

Simulation Results

2026-01-20

Simulation Setup

This simulation is performed with $n = 200$ and $d = 100$, using the 2-d lattice as the underlying graph. $s = 5$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.4$. The attached results are for a 10-replication simulation. The parameter vector θ has sparse components other than the following:

Parameter.Index	Value
6	-0.447
15	0.447
27	-0.447
62	0.447
90	0.447

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{6, 15, 72, 42\}$ elements of θ . Accordingly, **all statistics and visuals are indicative of performance only on the set \mathcal{C} .**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[6]	0.056	0.013
theta[15]	0.015	0.060
theta[72]	0.007	0.017
theta[42]	0.028	0.022
total	0.026	0.028

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[6]	0.162	0.021

	proposed	cgm
theta[15]	0.098	0.054
theta[72]	0.000	0.004
theta[42]	0.000	0.003
total	0.065	0.021

Mean absolute deviation comparison $\frac{1}{n.\text{sim}} \sum_{i=1}^{n.\text{sim}} \frac{1}{|\mathcal{C}|} \|\hat{\theta}_i - \theta\|$

Table 3: Mean Absolute Deviation of Parameter Estimates

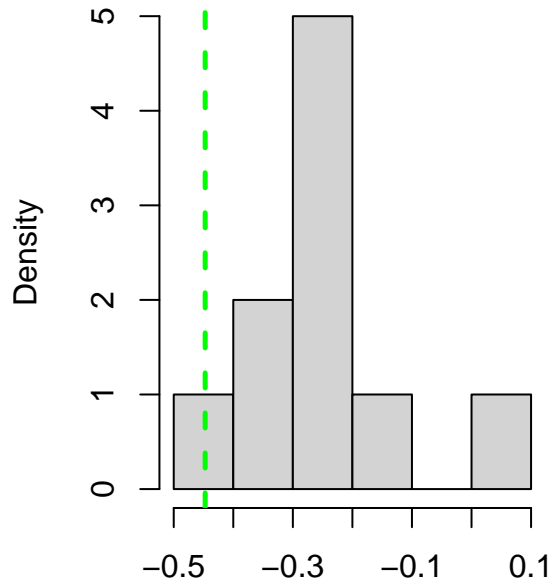
	proposed	cgm
theta[6]	0.201	0.099
theta[15]	0.118	0.190
theta[72]	0.058	0.101
theta[42]	0.142	0.121
total	0.130	0.128

Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

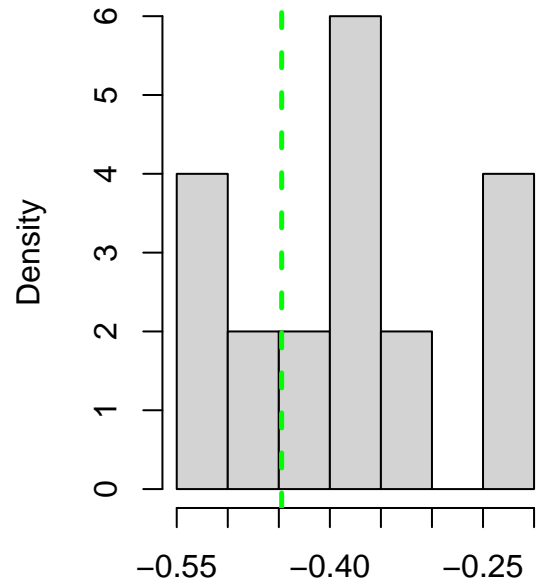
	proposed	cgm
theta[6]	0.401	0.114
theta[15]	0.283	0.189
theta[72]	0.000	0.027
theta[42]	0.006	0.025
total	0.172	0.089

Boxplots

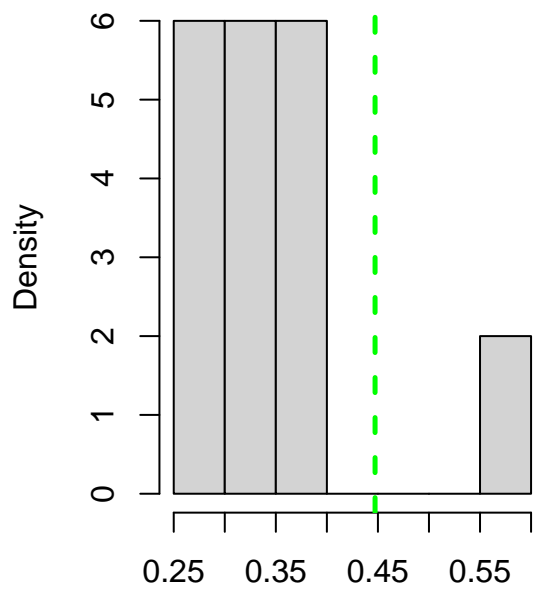
Histogram of proposed estimates for $\theta[6]=-0.447$



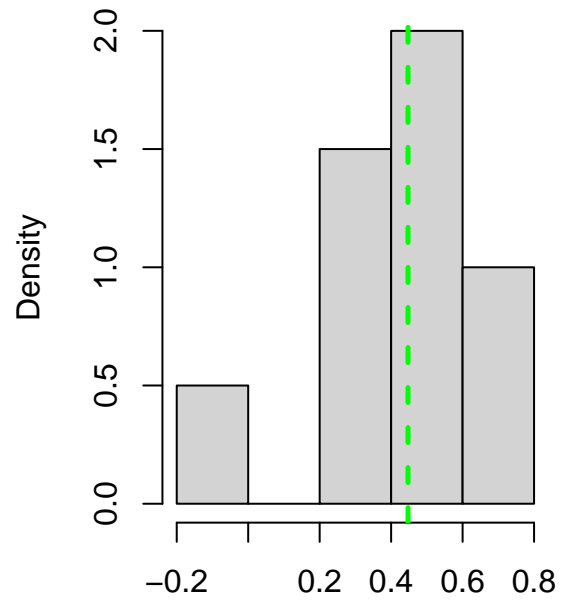
Histogram of cgm estimates for $\theta[6]=-0.447$



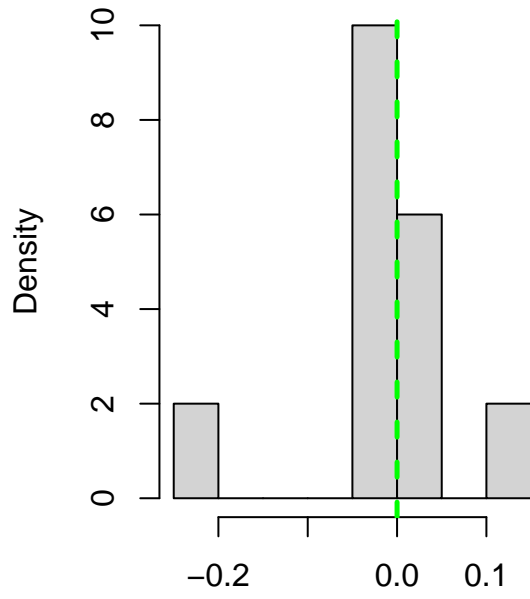
Histogram of proposed estimates for $\theta[15]=0.447$



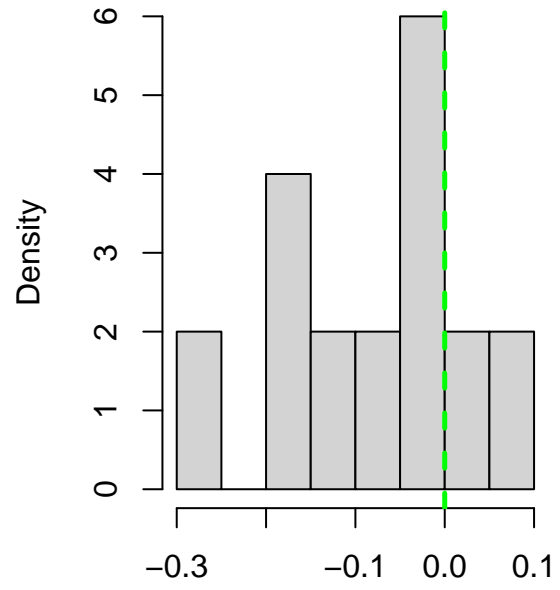
Histogram of cgm estimates for $\theta[15]=0.447$

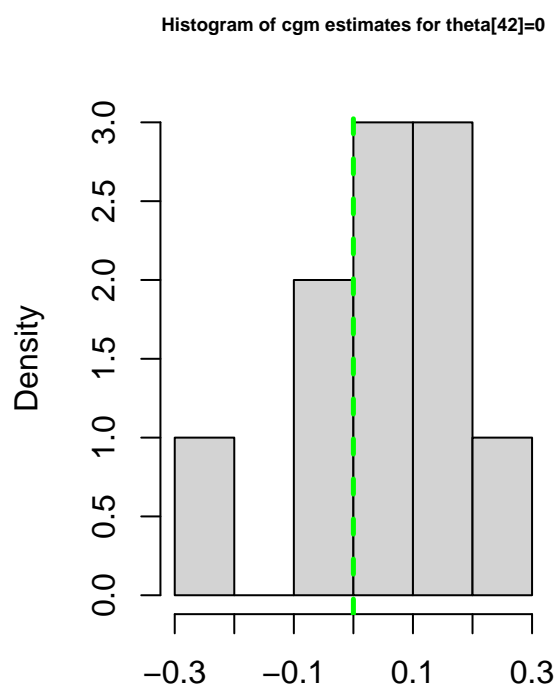
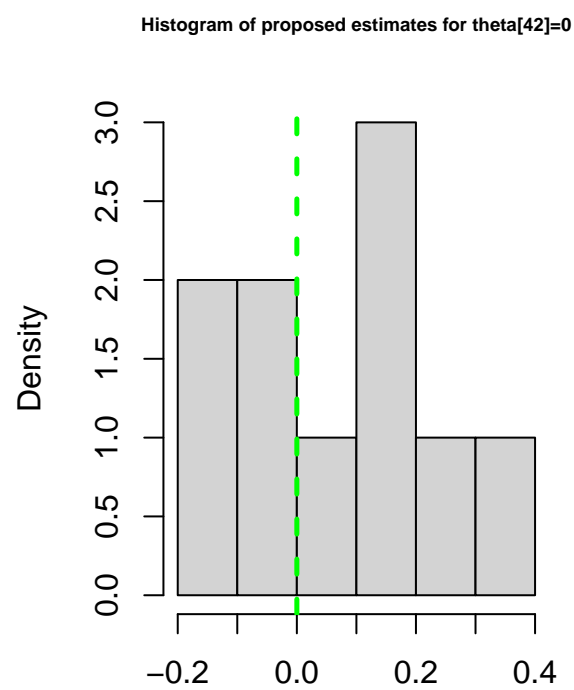


Histogram of proposed estimates for $\theta_{[72]}=0$



Histogram of cgm estimates for $\theta_{[72]}=0$





Statistics and 95% Confidence Intervals from per-Replicate Estimates

Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[6]	-0.247	0.128	-0.497	0.004	0.8
theta[15]	0.351	0.134	0.089	0.613	1.0
theta[72]	-0.024	0.134	-0.287	0.239	0.9
theta[42]	0.076	0.136	-0.190	0.343	1.0

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[6]	-0.391	0.125	-0.636	-0.145	1.0
theta[15]	0.429	0.142	0.151	0.708	0.8
theta[72]	-0.082	0.125	-0.328	0.164	1.0
theta[42]	0.046	0.125	-0.199	0.291	1.0