

# Simulation Results

2026-01-26

## Simulation Setup

This simulation is performed with  $n = 200$  and  $d = 200$ , using the 2-d lattice as the underlying graph.  $s = 5$  parameters are set to be nonzero, and the beta parameter is chosen to be  $\beta = 0.4$ . The attached results are for a 10-replication simulation. The parameter vector  $\theta$  has sparse components other than the following:

Parameter.Index	Value
1	0.447
44	-0.447
63	0.447
73	0.447
158	0.447

but for brevity, our simulation only estimates the indices of  $\theta$  in  $\mathcal{C} = \{1, 44, 71, 172\}$  elements of  $\theta$ . Accordingly, **all statistics and visuals are indicative of performance only on the set  $\mathcal{C}$ .**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

## Results

### Mean-squared error comparison

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[1]	0.069	0.022
theta[44]	0.039	0.039
theta[71]	0.019	0.009
theta[172]	0.025	0.011
total	0.038	0.020

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[1]	0.141	0.065

	proposed	cgm
theta[44]	0.140	0.082
theta[71]	0.001	0.000
theta[172]	0.000	0.000
total	0.071	0.037

### Mean absolute deviation comparison  $(\frac{1}{n.sim} \sum_{i=1}^{n.sim} \frac{1}{|\mathcal{C}|} \|\hat{\theta}_i - \theta\|)$

Table 3: Mean Absolute Deviation of Parameter Estimates

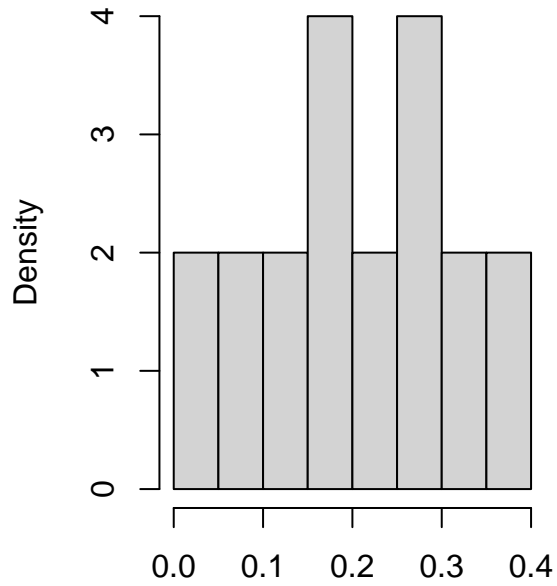
	proposed	cgm
theta[1]	0.244	0.130
theta[44]	0.169	0.178
theta[71]	0.104	0.081
theta[172]	0.121	0.085
total	0.159	0.118

Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

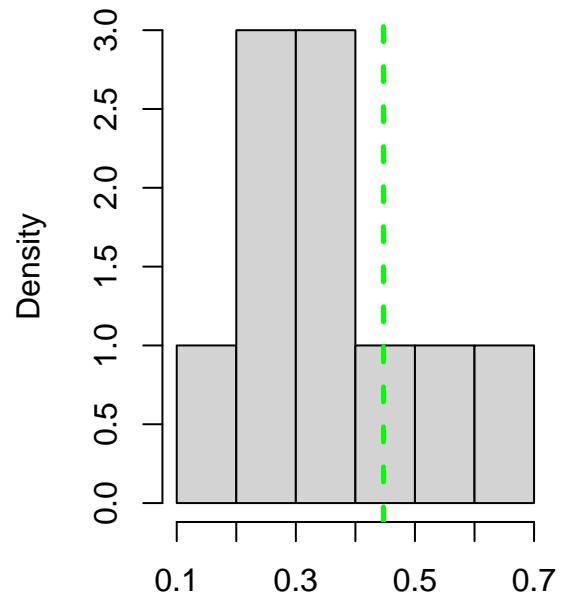
	proposed	cgm
theta[1]	0.343	0.220
theta[44]	0.370	0.261
theta[71]	0.011	0.000
theta[172]	0.007	0.000
total	0.183	0.120

## Boxplots

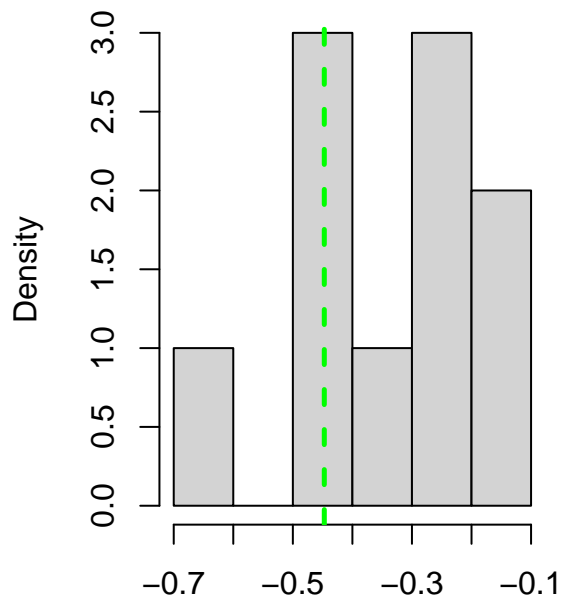
Histogram of proposed estimates for  $\theta[1]=0.447$



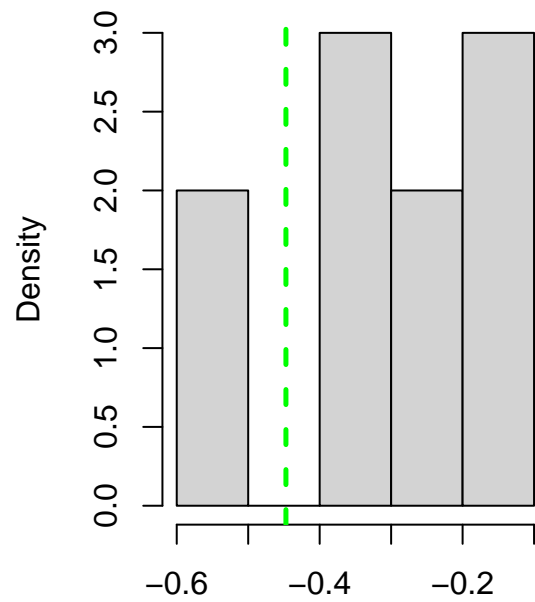
Histogram of cgm estimates for  $\theta[1]=0.447$

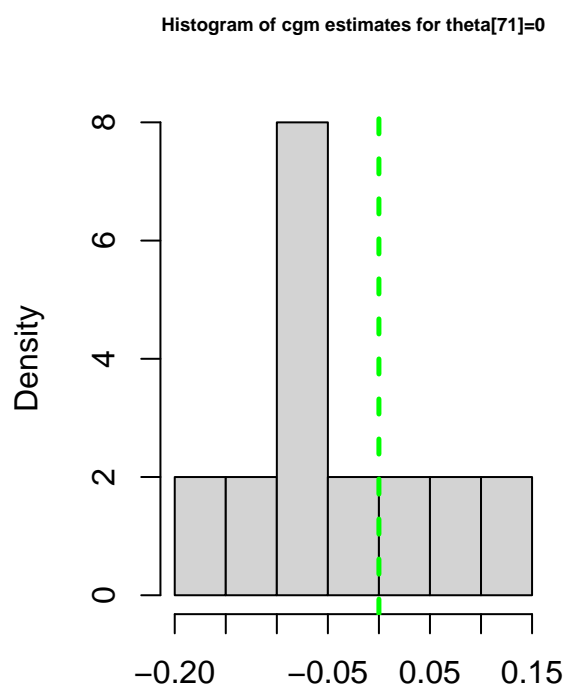
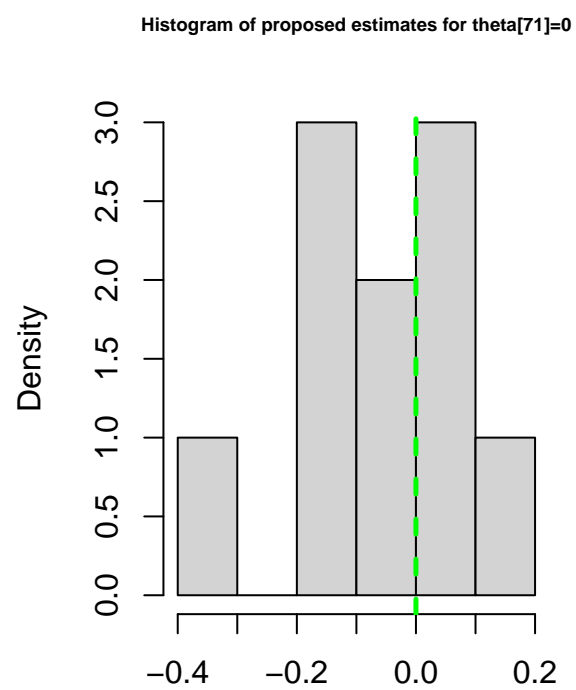


Histogram of proposed estimates for  $\theta[44]=-0.447$

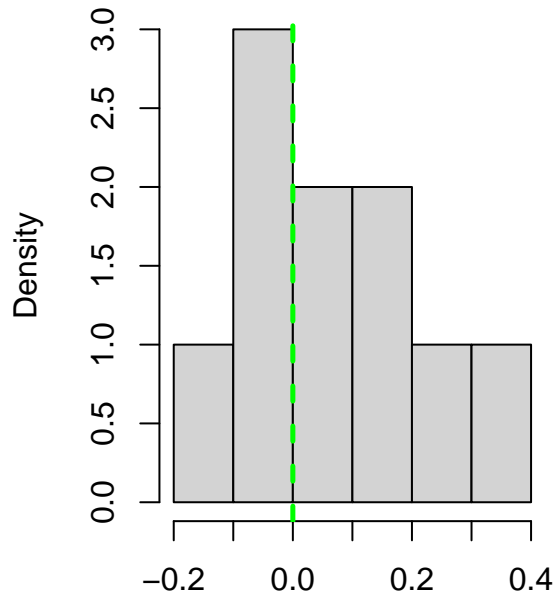


Histogram of cgm estimates for  $\theta[44]=-0.447$

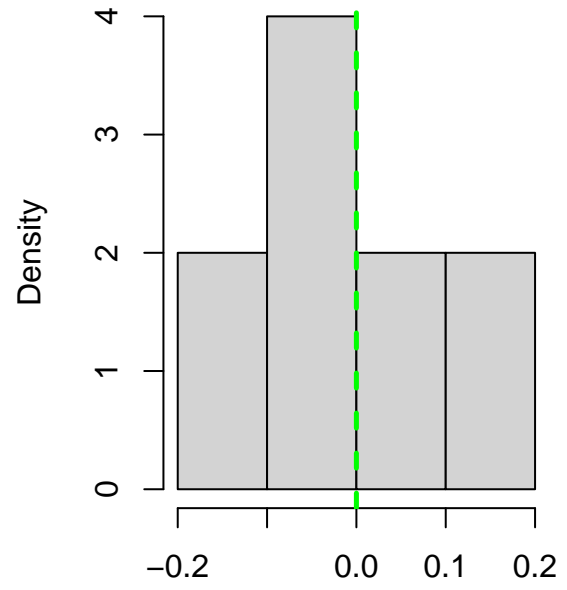




Histogram of proposed estimates for  $\theta_{172}=0$



Histogram of cgm estimates for  $\theta_{172}=0$



## Statistics and 95% Confidence Intervals from per-Replicate Estimates

### Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[1]	0.204	0.126	-0.044	0.451	0.6
theta[44]	-0.313	0.127	-0.562	-0.065	0.8
theta[71]	-0.053	0.122	-0.291	0.186	0.9
theta[172]	0.048	0.128	-0.202	0.299	0.8

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[1]	0.360	0.106	0.151	0.569	0.8
theta[44]	-0.310	0.100	-0.506	-0.114	0.6
theta[71]	-0.037	0.119	-0.269	0.196	1.0
theta[172]	-0.003	0.104	-0.206	0.201	1.0