= 1) but less than that of a plane (d =
2) . The Apollonian gasket was first described by Gottfried Leibniz in the 17th century , and is a curved precursor of the 20th @-@ century Sierpi?ski triangle . The Apollonian gasket also has deep connections to other fields of mathematics ; for example , it is the limit set of Kleinian groups .

The configuration of a circle tangent to four circles in the plane has special properties , which have been elucidated by Larmor (1891) and Lachlan (1893) . Such a configuration is also the basis for Casey 's theorem , itself a generalization of Ptolemy 's theorem .

The extension of Apollonius ' problem to three dimensions , namely , the problem of finding a fifth sphere that is tangent to four given spheres , can be solved by analogous methods . For example , the given and solution spheres can be resized so that one given sphere is shrunk to point while maintaining tangency . Inversion in this point reduces Apollonius ' problem to finding a plane that is tangent to three given spheres . There are in general eight such planes , which become the solutions to the original problem by reversing the inversion and the resizing . This problem was first considered by Pierre de Fermat , and many alternative solution methods have been developed over the centuries .

Apollonius ' problem can even be extended to d dimensions , to construct the hyperspheres tangent to a given set of d + 1 hyperspheres . Following the publication of Frederick Soddy 's re @-@ derivation of the Descartes theorem in 1936 , several people solved (independently) the mutually tangent case corresponding to Soddy 's circles in d dimensions .

= = Applications = =

The principal application of Apollonius ' problem , as formulated by Isaac Newton , is hyperbolic trilateration , which seeks to determine a position from the differences in distances to at least three points . For example , a ship may seek to determine its position from the differences in arrival times of signals from three synchronized transmitters . Solutions to Apollonius ' problem were used in World War I to determine the location of an artillery piece from the time a gunshot was heard at three different positions , and hyperbolic trilateration is the principle used by the Decca Navigator System and LORAN . Similarly , the location of an aircraft may be determined from the difference in arrival times of its transponder signal at four receiving stations . This multilateration problem is equivalent to the three @-@ dimensional generalization of Apollonius ' problem and applies to global positioning systems such as GPS . It is also used to determine the position of calling animals (such as birds and whales) , although Apollonius ' problem does not pertain if the speed of sound varies with direction (i.e. , the transmission medium not isotropic) .

Apollonius ' problem has other applications . In Book 1 , Proposition 21 in his Principia , Isaac Newton used his solution of Apollonius ' problem to construct an orbit in celestial mechanics from the center of attraction and observations of tangent lines to the orbit corresponding to instantaneous velocity . The special case of the problem of Apollonius when all three circles are tangent is used in the Hardy ? Littlewood circle method of analytic number theory to construct Hans Rademacher 's contour for complex integration , given by the boundaries of an infinite set of Ford circles each of which touches several others . Finally , Apollonius ' problem has been applied to some types of packing problems , which arise in disparate fields such as the error @-@ correcting codes used on DVDs and the design of pharmaceuticals that bind in a particular enzyme of a pathogenic bacterium .