= x . As an example , if we let a function f represent addition (a commutative operation) so that f (x , y) =
x + y then f is a symmetric function , which can be seen in the image on the right .
For relations , a symmetric relation is analogous to a commutative operation , in that if a relation R is symmetric , then <formula> .

= = Non @-@ commuting operators in quantum mechanics = =

In quantum mechanics as formulated by Schrödinger , physical variables are represented by linear operators such as x (meaning multiply by x) , and <formula> . These two operators do not commute as may be seen by considering the effect of their compositions <formula> and <formula> (also called products of operators) on a one @-@ dimensional wave function <formula> :
<formula>
According to the uncertainty principle of Heisenberg , if the two operators representing a pair of variables do not commute , then that pair of variables are mutually complementary , which means they cannot be simultaneously measured or known precisely . For example , the position and the linear momentum in the x @-@ direction of a particle are represented respectively by the operators <formula> and <formula> (where <formula> is the reduced Planck constant) . This is the same example except for the constant <formula> , so again the operators do not commute and the physical meaning is that the position and linear momentum in a given direction are complementary .

= = = Books = = =

Axler , Sheldon (1997) . Linear Algebra Done Right , 2e . Springer . ISBN 0 @-@ 387 @-@ 98258 @-@ 2 .
Abstract algebra theory . Covers commutativity in that context . Uses property throughout book .
Copi , Irving M. ; Cohen , Carl (2005) . Introduction to Logic . Prentice Hall .
Gallian , Joseph (2006) . Contemporary Abstract Algebra , 6e . Boston , Mass . : Houghton Mifflin . ISBN 0 @-@ 618 @-@ 51471 @-@ 6 .
Linear algebra theory . Explains commutativity in chapter 1 , uses it throughout .
Goodman , Frederick (2003) . Algebra : Abstract and Concrete , Stressing Symmetry , 2e . Prentice Hall . ISBN 0 @-@ 13 @-@ 067342 @-@ 0 .
Abstract algebra theory . Uses commutativity property throughout book .
Hurley , Patrick (1991) . A Concise Introduction to Logic 4th edition . Wadsworth Publishing .

= = = Articles = = =

http : / / web.archive.org / web / 20070713072942 / http : / / www.ethnomath.org / resources / lumpkin1997.pdf Lumpkin , B. (1997) . The Mathematical Legacy Of Ancient Egypt - A Response To Robert Palter . Unpublished manuscript .
Article describing the mathematical ability of ancient civilizations .
Robins , R. Gay , and Charles C. D. Shute . 1987 . The Rhind Mathematical Papyrus : An Ancient Egyptian Text . London : British Museum Publications Limited . ISBN 0 @-@ 7141 @-@ 0944 @-@ 4
Translation and interpretation of the Rhind Mathematical Papyrus .

= = = Online resources = = =

Hazewinkel , Michiel , ed . (2001) , " Commutativity " , Encyclopedia of Mathematics , Springer , ISBN 978 @-@ 1 @-@ 55608 @-@ 010 @-@ 4
Krowne , Aaron , Commutative at PlanetMath.org. , Accessed 8 August 2007 .
Definition of commutativity and examples of commutative operations
Weisstein , Eric W. , " Commute " , MathWorld . , Accessed 8 August 2007 .

Explanation of the term commute
Yark . Examples of non @-@ commutative operations at PlanetMath.org. , Accessed 8 August 2007
Examples proving some noncommutative operations
O 'Conner , J J and Robertson , E F. MacTutor history of real numbers , Accessed 8 August 2007
Article giving the history of the real numbers
Cabillón , Julio and Miller , Jeff . Earliest Known Uses Of Mathematical Terms , Accessed 22 November 2008
Page covering the earliest uses of mathematical terms
O 'Conner , J J and Robertson , E F. MacTutor biography of François Servois , Accessed 8 August 2007
Biography of Francois Servois , who first used the term