probabilities, giving both fraction and decimal answers, rounding to 4 decimal places:
 - (a) What is the probability that I get a blue pair of socks?

Answer to 5(a):
$$\frac{12}{1122} = 0.0107$$

$$\frac{4}{34} \cdot \frac{3}{33} = \frac{12}{1122} = 0.0107$$

(b) What is the probability that I do not get a blue pair of socks?

Answer to 5(b):
$$\frac{1110}{1122} = 0.9893$$

$$1 - \frac{12}{1122} = \frac{1110}{1122} = 0.9893$$

(c) What is the probability that I either get a white pair or a blue pair of socks?

Answer to 5(c):
$$\frac{194}{1122} = 0.1729$$

$$\frac{14}{34} \cdot \frac{13}{33} + \frac{12}{1122} = \frac{194}{1122} = 0.1729$$

(d) What is the probability that I get one black sock and one white sock?

Answer to 5(d):
$$\frac{280}{1122} = 0.2496$$

$$\frac{10}{34} \cdot \frac{14}{33} + \frac{14}{34} \cdot \frac{10}{33} = \frac{280}{1122} = 0.2496$$

6 Suppose a math class contains 25 students, 14 females (three of whom speak French) and 11 males (two of whom speak French). Compute the probability that a randomly selected student speaks French, given that the student is female.

Answer to 6:
$$\frac{3}{14} = 0.2143$$

(7) Suppose a math class contains 30 students, 18 females (four of whom speak French) and 12 males (three of whom speak French). Compute the probability that a randomly selected student is male, given that the student speaks French.

Answer to 7:
$$\frac{3}{7} = 0.4286$$

- 8 A certain virus infects one in every 400 people. A test used to detect the virus in a person is positive 90% of the time if the person has the virus and 10% of the time if the person does not have the virus.
 - (a) Find the probability that a person has the virus given that they have tested positive.

Answer to 8(a):
$$0.0221 = 2.21\%$$

$$P(virus|positive) = \frac{P(virus\;AND\;positive)}{P(positive)} = \frac{0.9}{0.9 + 39.9} = 0.0221$$

(b) Find the probability that a person does not have the virus given that they test negative.

Answer to 8(b):
$$0.9997 = 99.97\%$$

$$P(no\ virus|negative) = \frac{P(no\ virus\ AND\ negative)}{P(negative)} = \frac{399 \cdot 0.9}{399 \cdot 0.9 + 0.1} = 0.9997$$

(9) A poll was taken of 14,056 working adults aged 40-70 to determine their level of education. The participants were classified by sex and by level of education. The results were as follows.

Education Level	Male	Female	Total
High School or Less	3141	2434	5575
Bachelor's Degree	3619	3761	7380
Master's Degree	534	472	1006
Ph.D.	52	43	95
Total	7346	6710	14,056

A person is selected at random. Compute the following probabilities:

a) The probability that the selected person is male, given he has a Master's degree.

Answer to 9a:
$$\frac{534}{1006} = 0.5308$$

b) The probability that the selected person does not have a Master's degree, given it is a male.

Answer to 9b:
$$\boxed{\frac{6812}{7346} = 0.9273}$$

c) The probability that the selected person is female, given that she has a Bachelor's degree.

Answer to 9c:
$$\sqrt{\frac{3761}{7380}} = 0.0.5096$$

d) The probability that the selected person has a Ph.D, given it is a female.

Answer to 9d:
$$\frac{43}{6710} = 0.0064$$

In problems 1-4, use your calculator to evaluate the given expression.

(1) 12!

Answer to 1: | 479,001,600

Answer to 2: | 20

 $(3)_{8}P_{3}$

Answer to 3: | 336

 $_{22}C_{4}$

Answer to 4: | 7315

(5) A license plate is to have the following form: three letters followed by three numbers. An example of a license plate would be MTH 314. How many different license plates can be made, assuming that letters and numbers can be reused?

Answer to 5: | 17,576,000

 $26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 17,576,000$

(6) A bride is choosing a dress for her bridesmaids. If there are 4 styles, 3 colors, and 6 fabrics, in how many ways can she select the bridesmaids' dress?

Answer to 6: | 72

 $4 \cdot 3 \cdot 6 = 72$

(7) A quiz consists of 5 true-or-false questions. In how many ways can a student answer the quiz?

Answer to 7: | 32

 $2^5 = 32$

(8) A boy owns 2 pairs of pants, 3 shirts, 8 ties, and 2 jackets. How many different outfits can he wear to school if he must wear one of each item?

Answer to 8: 96

 $2 \cdot 3 \cdot 8 \cdot 2 = 96$

(9) Lisa is shopping for a new car. She has the following decisions to make: type (sedan, SUV, pick-up truck), make (domestic or import), color (black, white, silver). In how many ways can she select her new vehicle?

Answer to 9: 18

 $3 \cdot 2 \cdot 3 = 18$

A social security number contains nine digits, such as 999-04-6756. How many different social security numbers can be formed? Do you think we will ever run out?

Answer to 10: 1,000,000,000

 $10^9 = 1,000,000,000$

How many different ways can the letters of the word MATH be rearranged to form a four-letter code word?

Answer to 11: 24

How many ways can we select five door prizes from seven different ones and distribute them among five people?

Answer to 12: | 2520

 $7 \ nPr \ 5 = 2520$

Eight sprinters have made it to the Olympic finals in the 100-meter race. In how many different ways can the gold, silver, and bronze medals be awarded?

Answer to 13: | 336

 $8 \ nPr \ 3 = 336$

At a charity benefit with 25 people in attendance, three \$50 gift certificates are given away as door prizes. Assuming no person receives more than one prize, how many different ways can the gift certificates be awarded?

Answer to 14: 2300

 $25 \ nCr \ 3 = 2300$

Ten bands are to perform at a weekend festival. How many different ways are there to 15 schedule their appearances?

Answer to 15: 3,628,800

10! = 3,628,800

There are seven books in the Harry Potter series. In how many ways can you arrange the books on your shelf?

Answer to 16: 5040

7! = 5040

How many different four-card hands can be dealt from a 52 card deck?

Answer to 17: | 270,725

 $52 \ nCr \ 4 = 270,725$

There are 40 runners in a race, and no ties. In how many ways can the first three finishers be chosen from the 40 runners, regardless of how they are arranged?

Answer to 18: | 9880

 $40 \ nCr \ 3 = 9880$

19 There are 24 students in a MATH 120 class at FCC. How many ways are there to select 5 students for a group project?

Answer to 19: | 42,504

 $24 \ nCr \ 5 = 42,504$

A local children's center has 55 kids, and 6 are selected to take a picture for the center's advertisement. How many ways are there to select 6 children for the picture?

Answer to 20: 28,989,675

 $55 \ nCr \ 6 = 28,989,675$

Answer to 21:
$$\boxed{\frac{12,915}{12,271,512} = 0.0011}$$

Total possibilities: choose 6 numbers from 48: 48 nCr 6 = 12,271,512 Ways to win: choose 4 of the 6 to match, 2 of the 42 to not match:

Answer:
$$\frac{(6 \ nCr \ 4) \cdot (42 \ nCr \ 2) = 12,915}{12,271,512} = 0.0011$$

Answer to 22:
$$\boxed{\frac{24}{2,598,960} = 0.000009}$$

Total possibilities: choose 5 cards from 52: 52
$$nCr$$
 5 = 2,598,960 Ways to win: choose 3 out of 4 Aces, 2 out of 4 Kings: $(4 nCr$ 3) \cdot $(4 nCr$ 2) = 24

Answer:
$$\frac{24}{2,598,960} = 0.000009$$

Answer to 23:
$$\boxed{\frac{1}{120} = 0.0083}$$

NOTE: the 16 is irrelevant. Once you select 5 CDs, there is one correct alphabetical ordering out of all possible ways to arrange the 5.

Total possibilities: arrange 5:
$$5! = 120$$

Ways to arrange alphabetically: 1
Answer: $\frac{1}{120} = 0.0083$

Answer to 24:
$$\boxed{\frac{91}{17,383,860} = 0.000005}$$

Total possibilities: select 12 people from 27: 27
$$nCr$$
 12 = 17, 383, 860 All male: select 12 people out of 14: 14 nCr 12 = 91 Answer: $\frac{91}{17,383,860} = 0.000005$

Answer to 25:
$$\frac{1287}{2,598,960} = 0.0005$$

Total possibilities: select 5 cards from 52: 52
$$nCr$$
 5 = 2,598,960 All hearts: select 5 cards out of 13: 13 nCr 5 = 1287 Answer: $\frac{1287}{2,598,960} = 0.0005$

Answer to 26:
$$\frac{48}{2,598,960} = 0.00002$$

Total possibilities: select 5 cards from 52: 52
$$nCr$$
 5 = 2,598,960 Four Aces: select 4 cards out of 4 and 1 card out of 48: $(4 nCr$ 4) \cdot $(48 nCr$ 1) = 48 Answer: $\frac{48}{2,598,960} = 0.00002$

- (27) A jury pool has 18 men and 21 women, from which 12 jurors will be selected. Assuming that each person is equally likely to be selected and that the jury is selected at random, find the probability the jury consists of
 - (a) all men

Answer to 27(a):
$$\frac{18,564}{3,910,797,436} = 0.000005$$

$$\frac{18 \ nCr \ 12}{39 \ nCr \ 12} = \frac{18,564}{3,910,797,436} = 0.000005$$

(b) all women

Answer to 27(b):
$$\boxed{\frac{293,930}{3,910,797,436} = 0.00008}$$

$$\frac{21\ nCr\ 12}{39\ nCr\ 12} = \frac{293,930}{3,910,797,436} = 0.00008$$

(c) 8 men and 4 women

Answer to 27(c):
$$\boxed{\frac{261,891,630}{3,910,797,436} = 0.0670}$$

$$\frac{(18\ nCr\ 8)\cdot (21\ nCr\ 4)}{39\ nCr\ 12} = \frac{261,891,630}{3,910,797,436} = 0.0670$$

(d) 6 men and 6 women

Answer to 27(d):
$$\boxed{\frac{1,007,356,896}{3,910,797,436} = 0.2576}$$

$$\frac{(18 \ nCr \ 6) \cdot (21 \ nCr \ 6)}{39 \ nCr \ 12} = \frac{1,007,356,896}{3,910,797,436} = 0.2576$$

- (28) A race consisted of 8 women and 10 men. What is the probability that the top 3 finishers were:
 - (a) all men

Answer to 28(a):
$$\boxed{\frac{120}{816} = 0.1471}$$

$$\frac{10 \ nCr \ 3}{18 \ nCr \ 3} = \frac{120}{816} = 0.1471$$

(b) all women

Answer to 28(b):
$$\frac{56}{816} = 0.0686$$

$$\frac{8 \ nCr \ 3}{18 \ nCr \ 3} = \frac{56}{816} = 0.0686$$

(c) 2 men and 1 woman

Answer to 28(c):
$$\frac{360}{816} = 0.4412$$

$$\frac{(10 \ nCr \ 2) \cdot (8 \ nCr \ 1)}{18 \ nCr \ 3} = \frac{360}{816} = 0.4412$$

(d) 1 man and 2 women

Answer to 28(d):
$$\boxed{\frac{280}{816} = 0.3431}$$

$$\frac{(10\;nCr\;1)\cdot(8\;nCr\;2)}{18\;nCr\;3} = \frac{280}{816} = 0.3431$$

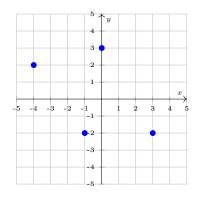
5

Linear Programming

SECTION 5.1 Linear Functions and Their Graphs

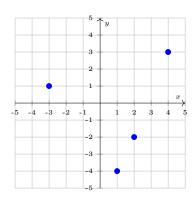
1. Plot the following points.

$$(0,3), (-4,2), (3,-2), (-1,-2)$$



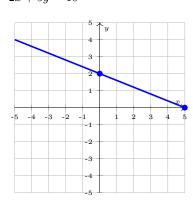
2. Plot the following points.

$$(-3,1), (4,3), (2,-2), (1,-4)$$



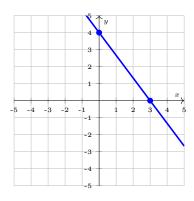
 ${\bf 3.}$ Graph the following line using the intercepts.

$$2x + 5y = 10$$



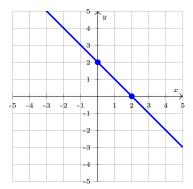
 ${\bf 4.}\,$ Graph the following line using the intercepts.

$$4x + 3y = 12$$



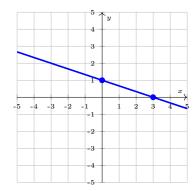
5. Graph the following line using the intercepts.

$$4x + 4y = 8$$



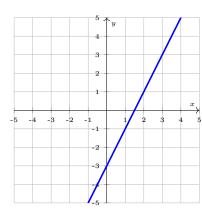
 ${\bf 6.}\,$ Graph the following line using the intercepts.

$$x + 3y = 3$$



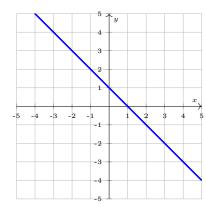
7. Graph the following line using the slope and *y*-intercept.

$$y = 2x - 3$$



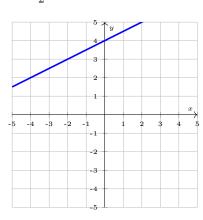
8. Graph the following line using the slope and y-intercept.

$$y = -x + 1$$



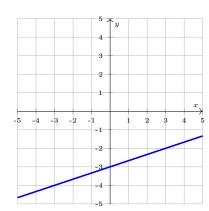
9. Graph the following line using the slope and *y*-intercept.

$$y = \frac{1}{2}x + 4$$



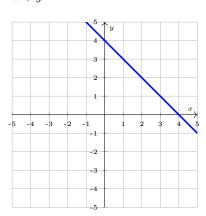
10. Graph the following line using the slope and y-intercept.

$$x - 3y = 9$$

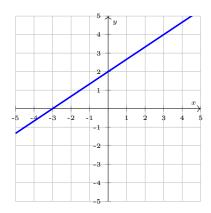


- 11. Graph the following line using any method.
- 12. Graph the following line using any method.

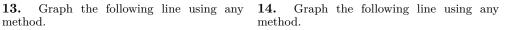
$$x + y = 4$$



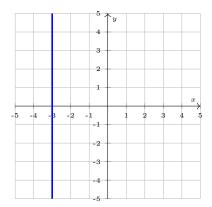
$$-2x + 3y = 6$$



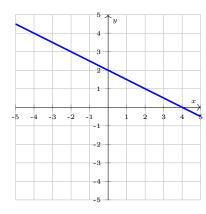
method.



$$x = -3$$



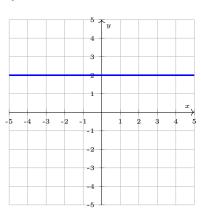
$$2y + x = 4$$



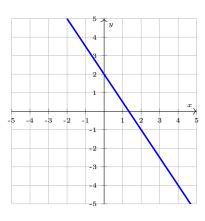
15. Graph the following line using any method.

16. Graph the following line using any method.

$$y = 2$$

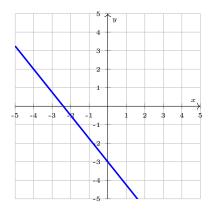


$$3x + 2y = 4$$

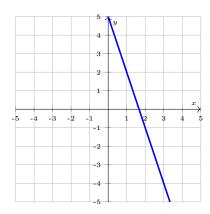


- **17.** method.
- Graph the following line using any 18. Graph the following line using any method.

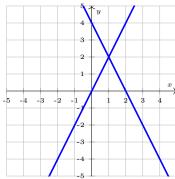
$$-5x - 4y = 12$$



$$y = -3x + 5$$



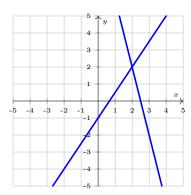




Solution: (1,2)

2. Solve the following system of equations by graphing.

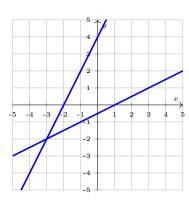
$$3x - 2y = 2$$
$$4x + y = 10$$



Solution: (2,2)

3. Solve the following system of equations by graphing.

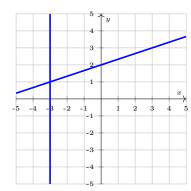
$$-4 = 2x - y$$
$$x = 2y + 1$$



Solution: (-3, -2)

4. Solve the following system of equations by graphing.

$$x - 3y = -6$$
$$x = -3$$



Solution: (-3,1)

In problems 5-13, solve each system of equations by substitution or elimination.

5.

3x + 5y = -12

x + 2y = -6

6.

x - y = 15y = -4x

7.

x + 2y = 13y + 7 = 4x

Solution: (6,-6)

Solution: (3,-12)

Solution: (3,5)

$$x - 3y = 0$$
$$2x - 3y = 6$$

$$x + y = 3$$
$$x - y = 7$$

$$3x + y = -3$$
$$4x + y = -4$$

Solution:
$$(6,2)$$

Solution:
$$(5,-2)$$

Solution:
$$(-1,0)$$

11.

$$2x + y = -2$$
$$5x + 3y = -6$$

$$5x + 2y = -1$$
$$4x - 5y = -14$$

13.
$$y = -3x + 7$$

4x + 2y = 11

Solution:
$$(0,-2)$$

Solution:
$$(-1,2)$$

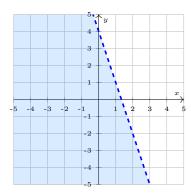
Solution:
$$\left(\frac{3}{2}, \frac{5}{2}\right)$$

SECTION 5.3 Systems of Linear Inequalities

In problems 1-4, graph the solution set for each linear inequality.

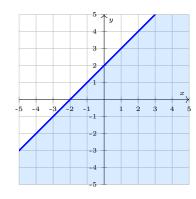
1.

$$3x + y < 4$$



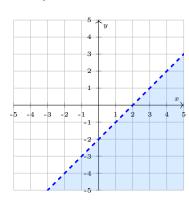
2.

$$x \geq y-2$$



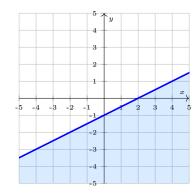
3.

$$2x - 2y > 4$$



4.

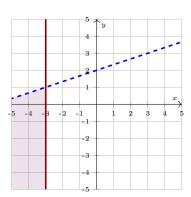
$$y \leq \frac{1}{2}x - 1$$



In problems 5–12, graph the solution set for each system of inequalities.

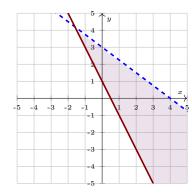
5.

$$x - 3y > -6$$
$$x \le -3$$



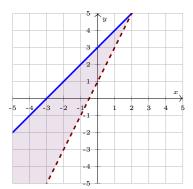
6.

$$3x + 4y < 12$$
$$y \ge -2x + 1$$



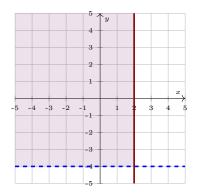
7.

$$x \ge y - 3$$
$$y > 2x + 1$$



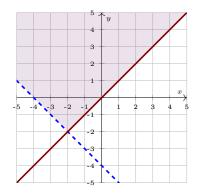
8.

$$y > -4$$
$$x \le 2$$



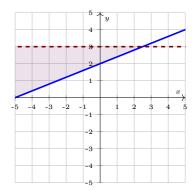
9.

$$x + y > -4$$
$$x - y \le 0$$



10.

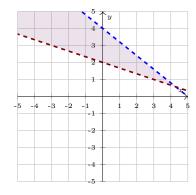
$$2x - 5y \le -10$$
$$y < 3$$



11.

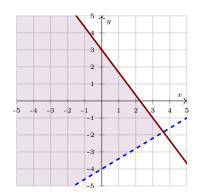
$$4x + 5y < 20$$

$$y > -\frac{1}{3}x + 2$$



12.

$$y > \frac{3}{5}x - 4$$
$$y \le -\frac{4}{3}x + 3$$



SECTION 5.4 Linear Programming

In problems 1–4, write an objective function for the given situation in terms of the variables defined.

 \bigcirc A fence company sells wooden and metal fences. Let x represent the number of wooden fences they sell and let y represent the number of metal fences. They make a profit of \$320 for each wooden fence and \$280 for each metal fence.

Answer to 1: p = 320x + 280y

2 You are placing mulch in your yard, and you find that pine chips cost \$2 per bag, while oak chips cost \$4 per bag. You want to minimize total cost. Let x be the number of bags of pine chips and y be the number of bags of oak chips.

Answer to 2: c = 2x + 4y

 \bigcirc An auto repair shop offers tire rotations and oil changes. They make a profit of \$25 on each oil change and a profit of \$18 on each tire rotation. Let x be the number of oil changes and y be the number of tire rotations.

Answer to 3: p = 25x + 18y

4 Taking a pill of Medicine A gives you 6 mg of an undesired substance, and Medicine B gives you 8 mg of the undesired substance (you want to minimize the amount of this substance). Let x be the number of A pills you take and y be the number of B pills you take.

Answer to 4: u = 6x + 8y

In problems 5-8, write an inequality to represent each constraint given.

(5) Manufacturing one chair (x) requires 6 ft of aluminum tube, and manufacturing one table (y) requires 12 ft of aluminum tube. There are 500 ft of aluminum tube available.

Answer to 5: $6x + 12y \le 500$

 \bigcirc Each oil change (x) takes 20 minutes and each tire rotation (y) takes 15 minutes. There are a total of 2400 minutes available.

Answer to 6: $20x + 15y \le 2400$

(7) You must take at least 3 pills of Medicine A (x) and at least 2 pills of Medicine B (y).

Answer to 7: $x \ge 3; y \ge 2$

(8) Mowing a large yard (x) uses 0.25 gallons of gasoline, and mowing a small yard (y) uses 0.1 gallons of gasoline. There are 5 gallons of gasoline available.

Answer to 8: $0.25x + 0.1y \le 5$

In problems 9–12, graph each feasible region and list the corner points.

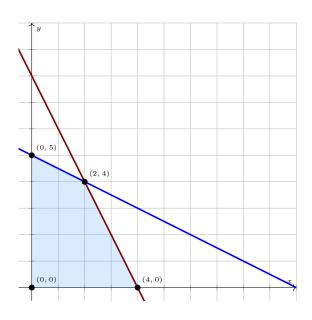
(9) Feasible region:

$$2x + 4y \le 20$$

$$4x + 2y \le 16$$

$$x \ge 0$$

$$y \ge 0$$



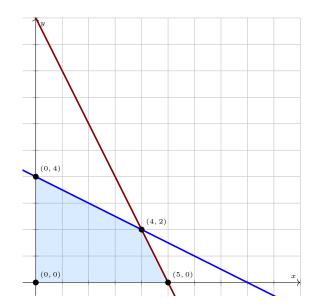
(10) Feasible region:

$$20x + 40y \le 160$$

$$18x + 9y \le 90$$

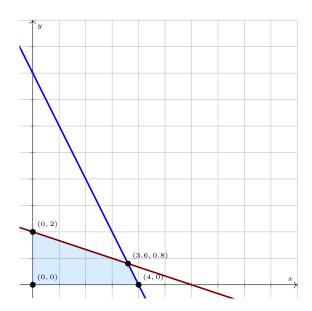
$$x \ge 0$$

$$y \ge 0$$



(11) Feasible region:

$$2x + y \le 8$$
$$x + 3y \le 6$$
$$x \ge 0$$
$$y \ge 0$$



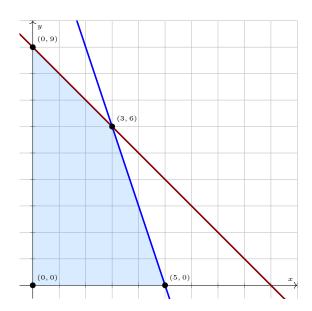
(12) Feasible region:

$$15x + 5y \le 75$$

$$9x + 9y \le 81$$

$$x \geq 0$$

$$y \ge 0$$



In problems 13–16, pick which of the given corner points maximizes the given objective function.

13) Objective function: p = 5x + 7y; Corners: (0,0), (8,4), (6,5), (0,8), and (12,0)

Answer to 13: (8,4)

(14) Objective function: p = 20x + 12y; Corners: (18, 9), (20, 0), (0, 36), and <math>(12, 10)

Answer to 14: (18,9)

(15) Objective function: p = 95x + 72y; Corners: (0,0), (5,7), (3,9), (0,10), and (7,0)

Answer to 15: (5,7)

(16) Objective function: p = 28x + 32y; Corners: (0,0), (12,9), (9,15), (0,13), and (15,0)

Answer to 16: (9,15)

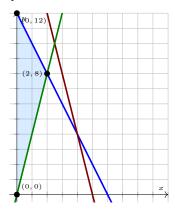
- A graphic designer can design a magazine cover or a logo. Her company makes a profit of \$800 for each magazine cover and \$500 for each logo. She estimates that it takes her 4 hours of brainstorming for a magazine cover and 2 hours of brainstorming for a logo. She'd like to keep the total brainstorming time under 24 hours a week. Further, she estimates that it takes her 2 hours to lay out a magazine cover and 0.5 hours to sketch up a logo, and she must fit this into 10 hours a week. Her boss requires her to design no more than 4 logos for each magazine cover she designs. How many of each should she design in order to maximize the company's profits? What is the maximum profit?
 - Variables: x = number of magazine covers; y = number of logos
 - Objective function: f = 800x + 500y
 - Constraints: (plus $x \ge 0$ and $y \ge 0$)

$$4x + 2y \le 24$$

$$2x + 0.5y \le 10$$

$$y \le 4x$$

• Feasible region and corner points:



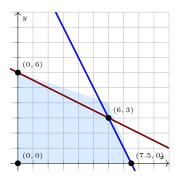
Answer to 17: x = 2 magazine covers; y = 8 logos; profit = \$5600

- A manufacturer of ski clothing makes ski pants and ski jackets. The profit on a pair of ski pants is \$2.00 and the profit on a jacket is \$1.50. Both pants and jackets require the work of sewing operators and cutters. There are 60 minutes of sewing operator time and 48 minutes of cutter time available. It takes 8 minutes to sew one pair of ski pants and 4 minutes to sew one jacket. Cutters take 4 minutes on pants and 8 minutes on a jacket. Find the maximum profit and the number of pants and jackets the manufacturer should make in order to maximize the profit.
 - Variables: x = number of ski pants; y = number of ski jackets

• Constraints: (plus $x \ge 0$ and $y \ge 0$)

$$8x + 4y \le 60$$
$$4x + 8y \le 48$$

• Feasible region and corner points:



Answer to 18: x = 6 ski pants; y = 3 ski jackets; profit = \$16.50

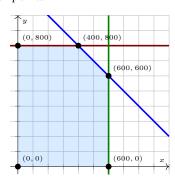
(19) An automotive plant makes the Quartz and the Pacer. The plant has a maximum production capacity of 1200 cars per week, and they can make at most 600 Quartz cars and 800 Pacers each week. If the profit on a Quartz is \$500 and the profit on a Pacer is \$800, find how many of each type of car the plant should produce. What is the maximum profit?

• Variables: x = number of Quartzes; y = number of Pacers

- Objective function: f = 500x + 800y
- Constraints: (plus $x \ge 0$ and $y \ge 0$)

$$x + y \le 1200$$
$$x \le 600$$
$$y \le 800$$

• Feasible region and corner points:



Answer to 19: x = 400 Quartzes; y = 800 Pacers; profit = \$840,000

(20) A farmer has a field of 70 acres in which he plants potatoes and corn. The seed for potatoes costs \$20/acre, the seed for corn costs \$60/acre, and the farmer has set aside \$3000 to spend on seed. The profit per acre of potatoes is \$150 and the profit for corn is \$50 an acre. How many acres of each should the farmer plant? What is the maximum profit?

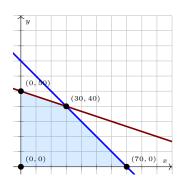
• Variables: x = acres of potatoes; y = acres of corn

• Objective function: f = 150x + 50y

• Constraints: (plus $x \ge 0$ and $y \ge 0$)

$$x + y \le 70$$
$$20x + 60y \le 3000$$

• Feasible region and corner points:



Answer to 20: x = 70 acres of potatoes; y = 0 acres of corn; profit = \$10,500

(21) A manufacturer produces two models of mountain bikes. The times (in hours) required for assembling and painting each model are given by the following table:

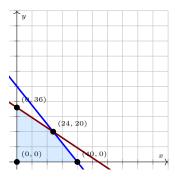
	Model A	Model B
Assembling	5	4
Painting	2	3

The maximum total weekly hours available in the assembly department and the painting department are 200 hours and 108 hours, respectively. The profits per unit are \$25 for Model A and \$15 for Model B. How many of each type should be produced to maximize profit? What is the maximum profit?

- Variables: x = number of Model A; y = number of Model B
- Objective function: f = 25x + 15y
- Constraints: (plus $x \ge 0$ and $y \ge 0$)

$$5x + 4y \le 200$$
$$2x + 3y \le 108$$

• Feasible region and corner points:



Answer to 21: x = 40 bikes of Model A; y = 0 bikes of Model B; profit = \$1000

(22) A student earns \$10 per hour for tutoring and \$7 per hour as a teacher's aide. To have enough free time for studies, he can work no more than 20 hours per week. The tutoring center requires that each tutor spends at least three hours per week tutoring, but no more than eight hours per week. How many hours should he work to maximize his earnings? What are the maximum earnings?

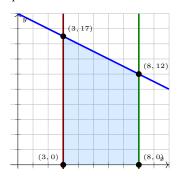
- Variables: x = hours tutoring; y = hours as TA
- Objective function: f = 10x + 7y
- Constraints: (plus $x \ge 0$ and $y \ge 0$)

$$x + y \le 20$$

$$x \ge 3$$

$$x \leq 8$$

• Feasible region and corner points:



Answer to 22: x = 8 hours tutoring; y = 12 hours as a TA; profit = \$164

Logic

SECTION 6.1 Statements and Logical Operations

In	$problems\ 1–10,\ determine\ whether\ or\ not\ each$	sentence	$is \ a \ s$	tatement.
1	1024 is the smallest 4-digit number that is a p	erfect squ	are.	
		Answer	to 1:	Yes, this is a statement.
(2)	Doc Brown's time machine requires 1.21 jigow.	atts of po	wer to	o operate.
		Answer	to 2:	Yes, this is a statement.
(3)	All elephants are purple.			
•		Answer	to 3:	Yes, this is a statement.
				,
4)	Don't do drugs.	NT (1:	• ,	1 1 1
	Answer to 4:	No, this	is not	a statement (command).
5	$64 = 2^5$			
		Answer	to 5:	Yes, this is a statement.
6)	Lizard people run the government.			
		Answer	to 6:	Yes, this is a statement.
	W/l4 l11:2			
\mathcal{O}	What are you holding?	No thi	. i	ot a statement (question).
_	Answer to 7	· No, till	s is iic	ot a statement (question).
8	Don't believe everything you read.			
	Answer to 8:	No, this	is not	a statement (command).
9	More soldiers died from sickness than combat	in the Re	volutio	onary War.
		Answer	to 9:	Yes, this is a statement.
10	This statement is a lie			
10)	This statement is a lie.	:- : 4 -		
				ment (self-contradictory).
In	problems 11–13, let p and q represent the follo	owing state	ement.	s:
	p: It will rain this weekend.			
	q: We can go to the mall on Saturday.			
Write	e each of the following statements in symbolic for	form.		
11)	"It won't rain this weekend."			
$\overline{}$				Answer to 11: $\sim p$
12	"Either it will rain this weekend, or we can go	to the me	all on	Saturday"
12)	Elimer it will rain this weekend, or we can go	to the ma	an on	Answer to 12: $p \lor q$
13)	"We can't go to the mall on Saturday, but at l	least it wo	on't ra	in this weekend."
			A	nswer to 13: $\sim q \land \sim p$
In	problems 14–16, let p and q represent the following	owing state	ement.	s:
	p: Interest rates are low.			
	q: It is not time to buy a house.			

Write each of the following statements in symbolic form.

(14) "It is time to buy a house; interest rates are low."
Answer to 14: $\boxed{ \sim q \wedge p }$
(15) "Interest rates are not low, and it is not time to buy a house."
Answer to 15: $\boxed{\sim p \land q}$
(16) "Either it is time to buy a house, or interest rates are not low."
Answer to 16: $\boxed{\sim q \lor \sim p}$
In problems 17–22, let p and q represent the following statements:
p: You study hard. q: You graduate with honors.
Write each of the following statements in symbolic form.
(17) "You study hard and you graduate with honors."
The study hard and you graduate with hollors. Answer to 17: $p \wedge q$
(18) "You don't study hard, but you graduate with honors anyway."
Answer to 18: $\boxed{\sim p \wedge q}$
(19) "You study hard, but you still don't graduate with honors."
Answer to 19: $p \land \sim q$
(20) "You study hard or you don't graduate with honors."
Answer to 20: $p \lor \sim q$
(21) "You don't study hard or you graduate with honors anyway."
Answer to 21: $\sim p \vee q$
(22) "You don't study hard, or you don't graduate with honors."
Answer to 22: $\boxed{\sim p \lor \sim q}$
In problems 23–26, let p and q represent the following statements:
p: Kevin Durant will win the MVP award.
q: The Lakers will win the NBA championship.
Write each of the following statements in words.
$egin{pmatrix} oldsymbol{23} & p ee q \end{pmatrix}$
Kevin Durant will win the MVP award, or the Lakers will win the NBA championship.
Kevin Durant will win the MVP award, and the Lakers will win the NBA championship.
$oxed{25}\sim pee q$
Answer to 25:
Kevin Durant will not win the MVP award, or the Lakers will win the NBA championship.
(26) $p \wedge \sim q$
Answer to 26: Kevin Durant will win the MVP award, and the Lakers will not win the NBA championship.

In problems 27–30, let p and q represent the following statements:

p: This is a dog.

q: This is a mammal.

Write each of the following statements in words.

$$(27) \sim p \wedge q$$

Answer to 27: This is not a dog, but it is a mammal.

$$(28)$$
 $p \wedge \sim q$

Answer to 28: This is a dog, and it is not a mammal.

$$(29) \sim p \lor \sim q$$

Answer to 29: This is not a dog, or it is not a mammal.

$$(30) \sim q$$

Answer to 30: This is not a mammal.

In problems 31–34, let p and q represent the following statements:

p: Eggs make a good breakfast.

q: Bacon is not healthy.

Write each of the following statements in words.

$$(31) \sim p$$

Answer to 31: Eggs do not make a good breakfast.

$$(32)$$
 $p \lor \sim q$

Answer to 32: Eggs make a good breakfast, or bacon is healthy.

$$(33) \sim p \wedge \sim q$$

Answer to 33: Eggs do not make a good breakfast, and bacon is healthy.

$$(34) \sim p \vee q$$

Answer to 34: Eggs do not make a good breakfast, or bacon is not healthy.

In problems 35–40, evaluate the truth value of the given statement for the given truth values of p, q, and r.

(35) $p \lor \sim q$, where p is T and q is F.

Answer to 35: T

$$T \vee \sim F = T \vee T = T$$

(36) $\sim p \wedge q$, where p is F and q is F.

Answer to 36: F

$$\sim F \wedge F = T \wedge F = F$$

(37) $p \wedge (q \vee \sim r)$, where p is T, q is F, and r is F.

Answer to 37: T

$$T \wedge (F \vee \sim F) = T \wedge (F \vee T) = T \wedge T = T$$

Answer to 38: T

$$\sim (F \land \sim F) = \sim (F \land T) = \sim F = T$$

(39) $(p \land q) \lor r$, where p is F, q is T, and r is T.

Answer to 39: T

$$(F \wedge T) \vee T = F \vee T = T$$

(40) $(p \lor (\sim p \lor q)) \land \sim (q \land r)$, where p is F, q is F, and r is T.

Answer to 40: T

$$(F \lor (\sim F \lor F)) \land \sim (F \land T) = (F \lor (T \lor F)) \land \sim (F) = (F \lor T) \land T = T \land T = T$$

In problems 41-46, fill in the blanks in each truth table.

	p	q	$\sim p$	$\sim p \wedge q$
41)	T T F	T F T F	F F T	F F T F

	p	q	$\sim p$	$p \wedge q$	$(p \land q) \lor \sim p$
42)	T	T	F	T	T
	T	F	F	F	F
	F	T	T	F	T

	p	q	$\sim p$	$\sim q$	$p \lor \sim q$	$\sim p \land (p \lor \sim q)$
43)	T T F	T F T	F F T	F T F	T T F	F F F
	T,	1,	1 1	1 1	1	1

	p	q	$\sim p$	$\sim q$	$\sim p \lor q$	$p \land \sim q$	$(p \land \sim q) \lor (\sim p \lor q)$
44)	T T F	T F T F	F F T	F T F T	T F T T	F T F F	T T T

$egin{array}{ c c c c c c c c c c c c c c c c c c c$	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	F F T F F T

(46)

p	q	r	$\sim p$	$\sim q$	$\sim r$	$ \hspace{.05cm} \sim p \hspace{.05cm} ee q \hspace{.05cm} $	$p \lor \sim r$	$\sim p \vee \sim q$	$(\sim p \ \lor q) \land (p \lor \sim r)$	$[(\sim p \ \lor q) \land (p \lor \sim r)] \land (\sim p \lor \sim q)$
T T F T T F	T F T F T F	T T T F F F	F F T F F T	F T F T F T	F F F T T	T F T T F T	T T F T T T	F T T F T T	T F F T T T	F F F F T T

In problems 47–50, write the negation of each quantified statement.

(47) All mammals are land-based.

Answer to 47: Some mammals are not land-based.

(48) Some horses are not domesticated.

Answer to 48: All horses are domesticated.

(49) Some trains are late.

Answer to 49: All trains are not late.

(50) All basketball players are not tall.

Answer to 50: Some basketball players are tall.

SECTION 6.2 Conditionals and Equivalence

In problems 1–8, let p and q represent the following statements:

p: You studied.

q: You passed this course.

Write each of the following statements in symbolic form.

1 If you passed this course, you studied.

Answer to 1: $q \to p$

2 You passed this course if and only if you studied.

Answer to 2: $q \leftrightarrow p$

(3) You didn't study if and only if you passed this course.

Answer to 3: $\sim p \leftrightarrow q$

4 You didn't study if you didn't pass this course.

Answer to 4: $\sim q \rightarrow \sim p$

Write each of the following statements in words.

 $\boxed{\mathbf{5}} \ p \to q$

Answer to 5: If you studied, then you passed this course.

Answer to 6: You didn't study if and only if you didn't pass this course.

 $(7) \sim q \rightarrow p$

Answer to 7: If you didn't pass this course, then you studied.

 $(8) \sim p \rightarrow q$

Answer to 8: If you didn't study, then you passed this course.

In problems 9–16, let p, q, and r represent the following statements:

p: The ice cream truck is here.

q: The pool is open.

r: It is summertime.

Write each of the following statements in symbolic form.

(9) If the ice cream truck is here, and the pool is open, then it is summertime.

Answer to 9: $(p \land q) \rightarrow r$

(10) It isn't summertime if the ice cream truck isn't here and the pool isn't open.

Answer to 10: $(\sim p \land \sim q) \rightarrow \sim r$

(11) It's summertime if and only if the pool is open.

Answer to 11: $r \leftrightarrow q$

(12) If it isn't summertime, the pool isn't open or the ice cream truck isn't here.

Write each of the following statements in words.

 $\overbrace{\mathbf{13}}) \ (p \lor q) \to r$

Answer to 13: If the ice cream truck is here or the pool is open, then it is summertime.

(14) $r \to (p \land q)$

Answer to 14: If it is summertime, then the ice cream truck is here and the pool is open.

(15) $p \leftrightarrow q$

Answer to 15: The ice cream truck is here if and only if the pool is open.

(16) $p \leftrightarrow (q \land r)$

Answer to 16:

The ice cream truck is here if and only if the pool is open and it is summertime.

In problems 17–24, fill in the blanks in each truth table.

	p	q	$q \rightarrow p$	$\sim (q \rightarrow p)$
17	T	T	T	F
	T	F	T	F
	F	T	F	T
	F	F	T	F

	p	q	$\sim p$	$\sim q$	$\sim p \leftrightarrow \sim q$
18	T	T	F	F	T
	T	F	F	T	F
	F	T	T	F	F

	p	q	$p \lor q$	$q \to (p \lor q)$
19	T T F	T F T F	T T T F	T T T

	p	q	$p \wedge q$	$\sim p$	$\sim q$	$\sim p \land \sim q$	$(p \land q) \to (\sim p \land \sim q)$
20	T	T	T	F	F	F	F
	T	F	F	F	T	F	T
	F	T	F	T	F	F	T

	p	q	$q \rightarrow p$	$p \rightarrow q$	$(q \to p) \land (p \to q)$
21	T T F	T F T	T T F T	T F T	T F F
21	T T F F	T F T F	T T F T	T F T T	T F F T

	p	q	$p \rightarrow q$	$\sim q$	$(p \to q) \land \sim q$
22	T T F F	T F T F	T F T T	F T F T	F F T

	p	q	r	$\sim p$	$\sim r$	$q \rightarrow \sim p$	$\sim r \wedge (q \rightarrow \sim p)$
24)	T T F T T F F	T F T F T F	T T T F F F F	F F T T F T	FFFTTT	F T T T T T	F F F F T T

In problems 25–28, determine if each statement is a tautology or not.

(25) $p \land \sim p$

Answer to 25: No, this is not a tautology.

(26) $p \lor \sim p$

Answer to 26: Yes, this is a tautology.

 $(27) (p \wedge q) \vee (\sim p \wedge q) \leftrightarrow q$

Answer to 27: Yes, this is a tautology.

(28) $(q \to p) \lor (\sim q \to \sim p)$

Answer to 28: Yes, this is a tautology.

In problems 29-30, select the statement that is equivalent to the one given.

(29) "Either the Cardinals or the Yankees will win the World Series."

Answer to 29:

- (a) If the Cardinals don't win the World Series, the Yankees will win it.
- (a) If the Cardinals don't win the World Series, the Yankees will win it.
- (b) The Cardinals and the Yankees will win the World Series.
- (c) If the Cardinals win the World Series, the Yankees will not win it.
- (d) If the Yankees win the World Series, the Cardinals will not.
- (30) "If the light is on, someone is home."

Answer to 30: (d) If someone is not home, the light is not on.

- (a) The light is on and someone is home.
- (b) If someone is home, the light is on.
- (c) Either someone is not home, or the light is on.
- (d) If someone is not home, the light is not on.

In problems 31-32, write the converse, inverse, and contrapositive of the given statement.

- (31) If you drive through the field, you see fireflies.
 - Converse: If you see fireflies, you drive through the field.
 - Inverse: If you don't drive through the field, you don't see fireflies.
 - Contrapositive: If you don't see fireflies, you don't drive through the field.
- (32) If you go to office hours, you pass the test.
 - Converse: If you pass the test, you go to office hours.
 - Inverse: If you don't go to office hours, you don't pass the test.
 - Contrapositive: If you don't pass the test, you don't go to office hours.

SECTION 6.3 Logic Rules

In problems 1-8, write the negation of each conditional statement.

1 If a fruit is blue, then it is not a banana.

A fruit is blue and it is a banana.

2 If the storm comes through, that awning will blow away.

The storm comes through and that awning does not blow away.

3 You'll catch a cold if you don't take Vitamin C.

You don't take Vitamin C and you don't catch a cold.

Logic You'll get a three percent return on your investment if you invest with us. You invest with us and you do not get a three percent return on your investment. (5) If you get an engineering degree, you'll be offered a job as soon as you graduate. You get an engineering degree and you are not offered a job as soon as you graduate. (6) If you pass this course, you will graduate this semester. You will pass this course, and you will not graduate this semester. (7) If your score is between 12 and 17, you will place into the first course. Your score is between 12 and 17 and you do not place into the first course. (8) If your GPA is over 3.7 and you live on campus, you are eligible for this scholarship. Your GPA is over 3.7 and you live on campus, but you are not eligible for this scholarship. In problems 9-12, use the distributive laws to write a statement that is logically equivalent to each given statement. Either the bridge will hold, or those cables will snap and the roadway will crack.

> The bridge will hold or those cables will snap, and the bridge will hold or the roadway will crack.

You either meet the job requirements or you don't, but you will not get the job.

Either you meet the job requirements and you will not get the job, or you don't meet them and you will not get the job.

Either get your grades up and get a job, or you won't get a car.

Get your grades up or you won't get a car, and get a job or you won't get a car.

This band is from Texas, and they have either three or four members.

This band is from Texas and they have three members, or they are from Texas and have four members.

In problems 13-16, use De Morgan's Laws to write a statement that is logically equivalent to each given statement.

(13) It is not true that North Dakota and East Dakota are both states.

North Dakota is not a state, or East Dakota is not a state.

(14) It is not true that this chapter covers logic and finance.

This chapter does not cover logic or it does not cover finance.

(15) It is not true that this book is entertaining or educational.

This book is not entertaining, and it is not educational.

(16) It is not true that today is Wednesday or Thursday.

Today is not Wednesday, and today is not Thursday.

In problems 17-20, use De Morgan's Laws to write the negation of each given statement.

(17) I pay taxes and I vote.

I don't pay taxes or I don't vote.

(18) Either the Packers or the Broncos will win the Super Bowl.

The Packers will not win the Super Bowl, and neither will the Broncos.

(19) Either that smoothie contains green vegetables, or it isn't as healthy as it looks.

That smoothie doesn't contain green vegetables, and it is as healthy as it looks.

(20) Class isn't over, and that clock is fast.

Class is over, or that clock isn't fast.

In problems 21–24, write the contrapositive of each conditional statement.

(21) If he is guilty, he won't testify at his trial.

If he testifies at his trial, he isn't guilty.

(22) If the cat is running, he either spotted a mouse or he spotted a squirrel.

If the cat didn't spot a mouse and he didn't spot a squirrel, he isn't running.

(23) If you do not report for jury duty, or you falsify your information, you will be prosecuted.

If you are not prosecuted, you reported for jury duty and you didn't falsify your information.

(24) If you give your plants water and sunlight, they will survive.

If your plants don't survive, you didn't give them water or you didn't give them sunlight.

SECTION 6.4 Arguments

In problems 1-4, determine whether each argument is inductive or deductive.

1 The last mayor was honest. The current mayor is honest. All mayors are honest.

Answer to 1: Inductive

(2) Every word has the letter e in it. Your name has the letter e in it.

Answer to 2: Deductive

(3) All deserts have some plant life. Some plants live in the Gobi Desert.

Answer to 3: Deductive

4 The sun rose yesterday, and it has risen every other day of my life. Therefore, the sun rises every day.

Answer to 4: Inductive

In problems 5-6, use a diagram (an Euler diagram) to evaluate whether each argument is valid.

(5) Every concert pianist can play *Chopsticks*. Ellen can play *Chopsticks*, so she must be a concert pianist.

Answer to 5: Invalid

(6) Every computer program has some bugs in it. Microsoft Excel is a computer program, so it has some bugs.

Answer to 6: Valid

In problems 7–14, use a truth table to determine whether the given argument is valid.

$$\begin{array}{c}
 p \to \sim q \\
 q \\
 \vdots \sim p
\end{array}$$

Answer to 7: Valid

$$\begin{array}{c}
 p \to q \\
 \sim p \\
 \hline
 \therefore q
\end{array}$$

Answer to 8: Invalid

$$\begin{array}{c}
 p \to q \\
 q \to p \\
 \vdots p \land q
\end{array}$$

Answer to 9: Invalid

$$\underbrace{ \begin{array}{c} \mathbf{10} \\ \mathbf{10} \\ \underline{} \\ \underline{} \\ \underline{} \\ \end{array} \begin{array}{c} p \rightarrow q \\ \underline{} \\ \underline{} \\ \\ \underline{} \\ \end{array} \begin{array}{c} p \rightarrow q \\ \underline{} \\ \\ \underline{} \\ \end{array} \begin{array}{c} \\ \\ \\ \\ \end{array} \begin{array}{c} \\ \\ \\ \end{array} \begin{array}{c$$

Answer to 10: Invalid

$$\underbrace{11} \begin{array}{c}
p \to q \\
q \land r \\
\vdots \quad p \lor r
\end{array}$$

Answer to 11: Valid

$$\underbrace{ \begin{array}{c} p \leftrightarrow q \\ q \rightarrow r \\ \hline \vdots \sim r \rightarrow \sim p \end{array} }$$

Answer to 12: Valid

Answer to 13: Invalid

Answer to 14: Invalid

In problems 15–22, determine which of the standard argument forms matches the given argument, and indicate whether this is a valid argument.

(15) If it is cold, my windows frost over. My windows are not frosted over, so it is not cold.

Answer to 15: Contrapositive Reasoning (Valid)

(16) You must eat well or you will not be healthy. I eat well, therefore I am healthy.

Answer to 16: Disjunctive Misuse (Invalid)

(17) We must build a hydroelectric plant or a nuclear plant. We won't build a nuclear plant, so we must build a hydroelectric plant.

Answer to 17: Disjunctive Reasoning (Valid)

(18) If we open the window, we will hear the birds. We hear the birds. Therefore, we opened the window.

Answer to 18: Fallacy of the Converse (Invalid)

(19) If I'm tired, I'm cranky. I'm tired. Therefore, I'm cranky.

Answer to 19: Direct Reasoning (Valid)

106 CHAPTER 6 Logic

(20) If everyone obeyed the law, no jails would be needed. Not everyone obeys the law, so some jails are needed.

Answer to 20: Fallacy of the Inverse (Invalid)

(21) If you pass this course, you will graduate. If you graduate, you will get a job. Therefore, if you get a job, you must have passed this course.

Answer to 21: Transitive Misuse (Invalid)

(22) If you study the old masters, your art will improve. If your art improves, you will get accepted to the art institute. Therefore, if you study the old masters, you will get accepted to the art institute.

Answer to 22: Transitive Reasoning (Valid)

Set Theory

SECTION 7.1 Basic Concepts

In problems 1-4, write each set using roster notation.

 \bigcirc A = the set of letters in the word "Mississippi."

Answer to 1: $A = \{m, i, s, p\}$

(2) B =the set of the four seasons in a year.

Answer to 2: $B = \{winter, spring, summer, fall\}$

(3) C =the set of natural numbers less than 6.

Answer to 3: $C = \{1, 2, 3, 4, 5\}$

(4) D = the set of even natural numbers between 7 and 13.

Answer to 4: $D = \{8, 10, 12\}$

In problems 5–13, fill in the blank with either \in or \notin to make each statement true.

5 6 _____{{2,4,6,8,10}}

Answer to 5: \in

(6) 7 ____{{2,4,...,12,14}}

Answer to 6: $\boxed{\notin}$

7 11 _____{{1,2,3,...,9}}

Answer to 7: \notin

8 37 _____{{1,2,3,...,50}}

Answer to 8: \in

(9) $3 = \{x \in \mathbb{Z} \mid x > -2\}$

Answer to 9: \in

(10) -2 _____N

Answer to 10: $|\notin|$

Answer to 11: \notin

Answer to 12: \subseteq

Answer to 13: \in

In problems 14–17, write each set using roster notation.

Answer to 14: $\{1, 3, 5, 7, 9\}$

 $(15) \{x \in \mathbb{N} \mid 2 \le x \le 5 \text{ and } x \text{ is even} \}.$

Answer to 15: $\{2,4\}$

 $(16) \{x \in \mathbb{N} \mid x < 4\}.$

Answer to 16: $\{1, 2, 3\}$

 $(17) \{ x \in \mathbb{N} \mid 3 < x < 7 \}.$

Answer to 17: [4, 5, 6]

In problems 18–23, fill in the blank with either \subseteq or $\not\subseteq$ to make each statement true.

(18) $\{1,3,6\}$ $_\{1,2,3,4\}$

Answer to 18: $\boxed{\cancel{\angle}}$

(19) {2,4,6} $_$ {1,2,...,10}

Answer to 19: \subseteq

(20) {-1,0,2} \mathbb{N}

(21) Ø \mathbb{N}

Answer to 21: \subseteq

22 $\{b, c, d, e\}$ _____ $\{a, b, c, d, e, f, g, h\}$

Answer to 22: \subseteq

 $(23) \{x \mid x \text{ is a cat}\} = \{x \mid x \text{ is a black cat}\}$

Answer to 23: $\boxed{\not\subseteq}$

In problems 24–29, fill in the blank with either \subset or \swarrow to make each statement true.

(24) $\{x, y, z\}$ _____{} $\{x, u, w, v, z, y, t\}$

Answer to 24: \subset

(25) {0,3,4,7,1} ____{1,3,0,7,4}

Answer to 25: \mathbb{Z}

- (28) $\{x \in \mathbb{N} \mid 5 < x < 12\}$ _____the set of natural numbers between 5 and 12
 - Answer to 28: Z

(29) {} ____Ø

Answer to 29: \mathbb{Z}

In problems 30-32, determine whether each set is empty or not.

 $\widehat{\mathbf{30}} \ A = \{0\}$

Answer to 30: Not empty

(31) $B = \{x \mid x \text{ is a month of the year whose name begins with the letter X} \}$

Answer to 31: Empty

(32) $C = \{x \mid x < 2 \text{ and } x > 7\}$

Answer to 32: Empty

In problems 33–38, find the cardinality of each set.

 $(33) A = \{12, 14, 16, 18, 20\}$

Answer to 33: n(A) = 5

 $\mathbf{(34)} \ B = \{1, 3, 5, \dots, 25\}$

Answer to 34: n(B) = 25

 $(35) \ C = \{ x \in \mathbb{N} \mid 3 \le x < 14 \}$

Answer to 35: n(C) = 11

 $(36) D = \{x \in \mathbb{N} \mid x < 2 \text{ and } x \ge 5\}$

Answer to 36: n(D) = 0

 $(37) E = \emptyset$

Answer to 37: n(E) = 0

(38) $F = \{x \mid x \text{ is a letter} \}$ in the word "elephant"

Answer to 38: n(F) = 7

SECTION 7.2 Set Operations

In problems 1-28, let $U = \{1, 2, 3, ..., 10\}$, $A = \{1, 3, 5, 7\}$, $B = \{1, 2, 3, 4\}$, and $C = \{3, 4, 6, 7, 9\}$.

Find each of the following sets.

 \bigcirc 1 A^c

Answer to 1: $\{2,4,6,8,9,10\}$

 \bigcirc B^c

Answer to 2: $\{5, 6, 7, 8, 9, 10\}$

 \bigcirc $A \cup C$

Answer to 3: $\{1, 3, 4, 5, 6, 7, 9\}$

(4) $A \cap B$

Answer to 4: $\{1,3\}$

 \bullet $B \cap C$

Answer to 5: [3, 4]

 \bigcirc $A \cup B$

Answer to 6: $\{1, 2, 3, 4, 5, 7\}$

(7) $A \setminus B$

Answer to 7: $\{5,7\}$

(8) $C \setminus A$

Answer to 8: [4, 6, 9]

Answer to 9: 6, 8, 9, 10

(10) $A^c \cup C$

Answer to 10: $\{2, 3, 4, 6, 7, 9, 10\}$

 $(11) B \cup C^c$

Answer to 11: $\{1, 2, 3, 4, 5, 8, 10\}$

Answer to 12: $\{5,7\}$

 $(\mathbf{13}) (A \cup C^c)^c$

Answer to 13: [4,6,9]

 $(14) (B^c \cap C)^c$

Answer to 14: $\{1, 2, 3, 4, 5, 8, 10\}$

 $(15) (A \cap B)^c$

Answer to 15: $\{2,4,5,6,7,8,9,10\}$

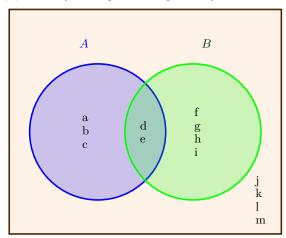
${\bf 112} \quad {\bf CHAPTER} \,\, {\bf 7} \quad {\bf Set \,\, Theory}$

 $\fbox{\textbf{28}} \ A \cup B \cup C$

$(16) (B \cup A)^c$	Answer to 16: $[\{6, 8, 9, 10\}]$
$\fbox{\textbf{17}} \ A \cup \varnothing$	Answer to 17: $[1,3,5,7]$
$\begin{array}{ c c c c } \hline \bf 18 & B \cup \varnothing \end{array}$	Answer to 18: $[1, 2, 3, 4]$
$\fbox{\textbf{19}} \ \ C \cup \varnothing$	Answer to 19: $[3,4,6,7,9]$
\bigcirc 20) $B \cap \varnothing$	Answer to 20: \varnothing
	Answer to 21: $[\{1, 3, 4, 6, 7, 9\}]$
	Answer to 22: $[\{1, 3, 4\}]$
	Answer to 23: $[\{1, 2, 3, 4, 7\}]$
$ (24) (A \cap B) \cup (C \cap B) $	Answer to 24: $[1, 3, 4]$
$25) (B \cup A) \cap (B \cup C)$	Answer to 25: $[\{1, 2, 3, 4, 7\}]$
$ (26) (A \cup C)^c \cap B^c $	Answer to 26: $[6, 9]$
	Answer to 27: [3]

Answer to 28: [1, 2, 3, 4, 5, 6, 7, 9]

In problems 29-40, use the following Venn diagram to find each set.



- $(29) A^{\alpha}$
- (30) $A \cup B$
- $(\mathbf{31}) (A \cap B)^c$
- (32) $A^c \cup B^c$
- (33) $A \setminus B$
- (34) $B \setminus A$
- $(35) A^c \cap B^c$
- $(36) (A \cup B)^c$
- (**37**) *U*
- $\widehat{\mathbf{38}}) \ A^c \cup B$
- $\overbrace{\mathbf{39}} A \cap B^c$
- (40) $A \setminus U$

Answer to 29: f, g, h, i, j, k, l, m

Answer to 30: $\{a, b, c, d, e, f, g, h, i\}$

Answer to 31: $\{a, b, c, f, g, h, i, j, k, l, m\}$

Answer to 32: $\{a, b, c, f, g, h, i, j, k, l, m\}$

Answer to 33: $\{a,b,c\}$

Answer to 34: f,g,h,i

Answer to 35: $\{j, k, l, m\}$

Answer to 36: $\{j, k, l, m\}$

Answer to 37: $\{a, b, c, d, e, f, g, h, i, j, k, l, m\}$

Answer to 38: $\{d, e, f, g, h, i, j, k, l, m\}$

Answer to 39: 6a, b, c

Answer to 40: \emptyset

SECTION 7.3 Properties of Set Operations

Suppose that $U = \{1, 2, ..., 10\}$, $A = \{1, 3, 5, 7\}$, $B = \{2, 3, 4, 5\}$, and $C = \{4, 5, 6, 7, 8\}$.

 $Associative \ Identities$

(1) Show that $(A \cup B) \cup C = A \cup (B \cup C)$.

Answer to 1: both are $\{1, 2, 3, 4, 5, 6, 7, 8\}$

(2) Show that $(A \cap B) \cap C = A \cap (B \cap C)$.

Answer to 2: both are $\{5\}$

Distributive Identities

(3) Show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Answer to 3: both are $\{3, 5, 7\}$

 \bigcirc Show that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Answer to 4: both are $\{1, 3, 4, 5, 7\}$

De Morgan's Laws

(5) Show that $(A \cup B)^c = A^c \cap B^c$.

Answer to 5: both are $\{6, 8, 9, 10\}$

(6) Show that $(A \cap B)^c = A^c \cup B^c$.

Answer to 6: both are $\{1, 2, 4, 6, 7, 8, 9, 10\}$

SECTION 7.4 Survey Problems

(1)	A survey asked 200 people what beverage they drink in the morning, and offered two
\sim	possible choices: tea and coffee. Suppose that 20 answered tea only, 80 answered coffee
	only, and 40 answered both.

(a) How many people drink tea in the morning?

Answer to 1(a): 60

(b) How many people drink neither tea nor coffee?

Answer to 1(b): 60

(2) A survey asked 100 people whether they used Twitter or Facebook in the last month. Of those surveyed, a total of 40 used Twitter, 70 used Facebook, and 20 used both.

(a) How many people used only Facebook?

Answer to 2(a): 50

(b) How many people used neither Facebook nor Twitter?

Answer to 2(b): 10

(3) Out of 100 customers of Domino's Pizza, 60 ordered pizza with onions and pepperoni, 80 ordered it with pepperoni, and 72 ordered it with onions.

(a) How many ordered onions but not pepperoni?

Answer to 3(a): 12

(b) How many ordered pepperoni but not onions?

Answer to 3(b): 20

(c) How many ordered neither onions nor pepperoni?

Answer to 3(c): 8

4 Out of 100 students surveyed, 24 rent movies, 20 rent movies and go the theater, and 15 do neither.

(a) How many students only rent movies?

Answer to 4(a): 4

(b) How many students only go to the theater?

Answer to 4(b): | 61

(c) How many students go to the theater or rent movies?

Answer to 4(c): | 85

(5) An independent survey agency was hired by the Metro to find out how many people commute to their school or job. The agency interviewed 1000 commuters and submitted the following report:

631 came by car 373: car and bus 554 came by bus 301: bus and metro 759 came by metro 231: all three types of transportation

The Metro refused to accept the report, stating that it was inaccurate. Why?

Answer to 5: The numbers are not consistent with 1000 surveyed (adds up 1233)

(6) One hundred fifty people were surveyed and asked if they believed in UFOs, ghosts, and Bigfoot.

```
43 believed in UFOs 8: ghosts and Bigfoot 25 believed in Bigfoot 10: UFOs and ghosts 44 believed in ghosts 5: UFOs and Bigfoot 2 believed in all three
```

(a) How many people surveyed believed in at least one of these things?

Answer to 6(a): 91

	Answer to $6(b)$: 6
(c) How many people didn't believe in any of the three?	
(1) II	Answer to $6(c)$: 59
(d) How many people believed in Bigfoot only?	Answer to 6(d): 14
(7) A survey asked students whether they had seen Star Wars, Th Rings (LotR).	e Matrix, or Lord of the
24 had seen Star Wars 18 had seen The Matrix 12: The Matrix and L 20 had seen LotR 14: Star Wars and Lo 6 had seen all three	otR
(a) How many students have seen exactly one of these movies?	Answer to 7(a): 8
(b) How many students have seen only Star Wars?	· / _
	Answer to 7(b): 6
(c) How many students have seen <i>Star Wars</i> , but not <i>LotR</i> ?	Answer to 7(c): 12
(d) How many students have not seen <i>The Matrix</i> ?	Answer to 7(d): 14
(8) A survey was given asking whether respondents watch movies Redbox, or Amazon Video.	s at home from Netflix,
53 only use Netflix 48: only Netflix and 62 only use Redbox 24 only use Amazon Video 10 use all three 48: only Netflix and 30: only Netflix and 25: none of these	d Amazon
(a) How many people use Redbox?	
(b) How many people use at least one of these?	Answer to 8(a): 136
	A nswer to 8(b) : 243
(c) How many people were surveyed?	Answer to 8(c): 268
(9) A survey asked buyers whether color, size, or brand influenced the The results are below.	heir choice of cell phone.
5 only said color 8 only said size 53: only size and brand 102 said all three 20: only color and siz 42: only color and brand 20: none of these	nd
(a) How many people were influenced by brand?	
	Answer to 9(a): 213
(b) How many people were influenced by color or size?	Answer to 9(b): 230
(c) How many people were surveyed?	Answer to 9(c): 266

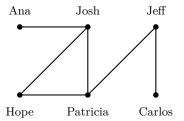
(b) How many people believed in ghosts and Bigfoot, but not UFOs?

8

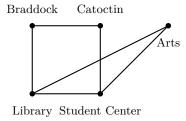
Graph Theory

SECTION 8.1 Introduction to Graphs

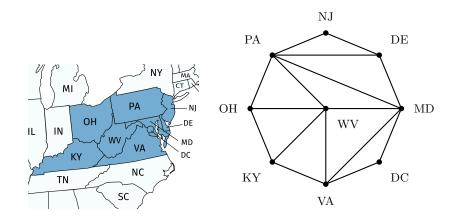
(1) There is a group of six people: Ana, Josh, Hope, Patricia, Jeff, and Carlos. Josh is friends with Patricia, Hope, and Ana; Jeff is friends with Patricia and Carlos; Patricia and Hope are friends. None of the other pairs of people are friends. Draw a graph to represent this group.



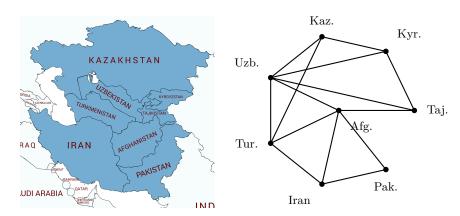
2 On campus, you travel between five buildings: Braddock Hall, Catoctin Hall, the library, the student center, and the arts building. There are walkways connecting Braddock Hall to Catoctin Hall and the library; the library is also connected to the arts building and the student center; there are also walkways connecting the student center to Catoctin Hall and the arts building. Draw a graph to represent this group of buildings.



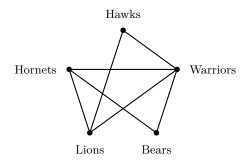
(3) The map below shows eight states and the District of Columbia highlighted in blue. Draw a graph to represent this map, where each node represents a state or district, and an edge represents a shared border between two regions.



4 The map below shows eight countries in Asia highlighted in blue. Draw a graph to represent this map, where each node represents a country, and an edge represents a shared border between two countries.



(5) The graph below represents a tournament; each edge marks a game between two teams.



(a) Did the Hornets and the Hawks play each other?

Answer to 5(a): No

(b) How many games do the Warriors play?

Answer to 5(b): $\boxed{4}$

(c) Which teams do the Bears play?

Answer to 5(c): Hornets and Warriors

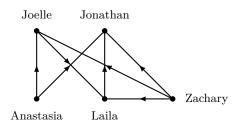
(d) Which team(s) played the most games?

Answer to 5(d): Warriors

(e) Which team(s) played the fewest games?

Answer to 5(e): Hawks and Bears (each played 2)

6 The graph below is an *influence graph*, where each edge represents the influence that one person has on another; the arrow goes from the influencer to the one they influence.



(a) Who does Laila influence?

Answer to 6(a): Jonathan

(b) Does Jonathan influence Anastasia?

Answer to 6(b): No

	(c)	How many people does Joelle influence?
		Answer to $6(c)$: 1
	(d)	Who is the most influential (influences the most people)?
		Answer to $6(d)$: Zachary (3)
	(e)	Who is the most influenced (influenced by the most people)?
		Answer to 6(e): Jonathan (3)
		elems 7–10, describe a graph that could be used to model the given application. Specifiver the following questions:
(a)	Are	pops allowed in this graph?
(b)	Are	nultiple edges allowed between the same pair of nodes?
(c)	Is th	s a simple graph or multigraph?
(d)	Is th	s graph directed or undirected?
(e)	Is th	s a complete graph (generally)?
7	_	ts between major cities, if each node represents a city and each edge describes a from one city to another (or from a city to itself, if there is a sightseeing or training).
	(a)	Are loops allowed in this graph?
		Answer to 7(a): Yes
	(b)	Are multiple edges allowed between the same pair of nodes?
	(a)	Answer to 7(b): Yes Is this a simple graph or multigraph?
	(c)	Answer to 7(c): Multigraph
	(d)	Is this graph directed or undirected?
		Answer to 7(d): Directed
	(e)	Is this a complete graph (generally)?
		Answer to $7(e)$: No
8		rty, where each node represents a person, and each edge represents whether one n knows the name of another.
	(a)	Are loops allowed in this graph?
		Answer to 8(a): Yes
	(b)	Are multiple edges allowed between the same pair of nodes?
	(a)	Answer to 8(b): Yes
	(c)	Is this a simple graph or multigraph? Answer to 8(c): Multigraph
	(d)	Is this graph directed or undirected?
	()	Answer to 8(d): Directed
	(e)	Is this a complete graph (generally)?
		Answer to 8(e): No
9	_	doorplan of a house, where each node represents a room or space (like a hallway), each edge represents a doorway.
	(a)	Are loops allowed in this graph?
		Answer to 9(a): No
	(b)	Are multiple edges allowed between the same pair of nodes?
	/ \	Answer to 9(b): Yes
	(c)	Is this a simple graph or multigraph? Answer to 9(c): Multigraph
	(4)	Answer to 9(c): Multigraph See this graph directed or undirected?
	(a)	Is this graph directed or undirected? Answer to 9(d): Undirected
		The work of the condition of the conditi

(e) Is this a complete graph (generally)?

Answer to 9(e): No

- (10) Courses offered at a college, where each node represents a course, and each edge represents a prerequisite requirement.
 - (a) Are loops allowed in this graph?

Answer to 10(a): No

(b) Are multiple edges allowed between the same pair of nodes?

Answer to 10(b): No

(c) Is this a simple graph or multigraph?

Answer to 10(c): Simple

(d) Is this graph directed or undirected?

Answer to 10(d): Directed

(e) Is this a complete graph (generally)?

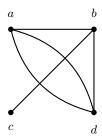
Answer to 10(e): No

For problems 11–14, answer the following questions for the given graph:

- (a) Is it a simple graph or multigraph?
- (b) Is it directed or undirected?

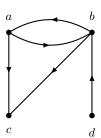
11

Answer to 11: (a) Multigraph (b) Undirected



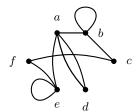
12

Answer to 12: (a) Multigraph (b) Directed

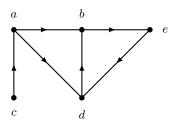


(13

Answer to 13: (a) Multigraph (b) Undirected

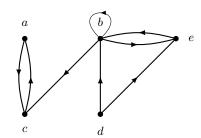


Answer to 14: (a) Simple graph (b) Directed



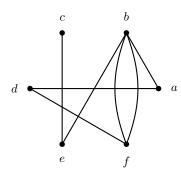
For problems 15–17, determine the degree of each node. For the directed graphs, determine both the in-degree and the out-degree of each node.



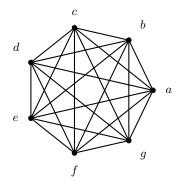


\mathbf{Node}	In-Degree	${\bf Out\text{-}Degree}$
a	1	1
b	3	3
c	2	1
d	0	2
e	2	1

(16)



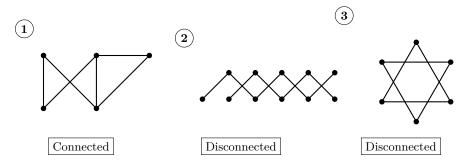
Node	Degree
\overline{a}	2
b	4
c	1
d	2
e	2
f	3



\mathbf{Node}	Degree
\overline{a}	6
b	6
c	6
d	6
e	6
f	6
g	6

SECTION 8.2 Euler and Hamilton Paths

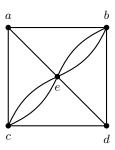
In problems 1-3, determine whether the given graph is connected or disconnected.



In problems 4–16, determine whether the given graph has an Euler circuit (and draw one if it exists). If not, determine whether it has an Euler path (and draw one if so).

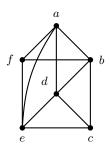
(4)

Answer to 4: Euler path: starting/ending at a/d



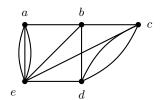
5

Answer to 5: Euler path: starting/ending at c/f



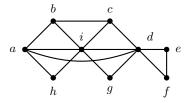
6

Answer to 6: Euler circuit



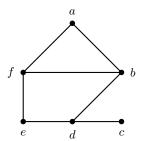
7

Answer to 7: Euler path: starting/ending at b/c



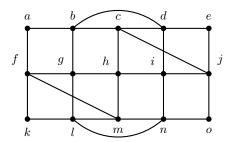
8

Answer to 8: No Euler circuit or path



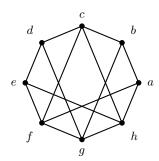
9

Answer to 9: Euler circuit



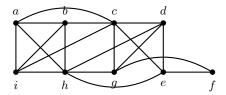
10

Answer to 10: No Euler circuit or path



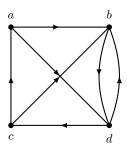
11

Answer to 11: Euler circuit



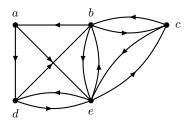
(12)

Answer to 12: No Euler circuit or path

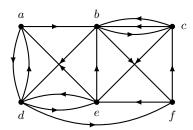


13

Answer to 13: Euler path: starting at a, ending at e

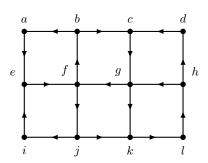


Answer to 14: Euler path: starting at c, ending at b



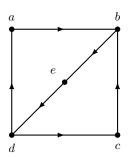
(15)

Answer to 15: No Euler circuit or path

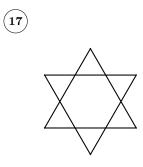


(16)

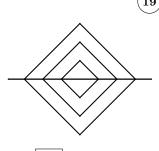
Answer to 16: Euler path: starting at d and ending at b

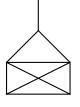


In problems 17–19, determine whether the picture shown could be drawn in one continuous motion without lifting the pencil or retracing part of the drawing.



Yes





Yes

No

(20) The map below shows ten states highlighted in blue. Is there a path that a traveler could take through these states in such a way that they cross each border between two states exactly once?



Answer to 20: Yes, starting/ending in MO/SC

21) France is divided into 18 administrative regions, of which twelve are contiguous. The map below shows seven of these regions highlighted in blue. Is there a path that a traveler could take through these regions in such a way that they cross each border between two regions exactly once?

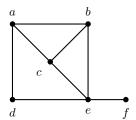


Answer to 21: Yes

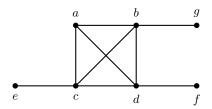
In problems 22–25, determine whether the given graph has a Hamilton circuit (and draw one if it does). If not, determine whether it has a Hamilton path (and draw one if so).



Answer to 22: Hamilton path (febcad, for instance)

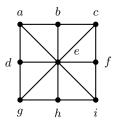


Answer to 23: No Hamilton circuit or path



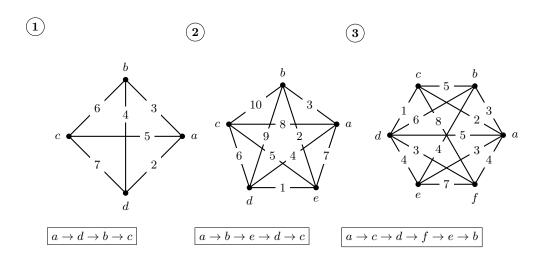
(24)

Answer to 24: Hamilton circuit (*abcefihgda*, for instance)

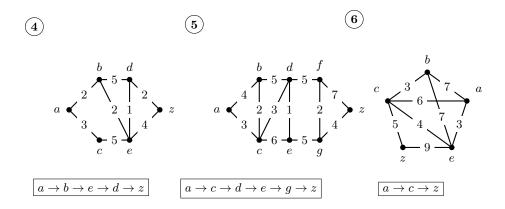


SECTION 8.3 Shortest Paths

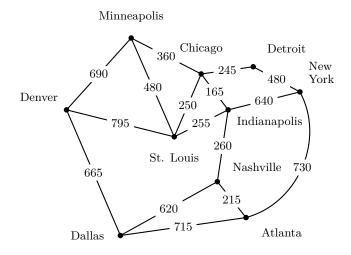
In problems 1-3, use the nearest neighbor algorithm to find a minimum possible circuit through each graph starting at a.



In problems 4–6, use Dijkstra's algorithm to find the shortest path through each graph between a and z.



(7) The graph below shows the distances between cities. Use the graph to answer the questions below.



(a) Use the nearest neighbor algorithm to find a path that starts in Chicago and visits all the cities shown, while trying to minimize distance traveled.

(b) What is the total length of the path found in part (a)?

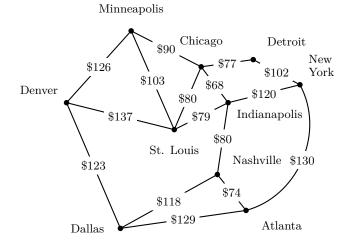
(c) Find the shortest path between Dallas and New York. What is the length of this path?

Answer to 7(c):
$$\overline{\mathrm{DAL} \to \mathrm{ATL} \to \mathrm{NYC} \ (1445 \ \mathrm{miles})}$$

(d) Find the shortest path between Minneapolis and Atlanta. What is the length of this path?

Answer to 7(d):
$$\boxed{\text{MIN} \rightarrow \text{CHI} \rightarrow \text{IND} \rightarrow \text{NAS} \rightarrow \text{ATL (1000 miles)}}$$

(8) The graph below shows the cost of flights between cities. Use the graph to answer the questions below.



(a) Use the nearest neighbor algorithm to find a path that starts in St. Louis and visits all the cities shown, while trying to minimize the cost of travel.

(b) What is the total cost of the path found in part (a)?

(c) Find the cheapest path between Nashville and Denver. What is the cost of this path?

Answer to 8(c):
$$\overline{\mathrm{NAS} \to \mathrm{DAL} \to \mathrm{DEN} \ (\$241)}$$

- (d) Find the cheapest path between Detroit and Denver. What is the cost of this path?

 Answer to 8(d): $\overline{DET \rightarrow CHI \rightarrow MIN \rightarrow DEN (\$293)}$
- 9 A salesperson has responsibility over four cities in Maryland and northern Virginia, and they compiled the distances between them; these distances are shown in the table below. If the salesperson needs to visit all four cities, and is currently in Ellicott City, use the nearest neighbor algorithm to plan their route. How far will they travel in total along this path?

	Annapolis	Alexandria	Ellicott City	Reston
Annapolis	_	45	26	46
Alexandria	45	_	37	19
Ellicott City	26	37	_	35
Reston	46	19	35	_

Answer to 9:
$$EC \rightarrow Ann. \rightarrow Alex. \rightarrow Res. (90 miles)$$

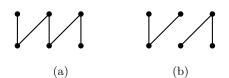
(10) An American tourist is traveling through Great Britain, and would like to visit five cities. The tourist has estimated the cost of a train ticket between pairs of these cities, and the results are shown in the table below. Plan a route that will take the tourist through all five cities with as little cost as possible, starting and ending in London. Use the nearest neighbor algorithm; what is the cost of this journey?

	London	Edinburgh	York	Cardiff	Chester
London	_	\$175	\$110	\$65	\$115
Edinburgh	\$175	_	\$95	\$195	\$105
York	\$110	\$95	_	\$145	\$60
Cardiff	\$65	\$195	\$145	_	\$85
Chester	\$115	\$105	\$60	\$85	-

Answer to 10: Lon.
$$\rightarrow$$
 Car. \rightarrow Ches. \rightarrow York \rightarrow Edin. \rightarrow Lon. (\$480)

SECTION 8.4 Trees

(1) Which of the following graphs are trees?







Answer to 1: Only (a) is a tree

(2) Which of the following graphs are trees?









Answer to 2: (a), (c), and (d) are trees

- (3) Build a binary search tree for the following numbers, sorted by value: 15, 29, 9, 11, 2, 31, 18, 3, 14, and 6. Add numbers to this tree in the order in which they are listed.
 - (a) How many comparisons are needed to locate 11 in this tree, starting from the top?

 Answer to 3(a): 2
 - (b) How many comparisons are needed to add 17 to this tree?

Answer to 3(b): $\boxed{4}$

(c) What is the parent of the node labeled 2?

Answer to 3(c): 9

(d) List the children of the node labeled 29.

Answer to 3(d): 18 and 31

- 4 Build a binary search tree for the following words, sorted alphabetically: gaffe, rebellion, fool, elaborate, spread, joke, freedom, stroke, guideline, and aware. Add words to this tree in the order in which they are listed.
 - (a) How many comparisons are needed to locate spread in this tree, starting from the top?

Answer to 4(a): 2

(b) How many comparisons are needed to add the word thorough to this tree?

Answer to 4(b): $\boxed{4}$

(c) What is the parent of the node labeled *elaborate*?

Answer to 4(c): fool

(d) List the children of the node labeled fool.

Answer to 4(d): elaborate and freedom

(5) Build a binary search tree for the following list of countries, sorting them by population. Add countries to this tree in the order in which they are listed.

Country	Population (millions)			
Philippines	108			
Vietnam	96			
Bangladesh	163			
France	65			
Mexico	128			
Germany	84			
Tanzania	58			
Nigeria	201			
Russia	146			
Italy	61			

(a) What is the parent node of Tanzania?

Answer to 5(a): France

(b) How many children does the Mexico node have?

Answer to 5(b): 1

(c) How many comparisons are needed to locate Nigeria in this tree, starting from the top?

Answer to 5(c): $\boxed{2}$

(6) Build a binary search tree for the following list of Major League Baseball teams, sorting them by total payroll. Add teams to this tree in the order in which they are listed.

Team	Payroll (millions)
<i>a</i>	
Cardinals	62
Cubs	70
Rangers	53
Reds	51
Phillies	63
Rockies	46
Red Sox	43
Brewers	37
Royals	32
Mariners	27

(a) What is the parent of the Phillies node?

Answer to 6(a): Cubs

(b) List the children of the Red Sox node.

Answer to 6(b): Brewers

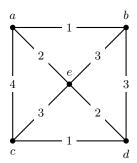
(c) How many comparisons are needed to locate the Royals in this tree, starting from the top?

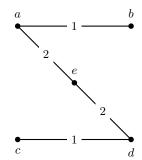
Answer to 6(c): $\boxed{6}$

In problems 7–9, use Kruskal's algorithm to find a minimum spanning tree for the given graph.

7

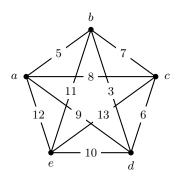
Answer to 7: shown below

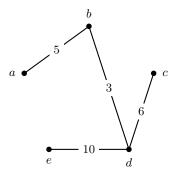




8

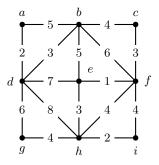
Answer to 8: shown below

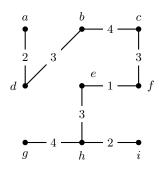




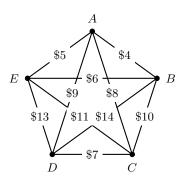
9

Answer to 9: shown below



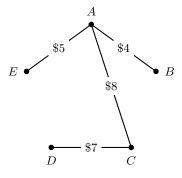


(10) A company requires reliable intranet and phone connectivity between their five offices (labeled A through E), so they decide to lease dedicated lines from the phone company. The phone company will charge for each link made. The graph below shows the costs, in thousands of dollars per year, for each link.

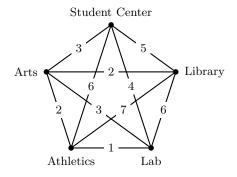


In order to save on costs, design a network that will connect these five offices with the lowest possible cost.

Answer to 10: shown below

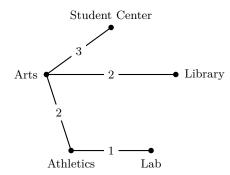


A maintenance team is responsible for a group of five buildings on campus. These buildings are shown in the graph below, with the distance given between each pair of buildings. After a blizzard, the team is tasked with clearing the snow, but there is not enough time to clear all the walkways.



Which walkways should the maintenance team plow in order to connect all the buildings, while minimizing the time needed to do so (really, by minimizing the distance)?

Answer to 11: shown below



(12) A power company needs to lay updated distribution lines connecting eight cities in Virginia to the power grid. The distances between these cities are given in the table below. Design a network that will minimize the amount of new line.

	Purcellville	Leesburg	Middleburg	Chantilly	Sterling	McLean	Arlington	Annandale
Purcellville	-	8	11	23	19	32	37	35
Leesburg	8	_	14	17	10	24	29	27
Middleburg	11	14	_	18	16	30	34	31
Chantilly	23	17	18	_	8	13	18	13
Sterling	19	10	16	8	_	15	20	17
McLean	32	24	30	13	15	_	5	7
Arlington	37	29	34	18	20	5	_	6
Annandale	35	27	31	13	17	7	6	_

What is the total required length of line that must be laid?

Answer to 12: 61 miles (minimal spanning tree shown below)

