Polar coordinate system

In mathematics , the polar coordinate system is a two @-@ dimensional coordinate system in which each point on a plane is determined by a distance from a reference point and an angle from a reference direction .

The reference point (analogous to the origin of a Cartesian system) is called the pole , and the ray from the pole in the reference direction is the polar axis . The distance from the pole is called the radial coordinate or radius , and the angle is called the angular coordinate , polar angle , or azimuth .

History

The concepts of angle and radius were already used by ancient peoples of the first millennium BC . The Greek astronomer and astrologer Hipparchus (190 ? 120 BC) created a table of chord functions giving the length of the chord for each angle , and there are references to his using polar coordinates in establishing stellar positions . In On Spirals , Archimedes describes the Archimedean spiral , a function whose radius depends on the angle . The Greek work , however , did not extend to a full coordinate system .

From the 8th century AD onward , astronomers developed methods for approximating and calculating the direction to Mecca (qibla) ? and its distance ? from any location on the Earth . From the 9th century onward they were using spherical trigonometry and map projection methods to determine these quantities accurately . The calculation is essentially the conversion of the equatorial polar coordinates of Mecca (i.e. its longitude and latitude) to its polar coordinates (i.e. its qibla and distance) relative to a system whose reference meridian is the great circle through the given location and the Earth 's poles , and whose polar axis is the line through the location and its antipodal point .

There are various accounts of the introduction of polar coordinates as part of a formal coordinate system . The full history of the subject is described in Harvard professor Julian Lowell Coolidge 's Origin of Polar Coordinates . Grégoire de Saint @-@ Vincent and Bonaventura Cavalieri independently introduced the concepts in the mid @-@ seventeenth century . Saint @-@ Vincent wrote about them privately in 1625 and published his work in 1647 , while Cavalieri published his in 1635 with a corrected version appearing in 1653 . Cavalieri first used polar coordinates to solve a problem relating to the area within an Archimedean spiral . Blaise Pascal subsequently used polar coordinates to calculate the length of parabolic arcs .

In Method of Fluxions (written 1671 , published 1736) , Sir Isaac Newton examined the transformations between polar coordinates , which he referred to as the " Seventh Manner ; For Spirals " , and nine other coordinate systems . In the journal Acta Eruditorum (1691) , Jacob Bernoulli used a system with a point on a line , called the pole and polar axis respectively . Coordinates were specified by the distance from the pole and the angle from the polar axis . Bernoulli 's work extended to finding the radius of curvature of curves expressed in these coordinates .

The actual term polar coordinates has been attributed to Gregorio Fontana and was used by 18th @-@ century Italian writers . The term appeared in English in George Peacock 's 1816 translation of Lacroix 's Differential and Integral Calculus . Alexis Clairaut was the first to think of polar coordinates in three dimensions , and Leonhard Euler was the first to actually develop them .

Conventions

The radial coordinate is often denoted by r or ? , and the angular coordinate by ? , ? , or t . The angular coordinate is specified as ? by ISO standard 31 @-@ 11 .

Angles in polar notation are generally expressed in either degrees or radians (2? rad being equal to 360 °) . Degrees are traditionally used in navigation , surveying , and many applied disciplines , while radians are more common in mathematics and mathematical physics .

In many contexts , a positive angular coordinate means that the angle ? is measured counterclockwise from the axis .
In mathematical literature , the polar axis is often drawn horizontal and pointing to the right .

= = = Uniqueness of polar coordinates = = =

Adding any number of full turns (360 °) to the angular coordinate does not change the corresponding direction . Also , a negative radial coordinate is best interpreted as the corresponding positive distance measured in the opposite direction . Therefore , the same point can be expressed with an infinite number of different polar coordinates (r , ? ± n × 360 °) or (? r , ? ± (2n + 1) 180 °) , where n is any integer . Moreover , the pole itself can be expressed as (0 , ?) for any angle ? .
Where a unique representation is needed for any point , it is usual to limit r to non @-@ negative numbers (r ? 0) and ? to the interval [0 , 360 °) or (? 180 ° , 180 °] (in radians , [0 , 2?) or (? ? , ?]) . One must also choose a unique azimuth for the pole , e.g. , ? = 0 .

= = Converting between polar and Cartesian coordinates = =

The polar coordinates r and ? can be converted to the Cartesian coordinates x and y by using the trigonometric functions sine and cosine :
<formula>
<formula>
The Cartesian coordinates x and y can be converted to polar coordinates r and ? with r ? 0 and ? in the interval (? ? , ?] by :
<formula> (as in the Pythagorean theorem or the Euclidean norm) , and
<formula> ,
where atan2 is a common variation on the arctangent function defined as
<formula>
The value of ? above is the principal value of the complex number function arg applied to x + iy . An angle in the range [0 , 2?) may be obtained by adding 2? to the value in case it is negative .

= = Polar equation of a curve = =

The equation defining an algebraic curve expressed in polar coordinates is known as a polar equation . In many cases , such an equation can simply be specified by defining r as a function of ? . The resulting curve then consists of points of the form (r (?) , ?) and can be regarded as the graph of the polar function r .