Simulation Results

2026-01-08

Simulation Setup

This simulation is performed with $n = 200$ and $d = 10$, using the 2-d lattice as the underlying graph. $s = 2$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0$. The attached results are for a 10-replication simulation. The true values of the parameter vector θ are

```
0 0 0 0 0.7071068 0 0 0 0 -0.7071068 ,
```

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{ 5, 10, 1, 4\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison ($\frac{1}{n.sim}\sum_{i=1}^{n.sim}\frac{1}{|\mathcal{C}|}\|\hat{\theta}_{i,\mathcal{C}} - \theta_{\mathcal{C}}\|^2$)

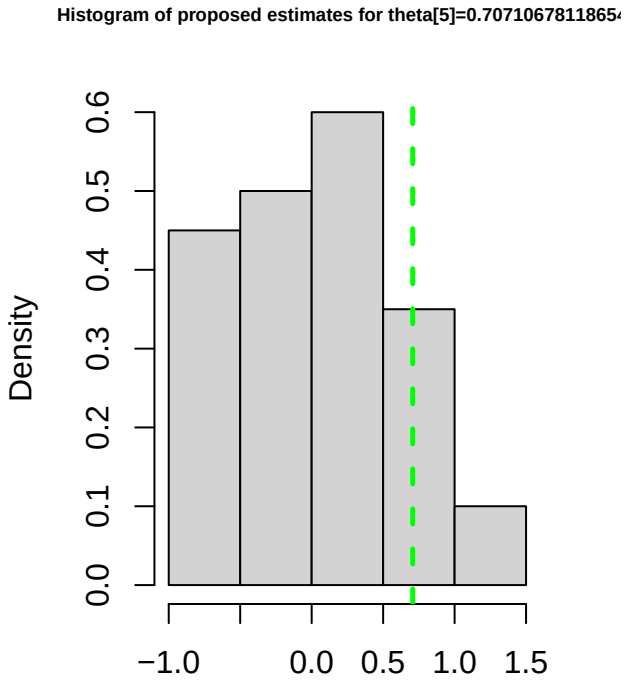
Table 1: Mean-Squared Error of Parameter Estimates

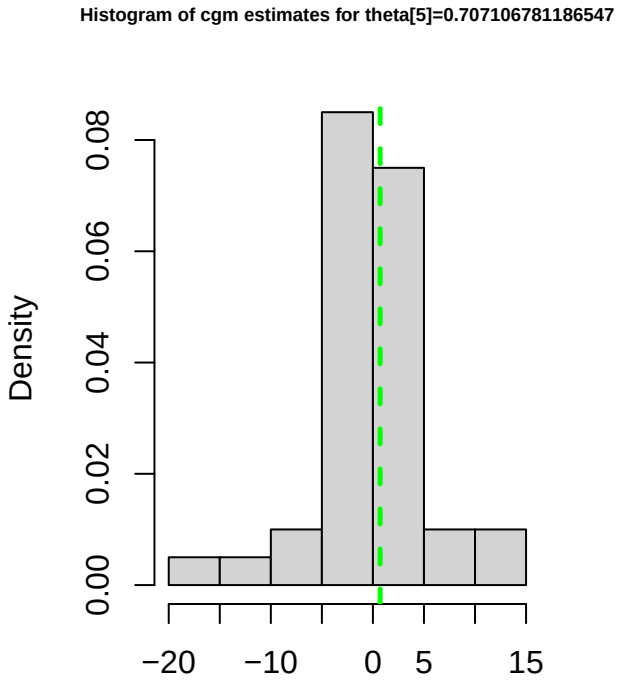
	proposed	cgm
theta[5]	0.061	62.823
theta[10]	0.036	41.416
theta[1]	0.029	0.449
theta[4]	0.041	1.478
total	0.042	26.542

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[5]	0.081	1.552
theta[10]	0.036	1.226
theta[1]	0.015	0.017
theta[4]	0.061	0.078
total	0.048	0.718

Boxplots


Histogram of proposed estimates for theta[5]=0.70710678118654
Density
0.0
0.1
0.2
0.3
0.4
0.5
0.6
−1.0
0.0
0.5
1.0
1.5


Histogram of cgm estimates for theta[5]=0.707106781186547
Density
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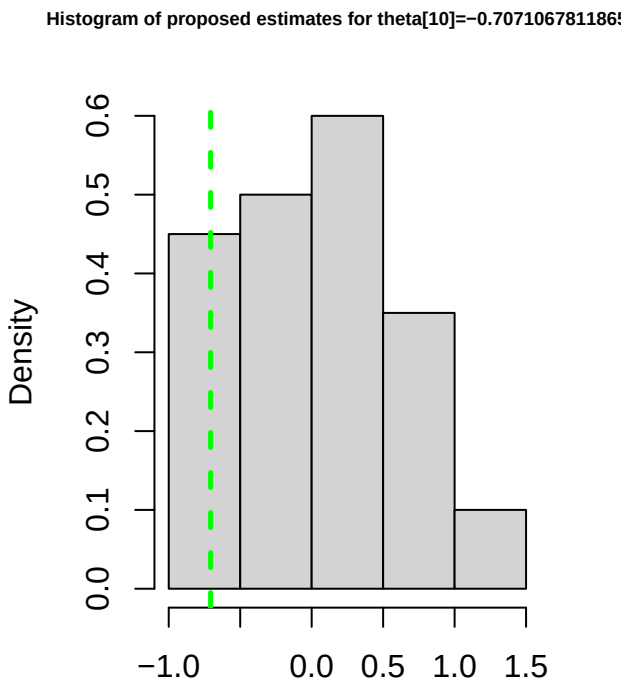
0.04
0.06
0.08
−20
−10
0
5
15


Histogram of proposed estimates for theta[10]=−0.707106781186
Density
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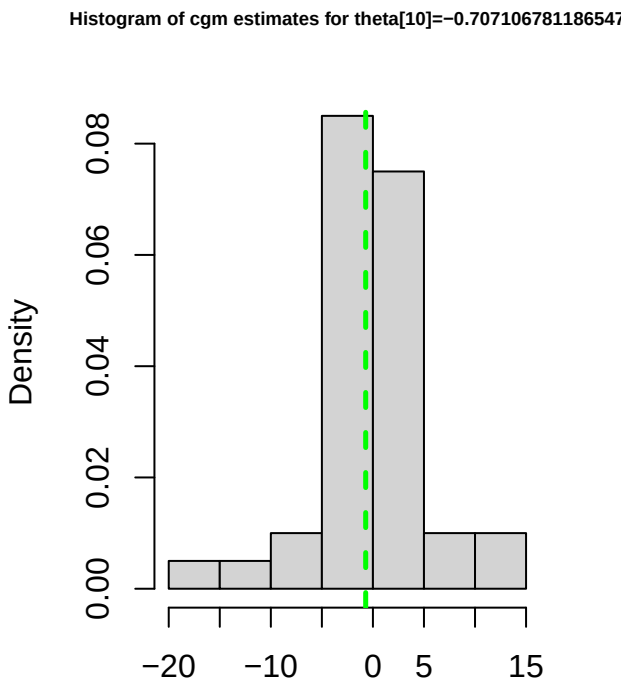
0.6
−1.0
0.0
0.5
1.0
1.5


Histogram of cgm estimates for theta[10]=−0.707106781186547
Density
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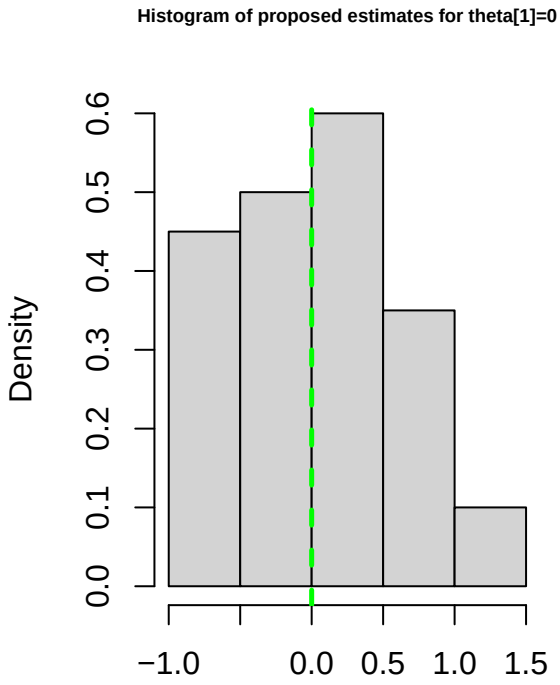
0.06
0.08
−20
−10
0
5
15


Histogram of proposed estimates for theta[1]=0
Density
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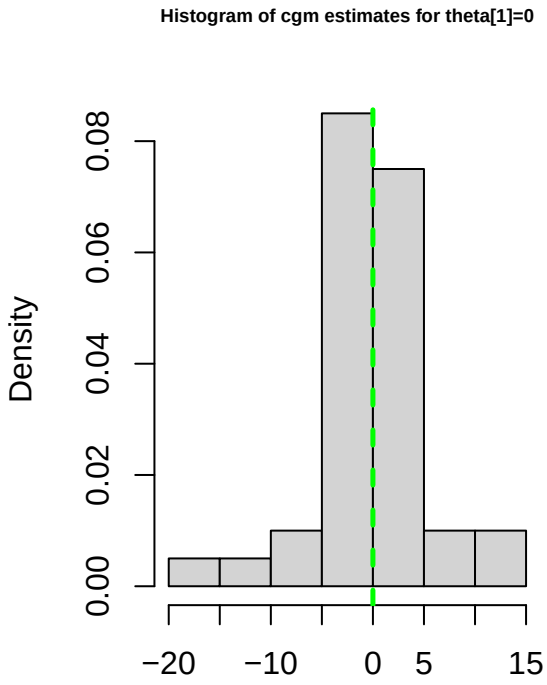
0.4
0.5
0.6
−1.0
0.0
0.5
1.0
1.5


Histogram of cgm estimates for theta[1]=0
Density
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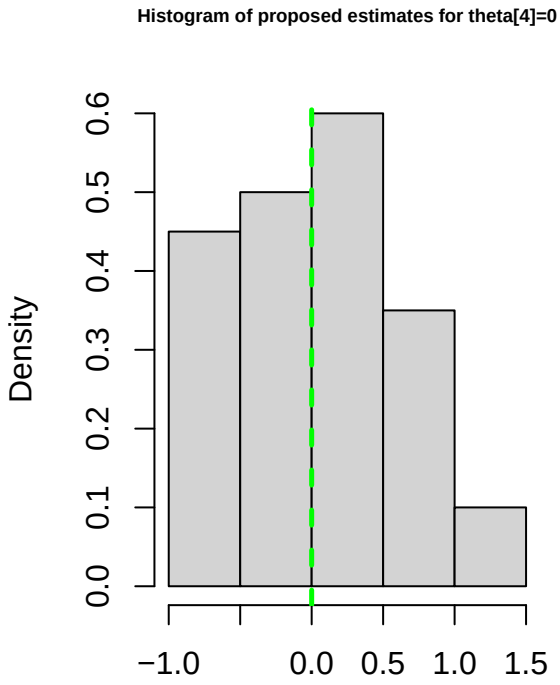
0.02
0.04
0.06
0.08
−20
−10
0
5
15


Histogram of proposed estimates for theta[4]=0
Density
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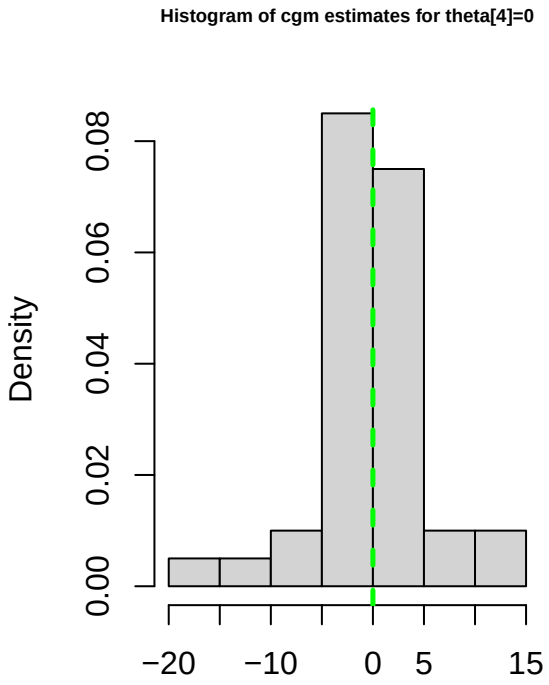
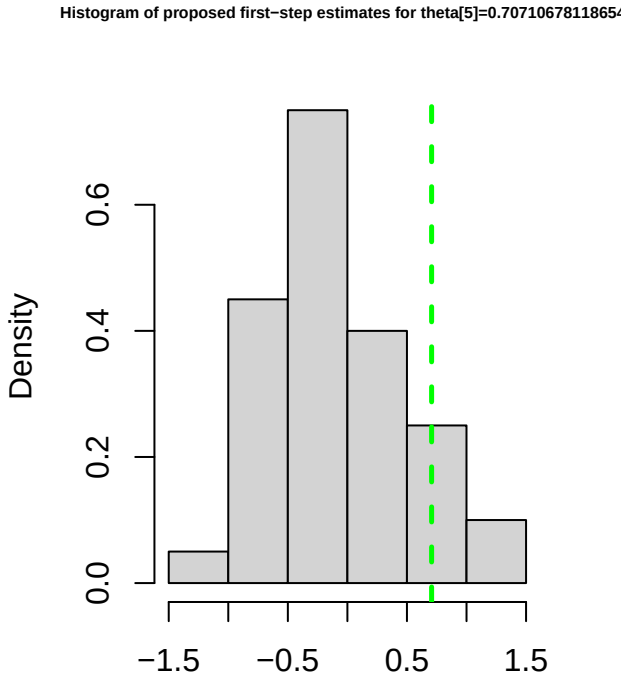
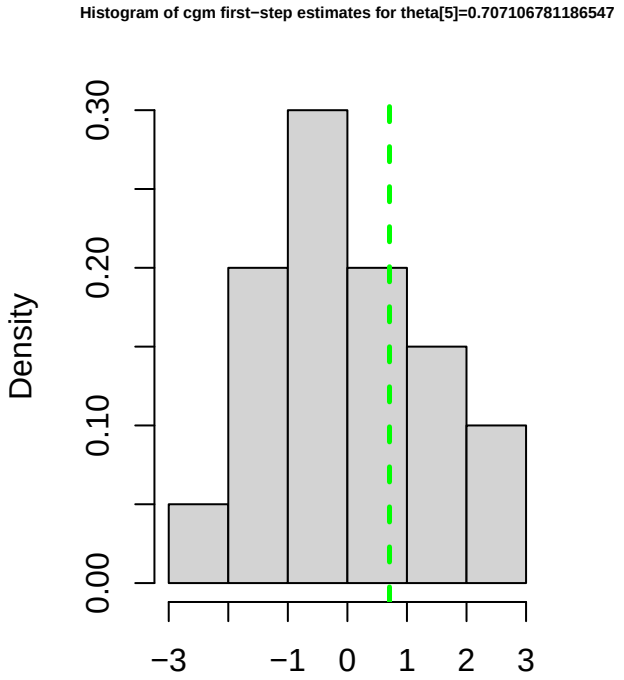
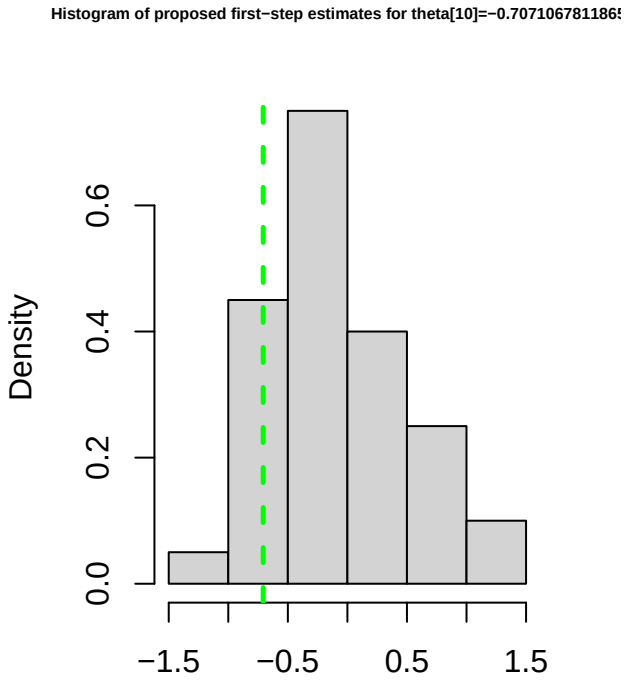
0.5
0.6
−1.0
0.0
0.5
1.0
1.5

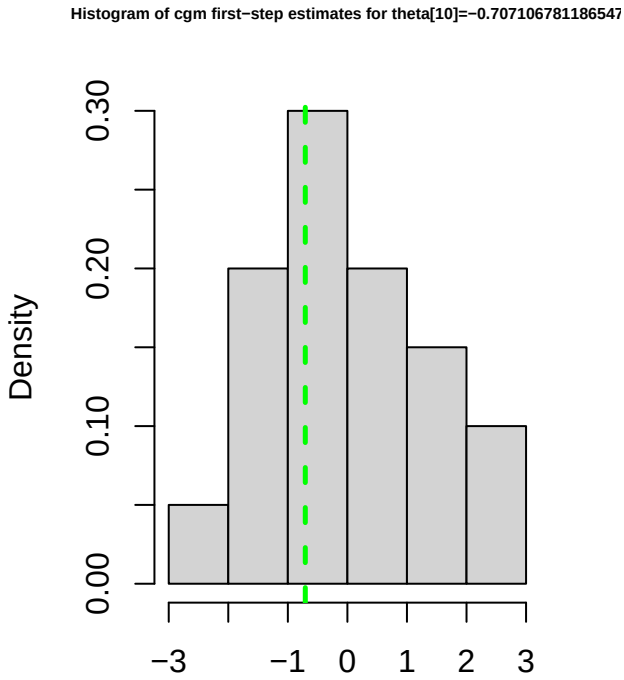

Histogram of cgm estimates for theta[4]=0
Density
0.00
0.02
0.04
0.06
0.08
−20
−10
0
5
15

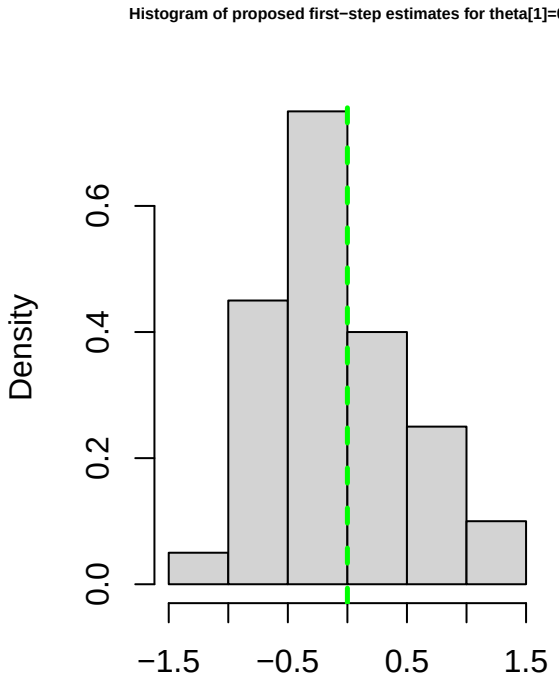
First Step Histograms

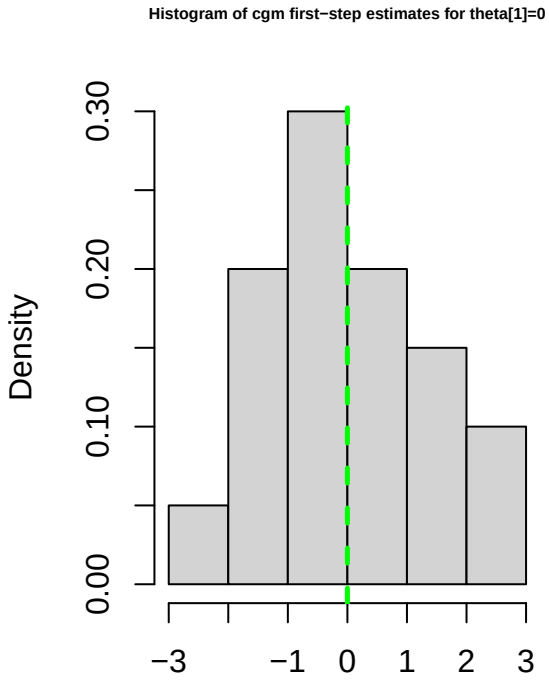


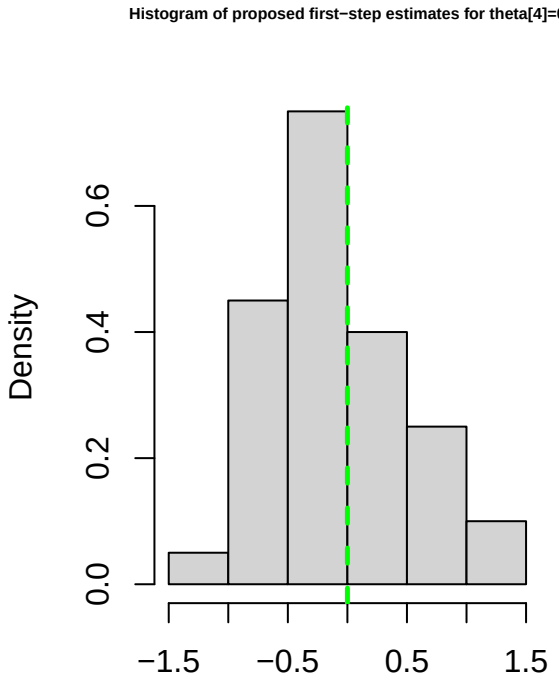


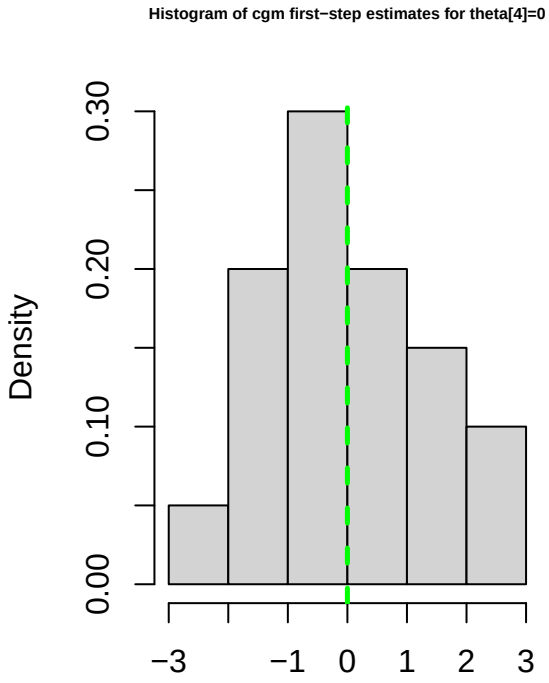





Histogram of proposed first−step estimates for theta[1]=0
Density
0.0
0.2
0.4
0.6
−1.5
−0.5
0.5
1.5


Histogram of cgm first−step estimates for theta[1]=0
Density
0.00
0.10
0.20
0.30
−3
−1
0
1
2
3


Histogram of proposed first−step estimates for theta[4]=0
Density
0.0
0.2
0.4
0.6
−1.5
−0.5
0.5
1.5


Histogram of cgm first−step estimates for theta[4]=0
Density
0.00
0.10
0.20
0.30
−3
−1
0
1
2
3

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 3: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[5]	0.306	0.723	1.248	0.367	1.198
theta[10]	-0.971	-0.748	-0.328	-0.967	-0.380
theta[1]	-0.359	0.068	0.174	-0.332	0.173
theta[4]	-0.312	-0.021	0.400	-0.288	0.343

Table 4: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[5]	-16.875	-2.953	0.006	-15.777	-0.085
theta[10]	-0.156	3.150	11.800	-0.043	11.670
theta[1]	-1.080	-0.203	1.545	-0.958	1.306
theta[4]	-3.690	0.244	0.688	-2.925	0.656

Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[5]	0.743	0.167	0.415	1.072	0.9
theta[10]	-0.750	0.167	-1.078	-0.422	0.9
theta[1]	-0.003	0.152	-0.300	0.294	0.9
theta[4]	-0.006	0.153	-0.305	0.293	0.9

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[5]	-5.069	0.506	-6.061	-4.077	0.0
theta[10]	4.202	0.470	3.280	5.124	0.1
theta[1]	-0.089	0.194	-0.470	0.292	0.6
theta[4]	-0.181	0.203	-0.580	0.217	0.6