= 1 , 2 , 3 . A fibration does exist for S1 (m =
0) , but not for S16 (m = 4) and beyond . Although generalizations of the relations to S16 are often true , they sometimes fail ; for example ,
<formula>
Thus there can be no fibration
<formula>
the first non @-@ trivial case of the Hopf invariant one problem , because such a fibration would imply that the failed relation is true .

= = = Framed cobordism = = =

Homotopy groups of spheres are closely related to cobordism classes of manifolds . In 1938 Lev Pontryagin established an isomorphism between the homotopy group ?n + k (Sn) and the group ?kframed (Sn + k) of cobordism classes of differentiable k @-@ submanifolds of Sn + k which are `framed ' , i.e. have a trivialized normal bundle . Every map ? : Sn + k ? Sn is homotopic to a differentiable map with <formula> a framed k @-@ dimensional submanifold . For example , ?n (Sn) = Z is the cobordism group of framed 0 @-@ dimensional submanifolds of Sn , computed by the algebraic sum of their points , corresponding to the degree of maps <formula> . The projection of the Hopf fibration <formula> represents a generator of ?3 (S2) = ?1framed (S3) = Z which corresponds to the framed 1 @-@ dimensional submanifold of S3 defined by the standard embedding <formula> with a nonstandard trivialization of the normal 2 @-@ plane bundle . Until the advent of more sophisticated algebraic methods in the early 1950s (Serre) the Pontrjagin isomorphism was the main tool for computing the homotopy groups of spheres . In 1954 the Pontrjagin isomorphism was generalized by René Thom to an isomorphism expressing other groups of cobordism classes (e.g. of all manifolds) as homotopy groups of spaces and spectra . In more recent work the argument is usually reversed , with cobordism groups computed in terms of homotopy groups (Scorpan 2005) .

= = = Finiteness and torsion = = =

In 1951 , Jean @-@ Pierre Serre showed that homotopy groups of spheres are all finite except for those of the form ?n (Sn) or ?4n ? 1 (S2n) (for positive n) , when the group is the product of the infinite cyclic group with a finite abelian group (Serre 1951) . In particular the homotopy groups are determined by their p @-@ components for all primes p . The 2 @-@ components are hardest to calculate , and in several ways behave differently from the p @-@ components for odd primes .