

$e = 1$  , it defines a parabola ; and if  $e < 1$  , it defines an ellipse . The special case  $e = 0$  of the latter results in a circle of radius  $\frac{1}{2e}$  .

== Intersection of two polar curves ==

The graphs of two polar functions  $r = f(\theta)$  and  $r = g(\theta)$  have possible intersections in 3 cases :

In the origin if the equations  $r = f(\theta)$  and  $r = g(\theta)$  have at least one solution each .

All the points  $(r, \theta)$  where  $\theta$  are the solutions to the equation  $f(\theta) = g(\theta)$  .

All the points  $(r, \theta)$  where  $\theta$  are the solutions to the equation  $f(\theta) = g(\theta + 2\pi k)$  where  $k$  is an integer .

== Complex numbers ==

Every complex number can be represented as a point in the complex plane , and can therefore be expressed by specifying either the point 's Cartesian coordinates ( called rectangular or Cartesian form ) or the point 's polar coordinates ( called polar form ) . The complex number  $z$  can be represented in rectangular form as

$z = x + yi$

where  $i$  is the imaginary unit , or can alternatively be written in polar form ( via the conversion formulae given above ) as

$z = r(\cos \theta + i \sin \theta)$

and from there as

$z = re^{i\theta}$

where  $e$  is Euler 's number , which are equivalent as shown by Euler 's formula . ( Note that this formula , like all those involving exponentials of angles , assumes that the angle  $\theta$  is expressed in radians . ) To convert between the rectangular and polar forms of a complex number , the conversion formulae given above can be used .

For the operations of multiplication , division , and exponentiation of complex numbers , it is generally much simpler to work with complex numbers expressed in polar form rather than rectangular form . From the laws of exponentiation :

Multiplication :

$z_1 z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}$

Division :

$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}$

Exponentiation ( De Moivre 's formula ) :

$z^n = r^n e^{in\theta}$

== Calculus ==

Calculus can be applied to equations expressed in polar coordinates .

The angular coordinate  $\theta$  is expressed in radians throughout this section , which is the conventional choice when doing calculus .

== Differential calculus ==