= 2p ? 3 . The case of 2 @-@ dimensional spheres is slightly different : the first p @-@ torsion occurs for k =

2p ? 3 + 1 . In the case of odd torsion there are more precise results ; in this case there is a big difference between odd and even dimensional spheres . If p is an odd prime and n = 2i + 1 , then elements of the p @-@ component of ?n + k (Sn) have order at most pi (Cohen , Moore & Neisendorfer 1979) . This is in some sense the best possible result , as these groups are known to have elements of this order for some values of k (Ravenel 2003 , p . 4) . Furthermore , the stable range can be extended in this case : if n is odd then the double suspension from ?k (Sn) to ?k + 2 (Sn + 2) is an isomorphism of p @-@ components if k < p (n + 1) ? 3 , and an epimorphism if equality holds (Serre 1952) . The p @-@ torsion of the intermediate group ?k + 1 (Sn + 1) can be strictly larger .

The results above about odd torsion only hold for odd @-@ dimensional spheres : for even @-@ dimensional spheres , the James fibration gives the torsion at odd primes p in terms of that of odd @-@ dimensional spheres ,

<formula>

(where (p) means take the p @-@ component) (Ravenel 2003 , p . 25) . This exact sequence is similar to the ones coming from the Hopf fibration ; the difference is that it works for all even @-@ dimensional spheres , albeit at the expense of ignoring 2 @-@ torsion . Combining the results for odd and even dimensional spheres shows that much of the odd torsion of unstable homotopy groups is determined by the odd torsion of the stable homotopy groups .

For stable homotopy groups there are more precise results about p @-@ torsion . For example , if k < 2p (p ? 1) ? 2 for a prime p then the p @-@ primary component of the stable homotopy group ?kS vanishes unless k + 1 is divisible by 2 (p ? 1) , in which case it is cyclic of order p (Fuks 2001) .

= = = The J @-@ homomorphism = = =

An important subgroup of ?n + k (Sn) , for k ? 2 , is the image of the J @-@ homomorphism J : ?k (SO (n)) ? ?n + k (Sn) , where SO (n) denotes the special orthogonal group (Adams 1966) . In the stable range n ? k + 2 , the homotopy groups ?k (SO (n)) only depend on k modulo 8 . This period 8 pattern is known as Bott periodicity , and it is reflected in the stable homotopy groups of spheres via the image of the J @-@ homomorphism which is :

a cyclic group of order 2 if k is congruent to 0 or 1 modulo 8 ;

trivial if k is congruent to 2 , 4 , 5 , or 6 modulo 8 ; and

a cyclic group of order equal to the denominator of B2n / 4n , where B2n is a Bernoulli number , if k ? 3 (mod 4) .