= c · f ? for a constant c) this assignment is linear , called a linear differential operator . In particular , the solutions to the differential equation D (f) =
0 form a vector space (over R or C) .

= = = Direct product and direct sum = = =

The direct product of vector spaces and the direct sum of vector spaces are two ways of combining an indexed family of vector spaces into a new vector space .
The direct product <formula> of a family of vector spaces Vi consists of the set of all tuples (vi) i ? I , which specify for each index i in some index set I an element vi of Vi . Addition and scalar multiplication is performed componentwise . A variant of this construction is the direct sum <formula> (also called coproduct and denoted <formula>) , where only tuples with finitely many nonzero vectors are allowed . If the index set I is finite , the two constructions agree , but in general they are different .

= = = Tensor product = = =

The tensor product V ? F W , or simply V ? W , of two vector spaces V and W is one of the central notions of multilinear algebra which deals with extending notions such as linear maps to several variables . A map g : V × W ? X is called bilinear if g is linear in both variables v and w . That is to say , for fixed w the map v ? g (v , w) is linear in the sense above and likewise for fixed v.
The tensor product is a particular vector space that is a universal recipient of bilinear maps g , as follows . It is defined as the vector space consisting of finite (formal) sums of symbols called tensors
v1 ? w1 + v2 ? w2 + ... + vn ? wn ,
subject to the rules
a · (v ? w)
= (a · v) ? w =
v ? (a · w) , where a is a scalar ,