

$= bx$. Because f takes arbitrarily large and arbitrarily small positive values , any number $y > 0$ lies between $f(x_0)$ and $f(x_1)$ for suitable x_0 and x_1 . Hence , the intermediate value theorem ensures that the equation $f(x) =$

y has a solution . Moreover , there is only one solution to this equation , because the function f is strictly increasing (for $b > 1$) , or strictly decreasing (for $0 < b < 1$) .

The unique solution x is the logarithm of y to base b , $\log_b(y)$. The function that assigns to y its logarithm is called logarithm function or logarithmic function (or just logarithm) .

The function $\log_b(x)$ is essentially characterized by the above product formula

<formula>

More precisely , the logarithm to any base $b > 1$ is the only increasing function f from the positive reals to the reals satisfying $f(b) = 1$ and

<formula>

== Inverse function ==

The formula for the logarithm of a power says in particular that for any number x ,

<formula>

In prose , taking the x @-@ th power of b and then the base @-@ b logarithm gives back x . Conversely , given a positive number y , the formula

<formula>