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1) , the ring is called a division ring . A field is defined as a commutative division ring .

Groups are frequently studied through group representations . In their most general form , these consist of a choice of group , a set , and an action of the group on the set , that is , an operation which takes an element of the group and an element of the set and returns an element of the set . Most often , the set is a vector space , and the group represents symmetries of the vector space . For example , there is a group which represents the rigid rotations of space . This is a type of symmetry of space , because space itself does not change when it is rotated even though the positions of objects in it do . Noether used these sorts of symmetries in her work on invariants in physics .

A powerful way of studying rings is through their modules . A module consists of a choice of ring , another set , usually distinct from the underlying set of the ring and called the underlying set of the module , an operation on pairs of elements of the underlying set of the module , and an operation which takes an element of the ring and an element of the module and returns an element of the module . The underlying set of the module and its operation must form a group . A module is a ring @-@ theoretic version of a group representation : Ignoring the second ring operation and the operation on pairs of module elements determines a group representation . The real utility of modules is that the kinds of modules that exist and their interactions , reveal the structure of the ring in ways that are not apparent from the ring itself . An important special case of this is an algebra . (The word algebra means both a subject within mathematics as well as an object studied in the subject of algebra .) An algebra consists of a choice of two rings and an operation which takes an element from each ring and returns an element of the second ring . This operation makes the second ring into a module over the first . Often the first ring is a field .

Words such as " element " and " combining operation " are very general , and can be applied to many real @-@ world and abstract situations . Any set of things that obeys all the rules for one (or two) operation (s) is , by definition , a group (or ring) , and obeys all theorems about groups (or rings) . Integer numbers , and the operations of addition and multiplication , are just one example . For example , the elements might be computer data words , where the first combining operation is exclusive or and the second is logical conjunction . Theorems of abstract algebra are powerful because they are general ; they govern many systems . It might be imagined that little could be concluded about objects defined with so few properties , but precisely therein lay Noether 's gift : to discover the maximum that could be concluded from a given set of properties , or conversely , to identify the minimum set , the essential properties responsible for a particular observation . Unlike most mathematicians , she did not make abstractions by generalizing from known examples ; rather , she worked directly with the abstractions . As van der Waerden recalled in his obituary of her ,

The maxim by which Emmy Noether was guided throughout her work might be formulated as follows : " Any relationships between numbers , functions , and operations become transparent , generally applicable , and fully productive only after they have been isolated from their particular objects and been formulated as universally valid concepts . "

This is the begriffliche Mathematik (purely conceptual mathematics) that was characteristic of Noether . This style of mathematics was consequently adopted by other mathematicians , especially in the (then new) field of abstract algebra .

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