Ars Conjectandi

Ars Conjectandi (Latin for " The Art of Conjecturing ") is a book on combinatorics and mathematical probability written by Jacob Bernoulli and published in 1713 , eight years after his death , by his nephew , Niklaus Bernoulli . The seminal work consolidated , apart from many combinatorial topics , many central ideas in probability theory , such as the very first version of the law of large numbers : indeed , it is widely regarded as the founding work of that subject . It also addressed problems that today are classified in the twelvefold way and added to the subjects ; consequently , it has been dubbed an important historical landmark in not only probability but all combinatorics by a plethora of mathematical historians . The importance of this early work had a large impact on both contemporary and later mathematicians ; for example , Abraham de Moivre .

Bernoulli wrote the text between 1684 and 1689 , including the work of mathematicians such as Christiaan Huygens , Gerolamo Cardano , Pierre de Fermat , and Blaise Pascal . He incorporated fundamental combinatorial topics such as his theory of permutations and combinations (the aforementioned problems from the twelvefold way) as well as those more distantly connected to the burgeoning subject : the derivation and properties of the eponymous Bernoulli numbers , for instance . Core topics from probability , such as expected value , were also a significant portion of this important work .

Background

In Europe , the subject of probability was first formally developed in the 16th century with the work of Gerolamo Cardano , whose interest in the branch of mathematics was largely due to his habit of gambling . He formalized what is now called the classical definition of probability : if an event has a possible outcomes and we select any b of those such that b ? a , the probability of any of the b occurring is <formula> . However , his actual influence on mathematical scene was not great ; he wrote only one light tome on the subject in 1525 titled Liber de ludo aleae (Book on Games of Chance) , which was published posthumously in 1663 .

The date which historians cite as the beginning of the development of modern probability theory is 1654 , when two of the most well @-@ known mathematicians of the time , Blaise Pascal and Pierre de Fermat , began a correspondence discussing the subject . The two initiated the communication because earlier that year , a gambler from Paris named Antoine Gombaud had sent Pascal and other mathematicians several questions on the practical applications of some of these theories ; in particular he posed the problem of points , concerning a theoretical two @-@ player game in which a prize must be divided between the players due to external circumstances halting the game . The fruits of Pascal and Fermat 's correspondence interested other mathematicians , including Christiaan Huygens , whose De ratiociniis in aleae ludo (Calculations in Games of Chance) appeared in 1657 as the final chapter of Van Schooten 's Exercitationes Matematicae . In 1665 Pascal posthumously published his results on the eponymous Pascal 's triangle , an important combinatorial concept . He referred to the triangle in his work Traité du triangle arithmétique (Traits of the Arithmetic Triangle) as the " arithmetic triangle " .

In 1662 , the book La Logique ou l ? Art de Penser was published anonymously in Paris . The authors presumably were Antoine Arnauld and Pierre Nicole , two leading Jansenists , who worked together with Blaise Pascal . The Latin title of this book is Ars cogitandi , which was a successful book on logic of the time . The Ars cogitandi consists of four books , with the fourth one dealing with decision @-@ making under uncertainty by considering the analogy to gambling and introducing explicitly the concept of a quantified probability .

In the field of statistics and applied probability , John Graunt published Natural and Political Observations Made upon the Bills of Mortality also in 1662 , initiating the discipline of demography . This work , among other things , gave a statistical estimate of the population of London , produced the first life table , gave probabilities of survival of different age groups , examined the different causes of death , noting that the annual rate of suicide and accident is constant , and commented on the level and stability of sex ratio . The usefulness and interpretation of Graunt 's tables were discussed in a series of correspondences by brothers Ludwig and Christiaan Huygens in 1667 , where they realized the difference between mean and median estimates and Christian even interpolated Graunt 's life table by a smooth curve , creating the first continuous probability distribution ; but their correspondences were not published . Later , Johan de Witt , the then prime minister of the Dutch Republic , published similar material in his 1671 work Waerdye van Lyf @-@ Renten (A Treatise on Life Annuities) , which used statistical concepts to determine life expectancy for practical political purposes ; a demonstration of the fact that this sapling branch of mathematics had significant pragmatic applications . De Witt 's work was not widely distributed beyond the Dutch Republic , perhaps due to his fall from power and execution by mob in 1672 . Apart from the practical contributions of these two work , they also exposed a fundamental idea that probability can be assigned to events that do not have inherent physical symmetry , such as the chances of dying at certain age , unlike say the rolling of a dice or flipping of a coin , simply by counting the frequency of occurrence . Thus probability could be more than mere combinatorics .

= = Development of Ars Conjectandi = =

In the wake of all these pioneers , Bernoulli produced much of the results contained in Ars Conjectandi between 1684 and 1689 , which he recorded in his diary Meditationes . When he began the work in 1684 at the age of 30 , while intrigued by combinatorial and probabilistic problems , Bernoulli had not yet read Pascal 's work on the " arithmetic triangle " nor de Witt 's work on the applications of probability theory : he had earlier requested a copy of the latter from his acquaintance Gottfried Leibniz , but Leibniz failed to provide it . The latter , however , did manage to provide Pascal 's and Huygen 's work , and thus it is largely upon these foundations that Ars Conjectandi is constructed . Apart from these works , Bernoulli certainly possessed or at least knew the contents from secondary sources of the La Logique ou l ? Art de Penser as well as Graunt 's Bills of Mortality , as he makes explicit reference to these two works .

Bernoulli ? s progress over time can be pursued by means of the Meditationes . Three working periods with respect to his " discovery " can be distinguished by aims and times . The first period , which lasts from 1684 to 1685 , is devoted to the study of the problems regarding the games of chance posed by Christiaan Huygens ; during the second period (1685 @-@ 1686) the investigations are extended to cover processes where the probabilities are not known a priori , but have to be determined a posteriori . Finally , in the last period (1687 @-@ 1689) , the problem of measuring the probabilities is solved .

Before the publication of his Ars Conjectandi , Bernoulli had produced a number of treaties related to probability :

Parallelismus ratiocinii logici et algebraici , Basel , 1685 .

In the Journal des Sçavans 1685 (26.VIII) , p . 314 there appear two problems concerning the probability each of two players may have of winning in a game of dice . Solutions were published in the Acta Eruditorum 1690 (May) , pp. 219 ? 223 in the article Quaestiones nonnullae de usuris , cum solutione Problematis de Sorte Alearum . In addition , Leibniz himself published a solution in the same journal on pages 387 @-@ 390 .

Theses logicae de conversione et oppositione enunciationum , a public lecture delivered at Basel , 12 February 1686 . Theses XXXI to XL are related to the theory of probability .

De Arte Combinatoria Oratio Inauguralis , 1692 .

The Letter à un amy sur les parties du jeu de paume , that is , a letter to a friend on sets in the game of Tennis , published with the Ars Conjectandi in 1713 .

Between 1703 to 1705 , Leibniz corresponded with Jakob after learning about his discoveries in probability from his brother Johann . Leibniz managed to provide thoughtful criticisms on Bernoulli 's law of large number , but failed to provide Bernoulli with de Witt 's work on annuities that he so desired . From the outset , Bernoulli wished for his work to demonstrate that combinatorics and probability theory would have numerous real @-@ world applications in all facets of society ? in the line of Graunt 's and de Witt 's work ? and would serve as a rigorous method of logical reasoning under insufficient evidence , as used in courtrooms and in moral judgements . It was also hoped that the theory of probability could provide comprehensive and consistent method of reasoning , where ordinary reasoning might be overwhelmed by the complexity of the situation . Thus the title Ars Conjectandi was chosen : a link to the concept of ars inveniendi from scholasticism , which provided the symbolic link to pragmatism he desired and also as an extension of the prior Ars Cogitandi .

In Bernoulli 's own words , the " art of conjecture " is defined in Chapter II of Part IV of his Ars Conjectandi as :

The art of measuring , as precisely as possible , probabilities of things , with the goal that we would be able always to choose or follow in our judgments and actions that course , which will have been determined to be better , more satisfactory , safer or more advantageous .

The development of the book was terminated by Bernoulli 's death in 1705 ; thus the book is essentially incomplete when compared with Bernoulli 's original vision . The quarrel with his younger brother Johann , who was the most competent person who could have fulfilled Jacob 's project , prevented Johann to get hold of the manuscript . Jacob 's own children were not mathematicians and were not up to the task of editing and publishing the manuscript . Finally Jacob 's nephew Niklaus , 7 years after Jacob 's death in 1705 , managed to publish the manuscript in 1713 .

= = Contents = =

Bernoulli 's work , originally published in Latin is divided into four parts . It covers most notably his theory of permutations and combinations ; the standard foundations of combinatorics today and subsets of the foundational problems today known as the twelvefold way . It also discusses the motivation and applications of a sequence of numbers more closely related to number theory than probability ; these Bernoulli numbers bear his name today , and are one of his more notable achievements .

The first part is an in @-@ depth expository on Huygens ' De ratiociniis in aleae ludo . Bernoulli provides in this section solutions to the five problems Huygens posed at the end of his work . He particularly develops Huygens ' concept of expected value ? the weighted average of all possible outcomes of an event . Huygens had developed the following formula :

<formula>

In this formula , E is the expected value , pi are the probabilities of attaining each value , and ai are the attainable values . Bernoulli normalizes the expected value by assuming that pi are the probabilities of all the disjoint outcomes of the value , hence implying that p0 + p1 + ... + pn = 1 . Another key theory developed in this part is the probability of achieving at least a certain number of successes from a number of binary events , today named Bernoulli trials , given that the probability of success in each event was the same . Bernoulli shows through mathematical induction that given a the number of favorable outcomes in each event , b the number of total outcomes in each event , d the desired number of successful outcomes , and e the number of events , the probability of at least d successes is

<formula>

The first part concludes with what is now known as the Bernoulli distribution .

The second part expands on enumerative combinatorics , or the systematic numeration of objects . It was in this part that two of the most important of the twelvefold ways ? the permutations and combinations that would form the basis of the subject ? were fleshed out , though they had been introduced earlier for the purposes of probability theory . He gives the first non @-@ inductive proof of the binomial expansion for integer exponent using combinatorial arguments . On a note more distantly related to combinatorics , the second section also discusses the general formula for sums of integer powers ; the free coefficients of this formula are therefore called the Bernoulli numbers , which influenced Abraham de Moivre 's work later , and which have proven to have numerous applications in number theory .

In the third part , Bernoulli applies the probability techniques from the first section to the common chance games played with playing cards or dice . Interestingly , he does not feel the necessity to describe the rules and objectives of the card games he analyzes . He presents probability problems related to these games and , once a method had been established , posed generalizations . For example , a problem involving the expected number of " court cards " ? jack , queen , and king ? one would pick in a five @-@ card hand from a standard deck of 52 cards containing 12 court cards could be generalized to a deck with a cards that contained b court cards , and a c @-@ card hand .

The fourth section continues the trend of practical applications by discussing applications of probability to civilibus , moralibus , and oeconomicis , or to personal , judicial , and financial decisions . In this section , Bernoulli differs from the school of thought known as frequentism , which defined probability in an empirical sense . As a counter , he produces a result resembling the law of large numbers , which he describes as predicting that the results of observation would approach theoretical probability as more trials were held ? in contrast , frequents defined probability in terms of the former . Bernoulli was very proud of this result , referring to it as his " golden theorem " , and remarked that it was " a problem in which I ? ve engaged myself for twenty years " . This early version of the law is known today as either Bernoulli 's theorem or the weak law of large numbers , as it is less rigorous and general than the modern version .

After these four primary expository sections , almost as an afterthought , Bernoulli appended to Ars Conjectandi a tract on calculus , which concerned infinite series . It was a reprint of five dissertations he had published between 1686 and 1704 .

= = Legacy = =

Ars Conjectandi is considered a landmark work in combinatorics and the founding work of mathematical probability . Among others , an anthology of great mathematical writings published by Elsevier and edited by historian Ivor Grattan @-@ Guinness describes the studies set out in the work " [occupying] mathematicians throughout 18th and 19th centuries " ? an influence lasting three centuries . Statistician Anthony Edwards praised not only the book 's groundbreaking content , writing that it demonstrated Bernoulli 's " thorough familiarity with the many facets [of combinatorics] , " but its form : " [Ars Conjectandi] is a very well @-@ written book , excellently constructed . " Perhaps most recently , notable popular mathematical historian and topologist William Dunham called the paper " the next milestone of probability theory [after the work of Cardano] " as well as " Jakob Bernoulli 's masterpiece " . It greatly aided what Dunham describes as " Bernoulli 's long @-@ established reputation " .

Bernoulli 's work influenced many contemporary and subsequent mathematicians . Even the afterthought @-@ like tract on calculus has been quoted frequently ; most notably by the Scottish mathematician Colin Maclaurin . Jacob 's program of applying his art of conjecture to the matters of practical life , which was terminated by his death in 1705 , was continued by his nephew Nicolaus Bernoulli , after having taken parts verbatim out of Ars Conjectandi , for his own dissertation entitled De Usu Artis Conjectandi in Jure which was published already in 1709 . Nicolas finally edited and assisted in the publication of Ars conjectandi in 1713 . Later Nicolaus also edited Jacob Bernoulli 's complete works and supplemented it with results taken from Jacob 's diary .

Pierre Rémond de Montmort , in collaboration with Nicolaus Bernoulli , wrote a book on probability Essay d 'analyse sur les jeux de hazard which appeared in 1708 , which can be seen as an extension of the Part III of Ars Conjectandi which applies combinatorics and probability to analyze games of chance commonly played at that time . Abraham de Moivre also wrote extensively on the subject in De mensura sortis : Seu de Probabilitate Eventuum in Ludis a Casu Fortuito Pendentibus of 1711 and its extension The Doctrine of Chances or , a Method of Calculating the Probability of Events in Play of 1718 . De Moivre 's most notable achievement in probability was the discovery of the first instance of central limit theorem , by which he was able to approximate the binomial distribution with the normal distribution . To achieve this De Moivre developed an asymptotic sequence for the factorial function ? - which we now refer to as Stirling 's approximation ? - and Bernoulli 's formula for the sum of powers of numbers . Both Montmort and de Moivre adopted the term probability from Jacob Bernoulli , which had not been used in all the previous publications on gambling , and both their works were enormously popular .

The refinement of Bernoulli 's Golden Theorem , regarding the convergence of theoretical probability and empirical probability , was taken up by many notable later day mathematicians like

De Moivre , Laplace , Poisson , Chebyshev , Markov , Borel , Cantelli , Kolmogorov and Khinchin . The complete proof of the Law of Large Numbers for the arbitrary random variables was finally provided during first half of 20th century .

A significant indirect influence was Thomas Simpson , who achieved a result that closely resembled de Moivre 's . According to Simpsons ' work 's preface , his own work depended greatly on de Moivre 's ; the latter in fact described Simpson 's work as an abridged version of his own . Finally , Thomas Bayes wrote an essay discussing theological implications of de Moivre 's results : his solution to a problem , namely that of determining the probability of an event by its relative frequency , was taken as a proof for the existence of God by Bayes . Finally in 1812 , Pierre @-@ Simon Laplace published his Théorie analytique des probabilités in which he consolidated and laid down many fundamental results in probability and statistics such as the moment generating function , method of least squares , inductive probability , and hypothesis testing , thus completing the final phase in the development of classical probability . Indeed , in light of all this , there is good reason Bernoulli 's work is hailed as such a seminal event ; not only did his various influences , direct and indirect , set the mathematical study of combinatorics spinning , but even theology was impacted .