= a · bx appear as straight lines with slope equal to the logarithm of b . Log @-@ log graphs scale both axes logarithmically , which causes functions of the form f (x) =
a · xk to be depicted as straight lines with slope equal to the exponent k . This is applied in visualizing and analyzing power laws .

= = = Psychology = = =

Logarithms occur in several laws describing human perception : Hick 's law proposes a logarithmic relation between the time individuals take to choose an alternative and the number of choices they have . Fitts 's law predicts that the time required to rapidly move to a target area is a logarithmic function of the distance to and the size of the target . In psychophysics , the Weber ? Fechner law proposes a logarithmic relationship between stimulus and sensation such as the actual vs. the perceived weight of an item a person is carrying . (This " law " , however , is less precise than more recent models , such as the Stevens ' power law .)
Psychological studies found that individuals with little mathematics education tend to estimate quantities logarithmically , that is , they position a number on an unmarked line according to its logarithm , so that 10 is positioned as close to 100 as 100 is to 1000 . Increasing education shifts this to a linear estimate (positioning 1000 10x as far away) in some circumstances , while logarithms are used when the numbers to be plotted are difficult to plot linearly .

= = = Probability theory and statistics = = =

Logarithms arise in probability theory : the law of large numbers dictates that , for a fair coin , as the number of coin @-@ tosses increases to infinity , the observed proportion of heads approaches one @-@ half . The fluctuations of this proportion about one @-@ half are described by the law of the iterated logarithm .
Logarithms also occur in log @-@ normal distributions . When the logarithm of a random variable has a normal distribution , the variable is said to have a log @-@ normal distribution . Log @-@ normal distributions are encountered in many fields , wherever a variable is formed as the product of many independent positive random variables , for example in the study of turbulence .
Logarithms are used for maximum @-@ likelihood estimation of parametric statistical models . For such a model , the likelihood function depends on at least one parameter that must be estimated . A maximum of the likelihood function occurs at the same parameter @-@ value as a maximum of the logarithm of the likelihood (the " log likelihood ") , because the logarithm is an increasing function . The log @-@ likelihood is easier to maximize , especially for the multiplied likelihoods for independent random variables .
Benford 's law describes the occurrence of digits in many data sets , such as heights of buildings . According to Benford 's law , the probability that the first decimal @-@ digit of an item in the data sample is d (from 1 to 9) equals log10 (d + 1) ? log10 (d) , regardless of the unit of measurement . Thus , about 30 % of the data can be expected to have 1 as first digit , 18 % start with 2 , etc . Auditors examine deviations from Benford 's law to detect fraudulent accounting .

= = = Computational complexity = = =

Analysis of algorithms is a branch of computer science that studies the performance of algorithms (computer programs solving a certain problem) . Logarithms are valuable for describing algorithms that divide a problem into smaller ones , and join the solutions of the subproblems .
For example , to find a number in a sorted list , the binary search algorithm checks the middle entry and proceeds with the half before or after the middle entry if the number is still not found . This algorithm requires , on average , log2 (N) comparisons , where N is the list 's length . Similarly , the merge sort algorithm sorts an unsorted list by dividing the list into halves and sorting these first before merging the results . Merge sort algorithms typically require a time approximately proportional to N · log (N) . The base of the logarithm is not specified here , because the result only changes by

a constant factor when another base is used . A constant factor is usually disregarded in the analysis of algorithms under the standard uniform cost model .
A function f (x) is said to grow logarithmically if f (x) is (exactly or approximately) proportional to the logarithm of x . (Biological descriptions of organism growth , however , use this term for an exponential function .) For example , any natural number N can be represented in binary form in no more than log2 (N) + 1 bits . In other words , the amount of memory needed to store N grows logarithmically with N.

= = = Entropy and chaos = = =

Entropy is broadly a measure of the disorder of some system . In statistical thermodynamics , the entropy S of some physical system is defined as
<formula>
The sum is over all possible states i of the system in question , such as the positions of gas particles in a container . Moreover , pi is the probability that the state i is attained and k is the Boltzmann constant . Similarly , entropy in information theory measures the quantity of information . If a message recipient may expect any one of N possible messages with equal likelihood , then the amount of information conveyed by any one such message is quantified as log2 (N) bits .
Lyapunov exponents use logarithms to gauge the degree of chaoticity of a dynamical system . For example , for a particle moving on an oval billiard table , even small changes of the initial conditions result in very different paths of the particle . Such systems are chaotic in a deterministic way , because small measurement errors of the initial state predictably lead to largely different final states . At least one Lyapunov exponent of a deterministically chaotic system is positive .

= = = Fractals = = =

Logarithms occur in definitions of the dimension of fractals . Fractals are geometric objects that are self @-@ similar : small parts reproduce , at least roughly , the entire global structure . The Sierpinski triangle (pictured) can be covered by three copies of itself , each having sides half the original length . This makes the Hausdorff dimension of this structure ln (3) / ln (2) ? 1 @.@ 58 . Another logarithm @-@ based notion of dimension is obtained by counting the number of boxes needed to cover the fractal in question .

= = = Music = = =

Logarithms are related to musical tones and intervals . In equal temperament , the frequency ratio depends only on the interval between two tones , not on the specific frequency , or pitch , of the individual tones . For example , the note A has a frequency of 440 Hz and B @-@ flat has a frequency of 466 Hz . The interval between A and B @-@ flat is a semitone , as is the one between B @-@ flat and B (frequency 493 Hz) . Accordingly , the frequency ratios agree :
<formula>
Therefore , logarithms can be used to describe the intervals : an interval is measured in semitones by taking the base @-@ 21 / 12 logarithm of the frequency ratio , while the base @-@ 21 / 1200 logarithm of the frequency ratio expresses the interval in cents , hundredths of a semitone . The latter is used for finer encoding , as it is needed for non @-@ equal temperaments .

= = = Number theory = = =

Natural logarithms are closely linked to counting prime numbers (2 , 3 , 5 , 7 , 11 , ...) , an important topic in number theory . For any integer x , the quantity of prime numbers less than or equal to x is denoted ? (x) . The prime number theorem asserts that ? (x) is approximately given by
<formula>

in the sense that the ratio of ? (x) and that fraction approaches 1 when x tends to infinity . As a consequence , the probability that a randomly chosen number between 1 and x is prime is inversely proportional to the number of decimal digits of x . A far better estimate of ? (x) is given by the offset logarithmic integral function Li (x) , defined by
<formula>
The Riemann hypothesis , one of the oldest open mathematical conjectures , can be stated in terms of comparing ? (x) and Li (x) . The Erd?s ? Kac theorem describing the number of distinct prime factors also involves the natural logarithm .
The logarithm of n factorial , n ! = 1 · 2 · ... · n , is given by
<formula>
This can be used to obtain Stirling 's formula , an approximation of n ! for large n .

= = Generalizations = =

= = = Complex logarithm = = =

The complex numbers a solving the equation
<formula>