Laplace ? Runge ? Lenz vector

In classical mechanics , the Laplace ? Runge ? Lenz vector (or simply the LRL vector) is a vector used chiefly to describe the shape and orientation of the orbit of one astronomical body around another , such as a planet revolving around a star . For two bodies interacting by Newtonian gravity , the LRL vector is a constant of motion , meaning that it is the same no matter where it is calculated on the orbit ; equivalently , the LRL vector is said to be conserved . More generally , the LRL vector is conserved in all problems in which two bodies interact by a central force that varies as the inverse square of the distance between them ; such problems are called Kepler problems .

The hydrogen atom is a Kepler problem , since it comprises two charged particles interacting by Coulomb 's law of electrostatics , another inverse square central force . The LRL vector was essential in the first quantum mechanical derivation of the spectrum of the hydrogen atom , before the development of the Schrödinger equation . However , this approach is rarely used today .

In classical and quantum mechanics , conserved quantities generally correspond to a symmetry of the system . The conservation of the LRL vector corresponds to an unusual symmetry ; the Kepler problem is mathematically equivalent to a particle moving freely on the surface of a four @-@ dimensional (hyper-) sphere , so that the whole problem is symmetric under certain rotations of the four @-@ dimensional space . This higher symmetry results from two properties of the Kepler problem : the velocity vector always moves in a perfect circle and , for a given total energy , all such velocity circles intersect each other in the same two points .

The Laplace ? Runge ? Lenz vector is named after Pierre @-@ Simon de Laplace , Carl Runge and Wilhelm Lenz . It is also known as the Laplace vector , the Runge ? Lenz vector and the Lenz vector . Ironically , none of those scientists discovered it . The LRL vector has been re @-@ discovered several times and is also equivalent to the dimensionless eccentricity vector of celestial mechanics . Various generalizations of the LRL vector have been defined , which incorporate the effects of special relativity , electromagnetic fields and even different types of central forces .

Context

A single particle moving under any conservative central force has at least four constants of motion , the total energy E and the three Cartesian components of the angular momentum vector L with respect to the origin . The particle 's orbit is confined to a plane defined by the particle 's initial momentum p (or , equivalently , its velocity v) and the vector r between the particle and the center of force (see Figure 1 , below) .

As defined below (see Mathematical definition) , the Laplace ? Runge ? Lenz vector (LRL vector) A always lies in the plane of motion for any central force . However , A is constant only for an inverse @-@ square central force . For most central forces , however , this vector A is not constant , but changes in both length and direction ; if the central force is approximately an inverse @-@ square law , the vector A is approximately constant in length , but slowly rotates its direction . A generalized conserved LRL vector <formula> can be defined for all central forces , but this generalized vector is a complicated function of position , and usually not expressible in closed form .