= Xqp ; Opq =
Oqp .
Further examples of commutative binary operations include addition and multiplication of complex numbers , addition and scalar multiplication of vectors , and intersection and union of sets .

= = = Noncommutative operations in everyday life = = =

Concatenation , the act of joining character strings together , is a noncommutative operation . For example ,
<formula>
Washing and drying clothes resembles a noncommutative operation ; washing and then drying produces a markedly different result to drying and then washing .
Rotating a book 90 ° around a vertical axis then 90 ° around a horizontal axis produces a different orientation than when the rotations are performed in the opposite order .
The twists of the Rubik 's Cube are noncommutative . This can be studied using group theory .
Also thought processes are noncommutative : A person asked a question (A) and then a question (B) may give different answers to each question than a person asked first (B) and then (A) , because asking a question may change the person 's state of mind .

= = = Noncommutative operations in mathematics = = =

Some non @-@ commutative binary operations :

= = History and etymology = =

Records of the implicit use of the commutative property go back to ancient times . The Egyptians used the commutative property of multiplication to simplify computing products . Euclid is known to have assumed the commutative property of multiplication in his book Elements . Formal uses of the commutative property arose in the late 18th and early 19th centuries , when mathematicians began to work on a theory of functions . Today the commutative property is a well known and basic property used in most branches of mathematics .
The first recorded use of the term commutative was in a memoir by François Servois in 1814 , which used the word commutatives when describing functions that have what is now called the commutative property . The word is a combination of the French word commuter meaning " to substitute or switch " and the suffix -ative meaning " tending to " so the word literally means " tending to substitute or switch . " The term then appeared in English in Philosophical Transactions of the Royal Society in 1844 .

= = Propositional logic = =

= = = Rule of replacement = = =

In truth @-@ functional propositional logic , commutation , or commutativity refer to two valid rules of replacement . The rules allow one to transpose propositional variables within logical expressions in logical proofs . The rules are :
<formula>
and
<formula>
where " <formula> " is a metalogical symbol representing " can be replaced in a proof with . "

= = = Truth functional connectives = = =

Commutativity is a property of some logical connectives of truth functional propositional logic . The following logical equivalences demonstrate that commutativity is a property of particular connectives . The following are truth @-@ functional tautologies .

Commutativity of conjunction

<formula>

Commutativity of disjunction

<formula>

Commutativity of implication (also called the law of permutation)

<formula>

Commutativity of equivalence (also called the complete commutative law of equivalence)

<formula>

= = Set theory = =

In group and set theory , many algebraic structures are called commutative when certain operands satisfy the commutative property . In higher branches of mathematics , such as analysis and linear algebra the commutativity of well @-@ known operations (such as addition and multiplication on real and complex numbers) is often used (or implicitly assumed) in proofs .

= = Mathematical structures and commutativity = =

A commutative semigroup is a set endowed with a total , associative and commutative operation .

If the operation additionally has an identity element , we have a commutative monoid

An abelian group , or commutative group is a group whose group operation is commutative .

A commutative ring is a ring whose multiplication is commutative . (Addition in a ring is always commutative .)

In a field both addition and multiplication are commutative .

= = Related properties = =

= = = Associativity = = =

The associative property is closely related to the commutative property . The associative property of an expression containing two or more occurrences of the same operator states that the order operations are performed in does not affect the final result , as long as the order of terms doesn 't change . In contrast , the commutative property states that the order of the terms does not affect the final result .

Most commutative operations encountered in practice are also associative . However , commutativity does not imply associativity . A counterexample is the function

<formula>

which is clearly commutative (interchanging x and y does not affect the result) , but it is not associative (since , for example , <formula> but <formula>) . More such examples may be found in Commutative non @-@ associative magmas .

= = = Symmetry = = =