= Primary line constants =

The primary line constants are parameters that describe the characteristics of conductive transmission lines , such as pairs of copper wires , in terms of the physical electrical properties of the line . The primary line constants are only relevant to such lines and are to be contrasted with the secondary line constants , which can be derived from them , and are more generally applicable . The secondary line constants can be used , for instance , to compare the characteristics of a waveguide to a copper line , whereas the primary constants have no meaning for a waveguide .

The constants are conductor resistance and inductance , and insulator capacitance and conductance , which are by convention given the symbols R , L , C , and G respectively . The constants are enumerated in terms of per unit length . The circuit representation of these elements requires a distributed element model and consequently calculus must be used to analyse the circuit . The secondary constants of characteristic impedance and propagation constant can be derived in this way .

A number of special cases have particularly simple solutions and important practical applications . Low loss cable requires only L and C to be included in the analysis , useful for short lengths of cable . Low frequency applications , such as twisted pair telephone lines , are dominated by R and C only . High frequency applications , such as RF co @-@ axial cable , are dominated by L and C. Lines loaded to prevent distortion need all four elements in the analysis , but have a simple , elegant solution .

= = The constants = =

There are four primary line constants , but in some circumstances some of them are small enough to be ignored and the analysis can be simplified . These four , and their symbols and units are as follows :

R and L are elements in series with the line ( because they are properties of the conductor ) and C and G are elements shunting the line ( because they are properties of the dielectric material between the conductors ) . G represents leakage current through the dielectric and in most cables is very small . The word loop is used to emphasise that the resistance and inductance of both conductors must be taken into account . For instance , if a line consists of two identical wires that have a resistance of 25 m? / m each , the loop resistance is double that , 50 m? / m . Because the values of the constants are quite small , it is common for manufacturers to quote them per kilometre rather than per metre ; in the English @-@ speaking world " per mile " can also be used .

The word " constant " can be misleading since there is some variation with frequency . In particular , R is heavily influenced by the skin effect . Furthermore , while G has virtually no effect at audio frequency , it can cause noticeable losses at high frequency with many of the dielectric materials used in cables due to a high loss tangent . Avoiding the losses caused by G is the reason many cables designed for use at UHF are air @-@ insulated or foam @-@ insulated ( which makes them virtually air @-@ insulated ) . The actual meaning of constant in this context is that the parameter is constant with distance . That is the line is assumed to be homogenous lengthwise . This condition is true for the vast majority of transmission lines in use today .

= = Typical values for some common cables = =

? Manufacturers commonly omit a value for inductance in their data sheets . Some of these values are estimated from the figures for capacitance and characteristic impedance by <formula> .

= = Circuit representation = =

The line constants cannot be simply represented as lumped elements in a circuit ; they must be described as distributed elements . For instance " pieces " of the capacitance are in between " pieces " of the resistance . However many pieces the R and C are broken into , it can always be

argued they should be broken apart further to properly represent the circuit , and after each division the number of meshes in the circuit is increased . This is shown diagramtically in figure 1 . To give a true representation of the circuit , the elements must be made infinitesimally small so that each element is distributed along the line . The infinitesimal elements in an infinitesimal distance <formula> are given by ;

<formula>
<formula>
<formula>
<formula>

It is convenient for the purposes of analysis to roll up these elements into general series impedance , Z , and shunt admittance , Y elements such that ;

<formula> and ,
<formula>

Analysis of this network ( figure 2 ) will yield the secondary line constants : the propagation constant , <formula> , ( whose real and imaginary parts are the attenuation constant , <formula> , and phase change constant , <formula> , respectively ) and the characteristic impedance , <formula> , which also , in general , will have real , <formula> , and imaginary , <formula> , parts , making a total of four secondary constants to be derived from the four primary constants . The term constant is even more misleading for the secondary constants as they all usually vary quite strongly with frequency , even if the frequency dependence of the primary constants is ignored . This is because the reactances in the circuit ( <formula> and <formula> ) introduce a dependence on <formula> . It is possible to choose specific values of the primary constants that result in <formula> and <formula> being constant ( the Heaviside condition ) but even in this case there is still <formula> which is directly proportional to <formula> . As with the primary constants , the meaning is that the secondary constants do not vary with distance along the line , not that they are independent of frequency .

= = Characteristic impedance = =

The characteristic impedance of a transmission line , <formula> , is defined as the impedance looking into an infinitely long line . Such a line will never return a reflection since the incident wave will never reach the end to be reflected . When considering a finite initial length of the line , the remainder of the line can be replaced by <formula> as its equivalent circuit . This is so because the remainder of the line is still infinitely long . Considering just the first section of the equivalent circuit of the line ( this is an L @-@ network consisting of one element each of <formula> and <formula> ) the remainder can be replaced by <formula> . This results in the network shown in figure 3 , which can be analysed for <formula> using the usual network analysis theorems ,

<formula>

which re @-@ arranges to ,

<formula>

Taking limits of both sides

<formula>

and since the line was assumed to be homogenous lengthwise ,

<formula>

= = Propagation constant = =

The ratio of the line input voltage to the voltage a distance <formula> further down the line ( that is , after one section of the equivalent circuit ) is given by a standard voltage divider calculation . The remainder of the line to the right , as in the characteristic impedance calculation , is replaced with <formula> ,

<formula>

Each infinitesimal section will multiply the voltage drop by the same factor . After <formula> sections the voltage ratio will be ,

<formula>
At a distance <formula> along the line , the number of sections is <formula> so that ,
<formula>
In the limit as <formula> ,
<formula>
The second order term <formula> will disappear in the limit , so we can write without loss of accuracy ,
<formula>
and comparing with the mathematical identity ,
<formula>
yields ,
<formula>
From the definition of propagation constant ,
<formula>
Hence ,
<formula>

== Special cases ==

An ideal transmission line will have no loss , which implies that the resistive elements are zero . It also results in a purely real ( resistive ) characteristic impedance . The ideal line cannot be realised in practice , but it is a useful approximation in many circumstances . This is especially true , for instance , when short pieces of line are being used as circuit components such as stubs . A short line has very little loss and this can then be ignored and treated as an ideal line . The secondary constants in these circumstances are ;
<formula>
<formula>
<formula>
<formula>

=== Twisted pair ===

Typically , twisted pair cable used for audio frequencies or low data rates has line constants dominated by R and C. The dielectric loss is usually negligible at these frequencies and G is close to zero . It is also the case that , at a low enough frequency , <formula> which means that L can also be ignored . In those circumstances the secondary constants become ,
<formula>
<formula>
<formula>
<formula>
The attenuation of this cable type increases with frequency , causing distortion of waveforms . Not so obviously , the variation of <formula> with frequency also causes a distortion of a type called dispersion . To avoid dispersion the requirement is that <formula> is directly proportional to <formula> . However , it is actually proportional to <formula> and dispersion results . <formula> also varies with frequency and is also partly reactive ; both these features will be the cause of reflections from a resistive line termination . This is another undesirable effect . The nominal impedance quoted for this type of cable is , in this case , very nominal , being valid at only one spot frequency , usually quoted at 800 Hz or 1 kHz .

=== Co @-@ axial cable ===

Cable operated at a high enough frequency ( VHF radio frequency or high data rates ) will meet the conditions <formula> and <formula> . This must eventually be the case as the frequency is

increased for any cable . Under those conditions R and G can both be ignored ( except for the purpose of calculating the cable loss ) and the secondary constants become ;

<formula>
<formula>
<formula>
<formula>

## Loaded line

Loaded lines are lines designed with deliberately increased inductance . This is done by adding iron or some other magnetic metal to the cable or adding coils . The purpose is to ensure that the line meets the Heaviside condition , which eliminates distortion caused by frequency @-@ dependent attenuation and dispersion , and ensures that <formula> is constant and resistive . The secondary constants are here related to the primary constants by ;

<formula>
<formula>
<formula>
<formula>

## Velocity

The velocity of propagation is given by ,
<formula>
Since ,
<formula> and <formula>
then ,
<formula>
In cases where ? can be taken as ,
<formula>
the velocity of propagation is given by ,
<formula>
The lower the capacitance the higher the velocity . With an air dielectric cable , which is approximated to with low @-@ loss cable , the velocity of propagation is very close to c , the speed of light in vacuo .