Simulation Results

2026-01-21

Simulation Setup

This simulation is performed with $n = 200$ and $d = 100$, using the 2-d lattice as the underlying graph. $s = 5$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.2$. The attached results are for a 10-replication simulation. The parameter vector θ has sparse components other than the following:

Parameter.Index	Value
7	-0.447
17	-0.447
44	-0.447
62	0.447
63	-0.447

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{$ 7, 17, 73, 41$\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[7]	0.065	0.023
theta[17]	0.053	0.036
theta[73]	0.009	0.009
theta[41]	0.014	0.013
total	0.035	0.020

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[7]	0.121	0.101

	proposed	cgm
theta[17]	0.148	0.104
theta[73]	0.000	0.000
theta[41]	0.001	0.000
total	0.067	0.051

Mean absolute deviation comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat

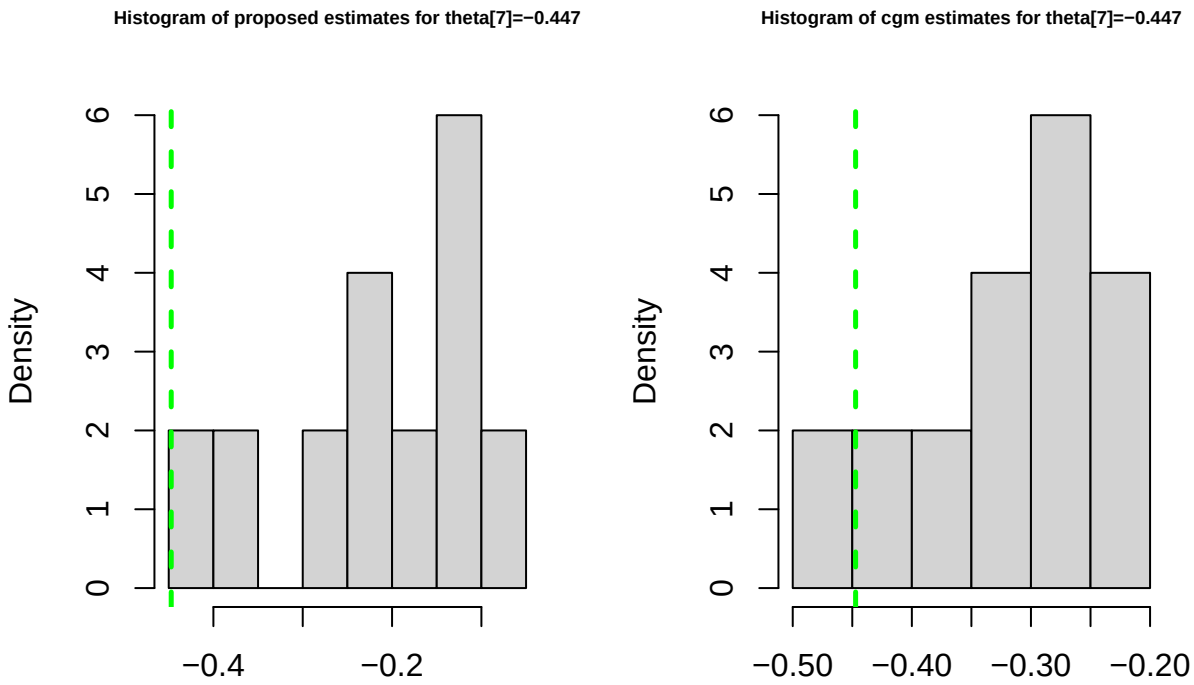
Table 3: Mean Absolute Deviation of Parameter Estimates

	proposed	cgm
theta[7]	0.230	0.132
theta[17]	0.182	0.161
theta[73]	0.075	0.075
theta[41]	0.100	0.091
total	0.147	0.115

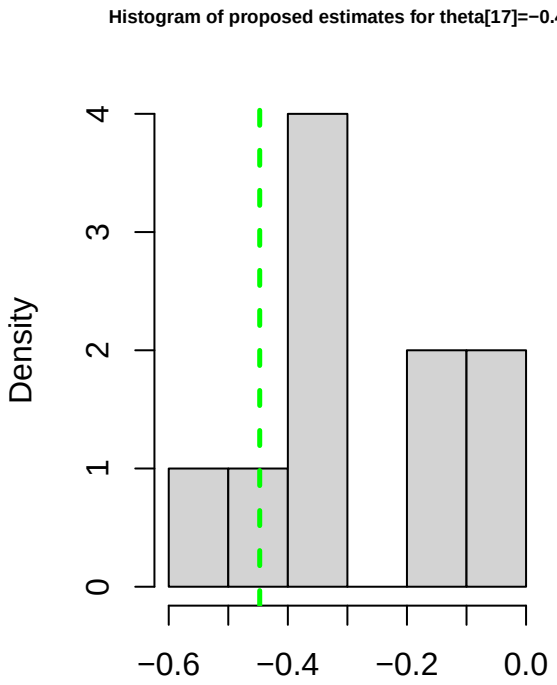
Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

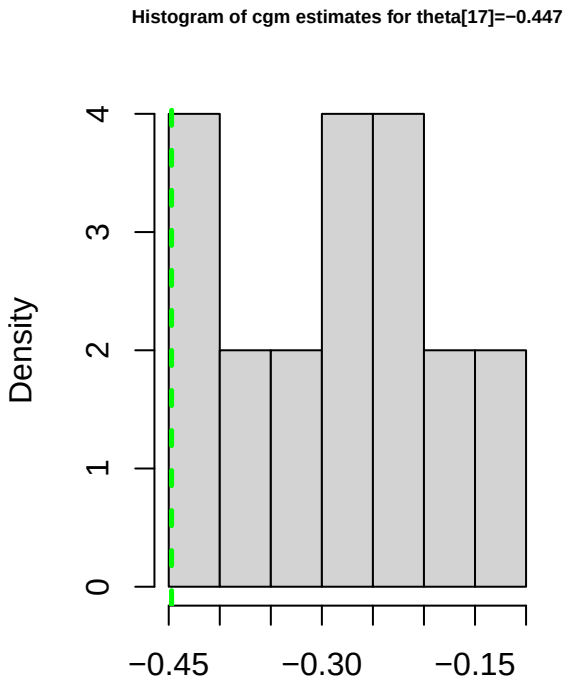
	proposed	cgm
theta[7]	0.332	0.278
theta[17]	0.378	0.308
theta[73]	0.000	0.000
theta[41]	0.007	0.000
total	0.179	0.146

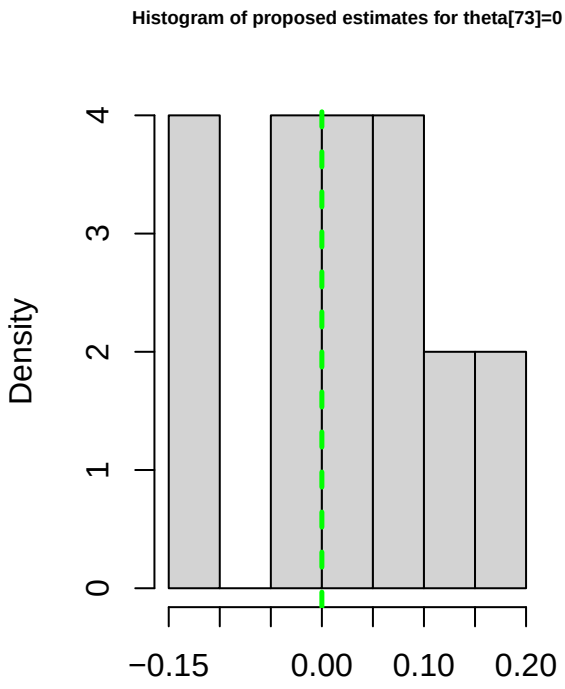
Boxplots

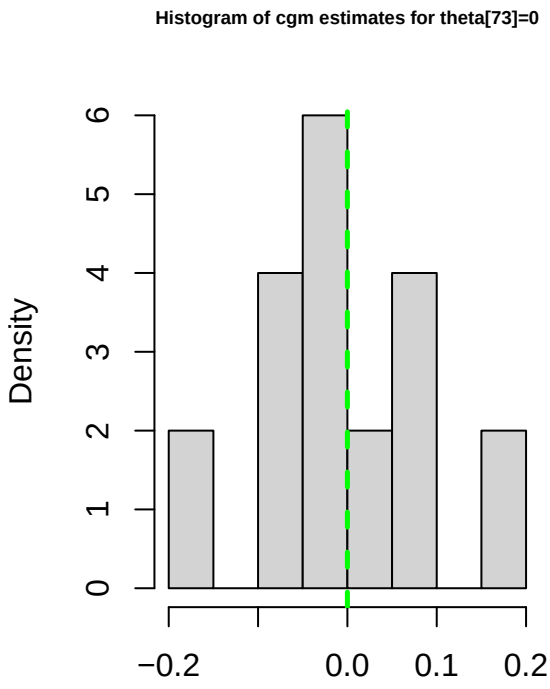

Histogram of proposed estimates for theta[7]=−0.447
Density
0
1
2
3
4
5
6
−0.4
−0.2

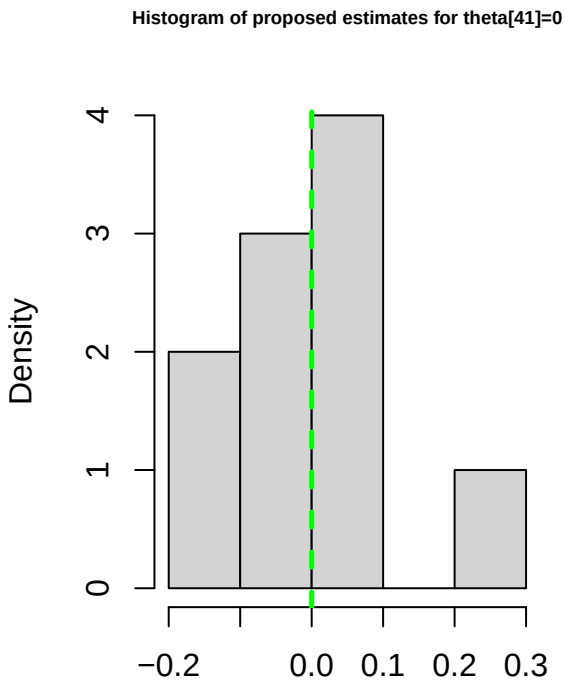
Histogram of cgm estimates for theta[7]=−0.447
Density
0
1
2
3
4
5
6
−0.50
−0.40
−0.30
−0.20

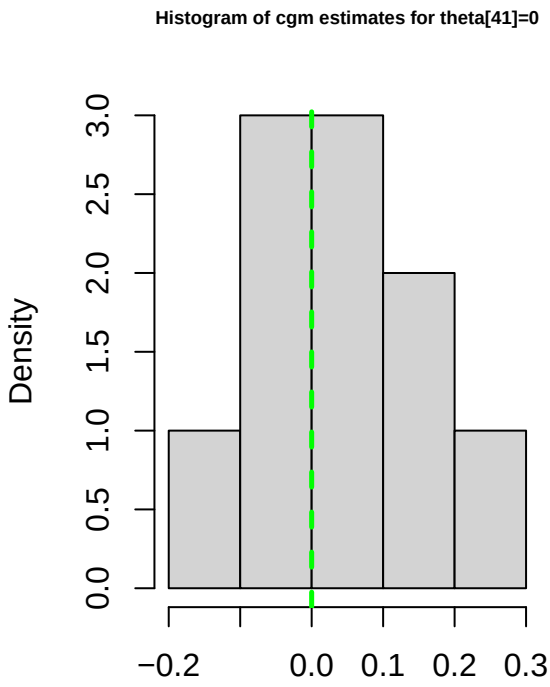

Histogram of proposed estimates for theta[17]=−0.447
Density
0
1
2
3
4
−0.6
−0.4
−0.2
0.0


Histogram of cgm estimates for theta[17]=−0.447
Density
0
1
2
3
4
−0.45
−0.30
−0.15


Histogram of proposed estimates for theta[73]=0
Density
0
1
2
3
4
−0.15
0.00
0.10
0.20


Histogram of cgm estimates for theta[73]=0
Density
0
1
2
3
4
5
6
−0.2
0.0
0.1
0.2


Histogram of proposed estimates for theta[41]=0
Density
0
1
2
3
4
−0.2
0.0
0.1
0.2
0.3


Histogram of cgm estimates for theta[41]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.2
0.0
0.1
0.2
0.3

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[7]	-0.218	0.131	-0.473	0.038	0.5
theta[17]	-0.285	0.141	-0.560	-0.009	0.7
theta[73]	0.022	0.119	-0.212	0.256	1.0
theta[41]	0.003	0.127	-0.246	0.252	0.9

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[7]	-0.324	0.098	-0.516	-0.132	0.8
theta[17]	-0.287	0.096	-0.476	-0.097	0.5
theta[73]	-0.008	0.094	-0.191	0.176	1.0
theta[41]	0.032	0.094	-0.153	0.217	0.9