= Pythagorean theorem =

In mathematics , the Pythagorean theorem , also known as Pythagoras ' theorem , is a fundamental relation in Euclidean geometry among the three sides of a right triangle . It states that the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides . The theorem can be written as an equation relating the lengths of the sides a , b and c , often called the " Pythagorean equation " :

<formula>

where c represents the length of the hypotenuse and a and b the lengths of the triangle 's other two sides .

Although it is often argued that knowledge of the theorem predates him , the theorem is named after the ancient Greek mathematician Pythagoras (c . 570 ? c . 495 BC) as it is he who , by tradition , is credited with its first recorded proof . There is some evidence that Babylonian mathematicians understood the formula , although little of it indicates an application within a mathematical framework . Mesopotamian , Indian and Chinese mathematicians all discovered the theorem independently and , in some cases , provided proofs for special cases .

The theorem has been given numerous proofs ? possibly the most for any mathematical theorem . They are very diverse , including both geometric proofs and algebraic proofs , with some dating back thousands of years . The theorem can be generalized in various ways , including higher @-@ dimensional spaces , to spaces that are not Euclidean , to objects that are not right triangles , and indeed , to objects that are not triangles at all , but n @-@ dimensional solids . The Pythagorean theorem has attracted interest outside mathematics as a symbol of mathematical abstruseness , mystique , or intellectual power ; popular references in literature , plays , musicals , songs , stamps and cartoons abound .

= = Pythagorean proof = =

The Pythagorean Theorem was known long before Pythagoras , but he may well have been the first to prove it . In any event , the proof attributed to him is very simple , and is called a proof by rearrangement .

The two large squares shown in the figure each contain four identical triangles , and the only difference between the two large squares is that the triangles are arranged differently . Therefore , the white space within each of the two large squares must have equal area . Equating the area of the white space yields the Pythagorean Theorem , Q.E.D.

That Pythagoras originated this very simple proof is sometimes inferred from the writings of the later Greek philosopher and mathematician Proclus . Several other proofs of this theorem are described below , but this is known as the Pythagorean one .

= = Other forms of the theorem = =

As pointed out in the introduction , if c denotes the length of the hypotenuse and a and b denote the lengths of the other two sides , the Pythagorean theorem can be expressed as the Pythagorean equation :

<formula>

If the length of both a and b are known , then c can be calculated as

<formula>

If the length of the hypotenuse c and of one side (a or b) are known , then the length of the other side can be calculated as

<formula>

or

<formula>

The Pythagorean equation relates the sides of a right triangle in a simple way , so that if the lengths of any two sides are known the length of the third side can be found . Another corollary of the

theorem is that in any right triangle , the hypotenuse is greater than any one of the other sides , but less than their sum .
A generalization of this theorem is the law of cosines , which allows the computation of the length of any side of any triangle , given the lengths of the other two sides and the angle between them . If the angle between the other sides is a right angle , the law of cosines reduces to the Pythagorean equation .

= = Other proofs of the theorem = =

This theorem may have more known proofs than any other (the law of quadratic reciprocity being another contender for that distinction) ; the book The Pythagorean Proposition contains 370 proofs .

= = = Proof using similar triangles = = =

This proof is based on the proportionality of the sides of two similar triangles , that is , upon the fact that the ratio of any two corresponding sides of similar triangles is the same regardless of the size of the triangles .
Let ABC represent a right triangle , with the right angle located at C , as shown on the figure . Draw the altitude from point C , and call H its intersection with the side AB . Point H divides the length of the hypotenuse c into parts d and e . The new triangle ACH is similar to triangle ABC , because they both have a right angle (by definition of the altitude) , and they share the angle at A , meaning that the third angle will be the same in both triangles as well , marked as ? in the figure . By a similar reasoning , the triangle CBH is also similar to ABC . The proof of similarity of the triangles requires the triangle postulate : the sum of the angles in a triangle is two right angles , and is equivalent to the parallel postulate . Similarity of the triangles leads to the equality of ratios of corresponding sides :
<formula>
The first result equates the cosines of the angles ? , whereas the second result equates their sines .

These ratios can be written as
<formula>
Summing these two equalities results in
<formula>
which , after simplification , expresses the Pythagorean theorem :
<formula>
The role of this proof in history is the subject of much speculation . The underlying question is why Euclid did not use this proof , but invented another . One conjecture is that the proof by similar triangles involved a theory of proportions , a topic not discussed until later in the Elements , and that the theory of proportions needed further development at that time .

= = = Euclid 's proof = = =

In outline , here is how the proof in Euclid 's Elements proceeds . The large square is divided into a left and right rectangle . A triangle is constructed that has half the area of the left rectangle . Then another triangle is constructed that has half the area of the square on the left @-@ most side . These two triangles are shown to be congruent , proving this square has the same area as the left rectangle . This argument is followed by a similar version for the right rectangle and the remaining square . Putting the two rectangles together to reform the square on the hypotenuse , its area is the same as the sum of the area of the other two squares . The details follow .
Let A , B , C be the vertices of a right triangle , with a right angle at A. Drop a perpendicular from A to the side opposite the hypotenuse in the square on the hypotenuse . That line divides the square on the hypotenuse into two rectangles , each having the same area as one of the two squares on

the legs .
For the formal proof , we require four elementary lemmata :
If two triangles have two sides of the one equal to two sides of the other , each to each , and the angles included by those sides equal , then the triangles are congruent (side @-@ angle @-@ side) .
The area of a triangle is half the area of any parallelogram on the same base and having the same altitude .
The area of a rectangle is equal to the product of two adjacent sides .
The area of a square is equal to the product of two of its sides (follows from 3) .
Next , each top square is related to a triangle congruent with another triangle related in turn to one of two rectangles making up the lower square .
The proof is as follows :
Let ACB be a right @-@ angled triangle with right angle CAB .
On each of the sides BC , AB , and CA , squares are drawn , CBDE , BAGF , and ACIH , in that order . The construction of squares requires the immediately preceding theorems in Euclid , and depends upon the parallel postulate .
From A , draw a line parallel to BD and CE . It will perpendicularly intersect BC and DE at K and L , respectively .
Join CF and AD , to form the triangles BCF and BDA .
Angles CAB and BAG are both right angles ; therefore C , A , and G are collinear . Similarly for B , A , and H.
Angles CBD and FBA are both right angles ; therefore angle ABD equals angle FBC , since both are the sum of a right angle and angle ABC .
Since AB is equal to FB and BD is equal to BC , triangle ABD must be congruent to triangle FBC .
Since A @-@ K @-@ L is a straight line , parallel to BD , then rectangle BDLK has twice the area of triangle ABD because they share the base BD and have the same altitude BK , i.e. , a line normal to their common base , connecting the parallel lines BD and AL . (lemma 2)
Since C is collinear with A and G , square BAGF must be twice in area to triangle FBC .