

$x = r \cos \theta$ and $y =$

$r \sin \theta$, one can derive a relationship between derivatives in Cartesian and polar coordinates. For a given function, $u(x, y)$, it follows that (by computing its total derivatives)

$\frac{du}{dr} =$

$\frac{du}{d\theta} =$

or

$\frac{du}{dr} =$

$\frac{du}{d\theta} =$

Hence, we have the following formulae:

$\frac{du}{dr} =$

$\frac{du}{d\theta} =$

Using the inverse coordinates transformation, an analogous reciprocal relationship can be derived between the derivatives. Given a function $u(r, \theta)$, it follows that

$\frac{du}{dr} =$

$\frac{du}{d\theta} =$

or

$\frac{du}{dr} =$

$\frac{du}{d\theta} =$

Hence, we have the following formulae:

$\frac{du}{dr} =$

$\frac{du}{d\theta} =$

To find the Cartesian slope of the tangent line to a polar curve $r(\theta)$ at any given point, the curve is first expressed as a system of parametric equations.

$x = r(\theta) \cos \theta$

$y = r(\theta) \sin \theta$

Differentiating both equations with respect to θ yields

$\frac{dx}{d\theta} =$

$\frac{dy}{d\theta} =$

Dividing the second equation by the first yields the Cartesian slope of the tangent line to the curve at the point $(r(\theta), \theta)$:

$\frac{dy}{dx} =$

For other useful formulas including divergence, gradient, and Laplacian in polar coordinates, see curvilinear coordinates.

Integral calculus (arc length) =