

Simulation Results

2026-01-20

Simulation Setup

This simulation is performed with $n = 200$ and $d = 100$, using the 2-d lattice as the underlying graph. $s = 5$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.2$. The attached results are for a 10-replication simulation. The parameter vector θ has sparse components other than the following:

| Parameter.Index | Value |
|-----------------|--------|
| 20 | -0.447 |
| 51 | -0.447 |
| 68 | 0.447 |
| 76 | 0.447 |
| 85 | 0.447 |

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{ 20, 51, 71, 40 \}$ elements of θ . Accordingly, **all statistics and visuals are indicative of performance only on the set \mathcal{C} .**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison

Table 1: Mean-Squared Error of Parameter Estimates

| | proposed | cgm |
|-----------|----------|-------|
| theta[20] | 0.040 | 0.050 |
| theta[51] | 0.071 | 0.035 |
| theta[71] | 0.015 | 0.028 |
| theta[40] | 0.020 | 0.041 |
| total | 0.036 | 0.038 |

Table 2: Mean-Squared Error of First-Step Parameter Estimates

| | proposed | cgm |
|-----------|----------|-------|
| theta[20] | 0.131 | 0.049 |

| | proposed | cgm |
|-----------|----------|-------|
| theta[51] | 0.117 | 0.035 |
| theta[71] | 0.000 | 0.005 |
| theta[40] | 0.000 | 0.003 |
| total | 0.062 | 0.023 |

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### Mean absolute deviation comparison $(\frac{1}{n.sim} \sum_{i=1}^{n.sim} \frac{1}{|\mathcal{C}|} |\hat{C}
```

Table 3: Mean Absolute Deviation of Parameter Estimates

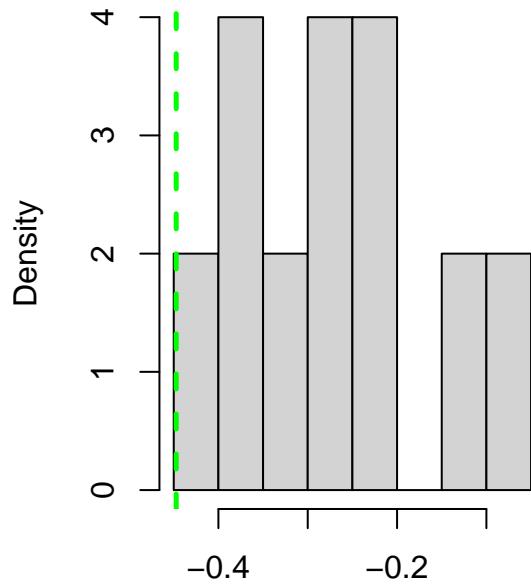
| | proposed | cgm |
|-----------|----------|-------|
| theta[20] | 0.168 | 0.180 |
| theta[51] | 0.233 | 0.169 |
| theta[71] | 0.108 | 0.126 |
| theta[40] | 0.087 | 0.170 |
| total | 0.149 | 0.161 |

Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

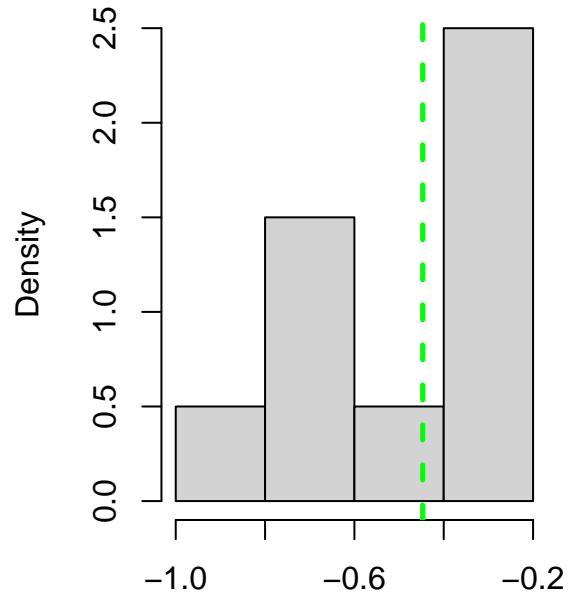
| | proposed | cgm |
|-----------|----------|-------|
| theta[20] | 0.352 | 0.189 |
| theta[51] | 0.317 | 0.151 |
| theta[71] | 0.006 | 0.033 |
| theta[40] | 0.005 | 0.023 |
| total | 0.170 | 0.099 |

Boxplots

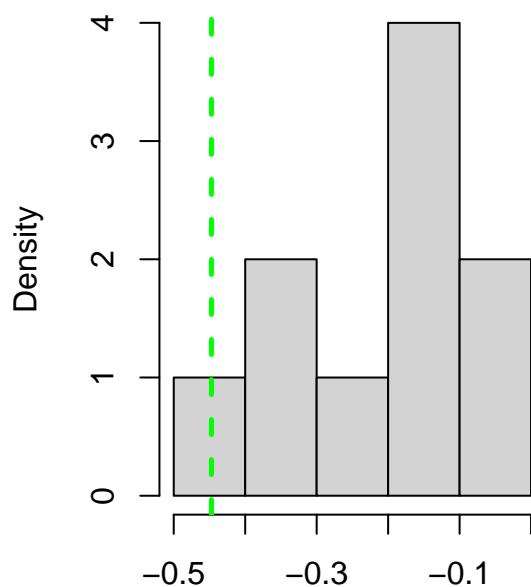
Histogram of proposed estimates for theta[20]=-0.447



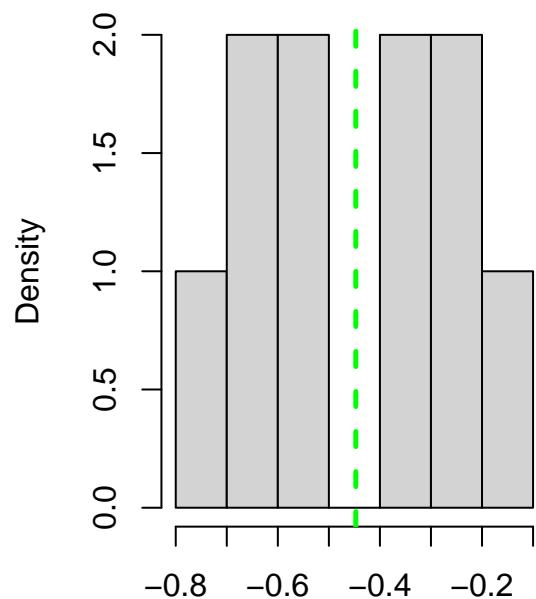
Histogram of cgm estimates for theta[20]=-0.447



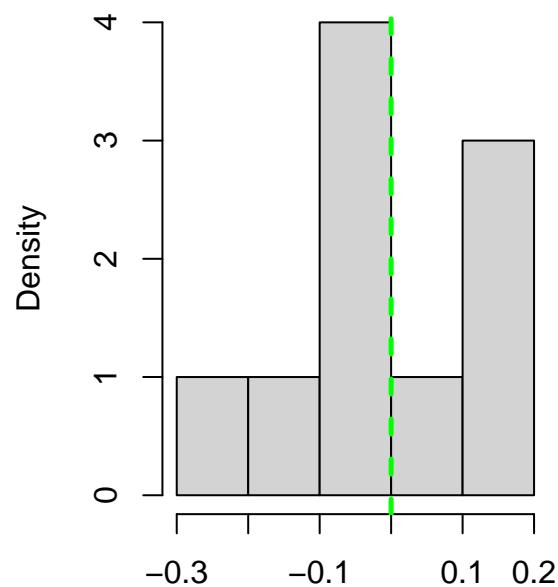
Histogram of proposed estimates for theta[51]=-0.447



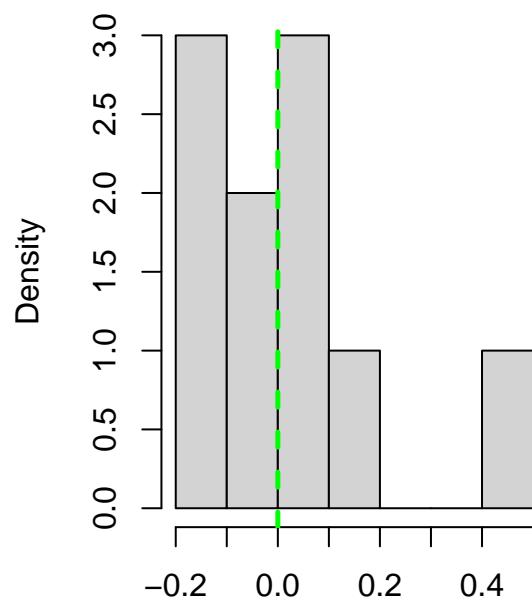
Histogram of cgm estimates for theta[51]=-0.447



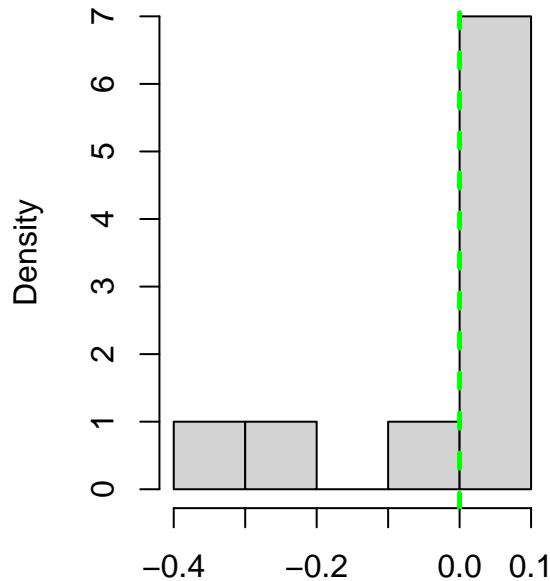
Histogram of proposed estimates for theta[71]=0



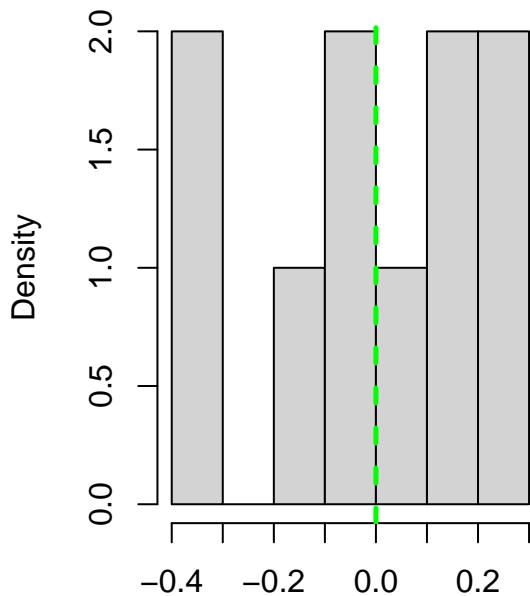
Histogram of cgm estimates for theta[71]=0



Histogram of proposed estimates for theta[40]=0



Histogram of cgm estimates for theta[40]=0



Statistics and 95% Confidence Intervals from per-Replicate Estimates

Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

| | Estimate | SE | lower.CI | upper.CI | cvg |
|-----------|----------|-------|----------|----------|-----|
| theta[20] | -0.279 | 0.142 | -0.557 | -0.001 | 0.9 |
| theta[51] | -0.218 | 0.131 | -0.474 | 0.038 | 0.5 |
| theta[71] | -0.019 | 0.131 | -0.276 | 0.238 | 1.0 |
| theta[40] | -0.040 | 0.129 | -0.292 | 0.212 | 0.9 |

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

| | Estimate | SE | lower.CI | upper.CI | cvg |
|-----------|----------|-------|----------|----------|-----|
| theta[20] | -0.489 | 0.129 | -0.743 | -0.235 | 0.9 |
| theta[51] | -0.452 | 0.132 | -0.711 | -0.192 | 0.9 |
| theta[71] | 0.013 | 0.133 | -0.249 | 0.274 | 0.9 |
| theta[40] | 0.002 | 0.141 | -0.273 | 0.278 | 0.8 |