= AL , BL =
AL + AS
(assuming the larger size convention for the B @-@ tiles) , which can be summarized in a substitution matrix equation :
<formula>
Combining this with the decomposition of enlarged ?A @-@ tiles into B @-@ tiles yields the substitution
<formula>
so that the enlarged tile ?AL decomposes into two AL tiles and one AS tiles . The matching rules force a particular substitution : the two AL tiles in a ?AL tile must form a kite ? thus a kite decomposes into two kites and a two half @-@ darts , and a dart decomposes into a kite and two half @-@ darts . Enlarged ?B @-@ tiles decompose into B @-@ tiles in a similar way (via ?A @-@ tiles) .
Composition and decomposition can be iterated , so that , for example
<formula>
The number of kites and darts in the nth iteration of the construction is determined by the nth power of the substitution matrix :
<formula>
where Fn is the nth Fibonacci number . The ratio of numbers of kites to darts in any sufficiently large P2 Penrose tiling pattern therefore approximates to the golden ratio ? . A similar result holds for the ratio of the number of thick rhombs to thin rhombs in the P3 Penrose tiling .

= = = = Deflation for P2 and P3 tilings = = = =

Starting with a collection of tiles from a given tiling (which might be a single tile , a tiling of the plane , or any other collection) , deflation proceeds with a sequence of steps called generations . In one generation of deflation , each tile is replaced with two or more new tiles that are scaled @-@ down versions of tiles used in the original tiling . The substitution rules guarantee that the new tiles will be arranged in accordance with the matching rules . Repeated generations of deflation produce a tiling of the original axiom shape with smaller and smaller tiles .
This rule for dividing the tiles is a subdivision rule .

= = = = Consequences and applications = = = =

Inflation and deflation yield a method for constructing kite and dart (P2) tilings , or rhombus (P3) tilings , known as up @-@ down generation .
The Penrose tilings , being non @-@ periodic , have no translational symmetry ? the pattern cannot be shifted to match itself over the entire plane . However , any bounded region , no matter how large , will be repeated an infinite number of times within the tiling . Therefore , a finite patch cannot differentiate between the uncountably many Penrose tilings , nor even determine which position within the tiling is being shown .
This shows in particular that the number of distinct Penrose tilings (of any type) is uncountably infinite . Up @-@ down generation yields one method to parameterize the tilings , but other methods use Ammann bars , pentagrids , or cut and project schemes .

= = Related tilings and topics = =

= = = Decagonal coverings and quasicrystals = = =

In 1996 , German mathematician Petra Gummelt demonstrated that a covering (so called to distinguish it from a non @-@ overlapping tiling) equivalent to the Penrose tiling can be constructed using a single decagonal tile if two kinds of overlapping regions are allowed . The decagonal tile is decorated with colored patches , and the covering rule allows only those overlaps compatible with the coloring . A suitable decomposition of the decagonal tile into kites and darts transforms such a covering into a Penrose (P2) tiling . Similarly , a P3 tiling can be obtained by inscribing a thick rhomb into each decagon ; the remaining space is filled by thin rhombs .

These coverings have been considered as a realistic model for the growth of quasicrystals : the overlapping decagons are ' quasi @-@ unit cells ' analogous to the unit cells from which crystals are constructed , and the matching rules maximize the density of certain atomic clusters .

= = = Related tilings = = =

The three variants of the Penrose tiling are mutually locally derivable . Selecting some subsets from the vertices of a P1 tiling allows to produce other non @-@ periodic tilings . If the corners of one pentagon in P1 are labeled in succession by 1 @,@ 3 @,@ 5 @,@ 2 @,@ 4 an unambiguous tagging in all the pentagons is established , the order being either clockwise or counterclockwise . Points with the same label define a tiling by Robinson triangles while points with the numbers 3 and 4 on them define the vertices of a Tie @-@ and @-@ Navette tiling .

There are also other related unequivalent tilings , such as the hexagon @-@ boat @-@ star and Mikulla ? Roth tilings . For instance , if the matching rules for the rhombus tiling are reduced to a specific restriction on the angles permitted at each vertex , a binary tiling is obtained . Its underlying symmetry is also fivefold but it is not a quasicrystal . It can be obtained either by decorating the rhombs of the original tiling with smaller ones , or by applying substitution rules , but not by de Bruijn 's cut @-@ and @-@ project method .

= = = Penrose tilings and art = = =

The aesthetic value of tilings has long been appreciated , and remains a source of interest in them ; here the visual appearance (rather than the formal defining properties) of Penrose tilings has attracted attention . The similarity with some decorative patterns used in the Middle East has been noted ; the physicists Peter J. Lu and Paul Steinhardt have presented evidence that a Penrose tiling underlies some examples of medieval Islamic geometric patterns , such as tilings at the Darb @-@ e Imam shrine in Isfahan .

Drop City artist Clark Richert used Penrose rhombs in artwork in 1970 - derived by projecting the rhombic triacontahedron shadow onto a plane observing the embedded " fat " rhombi and " skinny " rhombi which tile together to produce the non @-@ periodic tessellation .. Art historian Martin Kemp has observed that Albrecht Dürer sketched similar motifs of a rhombus tiling .

San Francisco 's new $ 4 @.@ 2 billion Transbay Transit Center is planned to have perforations in its exterior 's undulating white metal skin in the Penrose pattern .

The floor of the atrium of the Molecular and Chemical Sciences Building at the University of Western Australia is tiled with Penrose tiles .

The Andrew Wiles Building , the location of the Mathematics Department at the University of Oxford as of October 2013 , includes a section of Penrose tiling as the paving of its entrance . The pedestrian part of the street Keskuskatu in Helsinki is paved using a form of Penrose tiling . The work was finished in 2014 .