Simulation Results

2026-01-09

Simulation Setup

This simulation is performed with $n = 200$ and $d = 10$, using the 2-d lattice as the underlying graph. $s = 2$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.4$. The attached results are for a 10-replication simulation. The true values of the parameter vector θ are

```
0 0 0 0 0.7071068 0 0 0 0 -0.7071068 ,
```

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{ 5, 10, 1, 4\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison ($\frac{1}{n.sim}\sum_{i=1}^{n.sim}\frac{1}{|\mathcal{C}|}\|\hat{\theta}_{i,\mathcal{C}} - \theta_{\mathcal{C}}\|^2$)

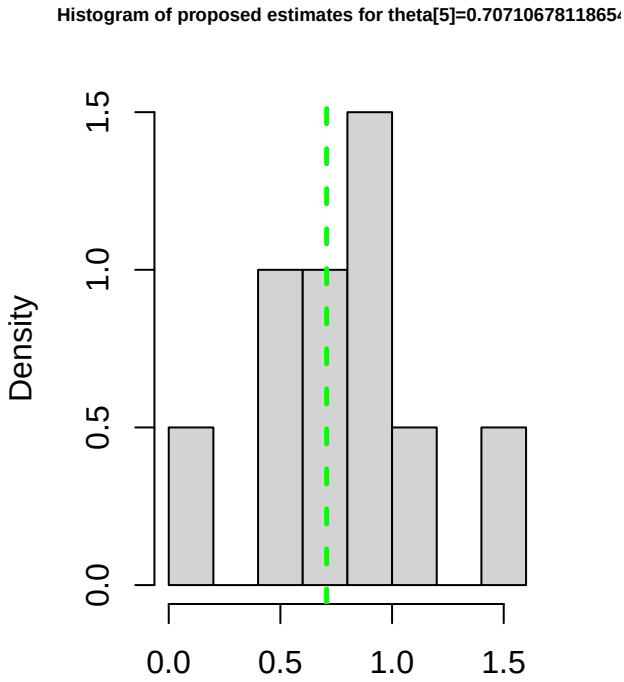
Table 1: Mean-Squared Error of Parameter Estimates

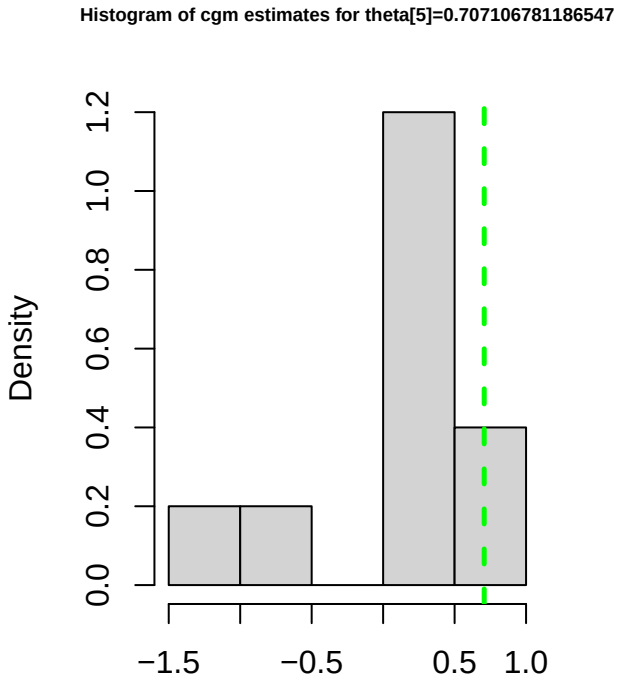
	proposed	cgm
theta[5]	0.119	0.593
theta[10]	0.144	0.256
theta[1]	0.070	0.046
theta[4]	0.051	0.020
total	0.096	0.228

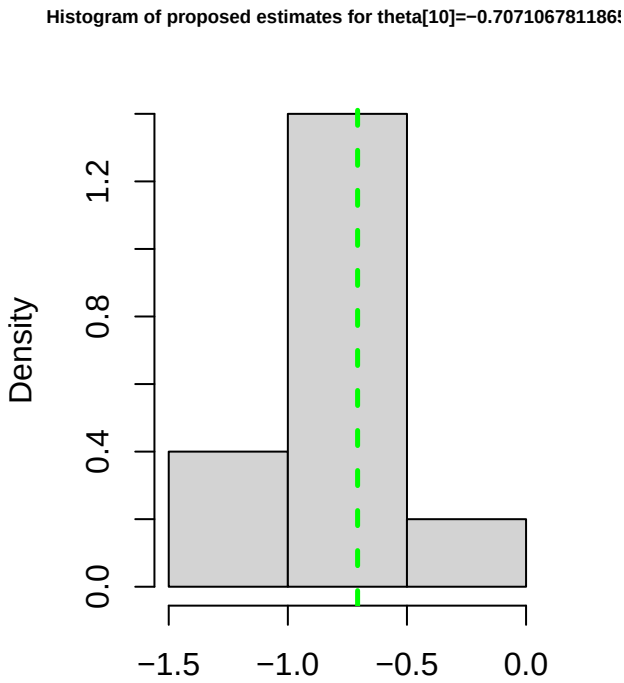
Table 2: Mean-Squared Error of First-Step Parameter Estimates

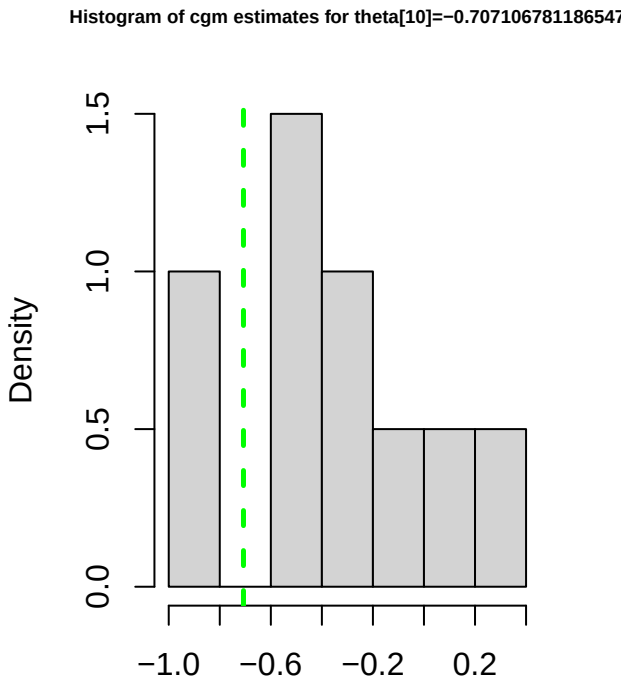
	proposed	cgm
theta[5]	0.152	0.171
theta[10]	0.214	0.109
theta[1]	0.038	0.008
theta[4]	0.026	0.003
total	0.107	0.073

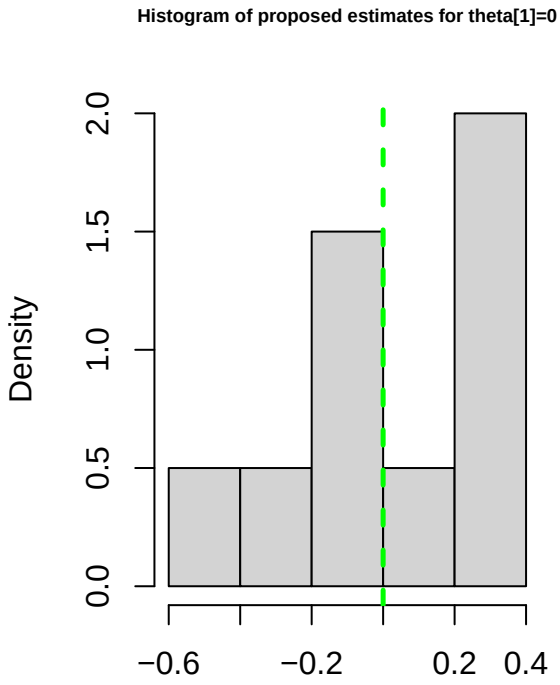
Boxplots

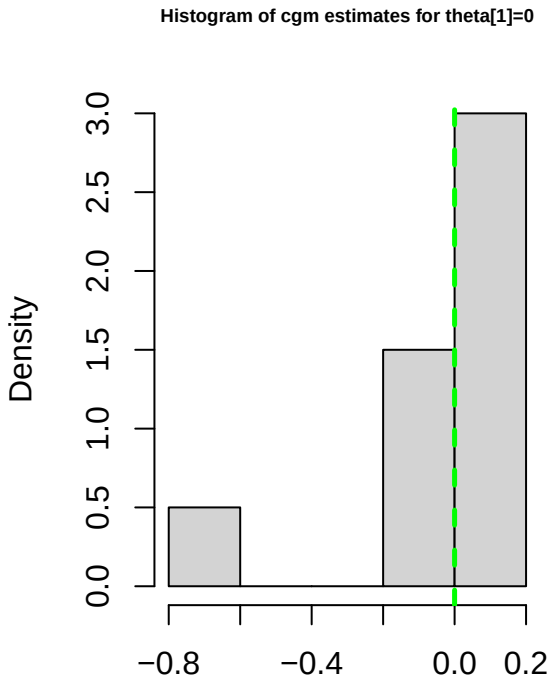

Histogram of proposed estimates for theta[5]=0.70710678118654
Density
0.0
0.5
1.0
1.5
0.0
0.5
1.0
1.5

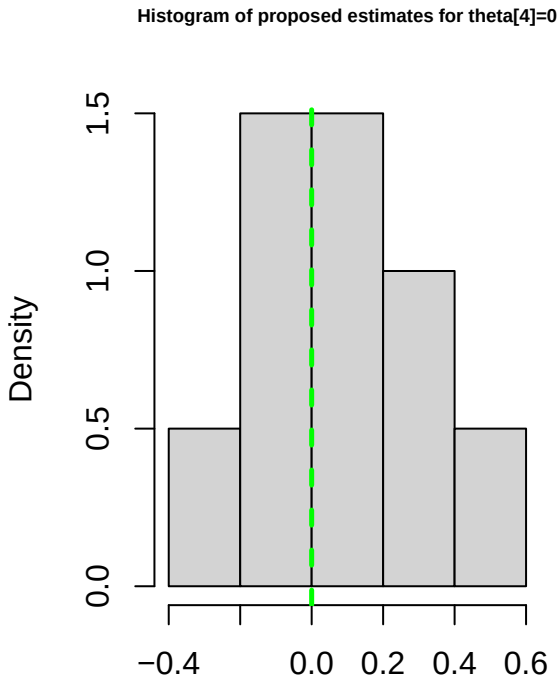

Histogram of cgm estimates for theta[5]=0.707106781186547
Density
0.0
0.2
0.4
0.6
0.8
1.0
1.2
−1.5
−0.5
0.5
1.0

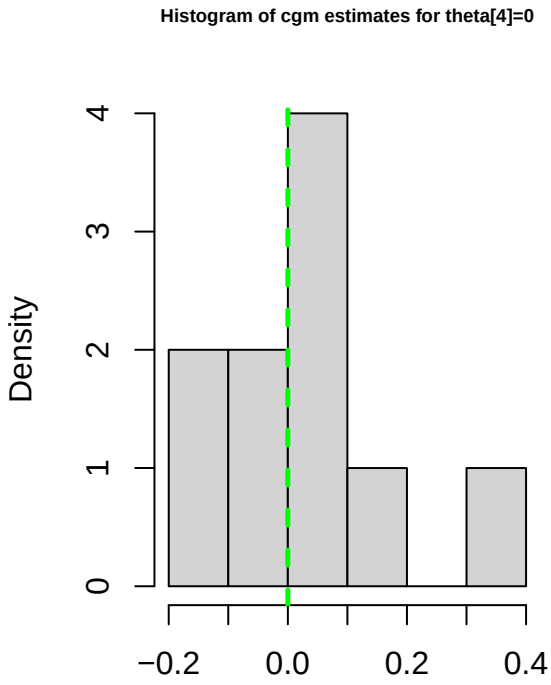

Histogram of proposed estimates for theta[10]=−0.707106781186
Density
0.0
0.4
0.8
1.2
−1.5
−1.0
−0.5
0.0


Histogram of cgm estimates for theta[10]=−0.707106781186547
Density
0.0
0.5
1.0
1.5
−1.0
−0.6
−0.2
0.2

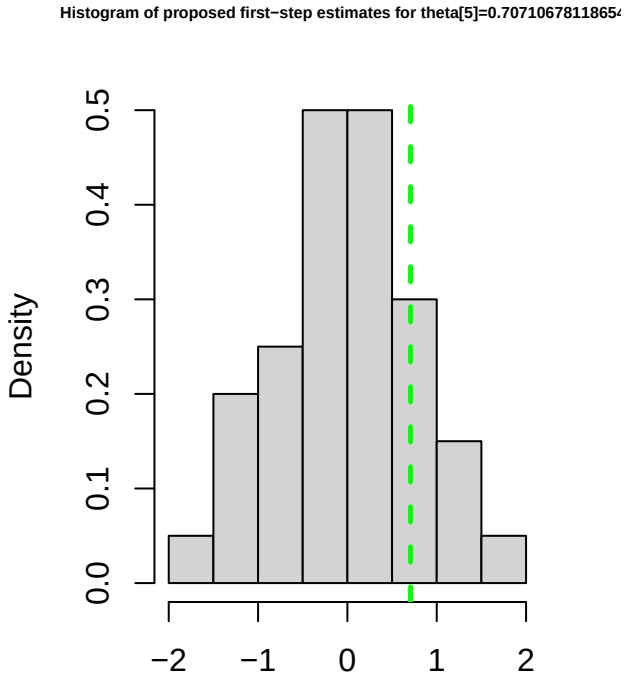

Histogram of proposed estimates for theta[1]=0
Density
0.0
0.5
1.0
1.5
2.0
−0.6
−0.2
0.2
0.4

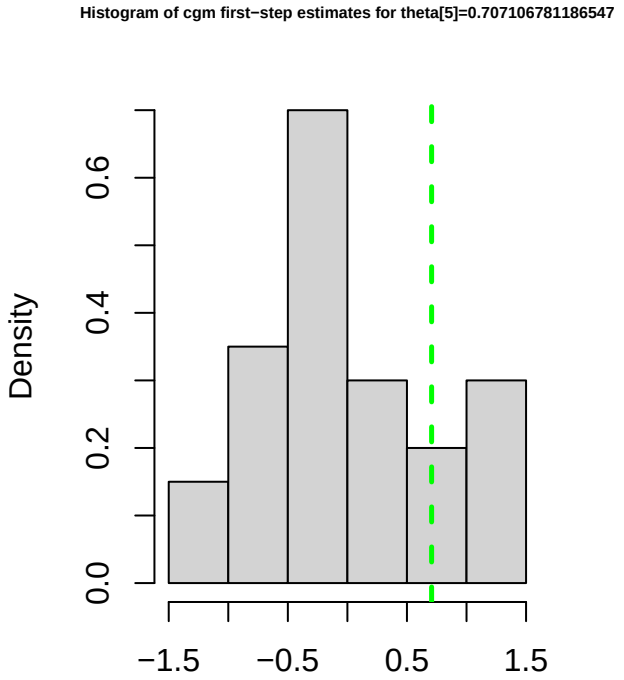

Histogram of cgm estimates for theta[1]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.8
−0.4
0.0
0.2

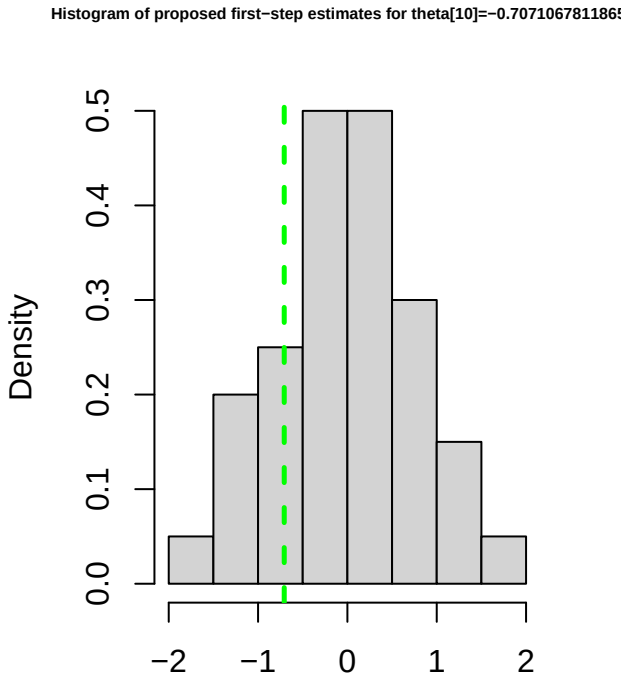

Histogram of proposed estimates for theta[4]=0
Density
0.0
0.5
1.0
1.5
−0.4
0.0
0.2
0.4
0.6

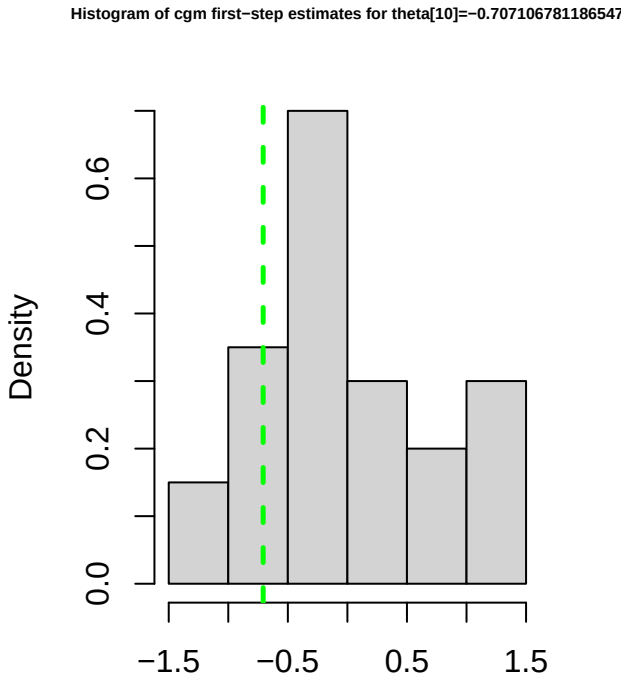

Histogram of cgm estimates for theta[4]=0
Density
0
1
2
3
4
−0.2
0.0
0.2
0.4

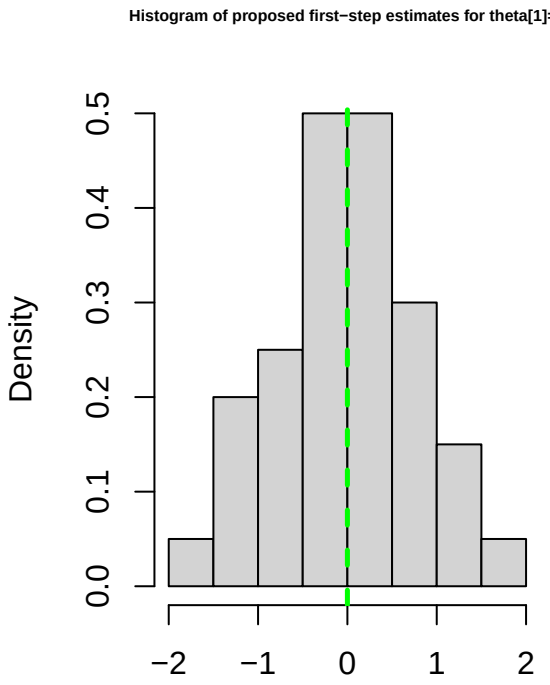
First Step Histograms

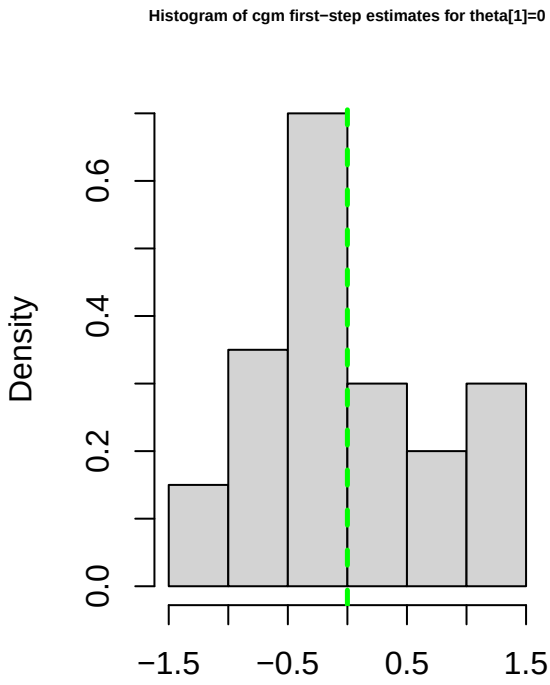


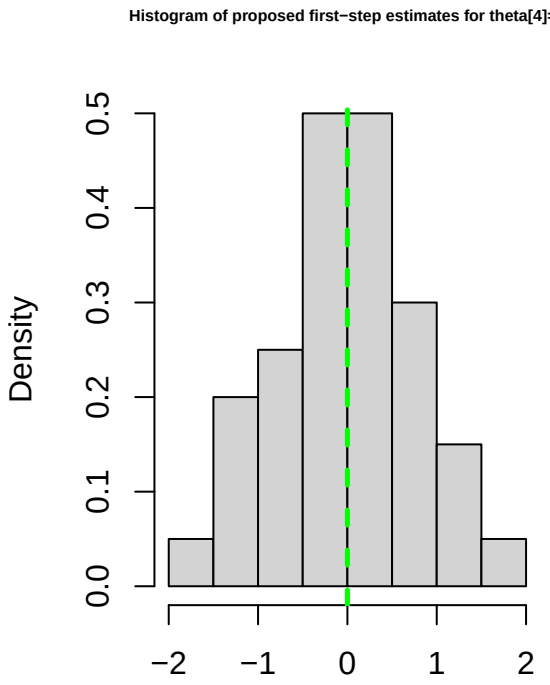


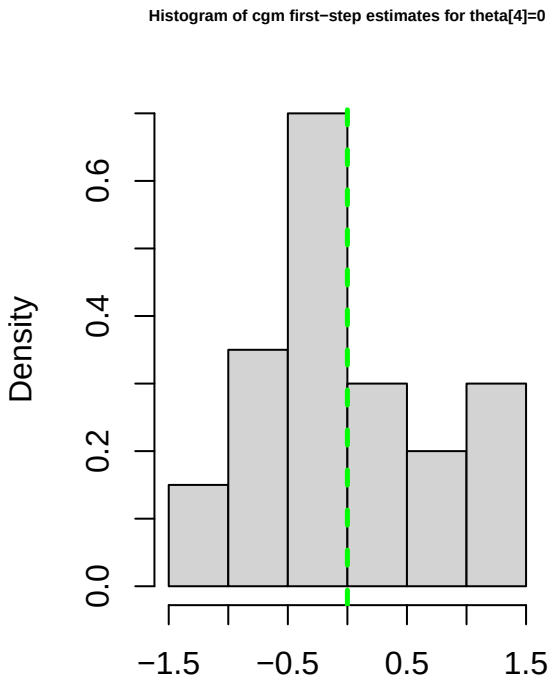





Histogram of proposed first−step estimates for theta[1]=0
Density
0.0
0.1
0.2
0.3
0.4
0.5
−2
−1
0
1
2


Histogram of cgm first−step estimates for theta[1]=0
Density
0.0
0.2
0.4
0.6
−1.5
−0.5
0.5
1.5


Histogram of proposed first−step estimates for theta[4]=0
Density
0.0
0.1
0.2
0.3
0.4
0.5
−2
−1
0
1
2


Histogram of cgm first−step estimates for theta[4]=0
Density
0.0
0.2
0.4
0.6
−1.5
−0.5
0.5
1.5

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 3: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[5]	0.196	0.782	1.489	0.243	1.397
theta[10]	-1.463	-0.822	-0.042	-1.402	-0.169
theta[1]	-0.497	0.021	0.337	-0.464	0.334
theta[4]	-0.231	0.048	0.545	-0.197	0.487

Table 4: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[5]	-1.059	0.194	0.723	-0.935	0.701
theta[10]	-0.932	-0.372	0.250	-0.909	0.220
theta[1]	-0.632	0.024	0.153	-0.517	0.136
theta[4]	-0.171	0.025	0.353	-0.156	0.301

Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[5]	0.759	0.198	0.371	1.147	0.8
theta[10]	-0.832	0.201	-1.226	-0.438	0.7
theta[1]	0.012	0.170	-0.320	0.345	0.9
theta[4]	0.085	0.177	-0.262	0.432	1.0

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[5]	0.132	0.249	-0.356	0.620	0.6
theta[10]	-0.356	0.231	-0.809	0.097	0.6
theta[1]	-0.045	0.148	-0.335	0.245	0.9
theta[4]	0.029	0.148	-0.262	0.319	0.9