

$a + b = b + a$ for any two integers (commutativity of addition) . Groups for which the commutativity equation $a \cdot b =$

$b \cdot a$ always holds are called abelian groups (in honor of Niels Henrik Abel) . The symmetry group described in the following section is an example of a group that is not abelian .

The identity element of a group G is often written as 1 or 1_G , a notation inherited from the multiplicative identity . If a group is abelian , then one may choose to denote the group operation by $+$ and the identity element by 0 ; in that case , the group is called an additive group . The identity element can also be written as id .

The set G is called the underlying set of the group (G , \cdot) . Often the group 's underlying set G is used as a short name for the group (G , \cdot) . Along the same lines , shorthand expressions such as " a subset of the group G " or " an element of group G " are used when what is actually meant is " a subset of the underlying set G of the group (G , \cdot) " or " an element of the underlying set G of the group (G , \cdot) " . Usually , it is clear from the context whether a symbol like G refers to a group or to an underlying set .

== Second example : a symmetry group ==

Two figures in the plane are congruent if one can be changed into the other using a combination of rotations , reflections , and translations . Any figure is congruent to itself . However , some figures are congruent to themselves in more than one way , and these extra congruences are called symmetries . A square has eight symmetries . These are :

the identity operation leaving everything unchanged , denoted id ;

rotations of the square around its center by 90° clockwise , 180° clockwise , and 270° clockwise , denoted by r_1 , r_2 and r_3 , respectively ;

reflections about the vertical and horizontal middle line (f_h and f_v) , or through the two diagonals (f_d and f_c) .

These symmetries are represented by functions . Each of these functions sends a point in the square to the corresponding point under the symmetry . For example , r_1 sends a point to its rotation 90° clockwise around the square 's center , and f_h sends a point to its reflection across the square 's vertical middle line . Composing two of these symmetry functions gives another symmetry function . These symmetries determine a group called the dihedral group of degree 4 and denoted D_4 . The underlying set of the group is the above set of symmetry functions , and the group operation is function composition . Two symmetries are combined by composing them as functions , that is , applying the first one to the square , and the second one to the result of the first application . The result of performing first a and then b is written symbolically from right to left as

$b \circ a$ (" apply the symmetry b after performing the symmetry a ") .

The right \circ - \circ to \circ - \circ left notation is the same notation that is used for composition of functions .

The group table on the right lists the results of all such compositions possible . For example , rotating by 270° clockwise (r_3) and then reflecting horizontally (f_h) is the same as performing a reflection along the diagonal (f_d) . Using the above symbols , highlighted in blue in the group table :