

$= Cr$, such as Hooke 's law , and inverse r^{-2} square forces , $F =$

C / r^2 , such as Newton 's law of universal gravitation and Coulomb 's law ? have a very unusual property . A particle moving under either type of force always returns to its starting place with its initial velocity , provided that it lacks sufficient energy to move out to infinity . In other words , the path of a bound particle is always closed and its motion repeats indefinitely , no matter what its initial position or velocity . As shown by Bertrand 's theorem , this property is not true for other types of forces ; in general , a particle will not return to its starting point with the same velocity .

However , Newton 's theorem shows that an inverse r^{-3} cubic force may be applied to a particle moving under a linear or inverse r^{-2} square force such that its orbit remains closed , provided that k equals a rational number . (A number is called " rational " if it can be written as a fraction m / n , where m and n are integers .) In such cases , the addition of the inverse r^{-3} cubic force causes the particle to complete m rotations about the center of force in the same time that the original particle completes n rotations . This method for producing closed orbits does not violate Bertrand 's theorem , because the added inverse r^{-3} cubic force depends on the initial velocity of the particle .