Simulation Results

2026-01-07

Simulation Setup

This simulation is performed with $n = 400$ and $d = 10$, using the 2-d lattice as the underlying graph. $s = 2$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0$. The attached results are for a 10-replication simulation. The true values of the parameter vector θ are

```
0 0 0 0 -0.7071068 0 0 0 0 -0.7071068 ,
```

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{$ 5, 10, 1, 4$\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison ($\frac{1}{n.sim}\sum_{i=1}^{n.sim}\frac{1}{|\mathcal{C}|}\|\hat{\theta}_{i,\mathcal{C}} - \theta_{\mathcal{C}}\|^2$)

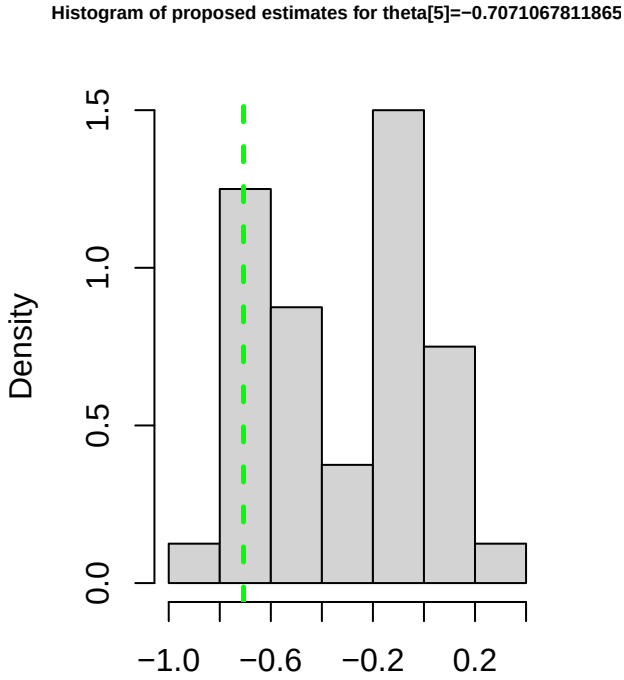
Table 1: Mean-Squared Error of Parameter Estimates

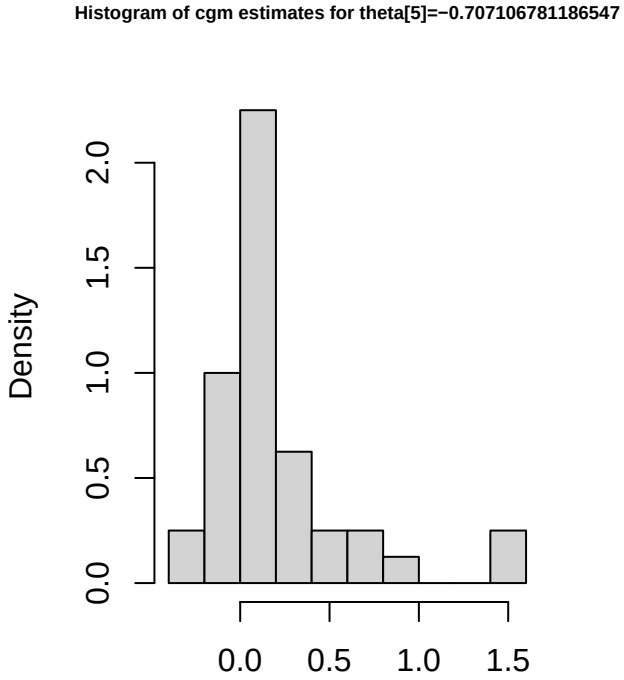
	proposed	cgm
theta[5]	0.028	1.424
theta[10]	0.024	1.381
theta[1]	0.011	0.018
theta[4]	0.023	0.019
total	0.022	0.710

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[5]	0.012	0.011
theta[10]	0.052	0.014
theta[1]	0.011	0.009
theta[4]	0.011	0.007
total	0.022	0.010

Boxplots


Histogram of proposed estimates for theta[5]=−0.7071067811865
Density
0.0
0.5
1.0
1.5
−1.0
−0.6
−0.2
0.2


Histogram of cgm estimates for theta[5]=−0.707106781186547
Density
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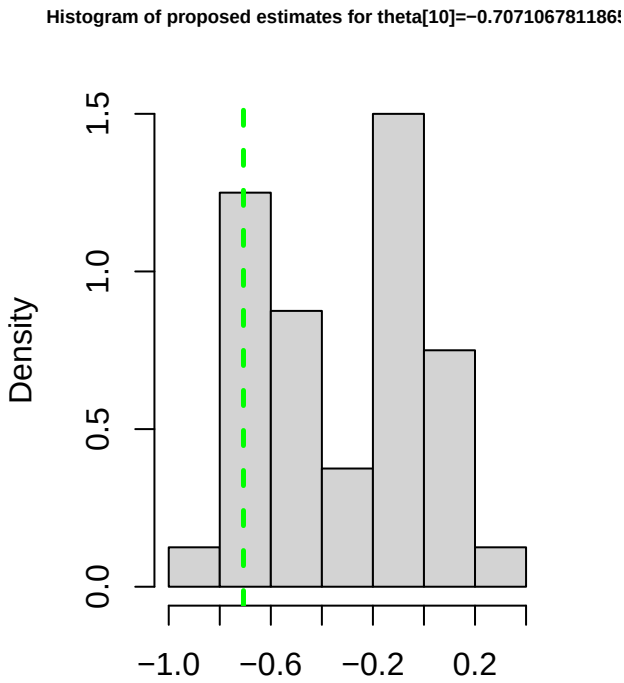
1.0
1.5
2.0
0.0
0.5
1.0
1.5


Histogram of proposed estimates for theta[10]=−0.707106781186
Density
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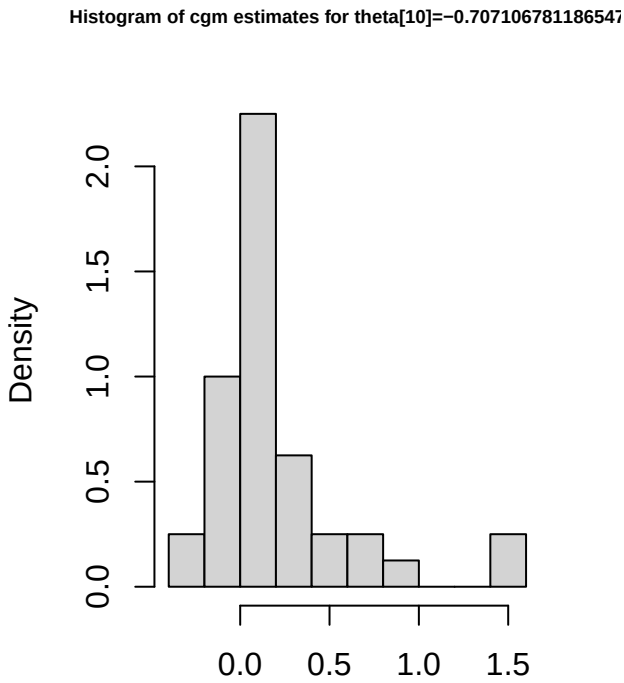
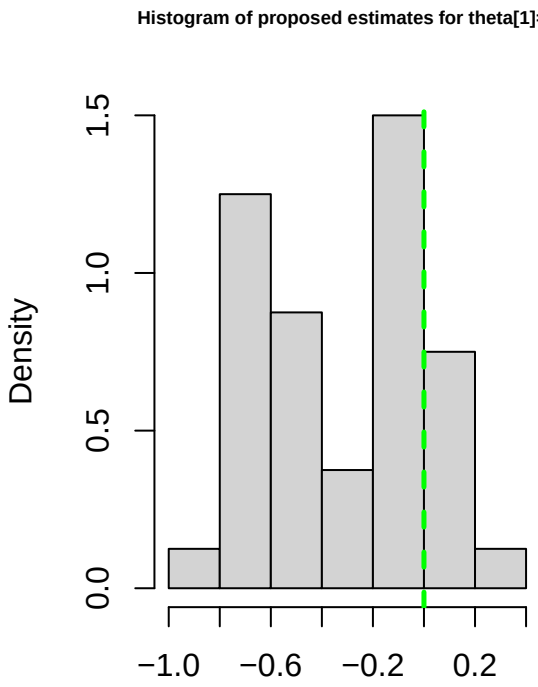
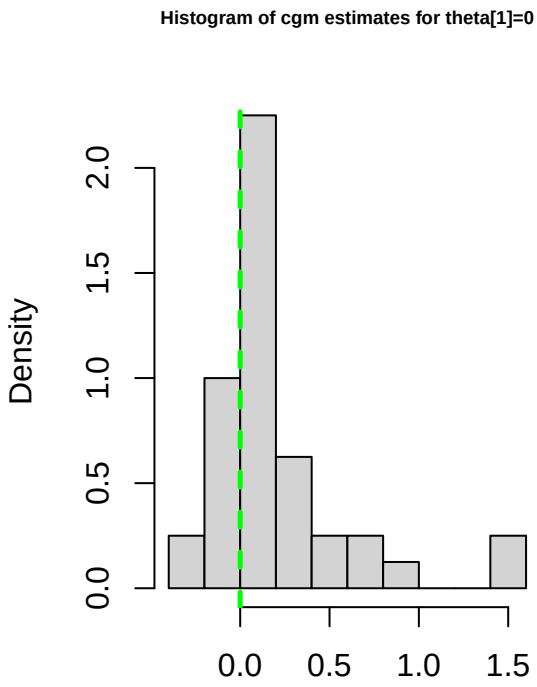
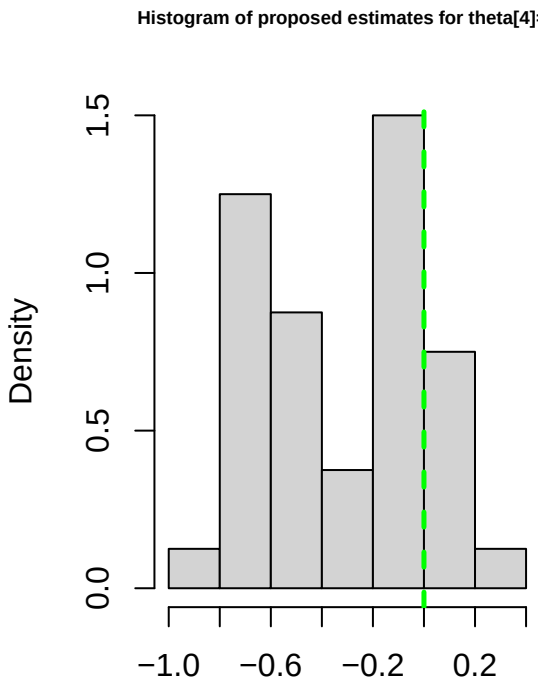
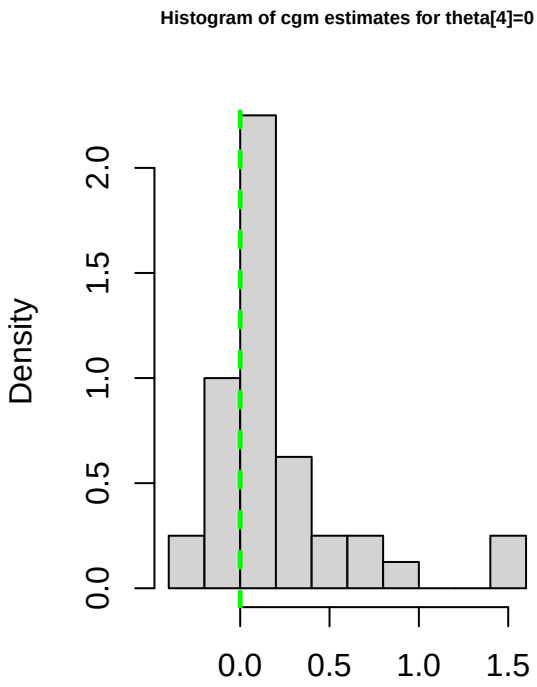
1.0
1.5
−1.0
−0.6
−0.2
0.2


Histogram of cgm estimates for theta[10]=−0.707106781186547
Density
0.0
0.5
1.0
1.5
2.0
0.0
0.5
1.0
1.5

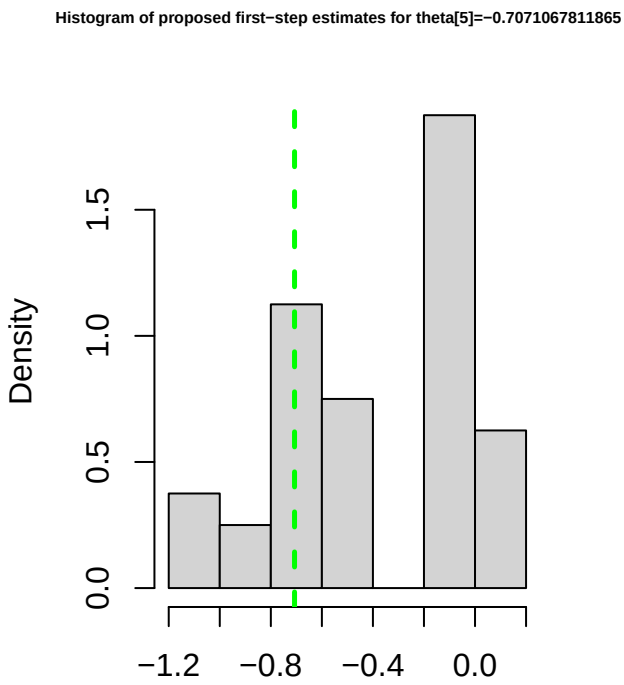

Histogram of proposed estimates for theta[1]=0
Density
0.0
0.5
1.0
1.5
−1.0
−0.6
−0.2
0.2

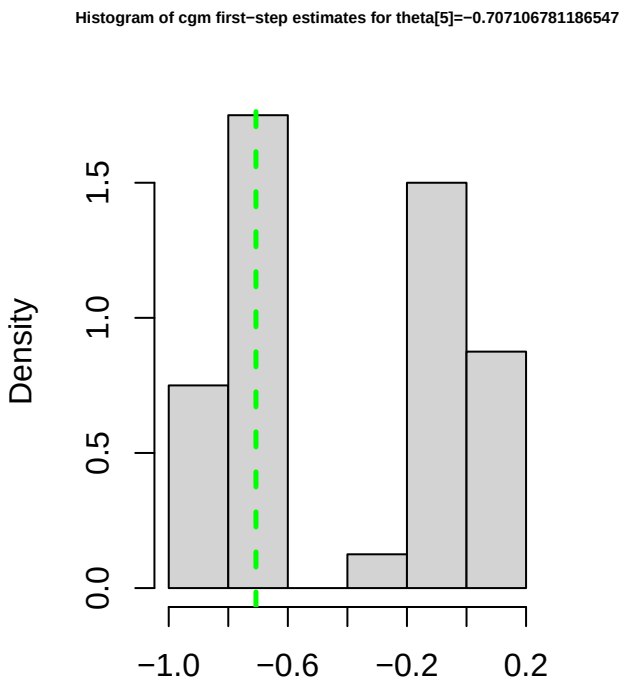

Histogram of cgm estimates for theta[1]=0
Density
0.0
0.5
1.0
1.5
2.0
0.0
0.5
1.0
1.5

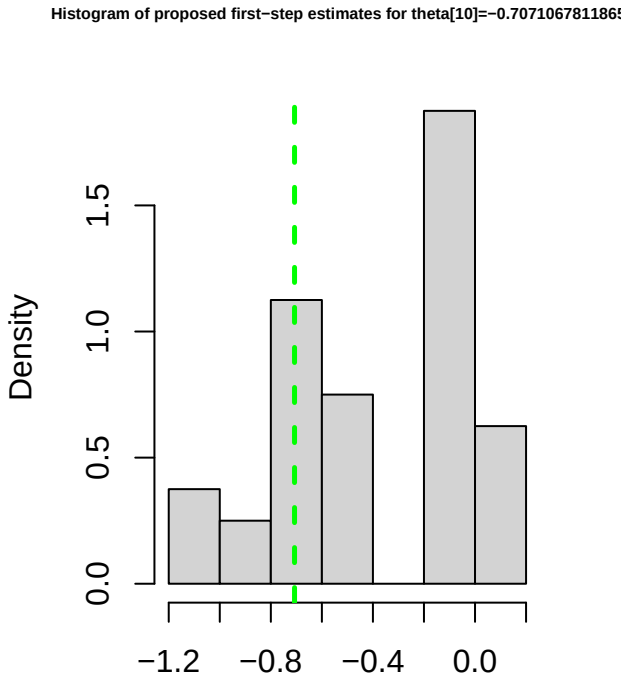

Histogram of proposed estimates for theta[4]=0
Density
0.0
0.5
1.0
1.5
−1.0
−0.6
−0.2
0.2

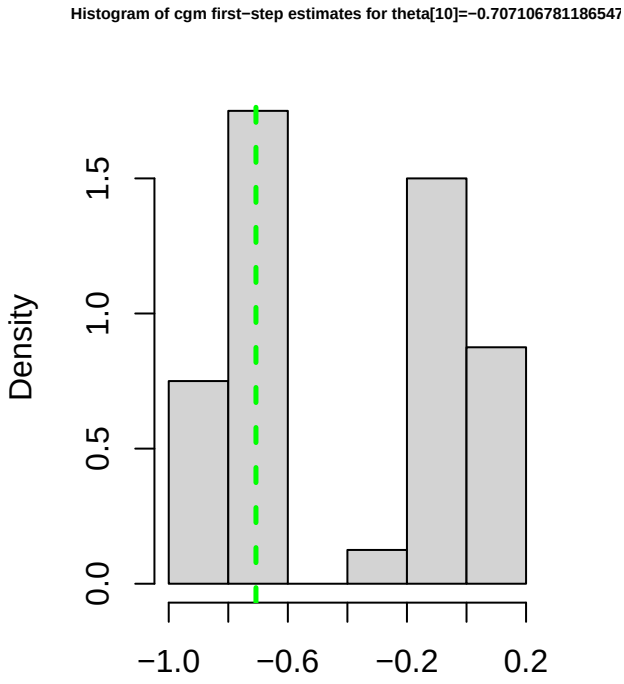

Histogram of cgm estimates for theta[4]=0
Density
0.0
0.5
1.0
1.5
2.0
0.0
0.5
1.0
1.5

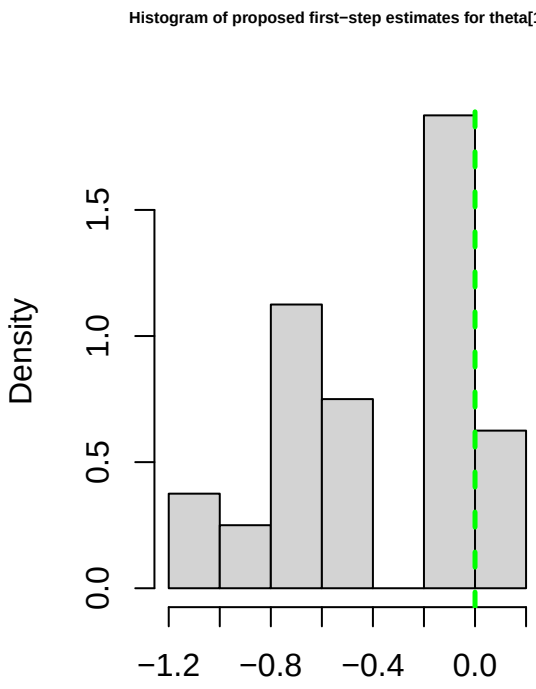
First Step Histograms

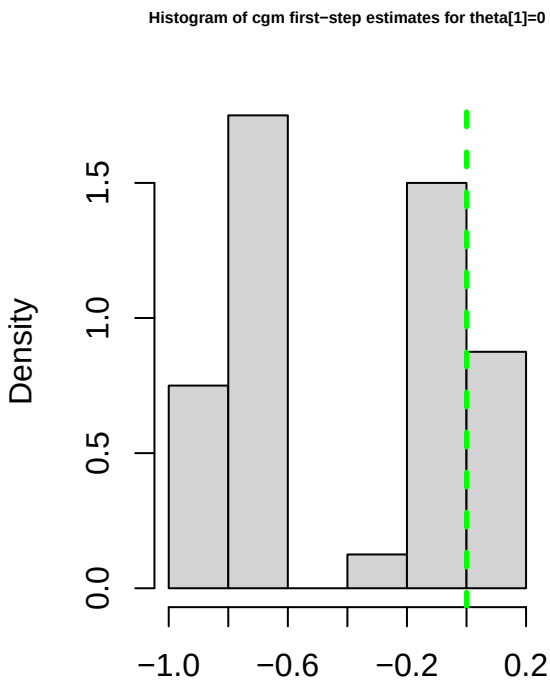


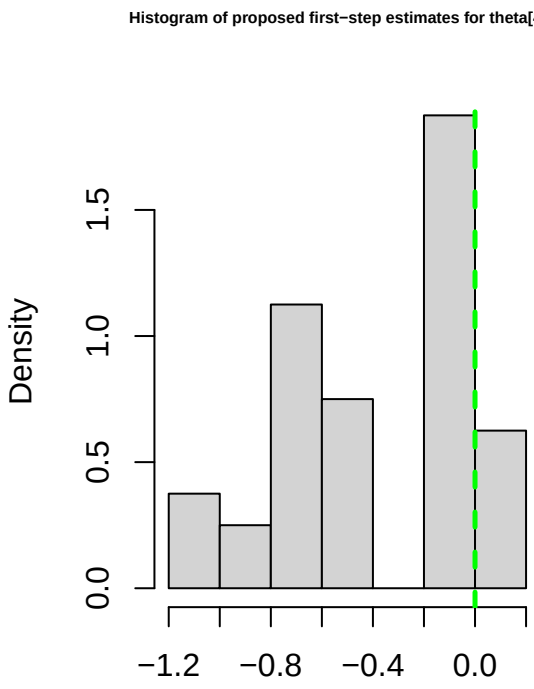


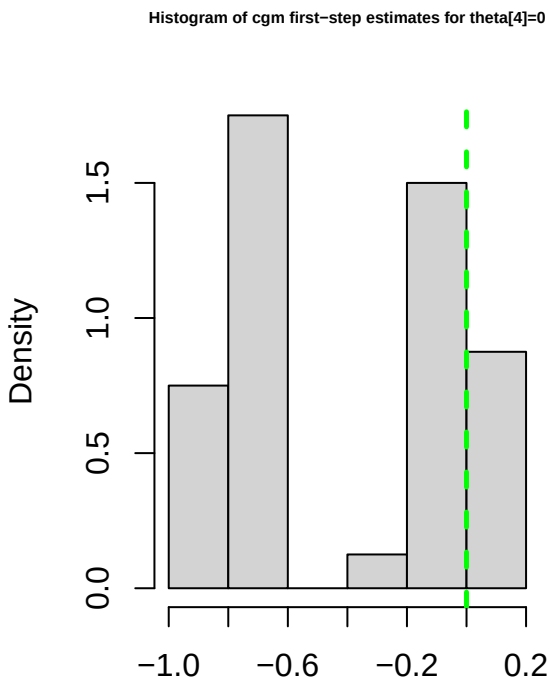





Histogram of proposed first−step estimates for theta[1]=0
Density
0.0
0.5
1.0
1.5
−1.2
−0.8
−0.4
0.0


Histogram of cgm first−step estimates for theta[1]=0
Density
0.0
0.5
1.0
1.5
−1.0
−0.6
−0.2
0.2


Histogram of proposed first−step estimates for theta[4]=0
Density
0.0
0.5
1.0
1.5
−1.2
−0.8
−0.4
0.0


Histogram of cgm first−step estimates for theta[4]=0
Density
0.0
0.5
1.0
1.5
−1.0
−0.6
−0.2
0.2

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 3: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[5]	-0.948	-0.640	-0.388	-0.897	-0.403
theta[10]	-0.757	-0.560	-0.368	-0.755	-0.406
theta[1]	-0.165	-0.011	0.171	-0.149	0.165
theta[4]	-0.301	-0.074	0.204	-0.266	0.192

Table 4: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[5]	0.017	0.333	1.448	0.023	1.269
theta[10]	0.000	0.252	1.418	0.002	1.284
theta[1]	-0.253	-0.062	0.105	-0.242	0.101
theta[4]	-0.062	0.055	0.362	-0.060	0.313

Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[5]	-0.626	0.171	-0.961	-0.290	0.9
theta[10]	-0.610	0.176	-0.955	-0.264	1.0
theta[1]	0.005	0.130	-0.249	0.258	1.0
theta[4]	-0.044	0.139	-0.317	0.228	0.9

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[5]	0.413	0.130	0.159	0.667	0.0
theta[10]	0.394	0.131	0.137	0.650	0.0
theta[1]	-0.072	0.063	-0.195	0.052	0.6
theta[4]	0.071	0.065	-0.056	0.198	0.7