Simulation Results

2025-11-07

Simulation Setup

This simulation is performed with $n = 200$ and $d = 10$, using the 2-d lattice as the underlying graph. $s = 2$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0$. The attached results are for a 5-replication simulation. The true values of the parameter vector θ are

```
 [1] 0.7071068 0.0000000 0.0000000 0.0000000 0.0000000 0.7071068 0.0000000 0.0000000
 [9] 0.0000000 0.0000000
```

The results from our code are not augmented with any comparison method here.

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

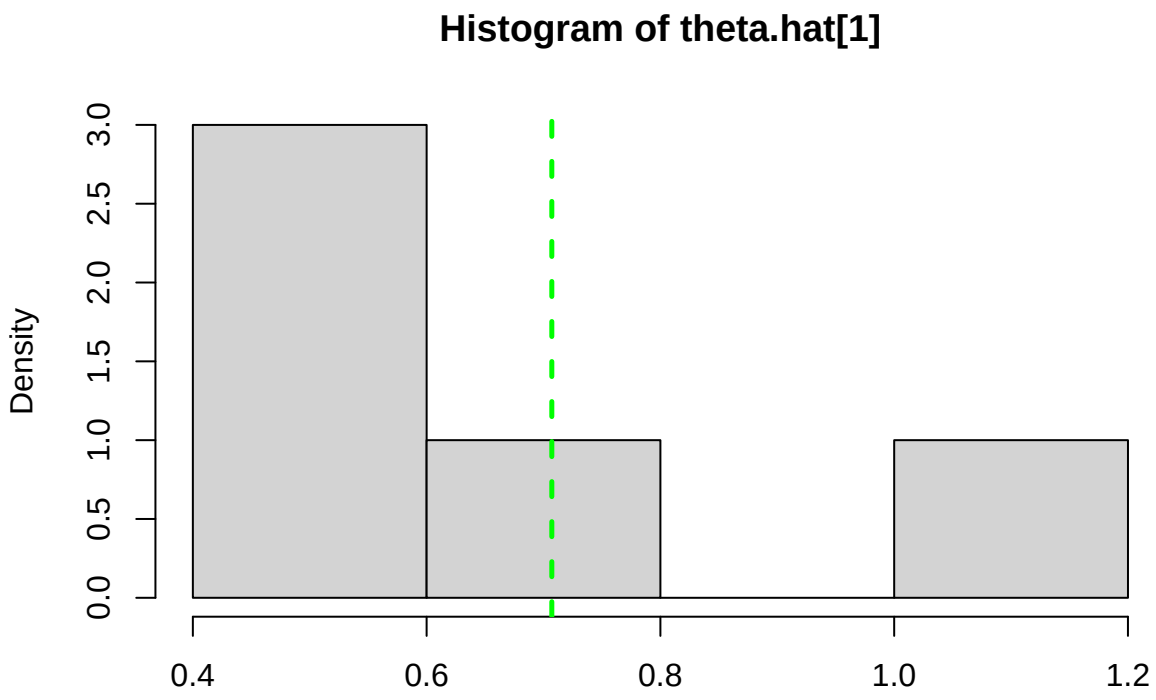
After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

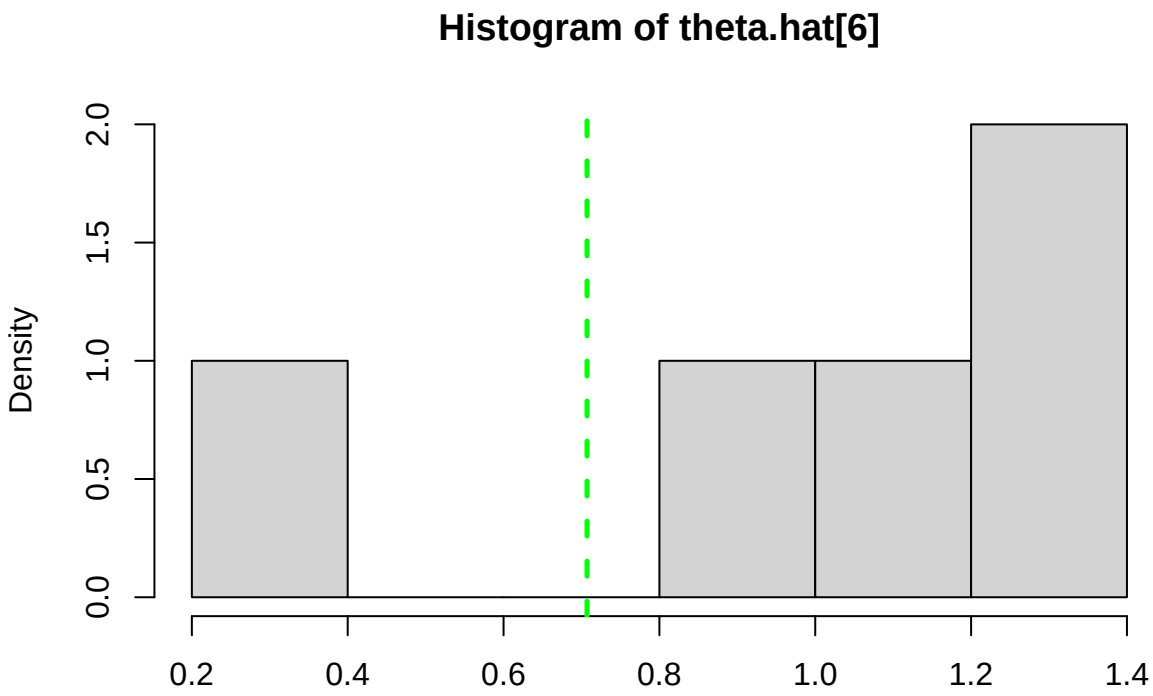
Mean-squared error comparison ($\frac{1}{n.sim}\sum_{i=1}^{n.sim}\frac{1}{d}\|\hat{\theta}_i - \theta\|^2$)

```
# A tibble: 1 x 2
  `MISLE (First-step) MSE` `MISLE MSE`
                     <dbl>       <dbl>
1                   0.0857      0.0361
```

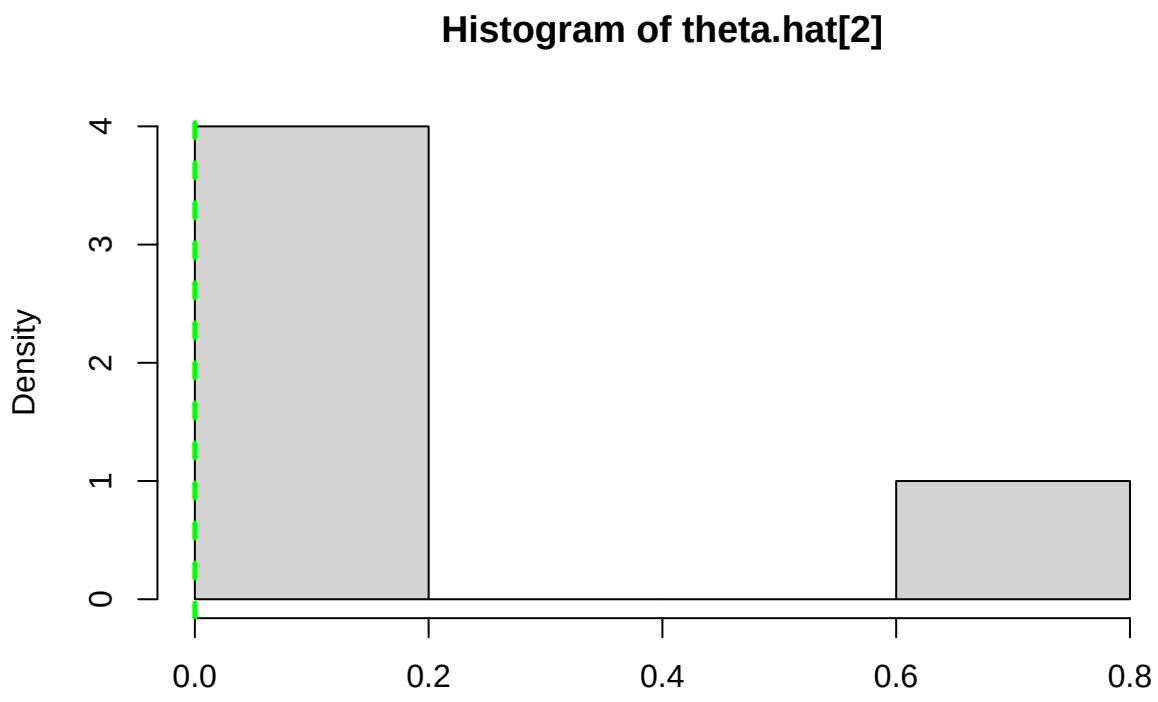
First Step Histograms



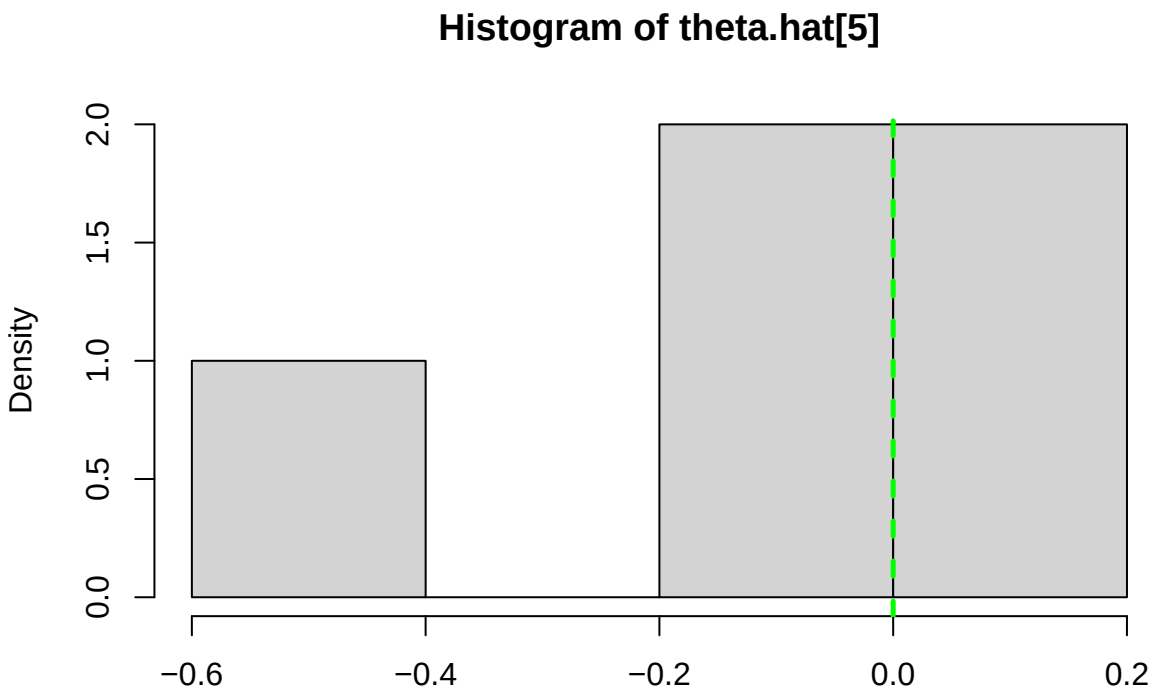

```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.5027  0.5158  0.5418  0.6560  0.6132  1.1064
[1] "95% CI based on bootstrap:"
      lower    upper
1 0.5039798 1.057049
```




```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.3704  0.8488  1.0095  0.9321  1.2057  1.2261
[1] "95% CI based on bootstrap:"
      lower   upper
1 0.4182478 1.22408
```

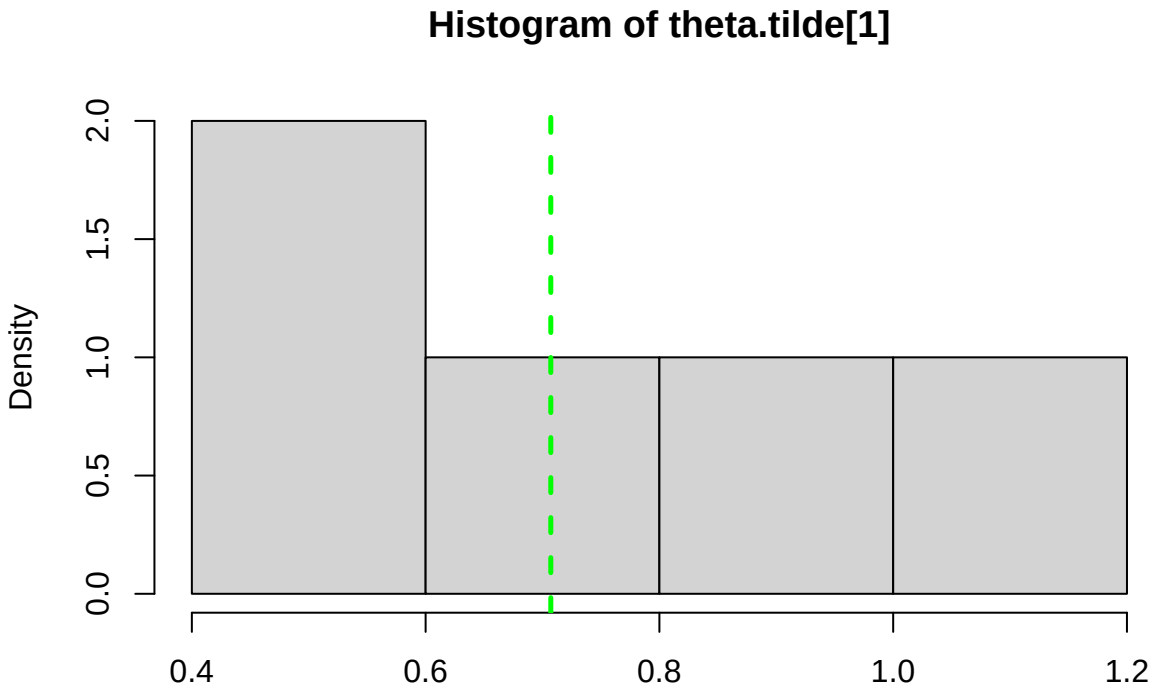



```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.0000  0.0000  0.0000  0.1584  0.1391  0.6530
[1] "95% CI based on bootstrap:"
  lower    upper
1     0 0.601575
```

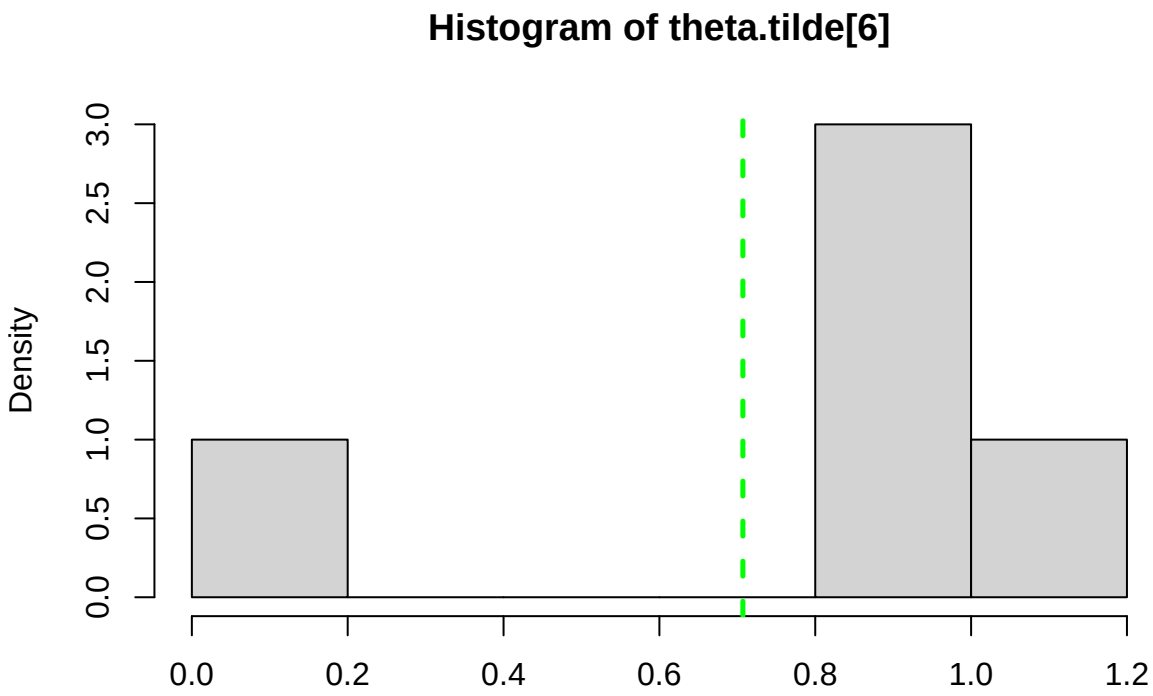



```
[1] "Summary statistics of bootstrap replicates:"
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-0.49693  0.00000  0.00000 -0.08290  0.01583  0.06658
[1] "95% CI based on bootstrap:"
      lower      upper
1 -0.447236 0.06150099
```

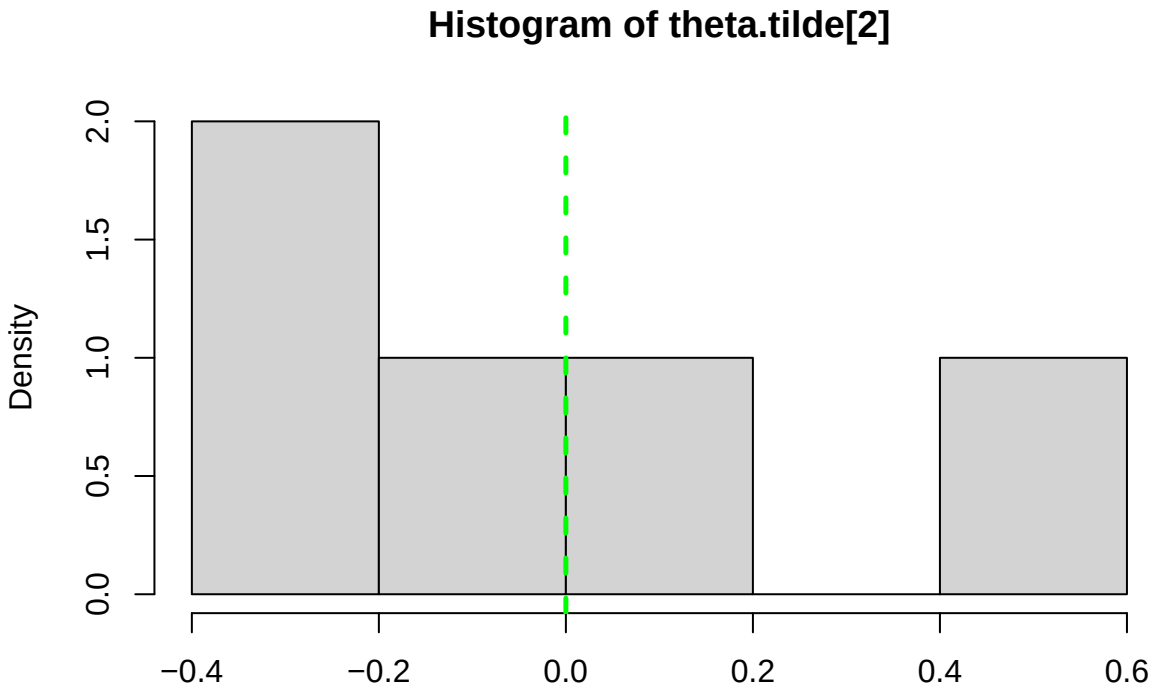
Statistics and 95% Confidence Intervals from per-Replicate Estimates



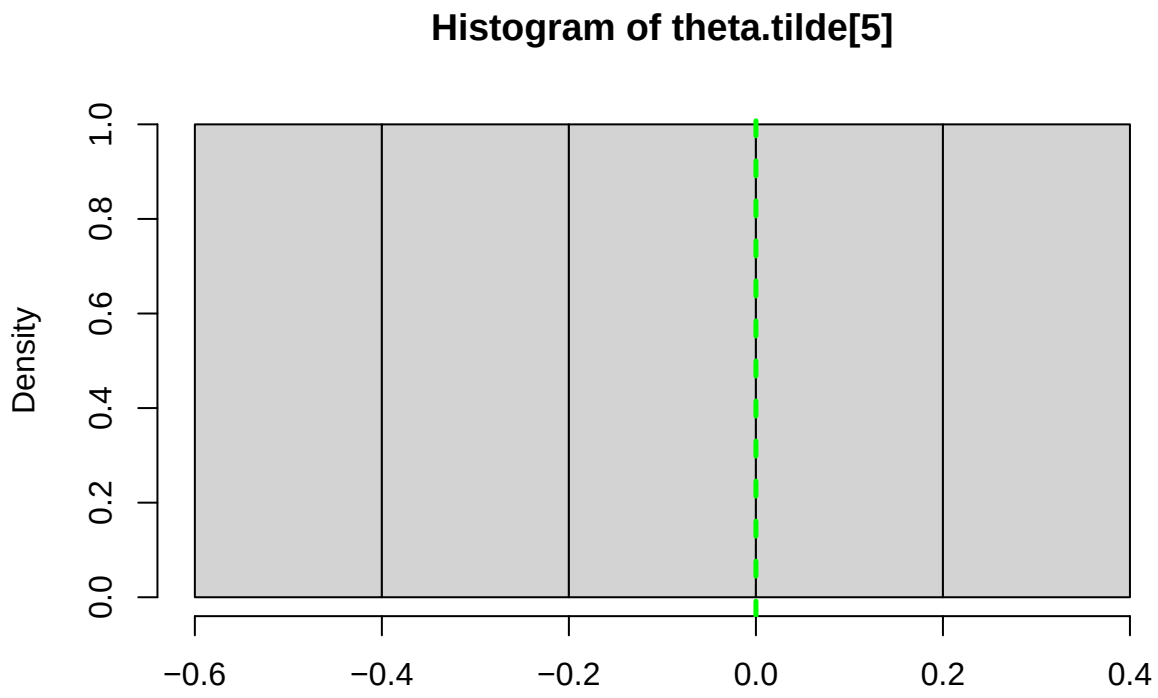

```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.4023  0.5422  0.7681  0.7377  0.8762  1.0999
[1] "95% CI based on bootstrap:"
      lower    upper
1 0.4163246 1.077505
```




```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
0.03036 0.80588 0.83571 0.71209 0.87883 1.00964
[1] "95% CI based on bootstrap:"
      lower     upper
1 0.1079104 0.9965627
```




```
[1] "Summary statistics of bootstrap replicates:"
     Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
-0.354171 -0.258374 -0.029823  0.006133  0.073943  0.599089
[1] "95% CI based on bootstrap:"
       lower     upper
1 -0.3445916 0.5465741
```




```
[1] "Summary statistics of bootstrap replicates:"
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-0.42743 -0.24921 -0.01959 -0.05434  0.19616  0.22838
[1] "95% CI based on bootstrap:"
       lower     upper
1 -0.4096093 0.2251606
```

Statistics for Theoretical 95% Confidence Intervals

```
[1] Length of Confidence Intervals for theta[1]
[1] Coverage proportion: 1
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.9387  0.9532  0.9718  0.9932  0.9791  1.1230
[1] Length of Confidence Intervals for theta[6]
[1] Coverage proportion: 0.8
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.8568  0.9439  1.0209  1.0382  1.1434  1.2260
[1] Length of Confidence Intervals for theta[2]
[1] Coverage proportion: 0.8
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.8892  0.9063  0.9395  0.9487  0.9451  1.0632
[1] Length of Confidence Intervals for theta[5]
[1] Coverage proportion: 1
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.8592  0.8788  0.9177  0.9056  0.9274  0.9451
```