= Emmy Noether =

Amalie Emmy Noether (German : [?nø?t?] ; 23 March 1882 ? 14 April 1935) was a German Jewish mathematician known for her landmark contributions to abstract algebra and theoretical physics .

She was described by Pavel Alexandrov , Albert Einstein , Jean Dieudonné , Hermann Weyl , and Norbert Wiener as the most important woman in the history of mathematics . As one of the leading mathematicians of her time , she developed the theories of rings , fields , and algebras . In physics , Noether 's theorem explains the connection between symmetry and conservation laws .

= = Introduction = =

Noether was born to a Jewish family in the Franconian town of Erlangen ; her father was a mathematician , Max Noether . She originally planned to teach French and English after passing the required examinations , but instead studied mathematics at the University of Erlangen , where her father lectured . After completing her dissertation in 1907 under the supervision of Paul Gordan , she worked at the Mathematical Institute of Erlangen without pay for seven years . At the time , women were largely excluded from academic positions . In 1915 , she was invited by David Hilbert and Felix Klein to join the mathematics department at the University of Göttingen , a world @-@ renowned center of mathematical research . The philosophical faculty objected , however , and she spent four years lecturing under Hilbert 's name . Her habilitation was approved in 1919 , allowing her to obtain the rank of Privatdozent .

Noether remained a leading member of the Göttingen mathematics department until 1933 ; her students were sometimes called the " Noether boys " . In 1924 , Dutch mathematician B. L. van der Waerden joined her circle and soon became the leading expositor of Noether 's ideas : her work was the foundation for the second volume of his influential 1931 textbook , Moderne Algebra . By the time of her plenary address at the 1932 International Congress of Mathematicians in Zürich , her algebraic acumen was recognized around the world . The following year , Germany 's Nazi government dismissed Jews from university positions , and Noether moved to the United States to take up a position at Bryn Mawr College in Pennsylvania . In 1935 she underwent surgery for an ovarian cyst and , despite signs of a recovery , died four days later at the age of 53 .

Noether 's mathematical work has been divided into three " epochs " . In the first (1908 ? 19) , she made contributions to the theories of algebraic invariants and number fields . Her work on differential invariants in the calculus of variations , Noether 's theorem , has been called " one of the most important mathematical theorems ever proved in guiding the development of modern physics " . In the second epoch (1920 ? 26) , she began work that " changed the face of [abstract] algebra " . In her classic paper Idealtheorie in Ringbereichen (Theory of Ideals in Ring Domains , 1921) Noether developed the theory of ideals in commutative rings into a tool with wide @-@ ranging applications . She made elegant use of the ascending chain condition , and objects satisfying it are named Noetherian in her honor . In the third epoch (1927 ? 35) , she published works on noncommutative algebras and hypercomplex numbers and united the representation theory of groups with the theory of modules and ideals . In addition to her own publications , Noether was generous with her ideas and is credited with several lines of research published by other mathematicians , even in fields far removed from her main work , such as algebraic topology .

= = Private life = =

Emmy 's father , Max Noether , was descended from a family of wholesale traders in Germany . At 14 , he had been paralyzed by polio . He regained mobility , but one leg remained affected . Largely self @-@ taught , he was awarded a doctorate from the University of Heidelberg in 1868 . After teaching there for seven years , he took a position in the Bavarian city of Erlangen , where he met and married Ida Amalia Kaufmann , the daughter of a prosperous merchant . Max Noether 's mathematical contributions were to algebraic geometry mainly , following in the footsteps of Alfred

Clebsch . His best known results are the Brill ? Noether theorem and the residue , or AF + BG theorem ; several other theorems are associated with him ; See Max Noether 's theorem .

Emmy Noether was born on 23 March 1882 , the first of four children . Her first name was " Amalie " , after her mother and paternal grandmother , but she began using her middle name at a young age . As a girl , Noether was well liked . She did not stand out academically although she was known for being clever and friendly . She was near @-@ sighted and talked with a minor lisp during childhood . A family friend recounted a story years later about young Noether quickly solving a brain teaser at a children 's party , showing logical acumen at that early age . She was taught to cook and clean , as were most girls of the time , and she took piano lessons . She pursued none of these activities with passion , although she loved to dance .

She had three younger brothers . The eldest , Alfred , was born in 1883 , was awarded a doctorate in chemistry from Erlangen in 1909 , but died nine years later . Fritz Noether , born in 1884 , is remembered for his academic accomplishments : after studying in Munich he made a reputation for himself in applied mathematics . The youngest , Gustav Robert , was born in 1889 . Very little is known about his life ; he suffered from chronic illness and died in 1928 .

= = Teaching = =

= = = University of Erlangen = = =

Noether showed early proficiency in French and English . In the spring of 1900 she took the examination for teachers of these languages and received an overall score of sehr gut (very good) . Her performance qualified her to teach languages at schools reserved for girls , but she chose instead to continue her studies at the University of Erlangen .

This was an unconventional decision ; two years earlier , the Academic Senate of the university had declared that allowing mixed @-@ sex education would " overthrow all academic order " . One of only two women students in a university of 986 , Noether was only allowed to audit classes rather than participate fully , and required the permission of individual professors whose lectures she wished to attend . Despite these obstacles , on 14 July 1903 she passed the graduation exam at a Realgymnasium in Nuremberg .

During the 1903 ? 04 winter semester , she studied at the University of Göttingen , attending lectures given by astronomer Karl Schwarzschild and mathematicians Hermann Minkowski , Otto Blumenthal , Felix Klein , and David Hilbert . Soon thereafter , restrictions on women 's participation in that university were rescinded .

Noether returned to Erlangen . She officially reentered the university on 24 October 1904 , and declared her intention to focus solely on mathematics . Under the supervision of Paul Gordan she wrote her dissertation , Über die Bildung des Formensystems der ternären biquadratischen Form (On Complete Systems of Invariants for Ternary Biquadratic Forms , 1907) . Although it had been well received , Noether later described her thesis as " crap " .

For the next seven years (1908 ? 15) she taught at the University of Erlangen 's Mathematical Institute without pay , occasionally substituting for her father when he was too ill to lecture . In 1910 and 1911 she published an extension of her thesis work from three variables to n variables .

Gordan retired in the spring of 1910 , but continued to teach occasionally with his successor , Erhard Schmidt , who left shortly afterward for a position in Breslau . Gordan retired from teaching altogether in 1911 with the arrival of Schmidt 's successor Ernst Fischer , and died in December 1912 .

According to Hermann Weyl , Fischer was an important influence on Noether , in particular by introducing her to the work of David Hilbert . From 1913 to 1916 Noether published several papers extending and applying Hilbert 's methods to mathematical objects such as fields of rational functions and the invariants of finite groups . This phase marks the beginning of her engagement with abstract algebra , the field of mathematics to which she would make groundbreaking contributions .

Noether and Fischer shared lively enjoyment of mathematics and would often discuss lectures long after they were over ; Noether is known to have sent postcards to Fischer continuing her train of mathematical thoughts .

= = = University of Göttingen = = =

In the spring of 1915 , Noether was invited to return to the University of Göttingen by David Hilbert and Felix Klein . Their effort to recruit her , however , was blocked by the philologists and historians among the philosophical faculty : women , they insisted , should not become privatdozent . One faculty member protested : " What will our soldiers think when they return to the university and find that they are required to learn at the feet of a woman ? " Hilbert responded with indignation , stating , " I do not see that the sex of the candidate is an argument against her admission as privatdozent . After all , we are a university , not a bath house . "

Noether left for Göttingen in late April ; two weeks later her mother died suddenly in Erlangen . She had previously received medical care for an eye condition , but its nature and impact on her death is unknown . At about the same time Noether 's father retired and her brother joined the German Army to serve in World War I. She returned to Erlangen for several weeks , mostly to care for her aging father .

During her first years teaching at Göttingen she did not have an official position and was not paid ; her family paid for her room and board and supported her academic work . Her lectures often were advertised under Hilbert 's name , and Noether would provide " assistance " .

Soon after arriving at Göttingen , however , she demonstrated her capabilities by proving the theorem now known as Noether 's theorem , which shows that a conservation law is associated with any differentiable symmetry of a physical system . American physicists Leon M. Lederman and Christopher T. Hill argue in their book Symmetry and the Beautiful Universe that Noether 's theorem is " certainly one of the most important mathematical theorems ever proved in guiding the development of modern physics , possibly on a par with the Pythagorean theorem " .

When World War I ended , the German Revolution of 1918 ? 19 brought a significant change in social attitudes , including more rights for women . In 1919 the University of Göttingen allowed Noether to proceed with her habilitation (eligibility for tenure) . Her oral examination was held in late May , and she successfully delivered her habilitation lecture in June .

Three years later she received a letter from the Prussian Minister for Science , Art , and Public Education , in which he conferred on her the title of nicht beamteter ausserordentlicher Professor (an untenured professor with limited internal administrative rights and functions) . This was an unpaid " extraordinary " professorship , not the higher " ordinary " professorship , which was a civil @-@ service position . Although it recognized the importance of her work , the position still provided no salary . Noether was not paid for her lectures until she was appointed to the special position of Lehrbeauftragte für Algebra a year later .

= = = Seminal work in abstract algebra = = =

Although Noether 's theorem had a profound effect upon physics , among mathematicians she is best remembered for her seminal contributions to abstract algebra . In his Introduction to Noether 's Collected Papers , Nathan Jacobson wrote that " The development of abstract algebra , which is one of the most distinctive innovations of twentieth century mathematics , is largely due to her ? in published papers , in lectures , and in personal influence on her contemporaries . "

Noether 's groundbreaking work in algebra began in 1920 . In collaboration with W. Schmeidler , she then published a paper about the theory of ideals in which they defined left and right ideals in a ring . The following year she published a landmark paper called Idealtheorie in Ringbereichen , analyzing ascending chain conditions with regard to (mathematical) ideals . Noted algebraist Irving Kaplansky called this work " revolutionary " ; the publication gave rise to the term " Noetherian ring " and the naming of several other mathematical objects as Noetherian .

In 1924 a young Dutch mathematician , B. L. van der Waerden , arrived at the University of

Göttingen . He immediately began working with Noether , who provided invaluable methods of abstract conceptualization . Van der Waerden later said that her originality was " absolute beyond comparison " . In 1931 he published Moderne Algebra , a central text in the field ; its second volume borrowed heavily from Noether 's work . Although Noether did not seek recognition , he included as a note in the seventh edition " based in part on lectures by E. Artin and E. Noether " . She sometimes allowed her colleagues and students to receive credit for her ideas , helping them develop their careers at the expense of her own .

Van der Waerden 's visit was part of a convergence of mathematicians from all over the world to Göttingen , which became a major hub of mathematical and physical research . From 1926 to 1930 Russian topologist Pavel Alexandrov lectured at the university , and he and Noether quickly became good friends . He began referring to her as der Noether , using the masculine German article as a term of endearment to show his respect . She tried to arrange for him to obtain a position at Göttingen as a regular professor , but was only able to help him secure a scholarship from the Rockefeller Foundation . They met regularly and enjoyed discussions about the intersections of algebra and topology . In his 1935 memorial address , Alexandrov named Emmy Noether " the greatest woman mathematician of all time " .

= = = Lecturing and students = = =

In Göttingen , Noether supervised more than a dozen doctoral students ; her first was Grete Hermann , who defended her dissertation in February 1925 . She later spoke reverently of her " dissertation @-@ mother " . Noether also supervised Max Deuring , who distinguished himself as an undergraduate and went on to contribute significantly to the field of arithmetic geometry ; Hans Fitting , remembered for Fitting 's theorem and the Fitting lemma ; and Zeng Jiongzhi (also rendered " Chiungtze C. Tsen " in English) , who proved Tsen 's theorem . She also worked closely with Wolfgang Krull , who greatly advanced commutative algebra with his Hauptidealsatz and his dimension theory for commutative rings .

In addition to her mathematical insight , Noether was respected for her consideration of others . Although she sometimes acted rudely toward those who disagreed with her , she nevertheless gained a reputation for constant helpfulness and patient guidance of new students . Her loyalty to mathematical precision caused one colleague to name her " a severe critic " , but she combined this demand for accuracy with a nurturing attitude . A colleague later described her this way : " Completely unegotistical and free of vanity , she never claimed anything for herself , but promoted the works of her students above all . "

Her frugal lifestyle at first was due to being denied pay for her work ; however , even after the university began paying her a small salary in 1923 , she continued to live a simple and modest life . She was paid more generously later in her life , but saved half of her salary to bequeath to her nephew , Gottfried E. Noether .

Mostly unconcerned about appearance and manners , biographers suggest she focused on her studies . A distinguished algebraist Olga Taussky @-@ Todd described a luncheon , during which Noether , wholly engrossed in a discussion of mathematics , " gesticulated wildly " as she ate and " spilled her food constantly and wiped it off from her dress , completely unperturbed " . Appearance @-@ conscious students cringed as she retrieved the handkerchief from her blouse and ignored the increasing disarray of her hair during a lecture . Two female students once approached her during a break in a two @-@ hour class to express their concern , but they were unable to break through the energetic mathematics discussion she was having with other students .

According to van der Waerden 's obituary of Emmy Noether , she did not follow a lesson plan for her lectures , which frustrated some students . Instead , she used her lectures as a spontaneous discussion time with her students , to think through and clarify important cutting @-@ edge problems in mathematics . Some of her most important results were developed in these lectures , and the lecture notes of her students formed the basis for several important textbooks , such as those of van der Waerden and Deuring .

Several of her colleagues attended her lectures , and she allowed some of her ideas , such as the

crossed product (verschränktes Produkt in German) of associative algebras , to be published by others . Noether was recorded as having given at least five semester @-@ long courses at Göttingen :

Winter 1924 / 25 : Gruppentheorie und hyperkomplexe Zahlen (Group Theory and Hypercomplex Numbers)

Winter 1927 / 28 : Hyperkomplexe Grössen und Darstellungstheorie (Hypercomplex Quantities and Representation Theory)

Summer 1928 : Nichtkommutative Algebra (Noncommutative Algebra)

Summer 1929 : Nichtkommutative Arithmetik (Noncommutative Arithmetic)

Winter 1929 / 30 : Algebra der hyperkomplexen Grössen (Algebra of Hypercomplex Quantities) .

These courses often preceded major publications in these areas .

Noether spoke quickly ? reflecting the speed of her thoughts , many said ? and demanded great concentration from her students . Students who disliked her style often felt alienated . Some pupils felt that she relied too much on spontaneous discussions . Her most dedicated students , however , relished the enthusiasm with which she approached mathematics , especially since her lectures often built on earlier work they had done together .

She developed a close circle of colleagues and students who thought along similar lines and tended to exclude those who did not . " Outsiders " who occasionally visited Noether 's lectures usually spent only 30 minutes in the room before leaving in frustration or confusion . A regular student said of one such instance : " The enemy has been defeated ; he has cleared out . "

Noether showed a devotion to her subject and her students that extended beyond the academic day . Once , when the building was closed for a state holiday , she gathered the class on the steps outside , led them through the woods , and lectured at a local coffee house . Later , after she had been dismissed by the Third Reich , she invited students into her home to discuss their plans for the future and mathematical concepts .

= = = Moscow = = =

In the winter of 1928 ? 29 Noether accepted an invitation to Moscow State University , where she continued working with P. S. Alexandrov . In addition to carrying on with her research , she taught classes in abstract algebra and algebraic geometry . She worked with the topologists , Lev Pontryagin and Nikolai Chebotaryov , who later praised her contributions to the development of Galois theory .

Although politics was not central to her life , Noether took a keen interest in political matters and , according to Alexandrov , showed considerable support for the Russian Revolution (1917) . She was especially happy to see Soviet advancements in the fields of science and mathematics , which she considered indicative of new opportunities made possible by the Bolshevik project . This attitude caused her problems in Germany , culminating in her eviction from a pension lodging building , after student leaders complained of living with " a Marxist @-@ leaning Jewess " .

Noether planned to return to Moscow , an effort for which she received support from Alexandrov . After she left Germany in 1933 he tried to help her gain a chair at Moscow State University through the Soviet Education Ministry . Although this effort proved unsuccessful , they corresponded frequently during the 1930s , and in 1935 she made plans for a return to the Soviet Union . Meanwhile , her brother , Fritz accepted a position at the Research Institute for Mathematics and Mechanics in Tomsk , in the Siberian Federal District of Russia , after losing his job in Germany .

= = = Recognition = = =

In 1932 Emmy Noether and Emil Artin received the Ackermann ? Teubner Memorial Award for their contributions to mathematics . The prize carried a monetary reward of 500 Reichsmarks and was seen as a long @-@ overdue official recognition of her considerable work in the field . Nevertheless , her colleagues expressed frustration at the fact that she was not elected to the Göttingen Gesellschaft der Wissenschaften (academy of sciences) and was never promoted to the position of

Ordentlicher Professor (full professor) .

Noether 's colleagues celebrated her fiftieth birthday in 1932 , in typical mathematicians ' style . Helmut Hasse dedicated an article to her in the Mathematische Annalen , wherein he confirmed her suspicion that some aspects of noncommutative algebra are simpler than those of commutative algebra , by proving a noncommutative reciprocity law . This pleased her immensely . He also sent her a mathematical riddle , the " m?? @-@ riddle of syllables " , which she solved immediately ; the riddle has been lost .

In November of the same year , Noether delivered a plenary address (großer Vortrag) on " Hyper @-@ complex systems in their relations to commutative algebra and to number theory " at the International Congress of Mathematicians in Zürich . The congress was attended by 800 people , including Noether 's colleagues Hermann Weyl , Edmund Landau , and Wolfgang Krull . There were 420 official participants and twenty @-@ one plenary addresses presented . Apparently , Noether 's prominent speaking position was a recognition of the importance of her contributions to mathematics . The 1932 congress is sometimes described as the high point of her career .

= = = Expulsion from Göttingen = = =

When Adolf Hitler became the German Reichskanzler in January 1933 , Nazi activity around the country increased dramatically . At the University of Göttingen the German Student Association led the attack on the " un @-@ German spirit " attributed to Jews and was aided by a privatdozent named Werner Weber , a former student of Noether . Antisemitic attitudes created a climate hostile to Jewish professors . One young protester reportedly demanded : " Aryan students want Aryan mathematics and not Jewish mathematics . "

One of the first actions of Hitler 's administration was the Law for the Restoration of the Professional Civil Service which removed Jews and politically suspect government employees (including university professors) from their jobs unless they had " demonstrated their loyalty to Germany " by serving in World War I. In April 1933 Noether received a notice from the Prussian Ministry for Sciences , Art , and Public Education which read : " On the basis of paragraph 3 of the Civil Service Code of 7 April 1933 , I hereby withdraw from you the right to teach at the University of Göttingen . " Several of Noether 's colleagues , including Max Born and Richard Courant , also had their positions revoked . Noether accepted the decision calmly , providing support for others during this difficult time . Hermann Weyl later wrote that " Emmy Noether ? her courage , her frankness , her unconcern about her own fate , her conciliatory spirit ? was in the midst of all the hatred and meanness , despair and sorrow surrounding us , a moral solace . " Typically , Noether remained focused on mathematics , gathering students in her apartment to discuss class field theory . When one of her students appeared in the uniform of the Nazi paramilitary organization Sturmabteilung (SA) , she showed no sign of agitation and , reportedly , even laughed about it later .

= = = Bryn Mawr = = =

As dozens of newly unemployed professors began searching for positions outside of Germany , their colleagues in the United States sought to provide assistance and job opportunities for them . Albert Einstein and Hermann Weyl were appointed by the Institute for Advanced Study in Princeton , while others worked to find a sponsor required for legal immigration . Noether was contacted by representatives of two educational institutions , Bryn Mawr College in the United States and Somerville College at the University of Oxford in England . After a series of negotiations with the Rockefeller Foundation , a grant to Bryn Mawr was approved for Noether and she took a position there , starting in late 1933 .

At Bryn Mawr , Noether met and befriended Anna Wheeler , who had studied at Göttingen just before Noether arrived there . Another source of support at the college was the Bryn Mawr president , Marion Edwards Park , who enthusiastically invited mathematicians in the area to " see Dr. Noether in action ! " Noether and a small team of students worked quickly through van der Waerden 's 1930 book Moderne Algebra I and parts of Erich Hecke 's Theorie der algebraischen Zahlen (Theory of

algebraic numbers) .

In 1934 , Noether began lecturing at the Institute for Advanced Study in Princeton upon the invitation of Abraham Flexner and Oswald Veblen . She also worked with and supervised Abraham Albert and Harry Vandiver . However , she remarked about Princeton University that she was not welcome at the " men 's university , where nothing female is admitted " .

Her time in the United States was pleasant , surrounded as she was by supportive colleagues and absorbed in her favorite subjects . In the summer of 1934 she briefly returned to Germany to see Emil Artin and her brother Fritz before he left for Tomsk . Although many of her former colleagues had been forced out of the universities , she was able to use the library as a " foreign scholar " .

= = Death = =

In April 1935 doctors discovered a tumor in Noether 's pelvis . Worried about complications from surgery , they ordered two days of bed rest first . During the operation they discovered an ovarian cyst " the size of a large cantaloupe " . Two smaller tumors in her uterus appeared to be benign and were not removed , to avoid prolonging surgery . For three days she appeared to convalesce normally , and she recovered quickly from a circulatory collapse on the fourth . On 14 April she fell unconscious , her temperature soared to 109 ° F (42 @.@ 8 ° C) , and she died . " [I] t is not easy to say what had occurred in Dr. Noether " , one of the physicians wrote . " It is possible that there was some form of unusual and virulent infection , which struck the base of the brain where the heat centers are supposed to be located . "

A few days after Noether 's death her friends and associates at Bryn Mawr held a small memorial service at College President Park 's house . Hermann Weyl and Richard Brauer traveled from Princeton and spoke with Wheeler and Taussky about their departed colleague . In the months that followed , written tributes began to appear around the globe : Albert Einstein joined van der Waerden , Weyl , and Pavel Alexandrov in paying their respects . Her body was cremated and the ashes interred under the walkway around the cloisters of the M. Carey Thomas Library at Bryn Mawr .

= = Contributions to mathematics and physics = =

Noether 's work in abstract algebra and topology was influential in mathematics , while in physics , Noether 's theorem has far @-@ ranging consequences for theoretical physics and dynamic systems . She showed an acute propensity for abstract thought , which allowed her to approach problems of mathematics in fresh and original ways . Her friend and colleague Hermann Weyl described her scholarly output in three epochs :

Emmy Noether 's scientific production fell into three clearly distinct epochs :

(1) the period of relative dependence , 1907 ? 1919 ;

(2) the investigations grouped around the general theory of ideals 1920 ? 1926 ;

(3) the study of the non @-@ commutative algebras , their representations by linear transformations , and their application to the study of commutative number fields and their arithmetics .

In the first epoch (1907 ? 19) , Noether dealt primarily with differential and algebraic invariants , beginning with her dissertation under Paul Gordan . Her mathematical horizons broadened , and her work became more general and abstract , as she became acquainted with the work of David Hilbert , through close interactions with a successor to Gordan , Ernst Sigismund Fischer . After moving to Göttingen in 1915 , she produced her seminal work for physics , the two Noether 's theorems .

In the second epoch (1920 ? 26) , Noether devoted herself to developing the theory of mathematical rings .

In the third epoch (1927 ? 35) , Noether focused on noncommutative algebra , linear transformations , and commutative number fields .

= = = Historical context = = =

In the century from 1832 to Noether 's death in 1935 , the field of mathematics ? specifically algebra ? underwent a profound revolution , whose reverberations are still being felt . Mathematicians of previous centuries had worked on practical methods for solving specific types of equations , e.g. , cubic , quartic , and quintic equations , as well as on the related problem of constructing regular polygons using compass and straightedge . Beginning with Carl Friedrich Gauss 's 1832 proof that prime numbers such as five can be factored in Gaussian integers , Évariste Galois 's introduction of permutation groups in 1832 (although , because of his death , his papers were only published in 1846 by Liouville) , William Rowan Hamilton 's discovery of quaternions in 1843 , and Arthur Cayley 's more modern definition of groups in 1854 , research turned to determining the properties of ever @-@ more @-@ abstract systems defined by ever @-@ more @-@ universal rules . Noether 's most important contributions to mathematics were to the development of this new field , abstract algebra .

= = = = Abstract algebra and begriffliche Mathematik (conceptual mathematics) = = = =

Two of the most basic objects in abstract algebra are groups and rings .
A group consists of a set of elements and a single operation which combines a first and a second element and returns a third . The operation must satisfy certain constraints for it to determine a group : It must be closed (when applied to any pair of elements of the associated set , the generated element must also be a member of that set) , it must be associative , there must be an identity element (an element which , when combined with another element using the operation , results in the original element , such as adding zero to a number or multiplying it by one) , and for every element there must be an inverse element .