Stochastic Processes in Biology

Course Overview

This is a graduate level introduction to stochastic modeling in biology. It is meant to both introduce you to the theory, as well as the essential numerical techniques used in stochastic modeling. In order to achieve this we will assign both ***regular homework***, and ***small-group challenges***. Groups will be assigned at random for each challenge.

Part of each class will be devoted to discussing the results of group challenges, and there will be a weekly review session where we will go into more detail.

A note on computation

To get started with Python you can use this nice introduction, which will also lead you through an implementation of an agent based model

https://github.com/Limor-Raviv/Tutorial_Agent_Based_Models

Please use the Jupyter environment to write your code. It makes it more readable.

https://jupyter.readthedocs.io/en/latest/install.html
https://365datascience.com/how-to-install-python/

Please also create a Github repository with your code, and keep it up to date. Generate a separate folder for each homework submission, and send me the link.

http://rogerdudler.github.io/git-guide/
https://drive.google.com/file/d/1N8bmZBat354DxCe9bRFJql0zDCAN5ZUK/view

We will create a Slack channel for the course. Please use it to ask me questions, and discuss the challenges and homework problems with each other.

Lesson Plan

The following is an overview of the course lesson plan, a sketch of the topics covered, and the associated computational challenges.

Introduction: Lectures 1-2 – *A quick review of probability*

- Basics of probability. Probability axioms, expectation, conditional probability, Bayes Theorem, univariate and multivariate Gaussian distribution, etc
- Generating pseudo-random numbers. Numerical generation of samples from different distributions, and essentials of stochastic simulations

Computational challenge 1: Rejection sampling

Part 1: Lectures 3-7 – *Markov processes*

- Markov chains, Markov chains in genetics, pure birth and pure death processes with uniform time increments, recurrence, transient states, master equations
- First passage times for simple Markov processes, invariant distributions, and quasi-steady state distributions
- The Poisson process and its properties, pure birth and death processes in continuous time, Yule Process
- Master equations and infinitesimal generators, computational implementation of birth and death processes using discretized time steps, and time steps drawn from the exponential distribution

Computational challenge 2: Simulating a birth-death process
Computational challenge 3: Tau-leaping and K-leaping

Part 2: Lectures 8-12 – *Biological networks*

- The Gillespie algorithm. Approximations of the Gillespie algorithm: tau-leaping and the chemical Langevin equation
- Examples of tau leaping and the Chem Langevin equation
- Representation of biochemical reaction networks. Biochemical reaction network models in gene regulatory networks
- Network motifs including switch and FFL. Intrinsic and extrinsic noise. Noise induced oscillations
- Parameter inference for stochastic processes

Computational challenge 4: Simulating a genetic switch and a repressilator
Computational challenge 5: Stochastic vs. deterministic models of gene circuits

Part 3: Lectures 13-17 – *Neuroscience*

- Simplified models of neurons. Integrate and fire models, and noise
- Generalized linear models of networks. Encoding and decoding, memory and decision making
- General derivation of FP equation. Example: Fokker-Planck equation for IF neurons
- Derivation of first passage time distributions
- Computing the distribution of firing times for linear and nonlinear IF neuron

Computational challenge 6: Integrate and fire models of neurons

Part 4: Lectures 18-22 – *Evolution*

- Wright-Fisher and Moran Models
- Derivation of diffusion approximation for a single locus Moran Model without selection
- Mutation, selection and stationary distribution for allele frequency
- Fixation probabilities and time to evolve
- Mapping between population genetic, ecological and linguistic models

Computational challenge 7: Wright-Fisher and Moran Models

Part 5: Lectures 23-27 – *Game theory*

- One-shot games. The Hawk-Dove game
- Iterated games. The Prisoner's Dilemma
- Stationary distributions for infinitely repeated games
- Replicator dynamics and infinite population models of evolution
- Agent-based models. Spatial Prisoner's Dilemma

Computational challenge 8: Evolutionary games
Computational challenge 9: Agent-based models

Final presentations: Lecture 28