Simulation Results

2026-01-14

Simulation Setup

This simulation is performed with $n = 200$ and $d = 20$, using the 2-d lattice as the underlying graph. $s = 2$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.4$. The attached results are for a 10-replication simulation. The true values of the parameter vector θ are

```
0 0 0 0 0 0.7071068 0 0 0 0 0 0.7071068 0 0 0 0 0 0 0 0 ,
```

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{$ 6, 12, 4, 8$\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

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### Mean-squared error comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat\theta
```

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[6]	0.586	0.117
theta[12]	0.177	0.102
theta[4]	0.029	0.019
theta[8]	0.107	0.025
total	0.225	0.066

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[6]	0.174	0.125
theta[12]	0.090	0.106
theta[4]	0.052	0.012
theta[8]	0.084	0.008
total	0.100	0.063

Mean absolute deviation comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat

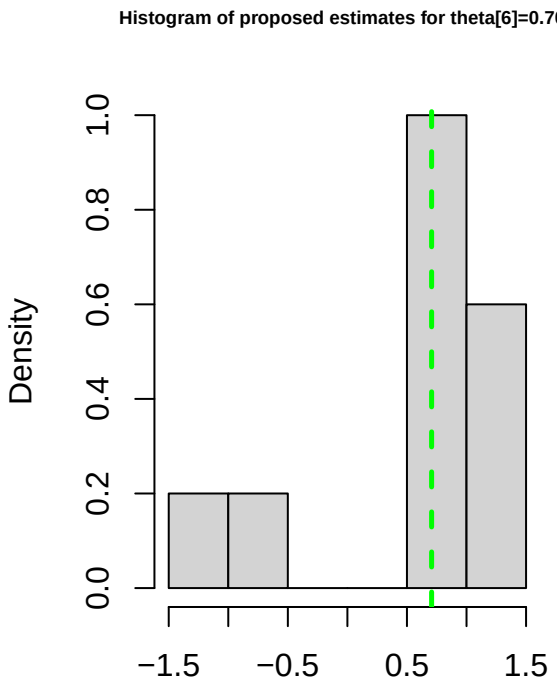
Table 3: Mean Absolute Deviation of Parameter Estimates

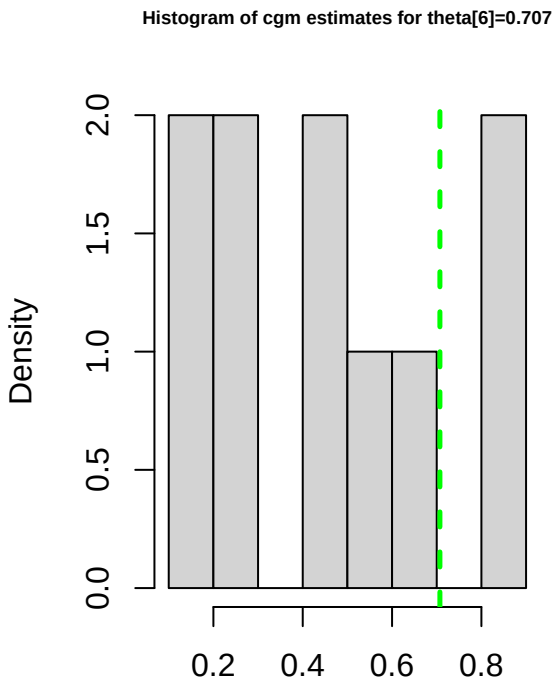
	proposed	cgm
theta[6]	0.479	0.293
theta[12]	0.295	0.232
theta[4]	0.136	0.111
theta[8]	0.269	0.117
total	0.295	0.188

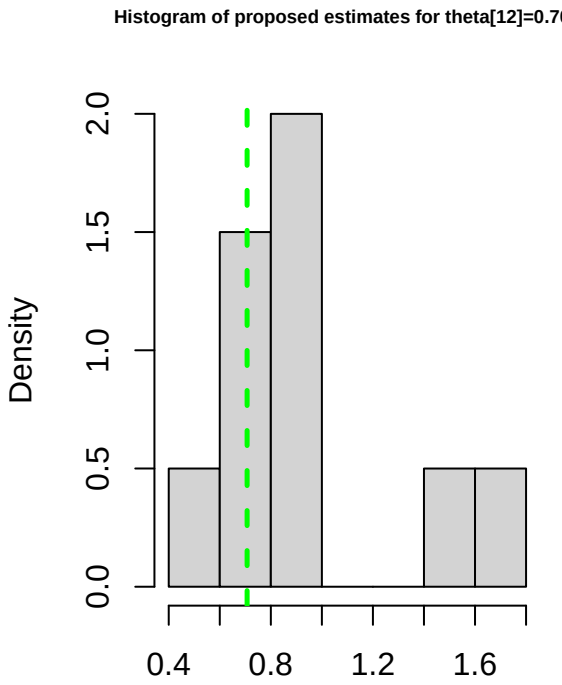
Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

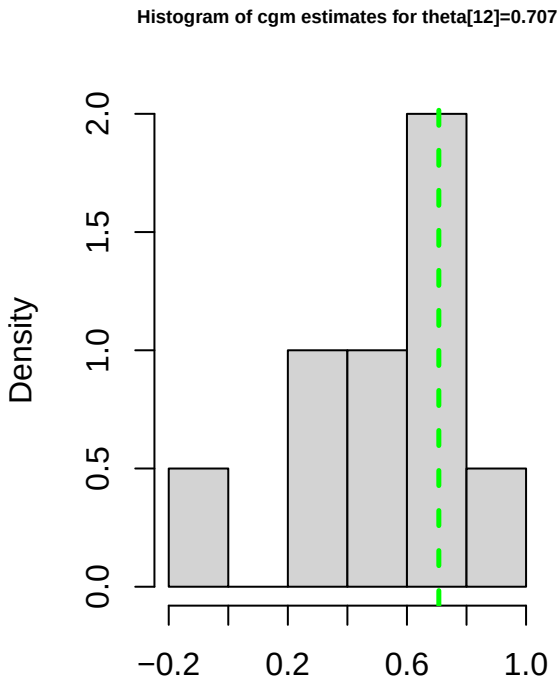
	proposed	cgm
theta[6]	0.299	0.300
theta[12]	0.257	0.277
theta[4]	0.193	0.050
theta[8]	0.214	0.055
total	0.241	0.171

Boxplots


Histogram of proposed estimates for theta[6]=0.707
Density
0.0
0.2
0.4
0.6
0.8
1.0
−1.5
−0.5
0.5
1.5


Histogram of cgm estimates for theta[6]=0.707
Density
0.0
0.5
1.0
1.5
2.0
0.2
0.4
0.6
0.8


Histogram of proposed estimates for theta[12]=0.707
Density
0.0
0.5
1.0
1.5
2.0
0.4
0.8
1.2
1.6


Histogram of cgm estimates for theta[12]=0.707
Density
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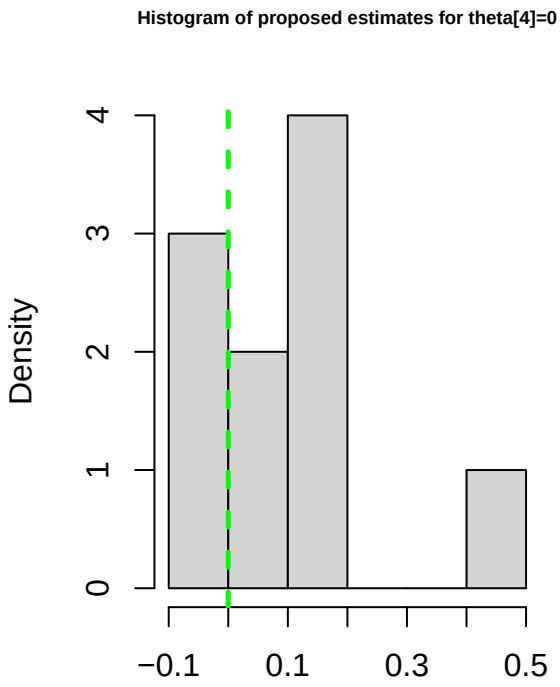
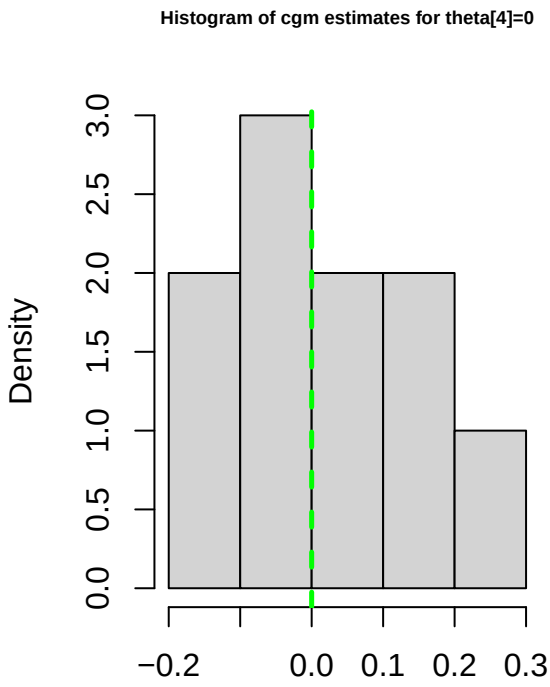
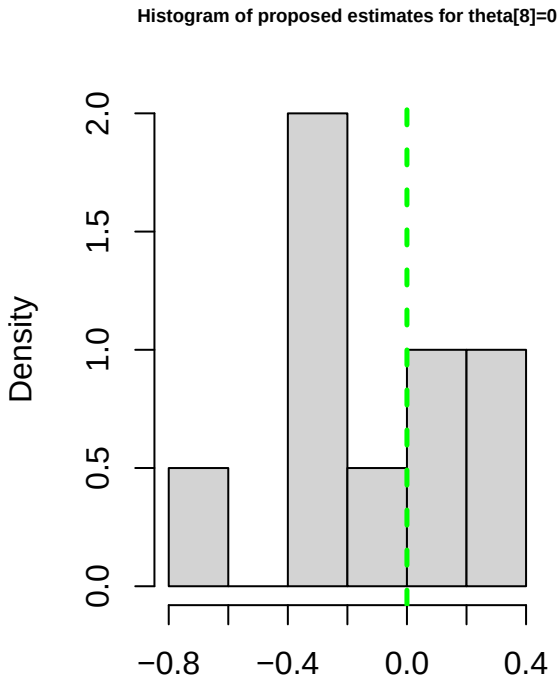
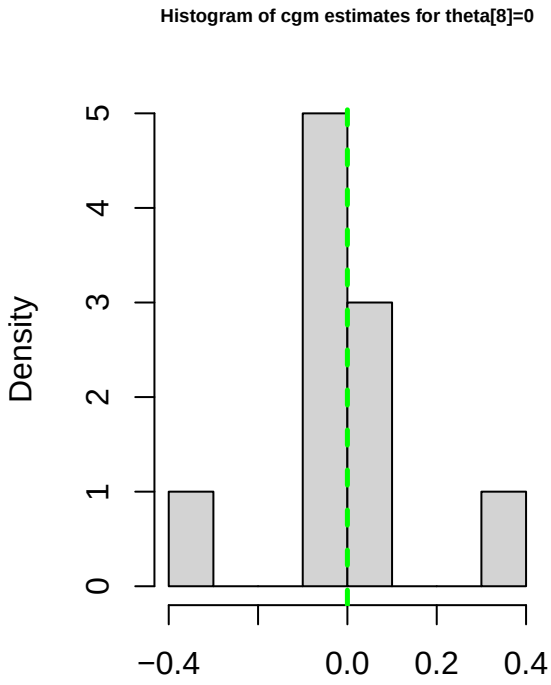
1.0
1.5
2.0
−0.2
0.2
0.6
1.0

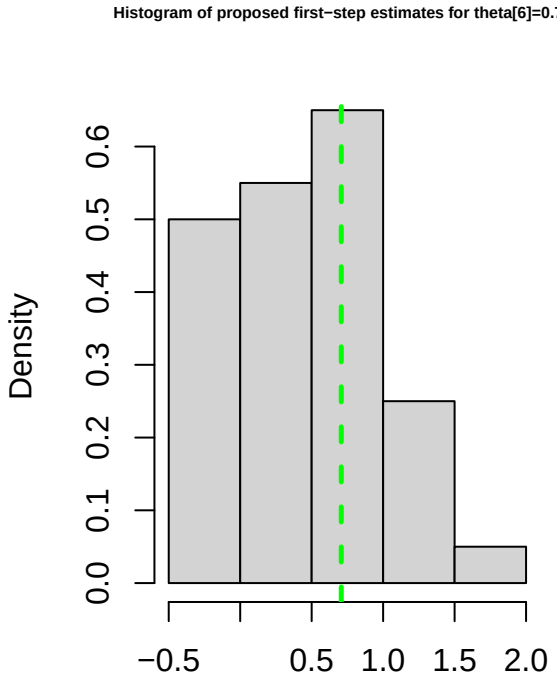

Histogram of proposed estimates for theta[4]=0
Density
0
1
2
3
4
−0.1
0.1
0.3
0.5

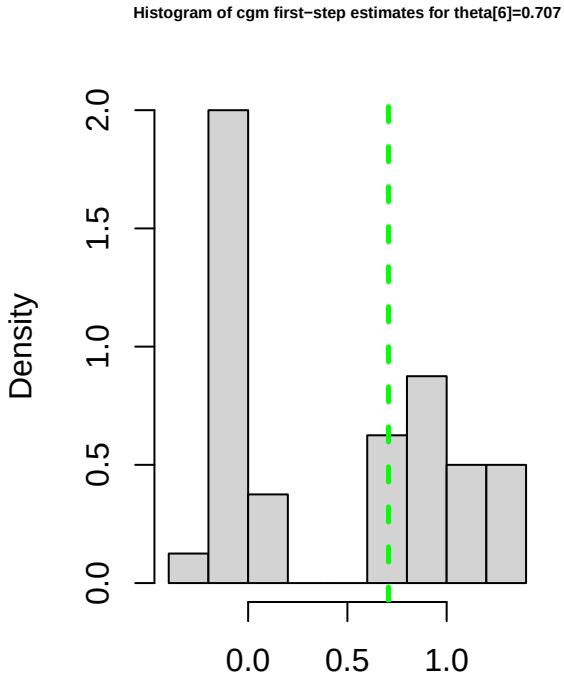

Histogram of cgm estimates for theta[4]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.2
0.0
0.1
0.2
0.3

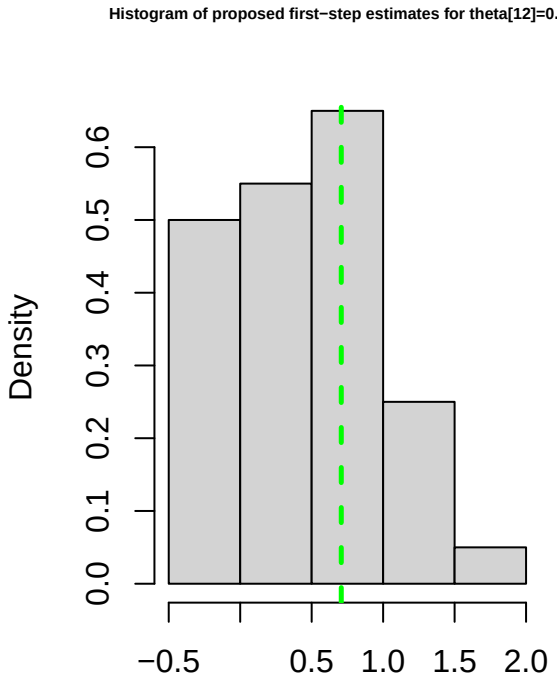

Histogram of proposed estimates for theta[8]=0
Density
0.0
0.5
1.0
1.5
2.0
−0.8
−0.4
0.0
0.4

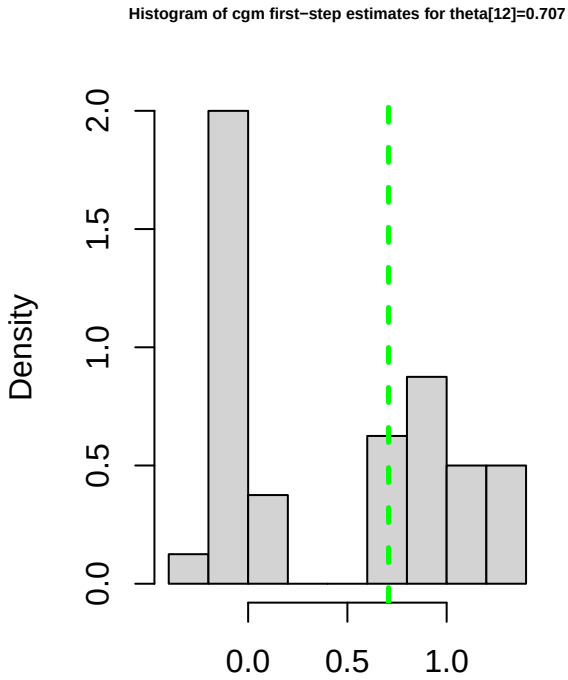

Histogram of cgm estimates for theta[8]=0
Density
0
1
2
3
4
5
−0.4
0.0
0.2
0.4

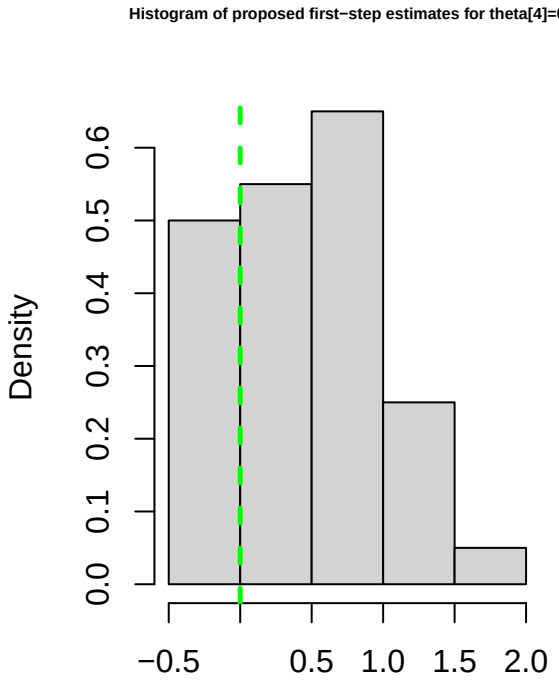
First Step Histograms

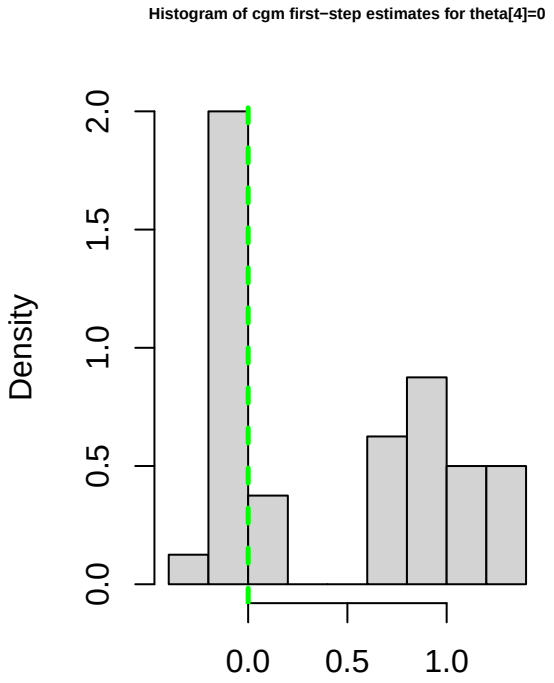


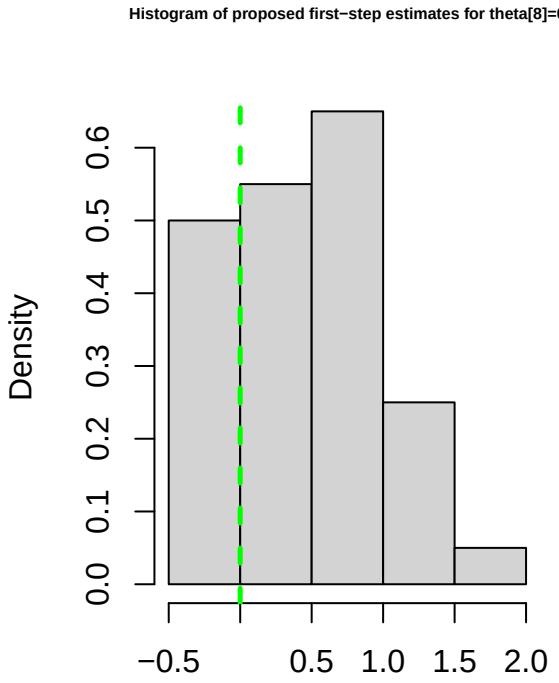


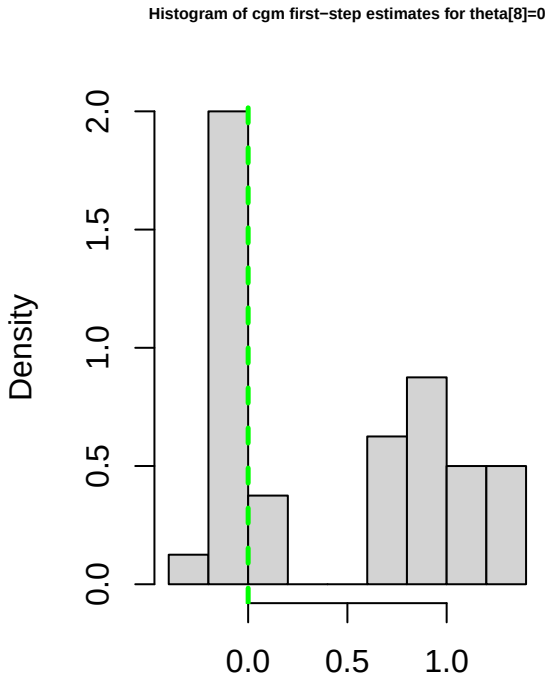





Histogram of proposed first−step estimates for theta[4]=0
Density
0.0
0.1
0.2
0.3
0.4
0.5
0.6
−0.5
0.5
1.0
1.5
2.0


Histogram of cgm first−step estimates for theta[4]=0
Density
0.0
0.5
1.0
1.5
2.0
0.0
0.5
1.0


Histogram of proposed first−step estimates for theta[8]=0
Density
0.0
0.1
0.2
0.3
0.4
0.5
0.6
−0.5
0.5
1.0
1.5
2.0


Histogram of cgm first−step estimates for theta[8]=0
Density
0.0
0.5
1.0
1.5
2.0
0.0
0.5
1.0

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 5: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[6]	-1.149	0.701	1.174	-1.045	1.148
theta[12]	0.507	0.899	1.661	0.541	1.622
theta[4]	-0.075	0.113	0.405	-0.071	0.354
theta[8]	-0.747	-0.131	0.323	-0.657	0.307

Table 6: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[6]	0.161	0.428	0.840	0.168	0.835
theta[12]	-0.073	0.559	0.828	0.014	0.814
theta[4]	-0.186	-0.006	0.267	-0.176	0.252
theta[8]	-0.342	-0.039	0.301	-0.285	0.254

Statistics for Theoretical 95% Confidence Intervals

Table 7: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[6]	0.483	0.216	0.059	0.907	0.6
theta[12]	0.952	0.199	0.561	1.343	0.8
theta[4]	0.101	0.191	-0.274	0.476	0.9
theta[8]	-0.113	0.178	-0.462	0.236	0.9

Table 8: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[6]	0.462	0.178	0.113	0.811	0.6
theta[12]	0.513	0.177	0.166	0.860	0.8
theta[4]	0.019	0.130	-0.234	0.273	1.0
theta[8]	-0.015	0.128	-0.266	0.237	0.8