

= Vector space =

A vector space (also called a linear space) is a collection of objects called vectors , which may be added together and multiplied (" scaled ") by numbers , called scalars in this context . Scalars are often taken to be real numbers , but there are also vector spaces with scalar multiplication by complex numbers , rational numbers , or generally any field . The operations of vector addition and scalar multiplication must satisfy certain requirements , called axioms , listed below .

Euclidean vectors are an example of a vector space . They represent physical quantities such as forces : any two forces (of the same type) can be added to yield a third , and the multiplication of a force vector by a real multiplier is another force vector . In the same vein , but in a more geometric sense , vectors representing displacements in the plane or in three @-@ dimensional space also form vector spaces . Vectors in vector spaces do not necessarily have to be arrow @-@ like objects as they appear in the mentioned examples : vectors are regarded as abstract mathematical objects with particular properties , which in some cases can be visualized as arrows .

Vector spaces are the subject of linear algebra and are well characterized by their dimension , which , roughly speaking , specifies the number of independent directions in the space . Infinite @-@ dimensional vector spaces arise naturally in mathematical analysis , as function spaces , whose vectors are functions . These vector spaces are generally endowed with additional structure , which may be a topology , allowing the consideration of issues of proximity and continuity . Among these topologies , those that are defined by a norm or inner product are more commonly used , as having a notion of distance between two vectors . This is particularly the case of Banach spaces and Hilbert spaces , which are fundamental in mathematical analysis .

Historically , the first ideas leading to vector spaces can be traced back as far as the 17th century 's analytic geometry , matrices , systems of linear equations , and Euclidean vectors . The modern , more abstract treatment , first formulated by Giuseppe Peano in 1888 , encompasses more general objects than Euclidean space , but much of the theory can be seen as an extension of classical geometric ideas like lines , planes and their higher @-@ dimensional analogs .

Today , vector spaces are applied throughout mathematics , science and engineering . They are the appropriate linear @-@ algebraic notion to deal with systems of linear equations ; offer a framework for Fourier expansion , which is employed in image compression routines ; or provide an environment that can be used for solution techniques for partial differential equations . Furthermore , vector spaces furnish an abstract , coordinate @-@ free way of dealing with geometrical and physical objects such as tensors . This in turn allows the examination of local properties of manifolds by linearization techniques . Vector spaces may be generalized in several ways , leading to more advanced notions in geometry and abstract algebra .

= = Introduction and definition = =

The concept of vector space will first be explained by describing two particular examples :

= = First example : arrows in the plane = =

The first example of a vector space consists of arrows in a fixed plane , starting at one fixed point . This is used in physics to describe forces or velocities . Given any two such arrows , v and w , the parallelogram spanned by these two arrows contains one diagonal arrow that starts at the origin , too . This new arrow is called the sum of the two arrows and is denoted $v + w$. In the special case of two arrows on the same line , their sum is the arrow on this line whose length is the sum or the difference of the lengths , depending on whether the arrows have the same direction . Another operation that can be done with arrows is scaling : given any positive real number a , the arrow that has the same direction as v , but is dilated or shrunk by multiplying its length by a , is called multiplication of v by a . It is denoted av . When a is negative , av is defined as the arrow pointing in the opposite direction , instead .