Group (mathematics)

In mathematics , a group is an algebraic structure consisting of a set of elements equipped with an operation that combines any two elements to form a third element . The operation satisfies four conditions called the group axioms , namely closure , associativity , identity and invertibility . One of the most familiar examples of a group is the set of integers together with the addition operation , but the abstract formalization of the group axioms , detached as it is from the concrete nature of any particular group and its operation , applies much more widely . It allows entities with highly diverse mathematical origins in abstract algebra and beyond to be handled in a flexible way while retaining their essential structural aspects . The ubiquity of groups in numerous areas within and outside mathematics makes them a central organizing principle of contemporary mathematics .

Groups share a fundamental kinship with the notion of symmetry . For example , a symmetry group encodes symmetry features of a geometrical object : the group consists of the set of transformations that leave the object unchanged and the operation of combining two such transformations by performing one after the other . Lie groups are the symmetry groups used in the Standard Model of particle physics ; Poincaré groups , which are also Lie groups , can express the physical symmetry underlying special relativity ; and Point groups are used to help understand symmetry phenomena in molecular chemistry .

The concept of a group arose from the study of polynomial equations , starting with Évariste Galois in the 1830s . After contributions from other fields such as number theory and geometry , the group notion was generalized and firmly established around 1870 . Modern group theory ? an active mathematical discipline ? studies groups in their own right . To explore groups , mathematicians have devised various notions to break groups into smaller , better @-@ understandable pieces , such as subgroups , quotient groups and simple groups . In addition to their abstract properties , group theorists also study the different ways in which a group can be expressed concretely (its group representations) , both from a theoretical and a computational point of view . A theory has been developed for finite groups , which culminated with the classification of finite simple groups , completed in 2004 . Since the mid @-@ 1980s , geometric group theory , which studies finitely generated groups as geometric objects , has become a particularly active area in group theory .

Definition and illustration

First example : the integers

One of the most familiar groups is the set of integers Z which consists of the numbers
... , ? 4 , ? 3 , ? 2 , ? 1 , 0 , 1 , 2 , 3 , 4 , ... , together with addition .

The following properties of integer addition serve as a model for the abstract group axioms given in the definition below .

For any two integers a and b , the sum a + b is also an integer . That is , addition of integers always yields an integer . This property is known as closure under addition .