Simulation Results

2026-01-20

Simulation Setup

This simulation is performed with $n = 200$ and $d = 200$, using the 2-d lattice as the underlying graph. $s = 5$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0$. The attached results are for a 10-replication simulation. The parameter vector θ has sparse components other than the following:

```
$\theta$$[$2$] = -0.447$\theta$$[$50$] = 0.447$\theta$$[$67$] = 0.447$\theta$$[$92$] = 0.447$\theta$$[$
```

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{$ 2, 50, 71, 172$\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

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### Mean-squared error comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat\theta
```

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[2]	0.070	2.048
theta[50]	0.062	0.514
theta[71]	0.007	0.366
theta[172]	0.005	0.050
total	0.036	0.745

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[2]	0.171	0.040
theta[50]	0.200	0.061
theta[71]	0.000	0.000
theta[172]	0.000	0.000
total	0.093	0.025

Mean absolute deviation comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat

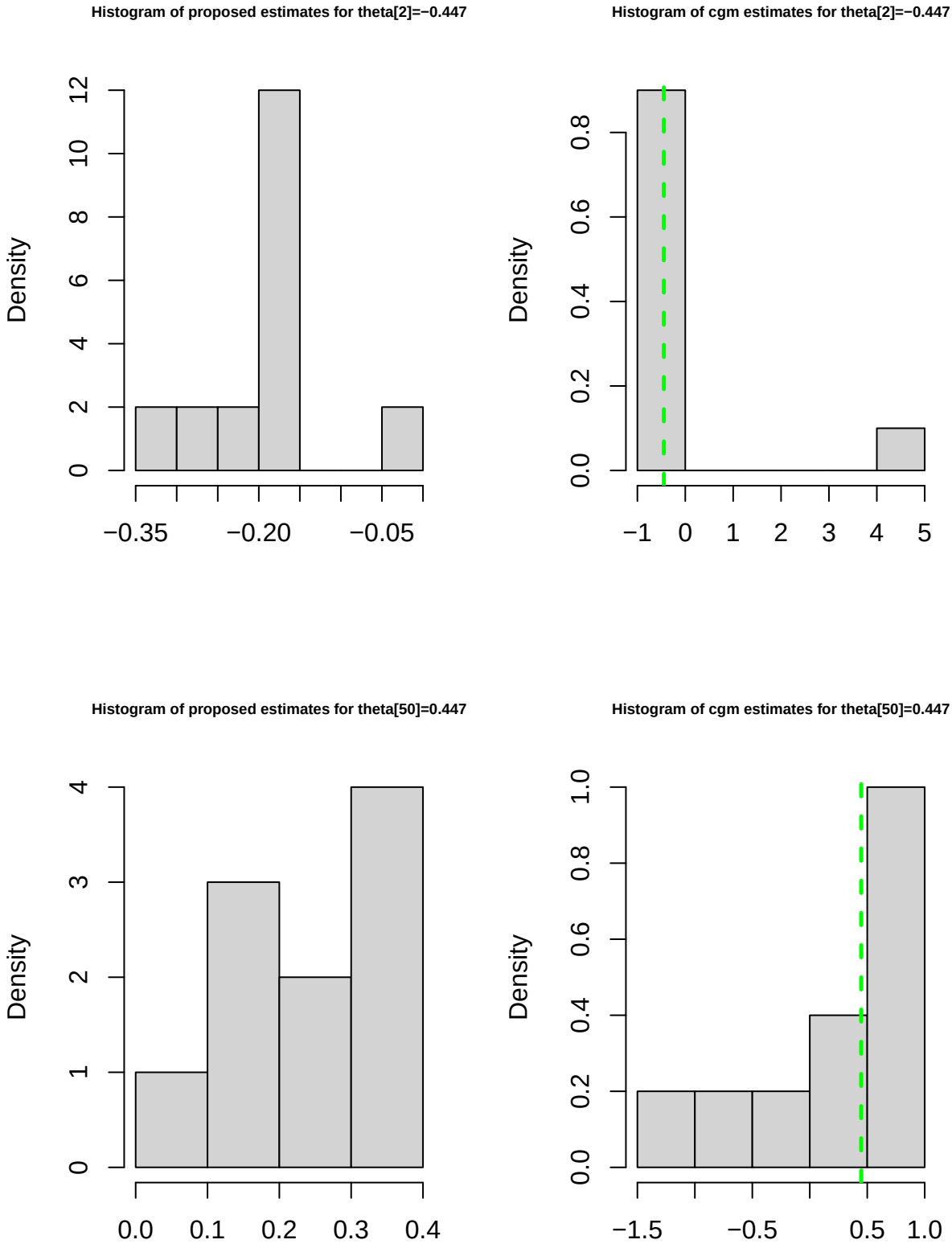
Table 3: Mean Absolute Deviation of Parameter Estimates

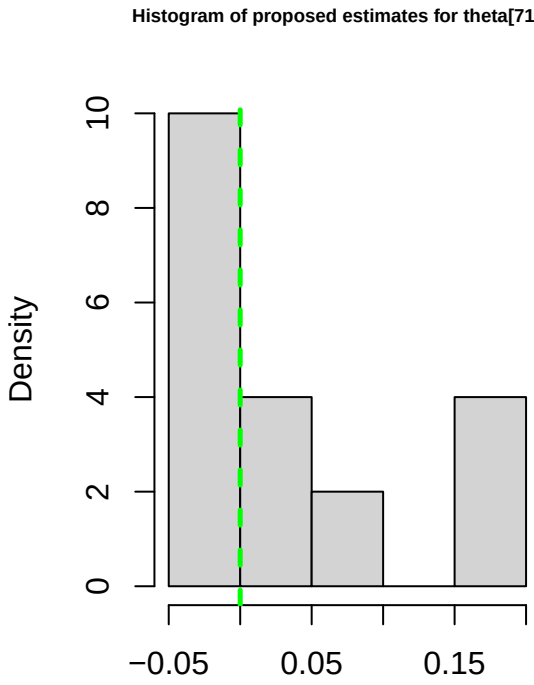
	proposed	cgm
theta[2]	0.254	0.579
theta[50]	0.224	0.504
theta[71]	0.055	0.360
theta[172]	0.056	0.165
total	0.147	0.402

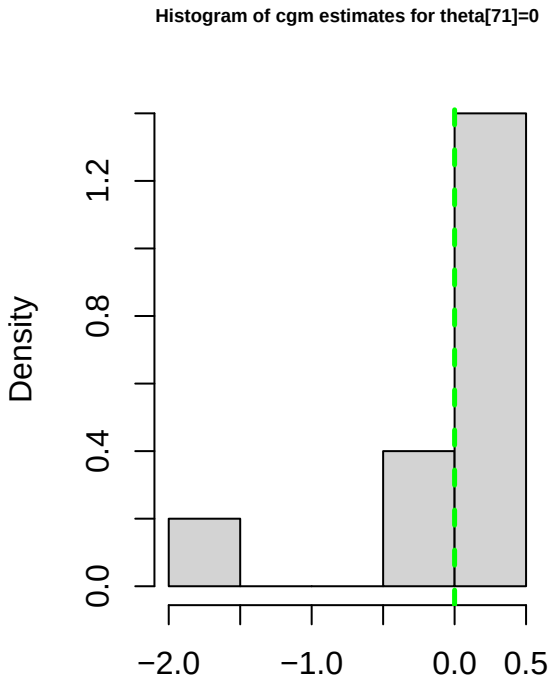
Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

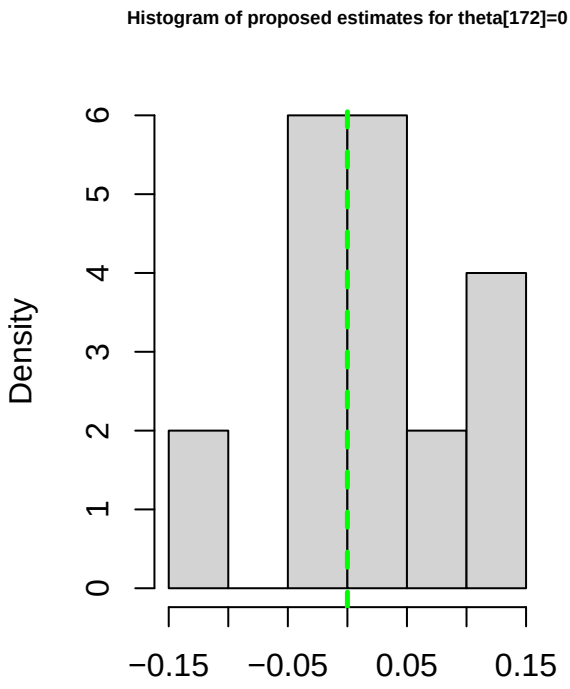
	proposed	cgm
theta[2]	0.407	0.154
theta[50]	0.447	0.215
theta[71]	0.000	0.005
theta[172]	0.000	0.000
total	0.214	0.094

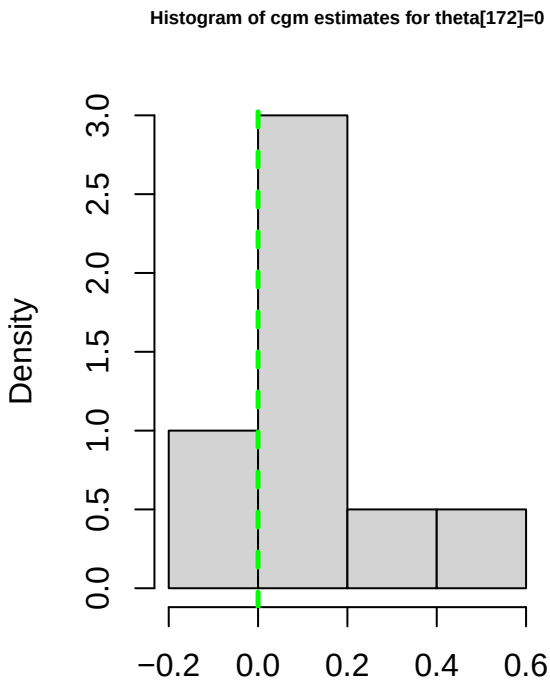
Boxplots


Histogram of proposed estimates for theta[2]=−0.447
Density
0
2
4
6
8
10
12
−0.35
−0.20
−0.05
Histogram of cgm estimates for theta[2]=−0.447
Density
0.0
0.2
0.4
0.6
0.8
−1
0
1
2
3
4
5
Histogram of proposed estimates for theta[50]=0.447
Density
0
1
2
3
4
0.0
0.1
0.2
0.3
0.4
Histogram of cgm estimates for theta[50]=0.447
Density
0.0
0.2
0.4
0.6
0.8
1.0
−1.5
−0.5
0.5
1.0

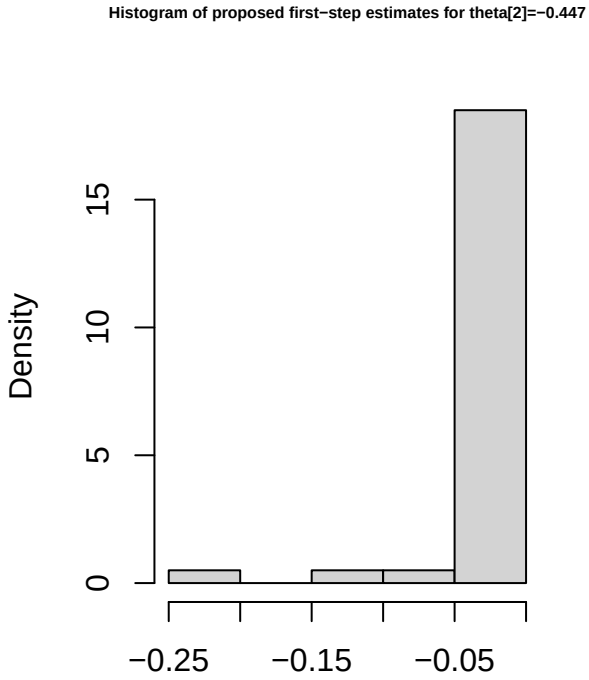

Histogram of proposed estimates for theta[71]=0
Density
0
2
4
6
8
10
−0.05
0.05
0.15

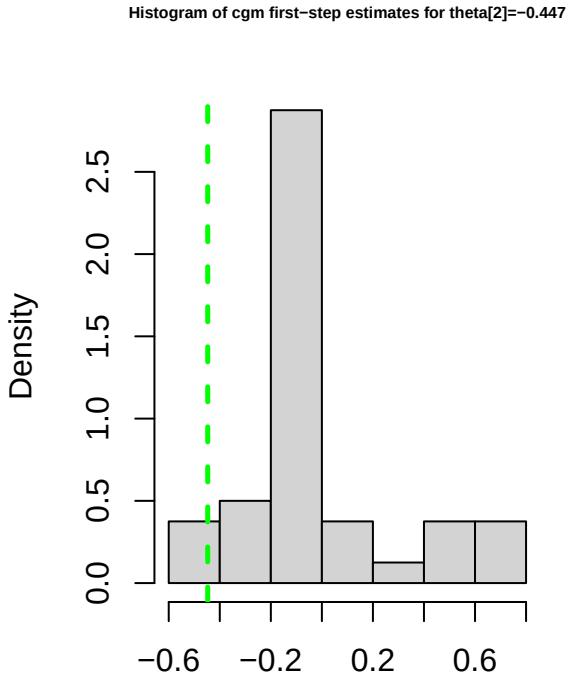

Histogram of cgm estimates for theta[71]=0
Density
0.0
0.4
0.8
1.2
−2.0
−1.0
0.0
0.5

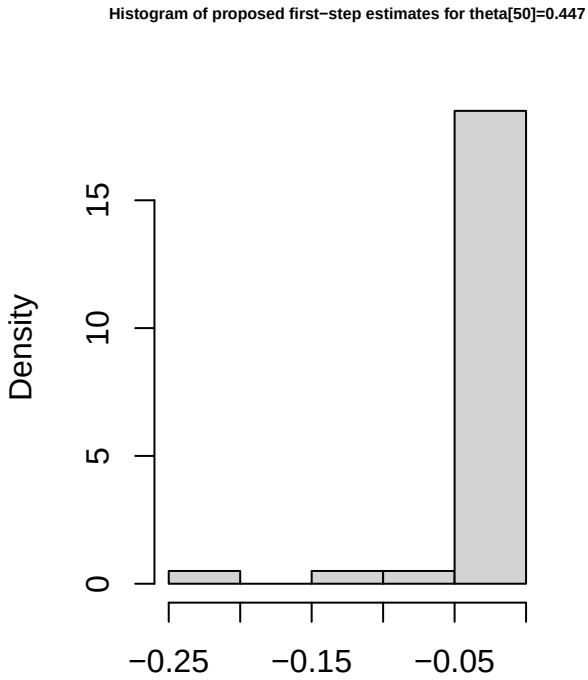

Histogram of proposed estimates for theta[172]=0
Density
0
1
2
3
4
5
6
−0.15
−0.05
0.05
0.15

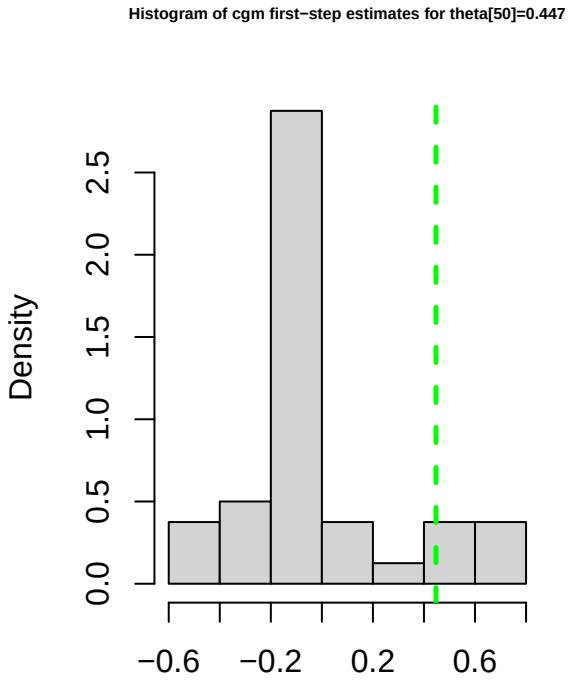

Histogram of cgm estimates for theta[172]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.2
0.0
0.2
0.4
0.6

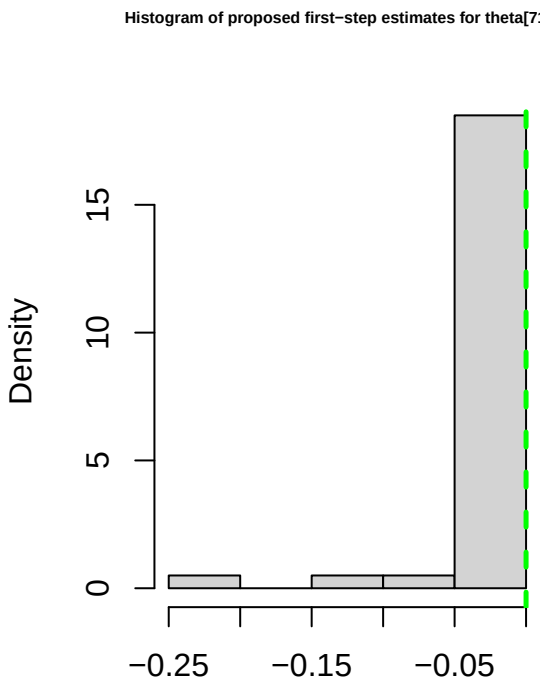
First Step Histograms

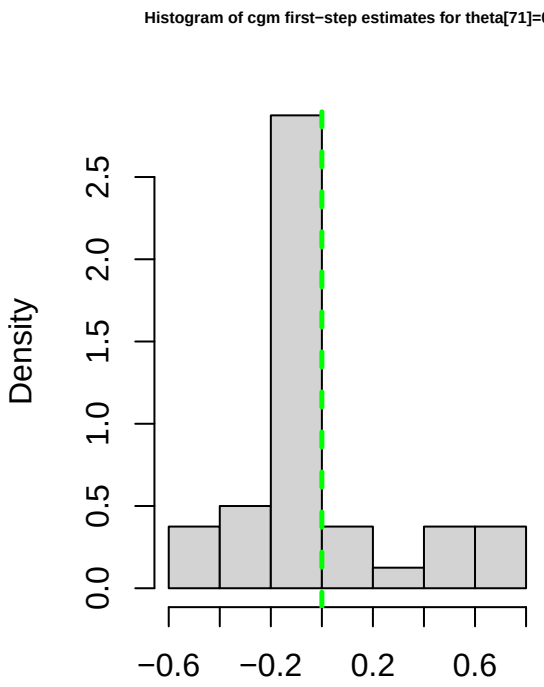


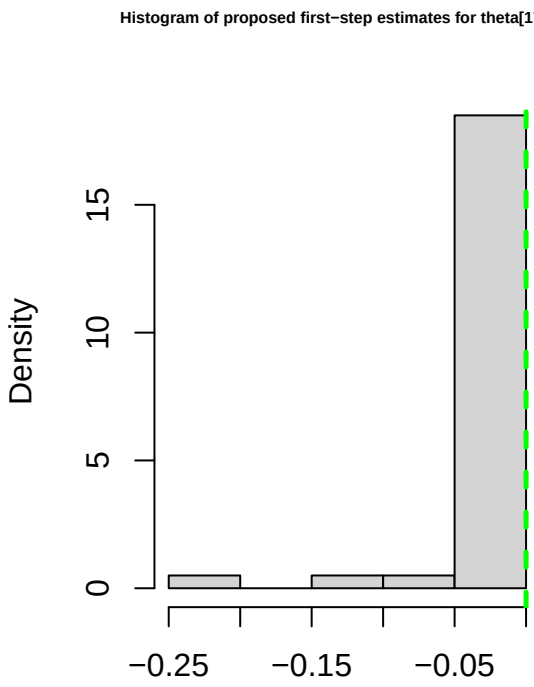


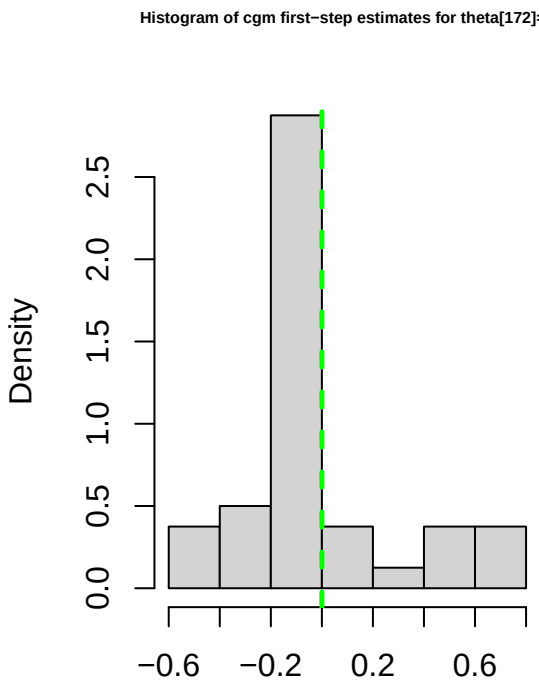





Histogram of proposed first−step estimates for theta[71]=0
Density
0
5
10
15
−0.25
−0.15
−0.05


Histogram of cgm first−step estimates for theta[71]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
−0.6
−0.2
0.2
0.6


Histogram of proposed first−step estimates for theta[172]=0
Density
0
5
10
15
−0.25
−0.15
−0.05


Histogram of cgm first−step estimates for theta[172]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
−0.6
−0.2
0.2
0.6

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 5: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[2]	-0.334	-0.185	-0.036	-0.319	-0.064
theta[50]	0.031	0.216	0.382	0.048	0.372
theta[71]	-0.032	-0.003	0.178	-0.030	0.175
theta[172]	-0.108	0.013	0.131	-0.093	0.130

Table 6: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[2]	-0.716	-0.392	4.047	-0.672	3.107
theta[50]	-1.386	0.435	0.686	-1.205	0.685
theta[71]	-1.762	0.108	0.401	-1.418	0.400
theta[172]	-0.153	0.075	0.556	-0.141	0.497

Statistics for Theoretical 95% Confidence Intervals

Table 7: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[2]	-0.193	0.103	-0.394	0.008	0.2
theta[50]	0.224	0.107	0.013	0.434	0.4
theta[71]	0.034	0.101	-0.164	0.232	1.0
theta[172]	0.020	0.103	-0.182	0.222	1.0

Table 8: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[2]	0.040	0.237	-0.426	0.505	0.9
theta[50]	0.158	0.189	-0.213	0.528	0.6
theta[71]	-0.043	0.183	-0.402	0.315	0.8
theta[172]	0.115	0.159	-0.197	0.427	0.9