= x1 / 3 is not differentiable at x =
0 .

In summary : for a function f to have a derivative it is necessary for the function f to be continuous , but continuity alone is not sufficient .

Most functions that occur in practice have derivatives at all points or at almost every point . Early in the history of calculus , many mathematicians assumed that a continuous function was differentiable at most points . Under mild conditions , for example if the function is a monotone function or a Lipschitz function , this is true . However , in 1872 Weierstrass found the first example of a function that is continuous everywhere but differentiable nowhere . This example is now known as the Weierstrass function . In 1931 , Stefan Banach proved that the set of functions that have a derivative at some point is a meager set in the space of all continuous functions . Informally , this means that hardly any continuous functions have a derivative at even one point .

= = = The derivative as a function = = =

Let f be a function that has a derivative at every point a in the domain of f . Because every point a has a derivative , there is a function that sends the point a to the derivative of f at a . This function is written f ? (x) and is called the derivative function or the derivative of f . The derivative of f collects all the derivatives of f at all the points in the domain of f .

Sometimes f has a derivative at most , but not all , points of its domain . The function whose value at a equals f ? (a) whenever f ? (a) is defined and elsewhere is undefined is also called the derivative of f . It is still a function , but its domain is strictly smaller than the domain of f .