

$= 1$ (in the reals) and $0.999 =$

0.1 (in the 10-adic), then by " blind faith and unabashed juggling of symbols " one may add the two equations and arrive at $0.999 + 0.1 = 1$. This equation does not make sense either as a 10-adic expansion or an ordinary decimal expansion , but it turns out to be meaningful and true if one develops a theory of " double-decimals " with eventually repeating left ends to represent a familiar system : the real numbers .

== Ultrafinitism ==

The philosophy of ultrafinitism rejects as meaningless concepts dealing with infinite sets , such as the summation of infinitely many numbers corresponding to the positional values of the decimal digits in $0.999 \dots$. In this approach to mathematics , only some particular (fixed) number of finite decimal digits are meaningful . Instead of " equality " , one has " approximate equality " , which is equality up to the number of decimal digits that one is permitted to compute . Doron Zeilberger , a proponent of this philosophy , calls the equality $0.999 \dots = 1$ " not even wrong " , for this reason . Instead , he argues that there is an algorithm that can check that the sequence obtained by truncating the decimal expansion of $0.999 \dots$ is approximately equal to 1 within a specified error . Although Katz and Katz (2010a) argue that ultrafinitism may capture the student intuition that $0.999 \dots$ ought to be less than 1 , the ideas of ultrafinitism do not enjoy widespread acceptance in the mathematical community , and the philosophy lacks a generally agreed upon formal mathematical foundation .

== Related questions ==

Zeno 's paradoxes , particularly the paradox of the runner , are reminiscent of the apparent paradox that $0.999 \dots$ and 1 are equal . The runner paradox can be mathematically modelled and then , like $0.999 \dots$, resolved using a geometric series . However , it is not clear if this mathematical treatment addresses the underlying metaphysical issues Zeno was exploring .

Division by zero occurs in some popular discussions of $0.999 \dots$, and it also stirs up contention . While most authors choose to define $0.999 \dots$, almost all modern treatments leave division by zero undefined , as it can be given no meaning in the standard real numbers . However , division by zero is defined in some other systems , such as complex analysis , where the extended complex plane , i.e. the Riemann sphere , has a " point at infinity " . Here , it makes sense to define $1/0$ to be infinity ; and , in fact , the results are profound and applicable to many problems in engineering and physics . Some prominent mathematicians argued for such a definition long before either number system was developed .

Negative zero is another redundant feature of many ways of writing numbers . In number systems , such as the real numbers , where " 0 " denotes the additive identity and is neither positive nor negative , the usual interpretation of " -0 " is that it should denote the additive inverse of 0 , which forces $-0 = 0$. Nonetheless , some scientific applications use separate positive and negative zeroes , as do some computing binary number systems (for example integers stored in the sign and magnitude or ones ' complement formats , or floating point numbers as specified by the IEEE floating point standard) .