

$a = 2$  is an integer, but the only solution to the equation  $a \cdot b = 1$  in this case is  $b = 1/2$ , which is a rational number, but not an integer. Hence not every element of  $\mathbb{Z}$  has a (multiplicative) inverse.

==== Rational numbers =====

The desire for the existence of multiplicative inverses suggests considering fractions  
 $\frac{a}{b}$