= g for any g in G and h in H. From an abstract point of view , isomorphic groups carry the same information . For example , proving that g ? g =
1G for some element g of G is equivalent to proving that a (g) ? a (g) = 1H , because applying a to the first equality yields the second , and applying b to the second gives back the first .

= = = Subgroups = = =

Informally , a subgroup is a group H contained within a bigger one , G. Concretely , the identity element of G is contained in H , and whenever h1 and h2 are in H , then so are h1 ? h2 and h1 ? 1 , so the elements of H , equipped with the group operation on G restricted to H , indeed form a group .
In the example above , the identity and the rotations constitute a subgroup R
= { id , r1 , r2 , r3 } , highlighted in red in the group table above : any two rotations composed are still a rotation , and a rotation can be undone by (i.e. is inverse to) the complementary rotations 270 ° for 90 ° , 180 ° for 180 ° , and 90 ° for 270 ° (note that rotation in the opposite direction is not defined) . The subgroup test is a necessary and sufficient condition for a nonempty subset H of a group G to be a subgroup : it is sufficient to check that g ? 1h ? H for all elements g , h ? H. Knowing the subgroups is important in understanding the group as a whole .
Given any subset S of a group G , the subgroup generated by S consists of products of elements of S and their inverses . It is the smallest subgroup of G containing S. In the introductory example above , the subgroup generated by r2 and fv consists of these two elements , the identity element id and fh =
fv ? r2 . Again , this is a subgroup , because combining any two of these four elements or their inverses (which are , in this particular case , these same elements) yields an element of this subgroup .

= = = Cosets = = =

In many situations it is desirable to consider two group elements the same if they differ by an element of a given subgroup . For example , in D4 above , once a reflection is performed , the square never gets back to the r2 configuration by just applying the rotation operations (and no further reflections) , i.e. the rotation operations are irrelevant to the question whether a reflection has been performed . Cosets are used to formalize this insight : a subgroup H defines left and right cosets , which can be thought of as translations of H by arbitrary group elements g . In symbolic terms , the left and right cosets of H containing g are
gH
= { g ? h : h ? H } and Hg =
{ h ? g : h ? H } , respectively .