= 1 , the extent to which they explain the equation depends on the audience . In introductory arithmetic , such proofs help explain why 0 @.@ 999 ? =
1 but 0 @.@ 333 ? < 0 @.@ 34 . In introductory algebra , the proofs help explain why the general method of converting between fractions and repeating decimals works . But the proofs shed little light on the fundamental relationship between decimals and the numbers they represent , which underlies the question of how two different decimals can be said to be equal at all .
Once a representation scheme is defined , it can be used to justify the rules of decimal arithmetic used in the above proofs . Moreover , one can directly demonstrate that the decimals 0 @.@ 999 ? and 1 @.@ 000 ? both represent the same real number ; it is built into the definition . This is done below .

= = Analytic proofs = =

Since the question of 0 @.@ 999 ? does not affect the formal development of mathematics , it can be postponed until one proves the standard theorems of real analysis . One requirement is to characterize real numbers that can be written in decimal notation , consisting of an optional sign , a finite sequence of one or more digits forming an integer part , a decimal separator , and a sequence of digits forming a fractional part . For the purpose of discussing 0 @.@ 999 ? , the integer part can be summarized as b0 and one can neglect negatives , so a decimal expansion has the form
<formula>
It should be noted that the fraction part , unlike the integer part , is not limited to a finite number of digits . This is a positional notation , so for example the digit 5 in 500 contributes ten times as much as the 5 in 50 , and the 5 in 0 @.@ 05 contributes one tenth as much as the 5 in 0 @.@ 5 .

= = = Infinite series and sequences = = =

Perhaps the most common development of decimal expansions is to define them as sums of infinite series . In general :
<formula>
For 0 @.@ 999 ? one can apply the convergence theorem concerning geometric series :
If <formula> then <formula>