

= Dirac delta function =

In mathematics, the Dirac delta function, or δ function, is a generalized function, or distribution, on the real number line that is zero everywhere except at zero, with an integral of one over the entire real line. The delta function is sometimes thought of as a hypothetical function whose graph is an infinitely high, infinitely thin spike at the origin, with total area one under the spike, and physically represents the density of an idealized point mass or point charge. It was introduced by theoretical physicist Paul Dirac.

From a purely mathematical viewpoint, the Dirac delta is not strictly a function, because any extended \mathbb{R} -valued real function that is equal to zero everywhere but a single point must have total integral zero. The delta function only makes sense as a mathematical object when it appears inside an integral. From this perspective the Dirac delta can usually be manipulated as though it were a function. The formal rules obeyed by this "function" are part of the operational calculus, a standard tool kit of physics and engineering. The operational calculus, and in particular the delta function, was viewed with suspicion by mathematicians of the early 20th century, until a satisfactory rigorous theory was introduced by Laurent Schwartz in the 1950s. Formally, the delta function must be defined as the distribution that corresponds to a probability measure supported at the origin. In many applications, the Dirac delta is regarded as a kind of limit (a weak limit) of a sequence of functions having a tall spike at the origin. The approximating functions of the sequence are thus "approximate" or "nascent" delta functions.

In the context of signal processing the delta function is often referred to as the unit impulse symbol (or function). Its discrete analog is the Kronecker delta function, which is usually defined on a discrete domain and takes values 0 and 1.

= Overview =

The graph of the delta function is usually thought of as following the whole x -axis and the positive y -axis. Despite its name, the delta function is not truly a function, at least not a usual one with range in real numbers. For example, the objects $f(x)$ and $g(x) =$