

$\log_b 1 = 0$, since $b^0 = 1$,
and $\log_b b = 1$, respectively.

Logarithmic identities

Several important formulas, sometimes called logarithmic identities or logarithmic laws, relate logarithms to one another.

Product, quotient, power and root

The logarithm of a product is the sum of the logarithms of the numbers being multiplied; the logarithm of the ratio of two numbers is the difference of the logarithms. The logarithm of the p th power of a number is p times the logarithm of the number itself; the logarithm of a p th root is the logarithm of the number divided by p . The following table lists these identities with examples. Each of the identities can be derived after substitution of the logarithm definitions $\log_b x = y \iff b^y = x$ or $x = b^y$ in the left hand sides.

Change of base

The logarithm $\log_b(x)$ can be computed from the logarithms of x and b with respect to an arbitrary base k using the following formula:

$$\log_b(x) = \frac{\log_k(x)}{\log_k(b)}$$

Typical scientific calculators calculate the logarithms to bases 10 and e . Logarithms with respect to any base b can be determined using either of these two logarithms by the previous formula:

$$\log_b(x) = \frac{\log_{10}(x)}{\log_{10}(b)}$$

Given a number x and its logarithm $\log_b(x)$ to an unknown base b , the base is given by:

$$b = x^{\frac{1}{\log_b(x)}}$$

Particular bases