Simulation Results

2026-01-16

Simulation Setup

This simulation is performed with $n = 200$ and $d = 50$, using the 2-d lattice as the underlying graph. $s = 5$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.4$. The attached results are for a 10-replication simulation. The true values of the parameter vector θ are

```
0 0 0 0 0 0 -0.4472136 0 0 0 0 0.4472136 -0.4472136 0 0 0 0 0 0 0 0 0 0 0.4472136 0 0 0 -0.4472136 0 0
```

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{ 7, 12, 4, 44\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

```
### Mean-squared error comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat\theta
```

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[7]	0.042	0.032
theta[12]	0.056	0.072
theta[4]	0.028	0.023
theta[44]	0.030	0.021
total	0.039	0.037

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[7]	0.101	0.075
theta[12]	0.135	0.050
theta[4]	0.000	0.000
theta[44]	0.001	0.002
total	0.059	0.032

Mean absolute deviation comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat

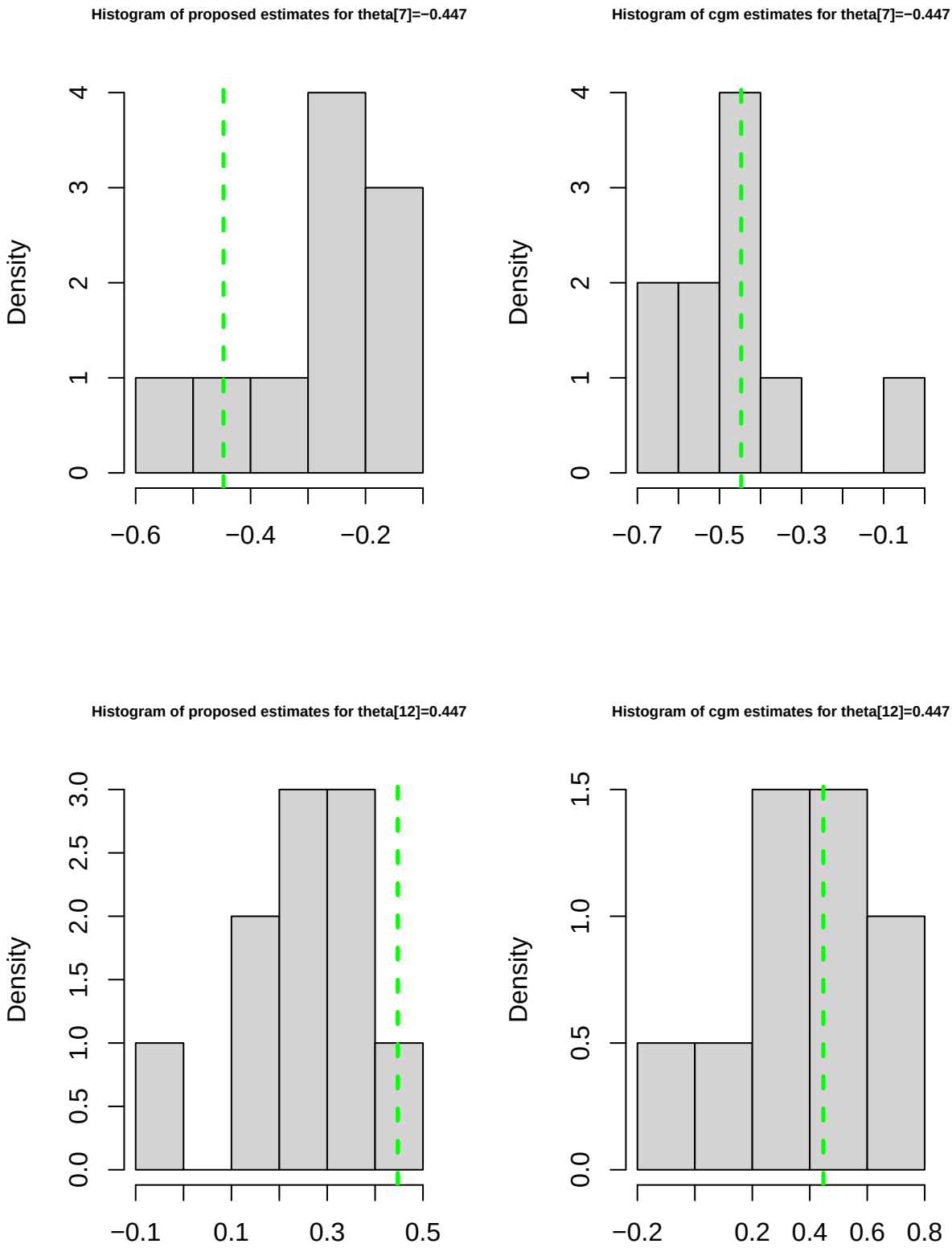
Table 3: Mean Absolute Deviation of Parameter Estimates

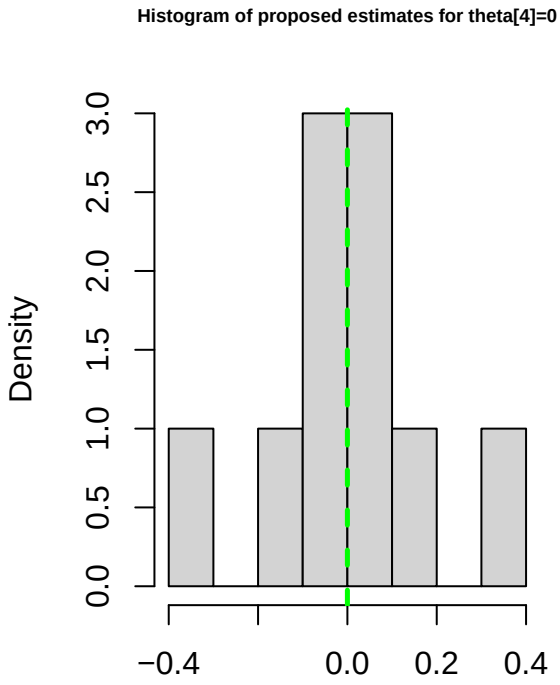
	proposed	cgm
theta[7]	0.187	0.126
theta[12]	0.199	0.211
theta[4]	0.129	0.122
theta[44]	0.157	0.120
total	0.168	0.145

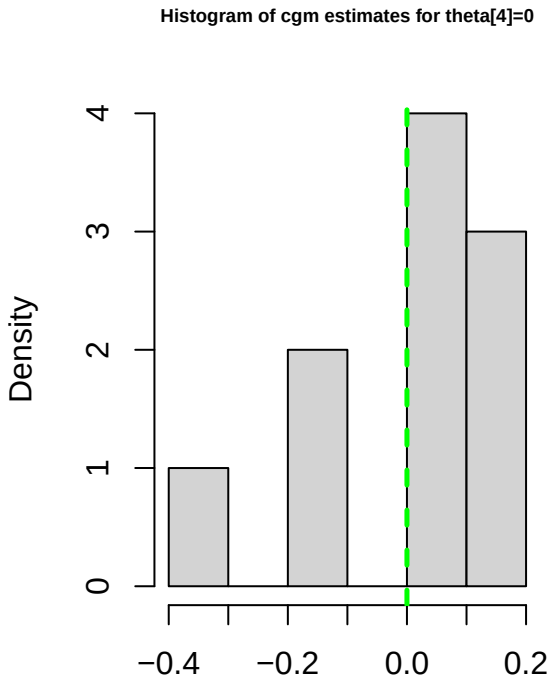
Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

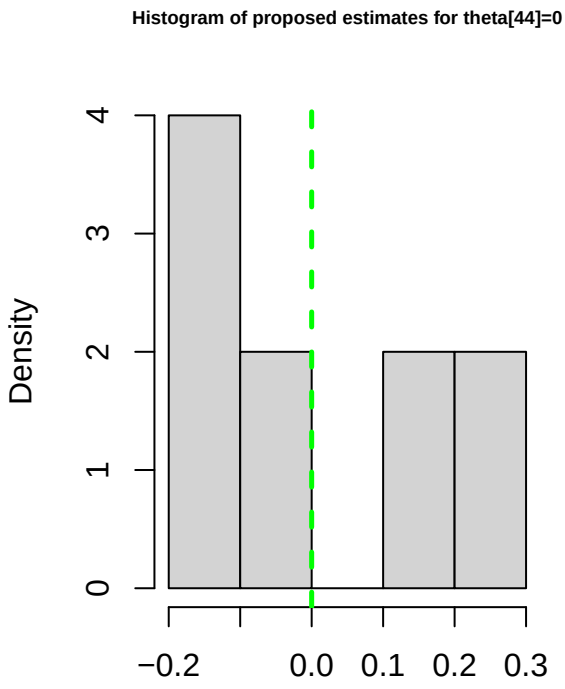
	proposed	cgm
theta[7]	0.296	0.224
theta[12]	0.328	0.172
theta[4]	0.000	0.013
theta[44]	0.010	0.014
total	0.159	0.106

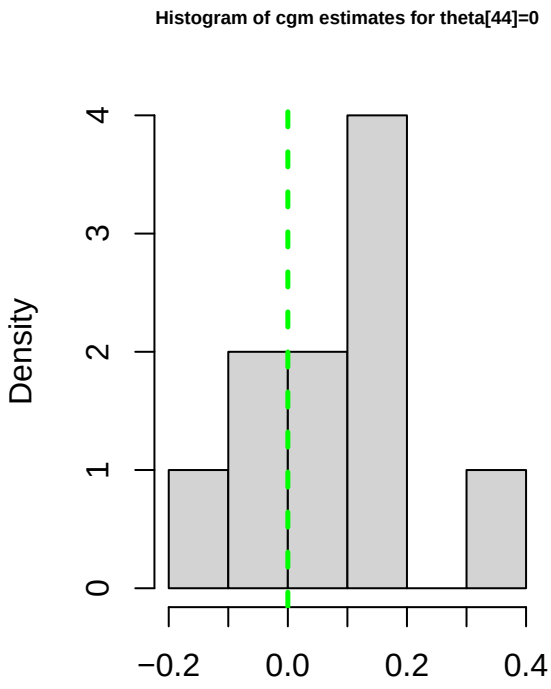
Boxplots


Histogram of proposed estimates for theta[7]=−0.447
Density
0
1
2
3
4
−0.6
−0.4
−0.2
Histogram of cgm estimates for theta[7]=−0.447
Density
0
1
2
3
4
−0.7
−0.5
−0.3
−0.1
Histogram of proposed estimates for theta[12]=0.447
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.1
0.1
0.3
0.5
Histogram of cgm estimates for theta[12]=0.447
Density
0.0
0.5
1.0
1.5
−0.2
0.2
0.4
0.6
0.8

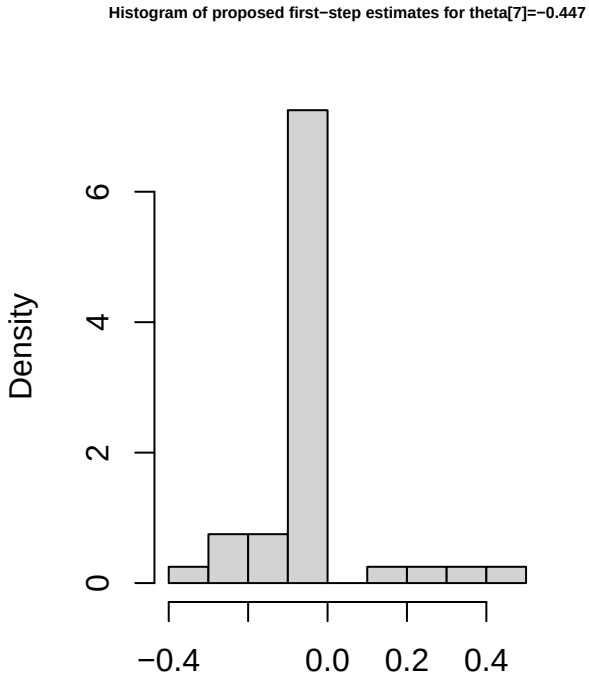

Histogram of proposed estimates for theta[4]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.4
0.0
0.2
0.4

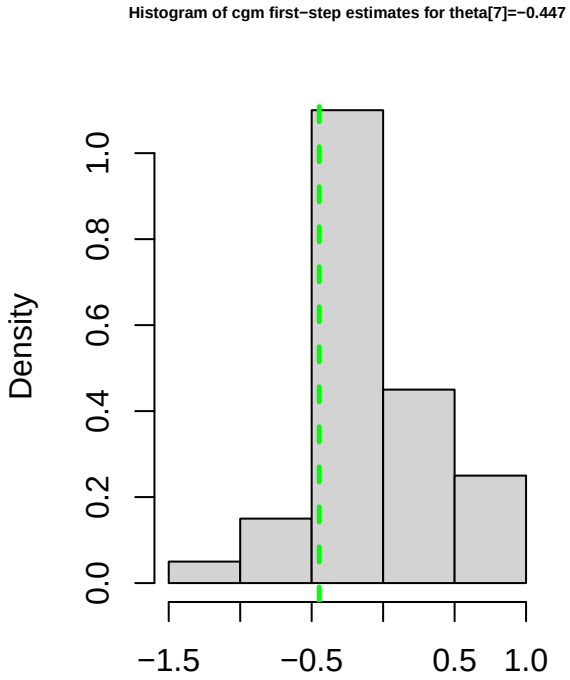

Histogram of cgm estimates for theta[4]=0
Density
0
1
2
3
4
−0.4
−0.2
0.0
0.2

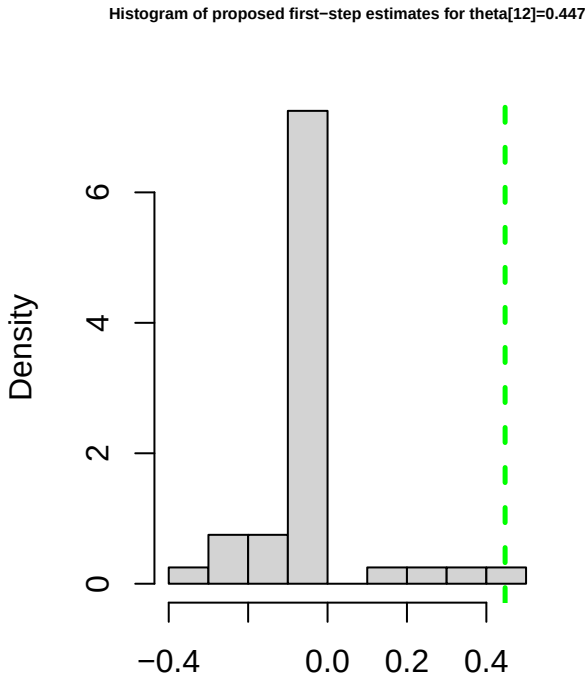

Histogram of proposed estimates for theta[44]=0
Density
0
1
2
3
4
−0.2
0.0
0.1
0.2
0.3

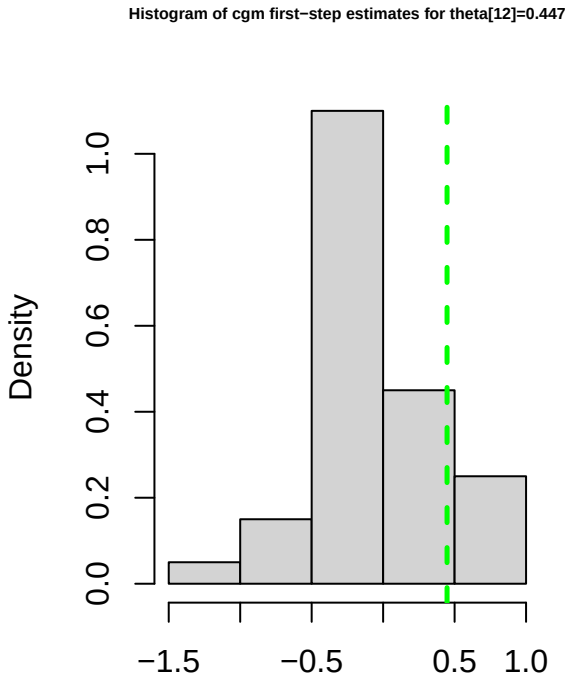

Histogram of cgm estimates for theta[44]=0
Density
0
1
2
3
4
−0.2
0.0
0.2
0.4

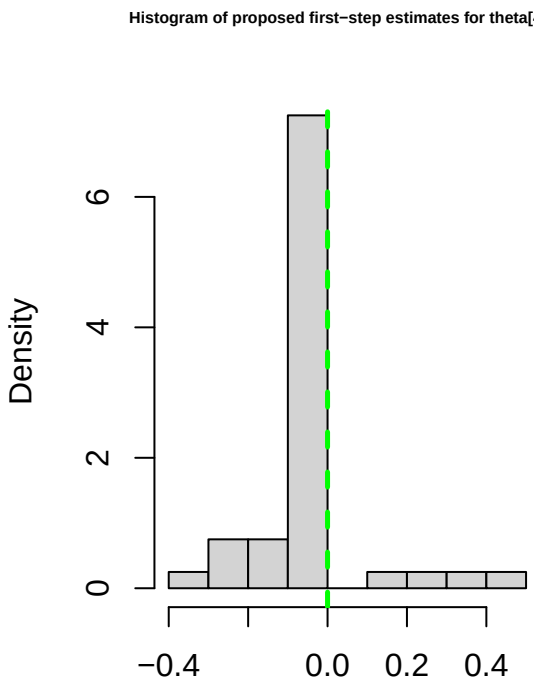
First Step Histograms

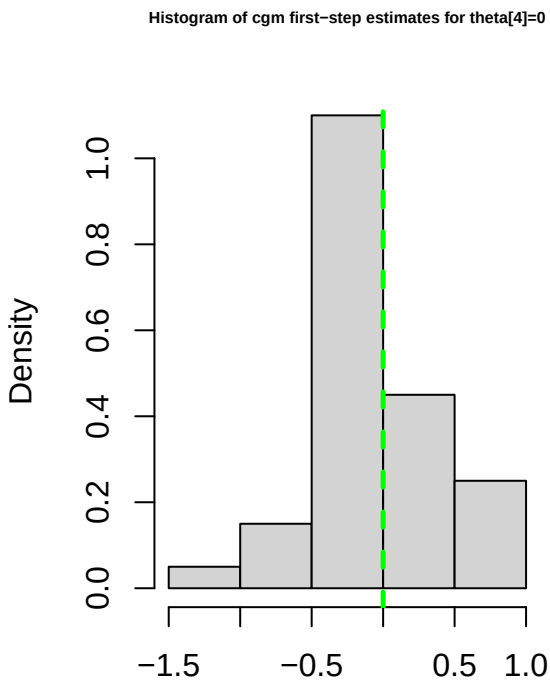


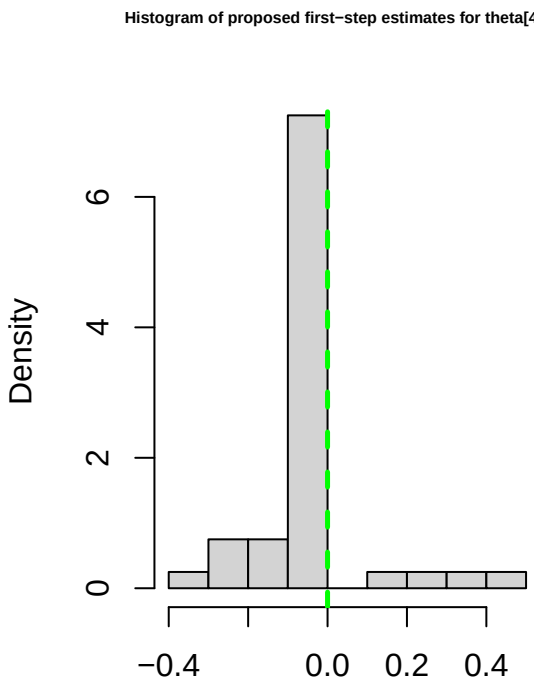


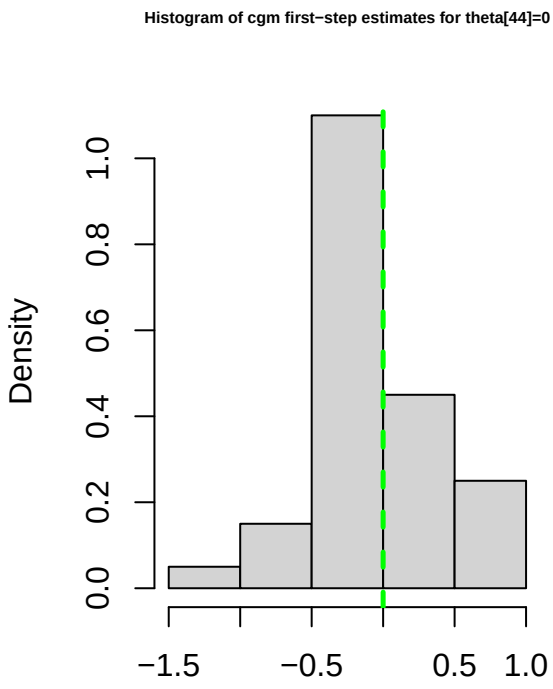





Histogram of proposed first−step estimates for theta[4]=0
Density
0
2
4
6
−0.4
0.0
0.2
0.4


Histogram of cgm first−step estimates for theta[4]=0
Density
0.0
0.2
0.4
0.6
0.8
1.0
−1.5
−0.5
0.5
1.0


Histogram of proposed first−step estimates for theta[44]=0
Density
0
2
4
6
−0.4
0.0
0.2
0.4


Histogram of cgm first−step estimates for theta[44]=0
Density
0.0
0.2
0.4
0.6
0.8
1.0
−1.5
−0.5
0.5
1.0

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 5: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[7]	-0.543	-0.259	-0.128	-0.515	-0.142
theta[12]	-0.036	0.259	0.432	0.002	0.419
theta[4]	-0.361	-0.007	0.310	-0.307	0.266
theta[44]	-0.186	-0.048	0.296	-0.184	0.283

Table 6: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[7]	-0.691	-0.466	-0.003	-0.682	-0.080
theta[12]	-0.181	0.372	0.702	-0.110	0.701
theta[4]	-0.343	0.043	0.181	-0.300	0.174
theta[44]	-0.104	0.097	0.310	-0.091	0.282

Statistics for Theoretical 95% Confidence Intervals

Table 7: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[7]	-0.280	0.133	-0.540	-0.019	0.8
theta[12]	0.248	0.122	0.009	0.488	0.6
theta[4]	-0.015	0.133	-0.275	0.245	0.8
theta[44]	0.017	0.125	-0.229	0.262	0.9

Table 8: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[7]	-0.450	0.136	-0.716	-0.185	0.9
theta[12]	0.366	0.133	0.105	0.627	0.6
theta[4]	0.002	0.120	-0.233	0.236	0.9
theta[44]	0.085	0.125	-0.159	0.330	0.9