

= Wieferich prime =

In number theory, a Wieferich prime is a prime number p such that p^2 divides $2^p - 1 - 1$, therefore connecting these primes with Fermat's little theorem, which states that every odd prime p divides $2^p - 1 - 1$. Wieferich primes were first described by Arthur Wieferich in 1909 in works pertaining to Fermat's last theorem, at which time both of Fermat's theorems were already well known to mathematicians.

Since then, connections between Wieferich primes and various other topics in mathematics have been discovered, including other types of numbers and primes, such as Mersenne and Fermat numbers, specific types of pseudoprimes and some types of numbers generalized from the original definition of a Wieferich prime. Over time, those connections discovered have extended to cover more properties of certain prime numbers as well as more general subjects such as number fields and the abc conjecture.

As of October 2014, the only known Wieferich primes are 1093 and 3511 (sequence A001220 in the OEIS).

= = Equivalent definitions = =

The stronger version of Fermat's little theorem, which a Wieferich prime satisfies, is usually expressed as a congruence relation $2^p - 1 \equiv 1 \pmod{p^2}$. From the definition of the congruence relation on integers, it follows that this property is equivalent to the definition given at the beginning. Thus if a prime p satisfies this congruence, this prime divides the Fermat quotient $q_p(2) = \frac{2^p - 2}{p}$. The following are two illustrative examples using the primes 11 and 1093: