Simulation Results

2026-01-21

Simulation Setup

This simulation is performed with $n = 200$ and $d = 400$, using the 2-d lattice as the underlying graph. $s = 5$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.4$. The attached results are for a 10-replication simulation. The parameter vector θ has sparse components other than the following:

Parameter.Index	Value
28	-0.447
237	-0.447
250	0.447
282	-0.447
399	-0.447

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{$ 28, 237, 328, 168$\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[28]	0.041	0.068
theta[237]	0.081	0.083
theta[328]	0.030	0.016
theta[168]	0.015	0.051
total	0.042	0.054

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[28]	0.184	0.047

	proposed	cgm
theta[237]	0.159	0.068
theta[328]	0.000	0.000
theta[168]	0.000	0.003
total	0.086	0.029

Mean absolute deviation comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat

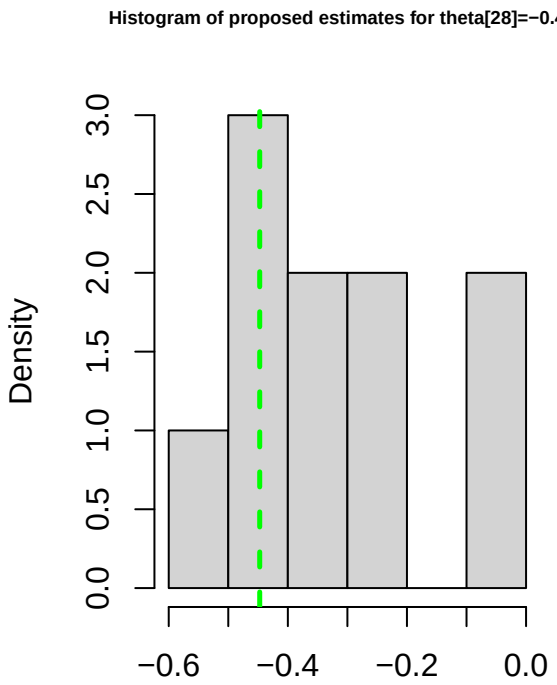
Table 3: Mean Absolute Deviation of Parameter Estimates

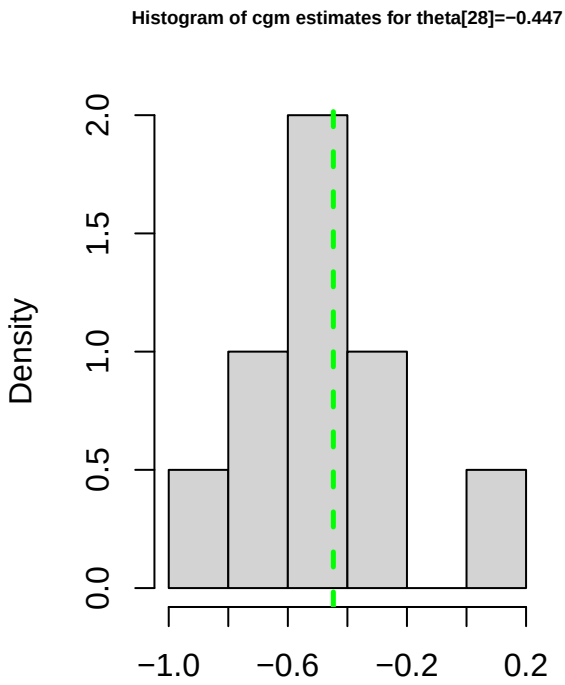
	proposed	cgm
theta[28]	0.157	0.209
theta[237]	0.257	0.233
theta[328]	0.147	0.110
theta[168]	0.098	0.202
total	0.165	0.188

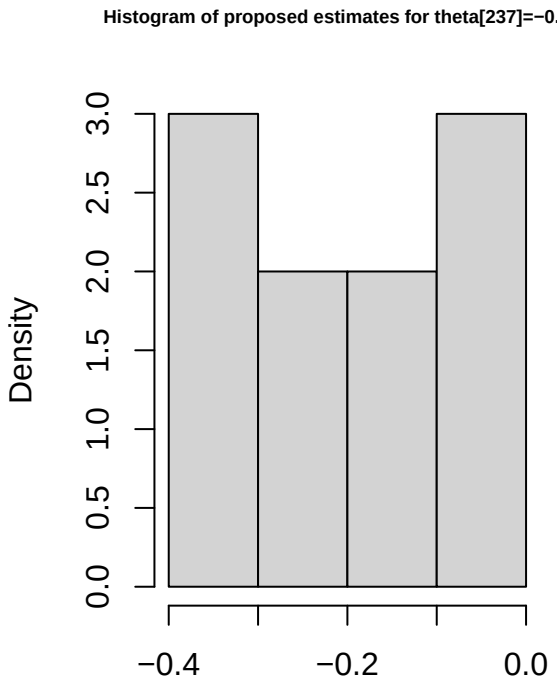
Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

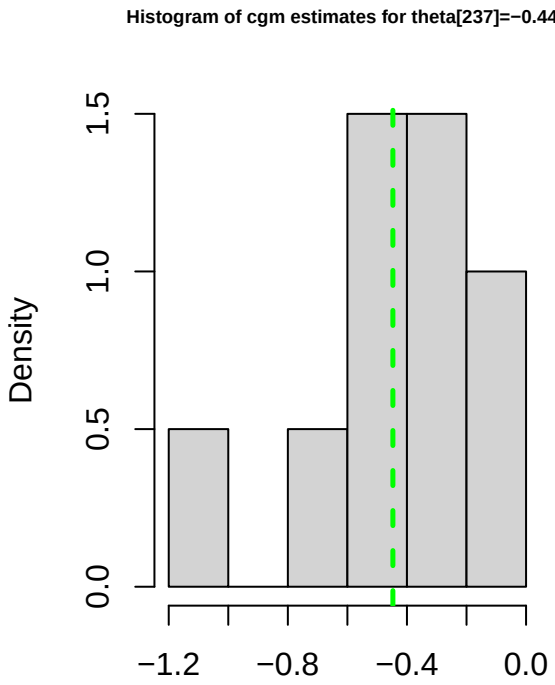
	proposed	cgm
theta[28]	0.422	0.187
theta[237]	0.389	0.241
theta[328]	0.000	0.000
theta[168]	0.000	0.022
total	0.203	0.112

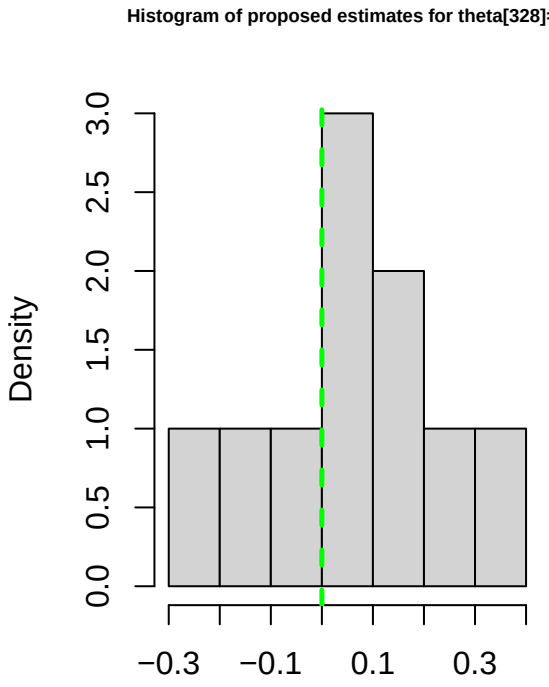
Boxplots

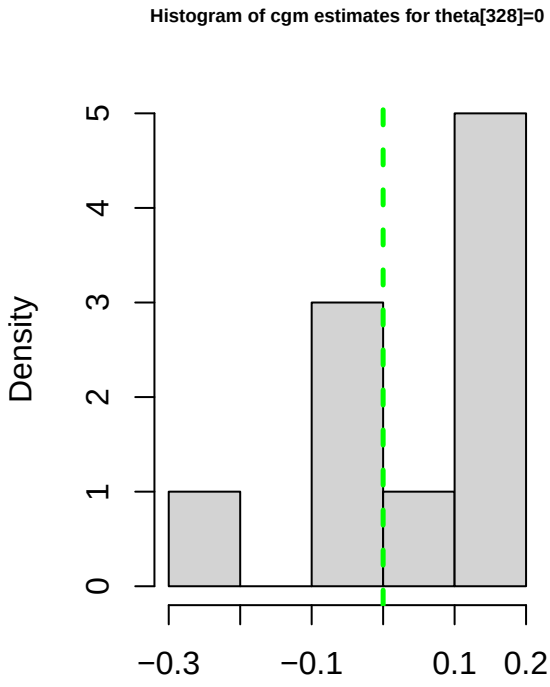

Histogram of proposed estimates for theta[28]=−0.447
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.6
−0.4
−0.2
0.0

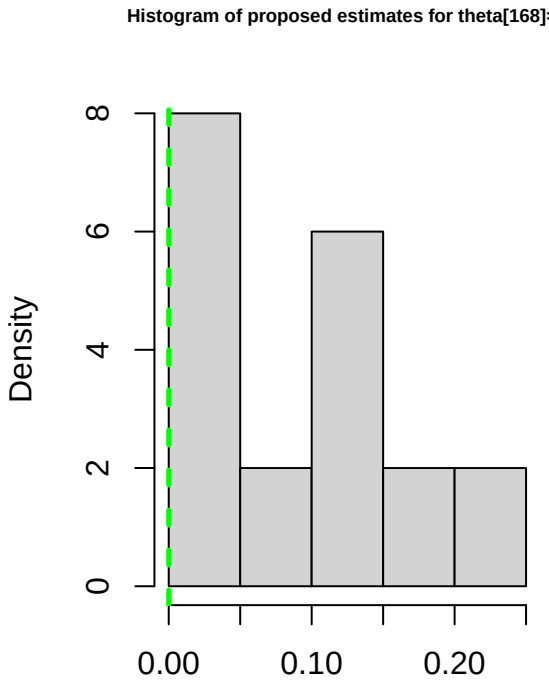

Histogram of cgm estimates for theta[28]=−0.447
Density
0.0
0.5
1.0
1.5
2.0
−1.0
−0.6
−0.2
0.2

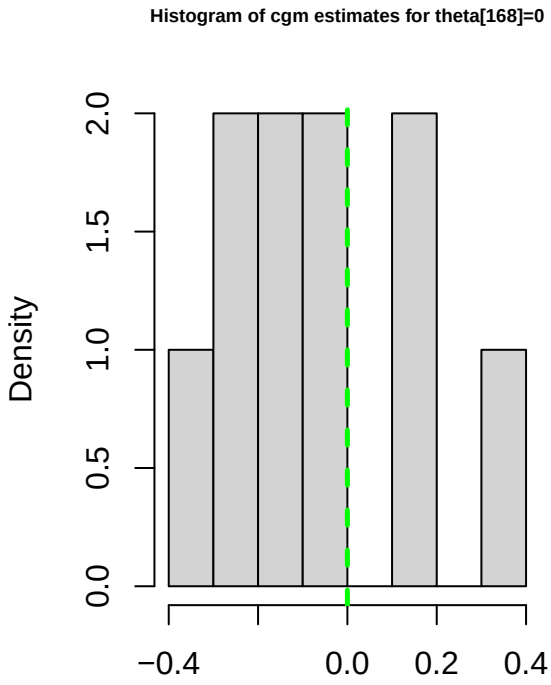

Histogram of proposed estimates for theta[237]=−0.447
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.4
−0.2
0.0


Histogram of cgm estimates for theta[237]=−0.447
Density
0.0
0.5
1.0
1.5
−1.2
−0.8
−0.4
0.0


Histogram of proposed estimates for theta[328]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.3
−0.1
0.1
0.3


Histogram of cgm estimates for theta[328]=0
Density
0
1
2
3
4
5
−0.3
−0.1
0.1
0.2


Histogram of proposed estimates for theta[168]=0
Density
0
2
4
6
8
0.00
0.10
0.20


Histogram of cgm estimates for theta[168]=0
Density
0.0
0.5
1.0
1.5
2.0
−0.4
0.0
0.2
0.4

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[28]	-0.308	0.115	-0.533	-0.082	0.7
theta[237]	-0.191	0.126	-0.437	0.056	0.4
theta[328]	0.065	0.111	-0.152	0.282	0.7
theta[168]	0.098	0.113	-0.124	0.319	1.0

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[28]	-0.459	0.132	-0.718	-0.200	0.7
theta[237]	-0.454	0.132	-0.713	-0.196	0.7
theta[328]	0.030	0.121	-0.207	0.266	1.0
theta[168]	-0.078	0.119	-0.312	0.156	0.6