

$= a ( \cdot )$  and  $b ( \cdot + \cdot ) =$

$\cdot b ( \cdot )$  ( so that the flow is invariant under point reflection ) , the resulting flow can be shown to obey the avoidance principle and an analog of the Gage ? Hamilton ? Grayson theorem .

The affine curve @-@ shortening flow was first investigated by Alvarez et al . ( 1993 ) and Sapiro & Tannenbaum ( 1993 ) . In this flow , the normal speed of the curve is proportional to the cube root of the curvature . The resulting flow is invariant ( with a corresponding time scaling ) under the affine transformations of the Euclidean plane , a larger symmetry group than the similarity transformations under which the curve @-@ shortening flow is invariant . Under this flow , an analogue of the Gage ? Hamilton ? Grayson theorem applies , under which any simple closed curve eventually becomes convex and then converges to an ellipse as it collapses to a point .

Transforming a curve with equal normal speeds at all points has been called the grassfire transform . Curves evolved in this way will in general develop sharp corners , the trace of which forms the medial axis of the curve . A closely related curve evolution which moves straight segments of a polygonal curve at equal speeds but allows concave corners to move more quickly than unit speed instead forms a different type of topological skeleton of the given curve , its straight skeleton .

For surfaces in higher dimensions , there is more than one definition of curvature , including extrinsic ( embedding @-@ dependent ) measures such as the mean curvature and intrinsic measures such as the Gaussian curvature and Ricci curvature . Correspondingly , there are several ways of defining geometric flows based on curvature , including the mean curvature flow ( in which the normal speed of an embedded surface is its mean curvature ) , the Ricci flow ( an intrinsic flow on the metric of a space based on its Ricci curvature ) and the Willmore flow ( the gradient flow for an energy functional combining the mean curvature and Gaussian curvature ) . The curve @-@ shortening flow is a special case of the mean curvature flow for one @-@ dimensional curves .

Inspired by the curve @-@ shortening flow on smooth curves , researchers have studied methods for flowing polygons so that they stay polygonal , with applications including pattern formation and synchronization in distributed systems of robots . Length @-@ preserving polygonal flows can be used to solve the carpenter 's rule problem .

In computer vision , the active contour model for edge detection and image segmentation is based on curve shortening , and evolves curves based on a combination of their curvature and the features of an image .