Andrew M. Gleason

Andrew Mattei Gleason (November 4 , 1921 ? October 17 , 2008) was an American mathematician who as a young World War II naval officer broke German and Japanese military codes , then over the succeeding sixty years made fundamental contributions to widely varied areas of mathematics , including the solution of Hilbert 's fifth problem , and was a leader in reform and innovation in math - e - mat - ics teaching at all levels . Gleason 's theorem in quantum logic and the Greenwood ? Gleason graph , an important example in Ramsey theory , are named for him .

Gleason 's entire academic career was at Harvard , from which he retired in 1992 . His numerous academic and scholarly leadership posts included chairmanship of the Harvard Mathematics Department and Harvard Society of Fellows , and presidency of the American Mathematical Society . He continued to advise the United States government on cryptographic security , and the Commonwealth of Massachusetts on math - e - mat - ics education for children , almost until the end of his life .

Gleason won the Newcomb Cleveland Prize in 1952 and the Gung ? Hu Distinguished Service Award of the American Mathematical Society in 1996 . He was a member of the National Academy of Sciences and of the American Philosophical Society , and held the Hollis Chair of Mathematics and Natural Philosophy at Harvard .

He was fond of saying that math - e - mat - ic - al proofs " really aren 't there to convince you that something is true ? ? ? they 're there to show you why it is true . " The Notices of the American Mathematical Society called him " one of the quiet giants of twentieth @-@ century mathematics , the consummate professor dedicated to scholarship , teaching , and service in equal measure . "

Biography

Gleason was born in Fresno , California , the youngest of three children ; his father Henry Gleason was a botanist and a member of the Mayflower Society , and his mother was the daughter of Swiss @-@ American winemaker Andrew Mattei . His older brother Henry , Jr. became a linguist . He grew up in Bronxville , New York , where his father was the curator of the New York Botanical Garden .

After briefly attending Berkeley High School (Berkeley , California) he graduated from Roosevelt High School in Yonkers , winning a scholarship to Yale University . Though Gleason 's mathematics education had gone only so far as some self @-@ taught calculus , Yale mathematician William Raymond Longley urged him to try a course in mechanics normally intended for juniors .

So I learned first year calculus and second year calculus and became the consultant to one end of the whole Old Campus ... I used to do all the homework for all the sections of [first @-@ year calculus] . I got plenty of practice in doing elementary calculus problems . I don 't think there exists a problem ? ? ? the classical kind of pseudo reality problem which first and second @-@ year students are given ? ? ? that I haven 't seen . "

One month later he enrolled in a differential equations course (" mostly full of seniors ") as well . When Einar Hille temporarily replaced the regular instructor , Gleason found Hille 's style " unbelievably different ... He had a view of mathematics that was just vastly different ... That was a very important experience for me . So after that I took a lot of courses from Hille " including , in his sophomore year , graduate @-@ level real analysis . " Starting with that course with Hille , I began to have some sense of what mathematics is about . "

While at Yale he competed three times (1940 , 1941 and 1942) in the recently founded William Lowell Putnam Mathematical Competition , always placing among the top five entrants in the country (making him the second three @-@ time Putnam Fellow) .

After the Japanese attacked Pearl Harbor during his senior year , Gleason applied for a commission in the US Navy , and on graduation joined the team working to break Japanese naval codes . (Others on this team included his future collaborator Robert E. Greenwood and Yale professor Marshall Hall , Jr .) He also collaborated with British researchers attacking the German Enigma cipher ; Alan Turing , who spent substantial time with Gleason while visiting Washington , called him " the brilliant young Yale graduate mathematician " in a report of his visit .

In 1946 , at the recommendation of Navy colleague Donald Howard Menzel , Gleason was appointed a Junior Fellow at Harvard . An early goal of the Junior Fellows program was to allow young scholars showing extraordinary promise to sidestep the lengthy PhD process ; four years later Harvard appointed Gleason an assistant professor of mathematics , though he was almost immediately recalled to Washington for cryptographic work related to the Korean War . He returned to Harvard in the fall of 1952 , and soon after published the most important of his results on Hilbert 's fifth problem (see below) . Harvard awarded him tenure the following year .

In January 1959 he married Jean Berko whom he had met at a party featuring the music of Tom Lehrer . Berko , a psycholinguist , worked for many years at Boston University . They had three daughters .

In 1969 Gleason took the Hollis Chair of Mathematics and Natural Philosophy , the oldest (est . 1727) scientific endowed professorship in the US . He retired from Harvard in 1992 but remained active in service to Harvard (as chair of the Society of Fellows , for example) and to mathematics : in particular , promoting the Harvard Calculus Reform Project and working with the Massachusetts Board of Education .

He died in 2008 from complications following surgery .

= = Teaching and education reform = =

Gleason said he " always enjoyed helping other people with math " ? ? ? a colleague said he " regarded teaching mathematics ? ? ? like doing mathematics ? ? ? as both important and also genuinely fun . " At fourteen , during his brief attendance at Berkeley High School , he found himself not only bored with first @-@ semester geometry , but also helping other students with their homework ? ? ? including those taking the second half of the course , which he soon began auditing .

At Harvard he " regularly taught at every level " , including administratively burdensome multisection courses . One class presented Gleason with a framed print of Picasso 's Mother and Child in recognition of his care for them .

In 1964 he created " the first of the ' bridge ' courses now ubiquitous for math majors , only twenty years before its time . " Such a course is designed to teach new students , accustomed to rote learning of mathematics in secondary school , how to reason abstractly and construct mathematical proofs . That effort led to publication of his Fundamentals of Abstract Analysis , of which one reviewer wrote :

This is a most unusual book ... Every working mathematician of course knows the difference between a lifeless chain of formalized propositions and the " feeling " one has (or tries to get) of a mathematical theory , and will probably agree that helping the student to reach that " inside " view is the ultimate goal of mathematical education ; but he will usually give up any attempt at successfully doing this except through oral teaching . The originality of the author is that he has tried to attain that goal in a textbook , and in the reviewer 's opinion , he has succeeded remarkably well in this all but impossible task . Most readers will probably be delighted (as the reviewer has been) to find , page after page , painstaking discussions and explanations of standard mathematical and logical procedures , always written in the most felicitous style , which spares no effort to achieve the utmost clarity without falling into the vulgarity which so often mars such attempts .

But Gleason 's " talent for exposition " did not always imply that the reader would be enlightened without effort of his own . Even in a wartime a memo on the urgently important decryption of the German Enigma cipher , Gleason and his colleagues wrote :

The reader may wonder why so much is left to the reader . A book on swimming strokes may be nice to read , but one must practice the strokes while actually in the water before one can claim to be a swimmer . So if the reader desires to actually possess the knowledge for recovering wiring from a depth , let the reader get his paper and pencils , using perhaps four colors to avoid confusion in the connecting links , and go to work .

His notes and exercises on probability and statistics , drawn up for his lectures to code @-@ breaking colleagues during the war (see below) remained in use in National Security Agency

training for several decades ; they were published openly in 1985 .
In a 1964 Science article , Gleason wrote of an apparent paradox arising in attempts to explain mathematics to nonmathematicians :
It is notoriously difficult to convey the proper impression of the frontiers of mathematics to nonspecialists . Ultimately the difficulty stems from the fact that mathematics is an easier subject than the other sciences . Consequently , many of the important primary problems of the subject ? ? ? that is , problems which can be understood by an intelligent outsider ? ? ? have either been solved or carried to a point where an indirect approach is clearly required . The great bulk of pure mathematical research is concerned with secondary , tertiary , or higher @-@ order problem , the very statement of which can hardly be understood until one has mastered a great deal of technical mathematics .
Gleason was part of the School Mathematics Study Group , which helped define the New Math of the 1960s ? ? ? ambitious changes in American elementary and high school mathematics teaching emphasizing understanding of concepts over rote algorithms . Gleason was " always interested in how people learn " ; as part of the New Math effort he spent most mornings over several months with second @-@ graders . Some years later he gave a talk in which he described his goal as having been :
to find out how much they could figure out for themselves , given appropriate activities and the right guidance . At the end of his talk , someone asked Andy whether he had ever worried that teaching math to little kids wasn 't how faculty at research institutions should be spending their time . [His] quick and decisive response : " No , I didn 't think about that at all . I had a ball ! "
In 1986 he helped found the Calculus Consortium , which has published a successful and influential series of " calculus reform " textbooks for college and high school , on precalculus , calculus , and other areas . His " credo for this program as for all of his teaching was that the ideas should be based in equal parts of geometry for visualization of the concepts , computation for grounding in the real world , and algebraic manipulation for power . " However , the program faced heavy criticism from the mathematics community for its omission of topics such as the mean value theorem , and for its perceived lack of mathematical rigor .

= = Cryptanalysis work = =

During World War II Gleason was part of OP @-@ 20 @-@ G , the U.S. Navy 's signals intelligence and cryptanalysis group , one task of which (in collaboration with British cryptographers at Bletchley Park such as Alan Turing) was to penetrate German Enigma machine communications networks . The British had great success with two of these networks , but the third , used for German @-@ Japanese naval coordination , remained unbroken because of a faulty assumption that it employed a simplified version of Enigma . After OP @-@ 20 @-@ G 's Marshall Hall observed that certain metadata in Berlin @-@ to @-@ Tokyo transmissions used letter sets disjoint from those used in Tokyo @-@ to @-@ Berlin metadata , Gleason hypothesized that the corresponding unencrypted letters sets were A @-@ M (in one direction) and N @-@ Z (in the other) , then devised novel statistical tests by which he confirmed this hypothesis . The result was routine decryption of this third network by 1944 . (This work also involved deeper math - e - mat - ics related to permutation groups and the graph isomorphism problem .)
OP @-@ 20 @-@ G then turned to the Japanese navy 's " Coral " cipher , a key tool for the attack on which was the " Gleason crutch " , a form of Chernoff bound on tail distributions of sums of independent random variables , but predating Chernoff 's work by a decade .
Toward the end of the war he concentrated on documenting the work of OP @-@ 20 @-@ G and developing systems for training new cryptographers .
In 1950 Gleason returned to active duty for the Korean War , serving as a Lieutenant Commander in the Nebraska Avenue Complex (which much later became the home of the DHS Cyber Security Division) . His cryptographic work from this period remains classified , but it is known that he recruited mathematicians and taught them cryptanalysis . He served on the advisory boards for the National Security Agency and the Institute for Defense Analyses , and he continued to recruit , and

to advise the military on cryptanalysis , almost to the end of his life .

Mathematics research

Gleason made fundamental contributions to widely varied areas of mathematics , including the theory of Lie groups , quantum mechanics , and combinatorics . According to Freeman Dyson 's famous classification of mathematicians as being either birds or frogs , Gleason was a frog : he worked as a problem solver rather than a visionary formulating grand theories .

Hilbert 's fifth problem

In 1900 David Hilbert posed 23 problems he felt would be central to next century of mathematics research . Hilbert 's fifth problem concerns the characterization of Lie groups by their actions on topological spaces : to what extent does their topology provide information sufficient to determine their geometry ?

The " restricted " version of Hilbert 's fifth problem (solved by Gleason) asks , more specifically , whether every locally Euclidean topological group is a Lie group . That is , if a group G has the structure of a topological manifold , can that structure be strengthened to a real analytic structure , so that within any neighborhood of an element of G , the group law is defined by a convergent power series , and so that overlapping neighborhoods have compatible power series definitions ? Prior to Gleason 's work , special cases of the problem had been solved by Luitzen Egbertus Jan Brouwer , John von Neumann , Lev Pontryagin , and Garrett Birkhoff , among others .

Gleason 's interest in the fifth problem began in the late 1940s , sparked by a course he took from George Mackey . In 1949 he published a paper introducing the " no small subgroups " property of Lie groups (the existence of a neighborhood of the identity within which no nontrivial subgroup exists) that would eventually be crucial to its solution . His 1952 paper on the subject , together with a paper published concurrently by Deane Montgomery and Leo Zippin , solves affirmatively the restricted version of Hilbert 's fifth problem , showing that indeed every locally Euclidean group is a Lie group . Gleason 's contribution was to prove that this is true when G has the no small subgroups property ; Montgomery and Zippin showed every locally Euclidean group has this property . As Gleason told the story , the key insight of his proof was to apply the fact that monotonic functions are differentiable almost everywhere . On finding the solution , he took a week of leave to write it up , and it was printed in the Annals of Mathematics alongside the paper of Montgomery and Zippin ; another paper a year later by Hidehiko Yamabe removed some technical side conditions from Gleason 's proof .

The " unrestricted " version of Hilbert 's fifth problem , closer to Hilbert 's original formulation , considers both a locally Euclidean group G and another manifold M on which G has a continuous action . Hilbert asked whether , in this case , M and the action of G could be given a real analytic structure . It was quickly realized that the answer was negative , after which attention centered on the restricted problem . However , with some additional smoothness assumptions on G and M , it might yet be possible to prove the existence of a real analytic structure on the group action . The Hilbert ? Smith conjecture , still unsolved , encapsulates the remaining difficulties of this case .

Quantum mechanics

The Born rule states that an observable property of a quantum system is defined by a Hermitian operator on a separable Hilbert space , that the only observable values of the property are the eigenvalues of the operator , and that the probability of the system being observed in a particular eigenvalue is the square of the absolute value of the complex number obtained by projecting the state vector (a point in the Hilbert space) onto the corresponding eigenvector . George Mackey had asked whether Born 's rule is a necessary consequence of a particular set of axioms for quantum mechanics , and more specifically whether every measure on the lattice of projections of a Hilbert space can be defined by a positive operator with unit trace . Though Richard Kadison proved this was false for two @-@ dimensional Hilbert spaces , Gleason 's theorem (published 1957) shows it to be true for higher dimensions .

Gleason 's theorem implies the nonexistence of certain types of hidden variable theories for quantum mechanics , strengthening a previous argument of John von Neumann . Von Neumann had claimed to show that hidden variable theories were impossible , but (as Grete Hermann pointed out) his demonstration made an assumption that quantum systems obeyed a form of additivity of expectation for noncommuting operators that might not hold a priori . In 1966 , John Stewart Bell showed that Gleason 's theorem could be used to remove this extra assumption from von Neumann 's argument .

= = = Ramsey theory = = =