= BD (BK + KC) =
BD × BC
Therefore , AB2 + AC2 = BC2 , since CBDE is a square .
This proof , which appears in Euclid 's Elements as that of Proposition 47 in Book 1 , demonstrates that the area of the square on the hypotenuse is the sum of the areas of the other two squares . This is quite distinct from the proof by similarity of triangles , which is conjectured to be the proof that Pythagoras used .

= = = Proofs by dissection and rearrangement = = =

We have already discussed the Pythagorean proof , which was a proof by rearrangement . The same idea is conveyed by the leftmost animation below , which consists of a large square , side a + b , containing four identical right triangles . The triangles are shown in two arrangements , the first of which leaves two squares a2 and b2 uncovered , the second of which leaves square c2 uncovered . The area encompassed by the outer square never changes , and the area of the four triangles is the same at the beginning and the end , so the black square areas must be equal , therefore a2 + b2 = c2 .
A second proof by rearrangement is given by the middle animation . A large square is formed with area c2 , from four identical right triangles with sides a , b and c , fitted around a small central square . Then two rectangles are formed with sides a and b by moving the triangles . Combining the smaller square with these rectangles produces two squares of areas a2 and b2 , which must have the same area as the initial large square .
The third , rightmost image also gives a proof . The upper two squares are divided as shown by the blue and green shading , into pieces that when rearranged can be made to fit in the lower square on the hypotenuse ? or conversely the large square can be divided as shown into pieces that fill the other two . This way of cutting one figure into pieces and rearranging them to get another figure is called dissection . This shows the area of the large square equals that of the two smaller ones .

= = = Einstein 's proof by dissection without rearrangement = = =

Albert Einstein gave a proof by dissection in which the pieces need not get moved . Instead of using a square on the hypotenuse and two squares on the legs , one can use any other shape that includes the hypotenuse , and two similar shapes that each include one of two legs instead of the hypotenuse . In Einstein 's proof , the shape that includes the hypotenuse is the right triangle itself . The dissection consists of dropping a perpendicular from the vertex of the right angle of the triangle to the hypotenuse , thus splitting the whole triangle into two parts . Those two parts have the same shape as the original right triangle , and have the legs of the original triangle as their hypotenuses , and the sum of their areas is that of the original triangle . Because the ratio of the area of a right triangle to the square of its hypotenuse is the same for similar triangles , the relationship between the areas of the three triangles holds for the squares of the sides of the large triangle as well .

= = = Algebraic proofs = = =

The theorem can be proved algebraically using four copies of a right triangle with sides a , b and c , arranged inside a square with side c as in the top half of the diagram . The triangles are similar with area <formula> , while the small square has side b ? a and area (b ? a) 2 . The area of the large square is therefore
<formula>
But this is a square with side c and area c2 , so
<formula>
A similar proof uses four copies of the same triangle arranged symmetrically around a square with side c , as shown in the lower part of the diagram . This results in a larger square , with side a + b and area (a + b) 2 . The four triangles and the square side c must have the same area as the larger

square ,
<formula>
giving
<formula>
A related proof was published by future U.S. President James A. Garfield (then a U.S. Representative) . Instead of a square it uses a trapezoid , which can be constructed from the square in the second of the above proofs by bisecting along a diagonal of the inner square , to give the trapezoid as shown in the diagram . The area of the trapezoid can be calculated to be half the area of the square , that is
<formula>
The inner square is similarly halved , and there are only two triangles so the proof proceeds as above except for a factor of <formula> , which is removed by multiplying by two to give the result .

= = = Proof using differentials = = =

One can arrive at the Pythagorean theorem by studying how changes in a side produce a change in the hypotenuse and employing calculus .
The triangle ABC is a right triangle , as shown in the upper part of the diagram , with BC the hypotenuse . At the same time the triangle lengths are measured as shown , with the hypotenuse of length y , the side AC of length x and the side AB of length a , as seen in the lower diagram part .
If x is increased by a small amount dx by extending the side AC slightly to D , then y also increases by dy . These form two sides of a triangle , CDE , which (with E chosen so CE is perpendicular to the hypotenuse) is a right triangle approximately similar to ABC . Therefore , the ratios of their sides must be the same , that is :
<formula>
This can be rewritten as <formula> , which is a differential equation that can be solved by direct integration :
<formula>
giving
<formula>