= f (x) + f (y) and f (a · x) =
a · f (x) for all x and y in V , all a in F.
An isomorphism is a linear map f : V ? W such that there exists an inverse map g : W ? V , which is a map such that the two possible compositions f ? g : W ? W and g ? f : V ? V are identity maps . Equivalently , f is both one @-@ to @-@ one (injective) and onto (surjective) . If there exists an isomorphism between V and W , the two spaces are said to be isomorphic ; they are then essentially identical as vector spaces , since all identities holding in V are , via f , transported to similar ones in W , and vice versa via g .
For example , the " arrows in the plane " and " ordered pairs of numbers " vector spaces in the introduction are isomorphic : a planar arrow v departing at the origin of some (fixed) coordinate system can be expressed as an ordered pair by considering the x- and y @-@ component of the arrow , as shown in the image at the right . Conversely , given a pair (x , y) , the arrow going by x to the right (or to the left , if x is negative) , and y up (down , if y is negative) turns back the arrow v.
Linear maps V ? W between two vector spaces form a vector space HomF (V , W) , also denoted L (V , W) . The space of linear maps from V to F is called the dual vector space , denoted V ? . Via the injective natural map V ? V ? ? , any vector space can be embedded into its bidual ; the map is an isomorphism if and only if the space is finite @-@ dimensional .
Once a basis of V is chosen , linear maps f : V ? W are completely determined by specifying the images of the basis vectors , because any element of V is expressed uniquely as a linear combination of them . If dim V = dim W , a 1 @-@ to @-@ 1 correspondence between fixed bases of V and W gives rise to a linear map that maps any basis element of V to the corresponding basis element of W. It is an isomorphism , by its very definition . Therefore , two vector spaces are isomorphic if their dimensions agree and vice versa . Another way to express this is that any vector space is completely classified (up to isomorphism) by its dimension , a single number . In particular , any n @-@ dimensional F @-@ vector space V is isomorphic to Fn . There is , however , no " canonical " or preferred isomorphism ; actually an isomorphism ? : Fn ? V is equivalent to the choice of a basis of V , by mapping the standard basis of Fn to V , via ? . The freedom of choosing a convenient basis is particularly useful in the infinite @-@ dimensional context , see below .

= = = Matrices = = =

Matrices are a useful notion to encode linear maps . They are written as a rectangular array of scalars as in the image at the right . Any m @-@ by @-@ n matrix A gives rise to a linear map from Fn to Fm , by the following
<formula> , where <formula> denotes summation ,
or , using the matrix multiplication of the matrix A with the coordinate vector x :
x ? Ax .
Moreover , after choosing bases of V and W , any linear map f : V ? W is uniquely represented by a matrix via this assignment .
The determinant det (A) of a square matrix A is a scalar that tells whether the associated map is an isomorphism or not : to be so it is sufficient and necessary that the determinant is nonzero . The linear transformation of Rn corresponding to a real n @-@ by @-@ n matrix is orientation preserving if and only if its determinant is positive .

= = = Eigenvalues and eigenvectors = = =