= 2k + 1 . One way to prove that zero is not odd is by contradiction : if 0 = 2k + 1 then k = ? 1 / 2 , which is not an integer . Since zero is not odd , if an unknown number is proven to be odd , then it cannot be zero . This apparently trivial observation can provide a convenient and revealing proof explaining why a number is nonzero .

A classic result of graph theory states that a graph of odd order (having an odd number of vertices) always has at least one vertex of even degree . (The statement itself requires zero to be even : the empty graph has an even order , and an isolated vertex has an even degree .) In order to prove the statement , it is actually easier to prove a stronger result : any odd @-@ order graph has an odd number of even degree vertices . The appearance of this odd number is explained by a still more general result , known as the handshaking lemma : any graph has an even number of vertices of odd degree . Finally , the even number of odd vertices is naturally explained by the degree sum formula .

Sperner 's lemma is a more advanced application of the same strategy . The lemma states that a certain kind of coloring on a triangulation of a simplex has a subsimplex that contains every color . Rather than directly construct such a subsimplex , it is more convenient to prove that there exists an odd number of such subsimplices through an induction argument . A stronger statement of the lemma then explains why this number is odd : it naturally breaks down as (n + 1) + n when one considers the two possible orientations of a simplex .

= = = Even @-@ odd alternation = = =

The fact that zero is even , together with the fact that even and odd numbers alternate , is enough to determine the parity of every other natural number . This idea can be formalized into a recursive definition of the set of even natural numbers :

0 is even .

(n + 1) is even if and only if n is not even .

This definition has the conceptual advantage of relying only on the minimal foundations of the natural numbers : the existence of 0 and of successors . As such , it is useful for computer logic systems such as LF and the Isabelle theorem prover . With this definition , the evenness of zero is not a theorem but an axiom . Indeed , " zero is an even number " may be interpreted as one of the Peano axioms , of which the even natural numbers are a model . A similar construction extends the definition of parity to transfinite ordinal numbers : every limit ordinal is even , including zero , and successors of even ordinals are odd .

The classic point in polygon test from computational geometry applies the above ideas . To determine if a point lies within a polygon , one casts a ray from infinity to the point and counts the number of times the ray crosses the edge of polygon . The crossing number is even if and only if the point is outside the polygon . This algorithm works because if the ray never crosses the polygon , then its crossing number is zero , which is even , and the point is outside . Every time the ray does cross the polygon , the crossing number alternates between even and odd , and the point at its tip alternates between outside and inside .

In graph theory , a bipartite graph is a graph whose vertices are split into two colors , such that neighboring vertices have different colors . If a connected graph has no odd cycles , then a bipartition can be constructed by choosing a base vertex v and coloring every vertex black or white , depending on whether its distance from v is even or odd . Since the distance between v and itself is 0 , and 0 is even , the base vertex is colored differently from its neighbors , which lie at a distance of 1 .

= = = Algebraic patterns = = =