Simulation Results

2026-01-13

Simulation Setup

This simulation is performed with $n = 200$ and $d = 20$, using the 2-d lattice as the underlying graph. $s = 2$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.2$. The attached results are for a 10-replication simulation. The true values of the parameter vector θ are

```
0 -0.7071068 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -0.7071068 0 0 ,
```

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{$ 2, 18, 5, 8$\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison ($\frac{1}{n.sim}\sum_{i=1}^{n.sim}\frac{1}{|\mathcal{C}|}\|\hat{\theta}_{i,\mathcal{C}} - \theta_{\mathcal{C}}\|^2$)

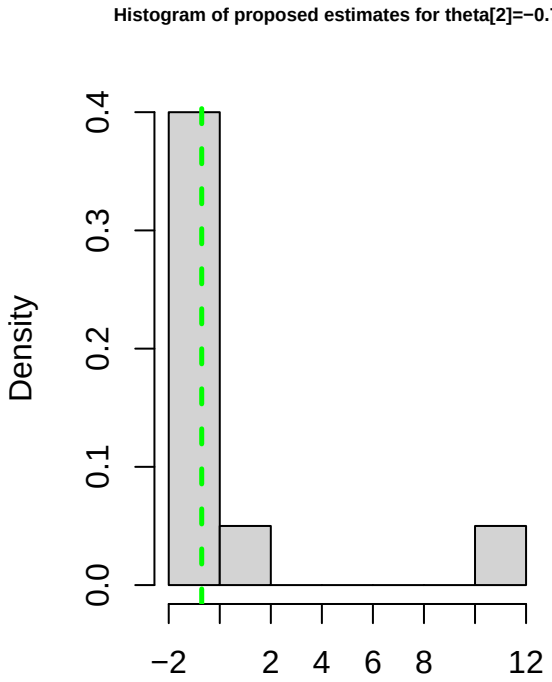
Table 1: Mean-Squared Error of Parameter Estimates

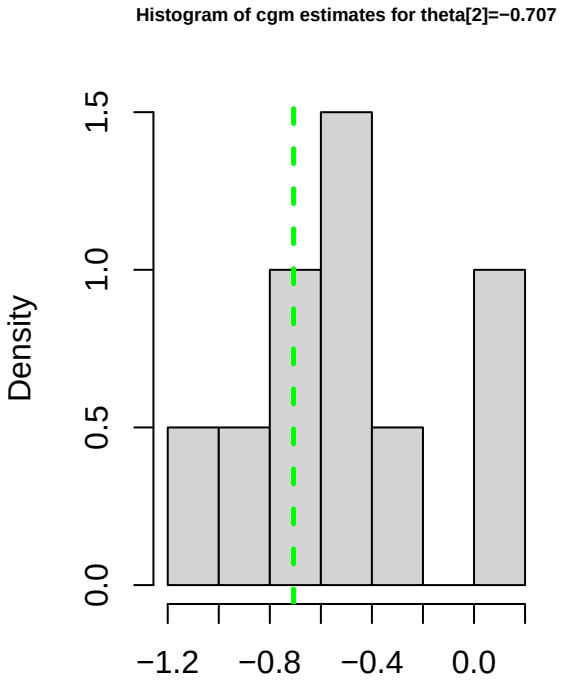
	proposed	cgm
theta[2]	13.640	0.171
theta[18]	2.324	0.139
theta[5]	0.188	0.023
theta[8]	0.213	0.026
total	4.091	0.090

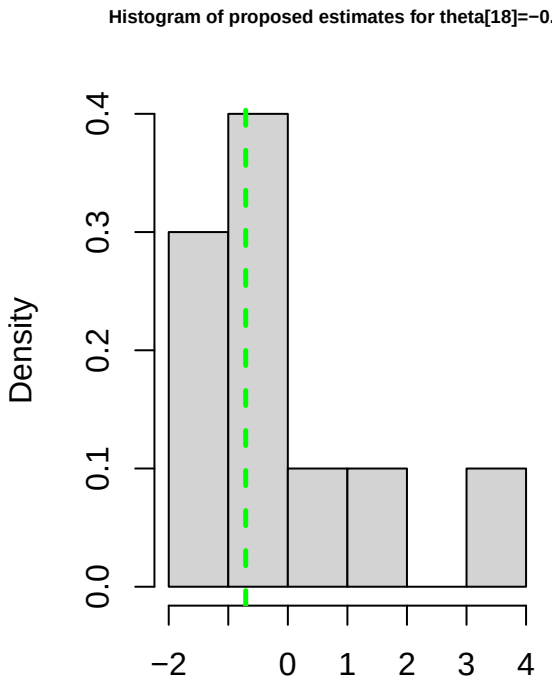
Table 2: Mean-Squared Error of First-Step Parameter Estimates

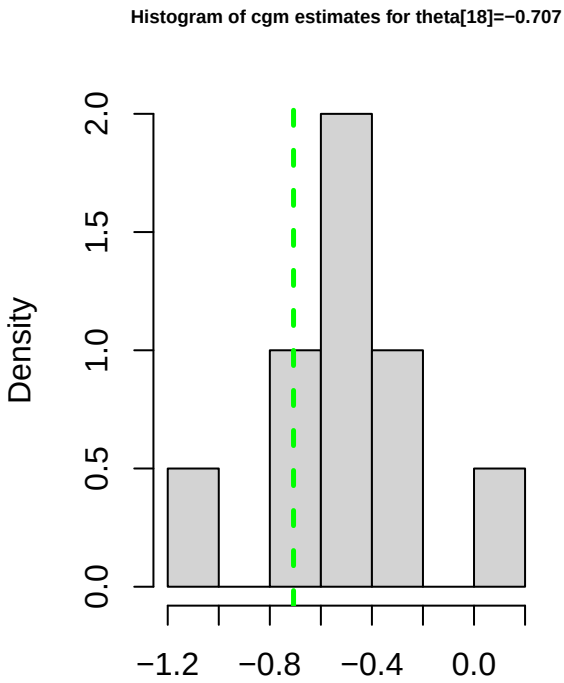
	proposed	cgm
theta[2]	0.277	0.159
theta[18]	0.293	0.153
theta[5]	0.037	0.023
theta[8]	0.111	0.007
total	0.180	0.086

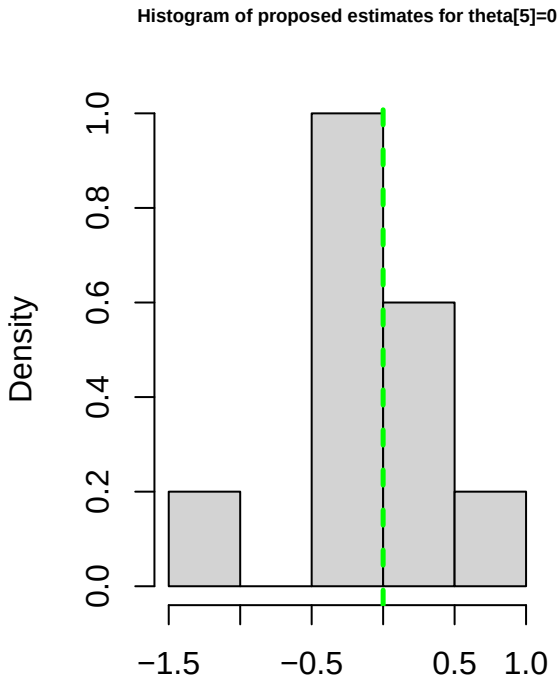
Boxplots

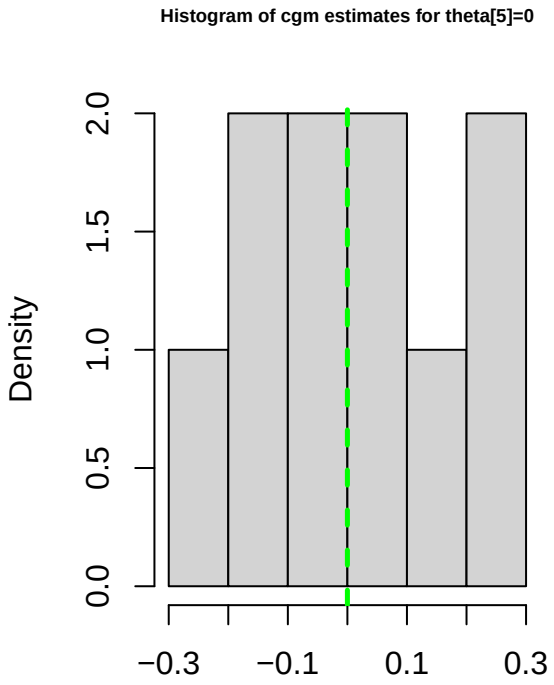

Histogram of proposed estimates for theta[2]=−0.707
Density
0.0
0.1
0.2
0.3
0.4
−2
2
4
6
8
12

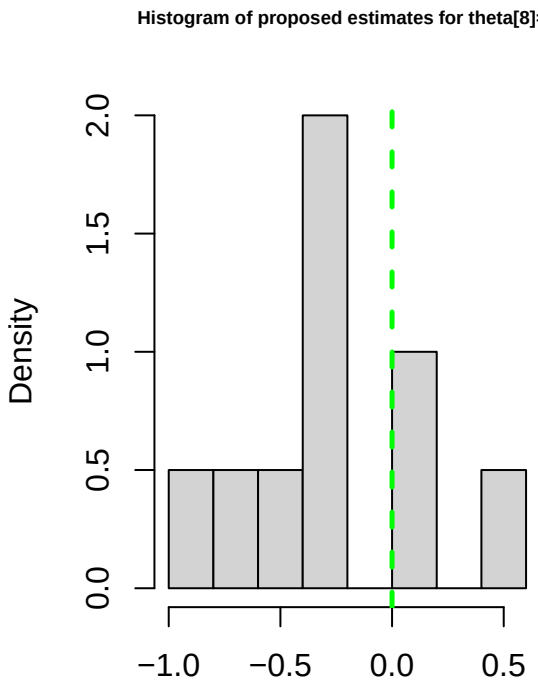

Histogram of cgm estimates for theta[2]=−0.707
Density
0.0
0.5
1.0
1.5
−1.2
−0.8
−0.4
0.0

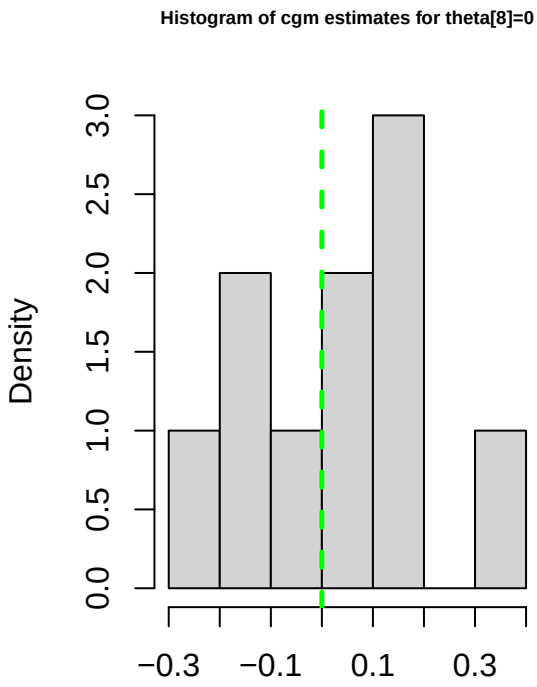

Histogram of proposed estimates for theta[18]=−0.707
Density
0.0
0.1
0.2
0.3
0.4
−2
0
1
2
3
4


Histogram of cgm estimates for theta[18]=−0.707
Density
0.0
0.5
1.0
1.5
2.0
−1.2
−0.8
−0.4
0.0

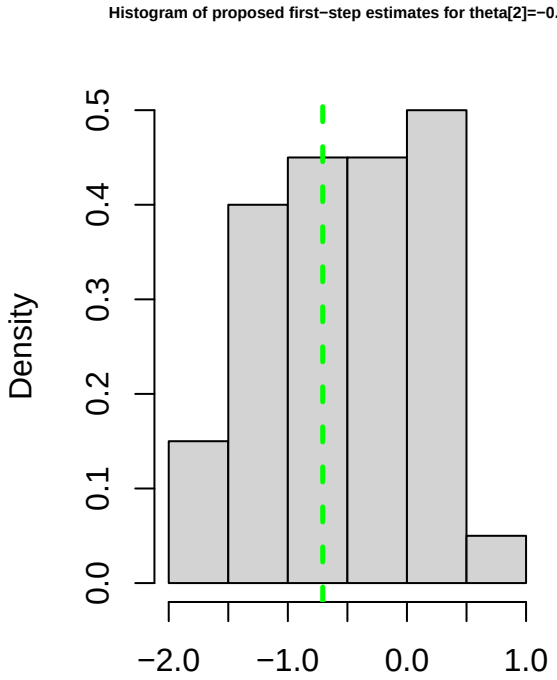

Histogram of proposed estimates for theta[5]=0
Density
0.0
0.2
0.4
0.6
0.8
1.0
−1.5
−0.5
0.5
1.0

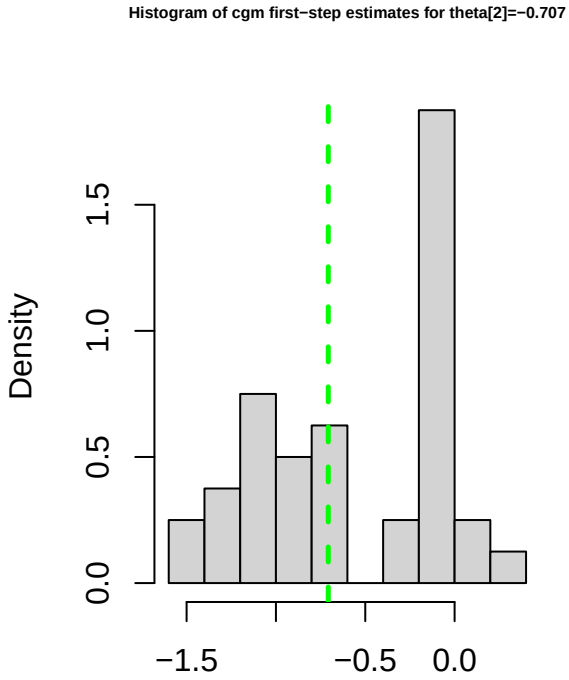

Histogram of cgm estimates for theta[5]=0
Density
0.0
0.5
1.0
1.5
2.0
−0.3
−0.1
0.1
0.3

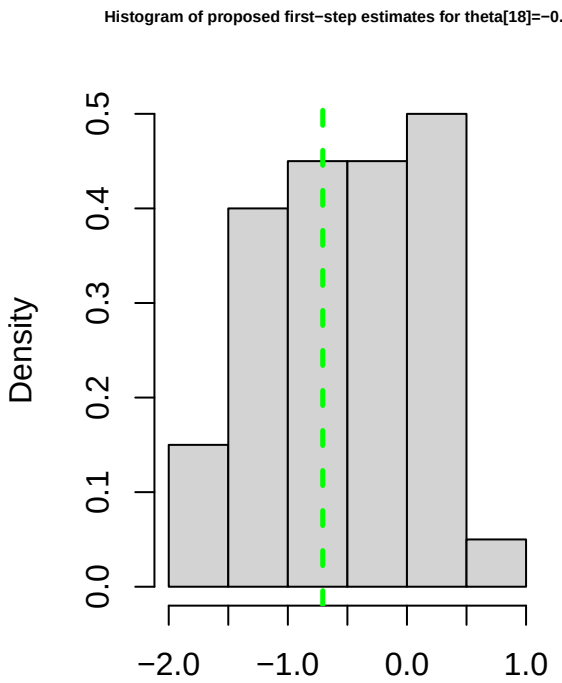

Histogram of proposed estimates for theta[8]=0
Density
0.0
0.5
1.0
1.5
2.0
−1.0
−0.5
0.0
0.5

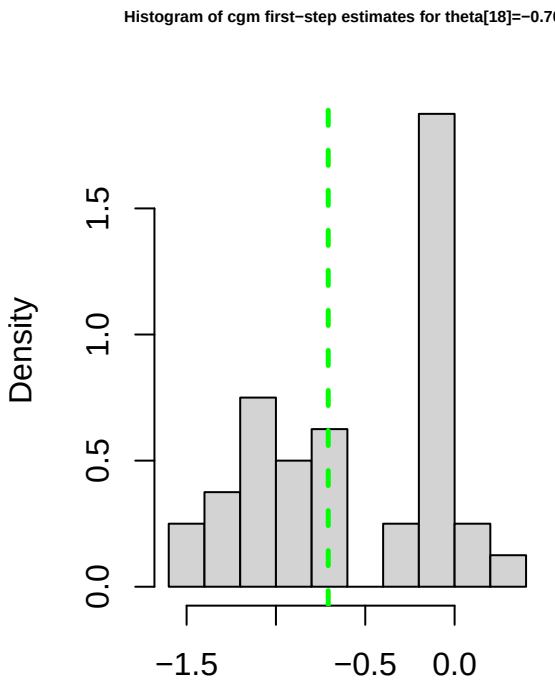

Histogram of cgm estimates for theta[8]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.3
−0.1
0.1
0.3

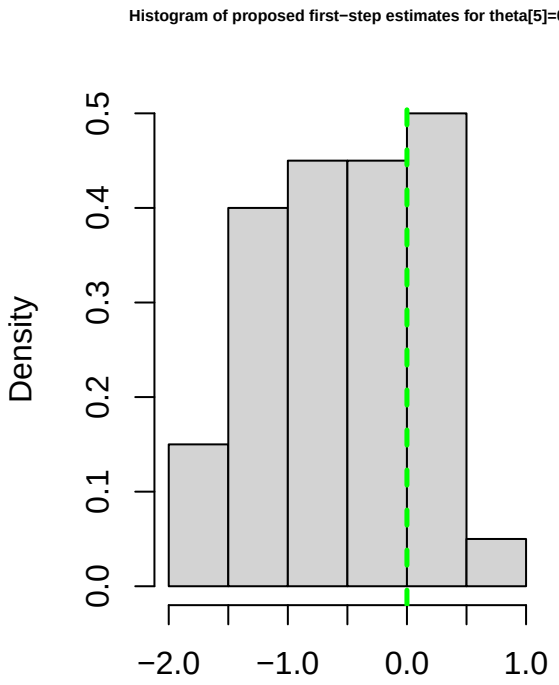
First Step Histograms

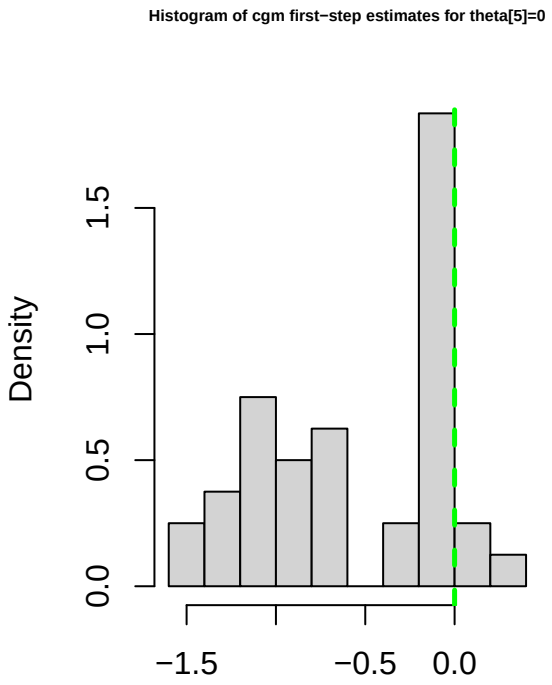


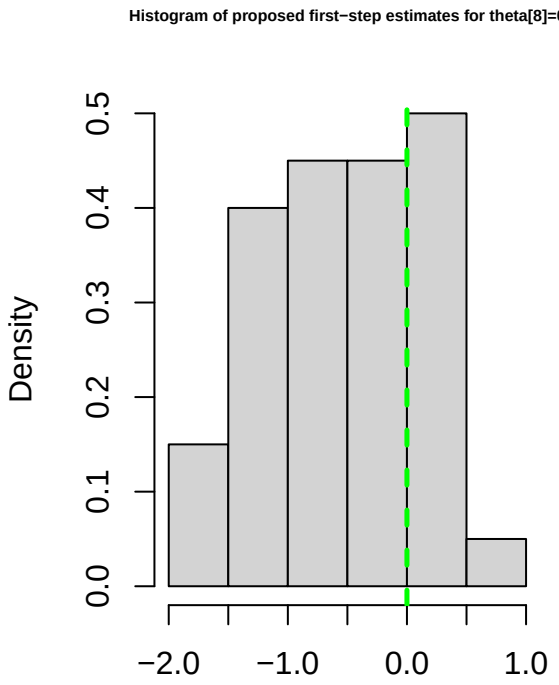


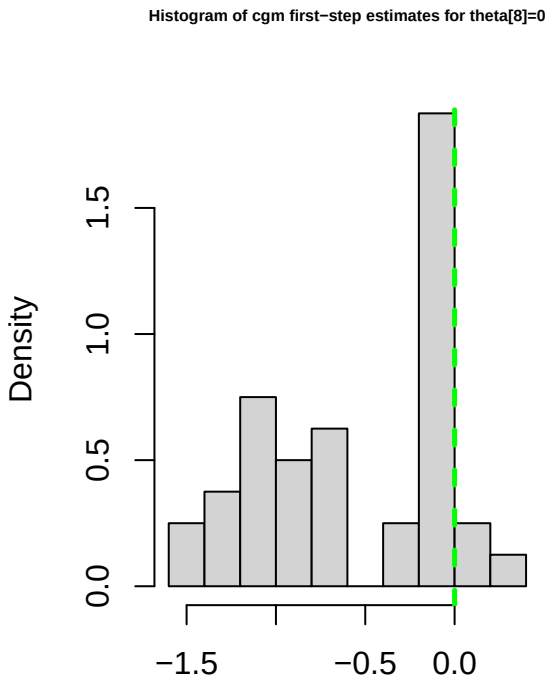





Histogram of proposed first−step estimates for theta[5]=0
Density
0.0
0.1
0.2
0.3
0.4
0.5
−2.0
−1.0
0.0
1.0


Histogram of cgm first−step estimates for theta[5]=0
Density
0.0
0.5
1.0
1.5
−1.5
−0.5
0.0


Histogram of proposed first−step estimates for theta[8]=0
Density
0.0
0.1
0.2
0.3
0.4
0.5
−2.0
−1.0
0.0
1.0


Histogram of cgm first−step estimates for theta[8]=0
Density
0.0
0.5
1.0
1.5
−1.5
−0.5
0.0

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 3: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[2]	-1.754	-0.630	10.818	-1.664	8.501
theta[18]	-1.259	-0.513	3.497	-1.258	2.943
theta[5]	-1.074	-0.054	0.637	-0.918	0.507
theta[8]	-0.999	-0.237	0.400	-0.943	0.323

Table 4: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[2]	-1.143	-0.534	0.058	-1.084	0.054
theta[18]	-1.101	-0.479	0.084	-1.024	0.009
theta[5]	-0.222	-0.011	0.203	-0.214	0.203
theta[8]	-0.225	0.062	0.306	-0.214	0.271

Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[2]	0.480	0.286	-0.080	1.040	0.4
theta[18]	-0.069	0.262	-0.583	0.446	0.3
theta[5]	-0.135	0.207	-0.542	0.271	0.8
theta[8]	-0.265	0.208	-0.672	0.142	0.7

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[2]	-0.496	0.190	-0.869	-0.124	0.7
theta[18]	-0.497	0.189	-0.868	-0.126	0.7
theta[5]	-0.003	0.131	-0.259	0.254	1.0
theta[8]	0.027	0.134	-0.236	0.290	0.9