Simulation Results

2026-01-13

Simulation Setup

This simulation is performed with $n = 200$ and $d = 40$, using the 2-d lattice as the underlying graph. $s = 5$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.4$. The attached results are for a 10-replication simulation. The true values of the parameter vector θ are

```
0 0 0 0 -3 0 -3 0 0 0 0 0 -3 0 0 0 0 0 -3 0 -3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ,
```

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{$ 5, 7, 4, 1$\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison ($\frac{1}{n.sim}\sum_{i=1}^{n.sim}\frac{1}{|\mathcal{C}|}\|\hat{\theta}_{i,\mathcal{C}} - \theta_{\mathcal{C}}\|^2$)

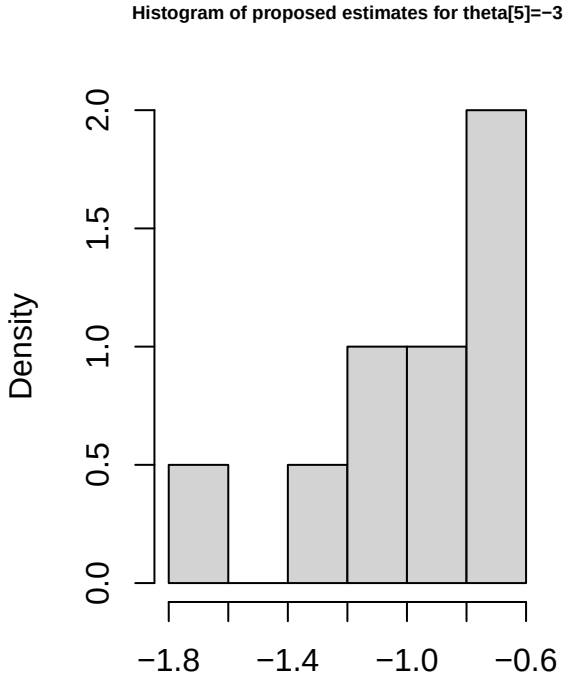
Table 1: Mean-Squared Error of Parameter Estimates

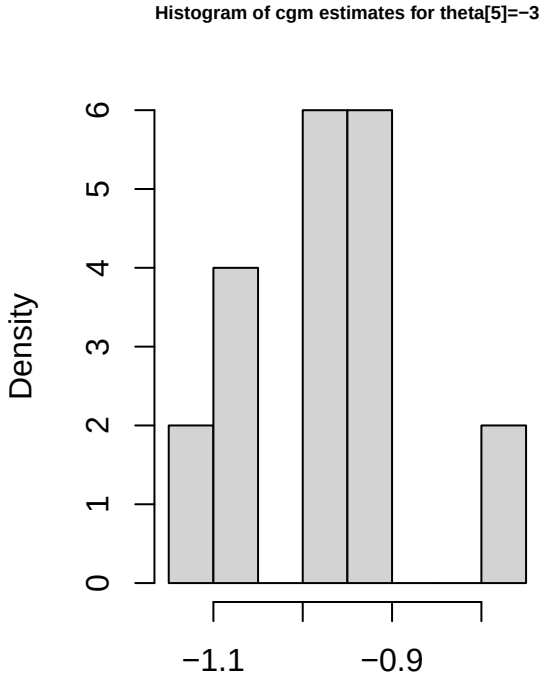
	proposed	cgm
theta[5]	4.341	4.106
theta[7]	4.166	4.131
theta[4]	0.059	0.003
theta[1]	0.098	0.007
total	2.166	2.062

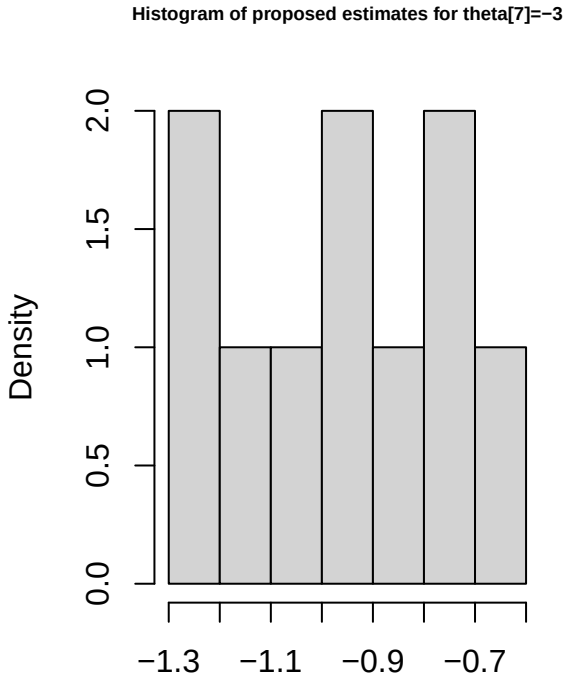
Table 2: Mean-Squared Error of First-Step Parameter Estimates

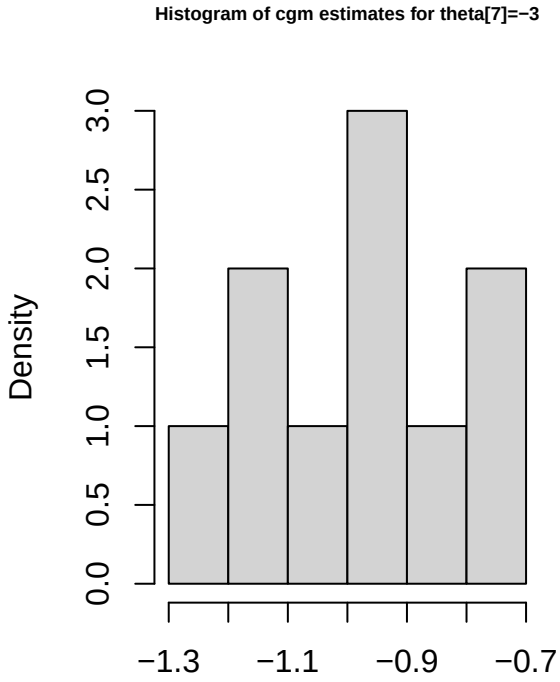
	proposed	cgm
theta[5]	4.972	4.705
theta[7]	5.033	4.824
theta[4]	0.045	0.003
theta[1]	0.016	0.001
total	2.517	2.383

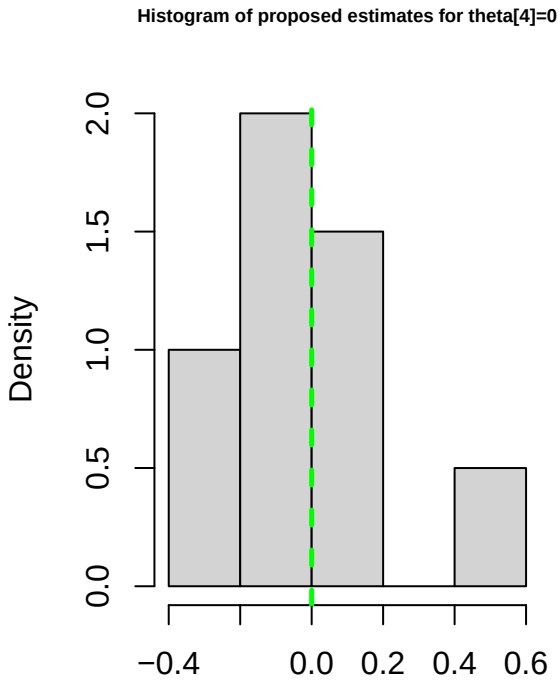
Boxplots

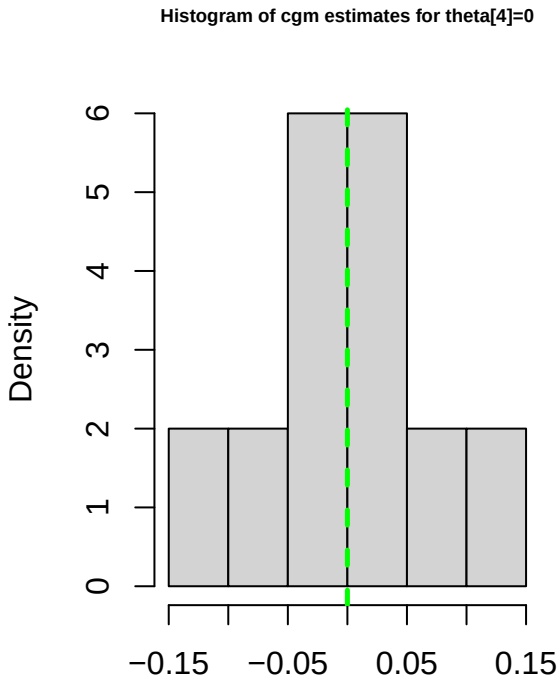

Histogram of proposed estimates for theta[5]=−3
Density
0.0
0.5
1.0
1.5
2.0
−1.8
−1.4
−1.0
−0.6

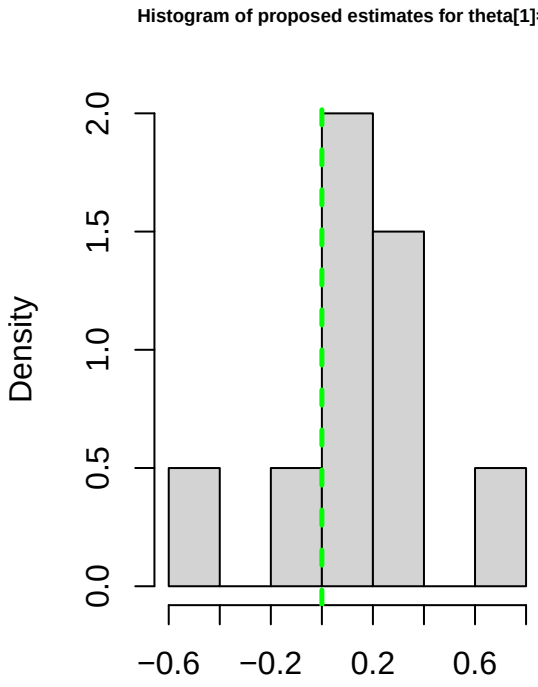

Histogram of cgm estimates for theta[5]=−3
Density
0
1
2
3
4
5
6
−1.1
−0.9

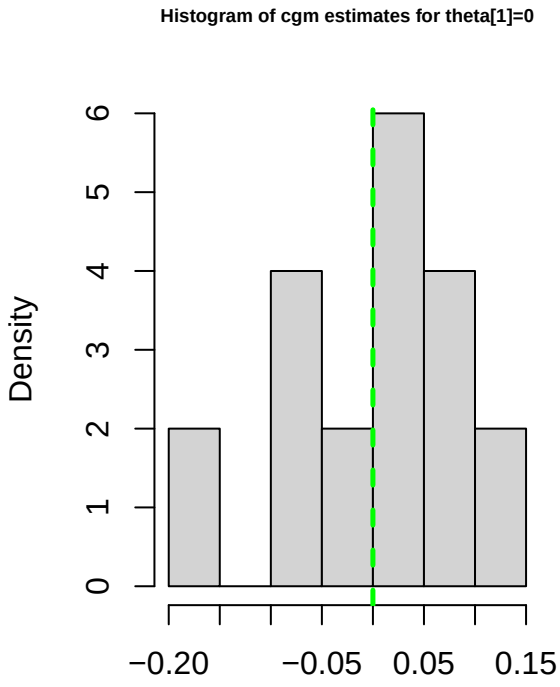

Histogram of proposed estimates for theta[7]=−3
Density
0.0
0.5
1.0
1.5
2.0
−1.3
−1.1
−0.9
−0.7


Histogram of cgm estimates for theta[7]=−3
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−1.3
−1.1
−0.9
−0.7

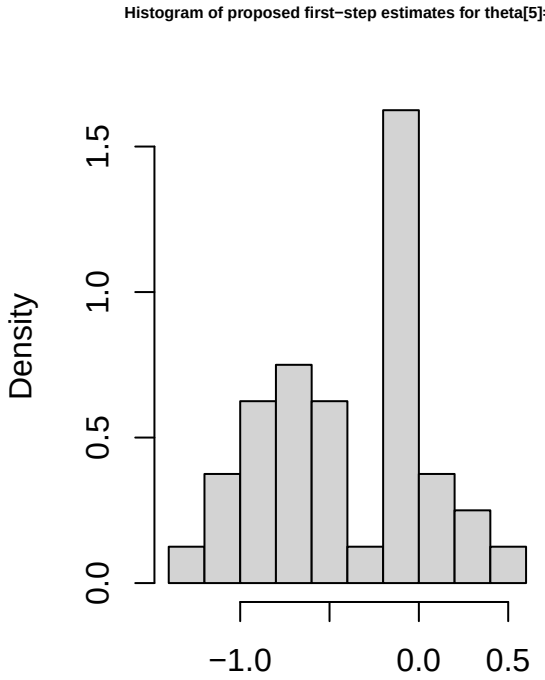

Histogram of proposed estimates for theta[4]=0
Density
0.0
0.5
1.0
1.5
2.0
−0.4
0.0
0.2
0.4
0.6

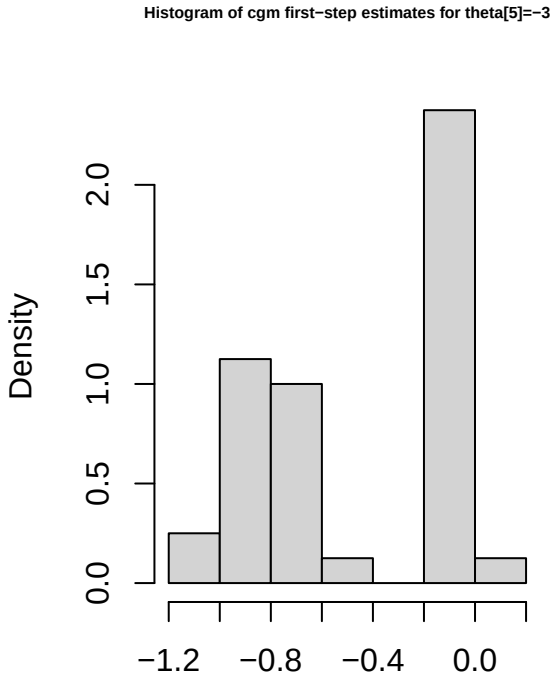

Histogram of cgm estimates for theta[4]=0
Density
0
1
2
3
4
5
6
−0.15
−0.05
0.05
0.15

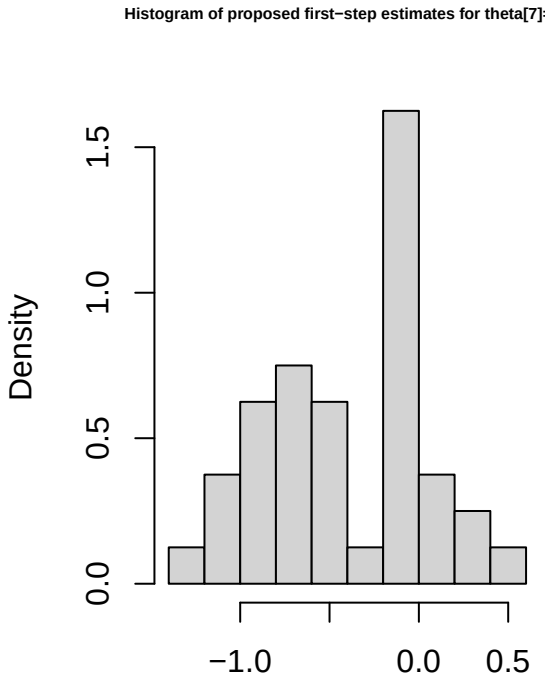

Histogram of proposed estimates for theta[1]=0
Density
0.0
0.5
1.0
1.5
2.0
−0.6
−0.2
0.2
0.6

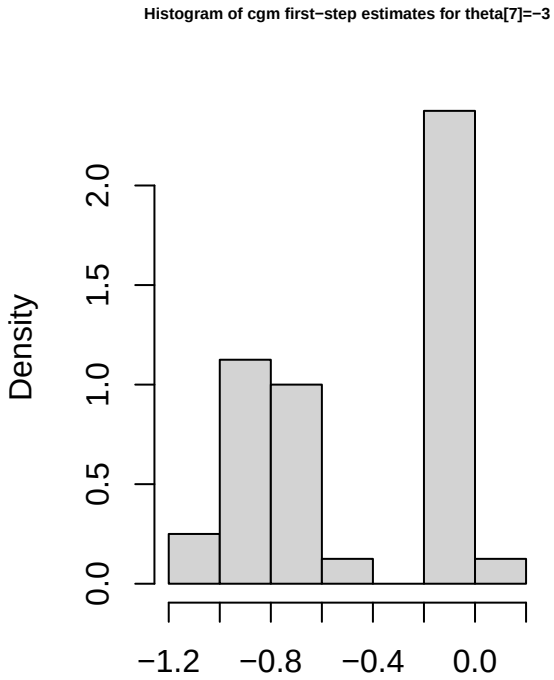

Histogram of cgm estimates for theta[1]=0
Density
0
1
2
3
4
5
6
−0.20
−0.05
0.05
0.15

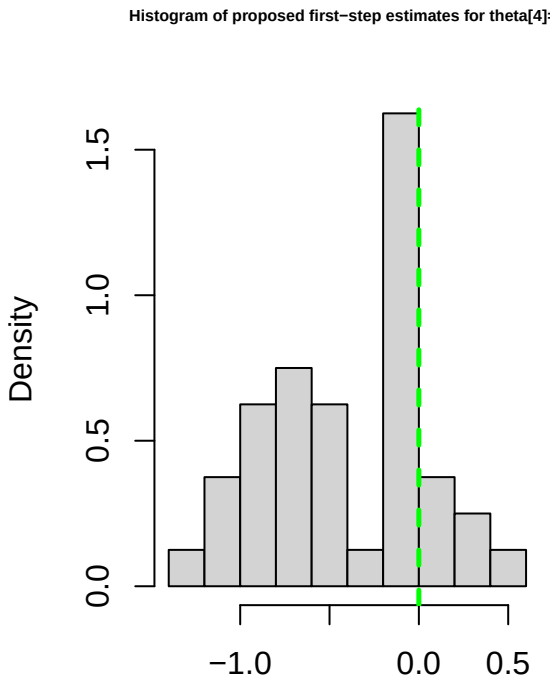
First Step Histograms

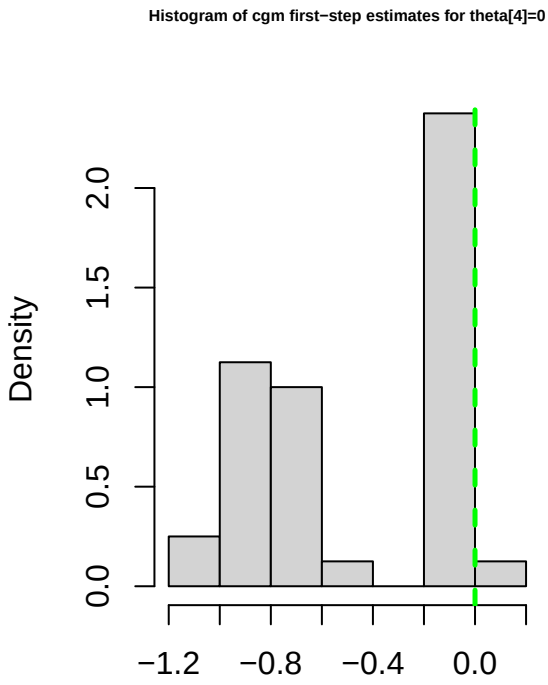


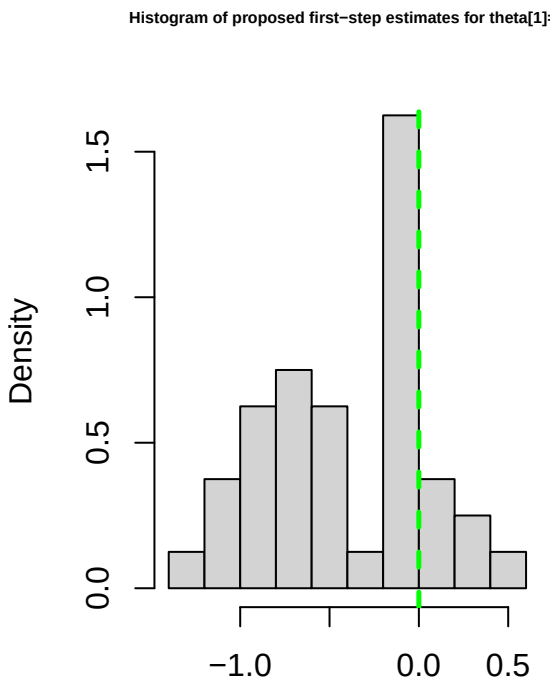


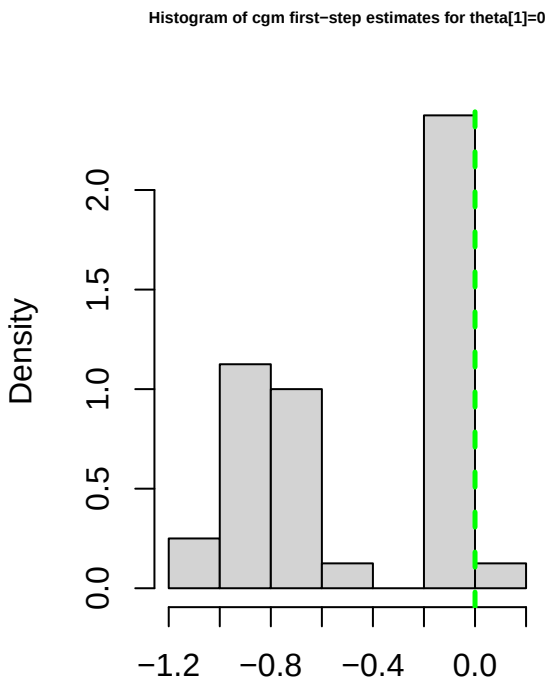





Histogram of proposed first−step estimates for theta[4]=0
Density
0.0
0.5
1.0
1.5
−1.0
0.0
0.5


Histogram of cgm first−step estimates for theta[4]=0
Density
0.0
0.5
1.0
1.5
2.0
−1.2
−0.8
−0.4
0.0


Histogram of proposed first−step estimates for theta[1]=0
Density
0.0
0.5
1.0
1.5
−1.0
0.0
0.5


Histogram of cgm first−step estimates for theta[1]=0
Density
0.0
0.5
1.0
1.5
2.0
−1.2
−0.8
−0.4
0.0

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 3: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[5]	-1.642	-0.875	-0.606	-1.574	-0.610
theta[7]	-1.264	-0.971	-0.674	-1.264	-0.698
theta[4]	-0.383	-0.029	0.541	-0.352	0.462
theta[1]	-0.485	0.150	0.728	-0.391	0.622

Table 4: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[5]	-1.122	-0.960	-0.778	-1.117	-0.808
theta[7]	-1.225	-0.955	-0.734	-1.205	-0.740
theta[4]	-0.102	-0.002	0.112	-0.091	0.099
theta[1]	-0.171	0.015	0.111	-0.152	0.104

Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[5]	-0.941	0.273	-1.476	-0.405	0.0
theta[7]	-0.968	0.277	-1.511	-0.426	0.0
theta[4]	-0.023	0.228	-0.470	0.425	1.0
theta[1]	0.125	0.246	-0.358	0.608	0.9

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[5]	-0.976	0.176	-1.320	-0.632	0
theta[7]	-0.973	0.182	-1.329	-0.617	0
theta[4]	0.000	0.162	-0.318	0.318	1
theta[1]	-0.002	0.151	-0.298	0.295	1