= a ? 1 ? b . If the ? operation is commutative , we get that x = y . If not , x may be different from y .

A consequence of this is that multiplying by a group element g is a bijection . Specifically , if g is an element of the group G , there is a bijection from G to itself called left translation by g sending h ? G to g ? h . Similarly , right translation by g is a bijection from G to itself sending h to h ? g . If G is abelian , left and right translation by a group element are the same .

## Basic concepts

To understand groups beyond the level of mere symbolic manipulations as above , more structural concepts have to be employed . There is a conceptual principle underlying all of the following notions : to take advantage of the structure offered by groups ( which sets , being " structureless " , do not have ) , constructions related to groups have to be compatible with the group operation . This compatibility manifests itself in the following notions in various ways . For example , groups can be related to each other via functions called group homomorphisms . By the mentioned principle , they are required to respect the group structures in a precise sense . The structure of groups can also be understood by breaking them into pieces called subgroups and quotient groups . The principle of " preserving structures " ? a recurring topic in mathematics throughout ? is an instance of working in a category , in this case the category of groups .

### Group homomorphisms

Group homomorphisms are functions that preserve group structure . A function a : G ? H between two groups ( G , ? ) and ( H , ? ) is called a homomorphism if the equation