

$x = a$  and  $x = b$

, where  $0 < b - a < \infty$ . Then, the area of  $R$  is

$\int_a^b f(x) dx$

This result can be found as follows. First, the interval  $[a, b]$  is divided into  $n$  subintervals, where  $n$  is an arbitrary positive integer. Thus, the length of each subinterval, is equal to  $\frac{b-a}{n}$  (the total length of the interval), divided by  $n$ , the number of subintervals. For each subinterval  $i = 1, 2, \dots, n$ , let  $x_i$  be the midpoint of the subinterval, and construct a sector with the center at the pole, radius  $r(x_i)$ , central angle  $\Delta\theta$  and arc length  $r(x_i) \Delta\theta$ . The area of each constructed sector is therefore equal to

$\frac{1}{2} r(x_i)^2 \Delta\theta$

Hence, the total area of all of the sectors is

$\sum_{i=1}^n \frac{1}{2} r(x_i)^2 \Delta\theta$

As the number of subintervals  $n$  is increased, the approximation of the area continues to improve. In the limit as  $n \rightarrow \infty$ , the sum becomes the Riemann sum for the above integral.

A mechanical device that computes area integrals is the planimeter, which measures the area of plane figures by tracing them out: this replicates integration in polar coordinates by adding a joint so that the 2-link element linkage effects Green's theorem, converting the quadratic polar integral to a linear integral.

== == Generalization == ==