Simulation Results

2025-10-14

Simulation Setup

This simulation is performed with $n = 400$ and $d = 10$, using the 2-d lattice as the underlying graph. $s = 2$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.1$. The attached results are for a 5-replication simulation. The true values of the parameter vector θ are

```
 [1]  0.0000000  0.7071068 -0.7071068  0.0000000  0.0000000  0.0000000  0.0000000
 [8]  0.0000000  0.0000000  0.0000000
```

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

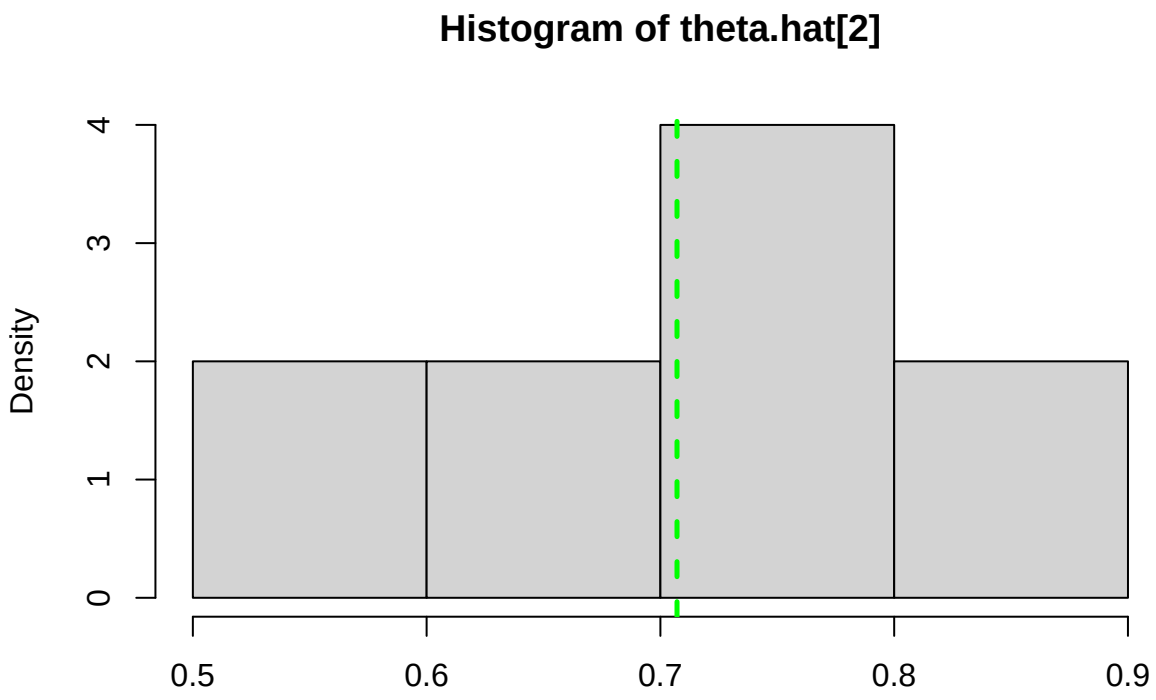
Results

Mean-squared error comparison ($\frac{1}{n.sim}\sum_{i=1}^{n.sim}\frac{1}{d}\|\hat{\theta}_i - \theta\|^2$)

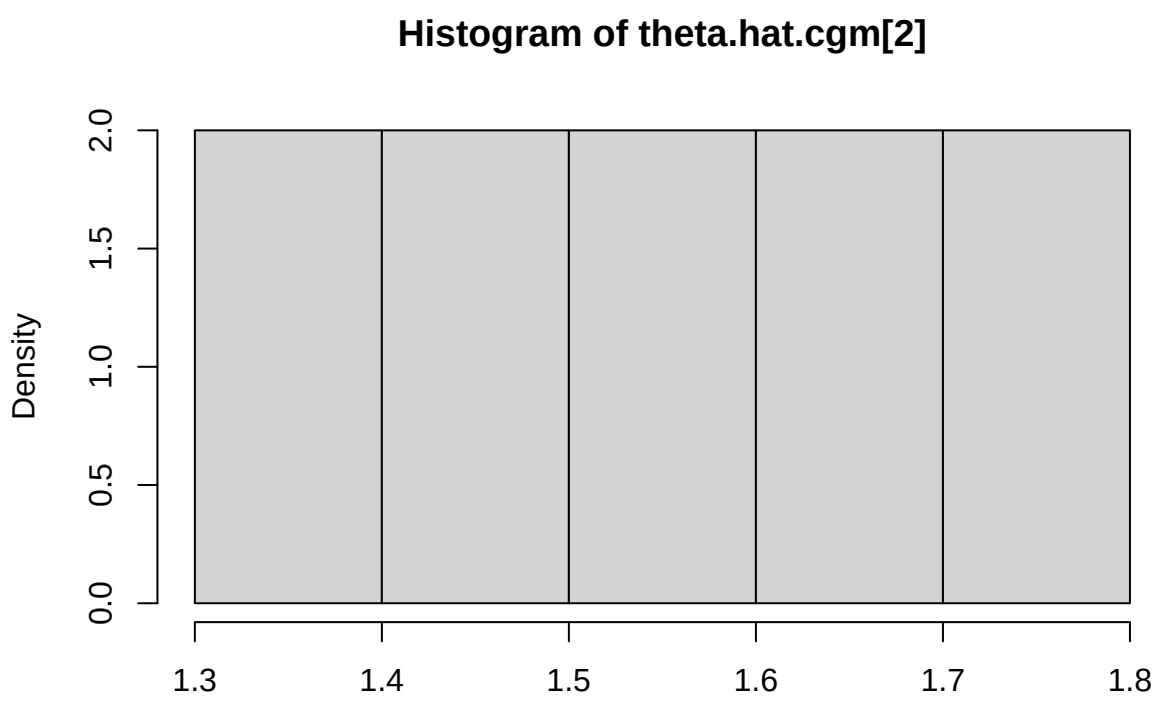
```
# A tibble: 1 x 2
  `MISLE (First-step) MSE` `MISLE MSE`
                     <dbl>       <dbl>
1                   0.0243      0.0109
```

```
# A tibble: 1 x 2
  `MISLE MSE` `CGM MSE`
        <dbl>     <dbl>
1      0.0109      1.01
```

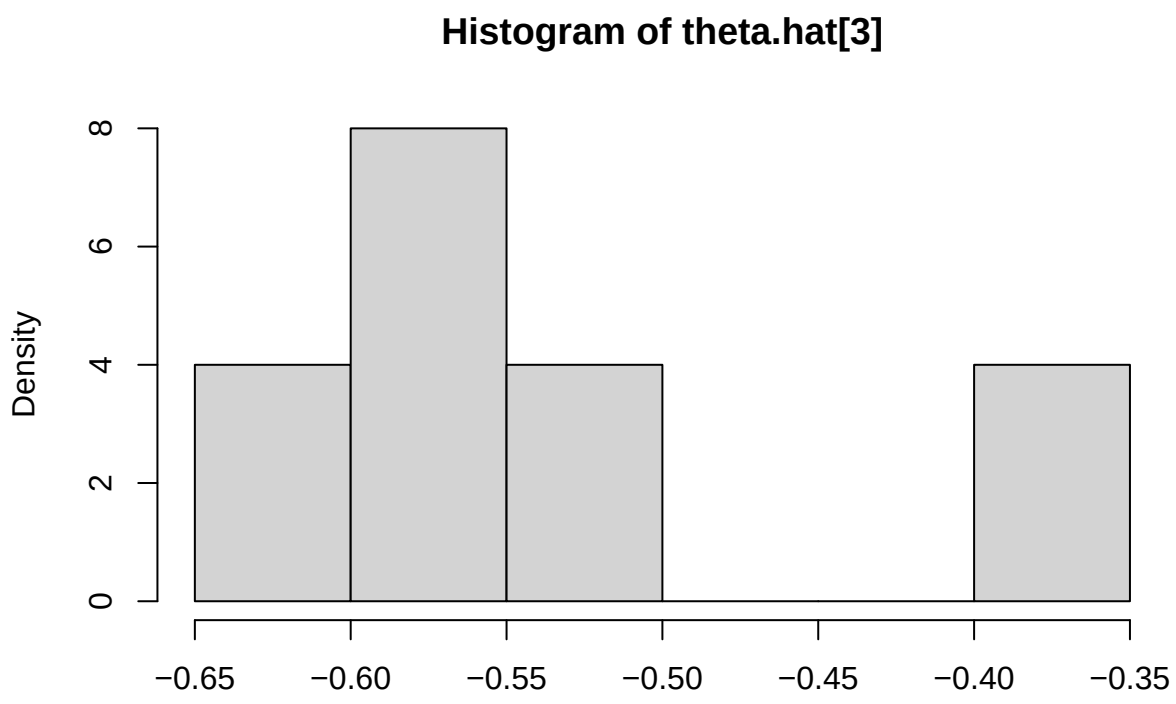
First Step Histograms



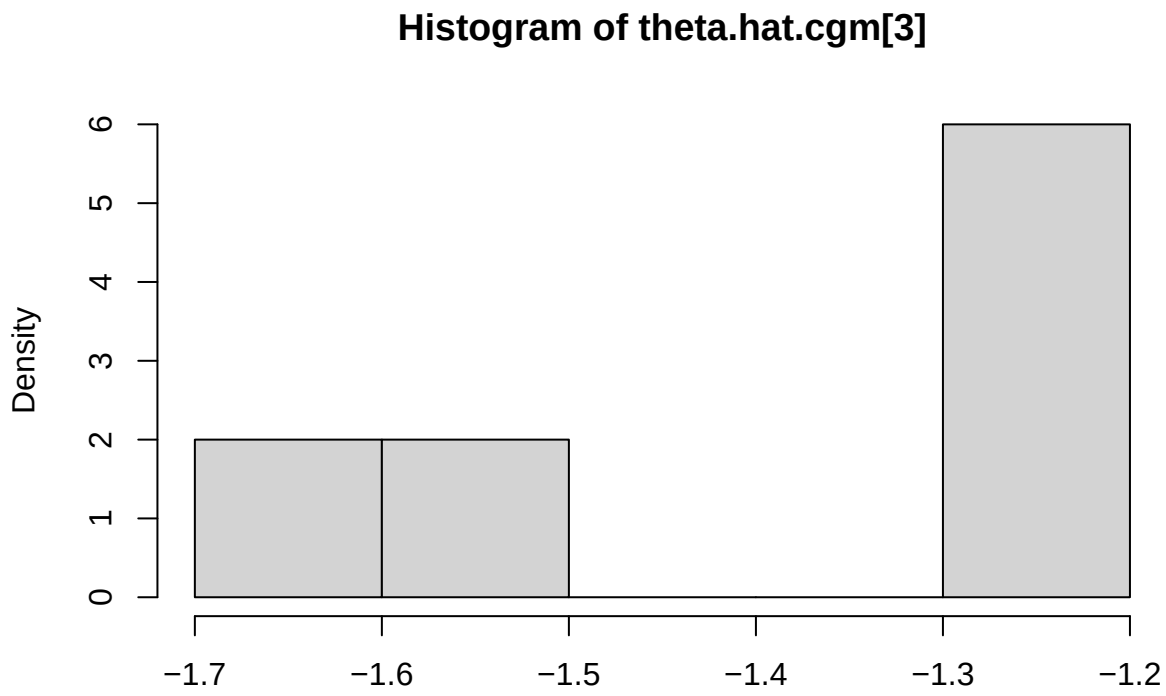

```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.5390  0.6807  0.7201  0.7167  0.7605  0.8831
[1] "95% CI based on bootstrap:"
      lower     upper
1 0.5532004 0.8708205
```



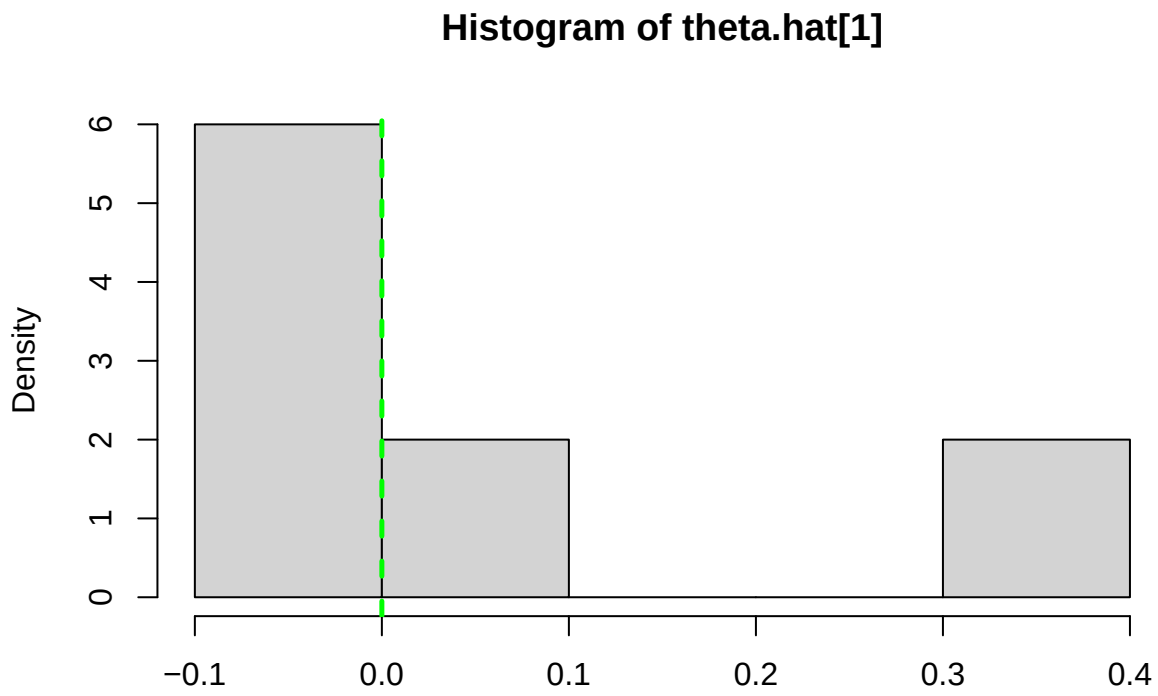

```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  1.346   1.414   1.560   1.533   1.600   1.744
[1] "95% CI based on bootstrap:"
  lower.cgm upper.cgm
1  1.352663  1.729283
```



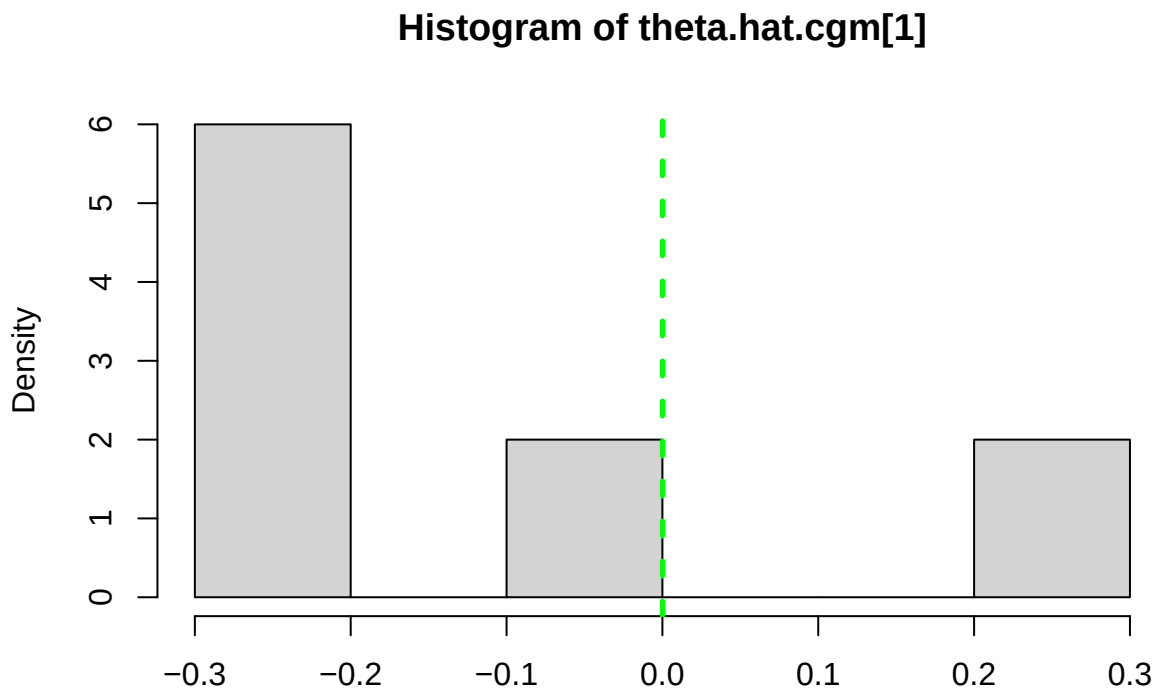

```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-0.6097 -0.5809 -0.5640 -0.5339 -0.5407 -0.3742
[1] "95% CI based on bootstrap:"
       lower     upper
1 -0.6068185 -0.390834
```



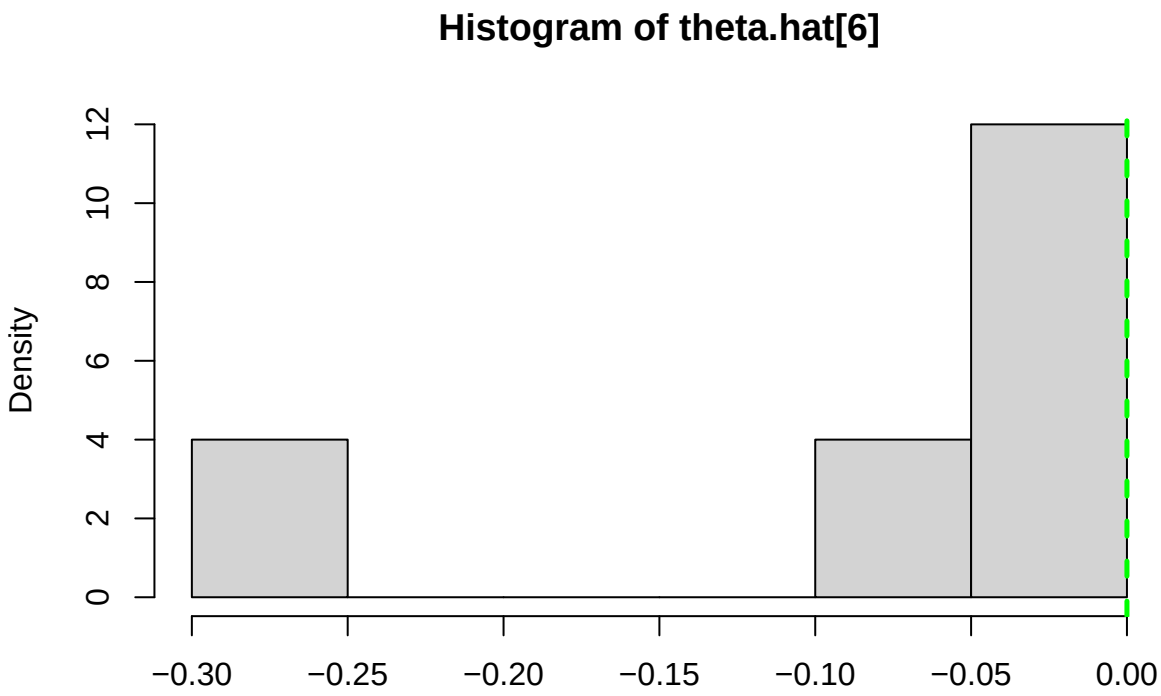

```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 -1.632  -1.539  -1.244  -1.375  -1.236  -1.223
[1] "95% CI based on bootstrap:"
  lower.cgm upper.cgm
1 -1.623049 -1.224183
```



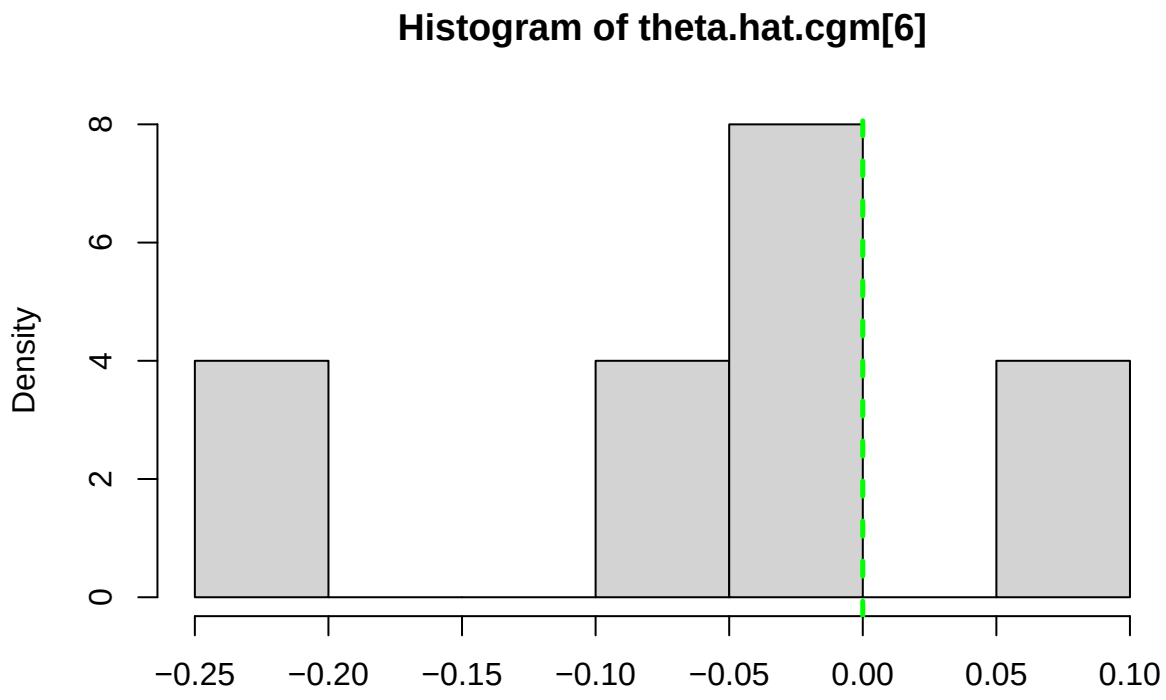

```
[1] "Summary statistics of bootstrap replicates:"
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-0.09803 -0.01828  0.00000  0.06620  0.05873  0.38861
[1] "95% CI based on bootstrap:"
        lower     upper
1 -0.09005993 0.3556189
```




```
[1] "Summary statistics of bootstrap replicates:"
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-0.29730 -0.25230 -0.23703 -0.13112 -0.08858  0.21960
[1] "95% CI based on bootstrap:"
   lower.cgm upper.cgm
1 -0.2928013 0.1887804
```

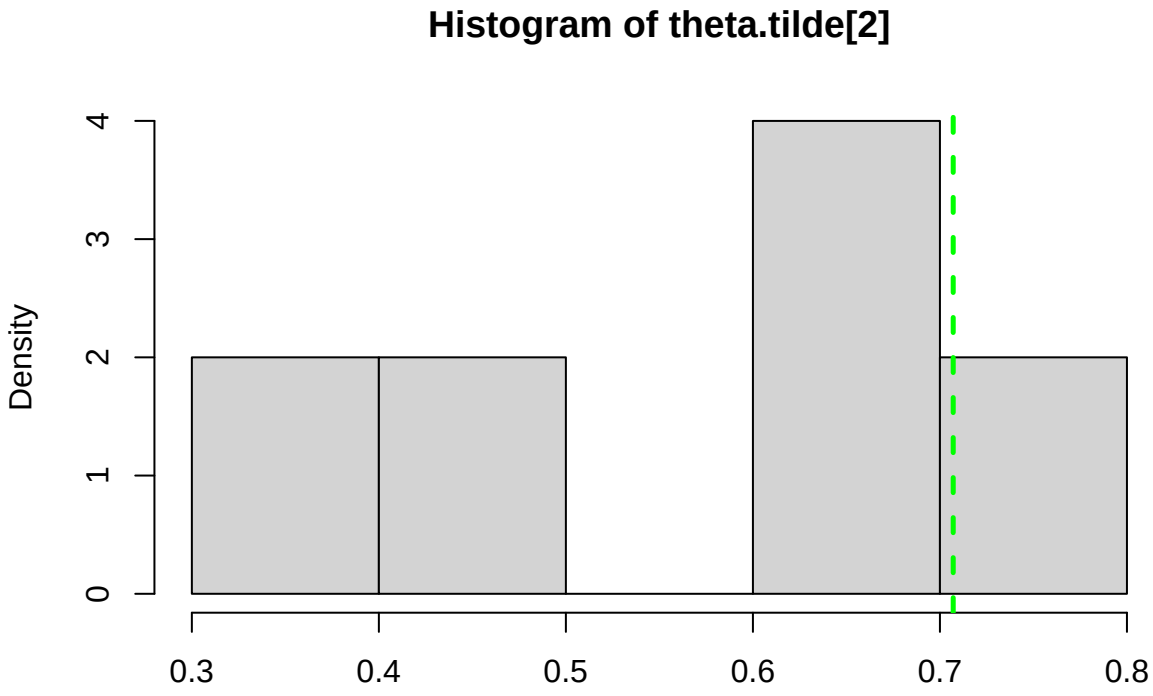



```
[1] "Summary statistics of bootstrap replicates:"
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-0.26187 -0.06485 -0.04425 -0.07419  0.00000  0.00000
[1] "95% CI based on bootstrap:"
       lower upper
1 -0.2421659     0
```

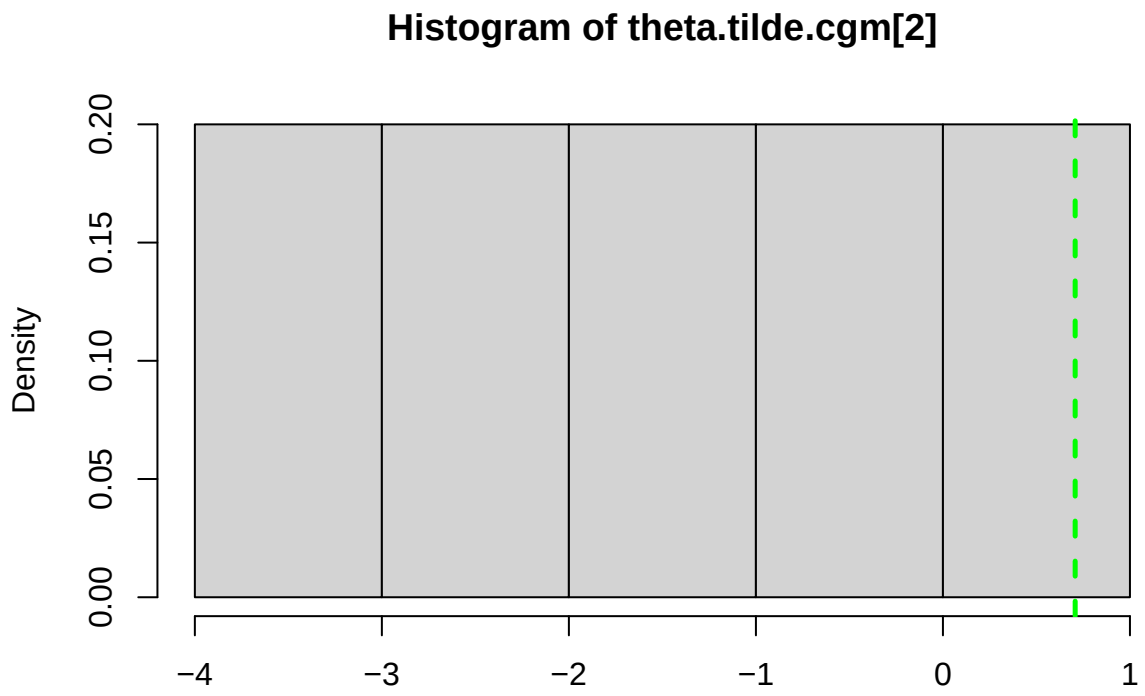



```
[1] "Summary statistics of bootstrap replicates:"
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-0.20558 -0.07572 -0.04363 -0.05694 -0.02219  0.06242
[1] "95% CI based on bootstrap:"
   lower.cgm  upper.cgm
1 -0.1925966 0.05396061
```

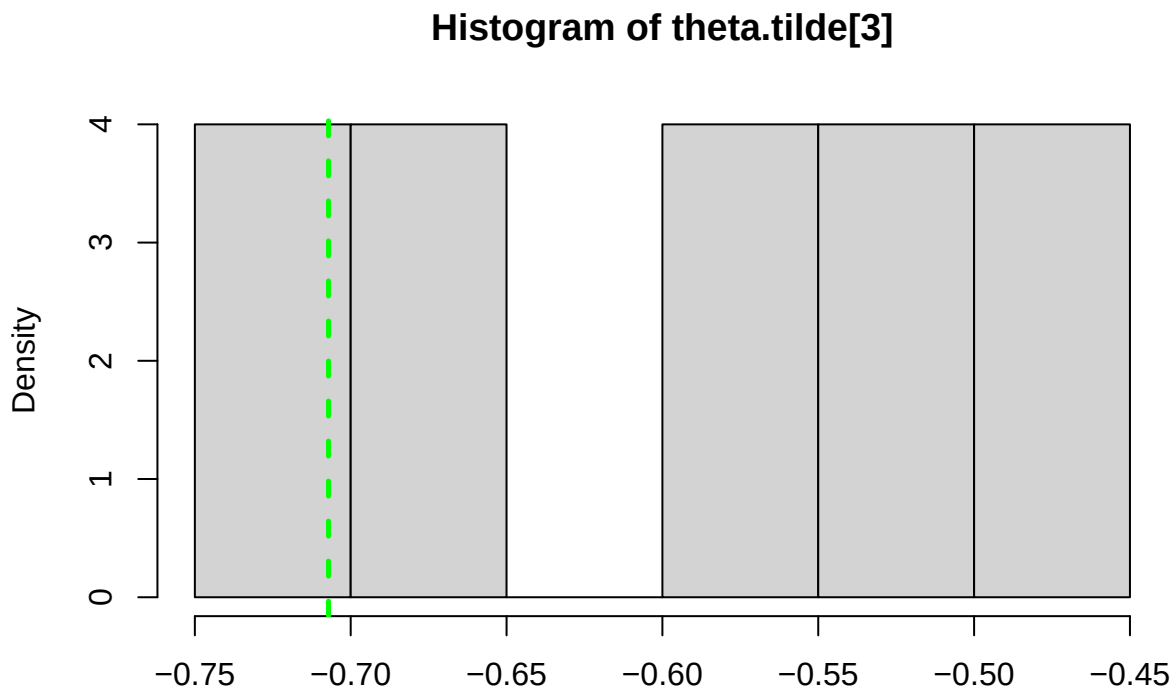
Statistics and 95% Confidence Intervals from per-Replicate Estimates



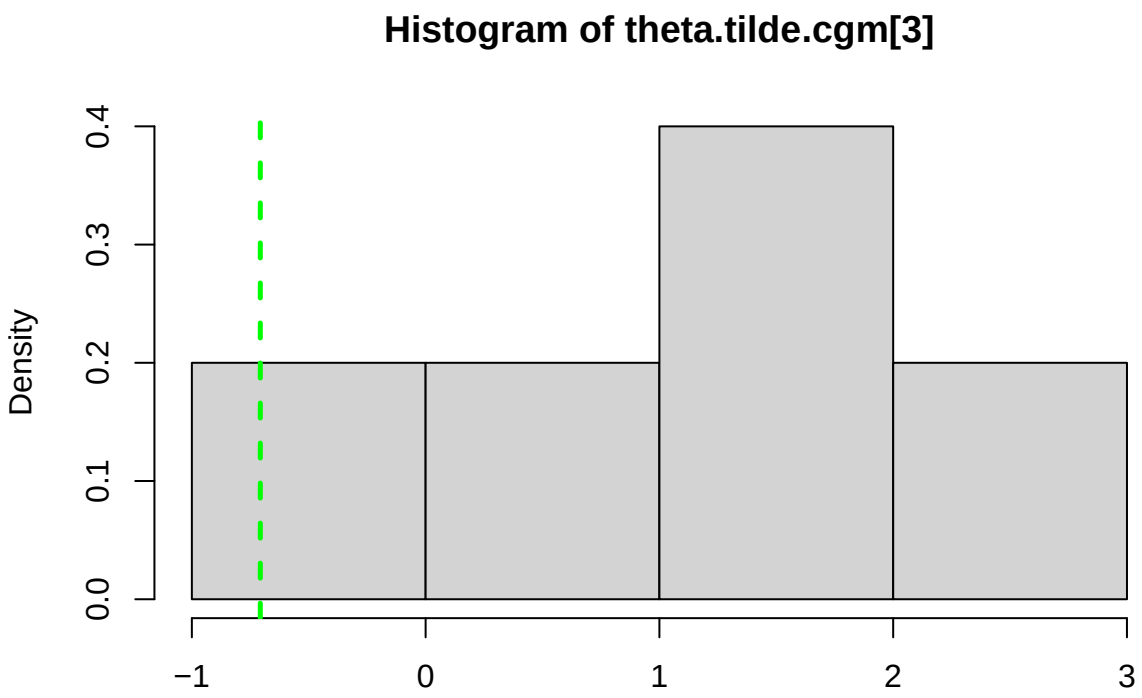

```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.3173  0.4621  0.6139  0.5548  0.6489  0.7316
[1] "95% CI based on bootstrap:"
      lower     upper
1 0.3317585 0.7233695
```



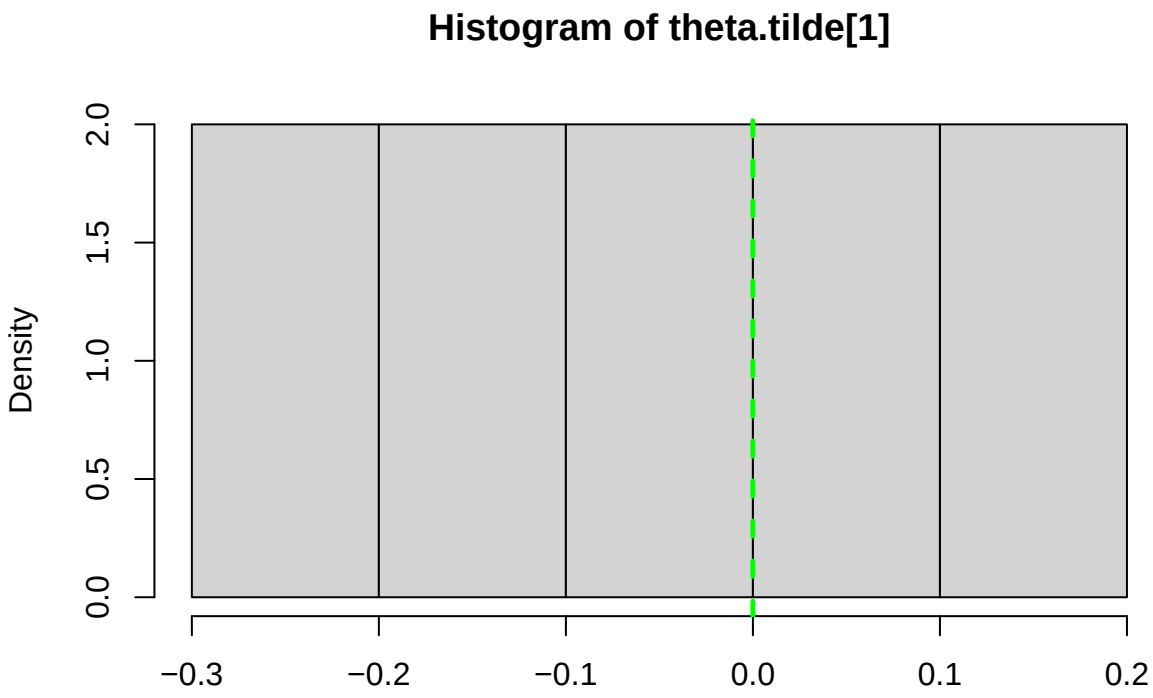

```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-3.0994 -2.0677 -1.2602 -1.2657 -0.3127  0.4114
[1] "95% CI based on bootstrap:"
  lower.cgm upper.cgm
1 -2.996213 0.3389843
```




```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-0.7184 -0.6771 -0.5771 -0.5996 -0.5465 -0.4790
[1] "95% CI based on bootstrap:"
       lower      upper
1 -0.7142406 -0.4857472
```

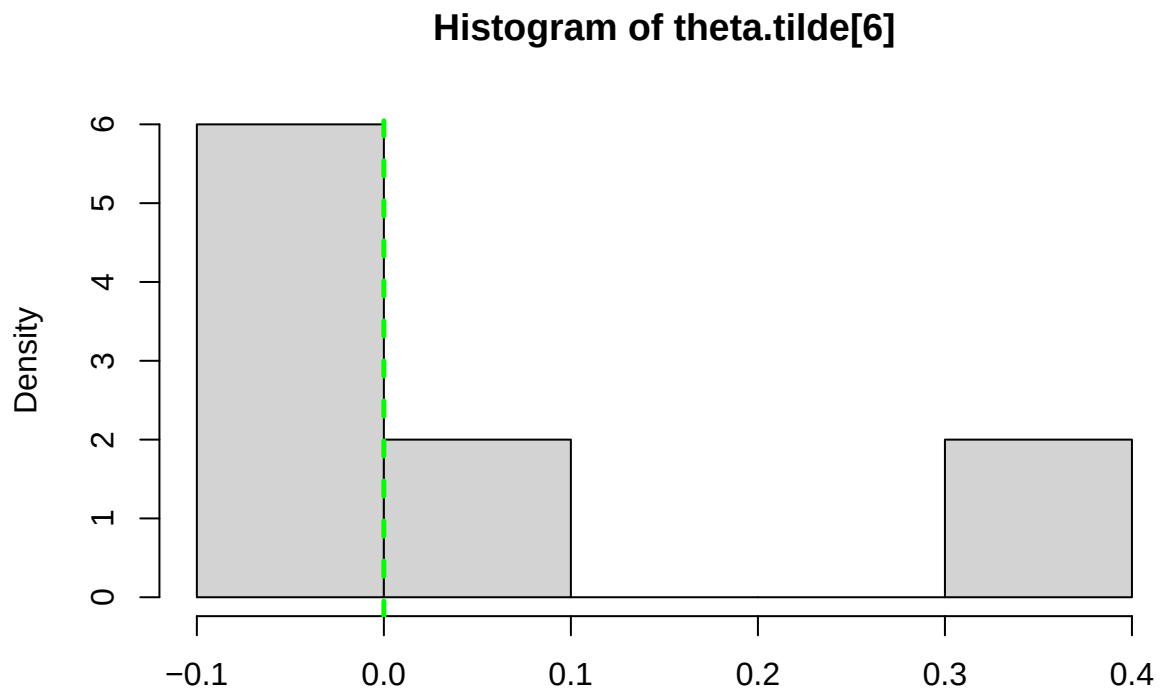



```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-0.4584  0.2191  1.1527  1.0233  1.2076  2.9958
[1] "95% CI based on bootstrap:"
   lower.cgm upper.cgm
1 -0.3906409  2.816974
```

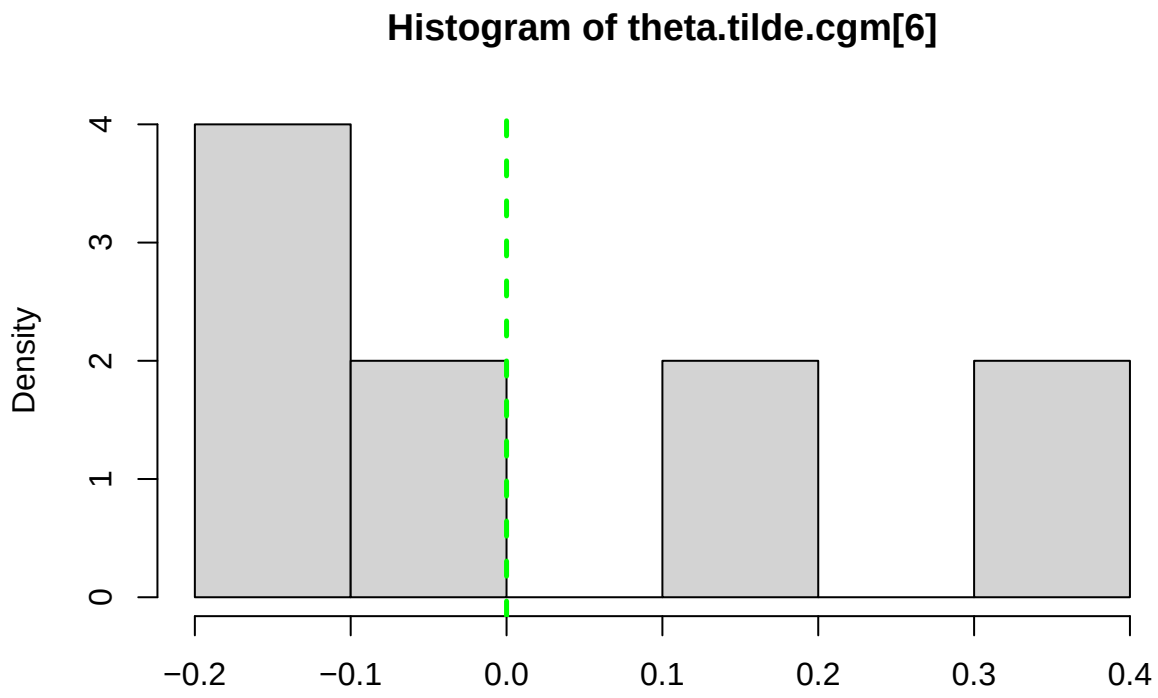



```
[1] "Summary statistics of bootstrap replicates:"
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-0.21376 -0.13282 -0.01617 -0.04215  0.04486  0.10715
[1] "95% CI based on bootstrap:"
       lower     upper
1 -0.2056686 0.1009201
```


```
[1] "Summary statistics of bootstrap replicates:"
     Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
-0.243821 -0.041621 -0.008646  0.248585  0.632445  0.904565
[1] "95% CI based on bootstrap:"
   lower.cgm upper.cgm
1 -0.2236006 0.8773534
```




```
[1] "Summary statistics of bootstrap replicates:"
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-0.09673 -0.03410 -0.02503  0.04823  0.02922  0.36776
[1] "95% CI based on bootstrap:"
        lower     upper
1 -0.09047007 0.3339079
```




```
[1] "Summary statistics of bootstrap replicates:"
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-0.15684 -0.10676 -0.03476  0.04965  0.15812  0.38848
[1] "95% CI based on bootstrap:"
   lower.cgm upper.cgm
1 -0.1518339 0.3654458
```

Statistics for Theoretical 95% Confidence Intervals

```
[1] Length of Confidence Intervals for theta[2]
[1] Coverage proportion: 0.8
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.5322  0.5574  0.5868  0.6216  0.7026  0.7289
[1] Length of Confidence Intervals for theta[2] (CGM Method)
[1] Coverage proportion: 1
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  4.872   5.216   5.746   5.595   5.968   6.174
[1] Length of Confidence Intervals for theta[3]
[1] Coverage proportion: 1
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.5116  0.5254  0.5381  0.5721  0.6264  0.6587
[1] Length of Confidence Intervals for theta[3] (CGM Method)
[1] Coverage proportion: 1
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  6.186   6.788   8.494   8.672   9.653  12.240
[1] Length of Confidence Intervals for theta[1]
[1] Coverage proportion: 1
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.4457  0.4768  0.5234  0.5148  0.5314  0.5964
[1] Length of Confidence Intervals for theta[1] (CGM Method)
[1] Coverage proportion: 1
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  3.348   3.677   3.977   4.086   4.708   4.722
[1] Length of Confidence Intervals for theta[6]
[1] Coverage proportion: 0.8
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.4844  0.4889  0.5448  0.5311  0.5539  0.5836
[1] Length of Confidence Intervals for theta[6] (CGM Method)
[1] Coverage proportion: 1
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  3.244   3.568   3.740   3.757   4.038   4.195
```