= Matrix (mathematics) =

In mathematics , a matrix (plural matrices) is a rectangular array of numbers , symbols , or expressions , arranged in rows and columns . The dimensions of matrix (1) are 2 × 3 (read " two by three ") , because there are two rows and three columns .

The individual items in a matrix are called its elements or entries . Provided that they are the same size (have the same number of rows and the same number of columns) , two matrices can be added or subtracted element by element . The rule for matrix multiplication , however , is that two matrices can be multiplied only when the number of columns in the first equals the number of rows in the second . Any matrix can be multiplied element @-@ wise by a scalar from its associated field . A major application of matrices is to represent linear transformations , that is , generalizations of linear functions such as f (x) = 4x . For example , the rotation of vectors in three dimensional space is a linear transformation which can be represented by a rotation matrix R : if v is a column vector (a matrix with only one column) describing the position of a point in space , the product Rv is a column vector describing the position of that point after a rotation . The product of two transformation matrices is a matrix that represents the composition of two linear transformations . Another application of matrices is in the solution of systems of linear equations . If the matrix is square , it is possible to deduce some of its properties by computing its determinant . For example , a square matrix has an inverse if and only if its determinant is not zero . Insight into the geometry of a linear transformation is obtainable (along with other information) from the matrix 's eigenvalues and eigenvectors .

Applications of matrices are found in most scientific fields . In every branch of physics , including classical mechanics , optics , electromagnetism , quantum mechanics , and quantum electrodynamics , they are used to study physical phenomena , such as the motion of rigid bodies . In computer graphics , they are used to project a 3D model onto a 2 dimensional screen . In probability theory and statistics , stochastic matrices are used to describe sets of probabilities ; for instance , they are used within the PageRank algorithm that ranks the pages in a Google search . Matrix calculus generalizes classical analytical notions such as derivatives and exponentials to higher dimensions .

A major branch of numerical analysis is devoted to the development of efficient algorithms for matrix computations , a subject that is centuries old and is today an expanding area of research . Matrix decomposition methods simplify computations , both theoretically and practically . Algorithms that are tailored to particular matrix structures , such as sparse matrices and near @-@ diagonal matrices , expedite computations in finite element method and other computations . Infinite matrices occur in planetary theory and in atomic theory . A simple example of an infinite matrix is the matrix representing the derivative operator , which acts on the Taylor series of a function .

= = Definition = =

A matrix is a rectangular array of numbers or other mathematical objects for which operations such as addition and multiplication are defined . Most commonly , a matrix over a field F is a rectangular array of scalars each of which is a member of F. Most of this article focuses on real and complex matrices , that is , matrices whose elements are real numbers or complex numbers , respectively . More general types of entries are discussed below . For instance , this is a real matrix :

<formula>

The numbers , symbols or expressions in the matrix are called its entries or its elements . The horizontal and vertical lines of entries in a matrix are called rows and columns , respectively .

= = = Size = = =

The size of a matrix is defined by the number of rows and columns that it contains . A matrix with m rows and n columns is called an m × n matrix or m @-@ by @-@ n matrix , while m and n are called its dimensions . For example , the matrix A above is a 3 × 2 matrix .

Matrices which have a single row are called row vectors , and those which have a single column are called column vectors . A matrix which has the same number of rows and columns is called a square matrix . A matrix with an infinite number of rows or columns (or both) is called an infinite matrix . In some contexts , such as computer algebra programs , it is useful to consider a matrix with no rows or no columns , called an empty matrix .

= = Notation = =

Matrices are commonly written in box brackets or parentheses :
<formula>
The specifics of symbolic matrix notation vary widely , with some prevailing trends . Matrices are usually symbolized using upper @-@ case letters (such as A in the examples above) , while the corresponding lower @-@ case letters , with two subscript indices (e.g. , a11 , or a1,1) , represent the entries . In addition to using upper @-@ case letters to symbolize matrices , many authors use a special typographical style , commonly boldface upright (non @-@ italic) , to further distinguish matrices from other mathematical objects . An alternative notation involves the use of a double @-@ underline with the variable name , with or without boldface style , (e.g. , <formula>) .
The entry in the i @-@ th row and j @-@ th column of a matrix A is sometimes referred to as the i , j , (i , j) , or (i , j) th entry of the matrix , and most commonly denoted as ai , j , or aij . Alternative notations for that entry are A [i , j] or Ai , j . For example , the (1 @,@ 3) entry of the following matrix A is 5 (also denoted a13 , a1,3 , A [1 @,@ 3] or A1,3) :
<formula>