= 1 if 0 ? A , and ? ( A ) =
0 otherwise . If the delta function is conceptualized as modeling an idealized point mass at 0 , then ? ( A ) represents the mass contained in the set A. One may then define the integral against ? as the integral of a function against this mass distribution . Formally , the Lebesgue integral provides the necessary analytic device . The Lebesgue integral with respect to the measure ? satisfies
<formula>
for all continuous compactly supported functions f . The measure ? is not absolutely continuous with respect to the Lebesgue measure ? in fact , it is a singular measure . Consequently , the delta measure has no Radon ? Nikodym derivative ? no true function for which the property
<formula>
holds . As a result , the latter notation is a convenient abuse of notation , and not a standard ( Riemann or Lebesgue ) integral .
As a probability measure on R , the delta measure is characterized by its cumulative distribution function , which is the unit step function
<formula>
This means that H ( x ) is the integral of the cumulative indicator function 1 ( ? ? , x ] with respect to the measure ? ; to wit ,
<formula>
Thus in particular the integral of the delta function against a continuous function can be properly understood as a Stieltjes integral :
<formula>
All higher moments of ? are zero . In particular , characteristic function and moment generating function are both equal to one .

= = = As a distribution = = =

In the theory of distributions a generalized function is thought of not as a function itself , but only in relation to how it affects other functions when it is " integrated " against them . In keeping with this philosophy , to define the delta function properly , it is enough to say what the " integral " of the delta function against a sufficiently " good " test function is . If the delta function is already understood as a measure , then the Lebesgue integral of a test function against that measure supplies the necessary integral .
A typical space of test functions consists of all smooth functions on R with compact support . As a distribution , the Dirac delta is a linear functional on the space of test functions and is defined by
for every test function ? .
For ? to be properly a distribution , it must be continuous in a suitable topology on the space of test functions . In general , for a linear functional S on the space of test functions to define a distribution , it is necessary and sufficient that , for every positive integer N there is an integer MN and a constant CN such that for every test function ? , one has the inequality
<formula>