= 1 ? 2 + 3 ? 4 + ? =

In mathematics , 1 ? 2 + 3 ? 4 + · · · is the infinite series whose terms are the successive positive integers , given alternating signs . Using sigma summation notation the sum of the first m terms of the series can be expressed as
<formula>
The infinite series diverges , meaning that its sequence of partial sums , (1 , ? 1 , 2 , ? 2 , ...) , does not tend towards any finite limit . Nonetheless , in the mid @-@ 18th century , Leonhard Euler wrote what he admitted to be a paradoxical equation :
<formula>
A rigorous explanation of this equation would not arrive until much later . Starting in 1890 , Ernesto Cesàro , Émile Borel and others investigated well @-@ defined methods to assign generalized sums to divergent series ? including new interpretations of Euler 's attempts . Many of these summability methods easily assign to 1 ? 2 + 3 ? 4 + ... a " sum " of 1 ? 4 after all . Cesàro summation is one of the few methods that do not sum 1 ? 2 + 3 ? 4 + ... , so the series is an example where a slightly stronger method , such as Abel summation , is required .
The series 1 ? 2 + 3 ? 4 + ... is closely related to Grandi 's series 1 ? 1 + 1 ? 1 + ... Euler treated these two as special cases of 1 ? 2n + 3n ? 4n + ... for arbitrary n , a line of research extending his work on the Basel problem and leading towards the functional equations of what are now known as the Dirichlet eta function and the Riemann zeta function .

= = Explanation of the paradox = =

In mathematics , if a set of rules is consistent with itself , then one can work with those rules . According to the definitions of " sum " and " equals " that most people are used to , it makes no sense to say that 1 ? 2 + 3 ? 4 + ... equals anything . However , there are other , somewhat more generous , ways of defining " sum " and " equals " that don 't contradict our ordinary , finite arithmetic , but which produce some additional surprising results with infinite sums . One way to see how that could possibly work is if the series (1 ? 2 + 3 ? 4 + ...) is added to itself four times in just the right way , causing all the positive terms and all the negative terms to cancel out , except for one of the initial 1 's . Thus , as four copies of the series add up to 1 , the series itself would equal 1 / 4 .
1 - 2 + 3 - 4 + 5 - 6 + ...
+ 1 - 2 + 3 - 4 + 5 - . . . . .
+ 1 - 2 + 3 - 4 + 5 - . . . . . .
+ 1 - 2 + 3 - 4 + ...