= z1 / 2 must be a solution to the equation w2 = z . But if w is a solution , then so is ? w , because (? 1) 2 = 1 . A unique but somewhat arbitrary solution called the principal value can be chosen using a general rule which also applies for nonrational powers .

Complex powers and logarithms are more naturally handled as single valued functions on a Riemann surface . Single valued versions are defined by choosing a sheet . The value has a discontinuity along a branch cut . Choosing one out of many solutions as the principal value leaves us with functions that are not continuous , and the usual rules for manipulating powers can lead us astray .

Any nonrational power of a complex number has an infinite number of possible values because of the multi @-@ valued nature of the complex logarithm . The principal value is a single value chosen from these by a rule which , amongst its other properties , ensures powers of complex numbers with a positive real part and zero imaginary part give the same value as does the rule defined above for the corresponding real base .

Exponentiating a real number to a complex power is formally a different operation from that for the corresponding complex number . However , in the common case of a positive real number the principal value is the same .

The powers of negative real numbers are not always defined and are discontinuous even where defined . In fact , they are only defined when the exponent is a rational number with the denominator being an odd integer . When dealing with complex numbers the complex number operation is normally used instead .

= = = Complex exponents with complex bases = = =

For complex numbers w and z with w ? 0 , the notation wz is ambiguous in the same sense that log w is .

To obtain a value of wz , first choose a logarithm of w ; call it log w . Such a choice may be the principal value Log w (the default , if no other specification is given) , or perhaps a value given by some other branch of log w fixed in advance . Then , using the complex exponential function one defines

<formula>

because this agrees with the earlier definition in the case where w is a positive real number and the (real) principal value of log w is used .

If z is an integer , then the value of wz is independent of the choice of log w , and it agrees with the earlier definition of exponentiation with an integer exponent .

If z is a rational number m / n in lowest terms with z > 0 , then the countably infinitely many choices of log w yield only n different values for wz ; these values are the n complex solutions s to the equation sn = wm .

If z is an irrational number , then the countably infinitely many choices of log w lead to infinitely many distinct values for wz .

The computation of complex powers is facilitated by converting the base w to polar form , as described in detail below .

A similar construction is employed in quaternions .

= = = Complex roots of unity = = =