= Science and technology of the Han dynasty =

The Han dynasty (206 BC ? 220 AD) of ancient China , divided between the eras of Western Han (206 BC ? 9 AD , when the capital was at Chang 'an) , Xin dynasty of Wang Mang (r . 9 ? 23 AD) , and Eastern Han (25 ? 220 AD , when the capital was at Luoyang , and after 196 AD at Xuchang) , witnessed some of the most significant advancements in premodern Chinese science and technology .

There were great innovations in metallurgy . In addition to Zhou @-@ dynasty China 's (c . 1050 BCE ? 256 BC) previous inventions of the blast furnace and cupola furnace to make pig iron and cast iron , respectively , the Han period saw the development of steel and wrought iron by use of the finery forge and puddling process . With the drilling of deep boreholes into the earth , the Chinese used not only derricks to lift brine up to the surface to be boiled into salt , but also set up bamboo @-@ crafted pipeline transport systems which brought natural gas as fuel to the furnaces . Smelting techniques were enhanced with inventions such as the waterwheel @-@ powered bellows ; the resulting widespread distribution of iron tools facilitated the growth of agriculture . For tilling the soil and planting straight rows of crops , the improved heavy @-@ moldboard plough with three iron plowshares and sturdy multiple @-@ tube iron seed drill were invented in the Han , which greatly enhanced production yields and thus sustained population growth . The method of supplying irrigation ditches with water was improved with the invention of the mechanical chain pump powered by the rotation of a waterwheel or draft animals , which could transport irrigation water up elevated terrains . The waterwheel was also used for operating trip hammers in pounding grain and in rotating the metal rings of the mechanical @-@ driven astronomical armillary sphere representing the celestial sphere around the Earth .

The quality of life was improved with many Han inventions . The Han Chinese had hempen @-@ bound bamboo scrolls to write on , yet by the 2nd century AD had invented the papermaking process which created a writing medium that was both cheap and easy to produce . The invention of the wheelbarrow aided in the hauling of heavy loads . The maritime junk ship and stern @-@ mounted steering rudder enabled the Chinese to venture out of calmer waters of interior lakes and rivers and into the open sea . The invention of the grid reference for maps and raised @-@ relief map allowed the Chinese to better navigate their terrain . In medicine , they used new herbal remedies to cure illnesses , calisthenics to keep physically fit , and regulated diets to avoid diseases . Authorities in the capital were warned ahead of time of the direction of sudden earthquakes with the invention of the seismometer that was tripped by a vibration @-@ sensitive pendulum device . To mark the passing of the seasons and special occasions , the Han Chinese used two variations of the lunisolar calendar , which were established due to efforts in astronomy and mathematics . Han @-@ era Chinese advancements in mathematics include the discovery of square roots , cube roots , the Pythagorean theorem , Gaussian elimination , the Horner scheme , improved calculations of pi , and negative numbers . Hundreds of new roads and canals were built to facilitate transport , commerce , tax collection , communication , and movement of military troops . The Han @-@ era Chinese also employed several types of bridges to cross waterways and deep gorges , such as beam bridges , arch bridges , simple suspension bridges , and pontoon bridges . Han ruins of defensive city walls made of brick or rammed earth still stand today .

= = Modern perspectives on science and technology during Han = =

Jin Guantao , a professor of the Institute of Chinese Studies at the Chinese University of Hong Kong , Fan Hongye , a research fellow with the Chinese Academy of Sciences ' Institute of Science Policy and Managerial Science , and Liu Qingfeng , a professor of the Institute of Chinese Culture at the Chinese University of Hong Kong , assert that the latter part of the Han dynasty was a unique period in the history of premodern Chinese science and technology . They compare it to the incredible pace of scientific and technological growth during the Song dynasty (960 ? 1279 AD) . However , they also argue that without the influence of proto @-@ scientific precepts in the ancient philosophy of Mohism , Chinese science continued to lack a definitive structure :

From the middle and late Eastern Han to the early Wei and Jin dynasties , the net growth of ancient Chinese science and technology experienced a peak (second only to that of the Northern Song dynasty) ... Han studies of the Confucian classics , which for a long time had hindered the socialization of science , were declining . If Mohism , rich in scientific thought , had rapidly grown and strengthened , the situation might have been very favorable to the development of a scientific structure . However , this did not happen because the seeds of the primitive structure of science were never formed . During the late Eastern Han , disastrous upheavals again occurred in the process of social transformation , leading to the greatest social disorder in Chinese history . One can imagine the effect of this calamity on science .

Joseph Needham (1900 ? 1995) , a late Professor from the University of Cambridge and author of the groundbreaking Science and Civilisation in China series , stated that the " Han time (especially the Later Han) was one of the relatively important periods as regards the history of science in China . " He noted the advancements during Han of astronomy and calendrical sciences , the " beginnings of systematic botany and zoology " , as well as the philosophical skepticism and rationalist thought embodied in Han works such as the Lunheng by the philosopher Wang Chong (27 ? 100 CE) .

= = Writing materials = =

The most common writing mediums found in archaeological digs from ancient sites predating the Han period are shells and bones as well as bronzewares . In the beginning of the Han period , the chief writing mediums were bamboo (Chinese : ??) and clay tablets , silk cloth , and rolled scrolls made of strips of bamboo sewn together with hempen string passed through drilled holes (?) and secured with clay stamps . The written characters on these narrow flat strips of bamboo were arranged into vertical columns .

While maps drawn in ink on flat silk cloths have been found in the tomb of the Marquess of Dai (interred in 168 BCE at Mawangdui , Hunan province) , the earliest known paper map found in China , dated 179 ? 41 BC and located at Fangmatan (near Tianshui , Gansu province) , is incidentally the oldest known piece of paper . Yet Chinese hempen paper of the Western Han and early Eastern Han eras was of a coarse quality and used primarily as wrapping paper . The papermaking process was not formally introduced until the Eastern Han court eunuch Cai Lun (50 ? 121 AD) created a process in 105 AD where mulberry tree bark , hemp , old linens , and fish nets were boiled together to make a pulp that was pounded , stirred in water , and then dunked with a wooden sieve containing a reed mat that was shaken , dried , and bleached into sheets of paper . The oldest known piece of paper with writing on it comes from the ruins of a Chinese watchtower at Tsakhortei , Alxa League , Inner Mongolia , dated precisely 110 AD when the Han garrison abandoned the area following a nomadic Xiongnu attack . By the 3rd century , paper became one of China 's chief writing mediums .

= = Ceramics = =

The Han ceramics industry was upheld by private businesses as well as local government agencies . Ceramics were used in domestic wares and utensils as well as construction materials for roof tiles and bricks .

Han dynasty grey pottery ? its color derived from the clay that was used ? was superior to earlier Chinese grey pottery due to the Han people 's use of larger kiln chambers , longer firing tunnels , and improved chimney designs . Kilns of the Han dynasty making grey pottery were able to reach firing temperatures above 1 @,@ 000 ° C (1 @,@ 830 ° F) . However , hard southern Chinese pottery made from a dense adhesive clay native only in the south (i.e. Guangdong , Guangxi , Hunan , Jiangxi , Fujian , Zhejiang , and southern Jiangsu) was fired at even higher temperatures than grey pottery during the Han . Glazed pottery of the Shang (c . 1600 ? c . 1050 BC) and Zhou (c . 1050 ? 256 BCE) dynasties were fired at high temperatures , but by the mid Western Han (206 BC ? 9 AD) , a brown @-@ glazed ceramic was made which was fired at the low temperature of 800 ° C (1 @,@ 470 ° F) , followed by a green @-@ glazed ceramic which became popular in the

Eastern Han (25 ? 220 CE) .

Wang Zhongshu states that the light @-@ green stoneware known as celadon was thought to exist only since the Three Kingdoms (220 ? 265 AD) period onwards , but argues that ceramic shards found at Eastern Han (25 ? 220 AD) sites of Zhejiang province can be classified as celadon . However , Richard Dewar argues that true celadon was not created in China until the early Song dynasty (960 ? 1279) when Chinese kilns were able to reach a minimum furnace temperature of 1 @,@ 260 ° C (2 @,@ 300 ° F) , with a preferred range of 1 @,@ 285 to 1 @,@ 305 ° C (2 @,@ 345 to 2 @,@ 381 ° F) for celadon .

= = Metallurgy = =

= = = Furnaces and smelting techniques = = =

A blast furnace converts raw iron ore into pig iron , which can be remelted in a cupola furnace to produce cast iron . The earliest specimens of cast iron found in China date to the 5th century BC during the late Spring and Autumn period , yet the oldest discovered blast furnaces date to the 3rd century BC and the majority date to the period after Emperor Wu of Han (r . 141 ? 87 BC) established a government monopoly over the iron industry in 117 BCE (most of the discovered iron works sites built before this date were merely foundries which recast iron that had been smelt elsewhere) . Iron ore smelted in blast furnaces during the Han was rarely if ever cast directly into permanent molds ; instead , the pig iron scraps were remelted in the cupola furnace to make cast iron . Cupola furnaces utilized a cold blast traveling through tuyere pipes from the bottom and over the top where the charge of charcoal and pig iron was introduced . The air traveling through the tuyere pipes thus became a hot blast once it reached the bottom of the furnace .

Although Chinese civilization lacked the bloomery , the Han Chinese were able to make wrought iron when they injected too much oxygen into the cupola furnace , causing decarburization . The Han @-@ era Chinese were also able to convert cast iron and pig iron into wrought iron and steel by using the finery forge and puddling process , the earliest specimens of such dating to the 2nd century BCE and found at Tieshengguo near Mount Song of Henan province . The semisubterranean walls of these furnaces were lined with refractory bricks and had bottoms made of refractory clay . Besides charcoal made of wood , Wang Zhongshu states that another furnace fuel used during the Han were " coal cakes " , a mixture of coal powder , clay , and quartz .

= = = Use of steel , iron , and bronze = = =

Donald B. Wagner writes that most domestic iron tools and implements produced during the Han were made of cheaper and more brittle cast iron , whereas the military preferred to use wrought iron and steel weaponry due to their more durable qualities . During the Han dynasty , the typical 0 @.@ 5 m (1 @.@ 6 ft) bronze sword found in the Warring States period was gradually replaced with an iron sword measuring roughly 1 m (3 @.@ 3 ft) in length . The ancient dagger @-@ axe (ge) made of bronze was still used by Han soldiers , although it was gradually phased out by iron spears and iron ji halberds . Even arrowheads , which were traditionally made of bronze , gradually only had a bronze tip and iron shaft , until the end of the Han when the entire arrowhead was made solely of iron . Farmers , carpenters , bamboo craftsmen , stonemasons , and rammed earth builders had at their disposal iron tools such as the plowshare , pickaxe , spade , shovel , hoe , sickle , axe , adze , hammer , chisel , knife , saw , scratch awl , and nails . Common iron commodities found in Han dynasty homes included tripods , stoves , cooking pots , belt buckles , tweezers , fire tongs , scissors , kitchen knives , fish hooks , and needles . Mirrors and oil lamps were often made of either bronze or iron . Coin money minted during the Han was made of either copper or copper and tin smelted together to make the bronze alloy .

= = Agriculture = =

= = = Tools and methods = = =

Modern archaeologists have unearthed Han iron farming tools throughout China , from Inner Mongolia in the north to Yunnan in the south . The spade , shovel , pick , and plow were used for tillage , the hoe for weeding , the rake for loosening the soil , and the sickle for harvesting crops . Depending on their size , Han plows were driven by either one ox or two oxen . Oxen were also used to pull the three @-@ legged iron seed drill (invented in Han China by the 2nd century BCE) , which enabled farmers to plant seeds in precise rows instead of casting them out by hand . While artwork of the Wei (220 ? 265 CE) and Jin (265 ? 420 CE) periods show use of the harrow for breaking up chunks of soil after plowing , it perhaps first appeared in China during the Eastern Han (25 ? 220 CE) . Irrigation works for agriculture included the use of water wells , artificial ponds and embankments , dams , canals , and sluice gates .

= = = Alternating fields = = =

During Emperor Wu 's (r . 141 ? 87 BCE) reign , the Grain Intendant Zhao Guo (??) invented the alternating fields system (daitianfa ???) . For every mou of land ? i.e. a thin but elongated strip of land measuring 1 @.@ 38 m (4 @.@ 5 ft) wide and 331 m (1 @,@ 086 ft) long , or an area of roughly 457 m2 (0 @.@ 113 acres) ? three low @-@ lying furrows (quan ?) that were each 0 @.@ 23 m (0 @.@ 75 ft) wide were sowed in straight lines with crop seed . While weeding in the summer , the loose soil of the ridges (long ?) on either side of the furrows would gradually fall into the furrows , covering the sprouting crops and protecting them from wind and drought . Since the position of the furrows and ridges were reversed by the next year , this process was called the alternating fields system .

This system allowed crops to grow in straight lines from sowing to harvest , conserved moisture in the soil , and provided a stable annual yield for harvested crops . Zhao Guo first experimented with this system right outside the capital Chang 'an , and once it proved successful , he sent out instructions for it to every commandery administrator , who were then responsible for disseminating these to the heads of every county , district , and hamlet in their commanderies . Sadao Nishijima speculates that the Imperial Counselor Sang Hongyang (d . 80 BCE) perhaps had a role in promoting this new system .

Rich families who owned oxen and large heavy moldboard iron plows greatly benefited from this new system . However , poorer farmers who did not own oxen resorted to using teams of men to move a single plow , which was exhausting work . The author Cui Shi (??) (d . 170 CE) wrote in his Simin yueling (????) that by the Eastern Han Era (25 ? 220 AD) an improved plow was invented which needed only one man to control it , two oxen to pull it , had three plowshares , a seed box for the drills , a tool which turned down the soil , and could sow roughly 45 @,@ 730 m2 (11 @.@ 30 acres) of land in a single day .

= = = Pit fields = = =

During the reign of Emperor Cheng of Han (r . 33 ? 7 BCE) , Fan Shengzhi wrote a manual (i.e. the Fan Shengzhi shu ????) which described the pit field system (aotian ??) . In this system , every mou of farmland was divided into 3 @,@ 840 grids which each had a small pit that was dug 13 @.@ 8 cm (5 @.@ 4 in) deep and 13 @.@ 8 cm (5 @.@ 4 in) wide and had good quality manure mixed into the soil . Twenty seeds were sowed into each pit , which allegedly produced 0 @.@ 6 L (20 oz) of harvested grain per pit , or roughly 2 @,@ 000 L (67 @,@ 630 oz) per mou . This system did not require oxen @-@ driven plows or the most fertile land , since it could be employed even on sloping terrains where supplying water was difficult for other methods of farming . Although this farming method was favored by the poor , it did require intensive labor , thus only large families could maintain such a system .

= = = Rice paddies = = =

Han farmers in the Yangzi River region of southern China often maintained paddy fields for growing rice . Every year , they would burn the weeds in the paddy field , drench it in water , sow rice by hand , and around harvest time cut the surviving weeds and drown them a second time . In this system , the field lays fallow for much of the year and thus did not remain very fertile . However , Han rice farmers to the north around the Huai River practiced the more advanced system of transplantation . In this system , individual plants were given intensive care (perhaps in the same location as the paddy field) , their offshoots separated so that more water could be conserved , and the field could be heavily fertilized since winter crops were grown while the rice seedlings were situated nearby in a plant nursery .

= = Mechanical and hydraulic engineering = =

= = = Literary sources and archaeological evidence = = =

Evidence of Han @-@ era mechanical engineering comes largely from the choice observational writings of sometimes disinterested Confucian scholars . Professional artisan @-@ engineers (jiang ?) did not leave behind detailed records of their work . Han scholars , who often had little or no expertise in mechanical engineering , sometimes provided insufficient information on the various technologies they described .

Nevertheless , some Han literary sources provide crucial information . As written by Yang Xiong in 15 BCE , the belt drive was first used for a quilling device which wound silk fibers onto the bobbins of weaver shuttles . The invention of the belt drive was a crucial first step in the development of later technologies during the Song dynasty , such as the chain drive and spinning wheel .

The inventions of the artisan @-@ engineer Ding Huan (??) are mentioned in the Miscellaneous Notes on the Western Capital . The official and poet Sima Xiangru (179 ? 117 BC) once hinted in his writings that the Chinese used a censer in the form of a gimbal , a pivot support made of concentric rings which allow the central gimbal to rotate on an axis while remaining vertically positioned . However , the first explicit mention of the gimbal used as an incense burner occurred around 180 CE when the artisan Ding Huan created his ' Perfume Burner for use among Cushions ' which allowed burning incense placed within the central gimbal to remain constantly level even when moved . Ding had other inventions as well . For the purpose of indoor air conditioning , he set up a large manually operated rotary fan which had rotating wheels that were 3 m (9 @.@ 8 ft) in diameter . He also invented a lamp which he called the ' nine @-@ storied hill @-@ censer ' , since it was shaped as a hillside . When the cylindrical lamp was lit , the convection of rising hot air currents caused vanes placed on the top to spin , which in turn rotated painted paper figures of birds and other animals around the lamp .

When Emperor Gaozu of Han (r . 202 ? 195 BC) came upon the treasury of Qin Shi Huang (r . 221 ? 210 BC) at Xianyang following the downfall of the Qin dynasty (221 ? 206 BC) , he found an entire miniature musical orchestra of puppets 1 m (3 @.@ 3 ft) tall who played mouth organs if one pulled on ropes and blew into tubes to control them . Zhang Heng wrote in the 2nd century AD that people could be entertained by theatrical plays of artificial fish and dragons . Later , the inventor Ma Jun (fl . 220 ? 265 AD) invented a theater of moving mechanical puppets powered by the rotation of a hidden waterwheel .

From literary sources it is known that the collapsible umbrella was invented during Wang Mang 's reign , although the simple parasol existed beforehand . This employed sliding levers and bendable joints that could be protracted and retracted .

Modern archaeology has led to the discovery of Han artwork portraying inventions which were otherwise absent in Han literary sources . This includes the crank handle . Han pottery tomb models of farmyards and gristmills possess the first known depictions of crank handles , which were used to

operate the fans of winnowing machines . The machine was used to separate chaff from grain , but the Chinese of later dynasties also employed the crank handle for silk @-@ reeling , hemp @-@ spinning , flour @-@ sifting , and drawing water from a well using the windlass . To measure distance traveled , the Han @-@ era Chinese also created the odometer cart . This invention is depicted in Han artwork by the 2nd century CE , yet detailed written descriptions were not offered until the 3rd century CE . The wheels of this device rotated a set of gears which in turn forced mechanical figures to bang gongs and drums that alerted the travelers of the distance traveled (measured in li) . From existing specimens found at archaeological sites , it is known that Han @-@ era craftsmen made use of the sliding metal caliper to make minute measurements . Although Han @-@ era calipers bear incised inscriptions of the exact day of the year they were manufactured , they are not mentioned in any Han literary sources .

= = = Uses of the waterwheel and water clock = = =

By the Han dynasty , the Chinese developed various uses for the waterwheel . An improvement of the simple lever @-@ and @-@ fulcrum tilt hammer device operated by one 's foot , the hydraulic @-@ powered trip hammer used for pounding , decorticating , and polishing grain was first mentioned in the Han dictionary Jijiupian of 40 BC . It was also mentioned in the Regional Speech (Fangyan) dictionary written by Yang Xiong (53 BC ? 18 AD) in 15 BC , the philosophical Xinlun ?? written by Huan Tan (43 BC ? 28 AD) in 20 AD , the poetry of Ma Rong (79 ? 166 CE) , and the writings of Kong Rong (153 ? 208 CE) .

In his Balanced Discourse (Lunheng) , the philosopher Wang Chong (27 ? 100 AD) was the first in China to describe the square @-@ pallet chain pump used to lift water (and other substances) . Although some models were operated manually by foot pedals , some chain pumps were powered by a horizontal waterwheel which rotated large toothed gears and a horizontal axis beam . Their primary use was for lifting water into irrigation ditches , but chain pumps were also used in public works programs , such as when Zhang Rang (d . 189 CE) had an engineer build several of them to lift water into pipes that provided the capital Luoyang and its palaces with clean water .

While acting as administrator of Nanyang in 31 CE , Du Shi (d . 38 CE) invented a water @-@ powered reciprocator which worked the bellows of the blast furnace and cupola furnace in smelting iron ; before this invention , intensive manual labor was required to work the bellows .

Although the astronomical armillary sphere (representing the celestial sphere) existed in China since the 1st century BCE , the mathematician and court astronomer Zhang Heng (78 ? 139 CE) provided it with motive power by using the constant pressure head of an inflow water clock to rotate a waterwheel that acted on a set of gears . Zhang Heng was also the first to address the problem of the falling pressure head in the inflow water clock (which gradually slowed the timekeeping) by setting up an additional tank between the reservoir and inflow vessel .

= = = Seismometer = = =

The Han court was responsible for the major efforts of disaster relief when natural disasters such as earthquakes devastated the lives of commoners . To better prepare for calamities , Zhang Heng invented a seismometer in 132 CE which provided instant alert to authorities in the capital Luoyang that an earthquake had occurred in a location indicated by a specific cardinal or ordinal direction . Although no tremors could be felt in the capital when Zhang told the court that an earthquake had just occurred in the northwest , a message came soon afterwards that an earthquake had indeed struck 400 to 500 km (250 to 310 mi) northwest of Luoyang (in what is now modern Gansu) . Zhang called his device the ' instrument for measuring the seasonal winds and the movements of the Earth ' (Houfeng didong yi ?????) , so @-@ named because he and others thought that earthquakes were most likely caused by the enormous compression of trapped air .

As described in the Book of the Later Han , the frame of the seismometer was a domed bronze vessel in the shape of a wine jar , although it was 1 @.@ 8 m (5 @.@ 9 ft) in diameter and decorated with scenes of mountains and animals . The trigger mechanism was an inverted

pendulum (which the Book of the Later Han calls the " central column ") that , if disturbed by the ground tremors of earthquakes located near or far away , would swing and strike one of eight mobile arms (representing the eight directions) , each with a crank and catch mechanism . The crank and a right angle lever would raise one of eight metal dragon heads located on the exterior , dislodging a metal ball from its mouth that dropped into the mouth of one of eight metal toads below arranged like the points on a compass rose , thus indicating the direction of the earthquake . The Book of the Later Han states that when the ball fell into any one of eight toad mouths , it produced a loud noise which gained the attention of those observing the device . While Wang Zhenduo (???) accepted the idea that Zhang 's seismometer had cranks and levers which were disturbed by the inverted pendulum , his contemporary Akitsune Imamura (1870 ? 1948) argued that the inverted pendulum could have had a pin at the top which , upon moving by force of the ground vibrations , would enter one of eight slots and expel the ball by pushing a slider . Since the Book of the Later Han states that the other seven dragon heads would not subsequently release the balls lodged up into their jaws after the first one had dropped , Imamura asserted that the pin of the pendulum would have been locked into the slot it had entered and thus immobilized the instrument until it was reset .

= = Mathematics and astronomy = =

= = = Mathematical treatises = = =

One of the earliest surviving mathematical treatises of ancient China is the Book on Numbers and Computation (Suan shu shu) , part of the Zhangjiashan Han bamboo texts dated 202 to 186 BCE and found in Jiangling County , Hubei . Another mathematical text compiled during the Han was The Arithmetical Classic of the Gnomon and the Circular Paths of Heaven (Zhoubi suanjing) , dated no earlier than the 1st century BCE (from perhaps multiple authors) and contained materials similar to those described by Yang Xiong in 15 BCE , yet the zhoubi school of mathematics was not explicitly mentioned until Cai Yong 's (132 ? 192 CE) commentary of 180 CE . A preface was added to the text by Zhao Shuang ?? in the 3rd century CE . There was also the Nine Chapters on the Mathematical Art (Jiuzhang Suanshu) ; its full title was found on two bronze standard measurers dated 179 CE (with speculation that its material existed in earlier books under different titles) and was provided with detailed commentary by Liu Hui (fl . 3rd century CE) in 263 CE . It is worth noting in this context that many of the documents excavated from Qin and Han sites contain evidence of the practical mathematics used by administrators for inventories and taxes , as well as for calculating labor needed for public works projects , just as described in the mathematical treatises .

= = = Innovations in the treatises = = =

The Suan shu shu presents basic mathematics problems and solutions . It was most likely a handbook for day @-@ to @-@ day business transactions or affairs of government administration . It contains problems and solutions for field measurements of area , proportional exchange rates for agricultural millet and rice , distribution by proportion , short width division , and excess and deficiency . Some of the problems found in the Suan shu shu appear in the later text Jiuzhang suanshu ; in five cases , the titles are exact matches . However , unlike the Jiuzhang suanshu , the Suan shu shu does not deal with problems involving right @-@ angle triangles , square roots , cube roots , and matrix methods , which demonstrates the significant advancements made in Chinese mathematics between the writings of these two texts .

The Zhoubi suanjing , written in dialogue form and with regularly presented problems , is concerned with the application of mathematics to astronomy . In one problem which sought to determine the height of the Sun from the Earth and the diameter of the Sun , Chen Zi (??) instructs Rong Fang (??) to wait until the shadow cast by the 8 chi tall gnomon is 6 chi (one chi during the Han was 33 cm) , so that a 3 @-@ 4 @-@ 5 right @-@ angle triangle can be constructed where the base is 60

@,@ 000 li (one li during the Han was the equivalent of 415 m or 1362 ft) , the hypotenuse leading towards the sun is 100 @,@ 000 li , and the height of the sun is 80 @,@ 000 li . Like the Jiuzhang suanshu , the Zhoubi suanjing also gives mathematical proof for the " Gougu Theorem " (???? ; i.e. where c is the length of the hypotenuse and a and b are the lengths of the other two sides , respectively , a2 + b2 = c2) , which is known as the Pythagorean theorem in the West after the Greek mathematician Pythagoras (fl . 6th century BCE) .

The Jiuzhang suanshu was perhaps the most groundbreaking of the three surviving Han treatises . It is the first known book to feature negative numbers , along with the Bakhshali manuscript (200 CE ? ? 600 CE ?) of India and the book of the Greek mathematician Diophantus (fl . 3rd century) written in about 275 CE . Negative numbers appeared as black counting rods , while positive numbers appeared as red counting rods . Although the decimal system existed in China since the Shang dynasty (c . 1600 ? c . 1050 BCE) , the earliest evidence of a decimal fraction (i.e. the denominator is a power of ten) is an inscription on a standard volume @-@ measuring vessel dated 5 CE and used by the mathematician and astronomer Liu Xin (46 BCE ? 23 CE) . Yet the first book to feature decimal fractions was the Jiuzhang suanshu , as a means to solve equations and represent measurements . Gaussian elimination , an algorithm used to solve linear equations , was known as the Array Rule in the Jiuzhang suanshu . While the book used continued fractions to find the roots of equations , Liu Hui built on this idea in the 3rd century when he increased the decimals to find the cube root of 1 @,@ 860 @,@ 867 (yielding the answer 123) , the same method used in the Horner scheme named after William George Horner (1786 ? 1837) .

= = = Approximations of pi = = =