

= 1 at $x = 0$ unless $00 = 1$.

Continuous exponents

Limits involving algebraic operations can often be evaluated by replacing subexpressions by their limits; if the resulting expression does not determine the original limit, the expression is known as an indeterminate form. In fact, when $f(t)$ and $g(t)$ are real-valued functions both approaching 0 (as t approaches a real number or $\pm\infty$), with $f(t) > 0$, the function $f(t)g(t)$ need not approach 1; depending on f and g , the limit of $f(t)g(t)$ can be any nonnegative real number or $+\infty$, or it can diverge. For example, the functions below are of the form $f(t)g(t)$ with $f(t), g(t) \rightarrow 0$ as $t \rightarrow 0^+$, but the limits are different:

$\lim_{t \rightarrow 0^+} t^x = 0$ for $x > 0$.

Thus, the two-variable function xy , though continuous on the set $\{(x, y) : x > 0\}$, cannot be extended to a continuous function on any set containing $(0, 0)$, no matter how one chooses to define 00 . However, under certain conditions, such as when f and g are both analytic functions and f is positive on the open interval $(0, b)$ for some positive b , the limit approaching from the right is always 1.

Complex exponents

In the complex domain, the function z^w may be defined for nonzero z by choosing a branch of $\log z$ and defining z^w as $e^{w \log z}$. This does not define 0^w since there is no branch of $\log z$ defined at $z = 0$, let alone in a neighborhood of 0.

History of differing points of view

The debate over the definition of 0^0 has been going on at least since the early 19th century. At that time, most mathematicians agreed that 0^0 was an indeterminate form, until in 1821 Cauchy listed 0^0 along with expressions like $\frac{0}{0}$ in a table of indeterminate forms. In the 1830s Libri published an unconvincing argument for $0^0 = 1$, and Möbius sided with him, erroneously claiming that $0^0 = 1$ whenever $0^x = 0$. A commentator who signed his name simply as "S" provided the counterexample of $0^0 = 0$, and this quieted the debate for some time. More historical details can be found in Knuth (1992).

More recent authors interpret the situation above in different ways:

Some argue that the best value for 0^0 depends on context, and hence that defining it once and for all is problematic. According to Benson (1999), "The choice whether to define 0^0 is based on convenience, not on correctness. If we refrain from defining 0^0 then certain assertions become unnecessarily awkward. The consensus is to use the definition $0^0 = 1$, although there are textbooks that refrain from defining 0^0 ."

Others argue that 0^0 should be defined as 1. Knuth (1992) contends strongly that 0^0 "has to be 1", drawing a distinction between the value 0^0 , which should equal 1 as advocated by Libri, and the limiting form 0^0 (an abbreviation for a limit of 0^x where $x \rightarrow 0$), which is necessarily an indeterminate form as listed by Cauchy: "Both Cauchy and Libri were right, but Libri and his defenders did not understand why truth was on their side."

Treatment on computers

IEEE floating point standard

The IEEE 754-2008 floating point standard is used in the design of most floating point libraries. It recommends a number of functions for computing a power:

pow treats 00 as 1 . This is the oldest defined version . If the power is an exact integer the result is the same as for pown , otherwise the result is as for powr (except for some exceptional cases) .
 pown treats 00 as 1 . The power must be an exact integer . The value is defined for negative bases ; e.g. , $\text{pown}(3, -5)$ is $1/243$.
 powr treats 00 as NaN (Not a Number ? undefined) . The value is also NaN for cases like $\text{powr}(3, -2)$ where the base is less than zero . The value is defined by $\text{epow} \times \log(\text{base})$.

=== Programming languages ===

Most programming language with a power function are implemented using the IEEE pow function and therefore evaluate 00 as 1 . The later C and C++ standards describe this as the normative behaviour . The Java standard mandates this behavior . The .NET Framework method System.Math.Pow also treats 00 as 1 .

=== Mathematics software ===

Sage simplifies b^0 to 1 , even if no constraints are placed on b . It takes 00 to be 1 , but does not simplify 0^x for other x .

Maple distinguishes between integers 0 , 1 , ... and the corresponding floats 0.0 , 1.0 , ... (usually denoted 0. , 1. , ...) . If x does not evaluates to a number , then x^0 and $x^{0.0}$ are respectively evaluated to 1 (integer) and 1.0 (float) ; on the other hand , 0^x is evaluated to the integer 0 , while 0.0^x is evaluated as 0.x. If both the base and the exponent are zero (or are evaluated to zero) , the result is Float (undefined) if the exponent is the float 0.0 ; with an integer as exponent , the evaluation of 00 results in the integer 1 , while that of $0^{0.0}$ results in the float 1.0 .

Macsyma also simplifies b^0 to 1 even if no constraints are placed on b , but issues an error for 00 . For $x > 0$, it simplifies 0^x to 0 .

Mathematica and Wolfram Alpha simplify b^0 into 1 , even if no constraints are placed on b . While Mathematica does not simplify 0^x , Wolfram Alpha returns two results , 0 for $x > 0$, and " indeterminate " for real x . Both Mathematica and Wolfram Alpha take 00 to be " (indeterminate) " .

Matlab , Python , Magma , GAP , singular , PARI / GP and the Google and iPhone calculators evaluate 00 as 1 .

== Limits of powers ==

The section § Zero to the power of zero gives a number of examples of limits that are of the indeterminate form 00 . The limits in these examples exist , but have different values , showing that the two-variable function xy has no limit at the point (0 , 0) . One may consider at what points this function does have a limit .