Simulation Results

2025-11-07

Simulation Setup

This simulation is performed with $n = 200$ and $d = 10$, using the 2-d lattice as the underlying graph. $s = 2$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.5$. The attached results are for a 5-replication simulation. The true values of the parameter vector θ are

```
 [1]  0.0000000  0.7071068  0.0000000  0.0000000  0.0000000  0.0000000  0.0000000
 [8]  0.0000000  0.0000000 -0.7071068
```

The results from our code are not augmented with any comparison method here.

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

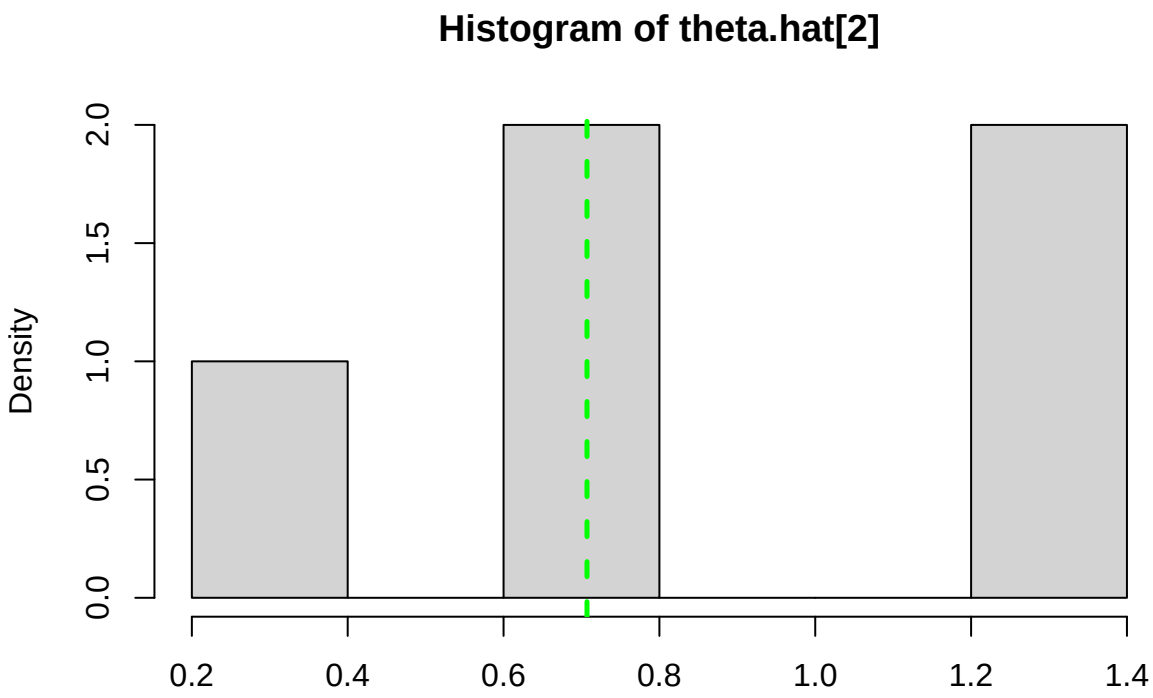
After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison ($\frac{1}{n.sim}\sum_{i=1}^{n.sim}\frac{1}{d}\|\hat{\theta}_i - \theta\|^2$)

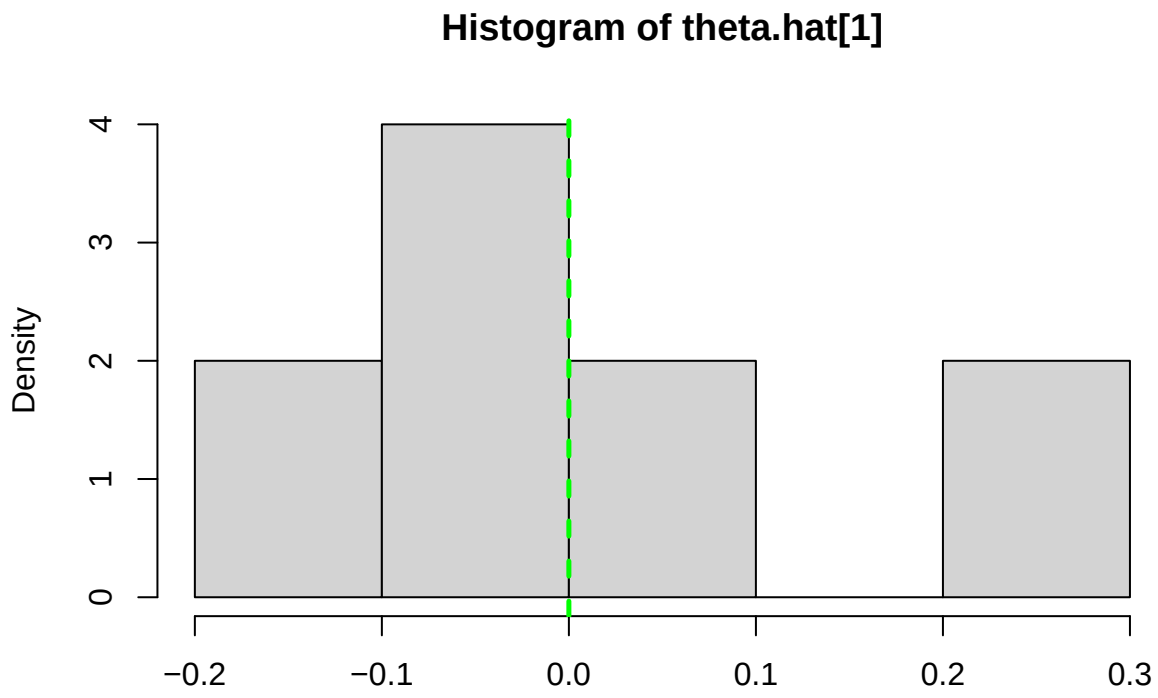
```
# A tibble: 1 x 2
  `MISLE (First-step) MSE` `MISLE MSE`
                     <dbl>       <dbl>
1                    0.142      0.0611
```

First Step Histograms

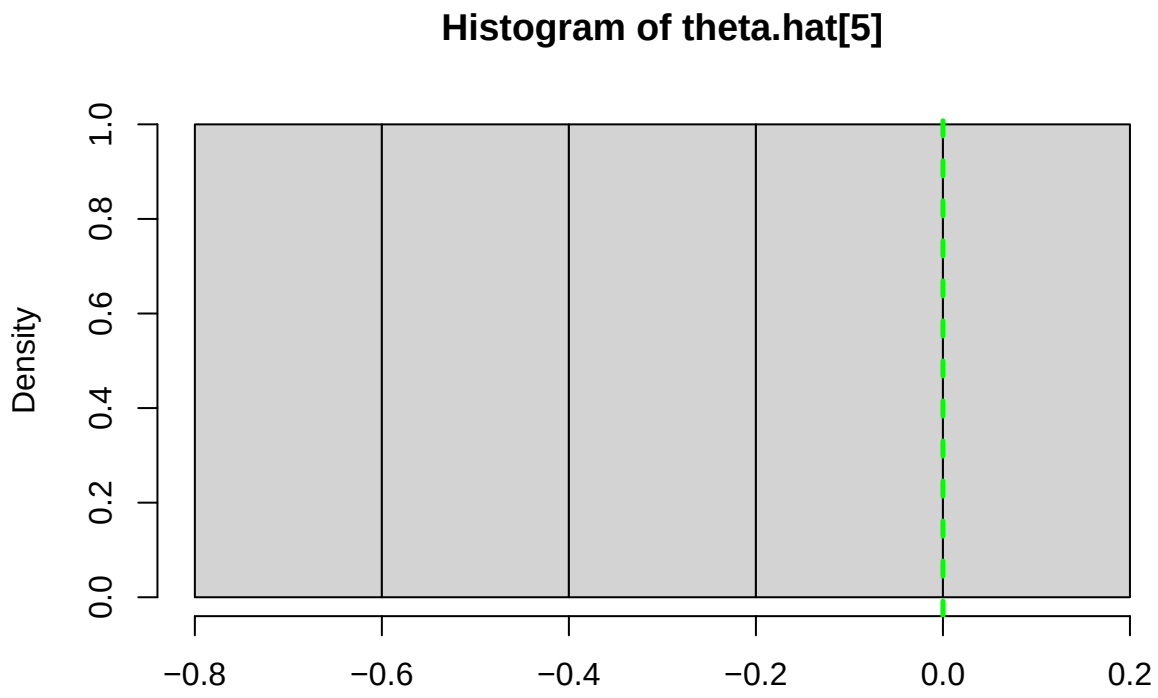



```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.3458  0.6500  0.7764  0.8559  1.2010  1.3064
[1] "95% CI based on bootstrap:"
      lower    upper
1 0.3761929 1.295857
```


```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-1.4788 -1.2346 -0.9822 -0.8638 -0.3995 -0.2239
[1] "95% CI based on bootstrap:"
      lower      upper
1 -1.454411 -0.2414313
```

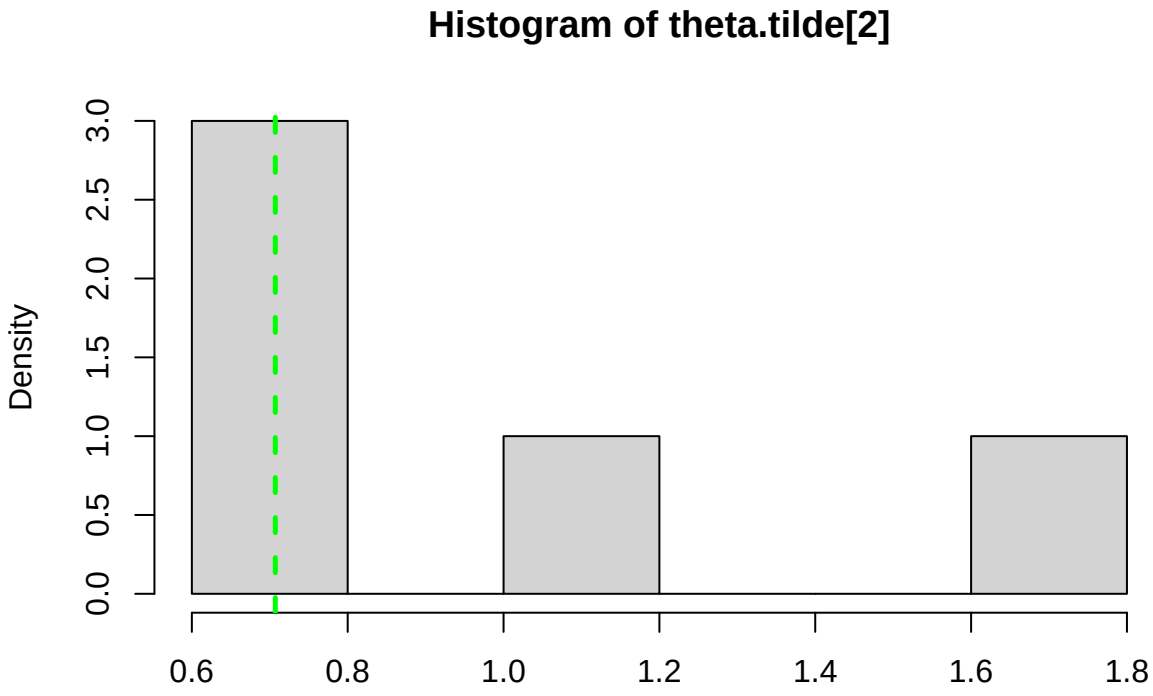



```
[1] "Summary statistics of bootstrap replicates:"
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-0.12515  0.00000  0.00000  0.02966  0.02023  0.25323
[1] "95% CI based on bootstrap:"
       lower    upper
1 -0.1126317 0.229933
```

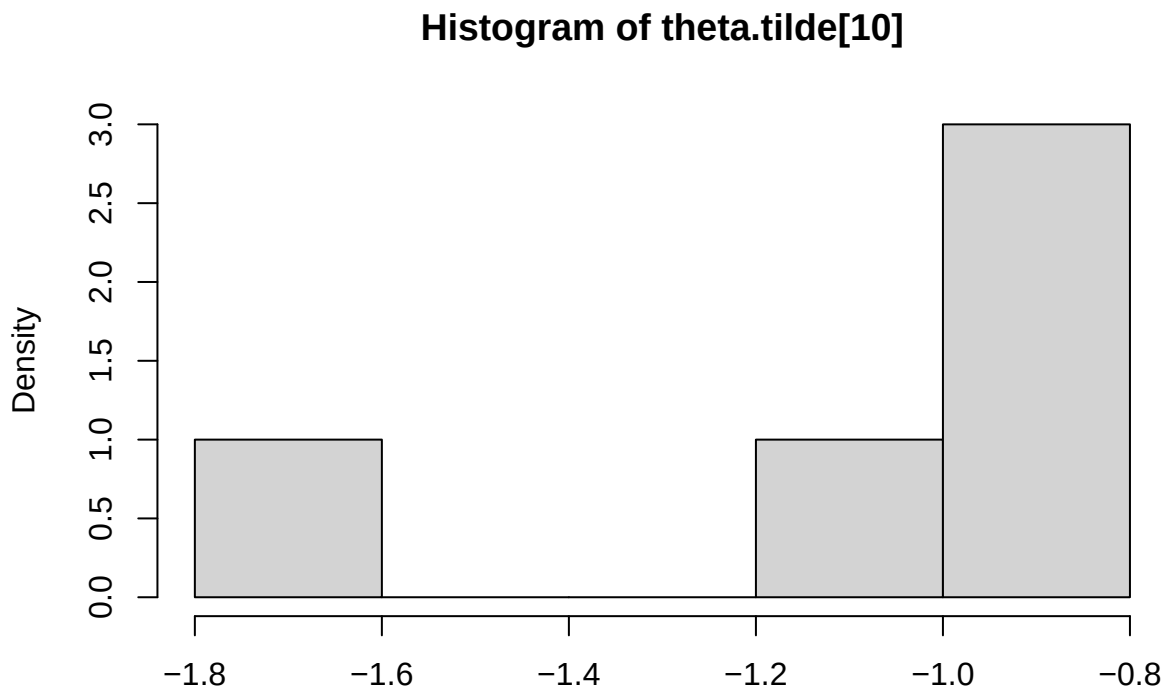



```
[1] "Summary statistics of bootstrap replicates:"
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-0.64126 -0.46747 -0.30226 -0.29753 -0.11428  0.03764
[1] "95% CI based on bootstrap:"
       lower      upper
1 -0.6238834 0.02244827
```

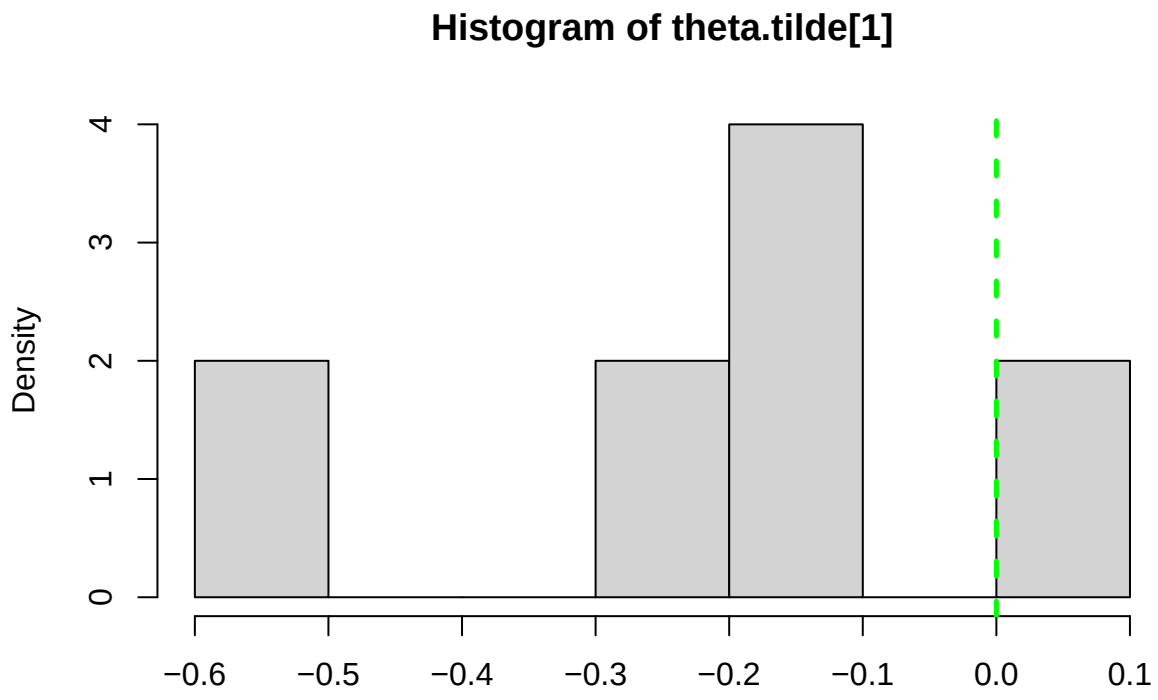
Statistics and 95% Confidence Intervals from per-Replicate Estimates



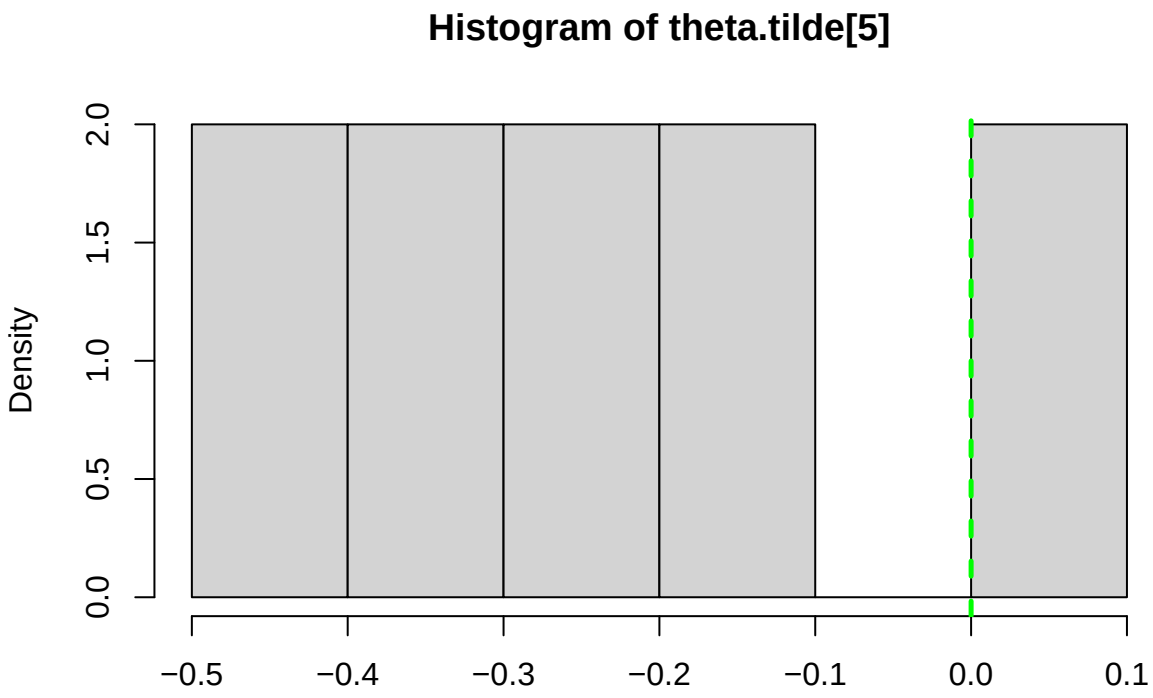

```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.6252  0.7686  0.7727  0.9920  1.1206  1.6727
[1] "95% CI based on bootstrap:"
      lower    upper
1 0.6395346 1.617507
```




```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-1.6851 -1.1314 -0.9259 -1.0781 -0.8374 -0.8110
[1] "95% CI based on bootstrap:"
      lower      upper
1 -1.629715 -0.8136053
```




```
[1] "Summary statistics of bootstrap replicates:"
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-0.50889 -0.26546 -0.18849 -0.21245 -0.11045  0.01104
[1] "95% CI based on bootstrap:"
       lower       upper
1 -0.4845439 -0.00111121
```




```
[1] "Summary statistics of bootstrap replicates:"
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-0.42081 -0.30502 -0.21929 -0.22201 -0.17757  0.01264
[1] "95% CI based on bootstrap:"
       lower        upper
1 -0.4092346 -0.006383149
```

Statistics for Theoretical 95% Confidence Intervals

```
[1] Length of Confidence Intervals for theta[2]
[1] Coverage proportion: 0.8
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.9207  1.0044  1.0629  1.0419  1.0766  1.1447
[1] Length of Confidence Intervals for theta[10]
[1] Coverage proportion: 0.8
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.8886  0.9747  0.9771  1.0119  1.0995  1.1196
[1] Length of Confidence Intervals for theta[1]
[1] Coverage proportion: 0.8
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.8921  0.9971  1.0199  1.0134  1.0470  1.1109
[1] Length of Confidence Intervals for theta[5]
[1] Coverage proportion: 1
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.8530  0.8859  0.9299  0.9791  1.0910  1.1357
```