= 0 and A2 =
m2k2 + 2 mEL2 , giving five independent constants of motion . (Since the magnitude of A , hence the eccentricity e of the orbit , can be determined from the total angular momentum L and the energy E , only the direction of A is conserved independently ; moreover , since A must be perpendicular to L , it contributes only one additional conserved quantity .)

This is consistent with the six initial conditions (the particle 's initial position and velocity vectors , each with three components) that specify the orbit of the particle , since the initial time is not determined by a constant of motion . The resulting 1 @-@ dimensional orbit in 6 @-@ dimensional phase space is thus completely specified .

A mechanical system with d degrees of freedom can have at most 2d ? 1 constants of motion , since there are 2d initial conditions and the initial time cannot be determined by a constant of motion . A system with more than d constants of motion is called superintegrable and a system with 2d ? 1 constants is called maximally superintegrable . Since the solution of the Hamilton ? Jacobi equation in one coordinate system can yield only d constants of motion , superintegrable systems must be separable in more than one coordinate system . The Kepler problem is maximally superintegrable , since it has three degrees of freedom (d = 3) and five independent constant of motion ; its Hamilton ? Jacobi equation is separable in both spherical coordinates and parabolic coordinates , as described below .

Maximally superintegrable systems follow closed , one @-@ dimensional orbits in phase space , since the orbit is the intersection of the phase @-@ space isosurfaces of their constants of motion . Consequently , the orbits are perpendicular to all gradients of all these independent isosurfaces , five in this specific problem , and hence are determined by the generalized cross products of all of these gradients . As a result , all superintegrable systems are automatically describable by Nambu mechanics , alternatively , and equivalently , to Hamiltonian mechanics .

Maximally superintegrable systems can be quantized using commutation relations , as illustrated below . Nevertheless , equivalently , they are also quantized in the Nambu framework , such as this classical Kepler problem into the quantum hydrogen atom .

= = Evolution under perturbed potentials = =

The Laplace ? Runge ? Lenz vector A is conserved only for a perfect inverse @-@ square central force . In most practical problems such as planetary motion , however , the interaction potential energy between two bodies is not exactly an inverse square law , but may include an additional central force , a so @-@ called perturbation described by a potential energy h (r) . In such cases , the LRL vector rotates slowly in the plane of the orbit , corresponding to a slow apsidal precession of the orbit .