= 0 @.@ 012987012987 ? and 012 + 987 =
999 .
E. Midy proved a general result about such fractions , now called Midy 's theorem , in 1836 . The publication was obscure , and it is unclear if his proof directly involved 0 @.@ 999 ? , but at least one modern proof by W. G. Leavitt does . If it can be proved that a decimal of the form 0.b1b2b3 ? is a positive integer , then it must be 0 @.@ 999 ? , which is then the source of the 9s in the theorem . Investigations in this direction can motivate such concepts as greatest common divisors , modular arithmetic , Fermat primes , order of group elements , and quadratic reciprocity .
Returning to real analysis , the base @-@ 3 analogue 0 @.@ 222 ? = 1 plays a key role in a characterization of one of the simplest fractals , the middle @-@ thirds Cantor set :
A point in the unit interval lies in the Cantor set if and only if it can be represented in ternary using only the digits 0 and 2 .
The nth digit of the representation reflects the position of the point in the nth stage of the construction . For example , the point 2 ? 3 is given the usual representation of 0 @.@ 2 or 0 @.@ 2000 ? , since it lies to the right of the first deletion and to the left of every deletion thereafter . The point 1 ? 3 is represented not as 0 @.@ 1 but as 0 @.@ 0222 ? , since it lies to the left of the first deletion and to the right of every deletion thereafter .
Repeating nines also turn up in yet another of Georg Cantor 's works . They must be taken into account to construct a valid proof , applying his 1891 diagonal argument to decimal expansions , of the uncountability of the unit interval . Such a proof needs to be able to declare certain pairs of real numbers to be different based on their decimal expansions , so one needs to avoid pairs like 0 @.@ 2 and 0 @.@ 1999 ? A simple method represents all numbers with nonterminating expansions ; the opposite method rules out repeating nines . A variant that may be closer to Cantor 's original argument actually uses base 2 , and by turning base @-@ 3 expansions into base @-@ 2 expansions , one can prove the uncountability of the Cantor set as well .

= = Skepticism in education = =

Students of mathematics often reject the equality of 0 @.@ 999 ? and 1 , for reasons ranging from their disparate appearance to deep misgivings over the limit concept and disagreements over the nature of infinitesimals . There are many common contributing factors to the confusion :
Students are often " mentally committed to the notion that a number can be represented in one and only one way by a decimal . " Seeing two manifestly different decimals representing the same number appears to be a paradox , which is amplified by the appearance of the seemingly well @-@ understood number 1 .
Some students interpret " 0 @.@ 999 ? " (or similar notation) as a large but finite string of 9s , possibly with a variable , unspecified length . If they accept an infinite string of nines , they may still expect a last 9 " at infinity " .
Intuition and ambiguous teaching lead students to think of the limit of a sequence as a kind of infinite process rather than a fixed value , since a sequence need not reach its limit . Where students accept the difference between a sequence of numbers and its limit , they might read " 0 @.@ 999 ? " as meaning the sequence rather than its limit .
These ideas are mistaken in the context of the standard real numbers , although some may be valid in other number systems , either invented for their general mathematical utility or as instructive counterexamples to better understand 0 @.@ 999 ?
Many of these explanations were found by David Tall , who has studied characteristics of teaching and cognition that lead to some of the misunderstandings he has encountered in his college students . Interviewing his students to determine why the vast majority initially rejected the equality , he found that " students continued to conceive of 0 @.@ 999 ? as a sequence of numbers getting closer and closer to 1 and not a fixed value , because ' you haven 't specified how many places there are ' or ' it is the nearest possible decimal below 1 ' " .