Final stellation of the icosahedron

In geometry , the complete or final stellation of the icosahedron is the outermost stellation of the icosahedron , and is " complete " and " final " because it includes all of the cells in the icosahedron 's stellation diagram .
This polyhedron is the seventeenth stellation of the icosahedron , and given as Wenninger model index 42 .
As a geometrical figure , it has two interpretations , described below :
As an irregular star (self @-@ intersecting) polyhedron with 20 identical self @-@ intersecting enneagrammic faces , 90 edges , 60 vertices .
As a simple polyhedron with 180 triangular faces (60 isosceles , 120 scalene) , 270 edges , and 92 vertices . This interpretation is useful for polyhedron model building .
Johannes Kepler researched stellations that create regular star polyhedra (the Kepler @-@ Poinsot polyhedra) in 1619 , but the complete icosahedron , with irregular faces , was first studied in 1900 by Max Brückner .

History

1619 : In Harmonices Mundi , Johannes Kepler first applied the stellation process , recognizing the small stellated dodecahedron and great stellated dodecahedron as regular polyhedra .
1809 : Louis Poinsot rediscovered Kepler 's polyhedra and two more , the great icosahedron and great dodecahedron as regular star polyhedra , now called the Kepler ? Poinsot polyhedra .
1812 : Augustin @-@ Louis Cauchy made a further enumeration of star polyhedra , proving there are only 4 regular star polyhedra .
1900 : Max Brückner extended the stellation theory beyond regular forms , and identified ten stellations of the icosahedron , including the complete stellation .
1924 : A.H. Wheeler in 1924 published a list of 20 stellation forms (22 including reflective copies) , also including the complete stellation .
1938 : In their 1938 book The Fifty Nine Icosahedra , H. S. M. Coxeter , P. Du Val , H. T. Flather and J. F. Petrie stated a set of stellation rules for the regular icosahedron and gave a systematic enumeration of the fifty @-@ nine stellations which conform to those rules . The complete stellation is referenced as the eighth in the book .
1974 : In Wenninger 's 1974 book Polyhedron Models , the final stellation of the icosahedron is included as the 17th model of stellated icosahedra with index number W42 .
1995 : Andrew Hume named it in his Netlib polyhedral database as the echidnahedron (the echidna , or spiny anteater is a small mammal that is covered with coarse hair and spines and which curls up in a ball to protect itself) .

Interpretations

As a stellation

The stellation of a polyhedron extends the faces of a polyhedron into infinite planes and generates a new polyhedron that is bounded by these planes as faces and the intersections of these planes as edges . The Fifty Nine Icosahedra enumerates the stellations of the regular icosahedron , according to a set of rules put forward by J. C. P. Miller , including the complete stellation . The Du Val symbol of the complete stellation is H , because it includes all cells in the stellation diagram up to and including the outermost " h " layer .

As a simple polyhedron

As a simple , visible surface polyhedron , the outward form of the final stellation is composed of 180 triangular faces , which are the outermost triangular regions in the stellation diagram . These join along 270 edges , which in turn meet at 92 vertices , with an Euler characteristic of 2 .

The 92 vertices lie on the surfaces of three concentric spheres . The innermost group of 20 vertices form the vertices of a regular dodecahedron ; the next layer of 12 form the vertices of a regular icosahedron ; and the outer layer of 60 form the vertices of a nonuniform truncated icosahedron . The radii of these spheres are in the ratio

<formula>

When regarded as a three @-@ dimensional solid object with edge lengths a , ?a , ?2a and ?2a ? 2 (where ? is the golden ratio) the complete icosahedron has surface area

<formula>

and volume

<formula>

= = = As a star polyhedron = = =