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Bluetooth Low Energy Ranging by Multi-Carrier Phase Difference

EESC 6353 Project
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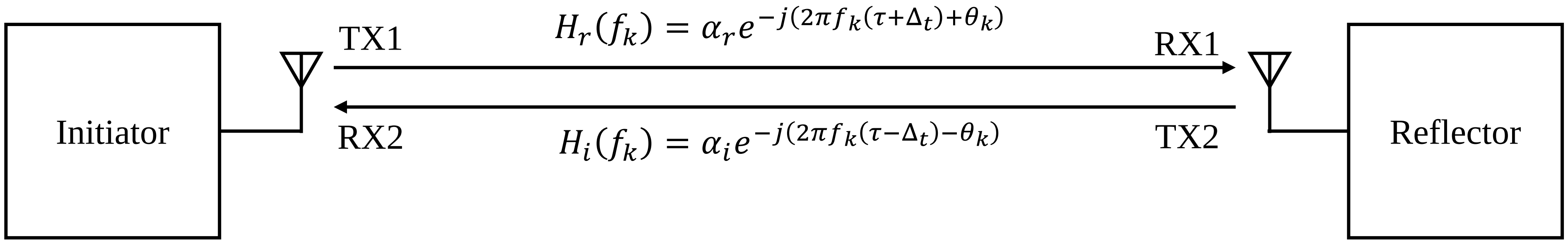
Bluetooth Low Energy (BLE) Ranging: Background and Motivations

- Background:
  - GPS does not work well indoors and has accuracy on the order of 3 meters.
  - Accurate estimation of distance between low energy devices has increased in interest for many applications with the emergence of the Internet-of-Things (IoT).
  - Most existing indoor positioning systems use Wi-Fi standards and are not low power. In addition, Wi-Fi positioning approaches often require more than one access point for accurate localization through trilateration (802.11az).
- Motivations:
  - Location-based targeted promotions for retail
  - Geo-fencing buildings and parking structures
  - Residential room occupancy detection
  - Digital car key ranging

Multi-Carrier Phase Difference (MCPD) Ranging



$$H_{2W}(f_k) = H_r(f_k)H_i(f_k) = \alpha e^{-j4\pi f_k \tau}$$

Recover time of flight τ from $H_{2W}(f_k)$, and estimate distance as $r = \tau \times c_0$.

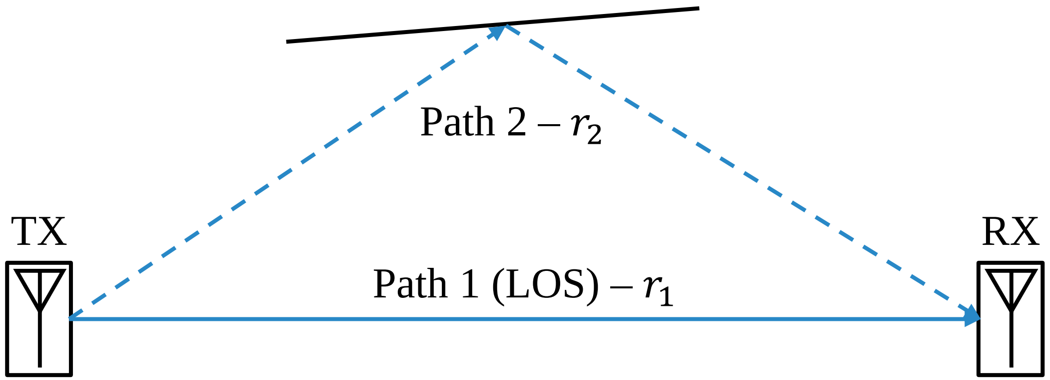
Key Fourier Transform Property: $\delta(t-\tau) \leftrightarrow e^{-j2\pi f\tau}$

Multipath Considerations

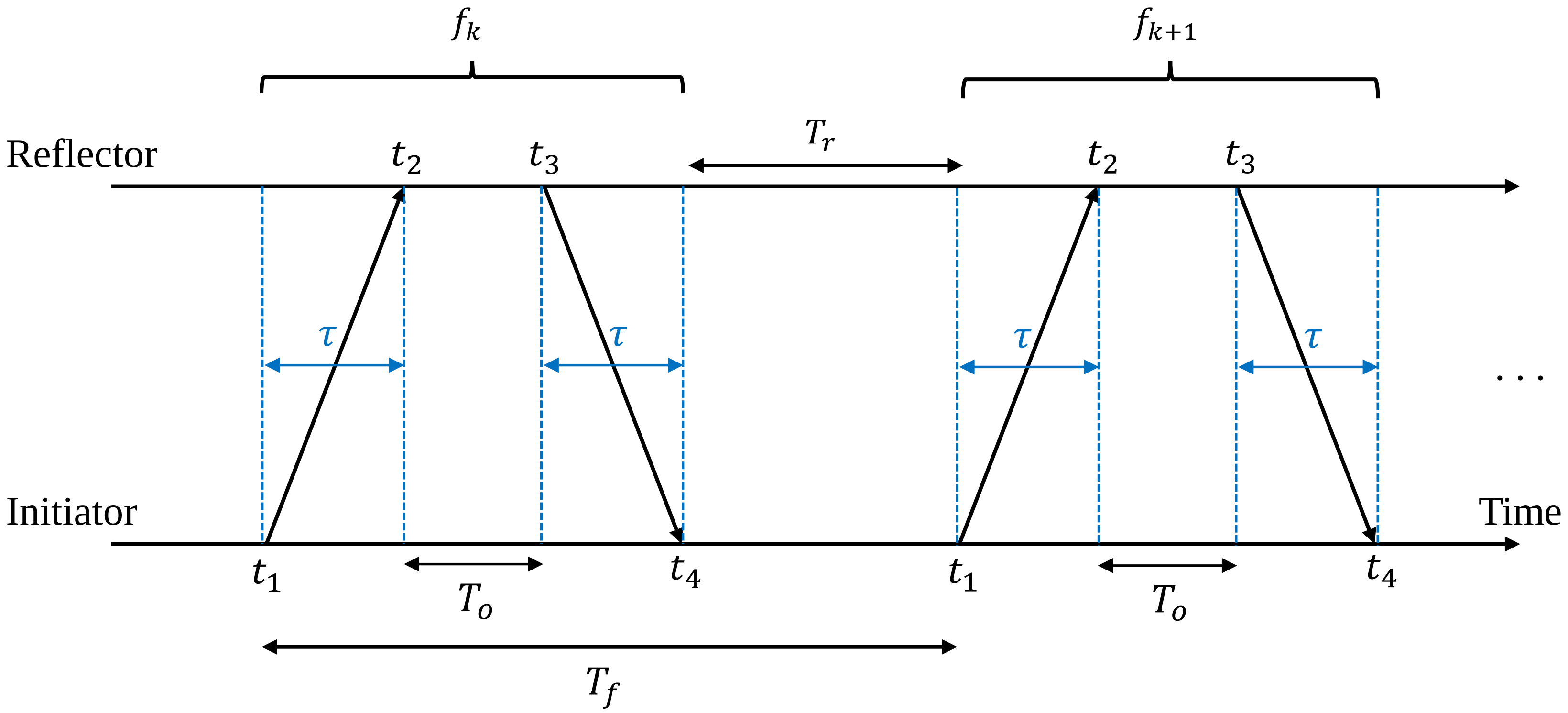
- Two-Way Signal Model For Two Paths:

$$H_{2W}(f_k) = \left(\alpha_1 e^{-j2\pi f_k\left(\frac{r_1}{c_0}\right)} + \alpha_2 e^{-j2\pi f_k\left(\frac{r_2}{c_0}\right)}\right)^2$$
$$= \alpha_1^2 e^{-j2\pi f_k\left(\frac{2r_1}{c_0}\right)} + \alpha_1\alpha_2 e^{-j2\pi f_k\left(\frac{r_1+r_2}{c_0}\right)} + \alpha_2^2 e^{-j2\pi f_k\left(\frac{2r_2}{c_0}\right)}$$

- Recovering the LOS distance r_1 from $H_{2W}(f_k)$ requires the use of spectral estimation techniques. One of the most well-known techniques is MUltiple SIgnal Classification (MUSIC) [2].



MCPD Timeline



The channel measurement timeline across frequencies f_k can be defined as the above figure where T_o is the intra-delay, T_r is the inter-delay, T_f is the total delay per channel measurement, and $\tau = \frac{r}{c_0}$ is the propagation delay related to distance. Max number of channels is $K_f = 78$ for BLE with 1 MHz channel spacing.

Crystal Offset

- The crystal in a BLE device is typically used for both LO (local oscillator) and clock-generation (sampling frequency).

- The actual frequencies and timing will scale by a factor of $(1+\eta)$ where η is the crystal offset, typically expressed in part-per-million (ppm or 10^{-6}).

- For BLE devices, the crystal offset can be assumed to be at most ± 20 ppm and varies per device.

Errors Due to Delay and Crystal Offset

Define the phase measured at the reflector and initiator respectively as

$$\phi_R(f_k, r) = -2\pi f_k^r \left(\frac{r}{c_0} - \Delta_t\right) - \theta$$

And

$$\phi_I(f_k, r) = -2\pi f_k^i \left(\frac{r}{c_0} + \Delta_t\right) + \theta$$

Where the frequencies and measurement timings defined as

$f_k^i = (1 + \eta_i) f_k$

$f_k^r = (1 + \eta_r) f_k$

$\Delta_t = T_k^i - T_k^r$

$T_k^r = (1 + \eta_r) t_k$

$T_k^i = (1 + \eta_i)(t_k + T_o)$

With T_o being the time difference at which measurement is taken at the initiator and reflector.

Errors Due to Delay and Crystal Offset

- Add the initiator and reflector phase responses to obtain

$$\phi_{2W}(f_k, r) = -2\pi\left(f_k^i + f_k^r\right)\frac{r}{c_0} + 2\pi\left(f_k^i - f_k^r\right)\left(T_k^r - T_k^i\right)$$

  and define $r = -\frac{c_0}{4\pi\Delta_f}\frac{\phi_{2W}\left(f_{K_f}, r\right) - \phi_{2W}(f_0, r)}{K_f - 1}$ as the LOS distance for K_f BLE channels with frequency spacing Δ_f.

- Compute the error with respect to the estimated distance $\tilde{r}$ as

$$\begin{aligned} e_r = \tilde{r} - r &= -\frac{c_0}{2}(\eta_r - \eta_i)^2\left(\frac{f_o}{\Delta_f} + K_f\right)T_f - \frac{c_0}{2}(1 + \eta_i)(\eta_i - \eta_r)T_o \\ &\approx -\frac{c_0}{2}(1 + \eta_i)(\eta_i - \eta_r)T_o \\ &\approx -\frac{c_0}{2}(\eta_i - \eta_r)T_o \end{aligned}$$

  The e_r approximations are made since the second term with respect to T_o dominates and $(1 + \eta_i) \approx 1$.

Doppler Effect for Mobility

- The effect of mobility is to shift the received carrier frequency as

$$f_r = \left(1 + \frac{\Delta_v}{c_0 + v_t}\right) f_t \approx \left(1 + \frac{\Delta_v}{c_0}\right) f_t$$

where Δ_v is the difference between the velocity of the receiver v_r and the velocity of the transmitter v_t, c_o is the propagation speed (speed of light), and f_t is the transmitted carrier frequency.

- The Doppler frequency shift can then be represented as

$$f_d = \frac{f_t}{c_0} \Delta_v = \frac{\Delta_v}{\lambda_t}$$

Effect of Mobility for BLE MCPD Ranging

- The Doppler frequency adds to the carrier frequencies as

$$\hat{f}_k^i = (1+\eta_i)f_k + f_{d,k}^i \quad \text{(where } f_{d,k}^i = \frac{f_k^r}{c_0}(v_i - v_r)\text{)}$$

and

$$\hat{f}_k^r = (1+\eta_r)f_k + f_{d,k}^r \quad \text{(where } f_{d,k}^r = \frac{f_k^i}{c_0}(v_r - v_i)\text{)}$$

where v_i and v_r are the velocities of the initiator and reflector respectively.

- Without crystal offset, the frequency offsets due to mobility cancel due to the sign change in $\Delta_v = v_i - v_r$ between initiator and reflector.

Effect of Mobility for BLE MCPD Ranging

- Including the frequency offset due to mobility with the previous derivation for e_r, the error due to crystal offset, we obtain the following new result

$$\hat{e}_r = \tilde{r} - r$$

$$= -\frac{c_0}{2\Delta_f}\left[(\eta_r - \eta_i)^2\left(f_o + K_f\Delta_f\right) - \left(f_{d,K_f}^{i} - f_{d,K_f}^{r}\right)(\eta_r - \eta_i)\right]T_f$$

$$-\frac{c_0}{2}\left[(1+\eta_i)(\eta_i - \eta_r) - \frac{1}{2}\left((1+\eta_i)^2 + (1+\eta_i)(1+\eta_r)\right)(v_i - v_r)\right]T_o$$

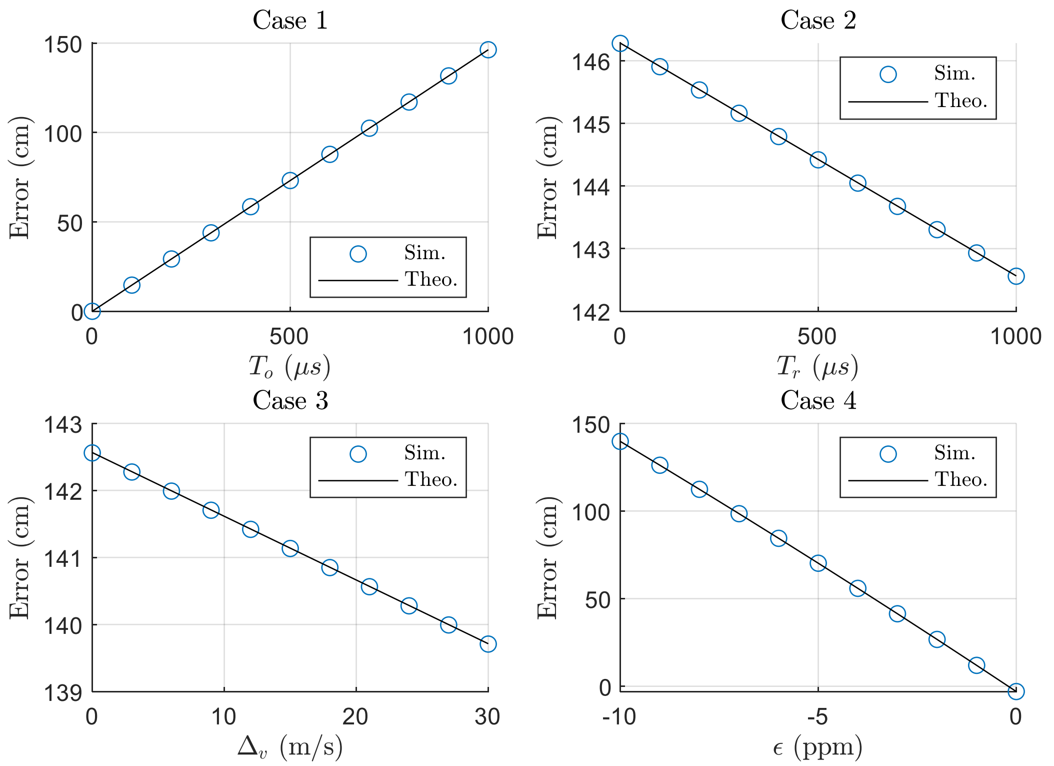
- The new terms are added due to mobility. For practical values for BLE, the error due to mobility is on the order of <1 cm.

Simulation: Effect of Crystal Offset and Mobility on Ranging Error

To illustrate the effect of crystal offset and mobility on ranging error (bias), we evaluate the following four cases:

Case	T_o	T_r	$\Delta_v = v_i - v_r$	$\epsilon = \eta_i - \eta_r$
1	0 to 1 ms	0	0	-10 ppm
2	1 ms	0 to 1 ms	0	-10 ppm
3	1 ms	1 ms	0 to 30 m/s	-10 ppm
4	1 ms	1 ms	30 m/s	0 to -10 ppm

Where $T_f = T_o + T_r$, the total channel measurement delay, is the sum of the intra- and inter-delay respectively.



The errors are most sensitive to intra-delay (T_o) and crystal offsets (ϵ), while being relatively insensitive to inter-delay (T_r) and mobility (Δ_v).

Fourier Transform-Based Distance Estimation

- The Fourier transform approach can be expressed as

$$h_{2W}(n) = \sum_{k=0}^{K-1} H_{2W}(f_k) e^{-j\frac{2\pi}{K}kn}$$

where $-\pi \leq n < \pi$ with spacing of $2\pi/K$, and the set of distance estimates is $d = -\frac{n \times c_0}{2\pi\Delta_f}$. The estimated LOS distance is then the peak in $h_{2W}(n)$ with lowest distance d.

- The Fourier transform approach has low complexity (can be implemented as FFT) but suffers from lack of resolution between two paths with similar distances.

MUSIC-Based Distance Estimation

- For MUSIC-based distance estimation, first define the smoothed sample correlation matrix as

$$\widehat{\mathbf{R}}_{xx} = \frac{1}{M}\sum_{k=1}^{M-1} \mathbf{x}(k)\mathbf{x}(k)^H$$

  where $\mathbf{x}(k) = [x(k) \quad \cdots \quad x(k+L-1)]^T$ is a subarray of the received noisy channel frequency response (CFR) measurements and $M = N - L + 1$ where N is the CFR snapshot length and L is the smoothing subarray size.

- Define the noise eigenvectors of $\widehat{\mathbf{R}}_{xx}$ as $\mathbf{q}_i$, $L_p \leq i \leq L-1$ where L_p is the number of signal paths.

- Define the steering vectors as $\mathbf{v}(\tau_i) = [1 \quad e^{-j2\pi\Delta_f\tau_i} \quad \ldots \quad e^{-j2\pi(L-1)\Delta_f\tau_i}]^T$ where the set of τ_i defines the set of MUSIC search delays ($\tau_i = \frac{r_i}{c_0}$) and Δ_f is the frequency measurement spacing (1 or 2 MHz for BLE).

MUSIC-Based Distance Estimation

- The MUSIC pseudospectrum is then calculated as

$$S_{MUSIC}(\tau) = \frac{1}{\sum_{i=L_p}^{L-1} \left|\mathbf{q}_i^H \mathbf{v}(\tau)\right|^2} = \frac{1}{\|\mathbf{Q}_w^H \mathbf{v}(\tau)\|^2}$$

  where $\mathbf{Q}_w = [\mathbf{q}_{L_p} \quad \mathbf{q}_{L_p+1} \quad \cdots \quad \mathbf{q}_{L-1}]$ is the matrix of noise eigenvectors. The estimated LOS distance is $r_1 = \tau_1 \times c_0$ where τ_1 is the smallest delay with a peak in $S_{MUSIC}(\tau)$.

- It has been well-studied that the estimated correlation matrix $\widehat{\mathbf{R}}_{xx}$ can be improved using a forward-backward approach [3]. The forward-backward correlation matrix is calculated as

$$\widehat{\mathbf{R}}_{xx}^{(FB)} = \frac{1}{2}\left(\widehat{\mathbf{R}}_{xx} + \mathbf{J}\widehat{\mathbf{R}}_{xx}^{*}\mathbf{J}\right)$$

  Where the superscript $*$ denotes conjugate and $\mathbf{J}$ is the $L \times L$ reversal matrix whose antidiagonal is all ones with zeros everywhere else.

Simulation: FFT vs. MUSIC Distance Estimation

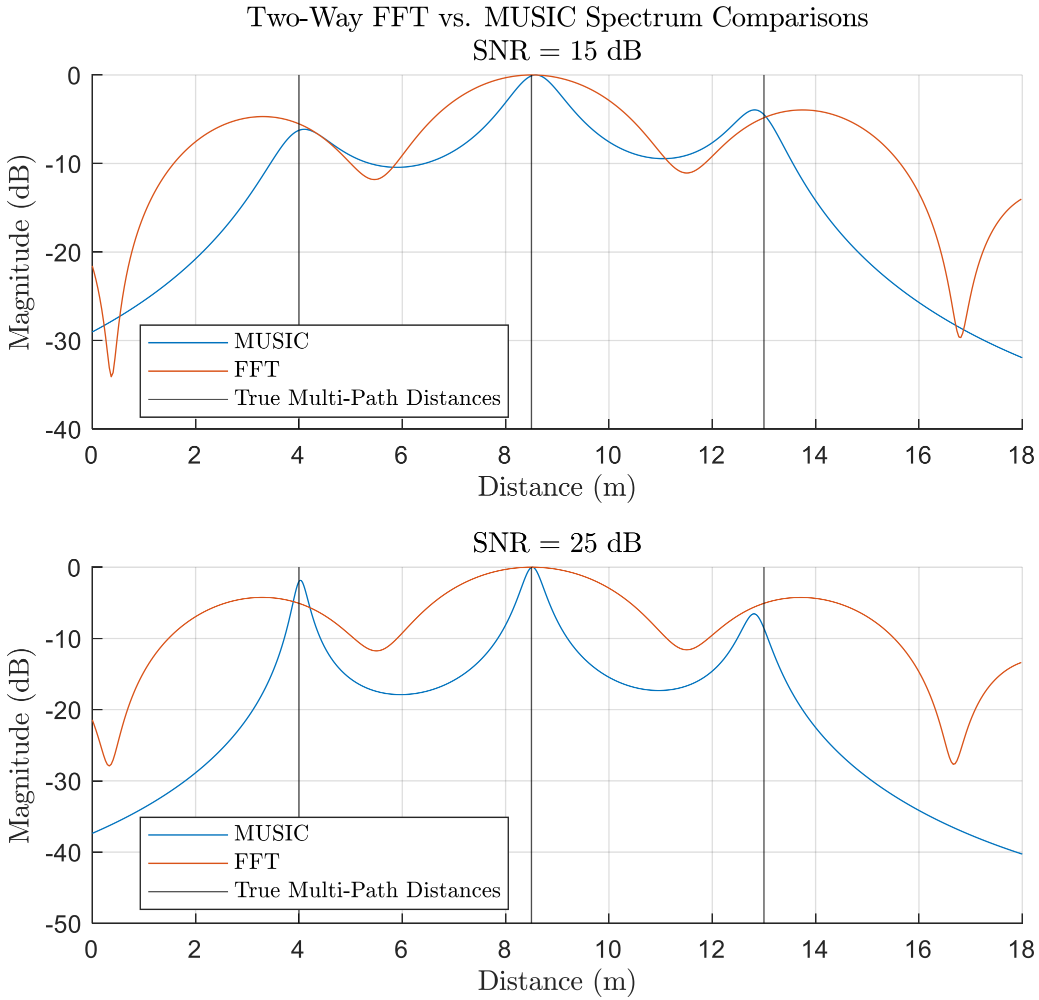
Simulated multi-path distances are 2 m and 6.5 m with equal attenuation. The two-way distances are then 4 m, 8.5 m, and 13 m as indicated by example iterations in the figure.

SNR = 15 dB over 1000 Iterations

Method	LOS Est. (m)	STD (m)	Mean Error (m)	RMSE (m)
FFT	1.642	0.026	-0.358	0.359
MUSIC	2.091	0.335	0.091	0.347

SNR = 25 dB over 1000 Iterations

Method	LOS Est. (m)	STD (m)	Mean Error (m)	RMSE (m)
FFT	1.642	0.009	-0.358	0.358
MUSIC	2.002	0.066	0.002	0.066



Residual Learning for Improved BLE Ranging

- Preliminaries of Residual Learning [4]
  - Additive Noise Model

$$\boldsymbol{z} = \boldsymbol{y} + \boldsymbol{n}$$

  - Goal: train a deep learning network (DNN) to predict the noise, e.g., $\hat{\boldsymbol{n}} \approx \boldsymbol{n}$, to improve SNR

$$\hat{\boldsymbol{y}} \approx \boldsymbol{z} - \hat{\boldsymbol{n}}$$

- Two-Way BLE Channel Frequency Response (CFR)
  - Element-wise multiplication of *Initiator* and *Reflector* CFR ($\odot$)

$$\boldsymbol{s}_{2W} = (\boldsymbol{s}_R + \boldsymbol{n}_R) \odot (\boldsymbol{s}_I + \boldsymbol{n}_I)$$
$$= \underbrace{\boldsymbol{s}_R \odot \boldsymbol{s}_I}_{\boldsymbol{\psi}} + \underbrace{(\boldsymbol{s}_R \odot \boldsymbol{n}_I + \boldsymbol{s}_I \odot \boldsymbol{n}_R + \boldsymbol{n}_R \odot \boldsymbol{n}_I)}_{\boldsymbol{\eta}}$$

  - Estimate the noiseless CFR by $\hat{\boldsymbol{\psi}} = \boldsymbol{s}_{2W} - \hat{\boldsymbol{\eta}}$

Steps:
1. Estimate $\hat{\boldsymbol{\psi}}$
2. Subtract from CFR
3. Improve SNR

DNN Architecture – Fully Convolutional Neural Network

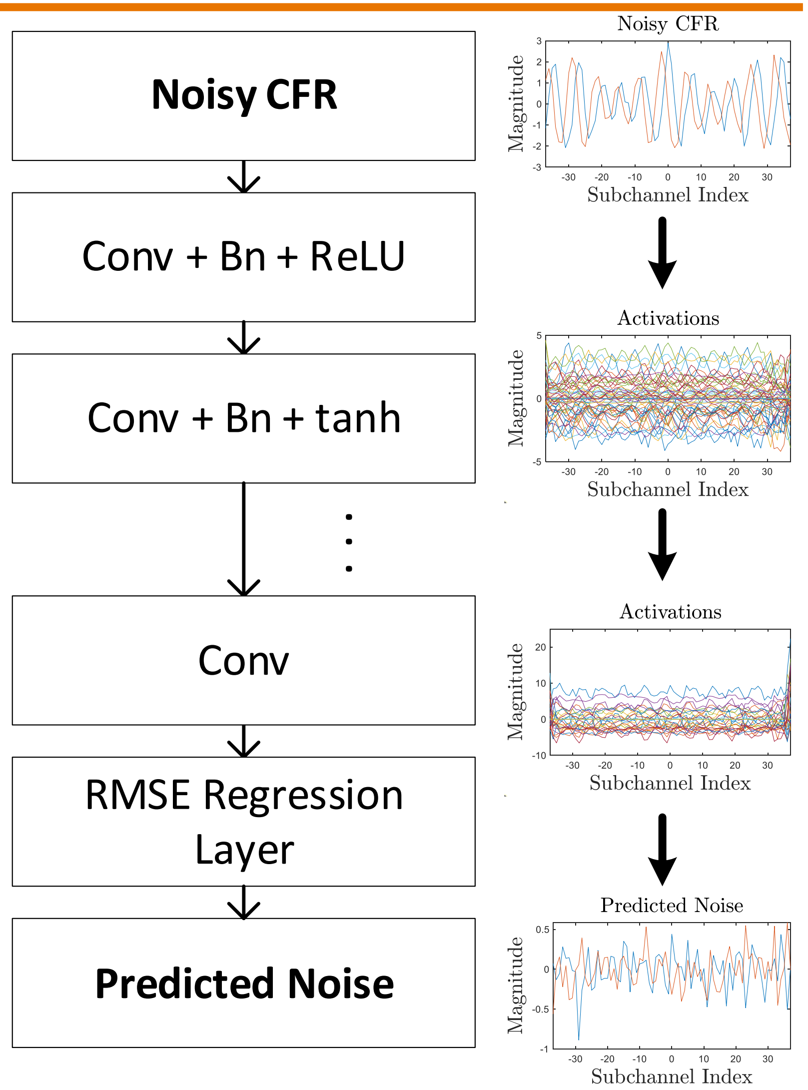
- Fully convolutional neural network (FCNN): input and output of network are identical in size, $\boldsymbol{\Theta}$ – parameters.

$$\hat{\boldsymbol{\eta}} = \boldsymbol{\mathcal{F}}(\boldsymbol{s}_{2W}, \boldsymbol{\Theta})$$

- Convolution layers
  - Zero-padding to preserve input size
  - Varying kernel size and decreasing number of layers
  - Alternating activation functions between ReLU and tanh
- RMSE regression, Adam optimizer

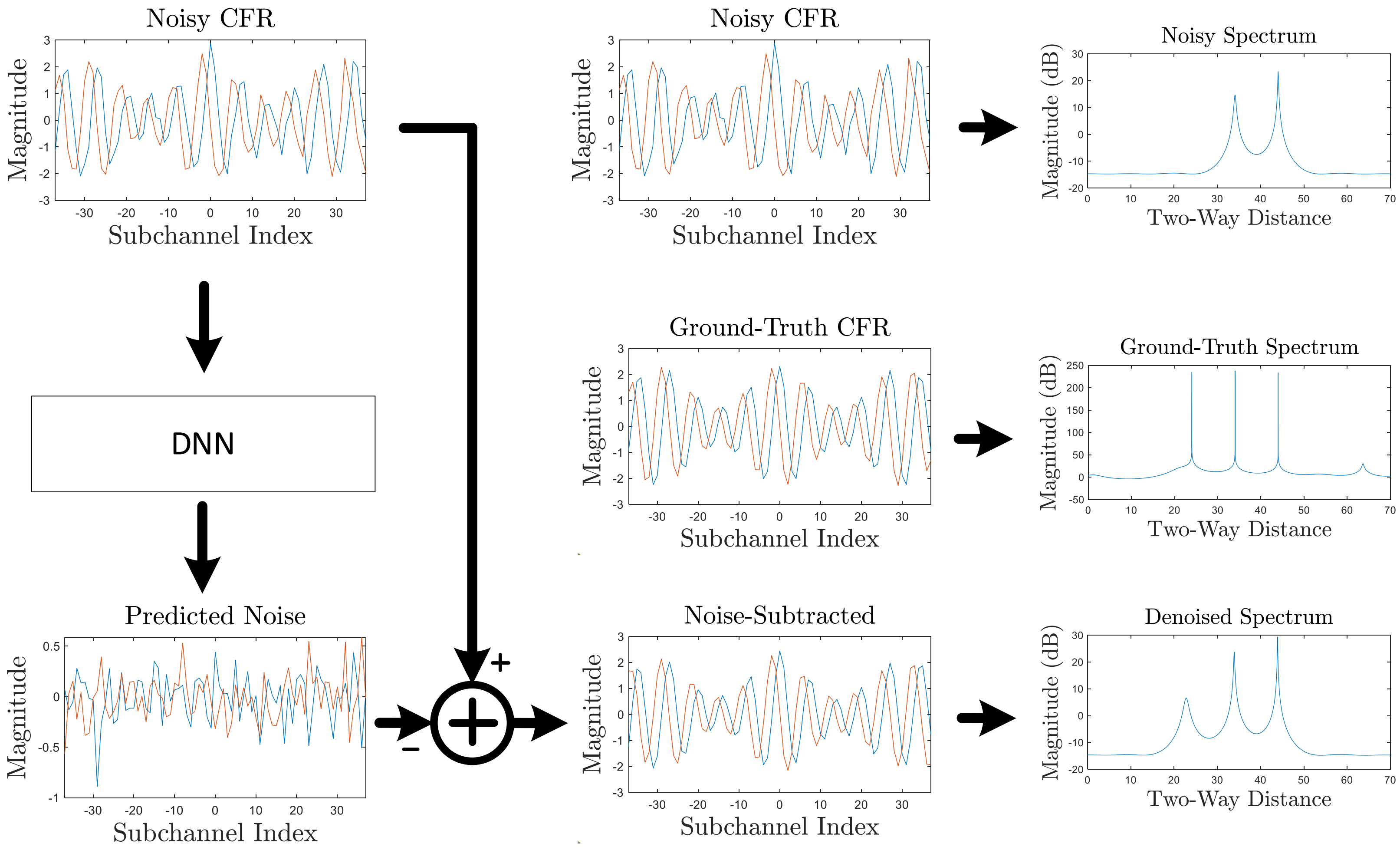
$$\mathcal{L}(\boldsymbol{\Theta}) = \sqrt{\frac{1}{N}\sum_{i=1}^{N} \|\boldsymbol{\eta}_i - \hat{\boldsymbol{\eta}}_i\|^2}$$

$$\boldsymbol{\Theta} = \arg\min \mathcal{L}(\boldsymbol{\Theta})$$



Residual DNN Demonstration

1. Direct MUSIC on noisy CFR
   - Missing LOS peak!
2. Ground Truth
   - Noiseless
   - Three ideal peaks
3. Noise-Subtracted
   - Predicted noise subtracted from noisy CFR
   - Higher SNR than 1
   - LOS peak is recovered!



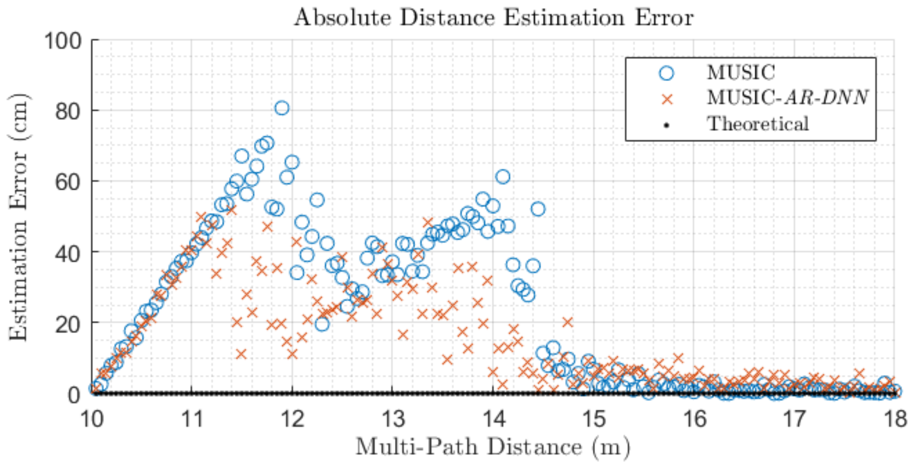
Comparison vs Classical MUSIC Algorithm

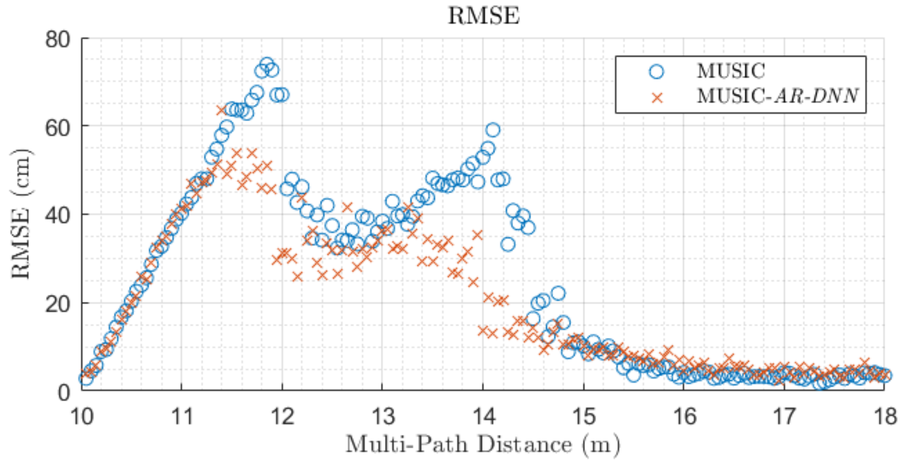
Train 2 Networks Offline

1. MUSIC-*A-DNN*: trained with AWGN fixed channel fading data
2. MUSIC-*AR-DNN*: trained with AWGN Rayleigh fading data

Test on both AWGN fixed and Rayleigh fading data

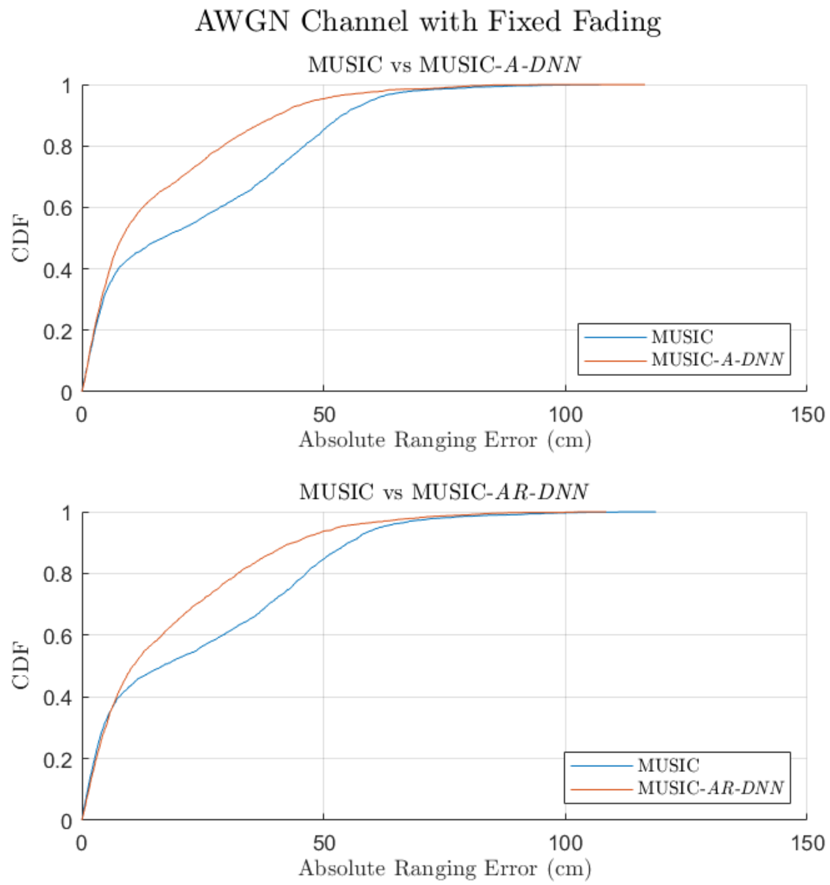
- ➢ Two-tap multi-path scenario
- ➢ 160 scenarios from 10 m – 18 m, step size 5 cm
    - Each scenario repeated 20 times with new noise/channel
- ➢ Compare empirical CDF of ranging error over all 3200 samples



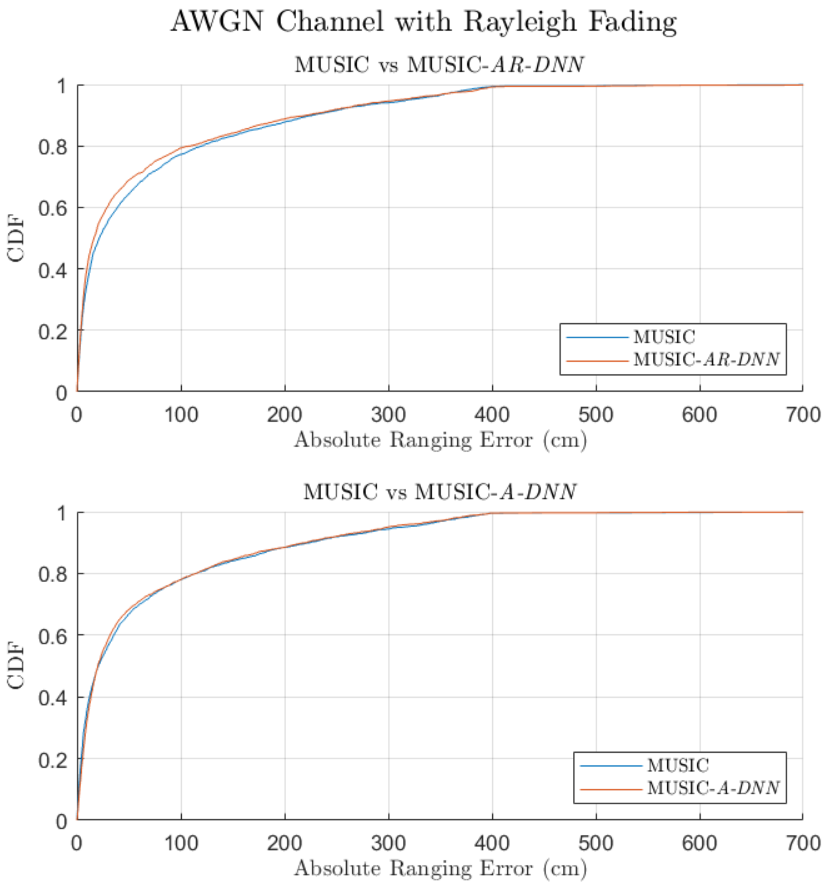


Comparison vs Classical MUSIC Algorithm – CDF

Fixed Fading Data



Rayleigh Fading Data



Comments

- Residual learning technique improves SNR
    - LOS peak is recovered more often than with traditional MUSIC
- Drawbacks:
    - Network must be retrained for various scenarios
    - Offline training time and online latency (~10 ms)
- Lower performance gain from MUSIC-*AR-DNN* than MUSIC-*A-DNN*
    - Residual learning does not address the impact of Rayleigh channel, only additive noise
    - SNR is still improved, but LOS may be lost
    - Alternative method (classical or deep learning) may yield better results for Rayleigh channel
    - Calls for further research into DNNs for sinusoidal-structured data
- Further investigation is likely to yield better results

Conclusions

- Investigated error introduced by crystal offset and timing errors
- Extended work of [1] by deriving analytical expression for ranging error due to mobility
  - For pedestrian velocities less than 3 m/s, error is less than 1 cm
- Considered multi-path and Rayleigh fading channel
- Introduced FFT and MUSIC spectral estimators for ranging
  - MUSIC has higher resolution, but requires high SNR
- Improved BLE ranging with residual learning technique
  - Novel residual learning framework specific to BLE ranging
  - FCNN architecture to estimate the noise for a given two-way CFR
  - Reduced average ranging error compared to direct MUSIC approach

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References

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