Simulation Results

2026-01-09

Simulation Setup

This simulation is performed with $n = 200$ and $d = 10$, using the 2-d lattice as the underlying graph. $s = 2$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0$. The attached results are for a 10-replication simulation. The true values of the parameter vector θ are

```
0 0 0 0 0 0 0.7071068 0 0.7071068 0 ,
```

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{$ 7, 9, 1, 4$\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison ($\frac{1}{n.sim}\sum_{i=1}^{n.sim}\frac{1}{|\mathcal{C}|}\|\hat{\theta}_{i,\mathcal{C}} - \theta_{\mathcal{C}}\|^2$)

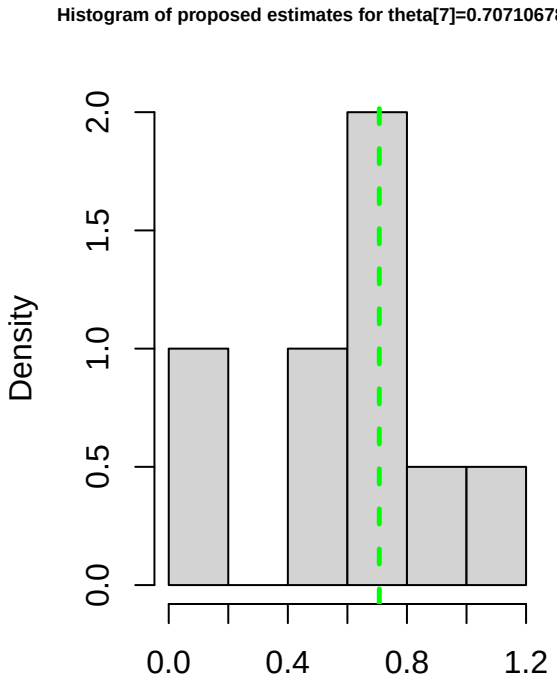
Table 1: Mean-Squared Error of Parameter Estimates

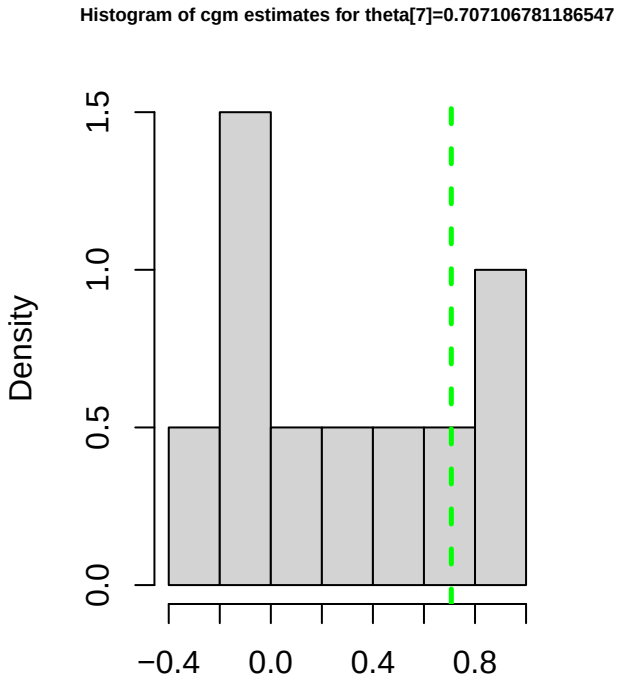
	proposed	cgm
theta[7]	0.111	0.337
theta[9]	0.044	0.236
theta[1]	0.049	0.045
theta[4]	0.059	0.027
total	0.066	0.161

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[7]	0.174	0.180
theta[9]	0.087	0.143
theta[1]	0.020	0.026
theta[4]	0.058	0.015
total	0.085	0.091

Boxplots


Histogram of proposed estimates for theta[7]=0.70710678118654
Density
0.0
0.5
1.0
1.5
2.0
0.0
0.4
0.8
1.2


Histogram of cgm estimates for theta[7]=0.707106781186547
Density
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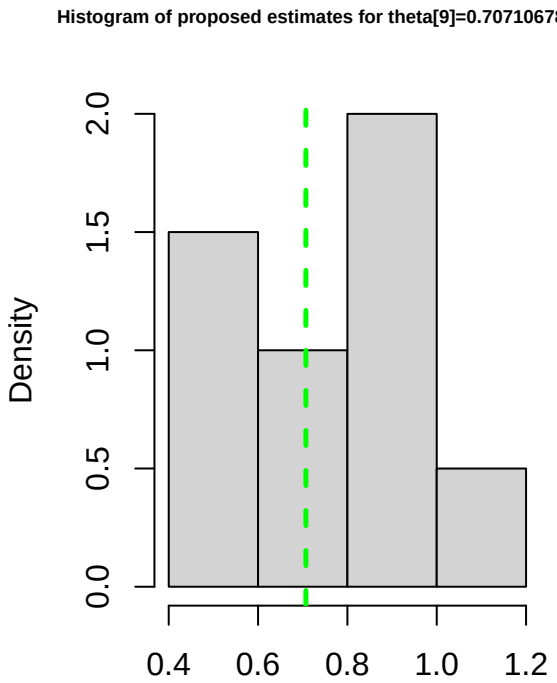
1.0
1.5
−0.4
0.0
0.4
0.8


Histogram of proposed estimates for theta[9]=0.70710678118654
Density
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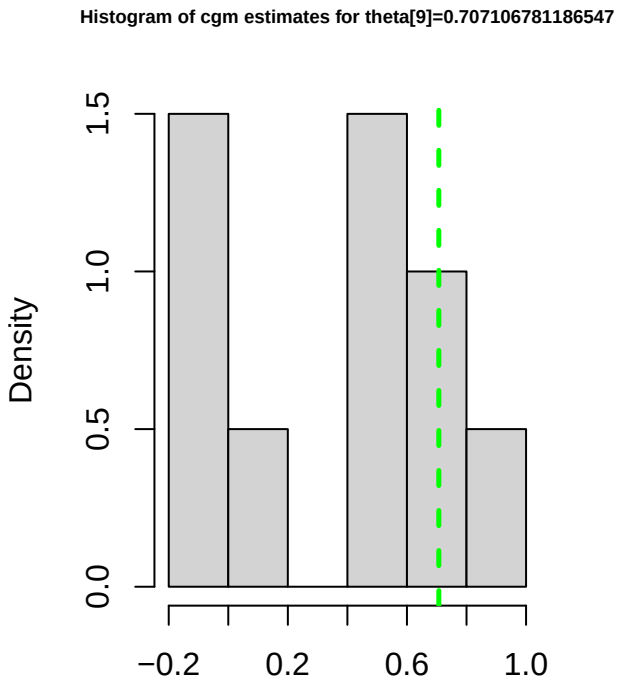
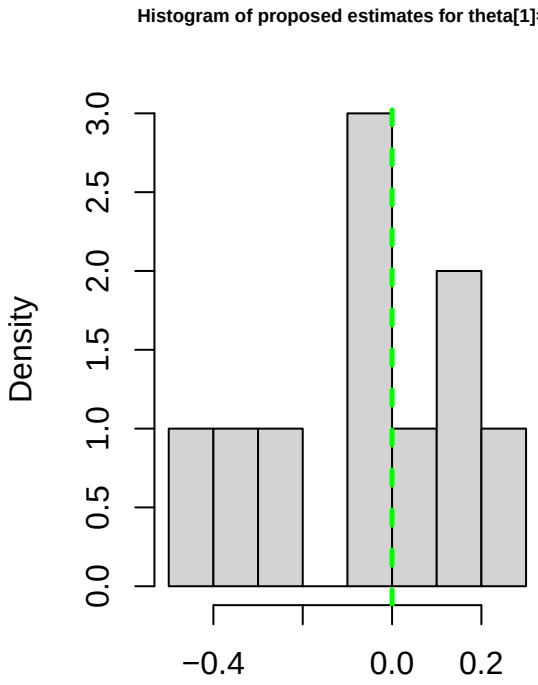
0.4
0.6
0.8
1.0
1.2


Histogram of cgm estimates for theta[9]=0.707106781186547
Density
0.0
0.5
1.0
1.5
−0.2
0.2
0.6
1.0


Histogram of proposed estimates for theta[1]=0
Density
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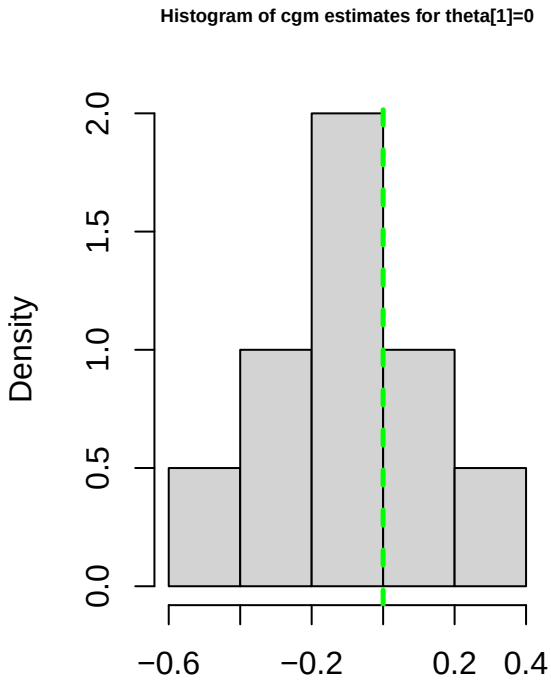
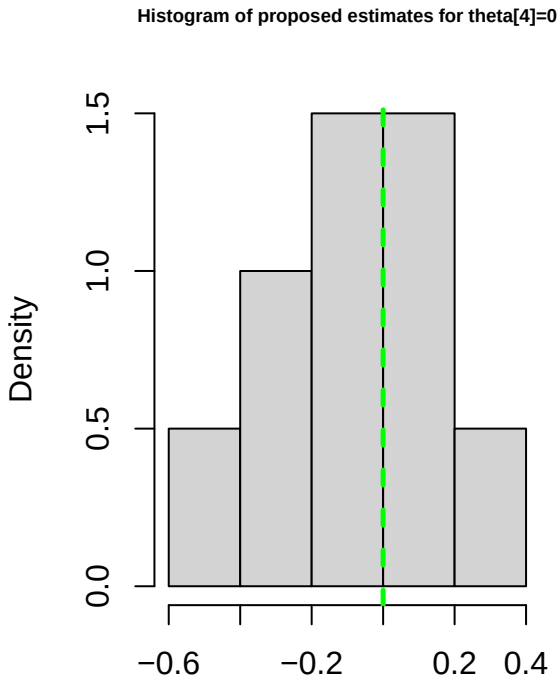
1.0
1.5
2.0
2.5
3.0
−0.4
0.0
0.2


Histogram of cgm estimates for theta[1]=0
Density
0.0
0.5
1.0
1.5
2.0
−0.6
−0.2
0.2
0.4


Histogram of proposed estimates for theta[4]=0
Density
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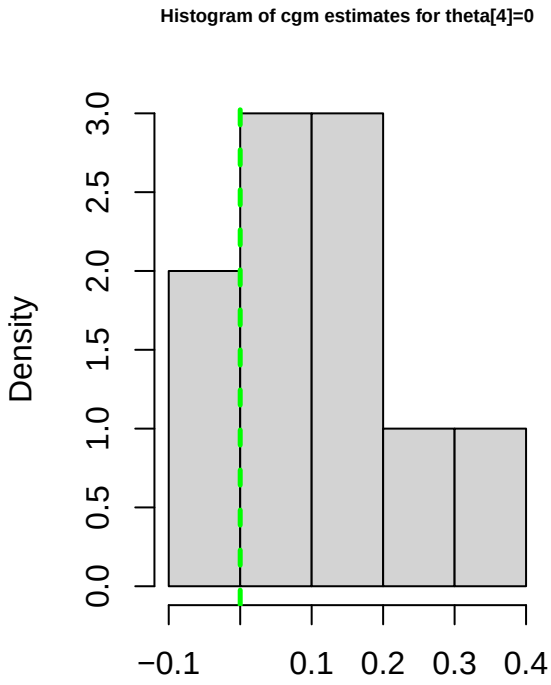
1.0
1.5
−0.6
−0.2
0.2
0.4


Histogram of cgm estimates for theta[4]=0
Density
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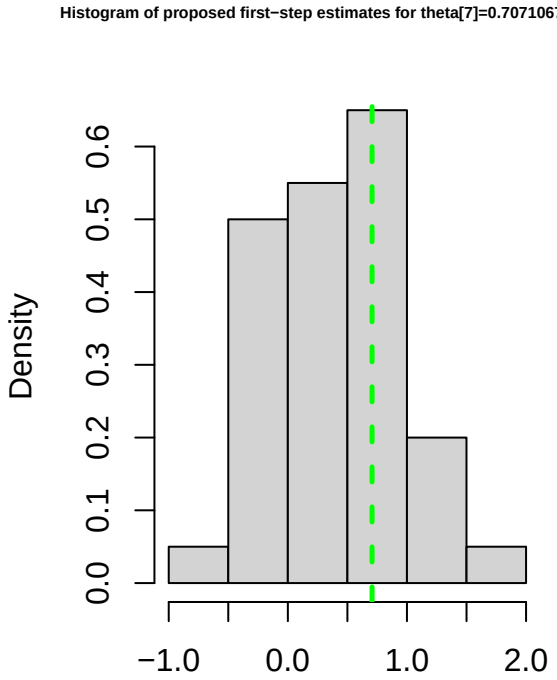
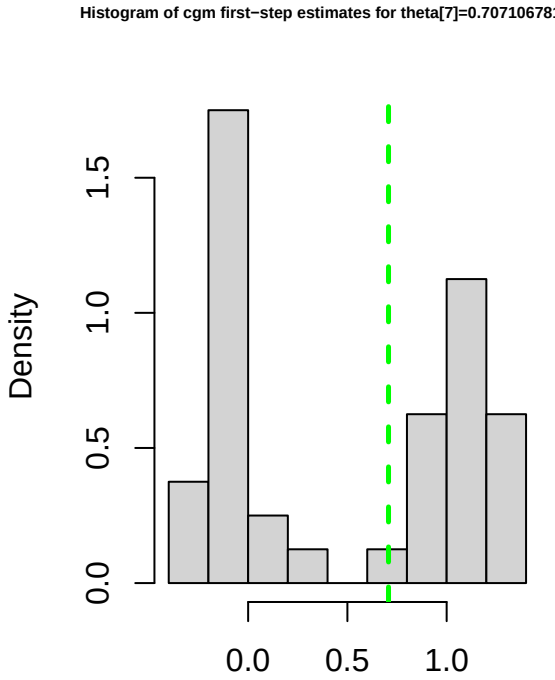
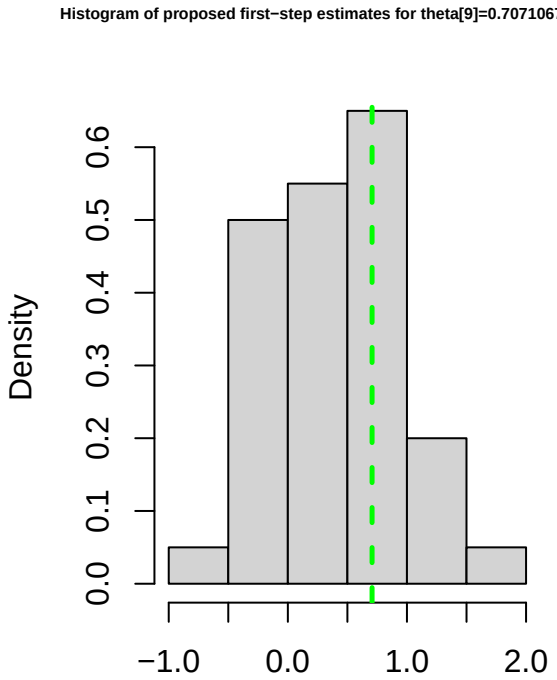
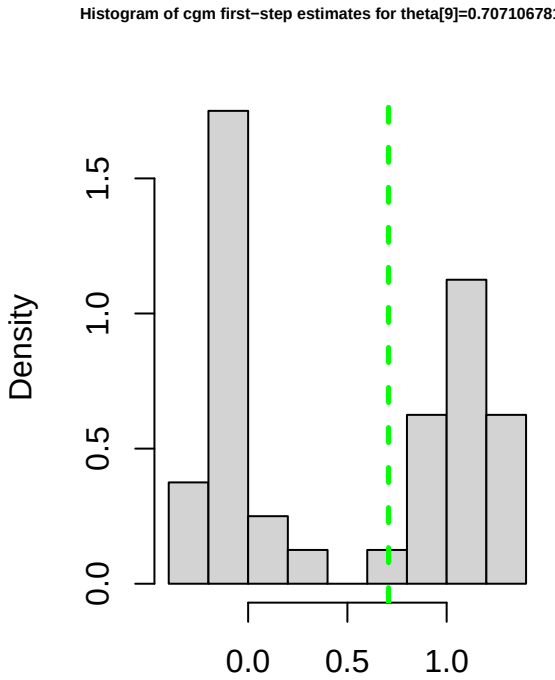
1.0
1.5
2.0
2.5
3.0
−0.1
0.1
0.2
0.3
0.4

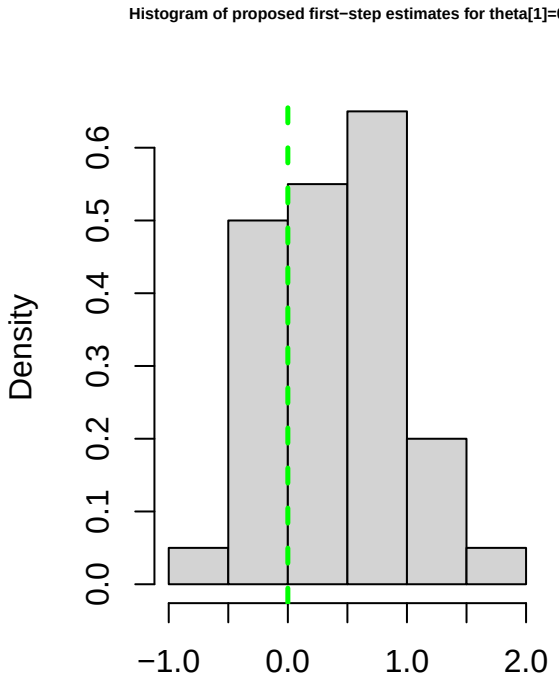
First Step Histograms

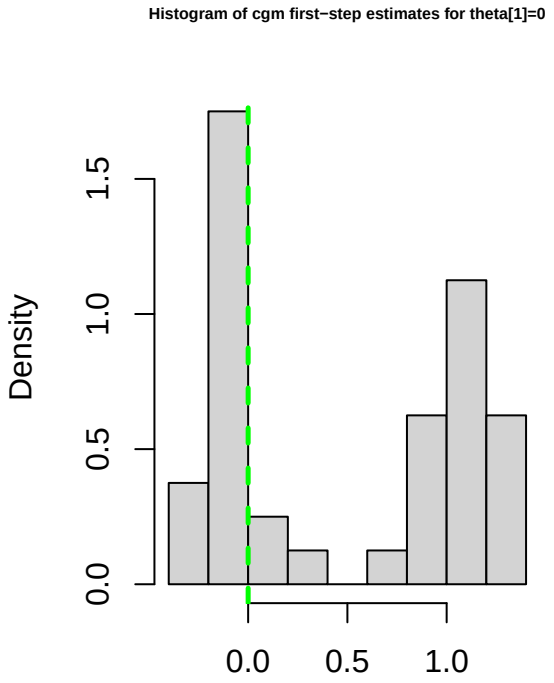


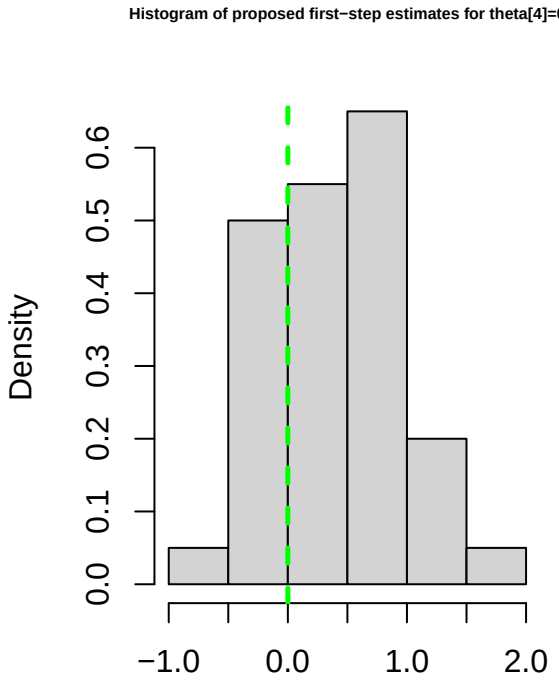


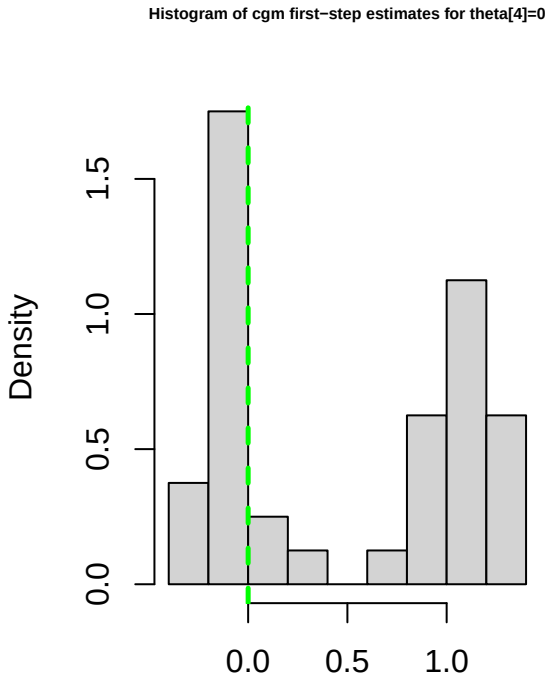





Histogram of proposed first−step estimates for theta[1]=0
Density
0.0
0.1
0.2
0.3
0.4
0.5
0.6
−1.0
0.0
1.0
2.0


Histogram of cgm first−step estimates for theta[1]=0
Density
0.0
0.5
1.0
1.5
0.0
0.5
1.0


Histogram of proposed first−step estimates for theta[4]=0
Density
0.0
0.1
0.2
0.3
0.4
0.5
0.6
−1.0
0.0
1.0
2.0


Histogram of cgm first−step estimates for theta[4]=0
Density
0.0
0.5
1.0
1.5
0.0
0.5
1.0

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 3: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[7]	0.036	0.676	1.079	0.050	1.061
theta[9]	0.423	0.801	1.136	0.436	1.082
theta[1]	-0.472	-0.044	0.215	-0.438	0.208
theta[4]	-0.471	-0.091	0.360	-0.439	0.320

Table 4: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[7]	-0.284	0.175	0.907	-0.235	0.905
theta[9]	-0.111	0.503	0.837	-0.095	0.807
theta[1]	-0.416	-0.064	0.331	-0.373	0.286
theta[4]	-0.079	0.101	0.340	-0.071	0.323

Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[7]	0.621	0.187	0.254	0.989	0.7
theta[9]	0.746	0.173	0.408	1.085	0.9
theta[1]	-0.062	0.154	-0.363	0.239	0.8
theta[4]	-0.059	0.155	-0.364	0.245	0.7

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[7]	0.286	0.257	-0.218	0.790	0.5
theta[9]	0.367	0.244	-0.112	0.846	0.6
theta[1]	-0.062	0.162	-0.378	0.255	0.9
theta[4]	0.110	0.160	-0.205	0.424	0.9