= 0 , and has all derivatives zero there . Consequently , the Taylor series of f (x) about x = 0 is identically zero . However , f (x) is not the zero function , so does not equal its Taylor series around the origin . Thus , f (x) is an example of a non @-@ analytic smooth function .

In real analysis , this example shows that there are infinitely differentiable functions f (x) whose Taylor series are not equal to f (x) even if they converge . By contrast , the holomorphic functions studied in complex analysis always possess a convergent Taylor series , and even the Taylor series of meromorphic functions , which might have singularities , never converge to a value different from the function itself . The complex function e ? z ? 2 , however , does not approach 0 when z approaches 0 along the imaginary axis , so it is not continuous in the complex plane and its Taylor series is undefined at 0 .

More generally , every sequence of real or complex numbers can appear as coefficients in the Taylor series of an infinitely differentiable function defined on the real line , a consequence of Borel 's lemma . As a result , the radius of convergence of a Taylor series can be zero . There are even infinitely differentiable functions defined on the real line whose Taylor series have a radius of convergence 0 everywhere .

Some functions cannot be written as Taylor series because they have a singularity ; in these cases , one can often still achieve a series expansion if one allows also negative powers of the variable x ; see Laurent series . For example , f (x) = e ? x ? 2 can be written as a Laurent series .

= = = Generalization = = =

There is , however , a generalization of the Taylor series that does converge to the value of the function itself for any bounded continuous function on (0 , ?) , using the calculus of finite differences . Specifically , one has the following theorem , due to Einar Hille , that for any t > 0 @,@

<formula>

Here ?nh is the n @-@ th finite difference operator with step size h . The series is precisely the Taylor series , except that divided differences appear in place of differentiation : the series is formally similar to the Newton series . When the function f is analytic at a , the terms in the series converge to the terms of the Taylor series , and in this sense generalizes the usual Taylor series .

In general , for any infinite sequence ai , the following power series identity holds :

<formula>

So in particular ,

<formula>

The series on the right is the expectation value of f (a + X) , where X is a Poisson distributed random variable that takes the value jh with probability e ? t / h (t / h) j / j ! . Hence ,

<formula>

The law of large numbers implies that the identity holds .

= = List of Maclaurin series of some common functions = =

See also List of mathematical series

Several important Maclaurin series expansions follow . All these expansions are valid for complex arguments x .

Exponential function :

<formula>

Natural logarithm :

<formula>

<formula>

Geometric series and its derivatives (see article for variants) :

<formula>

<formula>

<formula>

<formula>
<formula>