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Relay-based Estimation of Multiple Points on Process Frequency Response*

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Key Words—Frequency responses; relay feedback; identification; FFT

Abstract—In this paper, a new technique for process frequency-response identification is proposed, which can identify multiple points on a process frequency response from a single relay feedback test. A modified relay is proposed and it can effectively excite a process not only at the process critical frequency ω_c but also at $0.5\omega_c$ and $1.5\omega_c$. As a result, the process frequency responses at multiple frequencies around ω_c can be accurately estimated from one relay test using the FFT algorithm. The technique has been tested on various processes in real-time. The results show that the method is insensitive to noise and step-like load disturbance. The technique can also be easily extended to identify other frequency-response points of interest. © 1997 Elsevier Science Ltd.

1. *Introduction*

Process frequency-response identification is important in system analysis and control design. The frequency range of interest for such applications is usually from zero up to the process critical frequency ω_c (Hagglund, 1991). For traditional frequency-response identification, the frequencies of the exciting signals should be carefully selected based on the prior knowledge of process bandwidth. The relay feedback, however, can *automatically* excite an unknown stable process around this important frequency ω_c (Astrom and Hagglund, 1988). Furthermore, the traditional frequency response identification usually adopts an open-loop test while the relay feedback itself is a closed-loop controller. A closed-loop test is preferred to an open-loop one in control applications since it keeps the process close to the setpoint so that the process operates in a linear region where the frequency response is of interest. Relay-based process identification and controller tuning have received a great deal of attention (Li *et al.*, 1991; Leva, 1993; Palmor *et al.*, 1994; Shen *et al.*, 1996; Huang *et al.*, 1996). While the standard method of Astrom and Hagglund (1988) is successful in many process control applications, it also faces two major problems. (i) Due to the adoption of describing function approximation, the estimation of the critical point is not accurate enough for some kind of processes (Huang *et al.*, 1996). (ii) Only crude controller settings can be obtained based on this single point (Astrom, 1991). Within the context of the relay test, several modified identification methods have been reported (Li *et al.*, 1991; Leva, 1993; Palmor *et al.*, 1994; Shen *et al.*, 1996 Huang *et al.*, 1996). To identify two or more points on the process frequency response, additional linear components (or varying hysteresis width) have to be connected into the system and additional relay tests have to be performed (Li *et al.*, 1991; Leva, 1993; Palmor *et al.*, 1993, 1994, 1995). These methods are time consuming and the resultant estimations are still approximate in nature since they actually make repeated use of the standard method. Shen *et al.* (1996) and Huang *et al.* (1996) proposed a biased relay to obtain the static gain of a process together with the critical point. To identify *more* and *accurate* points on the process frequency response from one relay test is then appealing.

For a standard relay, it can excite the process only at ω_c, $3\omega_c$, ... frequencies. We know that the process frequency response around ω_c is most important for controller design. To estimate more points around ω_c in one relay test, a modified relay is then proposed. A parasitic relay is superimposed to the standard relay and the parasitic relay on–off period is twice as large as that of the standard relay. With this arrangement, besides the standard excitations at $\omega_c, 3\omega_c, \ldots$, we also have stimulation at $0.5\omega_c, 1.5\omega_c, \ldots$, enabling process frequency-response estimation at all these frequencies. To further improve the estimation accuracy, we avoid the description function method which is employed in standard relay-based identification. Instead, we employ the FFT algorithm to a period of stationary oscillations of the process input and output from the relay feedback. Real-time tests show that this modified relay test can produce accurate identification results at $0.5\omega_c$, ω_c and $1.5\omega_c$ under noisy environment and under step-like load disturbance.

The paper is organized as follows. In Section 2, the proposed method is presented. Some real-time identification examples are given in Section 3 to verify the method. Conclusions are in Section 4.

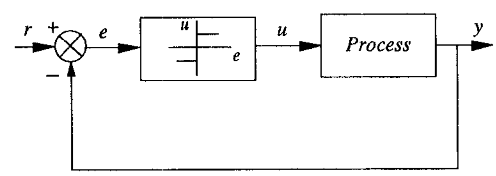

Fig. 1. Standard relay feedback system.

2. *The method*

2.1. *Standard relay method.* Consider a standard relay feedback system shown in Fig. 1, where the relay output amplitude is h. If the process has a phase lag of at least π radians, the relay feedback will usually drive the system to a stationary oscillation (Astrom and Hagglund, 1984). The resulting period T_c and amplitude a of this process output oscillation can be measured. The process critical point at

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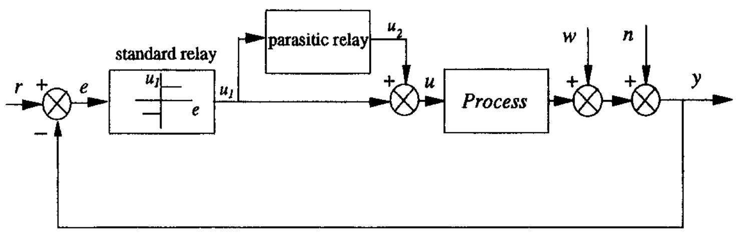

Fig. 2. Modified relay feedback system.

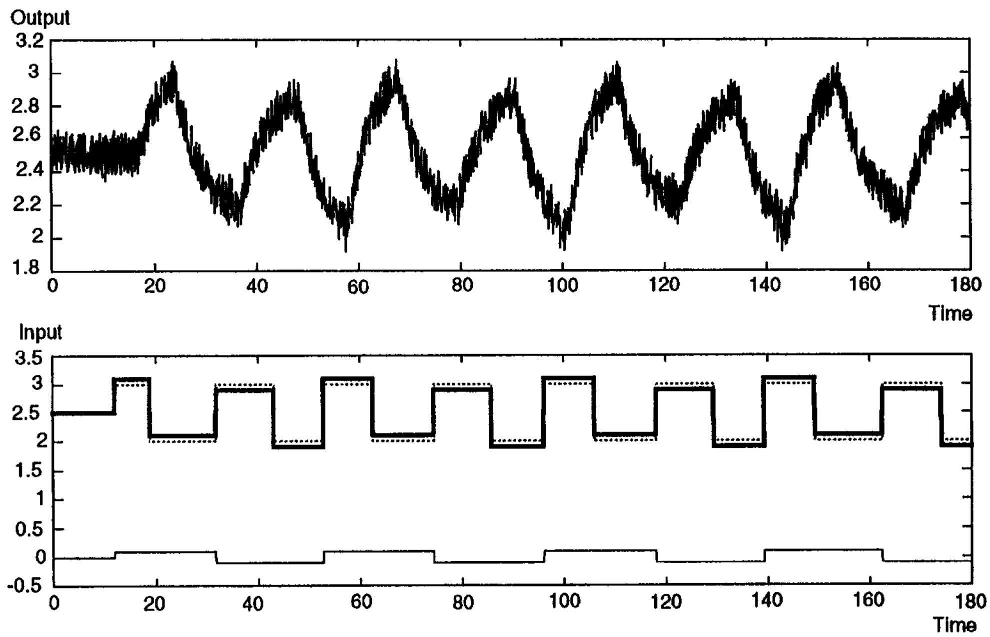

Fig. 3. Process input and output in the modified relay test. (——) u, (- - -) u_1 and (——) u_2.

phase-crossover in terms of the ultimate frequency ω_c and ultimate gain k_u can then be determined by the describing function method as

$$\omega_c = \frac{2\pi}{T_c}, \tag{1}$$

$$k_u = \frac{4h}{a\pi}. \tag{2}$$

The accuracy of the method is limited due to the adoption of describing function approximation. Besides, only one point on process frequency response is obtained from one relay test. To find more points, one has to perform more tests each for one point by cascading a different linear element to the process.

2.2. *Modified relay method.* The modified relay consists of a standard relay and a parasitic relay as shown in Fig. 2. The standard relay operates as usual with amplitude of the sampled output $u_1(k)$ being h, where $u_1(k)$ is the kth sample of $u_1(t)$. It is well known that this relay can excite process mainly at frequency ω_c. In order to additionally provide the effective excitation to the process at other frequencies than ω_c for use in control while maintaining the process output oscillation under such an arrangement, a parasitic relay with output amplitude $\alpha\ h$ and twice period of $u_1(k)$ is introduced and superimposed to $u_1(k)$. This implies that the output $u_2(k)$ of the parasitic relay flip-flops immediately once every period of oscillations in $u_1(k)$ is reached. The parasitic relay is realized by

$$\begin{cases} u_2(0) = \alpha h, \\ u_2(k) = -\alpha h \cdot \mathrm{sign}(u_2(k-1)) & \text{if } u_1(k-1) > 0 \text{ and } u_1(k) < 0, \\ u_2(k) = u_2(k-1) & \text{otherwise}, \end{cases} \tag{3}$$

where α is a constant coefficient. α should be large enough to have sufficient stimulation on the process while it should also be small enough such that the parasitic relay will not change the period of oscillation generated by the main relay too much. According to the extensive simulation, α is recommended to be 0.1–0.3. The output of the modified relay test is thus given by $u(k) = u_1(k) + u_2(k)$, and is sent as the input to process.

In this way, the process is stimulated by two different excitations whose periods are T_c and $2T_c$, respectively. The resultant process output y from the modified relay test is shown in Fig. 3 and will reach a stationary oscillation with the period being $2T_c$. Due to two excitations in u, y consists of frequency components at $(2\pi/T_c)$, (π/T_c) and their odd harmonics $(6\pi/T_c)$, $(10\pi/\ T_c), \ldots,$ and $(3\pi/T_c)$, $(5\pi/\ T_c)\ldots,$ respectively. For a linear process, the process frequency response can be obtained by

$$G(j\omega_i) = \frac{\int_0^{2T_c} y_s(t)e^{-j\omega_i t}dt}{\int_0^{2T_c} u_s(t)e^{-j\omega_i t}dt}, \quad i = 1, 2, \ldots, \tag{4}$$

where

$$\omega_l = \frac{(2i-1)2\pi}{2^l T_c}, \quad l = 0, 1, \ldots,$$

are the basic and its odd harmonic frequencies in u_s and y_s, u_s and y_s are a period $(2T_c)$ of the stationary

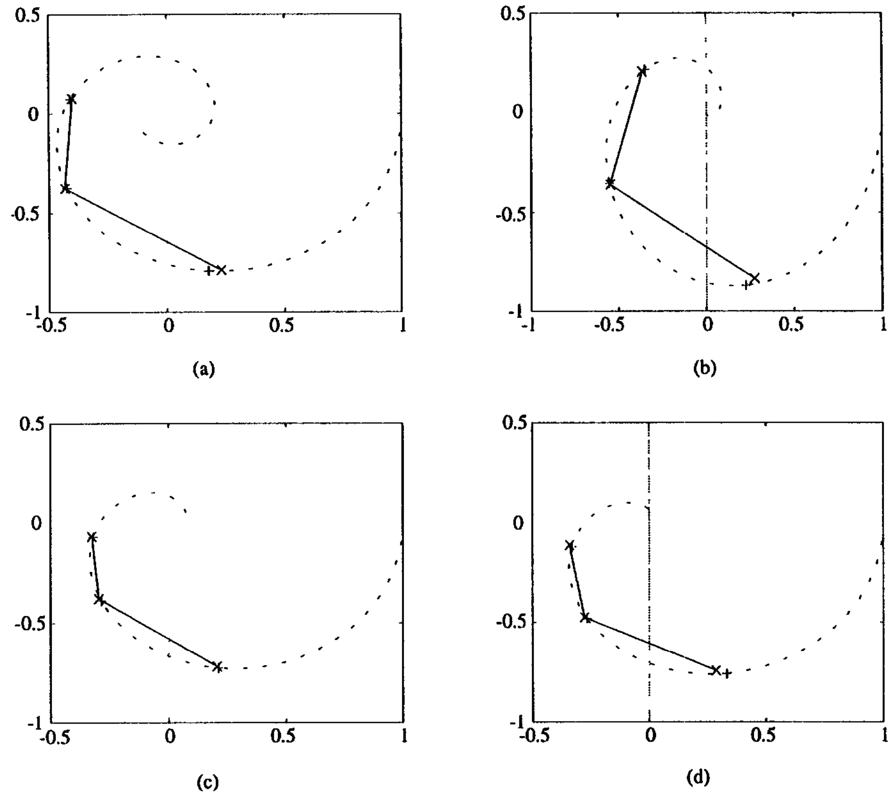

Fig. 4. Nyquist plots (- + -) Actual, (– × –) Estimated under N_1=10% noise.

oscillations of $u(k)$ and $y(k)$, respectively. $G(j\omega_i)$ in (4) can be computed using the FFT algorithm (Morrison, 1994) as

$$G(j\omega_i) = \frac{\mathrm{FFT}(y_s)}{\mathrm{FFT}(u_s)}. \tag{5}$$

Since the method adopts the spectrum analysis method instead of the describing function, it will lead to a more accurate process frequency-response estimation. The proposed method employs the FFT only once and the required computation burden is modest. It can identify multiple points on frequency response from a single relay test. Moreover, the method can be easily extended to find other points on the frequency response. You can flip-flop the parasitic relay at every 3 or 4 periods of the main oscillations generated by the standard relay and get other frequency points. You can also use more than one parasitic relay in a relay test and find more points on frequency response in one relay test. To estimate the static gain of a process, either a small set-point change at reference may be introduced or a bias may be added to our modified relay.

In a realistic environment, the major concerns for any identification method are disturbance and noise. It should be noted that our identification method is unaffected by a step-like load disturbance w in Fig. 2, which is a common case in practice. This can easily be shown from (4) as

$$\begin{aligned}\frac{\int_0^{2T_c} \tilde{y}_s(t)e^{-j\omega_i t}\,dt}{\int_0^{2T_c} \tilde{u}_s(t)e^{-j\omega_i t}dt} &= \frac{\int_0^{2T_c}[y_s(t)+w(t)]e^{-j\omega_i t}dt}{\int_0^{2T_c}(u_s(t)+\bar{u})e^{-j\omega_i t}dt}\\ &= \frac{\int_0^{2T_c} y_s(t)e^{-j\omega_i t}dt + w\int_0^{2T_c} e^{-j\omega_i t}dt}{\int_0^{2T_c} u_s(t)e^{-j\omega_i t}dt + \bar{u}\int_0^{2T_c} e^{-j\omega_i t}dt}\\ &= \frac{\int_0^{2T_c} y_s(t)e^{-j\omega_i t}dt}{\int_0^{2T_c} u_s(t)e^{-j\omega_i t}dt}\\ &= G(j\omega_i),\quad i = 1, 2, \ldots,\end{aligned} \tag{6}$$

where ω_i has the same definition as in (4). Our method is also insensitive to noise. As to the measurement of noise in the relay test, Astrom and Hagglund (1988) pointed out that a hysteresis in the relay is a simple way to reduce the influence of the measured noise. The width of hysteresis should be bigger than the noise band and is usually chosen 2 times larger than the noise band (Hang *et al.*, 1993). Filtering is another possibility (Astrom and Hagglund, 1984). To reduce noise effect further, especially in the case of large noise-to-signal ratio, we use the average of the last 2–4 periods of oscillations as the stationary oscillation period, depending on the noise level. With these anti-noise measures, the proposed method can reject noise quite effectively, and provide accurate frequency response estimation at frequencies of $0.5\omega_c$, ω_c and $1.5\omega_c$. It should be also noted that a non-zero initial condition of the process at the start of a relay test has no effect on our estimation because only stationary oscillations u_s and y_s after transient are used in the estimation.

3. *Real-time testing*

The proposed frequency-response identification method has been tested on several typical processes in real-time experiments. For assessment of accuracy, the identification error here is measured by the worst case error

$$\mathrm{ERR} = \max_i \left\{ \left| \frac{\hat{G}(j\omega_i) - G(j\omega_i)}{G(j\omega_i)} \right| \times 100\%,\ i = 1,2,3 \right\}, \tag{7}$$

where $G(j\omega_i)$ and $\hat{G}(j\omega_i)$ are the actual and the estimated process frequency responses respectively. The process frequency responses at (π/T_c), $(2\pi/T_c)$ and $(3\pi/T_c)$ are considered since the frequency response in these regions are especially important to controller design.

In order to test our method in a realistic environment, real-time relay tests were performed using the *Dual Process Simulator KI* 100 from *KentRidge Instruments*

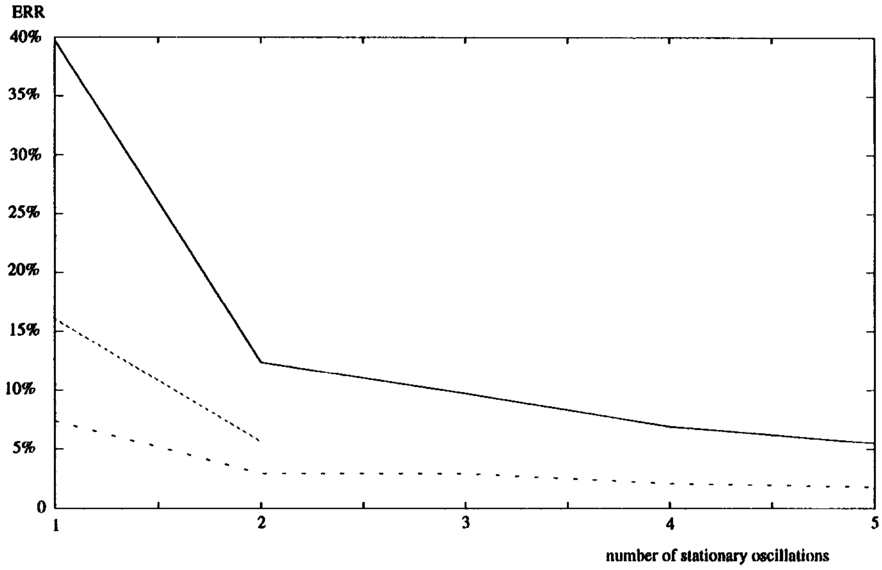

Fig. 5. ERR vs Number of stationary oscillations adopted. (- - -) N_1=0%, ($\cdots$) N_1=1% and (——) N_1=10%.

Table I Identification error (ERR)

Disturbance	$d=0$			$d=0.5$
Noise	N_1=0%	1%	10%	10%
Processes	N_2=4%	11%	31%	31%
$\frac{1}{5s+1}e^{-5s}$	2.57%	5.02%	6.83%	7.17%
$\frac{1}{(s+1)^8}$	2.93%	5.46%	6.90%	6.35%
$\frac{1}{(s+1)(5s+1)}e^{-2.5s}$	5.01%	5.08%	5.41%	5.16%
$\frac{1-s}{(2s+1)^2(5s+1)}e^{-0.5s}$	3.55%	5.17%	6.38%	5.13%

(KentRidge, 1992). The simulator is an analog process simulator and can be configured to simulate a wide range of industrial processes with different levels of noise and disturbance. The Simulator is connected to a PC-computer via an A/D and D/A board. The window-based *DT VEE* 3.0 (Data Translation, 1995) is used as the system control platform, on which the relay control code is written in C++. The fastest sampling time of the *VEE* system is 0.06 s. In the context of system identification, noise-to-signal ratio (Haykin, 1989) is usually defined as

$$\text{Noise-to-signal power spectrum ratio} = \frac{\text{mean power spectrum density of noise}}{\text{mean power spectrum density of signal}},$$

(denoted by N_1) or

$$\text{Noise–signal mean ratio} = \frac{\text{mean(abs(noise))}}{\text{mean(abs(signal))}},$$

(denoted by N_2). A few examples of real-time testing are presented as follows.

Example 1. Consider a first-order plus dead-time process

$$G(s)\frac{1}{5s+1}e^{-5s}.$$

In our relay test, the standard relay amplitude is chosen as 0.5 and the parasitic relay height is set to 20%×0.5. Without additional noise, the noise-to-signal ratio N_1 of the inherent noise in our test environment is 0.025% (N_2=4%). The identification error ERR is 2.57%. To see noise effects, extra noise is introduced with the noise source in the *Simulator*. Time sequences of $y(t)$ and $u(t)$ in a relay test under N_1=10% (N_2=31%) are shown in Fig. 3. The first part of the test in Fig. 3 (t=0–12) is the "listening period", in which the noise bands of $y(t)$ and $u(t)$ at steady state are measured. Under this noise, the hysteresis is chosen as 0.3. With averaging 4 periods of stationary oscillations, the estimated frequency-response points under this noise level are shown in Fig. 4(a). The result is pretty good.

To ensure estimation accuracy under different noise levels, the number of stationary oscillation periods adopted in average calculation should be different. The estimation error ERR vs the number of stationary oscillation periods adopted in average is plotted in Fig. 5, which can be used as a guide in deciding how many periods are enough to achieve a certain estimation accuracy under a given noise level.

Under noise level N_1=10% (N_2=31%), the real time testing on proposed method is also performed on other typical processes listed below.

Example 2. For a multi-lag high-order process

$$G(S)=\frac{1}{(s+1)^8},$$

the actual and the estimated frequency responses are plotted in Fig. 4(b), which is satisfactory.

Example 3. Consider a process which has different poles

$$G(s)=\frac{1}{(s+1)(5s+1)}e^{-2.5s}.$$

The actual and the estimated frequency responses are presented in Fig. 4(c). The accuracy of the estimated frequency response is excellent.

Example 4. For a non-minimum phase plus dead time process

$$G(s)=\frac{1-s}{(2s+1)^2(5s+1)}e^{-0.5s},$$

the actual and the estimated Nyquist curves are shown in Fig. 4(d). The accuracy of the proposed method is evident.

Table I shows the identification accuracy of the above four real-time examples under different noise and disturbance levels. Evidently, the identification results are satisfactory.

4. *Conclusions*

A new method for process frequency-response identification has been developed in the context of the relay feedback test. The method has several unique features. First, it can estimate multiple points on process frequency-response simultaneously with one single relay experiment and this saves testing time greatly. Second, the method is accurate since no approximation is made. The computations involved are simple so that it can be easily implemented on micro-processors. Third, the method is insensitive to noise and step-like load disturbance, and non-zero initial condition. Various processes have been employed to demonstrate the effectiveness of the method in real time. The identified process frequency response is useful for process analysis and controller design.

References

Astrom, K. J. and T. Hagglund (1984). Automatic tuning of simple regulators with specifications on phase and amplitude margins *Automatica*, **20**, 645–651.

Astrom, K. J. (1988). *Automatic Tuning of PID Controllers*. Instrument Society of America.

Astrom, K. J. (1991). Assessment of achievable performance of simple feedback loops. *Int. J. Adaptive Control Signal Process.*, **5**, 3–19.

Data Translation (1995). *Building an Operator Interface in DT VEE, User Manual*, Data Translation, Inc.

Hang, C. C., T. H. Lee and W. K. Ho (1993). *Adaptive Control*, Instrument Society of America.

Haykin, S. (1989). *An Introduction to Analog & Digital Communications*. Wiley, New York.

Huang, H. P. *et al.* (1996). Auto-tuning for model-based PID controllers. *A.I.Ch.E. J.*, **42**, 1174–1180.

Hagglund, T. (1991). *Process Control in Practice*. Studentlitteratur and Chartwell-Bratt.

KentRidge Instruments Pte Ltd (1992). *Dual Process Simulator KI 100, User Guide.*

Leva, A. (1993). PID Auto-tuning algorithm based on relay feedback. *IEE Proc.D*, **140**, 328–338.

Li, W. *et al.* (1991). An improved auto-tune identification method. *Ind. Eng. Chem. Res.*, **30**, 1530–1541.

Morrison, N. (1994). *Introduction to Fourier Analysis*. Wiley, New York.

Palmor, Z. J., Y. Halevi and N. Krasney (1993). Automatic tuning of decentralized PID controllers for TITO processes. *Proc. IFAC 12th World Congr., Sydney*, **2**, 311–314.

Palmor, Z. J. and M. Blau (1994). An auto-tuner for Smith dead time compensator. *Int. J. Control*, **60**, 117–135.

Palmor, Z. J., Y. Halevi and N. Krasney (1995). Automatic tuning of decentralized PID controllers for TITO processes. *Automatica*, **31**, 1001–1010.

Shen, S. H., J. S. Wu and C. C. Yu (1996). Use of biased-relay feedback for system identification. *A.I.Ch.E. J.*, **42**, 1174–1180.