= A1 ? B1 to X2 =
A2 ? B2 and the mapping ? satisfies ? (A1) ? A2 and ? (B1) ? B2 , then the connecting morphism ? ? of the Mayer ? Vietoris sequence commutes with ? ? . That is , the following diagram commutes (the horizontal maps are the usual ones) :
<formula>

= = = Cohomological versions = = =

The Mayer ? Vietoris long exact sequence for singular cohomology groups with coefficient group G is dual to the homological version . It is the following :
<formula>
where the dimension preserving maps are restriction maps induced from inclusions , and the (co-) boundary maps are defined in a similar fashion to the homological version . There is also a relative formulation .
As an important special case when G is the group of real numbers R and the underlying topological space has the additional structure of a smooth manifold , the Mayer ? Vietoris sequence for de Rham cohomology is
<formula>
where { U , V } is an open cover of X , ? denotes the restriction map , and ? is the difference . The map d ? is defined similarly as the map ? ? from above . It can be briefly described as follows . For a cohomology class [?] represented by closed form ? in U ? V , express ? as a difference of forms ?U - ?V via a partition of unity subordinate to the open cover { U , V } , for example . The exterior derivative d?U and d?V agree on U ? V and therefore together define an n + 1 form ? on X. One then has d ? ([?]) = [?] .

= = = Derivation = = =

Consider the long exact sequence associated to the short exact sequences of chain groups (constituent groups of chain complexes)
<formula>
where ? (x)
= (x , ? x) , ? (x , y) =
x + y , and Cn (A + B) is the chain group consisting of sums of chains in A and chains in B. It is a fact that the singular n @-@ simplices of X whose images are contained in either A or B generate all of the homology group Hn (X) . In other words , Hn (A + B) is isomorphic to Hn (X) . This gives the Mayer ? Vietoris sequence for singular homology .
The same computation applied to the short exact sequences of vector spaces of differential forms
<formula>
yields the Mayer ? Vietoris sequence for de Rham cohomology .
From a formal point of view , the Mayer ? Vietoris sequence can be derived from the Eilenberg ? Steenrod axioms for homology theories using the long exact sequence in homology .

= = = Other homology theories = = =

The derivation of the Mayer ? Vietoris sequence from the Eilenberg ? Steenrod axioms does not require the dimension axiom , so in addition to existing in ordinary cohomology theories , it holds in extraordinary cohomology theories (such as topological K @-@ theory and cobordism) .

= = = Sheaf cohomology = = =

From the point of view of sheaf cohomology , the Mayer ? Vietoris sequence is related to ?ech cohomology . Specifically , it arises from the degeneration of the spectral sequence that relates ?ech cohomology to sheaf cohomology (sometimes called the Mayer ? Vietoris spectral sequence) in the case where the open cover used to compute the ?ech cohomology consists of two open sets . This spectral sequence exists in arbitrary topoi .