

EESC6360

Project 2 – FIR and IIR Filtering

Josiah Smith

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Question 1:

Write a MATLAB Program to compute and display the poles and zeros, to compute and display the factor form, and to generate the pol-zero plot of the z-transform that is a ratio of two polynomials in z^{-1} . Using this program, analyze the z-transform $G(z)$ of Equation 1.

$$G(Z) = \frac{2 + 5z^{-1} + 9z^{-2} + 5z^{-3} + 3z^{-4}}{5 + 45z^{-1} + 2z^{-2} + z^{-3} + z^{-4}} \quad (1)$$

Solution:

% CODE:

```
num = [2,5,9,5,3];
den = [5,45,2,1,1];
[p_i1,z_i1] = question1(num,den,"G(z)");

function [poleG,zeroG] = question1(num,den,str)
syms z
poleG = roots(den);
zeroG = zeros(length(den)-1,1);
zeroG(1:length(num)-1,1) = roots(num);
disp("Zeros:")
disp(zeroG');
pause(1)
disp("Poles:");
disp(poleG');
pause(1)
denz = 1;
numz = 1;
for i = 1:length(zeroG)
    numz = numz * (1 - zeroG(i)*z^-1);
end
for j = 1:length(poleG)
    denz = denz * (1 - poleG(j)*z^-1);
end
disp("Factored G(z)")
pretty(numz);
if den ~= 1
    disp("-----")
    pretty(denz);
end
s = tf('s');
Gs = tf(num,den);
figure
pzplot(Gs*s^(length(den)-length(num)));
title("Zero-Pole Plot of " + str)
end
```

Output:

Zeros:

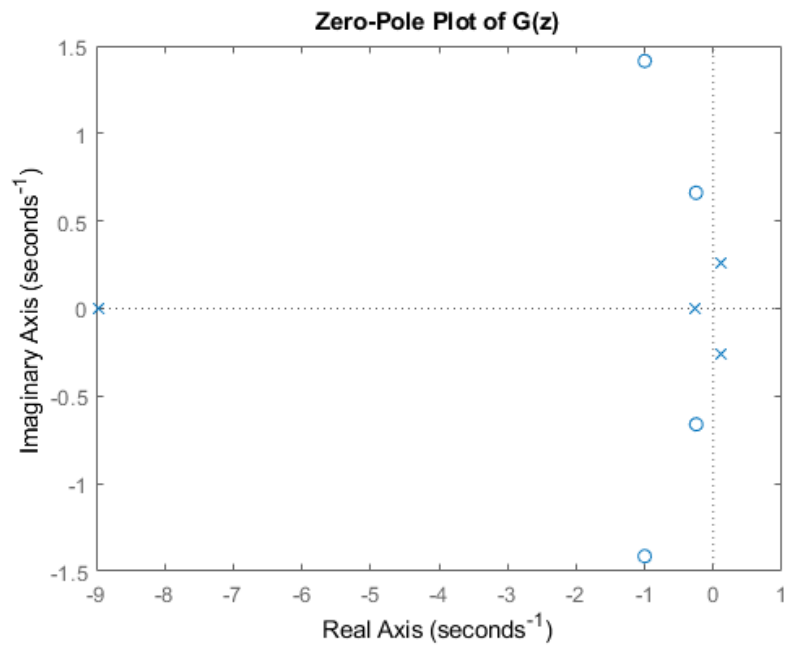
-1.0000 - 1.4142i -1.0000 + 1.4142i -0.2500 - 0.6614i -0.2500 + 0.6614i

Poles:

-8.9576 + 0.0000i -0.2718 + 0.0000i 0.1147 - 0.2627i 0.1147 + 0.2627i

Factored G(z)

$$\frac{\left(\frac{-1 + \sqrt{2}i}{z} - 1 \right) \left(\frac{-1 + \sqrt{2}i}{z} + 1 \right)}{\left(\frac{-8.9576 + 0.0000i}{z} - 1 \right) \left(\frac{-0.2718 + 0.0000i}{z} - 1 \right) \left(\frac{0.1147 - 0.2627i}{z} - 1 \right) \left(\frac{0.1147 + 0.2627i}{z} - 1 \right)}$$



Question 2:

Write a MATLAB program to compute and display the rational z-transform from its zeros, poles, and gain constant. Using this program, determine the rational form of a z-transform with the given zeros, poles, and gain constant.

Solution:

% CODE:

```
z_i = [0.3,2.5,-0.2+1j*0.4,-0.2-1j*0.4];  
p_i = [0.5,-0.75,0.6+1j*0.7,0.6-1j*0.7];  
b0 = 3.9;  
[b,a] = question2(z_i,p_i,b0)
```

```
function [b,a] = question2(z_i,pi,b0)  
syms z  
numz = 1;  
denz = 1;  
for i = 1:length(z_i)  
    numz = numz*(z-z_i(i));  
end  
numz = expand(numz);  
for j = 1:length(pi)  
    denz = denz*(z-pi(j));  
end  
denz = expand(denz);  
disp("G(z) = ");  
pretty(b0*numz/denz)  
b = sym2poly(numz);  
a = sym2poly(denz);  
end
```

Output:

G(z) =

$$\frac{39z^4}{10} + \frac{234z^3}{25} + \frac{663z^2}{1000} + \frac{507z}{500} - \frac{117}{200}$$
$$- \frac{z^4}{20} - \frac{19z^3}{40} + \frac{7z^2}{80} + \frac{53z}{160} - \frac{51}{160}$$

b =

3.9000 -9.3600 -0.6630 -1.0140 0.5850

a =

1.0000 -0.9500 0.1750 0.6625 -0.3187

Question 3:

Write a MATLAB program to compute and display the magnitude and phase response of a filter with rational z-transform. Use this program to display the magnitude and phase response for $G(z)$.

Solution:

% CODE:

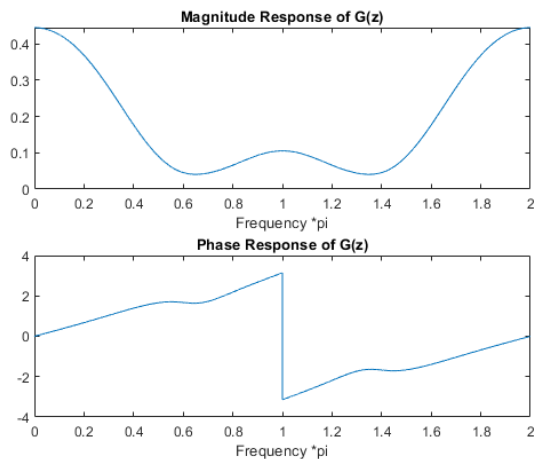
```
num = [2,5,9,5,3];
den = [5,45,2,1,1];
question3(num,den,"G(z)");

function question3(num,den,str)
w = 0:0.001:2*pi;
numw = 0;
denw = 0;
for i = 1:length(num)
    numw = numw + num(i).*exp(1j*w*(i-1));
end
for i = 1:length(den)
    denw = denw + den(i).*exp(1j*w*(i-1));
end

figure
subplot(211)
Hw = numw./denw;
plot(w/pi,abs(Hw))
xlabel('Frequency *pi')
title("Magnitude Response of " + str)

subplot(212)
plot(w/pi,angle(Hw))
xlabel('Frequency *pi')
title("Phase Response of " + str)

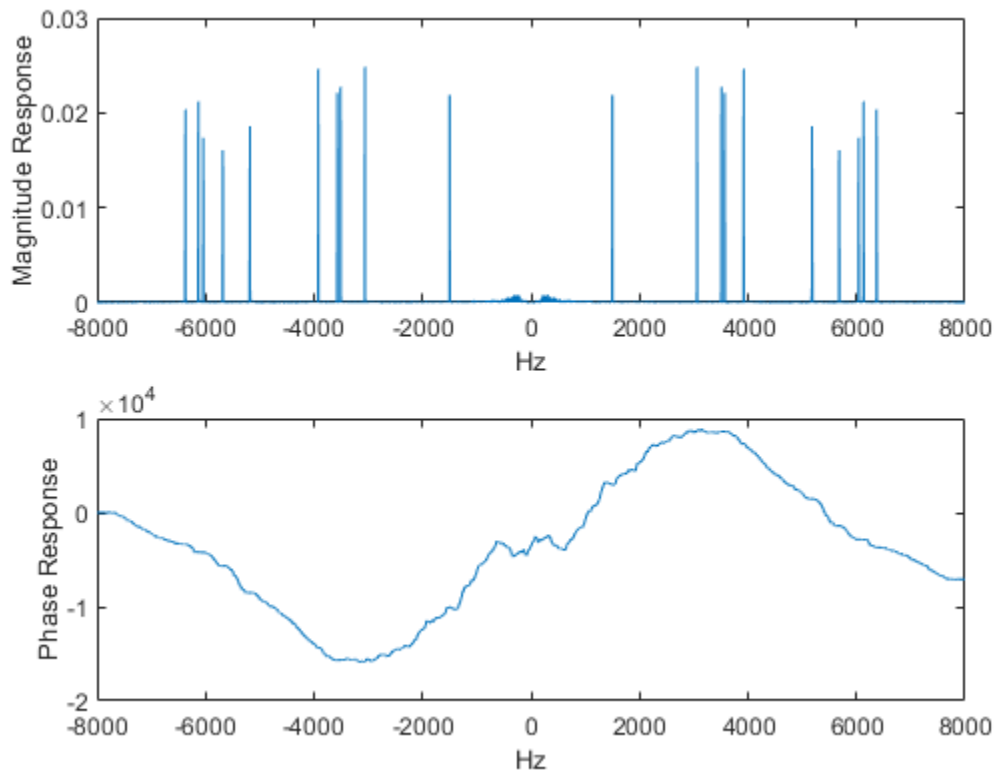
end
```



Question 4:

The audio signal in <http://tinyurl.com/ja5vxce> has been contaminated with multiple additive noisy signals with narrow spectral content. The goal of this project is to clean the audio by designing different notch filters

- a. Plot the magnitude and phase response of the audio file. Discuss the results.



We can see 10 highly distinct peaks in the positive portion of the frequency response. Since the human voice typically is around 400-4kHz and usually is wideband, we expect every one of these peaks to correspond to additive noise. We will design notch filters to cancel out these frequencies.

- b. Determine the spectral location associated with the noise. Give your results in the discrete time domain (normalized frequencies) and continuous time domain (in Hz).

Frequencies of Noise (kHz):

Columns 1 through 10

1.5000 3.0600 3.5100 3.5699 3.9200 5.1800 5.6799 6.0400 6.1299 6.3699

Discrete Normalized Frequencies of Noise:

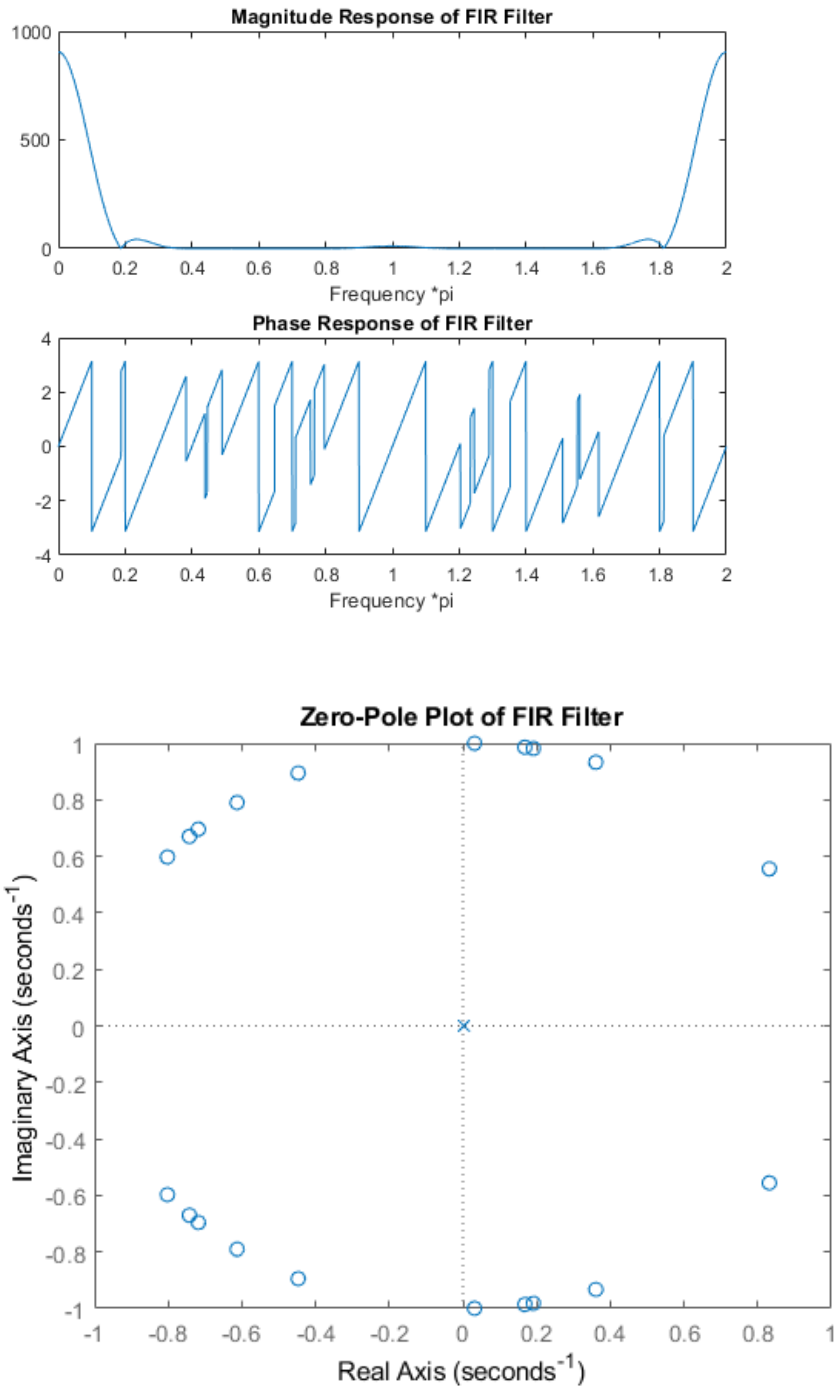
Columns 1 through 10

0.0937 0.1912 0.2194 0.2231 0.2450 0.3237 0.3550 0.3775 0.3831 0.3981

- c. Design a notch FIR filter with only zeros. Plot the magnitude and phase response of the filter.
Plot the magnitude response of the resulting signal. Create an audio file after filtering the signal.

Our FIR notch filter will have the following properties:

We will place zeros at the frequencies which contain noise and we desire to eliminate.



We can see that the zeros we placed at the normalized frequencies result in a 0 magnitude response of the filter at those normalized frequencies.

Zeros:

Columns 1 through 4

$$0.8315 - 0.5556i \quad 0.8315 + 0.5556i \quad 0.3608 - 0.9326i \quad 0.3608 + 0.9326i$$

Columns 5 through 8

$$0.1912 - 0.9815i \quad 0.1912 + 0.9815i \quad 0.1681 - 0.9858i \quad 0.1681 + 0.9858i$$

Columns 9 through 12

$$0.0314 - 0.9995i \quad 0.0314 + 0.9995i \quad -0.4470 - 0.8945i \quad -0.4470 + 0.8945i$$

Columns 13 through 16

$$-0.8020 - 0.5973i \quad -0.8020 + 0.5973i \quad -0.7423 - 0.6701i \quad -0.7423 + 0.6701i$$

Columns 17 through 20

$$-0.7181 - 0.6959i \quad -0.7181 + 0.6959i \quad -0.6129 - 0.7902i \quad -0.6129 + 0.7902i$$

Factored G(z)

$$\frac{\frac{566085731962705}{18014398509481984} - \frac{9002750976819333}{9007199254740992}i}{z} - 1$$

$$\frac{\frac{566085731962705}{18014398509481984} + \frac{9002750976819333}{9007199254740992}i}{z} - 1$$

$$\frac{\frac{5520343083264219}{9007199254740992} - \frac{3558632133896769}{4503599627370496}i}{z} + 1$$

$$\frac{\frac{5520343083264219}{9007199254740992} + \frac{3558632133896769}{4503599627370496}i}{z} + 1$$

$$\frac{\frac{1513968334038139}{9007199254740992} - \frac{4439525264545937}{4503599627370496}i}{z} - 1$$

$$\frac{\frac{1513968334038139}{9007199254740992} + \frac{4439525264545937}{4503599627370496}i}{z} - 1$$

$$\frac{\frac{234039014659009}{281474976710656} - \frac{2502038957673595}{4503599627370496}i}{z} - 1$$

$$\frac{\frac{234039014659009}{281474976710656} + \frac{2502038957673595}{4503599627370496}i}{z} - 1$$

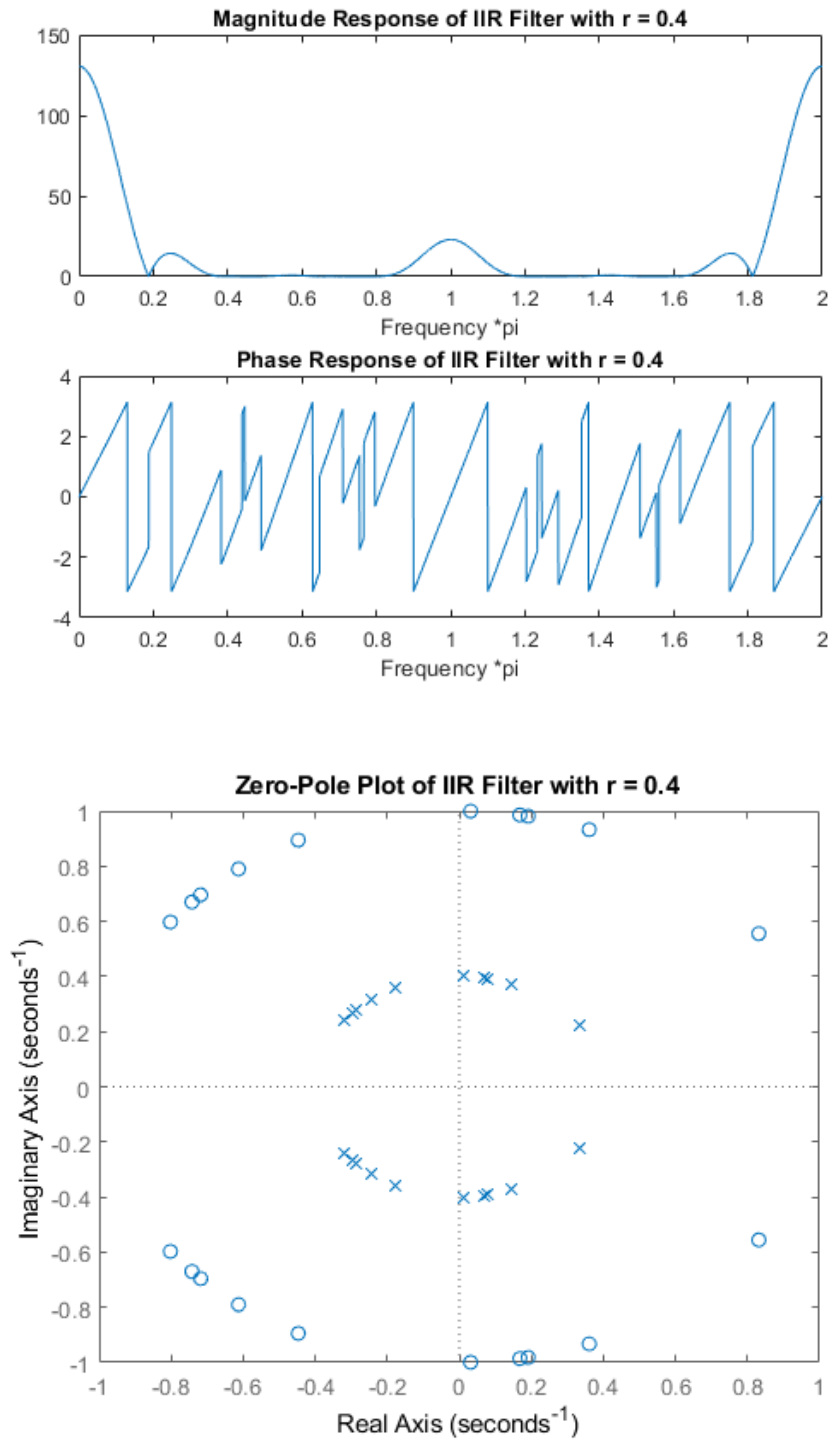
$$\frac{\frac{6685597574028107}{9007199254740992} - \frac{6035927724275377}{9007199254740992}i}{z} + 1$$

$$\begin{aligned}
& \left(\frac{6685597574028107}{9007199254740992} + \frac{6035927724275377}{9007199254740992}i}{z} + 1 \right) \\
& \left(\frac{3445177072878327}{18014398509481984} - \frac{8840946055072791}{9007199254740992}i}{z} - 1 \right) \\
& \left(\frac{3445177072878327}{18014398509481984} + \frac{8840946055072791}{9007199254740992}i}{z} - 1 \right) \\
& \left(\frac{6468296862995579}{9007199254740992} - \frac{6268235326380601}{9007199254740992}i}{z} + 1 \right) \\
& \left(\frac{6468296862995579}{9007199254740992} + \frac{6268235326380601}{9007199254740992}i}{z} + 1 \right) \\
& \left(\frac{2013000991858733}{4503599627370496} - \frac{8057353563157265}{9007199254740992}i}{z} + 1 \right) \\
& \left(\frac{2013000991858733}{4503599627370496} + \frac{8057353563157265}{9007199254740992}i}{z} + 1 \right) \\
& \left(\frac{812504570253995}{2251799813685248} - \frac{4200208911103973}{4503599627370496}i}{z} - 1 \right) \\
& \left(\frac{812504570253995}{2251799813685248} + \frac{4200208911103973}{4503599627370496}i}{z} - 1 \right) \\
& \left(\frac{3611975720343689}{4503599627370496} - \frac{5379978066609391}{9007199254740992}i}{z} + 1 \right) \\
& \left(\frac{3611975720343689}{4503599627370496} + \frac{5379978066609391}{9007199254740992}i}{z} + 1 \right)
\end{aligned}$$

- d. Design a notch IIR filter with zeros and poles. Set the magnitude of the poles to $r = 0.4$ ($p_k = r \cdot e^{j\omega_k}$). Plot the magnitude and phase response of the filter. Plot the magnitude response of the resulting signal. Create an audio file after filtering the signal.

Our IIR notch filter will have the following properties:

We will keep the zeros in the same locations as for the FIR filter, but we will add poles at the same frequencies to narrow the bandwidth of the notch. For this IIR filter, we will place our poles at a radial distance of 0.4 from the origin of the z-plane.



By adding poles at the same normalized frequencies as the zeroes, we increase the quality factor (decrease the bandwidth) of the notch filter at each of the normalized frequencies.

Zeros:

Columns 1 through 4

$$0.8315 - 0.5556i \quad 0.8315 + 0.5556i \quad 0.3608 - 0.9326i \quad 0.3608 + 0.9326i$$

Columns 5 through 8

$$0.1912 - 0.9815i \quad 0.1912 + 0.9815i \quad 0.1681 - 0.9858i \quad 0.1681 + 0.9858i$$

Columns 9 through 12

$$0.0314 - 0.9995i \quad 0.0314 + 0.9995i \quad -0.4470 - 0.8945i \quad -0.4470 + 0.8945i$$

Columns 13 through 16

$$-0.8020 - 0.5973i \quad -0.8020 + 0.5973i \quad -0.7423 - 0.6701i \quad -0.7423 + 0.6701i$$

Columns 17 through 20

$$-0.7181 - 0.6959i \quad -0.7181 + 0.6959i \quad -0.6129 - 0.7902i \quad -0.6129 + 0.7902i$$

Poles:

Columns 1 through 4

$$0.3326 - 0.2222i \quad 0.3326 + 0.2222i \quad 0.1443 - 0.3731i \quad 0.1443 + 0.3731i$$

Columns 5 through 8

$$0.0765 - 0.3926i \quad 0.0765 + 0.3926i \quad 0.0672 - 0.3943i \quad 0.0672 + 0.3943i$$

Columns 9 through 12

$$0.0126 - 0.3998i \quad 0.0126 + 0.3998i \quad -0.1788 - 0.3578i \quad -0.1788 + 0.3578i$$

Columns 13 through 16

$$-0.3208 - 0.2389i \quad -0.3208 + 0.2389i \quad -0.2969 - 0.2680i \quad -0.2969 + 0.2680i$$

Columns 17 through 20

$$-0.2873 - 0.2784i \quad -0.2873 + 0.2784i \quad -0.2452 - 0.3161i \quad -0.2452 + 0.3161i$$

Factored G(z)

$$\frac{\frac{566085731962705}{18014398509481984} - \frac{9002750976819333}{9007199254740992}i}{z} - 1$$

$$\frac{\frac{566085731962705}{18014398509481984} + \frac{9002750976819333}{9007199254740992}i}{z} - 1$$

$$\frac{\frac{5520343083264219}{9007199254740992} - \frac{3558632133896769}{4503599627370496}i}{z} + 1$$

$$\frac{\frac{5520343083264219}{9007199254740992} + \frac{3558632133896769}{4503599627370496}i}{z} + 1$$

$$\frac{\frac{1513968334038139}{9007199254740992} - \frac{4439525264545937}{4503599627370496}i}{z} - 1$$

$$\frac{\frac{1513968334038139}{9007199254740992} + \frac{4439525264545937}{4503599627370496}i}{z} - 1$$

$$\frac{\frac{234039014659009}{281474976710656} - \frac{2502038957673595}{4503599627370496}i}{z} - 1$$

$$\frac{\frac{234039014659009}{281474976710656} + \frac{2502038957673595}{4503599627370496}i}{z} - 1$$

$$\frac{\frac{6685597574028107}{9007199254740992} - \frac{6035927724275377}{9007199254740992}i}{z} + 1$$

$$\frac{\frac{6685597574028107}{9007199254740992} + \frac{6035927724275377}{9007199254740992}i}{z} + 1$$

$$\frac{\frac{3445177072878327}{18014398509481984} - \frac{8840946055072791}{9007199254740992}i}{z} - 1$$

$$\frac{\frac{3445177072878327}{18014398509481984} + \frac{8840946055072791}{9007199254740992}i}{z} - 1$$

$$\frac{\frac{6468296862995579}{9007199254740992} - \frac{6268235326380601}{9007199254740992}i}{z} + 1$$

$$\frac{\frac{6468296862995579}{9007199254740992} + \frac{6268235326380601}{9007199254740992}i}{z} + 1$$

$$\frac{\frac{2013000991858733}{4503599627370496} - \frac{8057353563157265}{9007199254740992}i}{z} + 1$$

$$\frac{\frac{2013000991858733}{4503599627370496} + \frac{8057353563157265}{9007199254740992}i}{z} + 1$$

$$\frac{\frac{812504570253995}{2251799813685248} - \frac{4200208911103973}{4503599627370496}i}{z} - 1$$

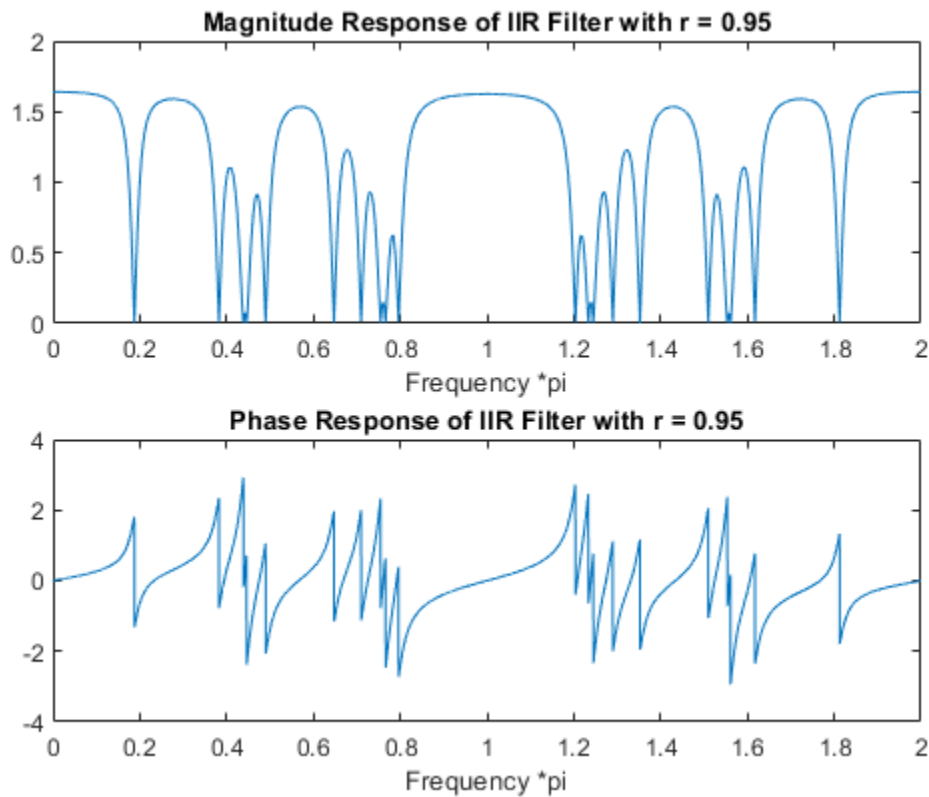
$$\frac{\frac{812504570253995}{2251799813685248} + \frac{4200208911103973}{4503599627370496}i}{z}$$

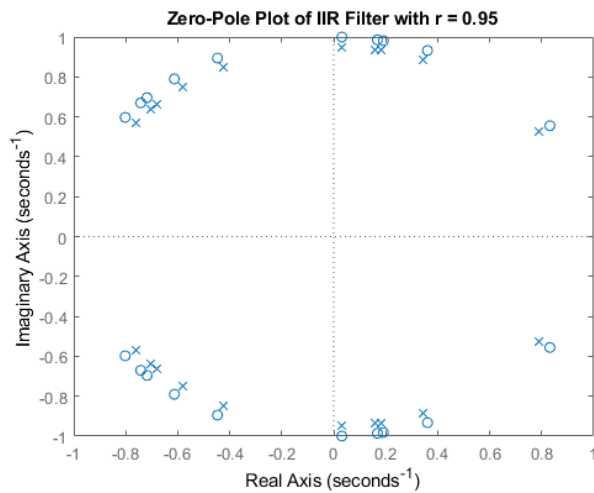
$$\frac{\left(\frac{3611975720343689}{4503599627370496} z^2 - \frac{5379978066609391}{9007199254740992} z + 1 \right)}{\left(\frac{3611975720343689}{4503599627370496} z^2 + \frac{5379978066609391}{9007199254740992} z + 1 \right)}$$

e. Repeat part (d) with $r = 0.95$

Our IIR notch filter will have the following properties:

We will keep the zeros in the same locations as for the FIR filter, but we will place poles at the same frequencies to narrow the bandwidth of the notch. For this IIR filter, we will place our poles at a radial distance of 0.4 from the origin of the z-plane.





By increasing the radial magnitude of the poles and placing them closer to the unit circle, we can see that the quality factor is drastically increased and the notch filter stops the desired normalized frequencies and allows other frequencies to pass.

Zeros:

Columns 1 through 4

$0.8315 - 0.5556i$ $0.8315 + 0.5556i$ $0.3608 - 0.9326i$ $0.3608 + 0.9326i$

Columns 5 through 8

$0.1912 - 0.9815i$ $0.1912 + 0.9815i$ $0.1681 - 0.9858i$ $0.1681 + 0.9858i$

Columns 9 through 12

$0.0314 - 0.9995i$ $0.0314 + 0.9995i$ $-0.4470 - 0.8945i$ $-0.4470 + 0.8945i$

Columns 13 through 16

$-0.8020 - 0.5973i$ $-0.8020 + 0.5973i$ $-0.7423 - 0.6701i$ $-0.7423 + 0.6701i$

Columns 17 through 20

$-0.7181 - 0.6959i$ $-0.7181 + 0.6959i$ $-0.6129 - 0.7902i$ $-0.6129 + 0.7902i$

Poles:

Columns 1 through 4

$0.7899 - 0.5278i$ $0.7899 + 0.5278i$ $0.3428 - 0.8860i$ $0.3428 + 0.8860i$

Columns 5 through 8

$0.1817 - 0.9325i$ $0.1817 + 0.9325i$ $0.1597 - 0.9365i$ $0.1597 + 0.9365i$

Columns 9 through 12

$0.0299 - 0.9495i$ $0.0299 + 0.9495i$ $-0.4246 - 0.8498i$ $-0.4246 + 0.8498i$

Columns 13 through 16

$-0.7619 - 0.5674i$ $-0.7619 + 0.5674i$ $-0.7051 - 0.6366i$ $-0.7051 + 0.6366i$

Columns 17 through 20

$-0.6822 - 0.6611i$ $-0.6822 + 0.6611i$ $-0.5822 - 0.7507i$ $-0.5822 + 0.7507i$

Factored $G(z)$

$$\begin{aligned}
& \left(\frac{\frac{566085731962705}{18014398509481984} - \frac{9002750976819333}{9007199254740992} i}{z} - 1 \right) \\
& \left(\frac{\frac{566085731962705}{18014398509481984} + \frac{9002750976819333}{9007199254740992} i}{z} - 1 \right) \\
& \left(\frac{\frac{5520343083264219}{9007199254740992} - \frac{3558632133896769}{4503599627370496} i}{z} + 1 \right) \\
& \left(\frac{\frac{5520343083264219}{9007199254740992} + \frac{3558632133896769}{4503599627370496} i}{z} + 1 \right) \\
& \left(\frac{\frac{1513968334038139}{9007199254740992} - \frac{4439525264545937}{4503599627370496} i}{z} - 1 \right) \\
& \left(\frac{\frac{1513968334038139}{9007199254740992} + \frac{4439525264545937}{4503599627370496} i}{z} - 1 \right) \\
& \left(\frac{\frac{234039014659009}{281474976710656} - \frac{2502038957673595}{4503599627370496} i}{z} - 1 \right) \\
& \left(\frac{\frac{234039014659009}{281474976710656} + \frac{2502038957673595}{4503599627370496} i}{z} - 1 \right) \\
& \left(\frac{\frac{6685597574028107}{9007199254740992} - \frac{6035927724275377}{9007199254740992} i}{z} + 1 \right) \\
& \left(\frac{\frac{6685597574028107}{9007199254740992} + \frac{6035927724275377}{9007199254740992} i}{z} + 1 \right) \\
& \left(\frac{\frac{3445177072878327}{18014398509481984} - \frac{8840946055072791}{9007199254740992} i}{z} - 1 \right) \\
& \left(\frac{\frac{3445177072878327}{18014398509481984} + \frac{8840946055072791}{9007199254740992} i}{z} - 1 \right) \\
& \left(\frac{\frac{6468296862995579}{9007199254740992} - \frac{6268235326380601}{9007199254740992} i}{z} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{\frac{6468296862995579}{9007199254740992} + \frac{6268235326380601}{9007199254740992}i}{z} + 1 \\
& \frac{\frac{2013000991858733}{4503599627370496} - \frac{8057353563157265}{9007199254740992}i}{z} + 1 \\
& \frac{\frac{2013000991858733}{4503599627370496} + \frac{8057353563157265}{9007199254740992}i}{z} + 1 \\
& \frac{\frac{812504570253995}{2251799813685248} - \frac{4200208911103973}{4503599627370496}i}{z} - 1 \\
& \frac{\frac{812504570253995}{2251799813685248} + \frac{4200208911103973}{4503599627370496}i}{z} - 1 \\
& \frac{\frac{3611975720343689}{4503599627370496} - \frac{5379978066609391}{9007199254740992}i}{z} + 1 \\
& \frac{\frac{3611975720343689}{4503599627370496} + \frac{5379978066609391}{9007199254740992}i}{z} + 1
\end{aligned}$$

Code for question 4:

```

[y,Fs] = audioread('project2audio.wav');
[y_new,y_new1,y_new2] = question4(y,Fs,0.4,0.95);
soundsc(y_new2,Fs)

function [y_new,y_new1,y_new2] = question4(y,Fs,r1,r2)
L = length(y);
yf = fftshift(fft(y))/L;
f = -Fs/2:Fs/L:Fs/2-Fs/L;
figure
subplot(211);
plot(f,abs(yf));
ylabel('Magnitude Response')
xlabel('Hz')
% plot phase
subplot(212);
plot(f,unwrap(angle(yf)));
ylabel('Phase Response')
xlabel('Hz')

```



```

[peaks,noi_loc] = findpeaks(abs(yf( (length(yf)+1)/2 : length(yf) ) ),f( (length(yf)+1)/2 :
length(yf) ),'MinPeakHeight',0.01);

disp("Frequencies of Noise (Hz):")
disp(noi_loc);
disp("Discrete Normalized Frequencies of Noise:")
disp(noi_loc/Fs);

syms z
% Make an FIR filter with zeros at noise frequencies
numz = 1;
%noi_loc = mean(noi_loc);
for i = 1:length(noi_loc)
    numz = numz .* (z^2-2*cos(2*pi*noi_loc(i)/Fs)*z+1);
end
numHz = sym2poly(numz);

question3(numHz,1,"FIR Filter");
question1(numHz,1,"FIR Filter");

y_new = filter(numHz,1,y);

% Make an IIR filter with r1
denz = 1;
for i = 1:length(noi_loc)
    denz = denz .* (z^2-2*r1*cos(2*pi*noi_loc(i)/Fs)*z+r1^2);
end
denHz1 = sym2poly(denz);

question3(numHz,denHz1,"IIR Filter with r = 0.4");
question1(numHz,denHz1,"IIR Filter with r = 0.4");

y_new1 = filter(numHz,denHz1,y);

% Make an IIR filter with r2
denz = 1;
for i = 1:length(noi_loc)
    denz = denz .* (z^2-2*r2*cos(2*pi*noi_loc(i)/Fs)*z+r2^2);
end
denHz2 = sym2poly(denz);

question3(numHz,denHz2,"IIR Filter with r = 0.95");
question1(numHz,denHz2,"IIR Filter with r = 0.95");

y_new2 = filter(numHz,denHz2,y);
end
function question3(num,den,str)
w = 0:0.001:2*pi;
numw = 0;
denw = 0;
for i = 1:length(num)
    numw = numw + num(i).*exp(1j*w*(i-1));
end

```

```

for i = 1:length(den)
    denw = denw + den(i).*exp(1j*w*(i-1));
end

figure
subplot(211)
Hw = numw./denw;
plot(w/pi,abs(Hw))
xlabel('Frequency *pi')
title("Magnitude Response of " + str)

subplot(212)
plot(w/pi,angle(Hw))
xlabel('Frequency *pi')
title("Phase Response of " + str)

end

function [poleG,zeroG] = question1(num,den,str)
syms z
poleG = roots(den);
zeroG = zeros(length(den)-1,1);
zeroG(1:length(num)-1,1) = roots(num);
disp("Zeros:")
disp(zeroG');
pause(1)
disp("Poles:");
disp(poleG');
pause(1)
denz = 1;
numz = 1;
for i = 1:length(zeroG)
    numz = numz * ( 1 - zeroG(i)*z^1);
end
for j = 1:length(poleG)
    denz = denz * ( 1 - poleG(j)*z^1);
end
disp("Factored G(z)")
pretty(numz);
if den ~= 1
    disp("-----")
    pretty(denz);
end
s = tf('s');
Gs = tf(num,den);
figure
pzplot(Gs*s^(length(den)-length(num)));
title("Zero-Pole Plot of " + str)
end

```

% Audiofiles written

Audiowrite("FIRaudio.wav",y_new/length(y_new),Fs)