= 0) , the remainders r ? 2 and r ? 1 equal a and b , the numbers for which the GCD is sought . In the next step (k =
1) , the remainders equal b and the remainder r0 of the initial step , and so on . Thus , the algorithm can be written as a sequence of equations
<formula>
If a is smaller than b , the first step of the algorithm swaps the numbers . For example , if a < b , the initial quotient q0 equals zero , and the remainder r0 is a . Thus , rk is smaller than its predecessor rk ? 1 for all k ? 0 .
Since the remainders decrease with every step but can never be negative , a remainder rN must eventually equal zero , at which point the algorithm stops . The final nonzero remainder rN ? 1 is the greatest common divisor of a and b . The number N cannot be infinite because there are only a finite number of nonnegative integers between the initial remainder r0 and zero .

= = = Proof of validity = = =