= 1) with MN =
0 for all N. Thus ? is a distribution of order zero . It is , furthermore , a distribution with compact support (the support being { 0 }) .
The delta distribution can also be defined in a number of equivalent ways . For instance , it is the distributional derivative of the Heaviside step function . This means that , for every test function ? , one has
<formula>
Intuitively , if integration by parts were permitted , then the latter integral should simplify to
<formula>
and indeed , a form of integration by parts is permitted for the Stieltjes integral , and in that case one does have
<formula>
In the context of measure theory , the Dirac measure gives rise to a distribution by integration . Conversely , equation (1) defines a Daniell integral on the space of all compactly supported continuous functions ? which , by the Riesz representation theorem , can be represented as the Lebesgue integral of ? with respect to some Radon measure .
Generally , when the term " Dirac delta function " is used , it is in the sense of distributions rather than measures , the Dirac measure being among several terms for the corresponding notion in measure theory . Some sources may also use the term Dirac delta distribution .

= = = Generalizations = = =

The delta function can be defined in n @-@ dimensional Euclidean space Rn as the measure such that
<formula>
for every compactly supported continuous function f . As a measure , the n @-@ dimensional delta function is the product measure of the 1 @-@ dimensional delta functions in each variable separately . Thus , formally , with x = (x1 , x2 , ... , xn) , one has
The delta function can also be defined in the sense of distributions exactly as above in the one @-@ dimensional case . However , despite widespread use in engineering contexts , (2) should be manipulated with care , since the product of distributions can only be defined under quite narrow circumstances .
The notion of a Dirac measure makes sense on any set . Thus if X is a set , x0 ? X is a marked point , and ? is any sigma algebra of subsets of X , then the measure defined on sets A ? ? by
<formula>
is the delta measure or unit mass concentrated at x0 .
Another common generalization of the delta function is to a differentiable manifold where most of its properties as a distribution can also be exploited because of the differentiable structure . The delta function on a manifold M centered at the point x0 ? M is defined as the following distribution :
for all compactly supported smooth real @-@ valued functions ? on M. A common special case of this construction is when M is an open set in the Euclidean space Rn .
On a locally compact Hausdorff space X , the Dirac delta measure concentrated at a point x is the Radon measure associated with the Daniell integral (3) on compactly supported continuous functions ? . At this level of generality , calculus as such is no longer possible , however a variety of techniques from abstract analysis are available . For instance , the mapping <formula> is a continuous embedding of X into the space of finite Radon measures on X , equipped with its vague topology . Moreover , the convex hull of the image of X under this embedding is dense in the space of probability measures on X.

= = Properties = =

= = = Scaling and symmetry = = =

The delta function satisfies the following scaling property for a non @-@ zero scalar ? :
<formula>
and so
In particular , the delta function is an even distribution , in the sense that
<formula>
which is homogeneous of degree ? 1 .

= = = Algebraic properties = = =

The distributional product of ? with x is equal to zero :
<formula>
Conversely , if xf (x) = xg (x) , where f and g are distributions , then
<formula>
for some constant c .

= = = Translation = = =

The integral of the time @-@ delayed Dirac delta is given by :
<formula>
This is sometimes referred to as the sifting property or the sampling property . The delta function is said to " sift out " the value at t = T.
It follows that the effect of convolving a function f (t) with the time @-@ delayed Dirac delta is to time @-@ delay f (t) by the same amount :
This holds under the precise condition that f be a tempered distribution (see the discussion of the Fourier transform below) . As a special case , for instance , we have the identity (understood in the distribution sense)
<formula>

= = = Composition with a function = = =

More generally , the delta distribution may be composed with a smooth function g (x) in such a way that the familiar change of variables formula holds , that
<formula>