= 27 , b2 =
9 , and c2 = 36 . Bringing the line segment together with its coordinate projections forms the traditional right triangle .

Similarly , for any two @-@ dimensional object in three @-@ dimensional space , the formula can be stated as :

<formula>

where D is the area of a specified two @-@ dimensional object , A is the area of the object 's projection onto the xy @-@ coordinate plane , B is the area of the object 's projection onto the xz @-@ coordinate plane , and C is the area of the object 's projection onto the yz @-@ coordinate plane .

The animation showing a blue three @-@ by @-@ three square object in three dimensions of space illustrates this application of the generalization to an object of more than one dimension . As the orientation of the object changes , the proportions of the green coordinate plane projections adjust accordingly , so the squares of the areas of the projections always add up to the same value : the square of the area of the original object . In this case , the sum of the squares of the projection areas always add up to 81 .

= = = = Applied to sets containing multiple objects = = = =

The generalization applies equally to sets of multiple objects , as long as they are in the same plane or parallel planes . The measures of the objects in such a set can be added together and essentially treated as a single object . The multiple line @-@ segment animation illustrates the generalization applied to a set of three one @-@ dimensional objects in three dimensions of space . In this case , two sequential line segments exist in parallel to a third line segment . Because lines are one @-@ dimensional , the coordinate subspaces onto which they are projected must also be one @-@ dimensional . Thus , projections appear on the coordinate axes rather than on the coordinate planes . The lengths of the projected line segments on a given axis are summed , then squared , then added to the total lengths squared on the other axes . The result is the squared sum of the lengths of the original line segments . For the sake of simplicity , when projections are single points of zero length , they are not shown , since they do not affect the calculations .

The generalization applies to flat objects of any shape , regular or irregular . The multi @-@ object animation illustrates the use of the generalization on a set of several different objects in different planes ? in this case , a triangle and a circle on one plane , and a flat cat on a parallel plane (shown in blue) . Projections of the set are shown in green on the coordinate plane subspaces . Objects shown initially upright in the yz @-@ plane are subsequently tilted in parallel . Again , regardless of set orientation , the result remains the same . On each coordinate plane subspace , the areas of object projections are calculated individually (to avoid miscalculations due to projection overlap) , then added together to produce the total projection area of the set on that plane . The projection set area is then squared for each coordinate plane . The sum of all projection set areas squared is always equal to the original set area squared .

= = = = Applied in any number of dimensions = = = =

This generalization holds regardless of the number of dimensions involved . The volume squared for a three @-@ dimensional object or set can be calculated by summing the squares of the volumes of the associated three @-@ dimensional projections onto three @-@ dimensional subspaces . Any number of dimensions is valid for the set as long as one uses the same number of dimensions for the coordinate subspaces and projections .

It is the built @-@ in symmetry of the Cartesian coordinate system where coordinates are orthogonal vectors of unit length in flat Euclidean space that allows this generalization to apply so broadly .

= = = Non @-@ Euclidean geometry = = =

The Pythagorean theorem is derived from the axioms of Euclidean geometry , and in fact , the Pythagorean theorem given above does not hold in a non @-@ Euclidean geometry . (The Pythagorean theorem has been shown , in fact , to be equivalent to Euclid 's Parallel (Fifth) Postulate .) In other words , in non @-@ Euclidean geometry , the relation between the sides of a triangle must necessarily take a non @-@ Pythagorean form . For example , in spherical geometry , all three sides of the right triangle (say a , b , and c) bounding an octant of the unit sphere have length equal to ? / 2 , and all its angles are right angles , which violates the Pythagorean theorem because a2 + b2 ? c2 .

Here two cases of non @-@ Euclidean geometry are considered ? spherical geometry and hyperbolic plane geometry ; in each case , as in the Euclidean case for non @-@ right triangles , the result replacing the Pythagorean theorem follows from the appropriate law of cosines .

However , the Pythagorean theorem remains true in hyperbolic geometry and elliptic geometry if the condition that the triangle be right is replaced with the condition that two of the angles sum to the third , say A + B = C. The sides are then related as follows : the sum of the areas of the circles with diameters a and b equals the area of the circle with diameter c .

= = = = Spherical geometry = = = =

For any right triangle on a sphere of radius R (for example , if ? in the figure is a right angle) , with sides a , b , c , the relation between the sides takes the form :

<formula>

This equation can be derived as a special case of the spherical law of cosines that applies to all spherical triangles :

<formula>

By expressing the Maclaurin series for the cosine function as an asymptotic expansion with the remainder term in big O notation ,

<formula>

it can be shown that as the radius R approaches infinity and the arguments a / R , b / R , and c / R tend to zero , the spherical relation between the sides of a right triangle approaches the Euclidean form of the Pythagorean theorem . Substituting the asymptotic expansion for each of the cosines into the spherical relation for a right triangle yields

<formula>

The constants a4 , b4 , and c4 have been absorbed into the big O remainder terms since they are independent of the radius R. This asymptotic relationship can be further simplified by multiplying out the bracketed quantities , cancelling the ones , multiplying through by ? 2 , and collecting all the error terms together :

<formula>

After multiplying through by R2 , the Euclidean Pythagorean relationship c2 = a2 + b2 is recovered in the limit as the radius R approaches infinity (since the remainder term tends to zero) :

<formula>

For small right triangles (a , b < < R) , the cosines can be eliminated to avoid loss of significance , giving

<formula>

= = = = Hyperbolic geometry = = = =

In a hyperbolic space with uniform curvature ? 1 / R2 , for a right triangle with legs a , b , and hypotenuse c , the relation between the sides takes the form :

<formula>

where cosh is the hyperbolic cosine . This formula is a special form of the hyperbolic law of cosines that applies to all hyperbolic triangles :

<formula>

with ? the angle at the vertex opposite the side c .
By using the Maclaurin series for the hyperbolic cosine , cosh x ? 1 + x2 / 2 , it can be shown that as a hyperbolic triangle becomes very small (that is , as a , b , and c all approach zero) , the hyperbolic relation for a right triangle approaches the form of Pythagoras 's theorem .
For small right triangles (a , b < < R) , the hyperbolic cosines can be eliminated to avoid loss of significance , giving
<formula>

= = = = Very small triangles = = = =

For any uniform curvature K (positive , zero , or negative) , in very small right triangles (| K | a2 , | K | b2 < < 1) with hypotenuse c , it can be shown that
<formula>

= = = Differential geometry = = =

On an infinitesimal level , in three dimensional space , Pythagoras 's theorem describes the distance between two infinitesimally separated points as :
<formula>
with ds the element of distance and (dx , dy , dz) the components of the vector separating the two points . Such a space is called a Euclidean space . However , in Riemannian geometry , a generalization of this expression useful for general coordinates (not just Cartesian) and general spaces (not just Euclidean) takes the form :
<formula>
which is called the metric tensor . (Sometimes , by abuse of language , the same term is applied to the set of coefficients gij .) It may be a function of position , and often describes curved space . A simple example is Euclidean (flat) space expressed in curvilinear coordinates . For example , in polar coordinates :
<formula>

= = History = =

There is debate whether the Pythagorean theorem was discovered once , or many times in many places , and the date of first discovery is uncertain , as is the date of the first proof . According to Joran Friberg , a historian of mathematics , evidence indicates that the Pythagorean Theorem was well @-@ known to the mathematicians of the First Babylonian Dynasty (20th to 16th centuries BC) , which would have been over a thousand years before Pythagoras was born . (Yale 's Institute for the Preservation of Cultural Heritage 's 3 @-@ D scan of a cuneiform tablet depicting the proof is one of their mostly widely used images .) Other sources , such as a book by Leon Lederman and Dick Teresi , mention that Pythagoras discovered the theorem , although Teresi subsequently stated that the Babylonians developed the theorem " at least fifteen hundred years before Pythagoras was born . " The history of the theorem can be divided into four parts : knowledge of Pythagorean triples , knowledge of the relationship among the sides of a right triangle , knowledge of the relationships among adjacent angles , and proofs of the theorem within some deductive system .
Bartel Leendert van der Waerden (1903 ? 1996) conjectured that Pythagorean triples were discovered algebraically by the Babylonians . Written between 2000 and 1786 BC , the Middle Kingdom Egyptian Berlin Papyrus 6619 includes a problem whose solution is the Pythagorean triple 6 : 8 : 10 , but the problem does not mention a triangle . The Mesopotamian tablet Plimpton 322 , written between 1790 and 1750 BC during the reign of Hammurabi the Great , contains many entries closely related to Pythagorean triples .
In India , the Baudhayana Sulba Sutra , the dates of which are given variously as between the 8th and 5th century BC , contains a list of Pythagorean triples discovered algebraically , a statement of the Pythagorean theorem , and a geometrical proof of the Pythagorean theorem for an isosceles

right triangle . The Apastamba Sulba Sutra (c . 600 BC) contains a numerical proof of the general Pythagorean theorem , using an area computation . Van der Waerden believed that " it was certainly based on earlier traditions " . Carl Boyer states that the Pythagorean theorem in ?ulba @-@ s?tram may have been influenced by ancient Mesopotamian math , but there is no conclusive evidence in favor or opposition of this possibility .

With contents known much earlier , but in surviving texts dating from roughly the 1st century BC , the Chinese text Zhou Bi Suan Jing (????) , (The Arithmetical Classic of the Gnomon and the Circular Paths of Heaven) gives a reasoning for the Pythagorean theorem for the (3 , 4 , 5) triangle ? in China it is called the " Gougu Theorem " (????) . During the Han Dynasty (202 BC to 220 AD) , Pythagorean triples appear in The Nine Chapters on the Mathematical Art , together with a mention of right triangles . Some believe the theorem arose first in China , where it is alternatively known as the " Shang Gao Theorem " (????) , named after the Duke of Zhou 's astronomer and mathematician , whose reasoning composed most of what was in the Zhou Bi Suan Jing .

Pythagoras , whose dates are commonly given as 569 ? 475 BC , used algebraic methods to construct Pythagorean triples , according to Proclus 's commentary on Euclid . Proclus , however , wrote between 410 and 485 AD . According to Thomas L. Heath (1861 ? 1940) , no specific attribution of the theorem to Pythagoras exists in the surviving Greek literature from the five centuries after Pythagoras lived . However , when authors such as Plutarch and Cicero attributed the theorem to Pythagoras , they did so in a way which suggests that the attribution was widely known and undoubted . " Whether this formula is rightly attributed to Pythagoras personally , [...] one can safely assume that it belongs to the very oldest period of Pythagorean mathematics . "

Around 400 BC , according to Proclus , Plato gave a method for finding Pythagorean triples that combined algebra and geometry . Around 300 BC , in Euclid 's Elements , the oldest extant axiomatic proof of the theorem is presented .

= = In popular culture = =

The Pythagorean theorem has arisen in popular culture in a variety of ways .

John Aubrey in his Brief Lives records of Thomas Hobbes that " He was forty years old before he looked on geometry ; which happened accidentally . Being in a gentleman 's library Euclid 's Elements lay open , and ' twas the forty @-@ seventh proposition * in the first book . He read the proposition . ' By G , ' said he , ' this is impossible ! ' So he reads the demonstration of it , which referred him back to such a proof ; which referred him back to another , which he also read . Et sic deinceps , that at last he was demonstratively convinced of that truth . This made him in love with geometry . "

Hans Christian Andersen wrote in 1831 a poem about the Pythagorean theorem : Formens Evige Magie (Et poetisk Spilfægteri) .

A verse of the Major @-@ General 's Song in the Gilbert and Sullivan comic opera The Pirates of Penzance , " About binomial theorem I 'm teeming with a lot o ' news , With many cheerful facts about the square of the hypotenuse " , makes an oblique reference to the theorem .

The Scarecrow in the film The Wizard of Oz makes a more specific reference to the theorem . Upon receiving his diploma from the Wizard , he immediately exhibits his " knowledge " by reciting a mangled and incorrect version of the theorem : " The sum of the square roots of any two sides of an isosceles triangle is equal to the square root of the remaining side . Oh , joy ! Oh , rapture ! I 've got a brain ! "

In 2000 , Uganda released a coin with the shape of an isosceles right triangle . The coin 's tail has an image of Pythagoras and the equation ?2 + ?2 = ?2 , accompanied with the mention " PYTHAGORAS MILLENNIUM " .

Greece , Japan , San Marino , Sierra Leone , and Suriname have issued postage stamps depicting Pythagoras and the Pythagorean theorem .

In Neal Stephenson 's speculative fiction Anathem , the Pythagorean theorem is referred to as ' the Adrakhonic theorem ' . A geometric proof of the theorem is displayed on the side of an alien ship to demonstrate the aliens ' understanding of mathematics .