

$= 2$, $f'(2) = 4$, and so on .

\dots Higher derivatives \dots

Let f be a differentiable function , and let $f'(x)$ be its derivative . The derivative of $f'(x)$ (if it has one) is written $f''(x)$ and is called the second derivative of f . Similarly , the derivative of a second derivative , if it exists , is written $f'''(x)$ and is called the third derivative of f . Continuing this process , one can define , if it exists , the n th derivative as the derivative of the $(n-1)$ th derivative . These repeated derivatives are called higher order derivatives . The n th derivative is also called the derivative of order n .

If $x(t)$ represents the position of an object at time t , then the higher order derivatives of x have physical interpretations . The second derivative of x is the derivative of $x'(t)$, the velocity , and by definition this is the object 's acceleration . The third derivative of x is defined to be the jerk , and the fourth derivative is defined to be the jounce .

A function f need not have a derivative , for example , if it is not continuous . Similarly , even if f does have a derivative , it may not have a second derivative . For example , let

$f(x) = \begin{cases} x^2 \sin(1/x) & x \neq 0 \\ 0 & x = 0 \end{cases}$

Calculation shows that f is a differentiable function whose derivative is

$f'(x) = 2x \sin(1/x) - \cos(1/x)$

$f'(x)$ is twice the absolute value function , and it does not have a derivative at zero . Similar examples show that a function can have k derivatives for any non negative integer k but no $(k+1)$ th order derivative . A function that has k successive derivatives is called k times differentiable . If in addition the k th derivative is continuous , then the function is said to be of differentiability class C^k . (This is a stronger condition than having k derivatives . For an example , see differentiability class .) A function that has infinitely many derivatives is called infinitely differentiable or smooth .

On the real line , every polynomial function is infinitely differentiable . By standard differentiation rules , if a polynomial of degree n is differentiated n times , then it becomes a constant function . All of its subsequent derivatives are identically zero . In particular , they exist , so polynomials are smooth functions .

The derivatives of a function f at a point x provide polynomial approximations to that function near x . For example , if f is twice differentiable , then

$f(x+h) \approx f(x) + f'(x)h + \frac{1}{2}f''(x)h^2$

in the sense that

$\frac{f(x+h) - [f(x) + f'(x)h + \frac{1}{2}f''(x)h^2]}{h^3} \rightarrow 0$

If f is infinitely differentiable , then this is the beginning of the Taylor series for f evaluated at $x+h$ around x .

\dots Inflection point \dots