= 1 / e for any n < 0 ; and the global minimum occurs at x = e ? 1 / n for any n > 0 .
The infinite tetration
<formula> or <formula>
converges if and only if e ? e ? x ? e1 / e (or approximately between 0 @.@ 0660 and 1 @.@ 4447) , due to a theorem of Leonhard Euler .

= = = Number theory = = =

The real number e is irrational . Euler proved this by showing that its simple continued fraction expansion is infinite . (See also Fourier 's proof that e is irrational .)
Furthermore , by the Lindemann ? Weierstrass theorem , e is transcendental , meaning that it is not a solution of any non @-@ constant polynomial equation with rational coefficients . It was the first number to be proved transcendental without having been specifically constructed for this purpose (compare with Liouville number) ; the proof was given by Charles Hermite in 1873 .
It is conjectured that e is normal , meaning that when e is expressed in any base the possible digits in that base are uniformly distributed (occur with equal probability in any sequence of given length) .

= = = Complex numbers = = =

The exponential function ex may be written as a Taylor series
<formula>
Because this series keeps many important properties for ex even when x is complex , it is commonly used to extend the definition of ex to the complex numbers . This , with the Taylor series for sin and cos x , allows one to derive Euler 's formula :
<formula>
which holds for all x . The special case with x = ? is Euler 's identity :
<formula>
from which it follows that , in the principal branch of the logarithm ,
<formula>
Furthermore , using the laws for exponentiation ,
<formula>
which is de Moivre 's formula .
The expression
<formula>
is sometimes referred to as cis (x) .

= = = Differential equations = = =

The general function
<formula>
is the solution to the differential equation :
<formula>

= = Representations = =

The number e can be represented as a real number in a variety of ways : as an infinite series , an infinite product , a continued fraction , or a limit of a sequence . The chief among these representations , particularly in introductory calculus courses is the limit
<formula>
given above , as well as the series
<formula>

given by evaluating the above power series for ex at x = 1 .
Less common is the continued fraction (sequence A003417 in the OEIS) .
<formula>
which written out looks like
<formula>
This continued fraction for e converges three times as quickly :
<formula>
which written out looks like
<formula>
Many other series , sequence , continued fraction , and infinite product representations of e have been developed .

= = = Stochastic representations = = =

In addition to exact analytical expressions for representation of e , there are stochastic techniques for estimating e . One such approach begins with an infinite sequence of independent random variables X1 , X2 ... , drawn from the uniform distribution on [0 , 1] . Let V be the least number n such that the sum of the first n observations exceeds 1 :
<formula>
Then the expected value of V is e : E (V) = e .

= = = Known digits = = =

The number of known digits of e has increased substantially during the last decades . This is due both to the increased performance of computers and to algorithmic improvements .
Since that time , the proliferation of modern high @-@ speed desktop computers has made it possible for amateurs to compute billions of digits of e .

= = In computer culture = =

In contemporary internet culture , individuals and organizations frequently pay homage to the number e .
For instance , in the IPO filing for Google in 2004 , rather than a typical round @-@ number amount of money , the company announced its intention to raise $ 2 @,@ 718 @,@ 281 @,@ 828 , which is e billion dollars rounded to the nearest dollar . Google was also responsible for a billboard that appeared in the heart of Silicon Valley , and later in Cambridge , Massachusetts ; Seattle , Washington ; and Austin , Texas . It read " { first 10 @-@ digit prime found in consecutive digits of e } .com " . Solving this problem and visiting the advertised (now defunct) web site led to an even more difficult problem to solve , which in turn led to Google Labs where the visitor was invited to submit a resume . The first 10 @-@ digit prime in e is 7427466391 , which starts at the 99th digit .
In another instance , the computer scientist Donald Knuth let the version numbers of his program Metafont approach e . The versions are 2 , 2 @.@ 7 , 2 @.@ 71 , 2 @.@ 718 , and so forth .