= Patterns in nature =

Patterns in nature are visible regularities of form found in the natural world . These patterns recur in different contexts and can sometimes be modelled mathematically . Natural patterns include symmetries , trees , spirals , meanders , waves , foams , tessellations , cracks and stripes . Early Greek philosophers studied pattern , with Plato , Pythagoras and Empedocles attempting to explain order in nature . The modern understanding of visible patterns developed gradually over time .

In the 19th century , Belgian physicist Joseph Plateau examined soap films , leading him to formulate the concept of a minimal surface . German biologist and artist Ernst Haeckel painted hundreds of marine organisms to emphasise their symmetry . Scottish biologist D 'Arcy Thompson pioneered the study of growth patterns in both plants and animals , showing that simple equations could explain spiral growth . In the 20th century , British mathematician Alan Turing predicted mechanisms of morphogenesis which give rise to patterns of spots and stripes . Hungarian biologist Aristid Lindenmayer and French American mathematician Benoît Mandelbrot showed how the mathematics of fractals could create plant growth patterns .

Mathematics , physics and chemistry can explain patterns in nature at different levels . Patterns in living things are explained by the biological processes of natural selection and sexual selection . Studies of pattern formation make use of computer models to simulate a wide range of patterns .

= = History = =

Early Greek philosophers attempted to explain order in nature , anticipating modern concepts . Plato (c 427 ? c 347 BC) ? looking only at his work on natural patterns ? argued for the existence of universals . He considered these to consist of ideal forms (????? eidos : " form ") of which physical objects are never more than imperfect copies . Thus , a flower may be roughly circular , but it is never a perfect mathematical circle . Pythagoras explained patterns in nature like the harmonies of music as arising from number , which he took to be the basic constituent of existence . Empedocles to an extent anticipated Darwin 's evolutionary explanation for the structures of organisms .

In 1202 , Leonardo Fibonacci (c 1170 ? c 1250) introduced the Fibonacci number sequence to the western world with his book Liber Abaci . Fibonacci gave an (unrealistic) biological example , on the growth in numbers of a theoretical rabbit population . In 1917 , D 'Arcy Wentworth Thompson (1860 ? 1948) published his book On Growth and Form . His description of phyllotaxis and the Fibonacci sequence , the mathematical relationships in the spiral growth patterns of plants , is classic . He showed that simple equations could describe all the apparently complex spiral growth patterns of animal horns and mollusc shells .

The Belgian physicist Joseph Plateau (1801 ? 1883) formulated the mathematical problem of the existence of a minimal surface with a given boundary , which is now named after him . He studied soap films intensively , formulating Plateau 's laws which describe the structures formed by films in foams .

The German psychologist Adolf Zeising (1810 ? 1876) claimed that the golden ratio was expressed in the arrangement of plant parts , in the skeletons of animals and the branching patterns of their veins and nerves , as well as in the geometry of crystals .

Ernst Haeckel (1834 ? 1919) painted beautiful illustrations of marine organisms , in particular Radiolaria , emphasising their symmetry to support his faux @-@ Darwinian theories of evolution .

The American photographer Wilson Bentley (1865 ? 1931) took the first micrograph of a snowflake in 1885 .

In 1952 , Alan Turing (1912 ? 1954) , better known for his work on computing and codebreaking , wrote The Chemical Basis of Morphogenesis , an analysis of the mechanisms that would be needed to create patterns in living organisms , in the process called morphogenesis . He predicted oscillating chemical reactions , in particular the Belousov ? Zhabotinsky reaction . These activator @-@ inhibitor mechanisms can , Turing suggested , generate patterns of stripes and spots in animals , and contribute to the spiral patterns seen in plant phyllotaxis .

In 1968 , the Hungarian theoretical biologist Aristid Lindenmayer (1925 ? 1989) developed the L @-@ system , a formal grammar which can be used to model plant growth patterns in the style of fractals . L @-@ systems have an alphabet of symbols that can be combined using production rules to build larger strings of symbols , and a mechanism for translating the generated strings into geometric structures . In 1975 , after centuries of slow development of the mathematics of patterns by Gottfried Leibniz , Georg Cantor , Helge von Koch , Wac?aw Sierpi?ski and others , Benoît Mandelbrot wrote a famous paper , How Long Is the Coast of Britain ? Statistical Self @-@ Similarity and Fractional Dimension , crystallising mathematical thought into the concept of the fractal .

= = Causes = =

Living things like orchids , hummingbirds , and the peacock 's tail have abstract designs with a beauty of form , pattern and colour that artists struggle to match . The beauty that people perceive in nature has causes at different levels , notably in the mathematics that governs what patterns can physically form , and among living things in the effects of natural selection , that govern how patterns evolve .
Mathematics seeks to discover and explain abstract patterns or regularities of all kinds . Visual patterns in nature find explanations in chaos theory , fractals , logarithmic spirals , topology and other mathematical patterns . For example , L @-@ systems form convincing models of different patterns of tree growth .
The laws of physics apply the abstractions of mathematics to the real world , often as if it were perfect . For example , a crystal is perfect when it has no structural defects such as dislocations and is fully symmetric . Exact mathematical perfection can only approximate real objects . Visible patterns in nature are governed by physical laws ; for example , meanders can be explained using fluid dynamics .
In biology , natural selection can cause the development of patterns in living things for several reasons , including camouflage , sexual selection , and different kinds of signalling , including mimicry and cleaning symbiosis . In plants , the shapes , colours , and patterns of flowers like the lily have evolved to optimise insect pollination (other plants may be pollinated by wind , birds , or bats) . European honey bees and other pollinating insects are attracted to flowers by a radial pattern of colours and stripes (some visible only in ultraviolet light) that serve as nectar guides that can be seen at a distance ; by scent ; and by rewards of sugar @-@ rich nectar and edible pollen .

= = Types of pattern = =

= = = Symmetry = = =

Symmetry is pervasive in living things . Animals mainly have bilateral or mirror symmetry , as do the leaves of plants and some flowers such as orchids . Plants often have radial or rotational symmetry , as do many flowers and some groups of animals such as sea anemones . Fivefold symmetry is found in the echinoderms , the group that includes starfish , sea urchins , and sea lilies .
Among non @-@ living things , snowflakes have striking sixfold symmetry : each flake is unique , its structure forming a record of the varying conditions during its crystallisation , with nearly the same pattern of growth on each of its six arms . Crystals in general have a variety of symmetries and crystal habits ; they can be cubic or octahedral , but true crystals cannot have fivefold symmetry (unlike quasicrystals) . Rotational symmetry is found at different scales among non @-@ living things including the crown @-@ shaped splash pattern formed when a drop falls into a pond , and both the spheroidal shape and rings of a planet like Saturn .
Symmetry has a variety of causes . Radial symmetry suits organisms like sea anemones whose adults do not move : food and threats may arrive from any direction . But animals that move in one direction necessarily have upper and lower sides , head and tail ends , and therefore a left and a

right . The head becomes specialised with a mouth and sense organs (cephalisation) , and the body becomes bilaterally symmetric (though internal organs need not be) . More puzzling is the reason for the fivefold (pentaradiate) symmetry of the echinoderms . Early echinoderms were bilaterally symmetrical , as their larvae still are . Sumrall and Wray argue that the loss of the old symmetry had both developmental and ecological causes .

Trees , fractals

Fractals are infinitely self @-@ similar , iterated mathematical constructs having fractal dimension . Infinite iteration is not possible in nature so all ' fractal ' patterns are only approximate . For example , the leaves of ferns and umbellifers (Apiaceae) are only self @-@ similar (pinnate) to 2 , 3 or 4 levels . Fern @-@ like growth patterns occur in plants and in animals including bryozoa , corals , hydrozoa like the air fern , Sertularia argentea , and in non @-@ living things , notably electrical discharges . Lindenmayer system fractals can model different patterns of tree growth by varying a small number of parameters including branching angle , distance between nodes or branch points (internode length) , and number of branches per branch point .

Fractal @-@ like patterns occur widely in nature , in phenomena as diverse as clouds , river networks , geologic fault lines , mountains , coastlines , animal coloration , snow flakes , crystals , blood vessel branching , and ocean waves .

Spirals

Spirals are common in plants and in some animals , notably molluscs . For example , in the nautilus , a cephalopod mollusc , each chamber of its shell is an approximate copy of the next one , scaled by a constant factor and arranged in a logarithmic spiral . Given a modern understanding of fractals , a growth spiral can be seen as a special case of self @-@ similarity .

Plant spirals can be seen in phyllotaxis , the arrangement of leaves on a stem , and in the arrangement (parastichy) of other parts as in composite flower heads and seed heads like the sunflower or fruit structures like the pineapple and snake fruit , as well as in the pattern of scales in pine cones , where multiple spirals run both clockwise and anticlockwise . These arrangements have explanations at different levels ? mathematics , physics , chemistry , biology ? each individually correct , but all necessary together . Phyllotaxis spirals can be generated mathematically from Fibonacci ratios : the Fibonacci sequence runs 1 , 1 , 2 , 3 , 5 , 8 , 13 ... (each subsequent number being the sum of the two preceding ones) . For example , when leaves alternate up a stem , one rotation of the spiral touches two leaves , so the pattern or ratio is 1 / 2 . In hazel the ratio is 1 / 3 ; in apricot it is 2 / 5 ; in pear it is 3 / 8 ; in almond it is 5 / 13 . In disc phyllotaxis as in the sunflower and daisy , the florets are arranged in Fermat 's spiral with Fibonacci numbering , at least when the flowerhead is mature so all the elements are the same size . Fibonacci ratios approximate the golden angle , 137 @.@ 508 ° , which governs the curvature of Fermat 's spiral .

From the point of view of physics , spirals are lowest @-@ energy configurations which emerge spontaneously through self @-@ organizing processes in dynamic systems . From the point of view of chemistry , a spiral can be generated by a reaction @-@ diffusion process , involving both activation and inhibition . Phyllotaxis is controlled by proteins that manipulate the concentration of the plant hormone auxin , which activates meristem growth , alongside other mechanisms to control the relative angle of buds around the stem . From a biological perspective , arranging leaves as far apart as possible in any given space is favoured by natural selection as it maximises access to resources , especially sunlight for photosynthesis .

Chaos , flow , meanders

In mathematics , a dynamical system is chaotic if it is (highly) sensitive to initial conditions (the so @-@ called " butterfly effect ") , which requires the mathematical properties of topological mixing and dense periodic orbits .

Alongside fractals , chaos theory ranks as an essentially universal influence on patterns in nature . There is a relationship between chaos and fractals ? the strange attractors in chaotic systems have a fractal dimension . Some cellular automata , simple sets of mathematical rules that generate patterns , have chaotic behaviour , notably Stephen Wolfram 's Rule 30 .

Vortex streets are zigzagging patterns of whirling vortices created by the unsteady separation of flow of a fluid , most often air or water , over obstructing objects . Smooth (laminar) flow starts to break up when the size of the obstruction or the velocity of the flow become large enough compared to the viscosity of the fluid .

Meanders are sinuous bends in rivers or other channels , which form as a fluid , most often water , flows around bends . As soon as the path is slightly curved , the size and curvature of each loop increases as helical flow drags material like sand and gravel across the river to the inside of the bend . The outside of the loop is left clean and unprotected , so erosion accelerates , further increasing the meandering in a powerful positive feedback loop .

=== Waves , dunes ===

Waves are disturbances that carry energy as they move . Mechanical waves propagate through a medium ? air or water , making it oscillate as they pass by . Wind waves are sea surface waves that create the characteristic chaotic pattern of any large body of water , though their statistical behaviour can be predicted with wind wave models . As waves in water or wind pass over sand , they create patterns of ripples . When winds blow over large bodies of sand , they create dunes , sometimes in extensive dune fields as in the Taklamakan desert . Dunes may form a range of patterns including crescents , very long straight lines , stars , domes , parabolas , and longitudinal or Seif (' sword ') shapes .

Barchans or crescent dunes are produced by wind acting on desert sand ; the two horns of the crescent and the slip face point downwind . Sand blows over the upwind face , which stands at about 15 degrees from the horizontal , and falls on to the slip face , where it accumulates up to the angle of repose of the sand , which is about 35 degrees . When the slip face exceeds the angle of repose , the sand avalanches , which is a nonlinear behaviour : the addition of many small amounts of sand causes nothing much to happen , but then the addition of a further small amount suddenly causes a large amount to avalanche . Apart from this nonlinearity , barchans behave rather like solitary waves .

=== Bubbles , foam ===

A soap bubble forms a sphere , a surface with minimal area ? the smallest possible surface area for the volume enclosed . Two bubbles together form a more complex shape : the outer surfaces of both bubbles are spherical ; these surfaces are joined by a third spherical surface as the smaller bubble bulges slightly into the larger one .

A foam is a mass of bubbles ; foams of different materials occur in nature . Foams composed of soap films obey Plateau 's laws , which require three soap films to meet at each edge at 120 ° and four soap edges to meet at each vertex at the tetrahedral angle of about 109 @.@ 5 ° . Plateau 's laws further require films to be smooth and continuous , and to have a constant average curvature at every point . For example , a film may remain nearly flat on average by being curved up in one direction (say , left to right) while being curved downwards in another direction (say , front to back) . Structures with minimal surfaces can be used as tents . Lord Kelvin identified the problem of the most efficient way to pack cells of equal volume as a foam in 1887 ; his solution uses just one solid , the bitruncated cubic honeycomb with very slightly curved faces to meet Plateau 's laws . No better solution was found until 1993 when Denis Weaire and Robert Phelan proposed the Weaire ? Phelan structure ; the Beijing National Aquatics Center adapted the structure for their outer wall in the 2008 Summer Olympics .

At the scale of living cells , foam patterns are common ; radiolarians , sponge spicules , silicoflagellate exoskeletons and the calcite skeleton of a sea urchin , Cidaris rugosa , all resemble

mineral casts of Plateau foam boundaries . The skeleton of the Radiolarian , Aulonia hexagona , a beautiful marine form drawn by Haeckel , looks as if it is a sphere composed wholly of hexagons , but this is mathematically impossible . The Euler characteristic states that for any convex polyhedron , the number of faces plus the number of vertices (corners) equals the number of edges plus two . A result of this formula is that any closed polyhedron of hexagons has to include exactly 12 pentagons , like a soccer ball , Buckminster Fuller geodesic dome , or fullerene molecule . This can be visualised by noting that a mesh of hexagons is flat like a sheet of chicken wire , but each pentagon that is added forces the mesh to bend (there are fewer corners , so the mesh is pulled in) .

= = = Tessellations = = =

Tessellations are patterns formed by repeating tiles all over a flat surface . There are 17 wallpaper groups of tilings . While common in art and design , exactly repeating tilings are less easy to find in living things . The cells in the paper nests of social wasps , and the wax cells in honeycomb built by honey bees are well @-@ known examples . Among animals , bony fish , reptiles or the pangolin , or fruits like the Salak are protected by overlapping scales or osteoderms , these form more @-@ or @-@ less exactly repeating units , though often the scales in fact vary continuously in size . Among flowers , the Snake 's Head Fritillary , Fritillaria meleagris , have a tessellated chequerboard pattern on their petals . The structures of minerals provide good examples of regularly repeating three @-@ dimensional arrays . Despite the hundreds of thousands of known minerals , there are rather few possible types of arrangement of atoms in a crystal , defined by crystal structure , crystal system , and point group ; for example , there are exactly 14 Bravais lattices for the 7 lattice systems in three @-@ dimensional space .

= = = Cracks = = =

Cracks are linear openings that form in materials to relieve stress . When an elastic material stretches or shrinks uniformly , it eventually reaches its breaking strength and then fails suddenly in all directions , creating cracks with 120 degree joints , so three cracks meet at a node . Conversely , when an inelastic material fails , straight cracks form to relieve the stress . Further stress in the same direction would then simply open the existing cracks ; stress at right angles can create new cracks , at 90 degrees to the old ones . Thus the pattern of cracks indicates whether the material is elastic or not . In a tough fibrous material like oak tree bark , cracks form to relieve stress as usual , but they do not grow long as their growth is interrupted by bundles of strong elastic fibres . Since each species of tree has its own structure at the levels of cell and of molecules , each has its own pattern of splitting in its bark .

= = = Spots , stripes = = =

Leopards and ladybirds are spotted ; angelfish and zebras are striped . These patterns have an evolutionary explanation : they have functions which increase the chances that the offspring of the patterned animal will survive to reproduce . One function of animal patterns is camouflage ; for instance , a leopard that is harder to see catches more prey . Another function is signalling ? for instance , a ladybird is less likely to be attacked by predatory birds that hunt by sight , if it has bold warning colours , and is also distastefully bitter or poisonous , or mimics other distasteful insects . A young bird may see a warning patterned insect like a ladybird and try to eat it , but it will only do this once ; very soon it will spit out the bitter insect ; the other ladybirds in the area will remain unmolested . The young leopards and ladybirds , inheriting genes that somehow create spottedness , survive . But while these evolutionary and functional arguments explain why these animals need their patterns , they do not explain how the patterns are formed .

= = = Pattern formation = = =

Alan Turing , and later the mathematical biologist James Murray , described a mechanism that spontaneously creates spotted or striped patterns : a reaction @-@ diffusion system . The cells of a young organism have genes that can be switched on by a chemical signal , a morphogen , resulting in the growth of a certain type of structure , say a darkly pigmented patch of skin . If the morphogen is present everywhere , the result is an even pigmentation , as in a black leopard . But if it is unevenly distributed , spots or stripes can result . Turing suggested that there could be feedback control of the production of the morphogen itself . This could cause continuous fluctuations in the amount of morphogen as it diffused around the body . A second mechanism is needed to create standing wave patterns (to result in spots or stripes) : an inhibitor chemical that switches off production of the morphogen , and that itself diffuses through the body more quickly than the morphogen , resulting in an activator @-@ inhibitor scheme . The Belousov ? Zhabotinsky reaction is a non @-@ biological example of this kind of scheme , a chemical oscillator .

Later research has managed to create convincing models of patterns as diverse as zebra stripes , giraffe blotches , jaguar spots (medium @-@ dark patches surrounded by dark broken rings) and ladybird shell patterns (different geometrical layouts of spots and stripes , see illustrations) . Richard Prum 's activation @-@ inhibition models , developed from Turing 's work , use six variables to account for the observed range of nine basic within @-@ feather pigmentation patterns , from the simplest , a central pigment patch , via concentric patches , bars , chevrons , eye spot , pair of central spots , rows of paired spots and an array of dots . More elaborate models simulate complex feather patterns in the Guinea fowl , Numida meleagris , in which the individual feathers feature transitions from bars at the base to an array of dots at the far (distal) end . These require an oscillation created by two inhibiting signals , with interactions in both space and time .

Patterns can form for other reasons in the vegetated landscape of tiger bush and fir waves . Tiger bush stripes occur on arid slopes where plant growth is limited by rainfall . Each roughly horizontal stripe of vegetation effectively collects the rainwater from the bare zone immediately above it . Fir waves occur in forests on mountain slopes after wind disturbance , during regeneration . When trees fall , the trees that they had sheltered become exposed and are in turn more likely to be damaged , so gaps tend to expand downwind . Meanwhile , on the windward side , young trees grow , protected by the wind shadow of the remaining tall trees . Natural patterns are sometimes formed by animals , as in the Mima mounds of the Northwestern United States and some other areas , which appear to be created over many years by the burrowing activities of pocket gophers .

In permafrost soils with an active upper layer subject to annual freeze and thaw , patterned ground can form , creating circles , nets , ice wedge polygons , steps , and stripes . Thermal contraction causes shrinkage cracks to form ; in a thaw , water fills the cracks , expanding to form ice when next frozen , and widening the cracks into wedges . These cracks may join up to form polygons and other shapes .

= = = Pioneering authors = = =

Fibonacci , Leonardo . Liber Abaci , 1202 .
----- translated by Sigler , Laurence E. Fibonacci 's Liber Abaci . Springer , 2002 .
Haeckel , Ernst . Kunstformen der Natur (Art Forms in Nature) , 1899 ? 1904 .
Thompson , D 'Arcy Wentworth . On Growth and Form . Cambridge , 1917 .

= = = General books = = =

Adam , John A. Mathematics in Nature : Modeling Patterns in the Natural World . Princeton University Press , 2006 .
Ball , Philip . Nature 's Patterns : a tapestry in three parts . 1 : Shapes . 2 : Flow . 3 : Branches . Oxford , 2009 .
Murphy , Pat and Neill , William . By Nature 's Design . Chronicle Books , 1993 .
Rothenberg , David . Survival of the Beautiful : Art , Science and Evolution . Bloomsbury Press ,

2011 .
 Stevens , Peter S. Patterns in Nature . Little , Brown & Co , 1974 .
 Stewart , Ian . What Shape is a Snowflake ? Magical Numbers in Nature . Weidenfeld & Nicolson , 2001 .

= = = Patterns from nature (as art) = = =

 Edmaier , Bernard . Patterns of the Earth . Phaidon Press , 2007 .
 Macnab , Maggie . Design by Nature : Using Universal Forms and Principles in Design . New Riders , 2012 .
 Nakamura , Shigeki . Pattern Sourcebook : 250 Patterns Inspired by Nature .. Books 1 and 2 . Rockport , 2009 .
 O 'Neill , Polly . Surfaces and Textures : A Visual Sourcebook . Black , 2008 .
 Porter , Eliot , and Gleick , James . Nature 's Chaos . Viking Penguin , 1990 .