= 99 × 99 ; 396 =
4 × 99) and is related to the fact that
<formula>
This might be compared to Heegner numbers , which have class number 1 and yield similar formulae .
Ramanujan 's series for ? converges extraordinarily rapidly (exponentially) and forms the basis of some of the fastest algorithms currently used to calculate ? . Truncating the sum to the first term also gives the approximation 9801 ? 2 / 4412 for ? , which is correct to six decimal places . See also the more general Ramanujan ? Sato series .
One of Ramanujan 's remarkable capabilities was the rapid solution of problems . Once , a roommate of his , P. C. Mahalanobis , posed the following problem :
" Imagine that you are on a street with houses marked 1 through n . There is a house in between (x) such that the sum of the house numbers to the left of it equals the sum of the house numbers to its right . If n is between 50 and 500 , what are n and x ? " This is a bivariate problem with multiple solutions . Ramanujan thought about it and gave the answer with a twist : He gave a continued fraction . The unusual part was that it was the solution to the whole class of problems . Mahalanobis was astounded and asked how he did it . ? It is simple . The minute I heard the problem , I knew that the answer was a continued fraction . Which continued fraction , I asked myself . Then the answer came to my mind ? , Ramanujan replied .
His intuition also led him to derive some previously unknown identities , such as
<formula>
for all ? , where ? (z) is the gamma function , and related to a special value of the Dedekind eta function . Expanding into series of powers and equating coefficients of ?0 , ?4 , and ?8 gives some deep identities for the hyperbolic secant .
In 1918 Hardy and Ramanujan studied the partition function P (n) extensively . They gave a non @-@ convergent asymptotic series that permits exact computation of the number of partitions of an integer . Hans Rademacher , in 1937 , was able to refine their formula to find an exact convergent series solution to this problem . Ramanujan and Hardy 's work in this area gave rise to a powerful new method for finding asymptotic formulae called the circle method .
In the last year of his life , Ramanujan discovered mock theta functions . For many years these functions were a mystery , but they are now known to be the holomorphic parts of harmonic weak Maass forms .

= = = The Ramanujan conjecture = = =

Although there are numerous statements that could have borne the name Ramanujan conjecture , there is one that was very influential on later work . In particular , the connection of this conjecture with conjectures of André Weil in algebraic geometry opened up new areas of research . That Ramanujan conjecture is an assertion on the size of the tau @-@ function , which has as generating function the discriminant modular form ? (q) , a typical cusp form in the theory of modular forms . It was finally proven in 1973 , as a consequence of Pierre Deligne 's proof of the Weil conjectures . The reduction step involved is complicated . Deligne won a Fields Medal in 1978 for that work .
In his paper On certain arithmetical functions , Ramanujan defined the so @-@ called Delta @-@ function whose coefficients are called ? (n) (the Ramanujan tau function) . He proved many congruences for these numbers such as ? (p) ? 1 + p11 mod 691 for primes p . This congruence (and others like it that Ramanujan proved) inspired Jean @-@ Pierre Serre (1954 Fields Medalist) to conjecture that there is a theory of Galois representations which " explains " these congruences and more generally all modular forms . Delta (z) is the first example of a modular form to be studied in this way . Pierre Deligne (in his Fields Medal winning work) proved Serre 's conjecture . The proof of Fermat 's Last Theorem proceeds by first reinterpreting elliptic curves and modular forms in terms of these Galois representations . Without this theory there would be no proof of Fermat 's Last Theorem .

Ramanujan 's notebooks

While still in Madras , Ramanujan recorded the bulk of his results in four notebooks of loose @-@ leaf paper . They were mostly written up without any derivations . This is probably the origin of the misperception that Ramanujan was unable to prove his results and simply thought up the final result directly . Mathematician Bruce C. Berndt , in his review of these notebooks and Ramanujan 's work , says that Ramanujan most certainly was able to prove most of his results , but chose not to .

That may have been for several reasons . Since paper was very expensive , Ramanujan would do most of his work and perhaps his proofs on slate , and then transfer just the results to paper . Using a slate was common for mathematics students in the Madras Presidency at the time . He was also quite likely to have been influenced by the style of G. S. Carr 's book , which stated results without proofs . Finally , it is possible that Ramanujan considered his workings to be for his personal interest alone and therefore recorded only the results .

The first notebook has 351 pages with 16 somewhat organised chapters and some unorganised material . The second notebook has 256 pages in 21 chapters and 100 unorganised pages , with the third notebook containing 33 unorganised pages . The results in his notebooks inspired numerous papers by later mathematicians trying to prove what he had found . Hardy himself created papers exploring material from Ramanujan 's work , as did G. N. Watson , B. M. Wilson , and Bruce Berndt . A fourth notebook with 87 unorganised pages , the so @-@ called " lost notebook " , was rediscovered in 1976 by George Andrews .

Notebooks 1 , 2 and 3 were published as a two @-@ volume set in 1957 by the Tata Institute of Fundamental Research (TIFR) , Mumbai , India . This was a photocopy edition of the original manuscripts , in his own handwriting .

In December 2011 , as part of the celebrations of the 125th anniversary of Ramanujan 's birth , TIFR republished the notebooks in a coloured two @-@ volume collector 's edition . These were produced from scanned and microfilmed images of the original manuscripts by expert archivists of Raja Muthiah Research Library , Chennai .

Hardy ? Ramanujan number 1729

The number 1729 is known as the Hardy ? Ramanujan number after a famous visit by Hardy to see Ramanujan at a hospital . In Hardy 's words :

I remember once going to see him when he was ill at Putney . I had ridden in taxi cab number 1729 and remarked that the number seemed to me rather a dull one , and that I hoped it was not an unfavorable omen . ' No ' , he replied , ' it is a very interesting number ; it is the smallest number expressible as the sum of two cubes in two different ways.'

Immediately before this anecdote , Hardy quoted Littlewood as saying , " Every positive integer was one of [Ramanujan 's] personal friends . "

The two different ways are