Course Title : Numerical Linear Algebra

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Course Status : Core

Course Aim: The aim of this course is to provide students with a solid foundation in numerical methods for solving linear algebra problems that arise in various scientific and engineering applications. Students will learn both the theory and practical algorithms used to compute solutions efficiently and accurately for large-scale systems, eigenvalue problems, and matrix factorizations.

Expected Learning Outcomes :

By the end of this course, students will be able to:

1. Understand and apply numerical methods for solving linear systems and matrix-related problems.
2. Analyze and implement algorithms for matrix factorizations, such as LU, QR, and Cholesky.
3. Solve eigenvalue problems and understand the implications of different numerical approaches.
4. Evaluate the stability and efficiency of numerical algorithms in the context of real-world applications.
5. Use software tools (e.g., MATLAB, Julia or Python) to implement and solve numerical linear algebra problems.
6. Critically assess the accuracy, convergence, and performance of numerical methods.

Prerequisite :

Linear Algebra
Basic knowledge of programming (e.g., Python, Julia or MATLAB)
Basic understanding of calculus and differential equations

Course Content :

0. Introduction and motivation through selected applications
Application examples of linear algebra methods in

- digital image processing and computer vision.
- digital signal processing and communication science.
- numerical solution of mathematical optimization problems.
- numerical solution of partial differential equations.

- document data representation and web-page ranking.

1. A Review on Basic Concepts:

1.1. Vectors

1.2. Matrices

2. Numerical solution methods for systems of linear equations

   2.1 Direct Methods for Linear Systems:
      - Gaussian elimination, pivoting strategies, and LU factorization.
      - Cholesky factorization for symmetric positive-definite matrices.

   2.2 Iterative Methods for Solving Linear Systems:
      - Jacobi, Gauss-Seidel, and Successive Over-Relaxation (SOR) methods.
      - Conjugate gradient and other Krylov subspace methods.
      - Preconditioning techniques.

3. Eigenvalue Problems:
   - Power method and inverse iteration.
   - QR algorithm for eigenvalue computations.
   - Applications to stability and vibrations.

4. Matrix Factorizations:
   - QR decomposition and its applications.
   - Singular Value Decomposition (SVD) and applications in data analysis.

5. Least Squares Problems:
   - Normal equations and QR decomposition for solving least squares problems.
   - Regularization methods for ill-conditioned systems.

6. Applications of Numerical Linear Algebra* (optional, depending on time):
   - Applications in machine learning, image processing, and optimization.
   - Large-scale computations and sparsity.

Software: for this course will be using Julia Language
Go here to install Julia: Julia installation
Go here to add Julia Kernel into jupyter notebook: add Julia kernel to Jupyter

Assessment Methods :

15 Minutes Quiz (each Wednesday)
Individual assignment (2) (First and second week)
Extended Quiz (1) (Last week on Friday/Saturday)

Reading List :

1. Reference Textbooks

- Demmel, J.W.: Applied Numerical Linear Algebra. SIAM.
- Data, B.N.: Numerical Linear Algebra and Applications. SIAM.
- Golub, G.H., Van Loan, C.F.: Matrix Computation (4th ed.). The Johns Hopkins UnviersityPress, 2013.
- Trefethen, L.N., Bau, D.: Numerical Linear Algebra. SIAM.
- Watkins, D.S.: Fundamentals of Matrix Computation (2nd ed.). Wiley-Interscience, 2002.

2.Selected Literature

- Cipra, B.A. The Best of the 20th Century: Editors Name Top 10 Algorithms. SIAM News, 33(4).
- Cui, Y.; Morikuni, K.; Tsuchiya, T.; Hayami,K., (2019). Implementation of interior- point methods for LP based on Krylov subspace iterative solvers with inner-iteration preconditioning. Computational Optimization and Applications. 74(3):143–176.
- D’Apuzzo, M.; De Simone, V.; di Serafino, D. (2008). On mutual impact of numerical linear algebra and large-scale optimization with focus on interior point methods. Computational Optimization and Applications, 45(2): 283–310.
- Demmel, J.; Kahan, W. (1990): Accurate singular values of bidiagonal matrices. SIAM Journal on Scientific and Statistical Computing, 11(5): 873-912.
- Eckart, C.; Young, G. (1936). The approximation of one matrix by another of lower rank. Psychometrika, 1(3):211–218.
- Elden, L.: Matrix Methods in Data Mining and Pattern Recognition. SIAM, 2019.
- Elden, L. (2006). Numerical linear algebra in data mining. Acta Numerica, (2006):1-57.
- Fletcher, R., Conjugate gradient methods for indefinite systems, in:Proc. Dundee Biennial
  Conf. on Numerical Analysis, G. Watson (editor), Springer, New York, 1975, pp. 73-89.

3.Further References

- Gentle, J.E.: Matrix Algebra: Theory, computations, and applications in statistics (2nd. ed.). Springer, 2017.
- Higham, N.J.: Accuracy and Stability of Numerical Algorithms (2nd. Ed.). SIAM, 2002.
- Horn, R. A.; Johnson, C. R.: Topics in Matrix Analysis (2nd ed.). Cambridge University
  Press, 2013.

- Petersen, K.B.; Pedersen, M.S.: The Matrix Cookbook. Technical University of Denmark.
  (available online at: https://www2.imm.dtu.dk/pubdb/pubs/3274-full.html)
- Cichocki, A.; Zdunek, R.; Phan, A.H; Amari, S.-i.: Nonnegative Matrix and Tensor Factorizations. John Wiley & Sons, Ltd, 2009.
- Davis, T. A.: Direct Methods for Sparse Linear Systems. SIAM, 2006.
- Davis, T. A.; Rajamanickam, S., Sid-Lakhdar, W.M., (2016): A survey of direct methods for sparse linear systems. Acta Numerica, 25:383-566.
- Dongarra, J.J.; Kontoghiorghes, E.J. (editors): Parallel Numerical Linear Algebra. Nova Science Pub Inc., 2002.
- Kolda, T.G.; Bader, B.W., (2009). Tensor decompositions and applications. SIAM Reviews, 51(3):455-500.
- O'Leary, D.P., (2000). Symbiosis between linear algebra and optimization. Journal of Computational and Applied Mathematics, 123(1-2): 447-465.