

= Leonhard Euler =

Leonhard Euler (/ ˈɒləˈr / OY @-@ lər ; Swiss Standard German [ˈɒləˈr] , German Standard German [ˈɒləˈr]) (15 April 1707 ? 18 September 1783) was a Swiss mathematician , physicist , astronomer , logician and engineer who made important and influential discoveries in many branches of mathematics like infinitesimal calculus and graph theory while also making pioneering contributions to several branches such as topology and analytic number theory . He also introduced much of the modern mathematical terminology and notation , particularly for mathematical analysis , such as the notion of a mathematical function . He is also known for his work in mechanics , fluid dynamics , optics , astronomy , and music theory .

Euler was one of the most eminent mathematicians of the 18th century , and is held to be one of the greatest in history . He is also widely considered to be the most prolific mathematician of all time . His collected works fill 60 to 80 quarto volumes , more than anybody in the field . He spent most of his adult life in St. Petersburg , Russia , and in Berlin , then the capital of Prussia .

A statement attributed to Pierre @-@ Simon Laplace expresses Euler 's influence on mathematics : " Read Euler , read Euler , he is the master of us all . "

= = Life = =

= = = Early years = = =

Leonhard Euler was born on 15 April 1707 , in Basel , Switzerland to Paul Euler , a pastor of the Reformed Church , and Marguerite née Brucker , a pastor 's daughter . He had two younger sisters : Anna Maria and Maria Magdalena , and a younger brother Johann Heinrich . Soon after the birth of Leonhard , the Eulers moved from Basel to the town of Riehen , where Euler spent most of his childhood . Paul Euler was a friend of the Bernoulli family ; Johann Bernoulli was then regarded as Europe 's foremost mathematician , and would eventually be the most important influence on young Leonhard .

Euler 's formal education started in Basel , where he was sent to live with his maternal grandmother . In 1720 , aged thirteen , he enrolled at the University of Basel , and in 1723 , he received a Master of Philosophy with a dissertation that compared the philosophies of Descartes and Newton . At that time , he was receiving Saturday afternoon lessons from Johann Bernoulli , who quickly discovered his new pupil 's incredible talent for mathematics . At that time Euler 's main studies included theology , Greek , and Hebrew at his father 's urging in order to become a pastor , but Bernoulli convinced his father that Leonhard was destined to become a great mathematician .

In 1726 , Euler completed a dissertation on the propagation of sound with the title De Sono . At that time , he was unsuccessfully attempting to obtain a position at the University of Basel . In 1727 , he first entered the Paris Academy Prize Problem competition ; the problem that year was to find the best way to place the masts on a ship . Pierre Bouguer , who became known as " the father of naval architecture " , won and Euler took second place . Euler later won this annual prize twelve times .

= = = Saint Petersburg = = =

Around this time Johann Bernoulli 's two sons , Daniel and Nicolaus , were working at the Imperial Russian Academy of Sciences in Saint Petersburg . On 31 July 1726 , Nicolaus died of fever after spending less than a year in Russia , and when Daniel assumed his brother 's position in the mathematics / physics division , he recommended that the post in physiology that he had vacated be filled by his friend Euler . In November 1726 Euler eagerly accepted the offer , but delayed making the trip to Saint Petersburg while he unsuccessfully applied for a physics professorship at the University of Basel .

Euler arrived in Saint Petersburg on 17 May 1727 . He was promoted from his junior post in the medical department of the academy to a position in the mathematics department . He lodged with

Daniel Bernoulli with whom he often worked in close collaboration . Euler mastered Russian and settled into life in Saint Petersburg . He also took on an additional job as a medic in the Russian Navy .

The Academy at Saint Petersburg , established by Peter the Great , was intended to improve education in Russia and to close the scientific gap with Western Europe . As a result , it was made especially attractive to foreign scholars like Euler . The academy possessed ample financial resources and a comprehensive library drawn from the private libraries of Peter himself and of the nobility . Very few students were enrolled in the academy in order to lessen the faculty 's teaching burden , and the academy emphasized research and offered to its faculty both the time and the freedom to pursue scientific questions .

The Academy 's benefactress , Catherine I , who had continued the progressive policies of her late husband , died on the day of Euler 's arrival . The Russian nobility then gained power upon the ascension of the twelve @-@ year @-@ old Peter II . The nobility were suspicious of the academy 's foreign scientists , and thus cut funding and caused other difficulties for Euler and his colleagues .

Conditions improved slightly after the death of Peter II , and Euler swiftly rose through the ranks in the academy and was made professor of physics in 1731 . Two years later , Daniel Bernoulli , who was fed up with the censorship and hostility he faced at Saint Petersburg , left for Basel . Euler succeeded him as the head of the mathematics department .

On 7 January 1734 , he married Katharina Gsell (1707 ? 1773) , a daughter of Georg Gsell , a painter from the Academy Gymnasium . The young couple bought a house by the Neva River . Of their thirteen children , only five survived childhood .

= = = Berlin = = =

Concerned about the continuing turmoil in Russia , Euler left St. Petersburg on 19 June 1741 to take up a post at the Berlin Academy , which he had been offered by Frederick the Great of Prussia . He lived for twenty @-@ five years in Berlin , where he wrote over 380 articles . In Berlin , he published the two works for which he would become most renowned : The Introductio in analysin infinitorum , a text on functions published in 1748 , and the Institutiones calculi differentialis , published in 1755 on differential calculus . In 1755 , he was elected a foreign member of the Royal Swedish Academy of Sciences .

In addition , Euler was asked to tutor Friederike Charlotte of Brandenburg @-@ Schwedt , the Princess of Anhalt @-@ Dessau and Frederick 's niece . Euler wrote over 200 letters to her in the early 1760s , which were later compiled into a best @-@ selling volume entitled Letters of Euler on different Subjects in Natural Philosophy Addressed to a German Princess . This work contained Euler 's exposition on various subjects pertaining to physics and mathematics , as well as offering valuable insights into Euler 's personality and religious beliefs . This book became more widely read than any of his mathematical works , and was published across Europe and in the United States . The popularity of the ' Letters ' testifies to Euler 's ability to communicate scientific matters effectively to a lay audience , a rare ability for a dedicated research scientist .

Despite Euler 's immense contribution to the Academy 's prestige , he eventually incurred the ire of Frederick and ended up having to leave Berlin . The Prussian king had a large circle of intellectuals in his court and he found the mathematician unsophisticated and ill @-@ informed on matters beyond numbers and figures . Euler was a simple , devoutly religious man who never questioned the existing social order or conventional beliefs , in many ways the polar opposite of Voltaire , who enjoyed a high place of prestige at Frederick 's court . Euler was not a skilled debater and often made it a point to argue subjects that he knew little about , making him the frequent target of Voltaire 's wit . Frederick also expressed disappointment with Euler 's practical engineering abilities :

I wanted to have a water jet in my garden : Euler calculated the force of the wheels necessary to raise the water to a reservoir , from where it should fall back through channels , finally spurting out in Sanssouci . My mill was carried out geometrically and could not raise a mouthful of water closer than fifty paces to the reservoir . Vanity of vanities ! Vanity of geometry !

== Eyesight deterioration ==

Euler's eyesight worsened throughout his mathematical career. In 1738, three years after nearly expiring from fever, he became almost blind in his right eye, but Euler rather blamed the painstaking work on cartography he performed for the St. Petersburg Academy for his condition. Euler's vision in that eye worsened throughout his stay in Germany, to the extent that Frederick referred to him as "Cyclops". Euler later developed a cataract in his left eye, which was discovered in 1766. Just a few weeks after its discovery, he was rendered almost totally blind. However, his condition appeared to have little effect on his productivity, as he compensated for it with his mental calculation skills and exceptional memory. For example, Euler could repeat the Aeneid of Virgil from beginning to end without hesitation, and for every page in the edition he could indicate which line was the first and which the last. With the aid of his scribes, Euler's productivity on many areas of study actually increased. He produced on average, one mathematical paper every week in the year 1775.

== Return to Russia and death ==

In 1760, with the Seven Years' War raging, Euler's farm in Charlottenburg was ransacked by advancing Russian troops. Upon learning of this event, General Ivan Petrovich Saltykov paid compensation for the damage caused to Euler's estate, later Empress Elizabeth of Russia added a further payment of 4000 roubles - an exorbitant amount at the time. The political situation in Russia stabilized after Catherine the Great's accession to the throne, so in 1766 Euler accepted an invitation to return to the St. Petersburg Academy. His conditions were quite exorbitant? a 3000 ruble annual salary, a pension for his wife, and the promise of high ranking appointments for his sons. All of these requests were granted. He spent the rest of his life in Russia. However, his second stay in the country was marred by tragedy. A fire in St. Petersburg in 1771 cost him his home, and almost his life. In 1773, he lost his wife Katharina after 40 years of marriage.

Three years after his wife's death, Euler married her half sister, Salome Abigail Gsell (1723? 1794). This marriage lasted until his death. In 1782 he was elected a Foreign Honorary Member of the American Academy of Arts and Sciences.

In St. Petersburg on 18 September 1783, after a lunch with his family, Euler was discussing the newly discovered planet Uranus and its orbit with a fellow academician Anders Johan Lexell, when he collapsed from a brain hemorrhage. He died a few hours later. Jacob von Staehlin Storcksburg wrote a short obituary for the Russian Academy of Sciences and Russian mathematician Nicolas Fuss, one of Euler's disciples, wrote a more detailed eulogy which he delivered at a memorial meeting. In his eulogy for the French Academy, French mathematician and philosopher Marquis de Condorcet, wrote:

il cessa de calculer et de vivre ? ... he ceased to calculate and to live.

Euler was buried next to Katharina at the Smolensk Lutheran Cemetery on Vasilievsky Island. In 1785, the Russian Academy of Sciences put a marble bust of Leonhard Euler on a pedestal next to the Director's seat and, in 1837, placed a headstone on Euler's grave. To commemorate the 250th anniversary of Euler's birth, the headstone was moved in 1956, together with his remains, to the 18th century necropolis at the Alexander Nevsky Monastery.

== Contributions to mathematics and physics ==

Euler worked in almost all areas of mathematics, such as geometry, infinitesimal calculus, trigonometry, algebra, and number theory, as well as continuum physics, lunar theory and other areas of physics. He is a seminal figure in the history of mathematics; if printed, his works, many of which are of fundamental interest, would occupy between 60 and 80 quarto volumes. Euler's name is associated with a large number of topics.

Euler is the only mathematician to have two numbers named after him: the important Euler's

number in calculus , e , approximately equal to 2.71828 , and the Euler-Mascheroni constant γ (gamma) sometimes referred to as just " Euler 's constant " , approximately equal to 0.57721 . It is not known whether γ is rational or irrational .

=== Mathematical notation ===

Euler introduced and popularized several notational conventions through his numerous and widely circulated textbooks . Most notably , he introduced the concept of a function and was the first to write $f(x)$ to denote the function f applied to the argument x . He also introduced the modern notation for the trigonometric functions , the letter e for the base of the natural logarithm (now also known as Euler 's number) , the Greek letter Σ for summations and the letter i to denote the imaginary unit . The use of the Greek letter π to denote the ratio of a circle 's circumference to its diameter was also popularized by Euler , although it did not originate with him .

=== Analysis ===

The development of infinitesimal calculus was at the forefront of 18th Century mathematical research , and the Bernoulli family friends of Euler were responsible for much of the early progress in the field . Thanks to their influence , studying calculus became the major focus of Euler 's work . While some of Euler 's proofs are not acceptable by modern standards of mathematical rigour (in particular his reliance on the principle of the generality of algebra) , his ideas led to many great advances . Euler is well known in analysis for his frequent use and development of power series , the expression of functions as sums of infinitely many terms , such as

<formula>

Notably , Euler directly proved the power series expansions for e and the inverse tangent function . (Indirect proof via the inverse power series technique was given by Newton and Leibniz between 1670 and 1680 .) His daring use of power series enabled him to solve the famous Basel problem in 1735 (he provided a more elaborate argument in 1741) :

<formula>

Euler introduced the use of the exponential function and logarithms in analytic proofs . He discovered ways to express various logarithmic functions using power series , and he successfully defined logarithms for negative and complex numbers , thus greatly expanding the scope of mathematical applications of logarithms . He also defined the exponential function for complex numbers , and discovered its relation to the trigonometric functions . For any real number x (taken to be radians) , Euler 's formula states that the complex exponential function satisfies

<formula>

A special case of the above formula is known as Euler 's identity ,

<formula>

called " the most remarkable formula in mathematics " by Richard P. Feynman , for its single uses of the notions of addition , multiplication , exponentiation , and equality , and the single uses of the important constants 0 , 1 , e , i and π . In 1988 , readers of the Mathematical Intelligencer voted it " the Most Beautiful Mathematical Formula Ever " . In total , Euler was responsible for three of the top five formulae in that poll .

De Moivre 's formula is a direct consequence of Euler 's formula .

In addition , Euler elaborated the theory of higher transcendental functions by introducing the gamma function and introduced a new method for solving quartic equations . He also found a way to calculate integrals with complex limits , foreshadowing the development of modern complex analysis . He also invented the calculus of variations including its best known result , the Euler-Lagrange equation .

Euler also pioneered the use of analytic methods to solve number theory problems . In doing so , he united two disparate branches of mathematics and introduced a new field of study , analytic number theory . In breaking ground for this new field , Euler created the theory of hypergeometric series , q -series , hyperbolic trigonometric functions and the analytic theory of continued

fractions . For example , he proved the infinitude of primes using the divergence of the harmonic series , and he used analytic methods to gain some understanding of the way prime numbers are distributed . Euler 's work in this area led to the development of the prime number theorem .

== Number theory ==

Euler 's interest in number theory can be traced to the influence of Christian Goldbach , his friend in the St. Petersburg Academy . A lot of Euler 's early work on number theory was based on the works of Pierre de Fermat . Euler developed some of Fermat 's ideas , and disproved some of his conjectures .

Euler linked the nature of prime distribution with ideas in analysis . He proved that the sum of the reciprocals of the primes diverges . In doing so , he discovered the connection between the Riemann zeta function and the prime numbers ; this is known as the Euler product formula for the Riemann zeta function .

Euler proved Newton 's identities , Fermat 's little theorem , Fermat 's theorem on sums of two squares , and he made distinct contributions to Lagrange 's four @-@ square theorem . He also invented the totient function $\phi(n)$, the number of positive integers less than or equal to the integer n that are coprime to n . Using properties of this function , he generalized Fermat 's little theorem to what is now known as Euler 's theorem . He contributed significantly to the theory of perfect numbers , which had fascinated mathematicians since Euclid . He proved that the relationship shown between perfect numbers and Mersenne primes earlier proved by Euclid was one @-@ to @-@ one , a result otherwise known as the Euclid ? Euler theorem . Euler also conjectured the law of quadratic reciprocity . The concept is regarded as a fundamental theorem of number theory , and his ideas paved the way for the work of Carl Friedrich Gauss . By 1772 Euler had proved that $2^{31} - 1 = 2^2 \cdot 43 \cdot 431 \cdot 967$ is a Mersenne prime . It may have remained the largest known prime until 1867 .

== Graph theory ==

In 1735 , Euler presented a solution to the problem known as the Seven Bridges of Königsberg . The city of Königsberg , Prussia was set on the Pregel River , and included two large islands that were connected to each other and the mainland by seven bridges . The problem is to decide whether it is possible to follow a path that crosses each bridge exactly once and returns to the starting point . It is not possible : there is no Eulerian circuit . This solution is considered to be the first theorem of graph theory , specifically of planar graph theory .

Euler also discovered the formula $V - E + F = 2$ relating the number of vertices , edges and faces of a convex polyhedron , and hence of a planar graph . The constant in this formula is now known as the Euler characteristic for the graph (or other mathematical object) , and is related to the genus of the object . The study and generalization of this formula , specifically by Cauchy and L 'Huillier , is at the origin of topology .

== Applied mathematics ==

Some of Euler 's greatest successes were in solving real @-@ world problems analytically , and in describing numerous applications of the Bernoulli numbers , Fourier series , Venn diagrams , Euler numbers , the constants e and γ , continued fractions and integrals . He integrated Leibniz 's differential calculus with Newton 's Method of Fluxions , and developed tools that made it easier to apply calculus to physical problems . He made great strides in improving the numerical approximation of integrals , inventing what are now known as the Euler approximations . The most notable of these approximations are Euler 's method and the Euler ? Maclaurin formula . He also facilitated the use of differential equations , in particular introducing the Euler ? Mascheroni constant :

<formula>

One of Euler 's more unusual interests was the application of mathematical ideas in music . In 1739 he wrote the *Tentamen novae theoriae musicae* , hoping to eventually incorporate musical theory as part of mathematics . This part of his work , however , did not receive wide attention and was once described as too mathematical for musicians and too musical for mathematicians .

== Physics and astronomy ==

Euler helped develop the Euler – Bernoulli beam equation , which became a cornerstone of engineering . Aside from successfully applying his analytic tools to problems in classical mechanics , Euler also applied these techniques to celestial problems . His work in astronomy was recognized by a number of Paris Academy Prizes over the course of his career . His accomplishments include determining with great accuracy the orbits of comets and other celestial bodies , understanding the nature of comets , and calculating the parallax of the sun . His calculations also contributed to the development of accurate longitude tables .

In addition , Euler made important contributions in optics . He disagreed with Newton 's corpuscular theory of light in the *Opticks* , which was then the prevailing theory . His 1740s papers on optics helped ensure that the wave theory of light proposed by Christiaan Huygens would become the dominant mode of thought , at least until the development of the quantum theory of light .

In 1757 he published an important set of equations for inviscid flow , that are now known as the Euler equations . In differential form , the equations are :

<formula>

where

ρ is the fluid mass density ,

\mathbf{u} is the fluid velocity vector , with components u , v , and w ,

E

$E = \rho e + \frac{1}{2} \rho (u^2 + v^2 + w^2)$ is the total energy per unit volume , with e being the internal energy per unit mass for the fluid ,

p is the pressure ,

$\nabla \cdot$ denotes the tensor product , and

$\mathbf{0}$ being the zero vector .

Euler is also well known in structural engineering for his formula giving the critical buckling load of an ideal strut , which depends only on its length and flexural stiffness :

<formula>

where

$F =$

maximum or critical force (vertical load on column) ,