= 1 / 2 and the infinite geometric series for ? =
? 1) :
<formula>
with generalized binomial coefficients
<formula>
For instance , with the first several terms written out explicitly for the common square root cases , is :
<formula>
<formula>
Trigonometric functions :
<formula>
<formula>
<formula>
<formula>
<formula>
<formula>
<formula>
Hyperbolic functions :
<formula>
<formula>
<formula>
<formula>
<formula>
The numbers Bk appearing in the summation expansions of tan (x) and tanh (x) are the Bernoulli numbers . The Ek in the expansion of sec (x) are Euler numbers .

= = Calculation of Taylor series = =

Several methods exist for the calculation of Taylor series of a large number of functions . One can attempt to use the definition of the Taylor series , though this often requires generalizing the form of the coefficients according to a readily apparent pattern . Alternatively , one can use manipulations such as substitution , multiplication or division , addition or subtraction of standard Taylor series to construct the Taylor series of a function , by virtue of Taylor series being power series . In some cases , one can also derive the Taylor series by repeatedly applying integration by parts . Particularly convenient is the use of computer algebra systems to calculate Taylor series .

= = = First example = = =

In order to compute the 7th degree Maclaurin polynomial for the function
<formula> ,
one may first rewrite the function as
<formula> .
The Taylor series for the natural logarithm is (using the big O notation)
<formula>
and for the cosine function
<formula> .
The latter series expansion has a zero constant term , which enables us to substitute the second series into the first one and to easily omit terms of higher order than the 7th degree by using the big O notation :
<formula>
Since the cosine is an even function , the coefficients for all the odd powers x , x3 , x5 , x7 , ... have to be zero .

= = = Second example = = =

Suppose we want the Taylor series at 0 of the function
<formula>
We have for the exponential function
<formula>
and , as in the first example ,
<formula>
Assume the power series is
<formula>
Then multiplication with the denominator and substitution of the series of the cosine yields
<formula>
Collecting the terms up to fourth order yields
<formula>
Comparing coefficients with the above series of the exponential function yields the desired Taylor series
<formula>

= = = Third example = = =

Here we employ a method called " Indirect Expansion " to expand the given function . This method uses the known Taylor expansion of the exponential function . In order to expand
<formula>
as a Taylor series in x , we use the known Taylor series of function ex :
<formula>
Thus ,
<formula>

= = Taylor series as definitions = =

Classically , algebraic functions are defined by an algebraic equation , and transcendental functions (including those discussed above) are defined by some property that holds for them , such as a differential equation . For example , the exponential function is the function which is equal to its own derivative everywhere , and assumes the value 1 at the origin . However , one may equally well define an analytic function by its Taylor series .

Taylor series are used to define functions and " operators " in diverse areas of mathematics . In particular , this is true in areas where the classical definitions of functions break down . For example , using Taylor series , one may define analytical functions of matrices and operators , such as the matrix exponential or matrix logarithm .

In other areas , such as formal analysis , it is more convenient to work directly with the power series themselves . Thus one may define a solution of a differential equation as a power series which , one hopes to prove , is the Taylor series of the desired solution .

= = Taylor series in several variables = =

The Taylor series may also be generalized to functions of more than one variable with
<formula>
For example , for a function that depends on two variables , x and y , the Taylor series to second order about the point (a , b) is
<formula>
where the subscripts denote the respective partial derivatives .

A second @-@ order Taylor series expansion of a scalar @-@ valued function of more than one variable can be written compactly as

<formula>
where <formula> is the gradient of <formula> evaluated at <formula> and <formula> is the Hessian matrix . Applying the multi @-@ index notation the Taylor series for several variables becomes
<formula>
which is to be understood as a still more abbreviated multi @-@ index version of the first equation of this paragraph , again in full analogy to the single variable case .

= = = Example = = =

Compute a second @-@ order Taylor series expansion around point (a , b) = (0 , 0) of a function
<formula>
Firstly , we compute all partial derivatives we need
<formula>
Now we evaluate these derivatives at the origin :
<formula>
The Taylor series is
<formula>
which in this case becomes
<formula>
Since log (1 + y) is analytic in | y | < 1 , we have
<formula>

= = Comparison with Fourier series = =

The trigonometric Fourier series enables one to express a periodic function (or a function defined on a closed interval [a , b]) as an infinite sum of trigonometric functions (sines and cosines) . In this sense , the Fourier series is analogous to Taylor series , since the latter allows one to express a function as an infinite sum of powers . Nevertheless , the two series differ from each other in several relevant issues :

Obviously the finite truncations of the Taylor series of f (x) about the point x = a are all exactly equal to f at a . In contrast , the Fourier series is computed by integrating over an entire interval , so there is generally no such point where all the finite truncations of the series are exact .

Indeed , the computation of Taylor series requires the knowledge of the function on an arbitrary small neighbourhood of a point , whereas the computation of the Fourier series requires knowing the function on its whole domain interval . In a certain sense one could say that the Taylor series is " local " and the Fourier series is " global . "

The Taylor series is defined for a function which has infinitely many derivatives at a single point , whereas the Fourier series is defined for any integrable function . In particular , the function could be nowhere differentiable . (For example , f (x) could be a Weierstrass function .)

The convergence of both series has very different properties . Even if the Taylor series has positive convergence radius , the resulting series may not coincide with the function ; but if the function is analytic then the series converges pointwise to the function , and uniformly on every compact subset of the convergence interval . Concerning the Fourier series , if the function is square @-@ integrable then the series converges in quadratic mean , but additional requirements are needed to ensure the pointwise or uniform convergence (for instance , if the function is periodic and of class C1 then the convergence is uniform) .

Finally , in practice one wants to approximate the function with a finite number of terms , let 's say with a Taylor polynomial or a partial sum of the trigonometric series , respectively . In the case of the Taylor series the error is very small in a neighbourhood of the point where it is computed , while it may be very large at a distant point . In the case of the Fourier series the error is distributed along the domain of the function .