Numerically controlled oscillator

A numerically controlled oscillator (NCO) is a digital signal generator which creates a synchronous (i.e. clocked) , discrete @-@ time , discrete @-@ valued representation of a waveform , usually sinusoidal . NCOs are often used in conjunction with a digital @-@ to @-@ analog converter (DAC) at the output to create a direct digital synthesizer (DDS) .

Numerically controlled oscillators offer several advantages over other types of oscillators in terms of agility , accuracy , stability and reliability . NCOs are used in many communications systems including digital up / down converters used in 3G wireless and software radio systems , digital PLLs , radar systems , drivers for optical or acoustic transmissions , and multilevel FSK / PSK modulators / demodulators .

Operation

An NCO generally consists of two parts :

A phase accumulator (PA) , which adds to the value held at its output a frequency control value at each clock sample .

A phase @-@ to @-@ amplitude converter (PAC) , which uses the phase accumulator output word (phase word) usually as an index into a waveform look @-@ up table (LUT) to provide a corresponding amplitude sample . Sometimes interpolation is used with the look @-@ up table to provide better accuracy and reduce phase error noise . Other methods of converting phase to amplitude , including mathematical algorithms such as power series can be used , particularly in a software NCO .

When clocked , the phase accumulator (PA) creates a modulo @-@ 2N sawtooth waveform which is then converted by the phase @-@ to @-@ amplitude converter (PAC) to a sampled sinusoid , where N is the number of bits carried in the phase accumulator . N sets the NCO frequency resolution and is normally much larger than the number of bits defining the memory space of the PAC look @-@ up table . If the PAC capacity is 2M , the PA output word must be truncated to M bits as shown in Figure 1 . However , the truncated bits can be used for interpolation . The truncation of the phase output word does not affect the frequency accuracy but produces a time @-@ varying periodic phase error which is a primary source of spurious products . Another spurious product generation mechanism is finite word length effects of the PAC output (amplitude) word .

The frequency accuracy relative to the clock frequency is limited only by the precision of the arithmetic used to compute the phase . NCOs are phase- and frequency @-@ agile , and can be trivially modified to produce a phase @-@ modulated or frequency @-@ modulated output by summation at the appropriate node , or provide quadrature outputs as shown in the figure .

Phase accumulator

A binary phase accumulator consists of an N @-@ bit binary adder and a register configured as shown in Figure 1 . Each clock cycle produces a new N @-@ bit output consisting of the previous output obtained from the register summed with the frequency control word (FCW) which is constant for a given output frequency . The resulting output waveform is a staircase with step size <formula> , the integer value of the FCW . In some configurations , the phase output is taken from the output of the register which introduces a one clock cycle latency but allows the adder to operate at a higher clock rate .

The adder is designed to overflow when the sum of the absolute value of its operands exceeds its capacity (2N ? 1) . The overflow bit is discarded so the output word width is always equal to its input word width . The remainder <formula> , called the residual , is stored in the register and the cycle repeats , starting this time from <formula> (see figure 2) . Since a phase accumulator is a finite state machine , eventually the residual at some sample K must return to the initial value <formula> . The interval K is referred to as the grand repetition rate (GRR) given by

<formula>

where GCD is the greatest common divisor function . The GRR represents the true periodicity for a given <formula> which for a high resolution NCO can be very long . Usually we are more interested in the operating frequency determined by the average overflow rate , given by

<formula> (1)

The frequency resolution , defined as the smallest possible incremental change in frequency , is given by

<formula> (2)

Equation (1) shows that the phase accumulator can be thought of as a programmable non @-@ integer frequency divider of divide ratio <formula> .

= = Phase @-@ to @-@ amplitude converter = =

The phase @-@ amplitude converter creates the sample @-@ domain waveform from the truncated phase output word received from the PA . The PAC can be a simple read only memory containing 2M contiguous samples of the desired output waveform which typically is a sinusoid . Often though , various tricks are employed to reduce the amount of memory required . This include various trigonometric expansions , trigonometric approximations and methods which take advantage of the quadrature symmetry exhibited by sinusoids . Alternatively , the PAC may consist of random access memory which can be filled as desired to create an arbitrary waveform generator .

= = Spurious products = =

Spurious products are the result of harmonic or non @-@ harmonic distortion in the creation of the output waveform due to non @-@ linear numerical effects in the signal processing chain . Only numerical errors are covered here . For other distortion mechanisms created in the digital @-@ to @-@ analog converter see the corresponding section in the direct @-@ digital synthesizer article .

= = = Phase truncation spurs = = =

The number of phase accumulator bits of an NCO (N) is usually between 16 and 64 . If the PA output word were used directly to index the PAC look @-@ up table an untenably high storage capacity in the ROM would be required . As such , the PA output word must be truncated to span a reasonable memory space . Truncation of the phase word causes phase modulation of the output sinusoid which introduces non @-@ harmonic distortion in proportion to the number of bits truncated . The number of spurious products created by this distortion is given by :

<formula> (3)

where W is the number of bits truncated .

In calculating the Spurious @-@ free dynamic range , we are interested in the spurious product with the largest amplitude relative to the carrier output level given by :

<formula>

where P is word width of the DAC . For W > 4 @,@

<formula>

Another related spurious generation method is the slight modulation due to the GRR outlined above . The amplitude of these spurs is low for large N and their frequency is generally too low to be detectable but they may cause issues for some applications .

= = = Amplitude truncation spurs = = =

Another source of spurious products is the amplitude quantization of the sampled waveform contained in the PAC look up table (s) . If the number of DAC bits is P , the AM spur level is approximately equal to ? 6 @.@ 02 P ? 1 @.@ 76 dBc .

= = = Mitigation techniques = = =

Phase truncation spurs can be reduced substantially by the introduction of white gaussian noise prior to truncation . The so @-@ called dither noise is summed into the lower W + 1 bits of the PA output word to linearize the truncation operation . Often the improvement can be achieved without penalty because the DAC noise floor tends to dominate system performance . Amplitude truncation spurs can not be mitigated in this fashion . Introduction of noise into the static values held in the PAC ROMs would not eliminate the cyclicality of the trunction error terms and thus would not achieve the desired effect .