= 1 / 2 and squared , the equation ? (1 / 2) 2 =
? reduces to the Wallis product formula . The gamma function is also connected to the Riemann zeta function and identities for the functional determinant , in which the constant ? plays an important role .
The gamma function is used to calculate the volume Vn (r) of the n @-@ dimensional ball of radius r in Euclidean n @-@ dimensional space , and the surface area Sn ? 1 (r) of its boundary , the (n ? 1) -dimensional sphere :
<formula>
<formula>
Further , it follows from the functional equation that
<formula>
The gamma function can be used to create a simple approximation to the factorial function n ! for large n : <formula> which is known as Stirling 's approximation . Equivalently ,
<formula>
As a geometrical application of Stirling 's approximation , let ?n denote the standard simplex in n @-@ dimensional Euclidean space , and (n + 1) ?n denote the simplex having all of its sides scaled up by a factor of n + 1 . Then
<formula>
Ehrhart 's volume conjecture is that this is the (optimal) upper bound on the volume of a convex body containing only one lattice point .

= = = Number theory and Riemann zeta function = = =

The Riemann zeta function ? (s) is used in many areas of mathematics . When evaluated at s = 2 it can be written as
<formula>
Finding a simple solution for this infinite series was a famous problem in mathematics called the Basel problem . Leonhard Euler solved it in 1735 when he showed it was equal to ?2 / 6 . Euler 's result leads to the number theory result that the probability of two random numbers being relatively prime (that is , having no shared factors) is equal to 6 / ?2 . This probability is based on the observation that the probability that any number is divisible by a prime p is 1 / p (for example , every 7th integer is divisible by 7 .) Hence the probability that two numbers are both divisible by this prime is 1 / p2 , and the probability that at least one of them is not is 1 ? 1 / p2 . For distinct primes , these divisibility events are mutually independent ; so the probability that two numbers are relatively prime is given by a product over all primes :
<formula>
This probability can be used in conjunction with a random number generator to approximate ? using a Monte Carlo approach .
The solution to the Basel problem implies that the geometrically derived quantity ? is connected in a deep way to the distribution of prime numbers . This is a special case of Weil 's conjecture on Tamagawa numbers , which asserts the equality of similar such infinite products of arithmetic quantities , localized at each prime p , and a geometrical quantity : the reciprocal of the volume of a certain locally symmetric space . In the case of the Basel problem , it is the hyperbolic 3 @-@ manifold SL2 (R) / SL2 (Z) .
The zeta function also satisfies Riemann 's functional equation , which involves ? as well as the gamma function :
<formula>
Furthermore , the derivative of the zeta function satisfies
<formula>
A consequence is that ? can be obtained from the functional determinant of the harmonic oscillator . This functional determinant can be computed via a product expansion , and is equivalent to the Wallis product formula . The calculation can be recast in quantum mechanics , specifically the variational approach to the spectrum of the hydrogen atom .

= = = Fourier series = = =

The constant ? also appears naturally in Fourier series of periodic functions . Periodic functions are functions on the group T = R / Z of fractional parts of real numbers . The Fourier decomposition shows that a complex @-@ valued function f on T can be written as an infinite linear superposition of unitary characters of T. That is , continuous group homomorphisms from T to the circle group U (1) of unit modulus complex numbers . It is a theorem that every character of T is one of the complex exponentials <formula> .

There is a unique character on T , up to complex conjugation , that is a group isomorphism . Using the Haar measure on the circle group , the constant ? is half the magnitude of the Radon ? Nikodym derivative of this character . The other characters have derivatives whose magnitudes are positive integral multiples of 2? . As a result , the constant ? is the unique number such that the group T , equipped with its Haar measure , is Pontrjagin dual to the lattice of integral multiples of 2? . This is a version of the one @-@ dimensional Poisson summation formula .

= = = Modular forms and theta functions = = =

The constant ? is connected in a deep way with the theory of modular forms and theta functions . For example , the Chudnovsky algorithm involves in an essential way the j @-@ invariant of an elliptic curve .

Modular forms are holomorphic functions in the upper half plane characterized by their transformation properties under the modular group <formula> (or its various subgroups) , a lattice in the group <formula> . An example is the Jacobi theta function

<formula>

which is a kind of modular form called a Jacobi form . This is sometimes written in terms of the nome <formula> .

The constant ? is the unique constant making the Jacobi theta function an automorphic form , which means that it transforms in a specific way . Certain identities hold for all automorphic forms . An example is

<formula>

which implies that ? transforms as a representation under the discrete Heisenberg group . General modular forms and other theta functions also involve ? , once again because of the Stone ? von Neumann theorem .

= = = Cauchy distribution and potential theory = = =

The Cauchy distribution

<formula>

is a probability density function . The total probability is equal to one , owing to the integral :

<formula>

The Shannon entropy of the Cauchy distribution is equal to log (4?) , which also involves ? .

The Cauchy distribution plays an important role in potential theory because it is the simplest Furstenberg measure , the classical Poisson kernel associated with a Brownian motion in a half @-@ plane . Conjugate harmonic functions and so also the Hilbert transform are associated with the asymptotics of the Poisson kernel . The Hilbert transform H is the integral transform given by the Cauchy principal value of the singular integral

<formula>

The constant ? is the unique (positive) normalizing factor such that H defines a linear complex structure on the Hilbert space of square @-@ integrable real @-@ valued functions on the real line . The Hilbert transform , like the Fourier transform , can be characterized purely in terms of its transformation properties on the Hilbert space L2 (R) : up to a normalization factor , it is the unique bounded linear operator that commutes with positive dilations and anticommutes with all reflections

of the real line . The constant ? is the unique normalizing factor that makes this transformation unitary .

= = = Complex dynamics = = =

An occurrence of ? in the Mandelbrot set fractal was discovered by David Boll in 1991 . He examined the behavior of the Mandelbrot set near the " neck " at (? 0 @.@ 75 , 0) . If points with coordinates (? 0 @.@ 75 , ?) are considered , as ? tends to zero , the number of iterations until divergence for the point multiplied by ? converges to ? . The point (0 @.@ 25 , ?) at the cusp of the large " valley " on the right side of the Mandelbrot set behaves similarly : the number of iterations until divergence multiplied by the square root of ? tends to ? .

= = Outside mathematics = =

= = = Describing physical phenomena = = =

Although not a physical constant , ? appears routinely in equations describing fundamental principles of the universe , often because of ? 's relationship to the circle and to spherical coordinate systems . A simple formula from the field of classical mechanics gives the approximate period T of a simple pendulum of length L , swinging with a small amplitude (g is the earth 's gravitational acceleration) :

<formula>

One of the key formulae of quantum mechanics is Heisenberg 's uncertainty principle , which shows that the uncertainty in the measurement of a particle 's position (?x) and momentum (?p) cannot both be arbitrarily small at the same time (where h is Planck 's constant) :

<formula>

The fact that ? is approximately equal to 3 plays a role in the relatively long lifetime of orthopositronium . The inverse lifetime to lowest order in the fine @-@ structure constant ? is

<formula>

where m is the mass of the electron .

? is present in some structural engineering formulae , such as the buckling formula derived by Euler , which gives the maximum axial load F that a long , slender column of length L , modulus of elasticity E , and area moment of inertia I can carry without buckling :

<formula>

The field of fluid dynamics contains ? in Stokes ' law , which approximates the frictional force F exerted on small , spherical objects of radius R , moving with velocity v in a fluid with dynamic viscosity ? :

<formula>

Under ideal conditions (uniform gentle slope on an homogeneously erodible substrate) , the sinuosity of a meandering river approaches ? . The sinuosity is the ratio between the actual length and the straight @-@ line distance from source to mouth . Faster currents along the outside edges of a river 's bends cause more erosion than along the inside edges , thus pushing the bends even farther out , and increasing the overall loopiness of the river . However , that loopiness eventually causes the river to double back on itself in places and " short @-@ circuit " , creating an ox @-@ bow lake in the process . The balance between these two opposing factors leads to an average ratio of ? between the actual length and the direct distance between source and mouth .

= = = Memorizing digits = = =

Piphilology is the practice of memorizing large numbers of digits of ? , and world @-@ records are kept by the Guinness World Records . The record for memorizing digits of ? , certified by Guinness World Records , is 70 @,@ 000 digits , recited in India by Rajveer Meena in 9 hours and 27 minutes

on 21 March 2015 . In 2006 , Akira Haraguchi , a retired Japanese engineer , claimed to have recited 100 @,@ 000 decimal places , but the claim was not verified by Guinness World Records .

One common technique is to memorize a story or poem in which the word lengths represent the digits of ? : The first word has three letters , the second word has one , the third has four , the fourth has one , the fifth has five , and so on . An early example of a memorization aid , originally devised by English scientist James Jeans , is " How I want a drink , alcoholic of course , after the heavy lectures involving quantum mechanics . " When a poem is used , it is sometimes referred to as a piem . Poems for memorizing ? have been composed in several languages in addition to English . Record @-@ setting ? memorizers typically do not rely on poems , but instead use methods such as remembering number patterns and the method of loci .

A few authors have used the digits of ? to establish a new form of constrained writing , where the word lengths are required to represent the digits of ? . The Cadaeic Cadenza contains the first 3835 digits of ? in this manner , and the full @-@ length book Not a Wake contains 10 @,@ 000 words , each representing one digit of ? .

= = = In popular culture = = =

Perhaps because of the simplicity of its definition and its ubiquitous presence in formulae , ? has been represented in popular culture more than other mathematical constructs .

In the 2008 Open University and BBC documentary co @-@ production , The Story of Maths , aired in October 2008 on BBC Four , British mathematician Marcus du Sautoy shows a visualization of the - historically first exact - formula for calculating ? when visiting India and exploring its contributions to trigonometry .

In the Palais de la Découverte (a science museum in Paris) there is a circular room known as the pi room . On its wall are inscribed 707 digits of ? . The digits are large wooden characters attached to the dome @-@ like ceiling . The digits were based on an 1853 calculation by English mathematician William Shanks , which included an error beginning at the 528th digit . The error was detected in 1946 and corrected in 1949 .

In Carl Sagan 's novel Contact it is suggested that the creator of the universe buried a message deep within the digits of ? . The digits of ? have also been incorporated into the lyrics of the song " Pi " from the album Aerial by Kate Bush .

In the United States , Pi Day falls on 14 March (written 3 / 14 in the US style) , and is popular among students. ? and its digital representation are often used by self @-@ described " math geeks " for inside jokes among mathematically and technologically minded groups . Several college cheers at the Massachusetts Institute of Technology include " 3 @.@ 14159 " . Pi Day in 2015 was particularly significant because the date and time 3 / 14 / 15 9 : 26 : 53 reflected many more digits of pi .

During the 2011 auction for Nortel 's portfolio of valuable technology patents , Google made a series of unusually specific bids based on mathematical and scientific constants , including ? .

In 1958 Albert Eagle proposed replacing ? by ?

= ? / 2 to simplify formulas . However , no other authors are known to use ? in this way . Some people use a different value , ? =

6 @.@ 283185 ... = 2? , arguing that ? , as the number of radians in one turn or as the ratio of a circle 's circumference to its radius rather than its diameter , is more natural than ? and simplifies many formulas . Celebrations of this number , because it approximately equals 6 @.@ 28 , by making 28 June " Tau Day " and eating " twice the pie " , have been reported in the media . However , this use of ? has not made its way into mainstream mathematics .

In 1897 , an amateur American mathematician attempted to persuade the Indiana legislature to pass the Indiana Pi Bill , which described a method to square the circle and contained text that implied various incorrect values for ? , including 3 @.@ 2 . The bill is notorious as an attempt to establish a value of scientific constant by legislative fiat . The bill was passed by the Indiana House of Representatives , but rejected by the Senate .