= 2 , the resulting vector aw has the same direction as w , but is stretched to the double length of w (right image below) . Equivalently 2w is the sum w + w . Moreover , (? 1) v =
? v has the opposite direction and the same length as v (blue vector pointing down in the right image) .

= = = Second example : ordered pairs of numbers = = =

A second key example of a vector space is provided by pairs of real numbers x and y . (The order of the components x and y is significant , so such a pair is also called an ordered pair .) Such a pair is written as (x , y) . The sum of two such pairs and multiplication of a pair with a number is defined as follows :
(x1 , y1) + (x2 , y2)
= (x1 + x2 , y1 + y2)
and
a (x , y) =
(ax , ay) .
The first example above reduces to this one if the arrows are represented by the pair of Cartesian coordinates of their end points .

= = = Definition = = =

A vector space over a field F is a set V together with two operations that satisfy the eight axioms listed below . Elements of V are commonly called vectors . Elements of F are commonly called scalars . The first operation , called vector addition or simply addition , takes any two vectors v and w and assigns to them a third vector which is commonly written as v + w , and called the sum of these two vectors . The second operation , called scalar multiplication takes any scalar a and any vector v and gives another vector av .
In this article , vectors are distinguished from scalars by boldface . In the two examples above , the field is the field of the real numbers and the set of the vectors consists of the planar arrows with fixed starting point and of pairs of real numbers , respectively .
To qualify as a vector space , the set V and the operations of addition and multiplication must adhere to a number of requirements called axioms . In the list below , let u , v and w be arbitrary vectors in V , and a and b scalars in F.
These axioms generalize properties of the vectors introduced in the above examples . Indeed , the result of addition of two ordered pairs (as in the second example above) does not depend on the order of the summands :
(xv , yv) + (xw , yw)
= (xw , yw) + (xv , yv) .
Likewise , in the geometric example of vectors as arrows , v + w =
w + v since the parallelogram defining the sum of the vectors is independent of the order of the vectors . All other axioms can be checked in a similar manner in both examples . Thus , by disregarding the concrete nature of the particular type of vectors , the definition incorporates these two and many more examples in one notion of vector space .
Subtraction of two vectors and division by a (non @-@ zero) scalar can be defined as