CMPSC 465 – Spring 2021 — Solutions to Homework 8

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1. Getting started

(a) I did not work in a group.

(b) I did not consult with any of my group members.

(c) I did not consult any non-class materials.

2. Connectivity Detection

```
function detectConnectivity(G):
        for each edge (u, v):
                if visited(v) == true:
                        return "yes"
                else:
                        visited(u) = true
        return("no")
```

This algorithm essentially detects a cycle using the structure of the *explore*() method. We know that there needs to exist a cycle in a graph for an edge to be removed and still stay connected. In a connected, undirected graph, you can have at most $|V| - 1$ amount of edges before a cycle is formed. Therefore, this algorithm will iterate, at the most, $|V|$ times if there happens to be a cycle. Therefore, the runtime of this algorithm will be $O(|V|)$.

3. Shortest Path and MST

(a) CLAIM: The MST changes when for all edge weights $w'_e := w_e - 1$.

EXAMPLE:
Say we have a graph G, with vertices $V = \{A, B, C, D\}$. The edge weights are as follows:
$w_{(A,B)} = 3$
$w_{(A,C)} = 1$
$w_{(C,D)} = 1$
$w_{(D,B)} = 2$

The MST would look like this: $A - B$, $A - C$, $C - D$ with a total cost of 5.

Decreasing all the edge weights, we get a graph with the following edge weights:
$w_{(A,B)} = 2$
$w_{(A,C)} = 0$
$w_{(C,D)} = 0$
$w_{(D,B)} = 1$

The new MST would look like this: $A - C$, $C - D$, $D - B$ with a total cost of 1.

(b) CLAIM: The shortest path changes when for all edge weights $w'_e := w_e - 1$.

EXAMPLE: Say we have a an edge, (u, v), with $w_{(u,v)} = 2$. There is also another path from u to v that have edges with weights of $w_{(u,s)} = 1$, $w_{(s,t)} = 1$, $w_{(t,v)} = 1$.

Without decresing the edge weights, the shortest path would be $u - v$ with a cost of 2. However, if all edge weights are decreased 1, the path $u - s - t - v$ would cost 0 and become the new shortest path.

4.MST and Cut Property

CLAIM: All the vertices and edges in both T and H are in an MST of H.
PROOF:
We can assume that, by being a subgraph of G, H is essentially a cut. Let $(S, V - S)$ be a cut that respects H. By the cut property, we are guaranteed that the edge of lowest weight, e, connecting the cut will be included in T. In other words, H $\cup\{e\}$ will be included in T. This means that $H \subset T$. Since $H \subset T$, it cannot contain any cycles because T is a tree which is acyclic. Since H is acyclic, the MST of H, Y, must be the same: $H = Y$. Therefore, we can say that all the vertices and edges within H is in Y. Futhermore, since $H \subset T$, all the vertices and edges within T must also be in Y.