= Georg Cantor =

Georg Ferdinand Ludwig Philipp Cantor (/ ?kænt??r / KAN @-@ tor ; German : [??e??k ?f??dinant ?lu?tv?ç ?f?l?p ?kant??] ; March 3 [O.S. February 19] 1845 ? January 6 , 1918) was a German mathematician . He invented set theory , which has become a fundamental theory in mathematics . Cantor established the importance of one @-@ to @-@ one correspondence between the members of two sets , defined infinite and well @-@ ordered sets , and proved that the real numbers are more numerous than the natural numbers . In fact , Cantor 's method of proof of this theorem implies the existence of an " infinity of infinities " . He defined the cardinal and ordinal numbers and their arithmetic . Cantor 's work is of great philosophical interest , a fact of which he was well aware .

Cantor 's theory of transfinite numbers was originally regarded as so counter @-@ intuitive ? even shocking ? that it encountered resistance from mathematical contemporaries such as Leopold Kronecker and Henri Poincaré and later from Hermann Weyl and L. E. J. Brouwer , while Ludwig Wittgenstein raised philosophical objections . Cantor , a devout Lutheran , believed the theory had been communicated to him by God . Some Christian theologians (particularly neo @-@ Scholastics) saw Cantor 's work as a challenge to the uniqueness of the absolute infinity in the nature of God ? on one occasion equating the theory of transfinite numbers with pantheism ? a proposition that Cantor vigorously rejected .

The objections to Cantor 's work were occasionally fierce : Henri Poincaré referred to his ideas as a " grave disease " infecting the discipline of mathematics , and Leopold Kronecker 's public opposition and personal attacks included describing Cantor as a " scientific charlatan " , a " renegade " and a " corrupter of youth . " Kronecker objected to Cantor 's proofs that the algebraic numbers are countable , and that the transcendental numbers are uncountable , results now included in a standard mathematics curriculum . Writing decades after Cantor 's death , Wittgenstein lamented that mathematics is " ridden through and through with the pernicious idioms of set theory , " which he dismissed as " utter nonsense " that is " laughable " and " wrong " . Cantor 's recurring bouts of depression from 1884 to the end of his life have been blamed on the hostile attitude of many of his contemporaries , though some have explained these episodes as probable manifestations of a bipolar disorder .

The harsh criticism has been matched by later accolades . In 1904 , the Royal Society awarded Cantor its Sylvester Medal , the highest honor it can confer for work in mathematics . David Hilbert defended it from its critics by declaring :

From his paradise that Cantor with us unfolded , we hold our breath in awe ; knowing , we shall not be expelled .

= = Life of Georg Cantor = =

= = = Youth and studies = = =

Georg Cantor was born in the western merchant colony in Saint Petersburg , Russia , and brought up in the city until he was eleven . Georg , the oldest of six children , was regarded as an outstanding violinist . His grandfather Franz Böhm (1788 ? 1846) (the violinist Joseph Böhm 's brother) was a well @-@ known musician and soloist in a Russian imperial orchestra . Cantor 's father had been a member of the Saint Petersburg stock exchange ; when he became ill , the family moved to Germany in 1856 , first to Wiesbaden then to Frankfurt , seeking winters milder than those of Saint Petersburg . In 1860 , Cantor graduated with distinction from the Realschule in Darmstadt ; his exceptional skills in mathematics , trigonometry in particular , were noted . In 1862 , Cantor entered the Swiss Federal Polytechnic . After receiving a substantial inheritance upon his father 's death in 1863 , Cantor shifted his studies to the University of Berlin , attending lectures by Leopold Kronecker , Karl Weierstrass and Ernst Kummer . He spent the summer of 1866 at the University of Göttingen , then and later a center for mathematical research .

= = = Teacher and researcher = = =

Cantor submitted his dissertation on number theory at the University of Berlin in 1867 . After teaching briefly in a Berlin girls ' school , Cantor took up a position at the University of Halle , where he spent his entire career . He was awarded the requisite habilitation for his thesis , also on number theory , which he presented in 1869 upon his appointment at Halle .

In 1874 , Cantor married Vally Guttmann . They had six children , the last (Rudolph) born in 1886 . Cantor was able to support a family despite modest academic pay , thanks to his inheritance from his father . During his honeymoon in the Harz mountains , Cantor spent much time in mathematical discussions with Richard Dedekind , whom he had met two years earlier while on Swiss holiday .

Cantor was promoted to Extraordinary Professor in 1872 and made full Professor in 1879 . To attain the latter rank at the age of 34 was a notable accomplishment , but Cantor desired a chair at a more prestigious university , in particular at Berlin , at that time the leading German university . However , his work encountered too much opposition for that to be possible . Kronecker , who headed mathematics at Berlin until his death in 1891 , became increasingly uncomfortable with the prospect of having Cantor as a colleague , perceiving him as a " corrupter of youth " for teaching his ideas to a younger generation of mathematicians . Worse yet , Kronecker , a well @-@ established figure within the mathematical community and Cantor 's former professor , disagreed fundamentally with the thrust of Cantor 's work . Kronecker , now seen as one of the founders of the constructive viewpoint in mathematics , disliked much of Cantor 's set theory because it asserted the existence of sets satisfying certain properties , without giving specific examples of sets whose members did indeed satisfy those properties . Cantor came to believe that Kronecker 's stance would make it impossible for him ever to leave Halle .

In 1881 , Cantor 's Halle colleague Eduard Heine died , creating a vacant chair . Halle accepted Cantor 's suggestion that it be offered to Dedekind , Heinrich M. Weber and Franz Mertens , in that order , but each declined the chair after being offered it . Friedrich Wangerin was eventually appointed , but he was never close to Cantor .

In 1882 , the mathematical correspondence between Cantor and Richard Dedekind came to an end , apparently as a result of Dedekind 's declining the chair at Halle . Cantor also began another important correspondence , with Gösta Mittag @-@ Leffler in Sweden , and soon began to publish in Mittag @-@ Leffler 's journal Acta Mathematica . But in 1885 , Mittag @-@ Leffler was concerned about the philosophical nature and new terminology in a paper Cantor had submitted to Acta . He asked Cantor to withdraw the paper from Acta while it was in proof , writing that it was " ... about one hundred years too soon . " Cantor complied , but then curtailed his relationship and correspondence with Mittag @-@ Leffler , writing to a third party :

Had Mittag @-@ Leffler had his way , I should have to wait until the year 1984 , which to me seemed too great a demand ! ... But of course I never want to know anything again about Acta Mathematica .

Cantor suffered his first known bout of depression in 1884 . Criticism of his work weighed on his mind : every one of the fifty @-@ two letters he wrote to Mittag @-@ Leffler in 1884 mentioned Kronecker . A passage from one of these letters is revealing of the damage to Cantor 's self @-@ confidence :

... I don 't know when I shall return to the continuation of my scientific work . At the moment I can do absolutely nothing with it , and limit myself to the most necessary duty of my lectures ; how much happier I would be to be scientifically active , if only I had the necessary mental freshness .

This crisis led him to apply to lecture on philosophy rather than mathematics . He also began an intense study of Elizabethan literature thinking there might be evidence that Francis Bacon wrote the plays attributed to Shakespeare (see Shakespearean authorship question) ; this ultimately resulted in two pamphlets , published in 1896 and 1897 .

Cantor recovered soon thereafter , and subsequently made further important contributions , including his diagonal argument and theorem . However , he never again attained the high level of his remarkable papers of 1874 ? 84 . He eventually sought , and achieved , a reconciliation with

Kronecker . Nevertheless , the philosophical disagreements and difficulties dividing them persisted .

In 1890 , Cantor was instrumental in founding the Deutsche Mathematiker @-@ Vereinigung and chaired its first meeting in Halle in 1891 , where he first introduced his diagonal argument ; his reputation was strong enough , despite Kronecker 's opposition to his work , to ensure he was elected as the first president of this society . Setting aside the animosity Kronecker had displayed towards him , Cantor invited him to address the meeting , but Kronecker was unable to do so because his wife was dying from injuries sustained in a skiing accident at the time .

= = = Late years = = =

After Cantor 's 1884 hospitalization , there is no record that he was in any sanatorium again until 1899 . Soon after that second hospitalization , Cantor 's youngest son Rudolph died suddenly (while Cantor was delivering a lecture on his views on Baconian theory and William Shakespeare) , and this tragedy drained Cantor of much of his passion for mathematics . Cantor was again hospitalized in 1903 . One year later , he was outraged and agitated by a paper presented by Julius König at the Third International Congress of Mathematicians . The paper attempted to prove that the basic tenets of transfinite set theory were false . Since the paper had been read in front of his daughters and colleagues , Cantor perceived himself as having been publicly humiliated . Although Ernst Zermelo demonstrated less than a day later that König 's proof had failed , Cantor remained shaken , and momentarily questioning God . Cantor suffered from chronic depression for the rest of his life , for which he was excused from teaching on several occasions and repeatedly confined in various sanatoria . The events of 1904 preceded a series of hospitalizations at intervals of two or three years . He did not abandon mathematics completely , however , lecturing on the paradoxes of set theory (Burali @-@ Forti paradox , Cantor 's paradox , and Russell 's paradox) to a meeting of the Deutsche Mathematiker ? Vereinigung in 1903 , and attending the International Congress of Mathematicians at Heidelberg in 1904 .

In 1911 , Cantor was one of the distinguished foreign scholars invited to attend the 500th anniversary of the founding of the University of St. Andrews in Scotland . Cantor attended , hoping to meet Bertrand Russell , whose newly published Principia Mathematica repeatedly cited Cantor 's work , but this did not come about . The following year , St. Andrews awarded Cantor an honorary doctorate , but illness precluded his receiving the degree in person .

Cantor retired in 1913 , living in poverty and suffering from malnourishment during World War I. The public celebration of his 70th birthday was canceled because of the war . He died on January 6 , 1918 in the sanatorium where he had spent the final year of his life .

= = Mathematical work = =

Cantor 's work between 1874 and 1884 is the origin of set theory . Prior to this work , the concept of a set was a rather elementary one that had been used implicitly since the beginning of mathematics , dating back to the ideas of Aristotle . No one had realized that set theory had any nontrivial content . Before Cantor , there were only finite sets (which are easy to understand) and " the infinite " (which was considered a topic for philosophical , rather than mathematical , discussion) . By proving that there are (infinitely) many possible sizes for infinite sets , Cantor established that set theory was not trivial , and it needed to be studied . Set theory has come to play the role of a foundational theory in modern mathematics , in the sense that it interprets propositions about mathematical objects (for example , numbers and functions) from all the traditional areas of mathematics (such as algebra , analysis and topology) in a single theory , and provides a standard set of axioms to prove or disprove them . The basic concepts of set theory are now used throughout mathematics .

In one of his earliest papers , Cantor proved that the set of real numbers is " more numerous " than the set of natural numbers ; this showed , for the first time , that there exist infinite sets of different sizes . He was also the first to appreciate the importance of one @-@ to @-@ one correspondences (hereinafter denoted " 1 @-@ to @-@ 1 correspondence ") in set theory . He used this concept to define finite and infinite sets , subdividing the latter into denumerable (or

countably infinite) sets and uncountable sets (nondenumerable infinite sets) .

Cantor developed important concepts in topology and their relation to cardinality . For example , he showed that the Cantor set is nowhere dense , but has the same cardinality as the set of all real numbers , whereas the rationals are everywhere dense , but countable .

Cantor introduced fundamental constructions in set theory , such as the power set of a set A , which is the set of all possible subsets of A. He later proved that the size of the power set of A is strictly larger than the size of A , even when A is an infinite set ; this result soon became known as Cantor 's theorem . Cantor developed an entire theory and arithmetic of infinite sets , called cardinals and ordinals , which extended the arithmetic of the natural numbers . His notation for the cardinal numbers was the Hebrew letter <formula> (aleph) with a natural number subscript ; for the ordinals he employed the Greek letter ? (omega) . This notation is still in use today .

The Continuum hypothesis , introduced by Cantor , was presented by David Hilbert as the first of his twenty @-@ three open problems in his address at the 1900 International Congress of Mathematicians in Paris . Cantor 's work also attracted favorable notice beyond Hilbert 's celebrated encomium . The US philosopher Charles Sanders Peirce praised Cantor 's set theory , and , following public lectures delivered by Cantor at the first International Congress of Mathematicians , held in Zurich in 1897 , Hurwitz and Hadamard also both expressed their admiration . At that Congress , Cantor renewed his friendship and correspondence with Dedekind . From 1905 , Cantor corresponded with his British admirer and translator Philip Jourdain on the history of set theory and on Cantor 's religious ideas . This was later published , as were several of his expository works .

= = = Number theory , trigonometric series and ordinals = = =

Cantor 's first ten papers were on number theory , his thesis topic . At the suggestion of Eduard Heine , the Professor at Halle , Cantor turned to analysis . Heine proposed that Cantor solve an open problem that had eluded Peter Gustav Lejeune Dirichlet , Rudolf Lipschitz , Bernhard Riemann , and Heine himself : the uniqueness of the representation of a function by trigonometric series . Cantor solved this difficult problem in 1869 . It was while working on this problem that he discovered transfinite ordinals , which occurred as indices n in the nth derived set Sn of a set S of zeros of a trigonometric series . Given a trigonometric series f (x) with S as its set of zeros , Cantor had discovered a procedure that produced another trigonometric series that had S1 as its set of zeros , where S1 is the set of limit points of S. If Sk + 1 is the set of limit points of Sk , then he could construct a trigonometric series whose zeros are Sk + 1 . Because the sets Sk were closed , they contained their Limit points , and the intersection of the infinite decreasing sequence of sets S , S1 , S2 , S3 , ... formed a limit set , which we would now call S? , and then he noticed that S? would also have to have a set of limit points S? + 1 , and so on . He had examples that went on forever , and so here was a naturally occurring infinite sequence of infinite numbers ? , ? + 1 , ? + 2 , ...

Between 1870 and 1872 , Cantor published more papers on trigonometric series , and also a paper defining irrational numbers as convergent sequences of rational numbers . Dedekind , whom Cantor befriended in 1872 , cited this paper later that year , in the paper where he first set out his celebrated definition of real numbers by Dedekind cuts . While extending the notion of number by means of his revolutionary concept of infinite cardinality , Cantor was paradoxically opposed to theories of infinitesimals of his contemporaries Otto Stolz and Paul du Bois @-@ Reymond , describing them as both " an abomination " and " a cholera bacillus of mathematics " . Cantor also published an erroneous " proof " of the inconsistency of infinitesimals .

= = = Set theory = = =

The beginning of set theory as a branch of mathematics is often marked by the publication of Cantor 's 1874 article , " Über eine Eigenschaft des Inbegriffes aller reellen algebraischen Zahlen " (" On a Property of the Collection of All Real Algebraic Numbers ") . This article was the first to provide a rigorous proof that there was more than one kind of infinity . Previously , all infinite collections had been implicitly assumed to be equinumerous (that is , of " the same size " or having

the same number of elements) . Cantor proved that the collection of real numbers and the collection of positive integers are not equinumerous . In other words , the real numbers are not countable . His proof differs from diagonal argument that he gave in 1891 . Cantor 's article also contains a new method of constructing transcendental numbers . Transcendental numbers were first constructed by Joseph Liouville in 1844 .

Cantor established these results using two constructions . His first construction shows how to write the real algebraic numbers as a sequence a1 , a2 , a3 , ... In other words , the real algebraic numbers are countable . Cantor starts his second construction with any sequence of real numbers . Using this sequence , he constructs nested intervals whose intersection contains a real number not in the sequence . Since every sequence of real numbers can be used to construct a real not in the sequence , the real numbers cannot be written as a sequence ? that is , the real numbers are not countable . By applying his construction to the sequence of real algebraic numbers , Cantor produces a transcendental number . Cantor points out that his constructions prove more ? namely , they provide a new proof of Liouville 's theorem : Every interval contains infinitely many transcendental numbers . Cantor 's next article contains a construction that proves the set of transcendental numbers has the same " power " (see below) as the set of real numbers .

Between 1879 and 1884 , Cantor published a series of six articles in Mathematische Annalen that together formed an introduction to his set theory . At the same time , there was growing opposition to Cantor 's ideas , led by Kronecker , who admitted mathematical concepts only if they could be constructed in a finite number of steps from the natural numbers , which he took as intuitively given . For Kronecker , Cantor 's hierarchy of infinities was inadmissible , since accepting the concept of actual infinity would open the door to paradoxes which would challenge the validity of mathematics as a whole . Cantor also introduced the Cantor set during this period .

The fifth paper in this series , " Grundlagen einer allgemeinen Mannigfaltigkeitslehre " (" Foundations of a General Theory of Aggregates ") , published in 1883 , was the most important of the six and was also published as a separate monograph . It contained Cantor 's reply to his critics and showed how the transfinite numbers were a systematic extension of the natural numbers . It begins by defining well @-@ ordered sets . Ordinal numbers are then introduced as the order types of well @-@ ordered sets . Cantor then defines the addition and multiplication of the cardinal and ordinal numbers . In 1885 , Cantor extended his theory of order types so that the ordinal numbers simply became a special case of order types .

In 1891 , he published a paper containing his elegant " diagonal argument " for the existence of an uncountable set . He applied the same idea to prove Cantor 's theorem : the cardinality of the power set of a set A is strictly larger than the cardinality of A. This established the richness of the hierarchy of infinite sets , and of the cardinal and ordinal arithmetic that Cantor had defined . His argument is fundamental in the solution of the Halting problem and the proof of Gödel 's first incompleteness theorem . Cantor wrote on the Goldbach conjecture in 1894 .

In 1895 and 1897 , Cantor published a two @-@ part paper in Mathematische Annalen under Felix Klein 's editorship ; these were his last significant papers on set theory . The first paper begins by defining set , subset , etc . , in ways that would be largely acceptable now . The cardinal and ordinal arithmetic are reviewed . Cantor wanted the second paper to include a proof of the continuum hypothesis , but had to settle for expositing his theory of well @-@ ordered sets and ordinal numbers . Cantor attempts to prove that if A and B are sets with A equivalent to a subset of B and B equivalent to a subset of A , then A and B are equivalent . Ernst Schröder had stated this theorem a bit earlier , but his proof , as well as Cantor 's , was flawed . Felix Bernstein supplied a correct proof in his 1898 PhD thesis ; hence the name Cantor ? Bernstein ? Schröder theorem .

= = = = One @-@ to @-@ one correspondence = = = =

Cantor 's 1874 Crelle paper was the first to invoke the notion of a 1 @-@ to @-@ 1 correspondence , though he did not use that phrase . He then began looking for a 1 @-@ to @-@ 1 correspondence between the points of the unit square and the points of a unit line segment . In an 1877 letter to Richard Dedekind , Cantor proved a far stronger result : for any positive integer n ,

there exists a 1 @-@ to @-@ 1 correspondence between the points on the unit line segment and all of the points in an n @-@ dimensional space . About this discovery Cantor wrote to Dedekind : " Je le vois , mais je ne le crois pas ! " (" I see it , but I don 't believe it ! ") The result that he found so astonishing has implications for geometry and the notion of dimension .

In 1878 , Cantor submitted another paper to Crelle 's Journal , in which he defined precisely the concept of a 1 @-@ to @-@ 1 correspondence , and introduced the notion of " power " (a term he took from Jakob Steiner) or " equivalence " of sets : two sets are equivalent (have the same power) if there exists a 1 @-@ to @-@ 1 correspondence between them . Cantor defined countable sets (or denumerable sets) as sets which can be put into a 1 @-@ to @-@ 1 correspondence with the natural numbers , and proved that the rational numbers are denumerable . He also proved that n @-@ dimensional Euclidean space Rn has the same power as the real numbers R , as does a countably infinite product of copies of R. While he made free use of countability as a concept , he did not write the word " countable " until 1883 . Cantor also discussed his thinking about dimension , stressing that his mapping between the unit interval and the unit square was not a continuous one .

This paper displeased Kronecker , and Cantor wanted to withdraw it ; however , Dedekind persuaded him not to do so and Weierstrass supported its publication . Nevertheless , Cantor never again submitted anything to Crelle .

= = = = Continuum hypothesis = = = =

Cantor was the first to formulate what later came to be known as the continuum hypothesis or CH : there exists no set whose power is greater than that of the naturals and less than that of the reals (or equivalently , the cardinality of the reals is exactly aleph @-@ one , rather than just at least aleph @-@ one) . Cantor believed the continuum hypothesis to be true and tried for many years to prove it , in vain . His inability to prove the continuum hypothesis caused him considerable anxiety .

The difficulty Cantor had in proving the continuum hypothesis has been underscored by later developments in the field of mathematics : a 1940 result by Gödel and a 1963 one by Paul Cohen together imply that the continuum hypothesis can neither be proved nor disproved using standard Zermelo ? Fraenkel set theory plus the axiom of choice (the combination referred to as " ZFC ") .

= = = = Paradoxes of set theory = = = =

Discussions of set @-@ theoretic paradoxes began to appear around the end of the nineteenth century . Some of these implied fundamental problems with Cantor 's set theory program . In an 1897 paper on an unrelated topic , Cesare Burali @-@ Forti set out the first such paradox , the Burali @-@ Forti paradox : the ordinal number of the set of all ordinals must be an ordinal and this leads to a contradiction . Cantor discovered this paradox in 1895 , and described it in an 1896 letter to Hilbert . Criticism mounted to the point where Cantor launched counter @-@ arguments in 1903 , intended to defend the basic tenets of his set theory .

In 1899 , Cantor discovered his eponymous paradox : what is the cardinal number of the set of all sets ? Clearly it must be the greatest possible cardinal . Yet for any set A , the cardinal number of the power set of A is strictly larger than the cardinal number of A (this fact is now known as Cantor 's theorem) . This paradox , together with Burali @-@ Forti paradox , led Cantor to formulate a concept called limitation of size , according to which the collection of all ordinals , or of all sets , was an " inconsistent multiplicity " that was " too large " to be a set . Such collections later became known as proper classes .

One common view among mathematicians is that these paradoxes , together with Russell 's paradox , demonstrate that it is not possible to take a " naive " , or non @-@ axiomatic , approach to set theory without risking contradiction , and it is certain that they were among the motivations for Zermelo and others to produce axiomatizations of set theory . Others note , however , that the paradoxes do not obtain in an informal view motivated by the iterative hierarchy , which can be seen as explaining the idea of limitation of size . Some also question whether the Fregean formulation of naive set theory (which was the system directly refuted by the Russell paradox) is really a faithful

interpretation of the Cantorian conception .

= = Philosophy , religion and Cantor 's mathematics = =

The concept of the existence of an actual infinity was an important shared concern within the realms of mathematics , philosophy and religion . Preserving the orthodoxy of the relationship between God and mathematics , although not in the same form as held by his critics , was long a concern of Cantor 's . He directly addressed this intersection between these disciplines in the introduction to his Grundlagen einer allgemeinen Mannigfaltigkeitslehre , where he stressed the connection between his view of the infinite and the philosophical one . To Cantor , his mathematical views were intrinsically linked to their philosophical and theological implications ? he identified the Absolute Infinite with God , and he considered his work on transfinite numbers to have been directly communicated to him by God , who had chosen Cantor to reveal them to the world .

Debate among mathematicians grew out of opposing views in the philosophy of mathematics regarding the nature of actual infinity . Some held to the view that infinity was an abstraction which was not mathematically legitimate , and denied its existence . Mathematicians from three major schools of thought (constructivism and its two offshoots , intuitionism and finitism) opposed Cantor 's theories in this matter . For constructivists such as Kronecker , this rejection of actual infinity stems from fundamental disagreement with the idea that nonconstructive proofs such as Cantor 's diagonal argument are sufficient proof that something exists , holding instead that constructive proofs are required . Intuitionism also rejects the idea that actual infinity is an expression of any sort of reality , but arrive at the decision via a different route than constructivism . Firstly , Cantor 's argument rests on logic to prove the existence of transfinite numbers as an actual mathematical entity , whereas intuitionists hold that mathematical entities cannot be reduced to logical propositions , originating instead in the intuitions of the mind . Secondly , the notion of infinity as an expression of reality is itself disallowed in intuitionism , since the human mind cannot intuitively construct an infinite set . Mathematicians such as Brouwer and especially Poincaré adopted an intuitionist stance against Cantor 's work . Citing the paradoxes of set theory as an example of its fundamentally flawed nature , Poincaré held that " most of the ideas of Cantorian set theory should be banished from mathematics once and for all . " Finally , Wittgenstein 's attacks were finitist : he believed that Cantor 's diagonal argument conflated the intension of a set of cardinal or real numbers with its extension , thus conflating the concept of rules for generating a set with an actual set .

Some Christian theologians saw Cantor 's work as a challenge to the uniqueness of the absolute infinity in the nature of God . In particular , Neo @-@ Thomist thinkers saw the existence of an actual infinity that consisted of something other than God as jeopardizing " God 's exclusive claim to supreme infinity " . Cantor strongly believed that this view was a misinterpretation of infinity , and was convinced that set theory could help correct this mistake :

... the transfinite species are just as much at the disposal of the intentions of the Creator and His absolute boundless will as are the finite numbers .

Cantor also believed that his theory of transfinite numbers ran counter to both materialism and determinism ? and was shocked when he realized that he was the only faculty member at Halle who did not hold to deterministic philosophical beliefs .

In 1888 , Cantor published his correspondence with several philosophers on the philosophical implications of his set theory . In an extensive attempt to persuade other Christian thinkers and authorities to adopt his views , Cantor had corresponded with Christian philosophers such as Tilman Pesch and Joseph Hontheim , as well as theologians such as Cardinal Johannes Franzelin , who once replied by equating the theory of transfinite numbers with pantheism . Cantor even sent one letter directly to Pope Leo XIII himself , and addressed several pamphlets to him .

Cantor 's philosophy on the nature of numbers led him to affirm a belief in the freedom of mathematics to posit and prove concepts apart from the realm of physical phenomena , as expressions within an internal reality . The only restrictions on this metaphysical system are that all mathematical concepts must be devoid of internal contradiction , and that they follow from existing

definitions , axioms , and theorems . This belief is summarized in his assertion that " the essence of mathematics is its freedom . " These ideas parallel those of Edmund Husserl , whom Cantor had met in Halle .

Meanwhile , Cantor himself was fiercely opposed to infinitesimals , describing them as both an " abomination " and " the cholera bacillus of mathematics " .

Cantor 's 1883 paper reveals that he was well aware of the opposition his ideas were encountering :

... I realize that in this undertaking I place myself in a certain opposition to views widely held concerning the mathematical infinite and to opinions frequently defended on the nature of numbers .

Hence he devotes much space to justifying his earlier work , asserting that mathematical concepts may be freely introduced as long as they are free of contradiction and defined in terms of previously accepted concepts . He also cites Aristotle , Descartes , Berkeley , Leibniz , and Bolzano on infinity .

= = Cantor 's ancestry = =

Cantor 's paternal grandparents were from Copenhagen , and fled to Russia from the disruption of the Napoleonic Wars . There is very little direct information on his grandparents . Cantor was sometimes called Jewish in his lifetime , but has also variously been called Russian , German , and Danish as well .

Jakob Cantor , Cantor 's grandfather , gave his children Christian saints ' names . Further , several of his grandmother 's relatives were in the Czarist civil service , which would not welcome Jews , unless they converted to Christianity . Cantor 's father , Georg Waldemar Cantor , was educated in the Lutheran mission in Saint Petersburg , and his correspondence with his son shows both of them as devout Lutherans . Very little is known for sure about George Woldemar 's origin or education . His mother , Maria Anna Böhm , was an Austro @-@ Hungarian born in Saint Petersburg and baptized Roman Catholic ; she converted to Protestantism upon marriage . However , there is a letter from Cantor 's brother Louis to their mother , stating :

Mögen wir zehnmal von Juden abstammen und ich im Princip noch so sehr für Gleichberechtigung der Hebräer sein , im socialen Leben sind mir Christen lieber ...

(" Even if we were descended from Jews ten times over , and even though I may be , in principle , completely in favour of equal rights for Hebrews , in social life I prefer Christians ... ") which could be read to imply that she was of Jewish ancestry .

There were documented statements , during the 1930s , that called this Jewish ancestry into question :

More often [i.e. , than the ancestry of the mother] the question has been discussed of whether Georg Cantor was of Jewish origin . About this it is reported in a notice of the Danish genealogical Institute in Copenhagen from the year 1937 concerning his father : " It is hereby testified that Georg Woldemar Cantor , born 1809 or 1814 , is not present in the registers of the Jewish community , and that he completely without doubt was not a Jew ... "

It is also later said in the same document :

Also efforts for a long time by the librarian Josef Fischer , one of the best experts on Jewish genealogy in Denmark , charged with identifying Jewish professors , that Georg Cantor was of Jewish descent , finished without result . [Something seems to be wrong with this sentence , but the meaning seems clear enough .] In Cantor 's published works and also in his Nachlass there are no statements by himself which relate to a Jewish origin of his ancestors . There is to be sure in the Nachlass a copy of a letter of his brother Ludwig from 18 November 1869 to their mother with some unpleasant antisemitic statements , in which it is said among other things : ...

(the rest of the quote is finished by the very first quote above) . In Men of Mathematics , Eric Temple Bell described Cantor as being " of pure Jewish descent on both sides , " although both parents were baptized . In a 1971 article entitled " Towards a Biography of Georg Cantor , " the British historian of mathematics Ivor Grattan @-@ Guinness mentions (Annals of Science 27 , pp.

345 ? 391 , 1971) that he was unable to find evidence of Jewish ancestry . (He also states that Cantor 's wife , Vally Guttmann , was Jewish) .

In a letter written by Georg Cantor to Paul Tannery in 1896 (Paul Tannery , Memoires Scientifique 13 Correspondence , Gauthier @-@ Villars , Paris , 1934 , p . 306) , Cantor states that his paternal grandparents were members of the Sephardic Jewish community of Copenhagen . Specifically , Cantor states in describing his father : " Er ist aber in Kopenhagen geboren , von israelitischen Eltern , die der dortigen portugisischen Judengemeinde ... " (" He was born in Copenhagen of Jewish (lit : " Israelite ") parents from the local Portuguese @-@ Jewish community . ")

In addition , Cantor 's maternal great uncle , a Hungarian violinist Josef Böhm , has been described as Jewish , which may imply that Cantor 's mother was at least partly descended from the Hungarian Jewish community .

In a letter to Bertrand Russell , Cantor described his ancestry and self @-@ perception as follows :

Neither my father nor my mother were of German blood , the first being a Dane , borne in Kopenhagen , my mother of Austrian Hungar descension . You must know , Sir , that I am not a regular just Germain , for I am born 3 March 1845 at Saint Peterborough , Capital of Russia , but I went with my father and mother and brothers and sister , eleven years old in the year 1856 , into Germany .

= = Historiography = =

Until the 1970s , the chief academic publications on Cantor were two short monographs by Schönflies (1927) ? largely the correspondence with Mittag @-@ Leffler ? and Fraenkel (1930) . Both were at second and third hand ; neither had much on his personal life . The gap was largely filled by Eric Temple Bell 's Men of Mathematics (1937) , which one of Cantor 's modern biographers describes as " perhaps the most widely read modern book on the history of mathematics " ; and as " one of the worst " . Bell presents Cantor 's relationship with his father as Oedipal , Cantor 's differences with Kronecker as a quarrel between two Jews , and Cantor 's madness as Romantic despair over his failure to win acceptance for his mathematics , and fills the picture with stereotypes . Grattan @-@ Guinness (1971) found that none of these claims were true , but they may be found in many books of the intervening period , owing to the absence of any other narrative . There are other legends , independent of Bell ? including one that labels Cantor 's father a foundling , shipped to Saint Petersburg by unknown parents . A critique of Bell 's book is contained in Joseph Dauben 's biography . Writes Dauben :

Cantor devoted some of his most vituperative correspondence , as well as a portion of the Beiträge , to attacking what he described at one point as the ' infinitesimal Cholera bacillus of mathematics ' , which had spread from Germany through the work of Thomae , du Bois Reymond and Stolz , to infect Italian mathematics ... Any acceptance of infinitesimals necessarily meant that his own theory of number was incomplete . Thus to accept the work of Thomae , du Bois @-@ Reymond , Stolz and Veronese was to deny the perfection of Cantor 's own creation . Understandably , Cantor launched a thorough campaign to discredit Veronese 's work in every way possible .