= Mechanical @-@ electrical analogies =

Mechanical @-@ electrical analogies are the representation of mechanical systems as electrical networks . At first , such analogies were used in reverse to help explain electrical phenomena in familiar mechanical terms . James Clerk Maxwell introduced analogies of this sort in the 19th century . However , as electrical network analysis matured it was found that certain mechanical problems could more easily be solved through an electrical analogy . Theoretical developments in the electrical domain that were particularly useful were the representation of an electrical network as an abstract topological diagram (the circuit diagram) using the lumped element model and the ability of network analysis to synthesise a network to meet a prescribed frequency function .

This approach is especially useful in the design of mechanical filters ? these use mechanical devices to implement an electrical function . However , the technique can be used to solve purely mechanical problems , and can also be extended into other , unrelated , energy domains . Nowadays , analysis by analogy is a standard design tool wherever more than one energy domain is involved . It has the major advantage that the entire system can be represented in a unified , coherent way . Electrical analogies are particularly used by transducer designers , by their nature they cross energy domains , and in control systems , whose sensors and actuators will typically be domain @-@ crossing transducers . A given system being represented by an electrical analogy may conceivably have no electrical parts at all . For this reason domain @-@ neutral terminology is preferred when developing network diagrams for control systems .

Mechanical @-@ electrical analogies are developed by finding relationships between variables in one domain that have a mathematical form identical to variables in the other domain . There is no one , unique way of doing this ; numerous analogies are theoretically possible , but there are two analogies that are widely used : the impedance analogy and the mobility analogy . The impedance analogy makes force and voltage analogous while the mobility analogy makes force and current analogous . By itself , that is not enough to fully define the analogy , a second variable must be chosen . A common choice is to make pairs of power conjugate variables analogous . These are variables which when multiplied together have units of power . In the impedance analogy , for instance , this results in force and velocity being analogous to voltage and current respectively .

Variations of these analogies are used for rotating mechanical systems , such as in electric motors . In the impedance analogy , instead of force , torque is made analogous to voltage . It is perfectly possible that both versions of the analogy are needed in , say , a system that includes rotating and reciprocating parts , in which case a force @-@ torque analogy is required within the mechanical domain and a force @-@ torque @-@ voltage analogy to the electrical domain . Another variation is required for acoustical systems ; here pressure and voltage are made analogous (impedance analogy) . In the impedance analogy , the ratio of the power conjugate variables is always a quantity analogous to electrical impedance . For instance force / velocity is mechanical impedance . The mobility analogy does not preserve this analogy between impedances across domains , but it does have another advantage over the impedance analogy . In the mobility analogy the topology of networks is preserved , a mechanical network diagram has the same topology as its analogous electrical network diagram .

= = Applications = =

Mechanical @-@ electrical analogies are used to represent the function of a mechanical system as an equivalent electrical system by drawing analogies between mechanical and electrical parameters . A mechanical system by itself can be so represented , but analogies are of greatest use in electromechanical systems where there is a connection between mechanical and electrical parts . Analogies are especially useful in analysing mechanical filters . These are filters constructed of mechanical parts but designed to work in an electrical circuit through transducers . Circuit theory is well developed in the electrical domain in general and in particular there is a wealth of filter theory available . Mechanical systems can make use of this electrical theory in mechanical designs through a mechanical @-@ electrical analogy .

Mechanical @-@ electrical analogies are useful in general where the system includes transducers between different energy domains . Another area of application is the mechanical parts of acoustic systems such as the pickup and tonearm of record players . This was of some importance in early phonographs where the audio is transmitted from the pickup needle to the horn through various mechanical components entirely without electrical amplification . Early phonographs suffered badly from unwanted resonances in the mechanical parts . It was found that these could be eliminated by treating the mechanical parts as components of a low @-@ pass filter which has the effect of flattening out the passband .

Electrical analogies of mechanical systems can be used just as a teaching aid , to help understand the behaviour of the mechanical system . In former times , up to about the early 20th century , it was more likely that the reverse analogy would be used ; mechanical analogies were formed of the then little understood electrical phenomena .

= = Forming an analogy = =

Electrical systems are commonly described by means of a circuit diagram . These are network diagrams that describe the topology of the electrical system using a specialised graph notation . The circuit diagram does not try and represent the true physical dimensions of the electrical components or their actual spatial relationship to each other . This is possible because the electrical components are represented as ideal lumped elements , that is , the element is treated as if it is occupying a single point (lumped at that point) . Non @-@ ideal components can be accommodated in this model by using more than one element to represent the component . For instance , a coil intended for use as an inductor has resistance as well as inductance . This can be represented on the circuit diagram as a resistor in series with an inductor . Thus , the first step in forming an analogy of a mechanical system is to describe it as a mechanical network in a similar way , that is , as a topological graph of ideal elements . Alternative , more abstract , representations to the circuit diagram are possible , for instance the bond graph .

In an electrical network diagram , limited to linear systems , there are three passive elements : resistance , inductance , and capacitance ; and two active elements : the voltage generator , and the current generator . The mechanical analogs of these elements can be used to construct a mechanical network diagram . What the mechanical analogs of these elements are depends on what variables are chosen to be the fundamental variables . There is a wide choice of variables that can be used , but most commonly used are a power conjugate pair of variables (described below) and the pair of Hamiltonian variables derived from these .

There is a limit to the applicability of this lumped element model . The model works well if the components are small enough that the time taken for a wave to cross them is insignificant , or equivalently , if there is no significant phase difference in the wave either side of the component . What amounts to significant depends on how accurate the model is required to be , but a common rule of thumb is to require components to be smaller than one sixteenth of a wavelength . Since wavelength decreases with frequency , this puts an upper limit on the frequency that can be covered in this kind of design . This limit is much lower in the mechanical domain than the equivalent limit in the electrical domain . This is because the much higher propagation speeds in the electrical domain lead to longer wavelengths (mechanical vibrations in steel propagate at about 6 @,@ 000 m / s , electromagnetic waves in common cable types propagate at about 2 x 108 m / s) . For instance , traditional mechanical filters are only made up to around 600 kHz (although MEMS devices can operate at much higher frequencies due to their very small size) . In the electrical domain , on the other hand , the transition from the lumped element model to the distributed element model occurs in the hundreds of megahertz region .

In some cases it is possible to continue using a topological network diagram even when components needing a distributed element analysis are present . In the electrical domain , a transmission line , a basic distributed element component , can be included in the model with the introduction of the additional element of electrical length . The transmission line is a special case because it is invariant along its length and hence the full geometry need not be modelled . Another

way of dealing with distributed elements is to use a finite element analysis whereaby the distributed element is approximated by a large number of small lumped elements . Just such an approach was used in one paper to model the cochlea of the human ear . Another condition required of electrical systems for the application of the lumped element model is that no significant fields exist outside the component since these can couple to other unrelated components . However , these effects can often be modelled by introducing some virtual lumped elements called strays or parasitics . An analog of this in mechanical systems is vibration in one component being coupled to an unrelated component .

Power conjugate variables

The power conjugate variables are a pair of variables whose product is power . In the electrical domain the power conjugate variables chosen are invariably voltage (v) and current (i) . Thus , the power conjugate variables in the mechanical domain are analogs . However , this is not enough to make the choice of mechanical fundamental variables unique . The usual choice for a translational mechanical system is force (F) and velocity (u) but it is not the only choice . A different pair may be more appropriate for a system with a different geometry , such as a rotational system .
Even after the mechanical fundamental variables have been chosen , there is still not a unique set of analogs . There are two ways that the two pairs of power conjugate variables can be associated with each other in the analogy . For instance the associations F with v and u with i can be made . However , the alternative associations u with v and F with i are also possible . This leads to two classes of analogies , the impedance analogies and the mobility analogies . These analogies are the dual of each other . The same mechanical network has analogs in two different electrical networks . These two electrical networks are the dual circuits of each other .

Hamiltonian variables

The Hamiltonian variables , also called the energy variables , are those variables which when time differentiated are equal to the power conjugate variables . The Hamiltonian variables are so called because they are the variables which usually appear in Hamiltonian mechanics . The Hamiltonian variables in the electrical domain are charge (q) and flux linkage (?) because ,
<formula> (Faraday 's law of induction) and , <formula>
In the translational mechanical domain the Hamiltonian variables are distance displacement (x) and momentum (p) because ,
<formula> (Newton 's second law of motion) and , <formula>
There is a corresponding relationship for other analogies and sets of variables . The Hamiltonian variables are also called the energy variables . The integrand of a power conjugate variable with respect to a Hamiltonian variable is a measure of energy . For instance ,
<formula> and , <formula>
are both expressions of energy . They can also be called generalised momentum and generalised displacement after their analogs in the mechanical domain . Some authors discourage this terminology because it is not domain neutral . Likewise , the use of the terms I @-@ type and V @-@ type (after current and voltage) is also discouraged .

Classes of analogy

There are two principle classes of analogy in use . The impedance analogy (also called the Maxwell analogy) preserves the analogy between mechanical , acoustical and electrical impedance but does not preserve the topology of networks . The mechanical network is arranged differently to its analogous electrical network . The mobility analogy (also called the Firestone analogy) preserves network topologies at the expense of losing the analogy between impedances across energy domains . There is also the through and across analogy , also called the Trent analogy . The through and across analogy between the electrical and mechanical domain is the same as in the

mobility analogy . However , the analogy between the electrical and acoustical domains is like the impedance analogy . Analogies between the mechanical and acoustical domain in the through and across analogy have a dual relationship with both the impedance analogy and mobility analogy .

Different fundamental variables are chosen for mechanical translation and rotational systems leading to two variants for each of the analogies . For instance , linear distance is the displacement variable in a translational system , but this is not so appropriate for rotating systems where angle is used instead . Acoustical analogies have also been included in the descriptions as a third variant . While acoustical energy is ultimately mechanical in nature , it is treated in the literature as an instance of a different energy domain , the fluid domain , and has different fundamental variables . Analogies between all three domains ? electrical , mechanical and acoustical ? are required to fully represent electromechanical audio systems .

Impedance analogies

Impedance analogies , also called the Maxwell analogy , classify the two variables making up the power conjugate pair as an effort variable and a flow variable . The effort variable in an energy domain is the variable analogous to force in the mechanical domain . The flow variable in an energy domain is the variable analogous to velocity in the mechanical domain . Power conjugate variables in the analog domain are chosen that bear some resemblance to force and velocity .
In the electrical domain , the effort variable is voltage and the flow variable is electrical current . The ratio of voltage to current is electrical resistance (Ohm 's law) . The ratio of the effort variable to the flow variable in other domains is also described as resistance . Oscillating voltages and currents give rise to the concept of electrical impedance when there is a phase difference between them . Impedance can be thought of as an extension to the concept of resistance . Resistance is associated with energy dissipation . Impedance encompasses energy storage as well as energy dissipation . The impedance analogy gives rise to the concept of impedance in other energy domains (but measured in different units) .

Translational mechanical impedance analogy

The translational impedance analogy describes mechanical systems moving in a single linear dimension and gives rise to the idea of mechanical impedance . The unit of mechanical impedance is the mechanical ohm ; in SI units this is N @-@ s / m , or Kg / s .

Rotational mechanical impedance analogy

The rotational impedance analogy describes rotating mechanical systems and gives rise to the idea of rotational impedance . The unit of rotational impedance in the SI system is N @-@ m @-@ s / rad .

Acoustical impedance analogy

The acoustical impedance analogy gives rise to the idea of acoustic impedance . The unit of acoustic impedance is the acoustic ohm ; in SI units this is N @-@ s / m5 .

Mobility analogies

Mobility analogies , also called the Firestone analogy , are the electrical duals of impedance analogies . That is , the effort variable in the mechanical domain is analogous to current (the flow variable) in the electrical domain , and the flow variable in the mechanical domain is analogous to voltage (the effort variable) in the electrical domain . The electrical network representing the mechanical system is the dual network of that in the impedance analogy .
The mobility analogy is characterised by admittance in the same way that the impedance analogy is

characterised by impedance . Admittance is the algebraic inverse of impedance . In the mechanical domain , mechanical admittance is more usually called mobility .

= = = = Translation mechanical mobility analogy = = = =

= = = = Rotational mechanical mobility analogy = = = =

= = = = Acoustical mobility analogy = = = =

= = = Through and across analogies = = =

Through and across analogies , also called the Trent analogy , classify the two variables making up the power conjugate pair as an across variable and a through variable . The across variable is a variable that appears across the two terminals of an element . The across variable is measured relative to the element terminals . The through variable is a variable that passes through , or acts through an element , that is , it has the same value at both terminals of the element . The through variable is not a relative measure . Thus , in the electrical domain the across variable is voltage and the through variable is current . In the mechanical domain the analogous variables are velocity and force , as in the mobility analogy .

= = = = Translational mechanical through and across analogy = = = =

= = = = Rotational mechanical through and across analogy = = = =

= = = = Acoustical through and across analogy = = = =

Pressure is an across variable because pressure is measured relative to the two terminals of an element , not as an absolute pressure . It is thus not analogous to force which is a through variable , even though pressure is in units of force per area . Forces act through an element ; a rod with a force applied to the top will transmit the same force to an element connected to its bottom . Thus , in the through and across analogy the mechanical domain is analogous to the electrical domain like the mobility analogy , but the acoustical domain is analogous to the electrical domain like the impedance analogy .

= = = Other energy domains = = =

The electrical analogy can be extended to many other energy domains . In the field of sensors and actuators , and for control systems using them , it is a common method of analysis to develop an electrical analogy of the entire system . Since sensors can be sensing a variable in any energy domain , and likewise outputs from the system can be in any energy domain , analogies for all energy domains are required . The following table gives a summary of the most common power conjugate variables used to form analogies .
It is perhaps more common in the thermal domain to choose temperature and thermal power as the fundamental variables because , unlike entropy , they can be measured directly . The concept of thermal resistance is based on this analogy . However , these are not power conjugate variables and are not fully compatible with the other variables in the table . An integrated electrical analogy across multiple domains that includes this [Which one ? The conjugate pair in the table or the non @-@ conjugate pair described in this paragraph ?] thermal analogy will not correctly model energy

flows .

 Similarly , the commonly seen analogy using mmf and magnetic flux as the fundamental variables , which gives rise to the concept of magnetic reluctance , does not correctly model energy flow . The variable pair mmf and magnetic flux is not a power conjugate pair . The analogy shown in the table , which does use a power conjugate pair is sometimes called the gyrator @-@ capacitor model .

= = Transducers = =

 A transducer is a device that takes energy from one domain as input and converts it to another energy domain as output . They are often reversible , but are rarely used in that way . Transducers have many uses and there are many kinds , in electromechanical systems they can be used as actuators and sensors . In audio electronics they provide the conversion between the electrical and acoustical domains . The transducer provides the link between the mechanical and electrical domains and thus a network representation is required for it in order to develop a unified electrical analogy . To do this the concept of port from the electrical domain is extended into other domains .
 Transducers have (at least) two ports , one port in the mechanical domain and one in the electrical domain , and are analogous to electrical two @-@ port networks . This is to be compared to the elements discussed so far which are all one @-@ ports . Two @-@ port networks can be represented as a 2 × 2 matrix , or equivalently , as a netork of two dependent generators and two impedances or admittances . There are six canonical forms of these representations : impedance parameters , chain parameters , hybrid parameters and their inverses . Any of them can be used . However , the representation of a passive transducer converting between analogous variables (for instance an effort variable to another effort variable in the impedance analogy) can be simplified by replacing the dependent generators with a transformer .
 On the other hand , a transducer converting non @-@ analogous power conjugate variables cannot be represented by a transformer . The two @-@ port element in the electrical domain that does this is called a gyrator . This device converts voltages to currents and currents to voltages . By analogy , a transducer that converts non @-@ analogous variables between energy domains is also called a gyrator . For instance , electromagnetic transducers convert current to force and velocity to voltage . In the impedance analogy such a transducer is a gyrator . Whether a transducer is a gyrator or a transformer is analogy related ; the same electromagnetic transducer in the mobility analogy is a transformer because it is converting between analogous variables .

= = History = =

 James Clerk Maxwell developed very detailed mechanical analogies of electrical phenomena . He was the first to associate force with voltage (1873) and consequently is usually credited with founding the impedance analogy . This was the earliest mechanical @-@ electrical analogy . However , the term impedance was not coined until 1886 , long after Maxwell 's death , by Oliver Heaviside . The idea of complex impedance was introduced by Arthur E. Kennelly in 1893 , and the concept of impedance was not extended into the mechanical domain until 1920 by Kennelly and Arthur Gordon Webster .
 Maxwell 's purpose in constructing this analogy was not to represent mechanical systems in terms of electrical networks . Rather , it was to explain electrical phenomena in more familiar mechanical terms . As electrical phenomena became better understood the reverse of this analogy , using electrical analogies to explain mechanical systems , started to become more common . Indeed , the lumped element abstract topology of electrical analysis has much to offer problems in the mechanical domain , and other energy domains for that matter . By 1900 the electrical analogy of the mechanical domain was becoming commonplace . From about 1920 the electrical analogy became a standard analysis tool . Vannevar Bush was a pioneer of this kind of modelling in his development of analogue computers , and a coherent presentation of this method was presented in a 1925 paper by Clifford A. Nickle .
 In 1933 Floyd A. Firestone proposed a new analogy , the mobility analogy , in which force is

analogous to current instead of voltage . Firestone introduced the concept of across and through variables in this paper and presented a structure for extending the analogy into other energy domains . A variation of the force @-@ current analogy was proposed by Horace M. Trent in 1955 and it is this version that is generally meant by the through and across analogy . Trent used a linear graph method of representing networks which has resulted in the force @-@ current analogy historically being associated with linear graphs . The force @-@ voltage analogy is historically used with bond graph representations , introduced in 1960 by Henry M. Paynter , however , it is possible to use either analogy with either representation if desired .