Simulation Results

2026-01-07

Simulation Setup

This simulation is performed with $n = 400$ and $d = 10$, using the 2-d lattice as the underlying graph. $s = 2$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.2$. The attached results are for a 10-replication simulation. The true values of the parameter vector θ are

```
0.7071068 0 0 0 0 0 -0.7071068 0 0 0 ,
```

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{\, 1, 7, 2, 5\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison $(\frac{1}{n.sim} \sum_{i=1}^{n.sim} \frac{1}{|\mathcal{C}|} \|\hat{\theta}_{i,\mathcal{C}} - \theta_{\mathcal{C}}\|^2)$

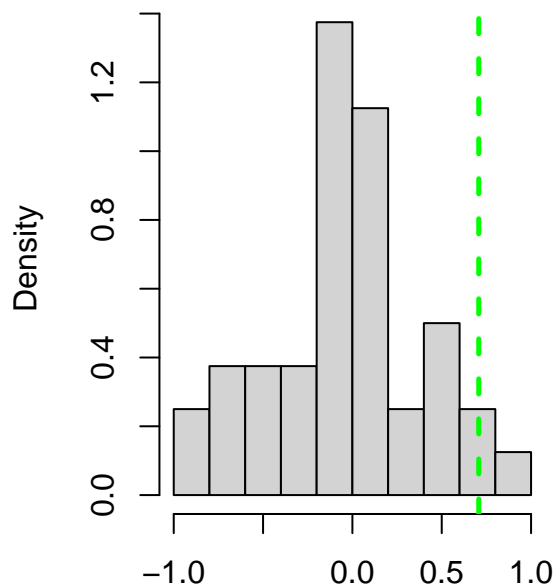Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[1]	0.079	4.895
theta[7]	0.101	5.176
theta[2]	0.011	0.116
theta[5]	0.019	0.157
total	0.052	2.586

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[1]	0.051	0.031
theta[7]	0.036	0.042
theta[2]	0.003	0.011
theta[5]	0.007	0.008
total	0.024	0.023

Boxplots

Histogram of proposed estimates for theta[1]=0.70710678118654

Density

Histogram of cgm estimates for theta[1]=0.707106781186547

Density

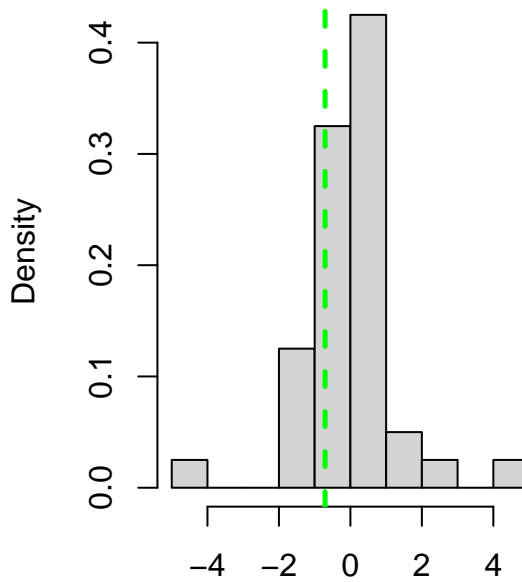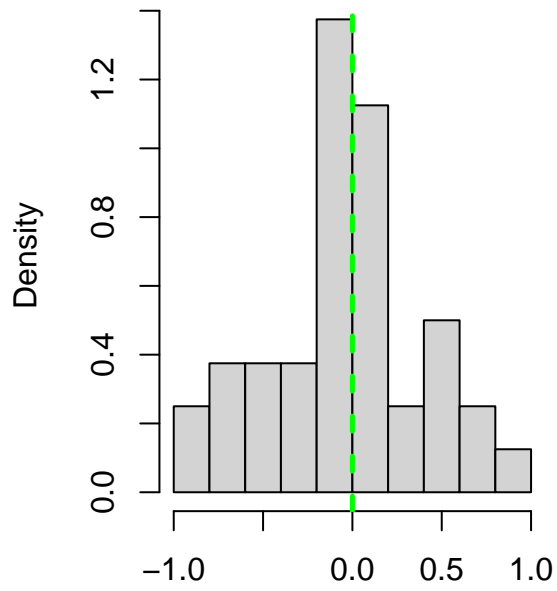**Histogram of proposed estimates for theta[7]=−0.7071067811865**
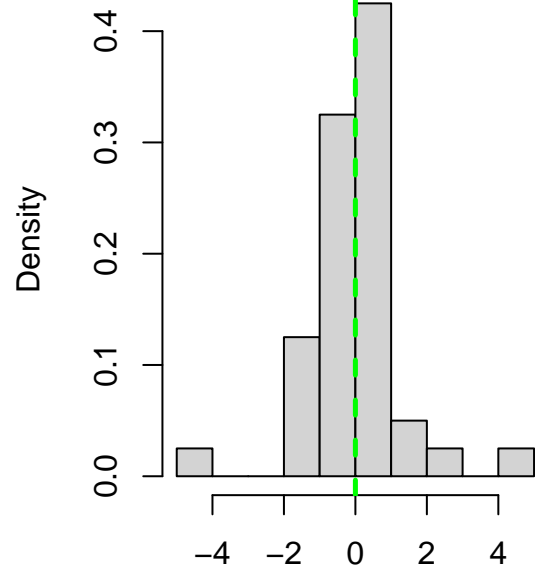

Density

Histogram of cgm estimates for theta[7]=−0.707106781186547

Density

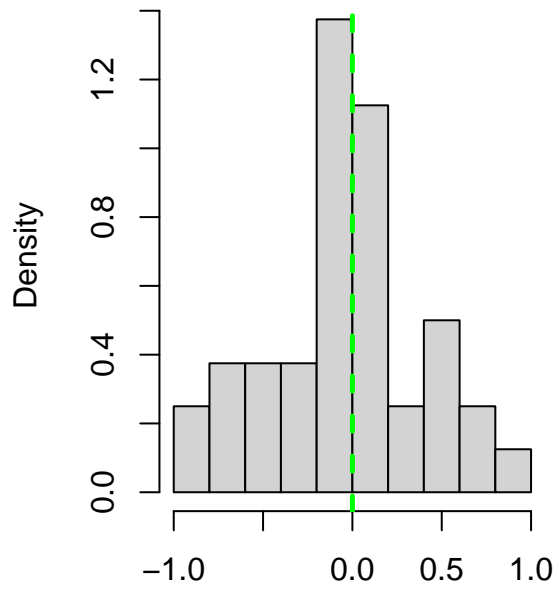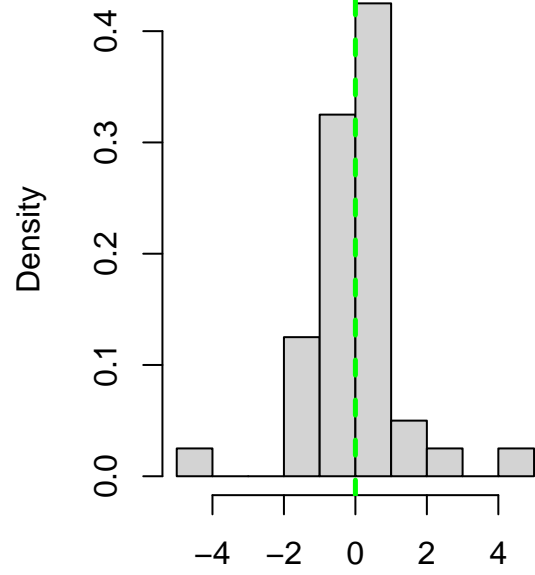**Histogram of proposed estimates for theta[2]=0**

Histogram of cgm estimates for theta[2]=0

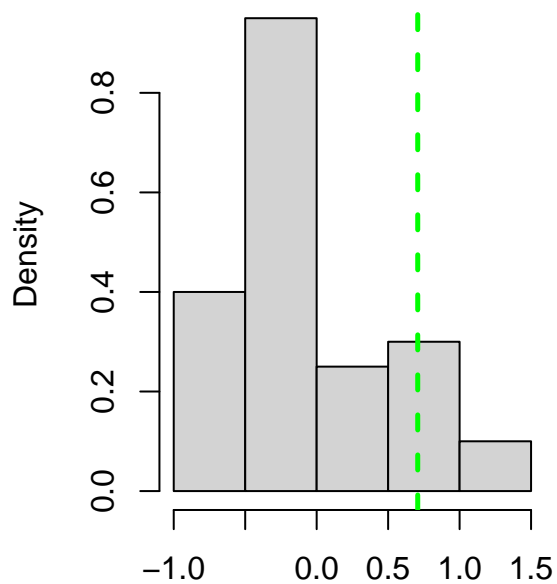**Histogram of proposed estimates for theta[5]=0**

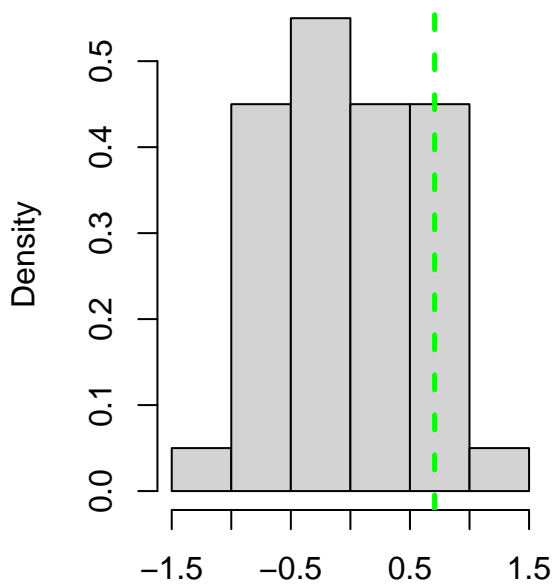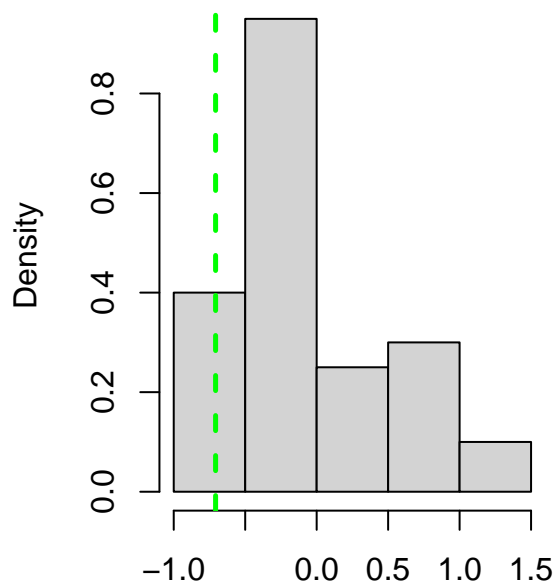**Histogram of cgm estimates for theta[5]=0**

First Step Histograms

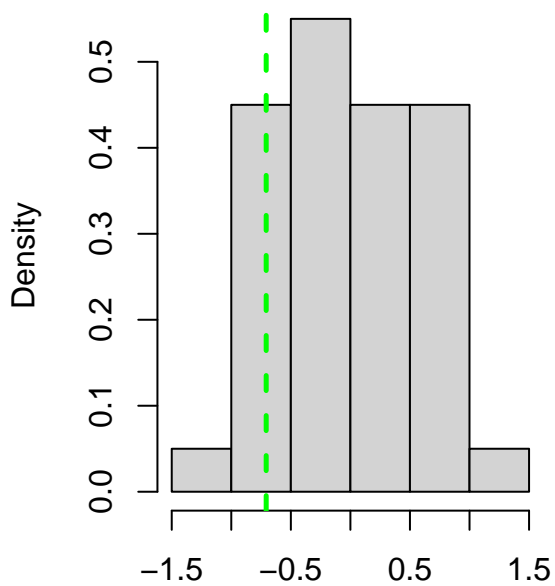### Histogram of proposed first-step estimates for theta[1]=0.70710678118654

Histogram of cgm first-step estimates for theta[1]=0.707106781186547

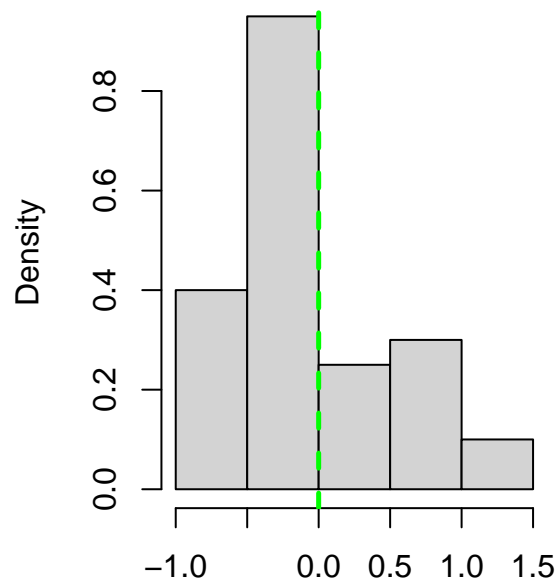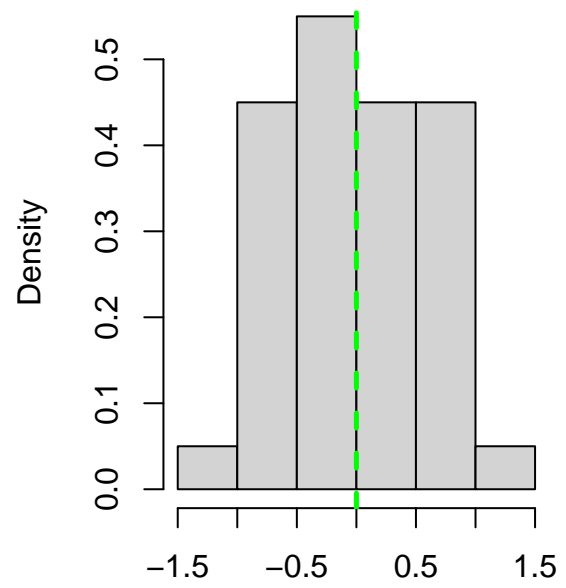### Histogram of proposed first-step estimates for theta[7]=-0.7071067811865

Histogram of cgm first-step estimates for theta[7]=-0.707106781186547
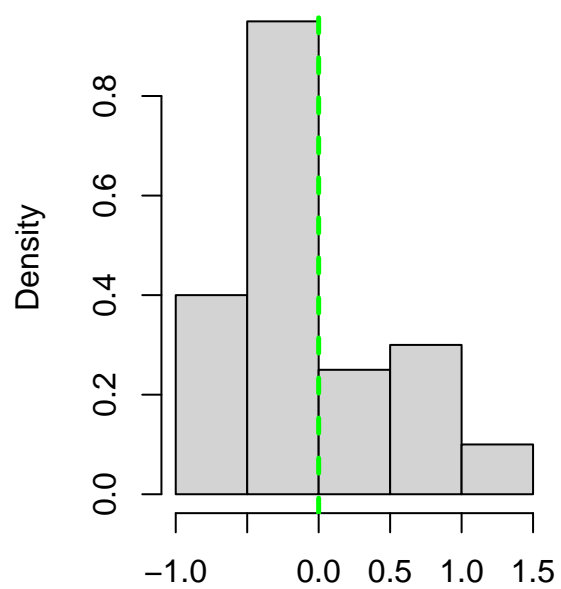
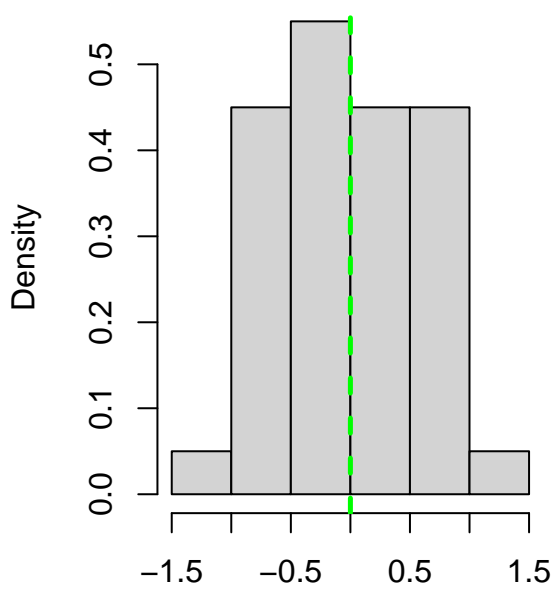**Histogram of proposed first−step estimates for theta[2]=0**

Histogram of cgm first−step estimates for theta[2]=0

Histogram of proposed first−step estimates for theta[5]=0

Histogram of cgm first−step estimates for theta[5]=0

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 3: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[1]	0.161	0.542	0.947	0.198	0.909
theta[7]	-0.886	-0.551	0.070	-0.881	-0.019
theta[2]	-0.209	-0.051	0.117	-0.199	0.111
theta[5]	-0.308	-0.032	0.169	-0.276	0.159

Table 4: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[1]	-4.343	-0.927	0.129	-3.743	0.100
theta[7]	-0.133	0.846	4.442	-0.084	3.921
theta[2]	-0.439	0.065	0.893	-0.407	0.725
theta[5]	-1.135	-0.008	0.378	-0.953	0.311

Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[1]	0.543	0.175	0.201	0.886	0.9
theta[7]	-0.549	0.172	-0.886	-0.211	0.8
theta[2]	-0.038	0.132	-0.297	0.221	1.0
theta[5]	-0.023	0.130	-0.277	0.231	0.9

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[1]	-1.136	0.141	-1.413	-0.860	0.0
theta[7]	1.169	0.143	0.888	1.450	0.0
theta[2]	0.053	0.063	-0.072	0.177	0.4
theta[5]	-0.108	0.063	-0.232	0.016	0.6