= 10 , b =
e (the irrational mathematical constant ? 2 @.@ 71828) , and b = 2 . In mathematical analysis , the logarithm to base e is widespread because of its particular analytical properties explained below . On the other hand , base @-@ 10 logarithms are easy to use for manual calculations in the decimal number system :
<formula>
Thus , log10 (x) is related to the number of decimal digits of a positive integer x : the number of digits is the smallest integer strictly bigger than log10 (x) . For example , log10 (1430) is approximately 3 @.@ 15 . The next integer is 4 , which is the number of digits of 1430 . Both the natural logarithm and the logarithm to base two are used in information theory , corresponding to the use of nats or bits as the fundamental units of information , respectively . Binary logarithms are also used in computer science , where the binary system is ubiquitous , in music theory , where a pitch ratio of two (the octave) is ubiquitous and the cent is the binary logarithm (scaled by 1200) of the ratio between two adjacent equally @-@ tempered pitches , and in photography to measure exposure values .
The following table lists common notations for logarithms to these bases and the fields where they are used . Many disciplines write log (x) instead of logb (x) , when the intended base can be determined from the context . The notation blog (x) also occurs . The " ISO notation " column lists designations suggested by the International Organization for Standardization (ISO 31 @-@ 11) .

= = History = =

The history of logarithm in seventeenth century Europe is the discovery of a new function that extended the realm of analysis beyond the scope of algebraic methods . The method of logarithms was publicly propounded by John Napier in 1614 , in a book titled Mirifici Logarithmorum Canonis Descriptio (Description of the Wonderful Rule of Logarithms) . Prior to Napier 's invention , there had been other techniques of similar scopes , such as the prosthaphaeresis or the use of tables of progressions , extensively developed by Jost Bürgi around 1600 .
The common logarithm of a number is the index of that power of ten which equals the number . Speaking of a number as requiring so many figures is a rough allusion to common logarithm , and was referred to by Archimedes as the " order of a number " . The first real logarithms were heuristic methods to turn multiplication into addition , thus facilitating rapid computation . Some of these methods used tables derived from trigonometric identities . Such methods are called prosthaphaeresis .
Invention of the function now known as natural logarithm began as an attempt to perform a quadrature of a rectangular hyperbola by Gregoire de Saint Vincent , a Belgian Jesuit residing in Prague . Archimedes had written The Quadrature of the Parabola in the third century BC , but a quadrature for the hyperbola eluded all efforts until Saint @-@ Vincent published his results in 1647 . The relation that the logarithm provides between a geometric progression in its argument and an arithmetic progression of values , prompted A. A. de Sarasa to make the connection of Saint @-@ Vincent ? s quadrature and the tradition of logarithms in prosthaphaeresis , leading to the term " hyperbolic logarithm " , a synonym for natural logarithm . Soon the new function was appreciated by Christiaan Huygens , Patavii , and James Gregory . The notation Log y was adopted by Leibniz in 1675 , and the next year he connected it to the integral <formula>

= = Logarithm tables , slide rules , and historical applications = =

By simplifying difficult calculations , logarithms contributed to the advance of science , especially astronomy . They were critical to advances in surveying , celestial navigation , and other domains . Pierre @-@ Simon Laplace called logarithms
" ... [a] n admirable artifice which , by reducing to a few days the labour of many months , doubles the life of the astronomer , and spares him the errors and disgust inseparable from long calculations . "