

$\ln(b)$ is the unique real solution to the equation $e^x = b$. So the same method working for real exponents also works for complex exponents.

For example :

$e^{i\pi} = -1$

$e^{i2\pi} = 1$

$e^{i3\pi} = -1$

The identity $(e^{bz})^u = e^{bzu}$ is not generally valid for complex powers. The power bz is a complex number and any power of it has to follow the rules for powers of complex numbers below. A simple counterexample is given by :

$e^{i\pi} = -1$

The identity is, however, valid for arbitrary complex z when u is an integer.

Powers of complex numbers

Integer powers of nonzero complex numbers are defined by repeated multiplication or division as above. If i is the imaginary unit and n is an integer, then i^n equals 1 , i , -1 , or $-i$, according to whether the integer n is congruent to 0 , 1 , 2 , or 3 modulo 4 . Because of this, the powers of i are useful for expressing sequences of period 4 .

Complex powers of positive reals are defined via e^x as in section Complex exponents with positive real bases above. These are continuous functions.

Trying to extend these functions to the general case of noninteger powers of complex numbers that are not positive reals leads to difficulties. Either we define discontinuous functions or multivalued functions. Neither of these options is entirely satisfactory.