EESC6343 Project: A Support Vector Machine MUSIC Algorithm

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***Abstract*—In this project, we will examine the work of El-Gonnouni et al. [1] towards developing robust direction-of-arrival (DoA) estimation. First, a mathematical relationship between the Minimum Variance Distortionless Response (MVDR) and the MUltiple SIgnal Classification (MUSIC) DoA algorithms. Then, a support vector machine (SVM) extension of these techniques is introduced and the results between all four variants are compared.**

I. INTRODUCTION

In the last decade, sensor array processing has been studied extensively in the literature. So much attention has been given to such topic because of its wide spectrum of applications. For instance, it can be utilized in the problems of source localization, target detection, and spatial spectrum estimation.

The study of this topic is still showing great significance because firstly, there are new challenges arising that need to be addressed. The new challenges include, but are not limited to, dealing with system nonlinearities, equations regularization, and model compatibility with different noise types and sources. Secondly, in the 5G era that deals extensively with the concepts of Internet of Things (IoT) and dense pico cells, interference has become a significant problem. One promising solution for such a problem is to make use of sensor array processing in a source localization context in order to limit omnidirectional transmission for the sake of interference limitation. This also has some other communication system performance enhancement considerations, in addition to dealing with the crucial concern about possible health hazards due to the significantly increased electromagnetic waves transmitting nodes.

The source localization problem is mainly concerned by estimating a received signal Direction of Arrival (DoA). However, multiple DoA estimation techniques has been discussed in the literature, the focus of this paper will be directed to the MUltiple SIgnal Classification (MUSIC) algorithm. Multiple variants of the MUISIC algorithm will be discussed and the similarities and differences will be highlighted. The discussion will start by the Minimum Variance Distortionless Response (MVDR) algorithm which is considered as a generalization of the MUSIC algorithm. After highlighting the differences between the classical MUSIC and MVDR techniques, a Support Vector Machine (SVM) version of both techniques is presented.

The rest of the paper will be organized as follows: Section II is going to provide a general explanation of the problem and a definition of the system parameters. The classical MVDR and MUSIC algorithms are discussed in section III. This is followed by an illustration of how these two techniques can be formulated in an SVM form in section IV. Next, a thorough investigation of the performance of both techniques in the classical form as well as the SVM form is presented in section V. Finally, section VI will conclude the paper.

II. PROBLEM FORMULATION

The work presented in this paper is going to consider both cases of Uniform Linear Array (ULA) antenna as well as NonUniform ones (NULA). However, without loss of generality, the analysis in this section will be limited to ULA antennas and later generalized for the nonuniform case in section IV. The only difference will be found in the array response vector $\boldsymbol{a}$ explained later in this section.

The use of such array antennas enables the receiving node to have access to multiple versions of the received signal that have experienced different propagation phases. Those differences in the propagation phase are function of the signal angle of incidence, and therefore can be exploited for DoA estimation. The received signal by the ULA antenna at time instant n is denoted by

$$\boldsymbol{x}[n] = \begin{bmatrix} x_1[n] & x_2[n] & \dots & x_L[n] \end{bmatrix}^T \tag{1}$$

where L is the number of antenna elements in the ULA, and n is the time index where $1 \leq n \leq N$.

A demonstration of a ULA antenna is depicted in Fig. 1, where it can be observed that the uniform antenna spacing d leads to a uniform propagation difference between signals incident on consecutive antenna elements. Therefore a linear progressive phase takes place among the elements of the signal vector $x[n]$. The phase difference between consecutive antenna elements is given by

$$\omega_m = 2\pi \frac{d}{\lambda} sin(\theta_m). \tag{2}$$

The linear phase among different antenna elements is given by

$$\boldsymbol{a}_m = \begin{bmatrix} 1 & e^{2\pi \frac{d}{\lambda} sin(\theta_m)} & \dots & e^{2\pi \frac{d}{\lambda} (L-1) sin(\theta_m)} \end{bmatrix}^T \tag{3}$$

The subscript m here represents different transmitting nodes that are subject to the localization problem. The problem here assumes having M independent signals simultaneously transmitted from M nodes. The vector a_k is considered as the functional form of the MVDR and MUSIC DoA estimation techniques discussed in this paper. It fully captures the received signal spatial footprint and therefore is pivotal in DoA estimation.

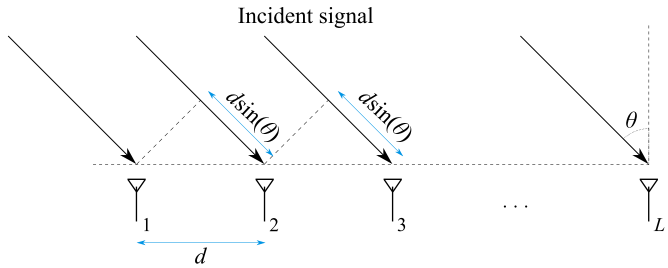


Fig. 1: Incident signal on a ULA antenna.

III. Classical DoA Estimation Techniques

A. MVDR Algorithm

The objective of the MVDR algorithm is to design a set of K filters $\{\boldsymbol{w}_1, \boldsymbol{w}_2, ..., \boldsymbol{w}_K\}$ that should be applied to the received signal to estimate the signal DoA. The number K depends on the aimed estimation resolution, where the larger K is, the higher estimation resolution is achieved. Each filter $\boldsymbol{w}_k$ within the designed bank of filters is said to be matched to a specific angle of incidence, and the MVDR algorithm tries to minimize its response to other angles.

The optimization function used for the filters design and optimization is set to be the filter output power defined by

$$S_x(k) = \frac{1}{N}\boldsymbol{w}_k^H \sum_n \boldsymbol{x}[n]\boldsymbol{x}^H[n]\boldsymbol{w}_k = \boldsymbol{w}_k^H \boldsymbol{R}\boldsymbol{w}_k, \tag{4}$$

where $\boldsymbol{R}$ is the received signal spatial correlation matrix and defined by

$$\boldsymbol{R} = \frac{1}{N}\boldsymbol{X}\boldsymbol{X}^H. \tag{5}$$

The signal matrix $\boldsymbol{X}$ is an $L \times N$ matrix representing the received signal from L antennas during time instants 1 through N. The signal $\boldsymbol{X}$ is a superposition between M received signals transmitted by M nodes subject to localization, and is given by

$$\begin{aligned}\boldsymbol{X} &= \boldsymbol{A}\boldsymbol{s} + \boldsymbol{n} \\ &= \begin{bmatrix}\boldsymbol{a}_1 & \dots & \boldsymbol{a}_M\end{bmatrix}_{L\times M} \begin{bmatrix}\boldsymbol{S}_1 \\ \vdots \\ \boldsymbol{S}_M\end{bmatrix}_{M\times N} + \begin{bmatrix}\boldsymbol{n}_1 \\ \vdots \\ \boldsymbol{n}_M\end{bmatrix}_{M\times N}\end{aligned} \tag{6}$$

where $\boldsymbol{a}_m$ is defined in (3), and $\boldsymbol{S}_m$ and $\boldsymbol{n}_m$ are the transmitted signal vector and noise vector at the receiver, respectively.

Next, the MVDR filter design process is formulated in a classical constrained minimization optimization problem as follows:

$$\begin{aligned}\min_{\boldsymbol{w}_k} \quad & \boldsymbol{w}_k^H \boldsymbol{R}\boldsymbol{w}_k \\ \text{s.t.} \quad & \boldsymbol{w}_k^H \boldsymbol{a}_k = 1.\end{aligned} \tag{7}$$

Solving this optimization problem using the Lagrangian multipliers method yields

$$\boldsymbol{w}_k = \frac{\boldsymbol{R}^{-1}\boldsymbol{a}_k}{\boldsymbol{a}_k^H \boldsymbol{R}^{-1}\boldsymbol{a}_k}. \tag{8}$$

By applying the filter $\boldsymbol{w}_k$ in (8), the filter output power will be given by

$$S_x^{MVDR}(k) = \frac{1}{\boldsymbol{a}_k^H \boldsymbol{R}^{-1}\boldsymbol{a}_k}. \tag{9}$$

B. A Relationship Between MVDR and MUSIC

The MUSIC algorithm can be seen as a variant of MVDR. The relation is well understood by doing an eigen value decomposition of the correlation matrix R and separating the signal term from the noise term [1]. By doing so, the correlation matrix R will have the form

$$\boldsymbol{R} = \boldsymbol{V}\boldsymbol{\Lambda}\boldsymbol{V}^H, \tag{10}$$

where

$$\boldsymbol{V} = \begin{bmatrix}\boldsymbol{V}_s & \boldsymbol{V}_n\end{bmatrix}, \tag{11}$$

$$\boldsymbol{\Lambda} = \begin{bmatrix}\boldsymbol{\Lambda}_s & 0 \\ 0 & \boldsymbol{\Lambda}_n\end{bmatrix}. \tag{12}$$

The matrices $\boldsymbol{V}_s$ and $\boldsymbol{V}_n$ consist of the eigen vectors of the signal subspace and noise subspace, respectively, while $\boldsymbol{\Lambda}_s$ and $\boldsymbol{\Lambda}_n$ contain their equivalent eigen values.

Using (10), (11), and (12), we can find an expression for $\boldsymbol{R}^{-1}$ and substitute it back in (9) to get

$$S_x^{MVDR}(k) = \frac{1}{\boldsymbol{a}_k^H(\boldsymbol{V}_s\boldsymbol{\Lambda}_s^{-1}\boldsymbol{V}_s^H + \boldsymbol{V}_n\boldsymbol{\Lambda}_n^{-1}\boldsymbol{V}_n^H)\boldsymbol{a}_k}. \tag{13}$$

As far as the MUSIC algorithm is concerned, only the noise term in the denominator of (13) is considered. The signal term of $\boldsymbol{R}^{-1}$ is neglected, which is a valid assumption at high SNR. And the eigen values of the noise are further assumed to be equal to 1, transforming $\boldsymbol{\Lambda}_n$ to $\boldsymbol{I}_n$, which is also a valid assumption for uncorrelated wide sense stationary noise [2], [3]. This transforms (13) to

$$S_x^{MUSIC}(k) = \frac{1}{\boldsymbol{a}_k^H \boldsymbol{V}_n \boldsymbol{V}_n^H \boldsymbol{a}_k}. \tag{14}$$

The previous form in (14) for the filter output power can be achieved by formulating a constrained minimization problem that is only concerned by minimizing the signal contribution with unmatched filters. In other words, minimizing

$$S_x^{MUSIC}(k) = \boldsymbol{w}_k^H \boldsymbol{V}_s \boldsymbol{V}_s^H \boldsymbol{w}_k. \tag{15}$$

Nevertheless, this optimization problem cannot be solved since the matrix $\boldsymbol{V}_s\boldsymbol{V}_s^H$ is rank deficient. Alternatively, the following form is used.

$$\begin{aligned}\min_{\boldsymbol{w}_k} \quad & \boldsymbol{w}_k^H \boldsymbol{V} \begin{bmatrix}\alpha\boldsymbol{I}_s & 0 \\ 0 & \beta\boldsymbol{I}_n\end{bmatrix} \boldsymbol{V}^H \boldsymbol{w}_k \\ \text{s.t.} \quad & \boldsymbol{w}_k^H \boldsymbol{a}_k = 1.\end{aligned} \tag{16}$$

where α is assumed to be much greater than β ($\alpha \gg \beta$) in order for this full rank approximation to be accurate and to finally lead to (14). Again the assumption of $\alpha \gg \beta$ is quiet reasonable in the case of high operating SNR. Finally, the solution to the optimization problem is

$$\boldsymbol{w}_k = \frac{\boldsymbol{V}\begin{bmatrix}\alpha^{-1}\boldsymbol{I}_s & 0 \\ 0 & \beta^{-1}\boldsymbol{I}_n\end{bmatrix}\boldsymbol{V}^H\boldsymbol{e}_k}{\boldsymbol{a}_k^H\boldsymbol{V}\begin{bmatrix}\alpha^{-1}\boldsymbol{I}_s & 0 \\ 0 & \beta^{-1}\boldsymbol{I}_n\end{bmatrix}\boldsymbol{V}^H\boldsymbol{e}_k} \tag{17}$$

IV. SVM-MVDR AND SVM-MUSIC ESTIMATORS

The classical MVDR and MUSIC algorithms can be extended to include an SVM optimization criterion [4]. In this project, we will employ the complex SVM formulation presented in [5] and [6] and use the cost function proposed in [7]. Additionally, it can be shown that the MVDR and MUSIC algorithms are particular cases of the SVM-MVDR and SVM-MUSIC, respectively [1]. The methods presented in [8] are generalized in this section. In this report, we hope to clarify and organize the analysis of [1], which omits several important assumptions and definitions integral for proper implementation.

A. SVM-MVDR Algorithm

First, J constraints are declared as complex column vectors of length L, $\boldsymbol{u}_k[j]$. Ideally, the array outputs of $\boldsymbol{u}_k[j]$ are the corresponding J labels $r_k[j]$ as

$$r_k[j] = \boldsymbol{w}_k^H \boldsymbol{u}_k[j]. \tag{18}$$

However, in a practical implementation, optimization is introduced to minimize the errors $e_k[j]$ for all $\boldsymbol{u}_k[j]$ expressed by

$$e_k[j] = r_k[j] - \boldsymbol{w}_k^H \boldsymbol{u}_k[j]. \tag{19}$$

To train the SVM, the constraints and labels are declared as follows. The first constraint $\boldsymbol{u}_k[1]$ and its corresponding label $r_k[n]$ are declared in attempt to match the desired DoA fully to the filter $\boldsymbol{w}_k$. The first constraint is equal to the spatial response of the array to the an incident signal at the angle corresponding to the spatial frequency $\omega_k = 2\pi \frac{d}{\lambda} \sin\theta_k$ and the label $r_k[1]$ is set to 1. The other constraints are declared in the same manner, but centered at spatial frequencies across the spectrum corresponding to other angles of arrival and their corresponding $r_k[j]$ values are all set to 0.

$$\boldsymbol{u}_k[1] = \begin{bmatrix} 1 & e^{j\omega_k} & \dots & e^{j(L-1)\omega_k} \end{bmatrix}^T \tag{20}$$

The errors, constraints, and labels are defined in matrix form as

$$\begin{aligned} \boldsymbol{e}_k &= \boldsymbol{r}_k - \boldsymbol{w}_k^H \boldsymbol{U}_k, && (21)\\ \boldsymbol{e}_k &= \begin{bmatrix} e_k[1] & \dots & e_k[J] \end{bmatrix}, && (22)\\ \boldsymbol{r}_k &= \begin{bmatrix} 1 & 0 & \dots & 0 \end{bmatrix}, && (23) \end{aligned}$$

$$\boldsymbol{U}_k = \begin{bmatrix} \boldsymbol{u}_k[1] & \dots & \boldsymbol{u}_k[J] \end{bmatrix} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ e^{j\omega_k} & e^{j\omega_{k_2}} & \dots & e^{j\omega_{k_3}} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j(L-1)\omega_k} & e^{j(L-1)\omega_{k_2}} & \dots & e^{j(L-1)\omega_{k_3}} \end{bmatrix}, \tag{24}$$

where $\omega_{k_j} \neq \omega_k \forall j \in [2, J]$. According to these declarations, during the SVM training, the filters at each angle index k, $\boldsymbol{w}_k$, are matched to the spatial response from an incident angle ω_k and unmatched from $J-1$ other angles of arrival across the spectrum, as optimally as possible. To optimize this matching, the errors are to be minimized and the robust cost function in [9] is applied:

$$\mathscr{L}_R(e_k[n]) = \begin{cases} 0 & |e_k[j] < \epsilon \\ \frac{1}{2\gamma}(|e_k[j]| - \epsilon)^2 - \epsilon & \epsilon \leq |e_k[j]| \leq \epsilon + \gamma C \\ C(|e_k[j]| - \epsilon) - \frac{1}{2}\gamma C^2 & \epsilon + \gamma C \leq |e_k[j]| \end{cases} \tag{25}$$

This cost function is 0 for when the magnitude of $e_k[j]$ is small, quadratic in the intervals $\epsilon \leq |e_k[j]| \leq \epsilon + \gamma C$, and linear for for errors above $\epsilon + \gamma C$. [1] discusses the adjustment of γC to apply a quadratic cost to the samples affected by thermal noise. Then for errors whose error is large, and likely to not have a Guassian distribution, a linear cost is applied [10].

Now, the corresponding functional to minimize the cost function in (25) must contain the filter output power, a regularization term, and the cost function applied to all errors and can be expressed as

$$\begin{aligned} L_p = \frac{1}{2}\boldsymbol{w}_k^H \boldsymbol{R} \boldsymbol{w}_k + \frac{\nu}{2}||\boldsymbol{w}_k||^2 \\ + \sum_{j=1}^{J} \left[\mathscr{L}_R(\xi_k[j] + \xi_k'[j]) + \mathscr{L}_R(\zeta_k[j] + \zeta_k'[j]) \right] \end{aligned} \tag{26}$$

with the following constraints, equivalent to those applied to the classical MVDR:

$$\begin{aligned} \Re(r_k[j] - \boldsymbol{w}_k^H \boldsymbol{u}_k[j]) &\leq \epsilon - \xi_k[j], && (27)\\ \Im(r_k[j] - \boldsymbol{w}_k^H \boldsymbol{u}_k[j]) &\leq \epsilon - \xi_k'[j], && (28)\\ \Re(-r_k[j] + \boldsymbol{w}_k^H \boldsymbol{u}_k[j]) &\leq \epsilon - \zeta_k[j], && (29)\\ \Im(-r_k[j] + \boldsymbol{w}_k^H \boldsymbol{u}_k[j]) &\leq \epsilon - \zeta_k'[j], && (30) \end{aligned}$$

where $\Re$ and $\Im$ indicate the real and imaginary parts of the complex numbers, respectively. These constraints take into account both the real and imaginary and the positive and negative parts of the errors. Note that ϵ is the error tolerance and $\xi_k[j], \xi_k'[j], \zeta_k[j], \zeta_k'[j]$ are the SVM slack variables, allowing for some tolerance in the boundary selection. For the remaining derivations, the error tolerance, ϵ, is considered to be 0, as it is defined for our implementation and the paper on which this report is based [1]. By optimizing the Lagrange function in (26), the optimal filter can be expressed as

$$\boldsymbol{w}_k = \boldsymbol{R}^{-1} \boldsymbol{U}_k \boldsymbol{\phi}_k, \tag{31}$$

where $\boldsymbol{\phi}_k$ are the Lagrange multipliers. The result in (31) can be substituted into (26) to yield the following functional.

$$L_d = -\frac{1}{2}\boldsymbol{\phi}_k^H \left[\boldsymbol{U}_k^H \boldsymbol{R}^{-1} \boldsymbol{U}_k + \gamma \boldsymbol{I} \right] \boldsymbol{\phi}_k - \Re(\boldsymbol{r}_k \boldsymbol{\phi}_k). \tag{32}$$

Now, the functional in (32) can be optimized by a quadratic programming (QP) package to obtain the optimal Lagrange multipliers $\boldsymbol{\phi}_k$. Then, using (31), the optimal filters can be easily computed. Alternatively, the spectrum at each k DoA index can be approximated by

$$S_k = \boldsymbol{\phi}_k^H \boldsymbol{U}_k^H \boldsymbol{R}^{-1} \boldsymbol{U}_k \boldsymbol{\phi}_k. \tag{33}$$

We want to make two remarks regarding the SVM-MVDR algorithm. First, as $\gamma C \to \infty$ and $\epsilon = 0$, the SVM-MVDR approaches the classical MVDR algorithm [1]. Second, the SVM optimization described in this section is a linear SVM model. But, a nonlinear version can be easily produced using kernel functions and a similar analysis. For the purposes of this project, however, we will only consider the linear case.

This optimization uses similar analysis to that of SVM classifiers; however, the implementation is quite different from the typical training and testing operation of most modern SVM classifiers. For the SVM-MVDR algorithm, an SVM is trained at each k angle index after the data matrix $\boldsymbol{X}$ is collected. This training does not rely on a prior training dataset, but purely relies on the information inherent to the sample spatial autocorrelation matrix $\boldsymbol{R}$ and the prior information on the array topology required in defining the constraints $\boldsymbol{u}_k[j]$. Therefore, the SVM-MVDR algorithm generalizes quite well to various signal-to-noise-ratios (SNR), number of DoAs, and array sizes.

B. SVM-MUSIC Algorithm

In a similar analysis to subsection III-B, the relationship between the SVM-MVDR and SVM-MUSIC algorithms is simply the substitution $\boldsymbol{R}^{-1} \approx \boldsymbol{V}_n \boldsymbol{V}_n^H$. Using this approximation, the same analysis can be performed to yield the functional

$$L_d = -\frac{1}{2} \boldsymbol{\phi}_k^H \left[\boldsymbol{U}_k^H \boldsymbol{V}_n \boldsymbol{V}_n^H \boldsymbol{U}_k + \gamma \boldsymbol{I} \right] \boldsymbol{\phi}_k - \Re(\boldsymbol{r}_k \boldsymbol{\phi}_k). \tag{34}$$

Again, this QP problem can also be easily solved by an optimization package. Then, the optimal filters can be obtained by

$$\boldsymbol{w}_k = \boldsymbol{V}_n \boldsymbol{V}_n^H \boldsymbol{U}_k \boldsymbol{\phi}_k, \tag{35}$$

and the spectrum and be estimated by

$$S_k = \boldsymbol{\phi}_k^H \boldsymbol{U}_k^H \boldsymbol{V}_n \boldsymbol{V}_n^H \boldsymbol{U}_k \boldsymbol{\phi}_k. \tag{36}$$

As described in subsection III-B, the MUSIC algorithm produces a higher resolution spatial spectrum by relying on key assumptions about the nature of the signal and prior knowledge on the number of incident signals, but has a simple mathematical connection to the MVDR algorithm.

Again, we want to emphasize the nature of this optimization being related to the SVM constrained optimization problem, but solved using a QP solver to obtain optimal Lagrange multipliers, from which optimal filters can be computed, rather than traditional SVM techniques involving training an SVM model and verifying it by classifying a testing dataset.

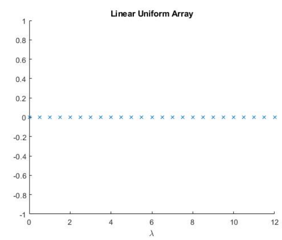
C. Algorithm Summary

The routine for high-resolution DoA estimation using the SVM-MVDR and SVM-MUSIC is summarized below.

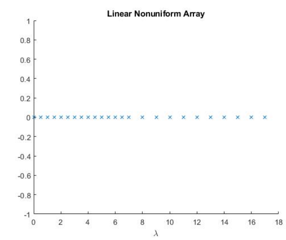
SVM-MVDR and SVM-MUSIC Summary

1) The data matrix $\boldsymbol{X}$ is collected and the sample spatial autocorrelation matrix $\boldsymbol{R}$ is computed by (5).
2) For each index k corresponding to spatial frequency ω_k, a QP optimization package is employed to optimize (32) or (34) to obtain the Lagrange multipliers $\boldsymbol{\phi}_k$ for the SVM-MVDR and SVM-MUSIC algorithms, respectively. In this step, $\boldsymbol{U}_k$ must be defined as described above.
3) Using $\boldsymbol{\phi}_k$, the SVM-MVDR spectrum is estimated by (33) and the SVM-MUSIC spectrum is equally estimated by (36).

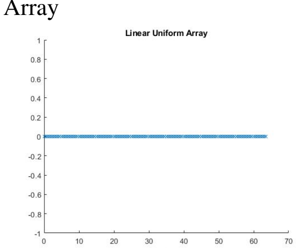
V. Results and Analysis



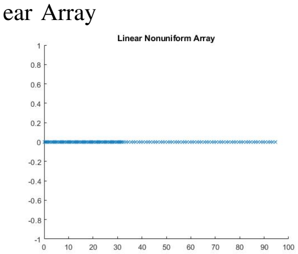
(a) 25 Element Uniform Linear Array



(b) 25 Element Nonuniform Linear Array



(c) 128 Element Uniform Linear Array



(d) 128 Element Nonuniform Linear Array

Fig. 2: Uniform and Nonuniform Linear Arrays: (a,c) elements equally spaced by $\lambda/2$, (b,d) first half of the elements are spaced by $\lambda/2$ and second half are spaced by λ.

A. Experimental Setup

We will compare the performance of the MVDR and MUSIC algorithms to their SVM counterparts as derived in section IV. We will consider a multiuser environment with users transmitting from unique angles from across the spectrum from -90° to 90°. Each user is transmitting a QPSK signal with unit power that experiences AWGN interference with power of σ_n. First, a uniform linear array is tested, followed by a nonuniform linear array.

For the SVM training, to acquire the optimal filters for each angle index k, ten constraints ($J = 10$) have been used across the spectrum at 256 equally spaced frequencies across the spectrum. This implies that 256 SVMs are trained every time the SVM-MVDR or SVM-MUSIC algorithm is run. For all the experiments, the SVM parameters are set as $\gamma = 0.01$, $C = 100$, and $\epsilon = 0$.

B. Uniform Linear Array

First, a received signal is simulated with three users whose DoAs are $-40°$, $40°$, and $60°$. Each signal consists of 50 QPSK snapshots. A uniform linear array with 25 elements, as shown in Fig. 2a, is used and the incoming signals are corrupted by AWGN with power $\sigma_n = 10$, so the SNR is -10 dB.

Fig. 3 shows the performance comparison among all four algorithms discuss in the previous sections. As visible in the figure, the MVDR algorithm shows a similar performance to the SVM-MVDR algorithm across the spectrum. But, the notable performance gain is visible in the SVM-MUSIC algorithm over the classical MUSIC algorithm. We notice both an improved peak resolution for the SVM-MUSIC algorithm as well as a significantly reduced noise floor. Thus, the SNR between the peak power at known DoAs and the noise floor is improved by the SVM-MUSIC algorithm.

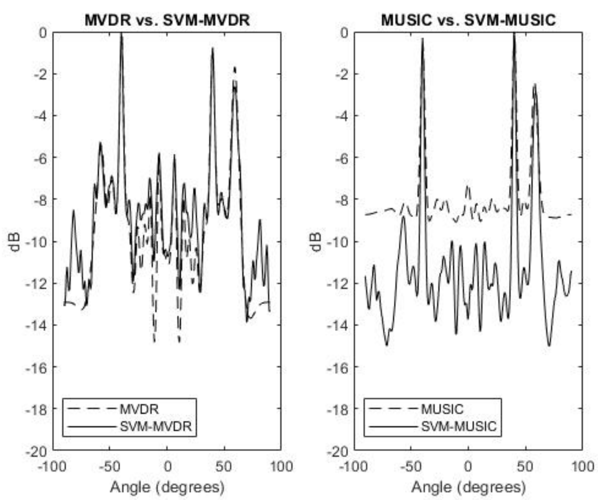


Fig. 3: Performance Comparison of Classical MVDR and MUSIC against their SVM Counterparts for SNR = -10 dB, 25 Element Uniform Linear Array

The experiment is repeated again with equal noise and signal powers, SNR = 0 dB and a 25 element uniform linear array. As expected, the DoA peaks are better resolved as the SNR is increased by 10 dB. Results are shown in Fig. 4. Again the MVDR and SVM-MVDR algorithms show similar performance with 25 elements, and the SVM-MUSIC algorithm shows a reduced noise floor in comparison to the MUSIC algorithm.

As shown in Fig. 5, increasing the SNR to 10 dB and keeping the same array topology the results imitate those for the -10 dB and 0 dB cases. The MVDR and SVM-MVDR algorithms yield similar DoA estimation and the MUSIC algorithm has an increased noise floor compared to the SVM-MUSIC algorithm.

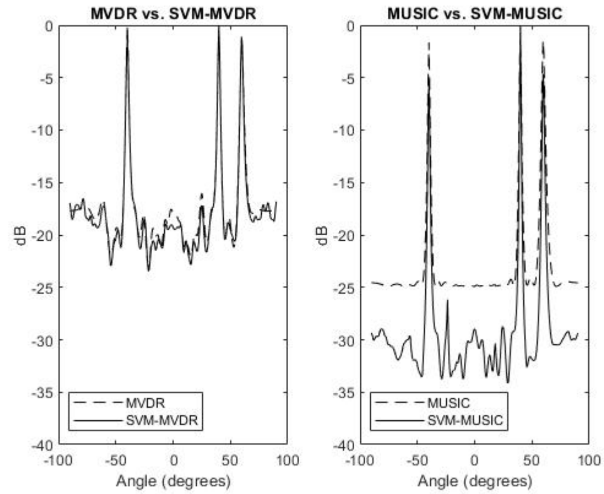


Fig. 4: Performance Comparison of Classical MVDR and MUSIC against their SVM Counterparts for SNR = 0 dB, 25 Element Uniform Linear Array

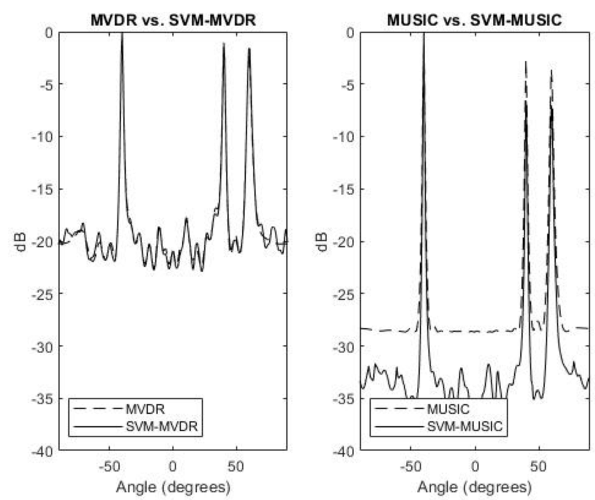


Fig. 5: Performance Comparison of Classical MVDR and MUSIC against their SVM Counterparts for SNR = 10 dB, 25 Element Uniform Linear Array

Now, when the uniform linear array is increased in size to 128 elements, Fig. 2c, the previous three experiments are repeated with SNRs of -10 dB, 0 dB, and 10 dB, as shown in Fig. 6, Fig. 7, and Fig. 8, respectively. For all three cases, an interesting phenomenon is observed. Now, the MVDR algorithm is wildly outperformed by the SVM-MVDR algorithm in both DoA peak resolution and noise floor. From these results, we can conclude that the SVM versions of the MVDR and MUSIC algorithms have outstanding performance when the number of array elements is large, showing increased performance over their classical equivalents in various noise environments. This further demonstrates the strengths of the SVM-MVDR and SVM-MUSIC algorithm in their generalizability and flexibility. For all the scenarios in this section and the following, the same SVM-MVDR and SVM-MUSIC routines can be applied without any changes or prior knowledge on the channel. The only requisite information is the topology of the receiver array, which is known. Next we will consider a nonuniform linear array and analyze the performance of all four algorithms.

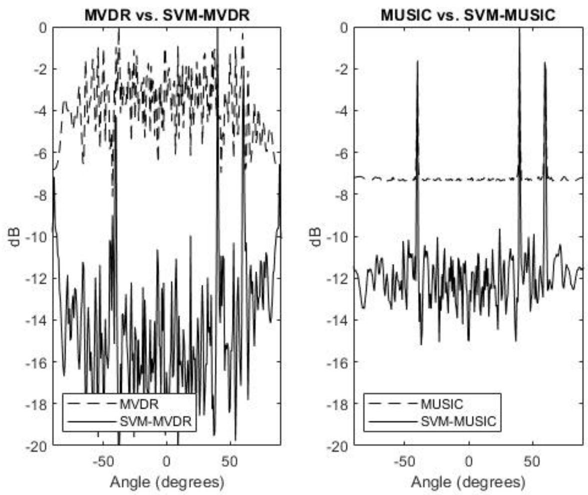


Fig. 6: Performance Comparison of Classical MVDR and MUSIC against their SVM Counterparts for SNR = -10 dB, 128 Element Uniform Linear Array

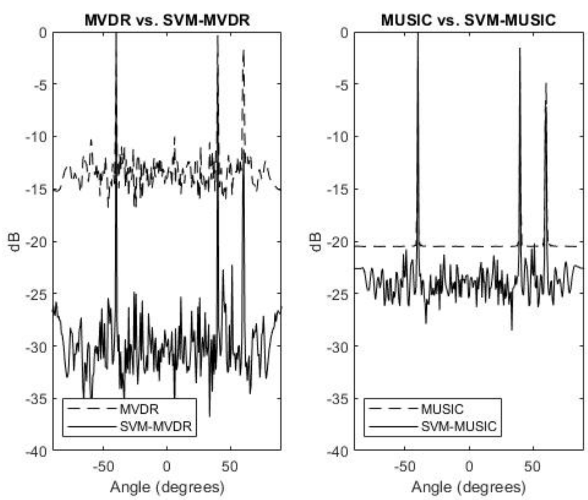


Fig. 7: Performance Comparison of Classical MVDR and MUSIC against their SVM Counterparts for SNR = 0 dB, 128 Element Uniform Linear Array

C. Nonuniform Linear Array

Now, simulations with the same DoAs are conducted this time with a nonuniform linear array with 25 elements, as shown in Fig. 2b, and 128 elements, as shown in Fig. 2d. Performance is evaluated at SNRs of -10 dB, 0 dB and

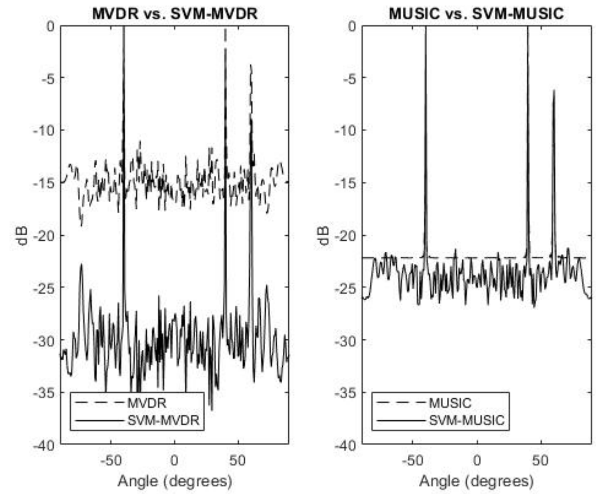


Fig. 8: Performance Comparison of Classical MVDR and MUSIC against their SVM Counterparts for SNR = 10 dB, 128 Element Uniform Linear Array

10 dB, although only the -10 dB results are shown in this report to demonstrate robustness. Similarly to the uniform case, for the smaller array topology, the MVDR and SVM-MVDR algorithms show nearly identical results while the SVM-MUSIC algorithm shows the highest peak resolution and lowest noise floor in comparison to all three other algorithms, Fig. 9. This trend has been validated for other SNRs not shown in this report. Again, increasing the array size to 128 elements results in a substantial performance gain in the SVM-MVDR algorithm in comparison to the classical MVDR algorithm, Fig. 10. However, under the same conditions, the MUSIC algorithm shows an increased performance yielding results similar in quality to those of the SVM-MUSIC algorithm. From these results, we can deduce that the SVM-MVDR algorithm benefits tremendously from an increased number of array elements, while the SVM-MUSIC algorithm displays excellent performance on systems with smaller array topologies.

VI. Conclusion

In this project, we were able to validate the work done in [1] by examining the MVDR DoA algorithm, its connection to the MUSIC DoA algorithm, and extensions of both to SVM-MVDR and SVM-MUSIC algorithms, respectively. Then, all four algorithms are validated in simulation and their performance is discussed. While the SVM algorithms display superior performance in comparison to their classical counterparts, their only drawback is the increased computational expense required to solve the QP optimization. However, most modern devices are capable of computing the classical and SVM versions of the algorithms at relatively comparable speeds. In a discussion of the results from the described algorithms, an interesting phenomenon can be observed for both the small array and large array cases in the presence of AWGN at various noise power levels. First, for smaller array sizes, the MVDR

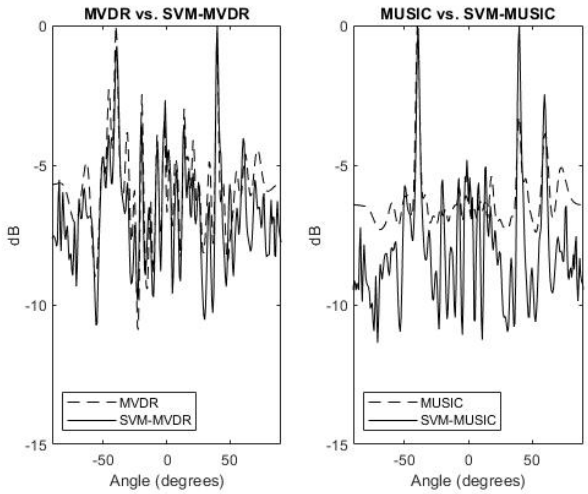


Fig. 9: Performance Comparison of Classical MVDR and MUSIC against their SVM Counterparts for SNR = -10 dB, 25 Element Nonuniform Linear Array

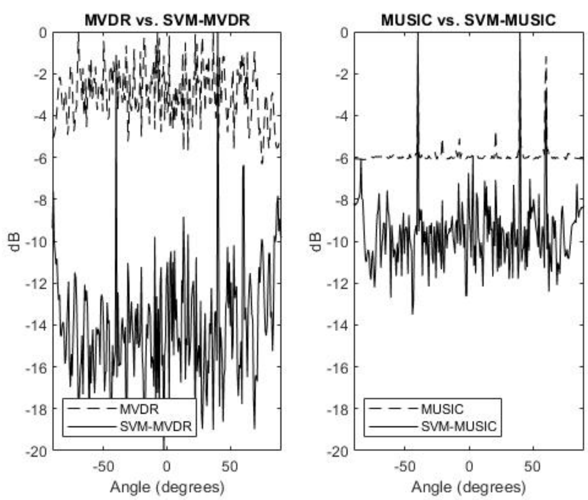


Fig. 10: Performance Comparison of Classical MVDR and MUSIC against their SVM Counterparts for SNR = -10 dB, 128 Element Nonuniform Linear Array

and SVM-MVDR perform quite similarly, but the MUSIC is significantly outperformed by the SVM-MUSIC algorithm when it comes to spatial spectral noise floor level, offering an improved noise floor. But for larger arrays, the SVM-MVDR algorithm now has a lower noise floor than the classical MVDR. And, the MUSIC algorithm performance approaches that of the SVM-MUSIC as the array size increases. From our analysis, we can conclude that for smaller arrays, the SVM-MUSIC shows the best performance among the algorithms considered in this report. For larger arrays, the SVM-MVDR algorithm may outperform the SVM-MUSIC and either can be selected at the designers' discretion. While discussion on this report centers around the performance comparisons of all four algorithms under various SNR channel conditions and array sizes, future work could consider other factors such as signal coherence, other SVM implementations (non-linear kernels etc.), and other subspace-based DoA algorithms (ESPIRT). In conclusion, we demonstrated the implementation of highly-adaptable SVM-MVDR and SVM-MUSIC algorithms that are easily generalized to various channel conditions and array sizes without requiring additional training, datasets, and tumult. These SVM-based DoA estimation techniques are shown to outperform their classical equivalents in both large and small array sizes.

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