Znám 's problem

In number theory , Znám 's problem asks which sets of k integers have the property that each integer in the set is a proper divisor of the product of the other integers in the set , plus 1 . Znám 's problem is named after the Slovak mathematician ?tefan Znám , who suggested it in 1972 , although other mathematicians had considered similar problems around the same time . One closely related problem drops the assumption of properness of the divisor , and will be called the improper Znám problem hereafter .

One solution to the improper Znám problem is easily provided for any k : the first k terms of Sylvester 's sequence have the required property . Sun (1983) showed that there is at least one solution to the (proper) Znám problem for each k ? 5 . Sun 's solution is based on a recurrence similar to that for Sylvester 's sequence , but with a different set of initial values .

The Znám problem is closely related to Egyptian fractions . It is known that there are only finitely many solutions for any fixed k . It is unknown whether there are any solutions to Znám 's problem using only odd numbers , and there remain several other open questions .

The problem

Znám 's problem asks which sets of integers have the property that each integer in the set is a proper divisor of the product of the other integers in the set , plus 1 . That is , given k , what sets of integers

<formula>

are there , such that , for each i , ni divides but is not equal to

<formula>

A closely related problem concerns sets of integers in which each integer in the set is a divisor , but not necessarily a proper divisor , of one plus the product of the other integers in the set . This problem does not seem to have been named in the literature , and will be referred to as the improper Znám problem . Any solution to Znám 's problem is also a solution to the improper Znám problem , but not necessarily vice versa .

History