

Simulation Results

2026-01-20

Simulation Setup

This simulation is performed with $n = 200$ and $d = 200$, using the 2-d lattice as the underlying graph. $s = 5$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0$. The attached results are for a 10-replication simulation. The parameter vector θ has sparse components other than the following:

Parameter.Index	Value
10	0.447
30	-0.447
103	-0.447
122	-0.447
175	-0.447

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{10, 30, 70, 171\}$ elements of θ . Accordingly, **all statistics and visuals are indicative of performance only on the set \mathcal{C} .**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[10]	0.042	0.083
theta[30]	0.033	0.080
theta[70]	0.017	0.099
theta[171]	0.008	0.097
total	0.025	0.090

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[10]	0.163	0.034

	proposed	cgm
theta[30]	0.139	0.037
theta[70]	0.000	0.002
theta[171]	0.000	0.000
total	0.076	0.018

Mean absolute deviation comparison $\frac{1}{n.\text{sim}} \sum_{i=1}^{n.\text{sim}} \frac{1}{|\mathcal{C}|} \|\hat{\theta}_i - \theta\|$

Table 3: Mean Absolute Deviation of Parameter Estimates

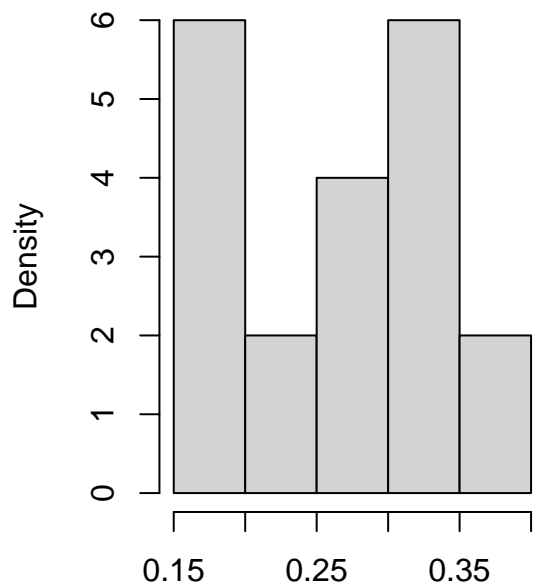
	proposed	cgm
theta[10]	0.192	0.234
theta[30]	0.162	0.212
theta[70]	0.096	0.238
theta[171]	0.080	0.211
total	0.133	0.224

Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

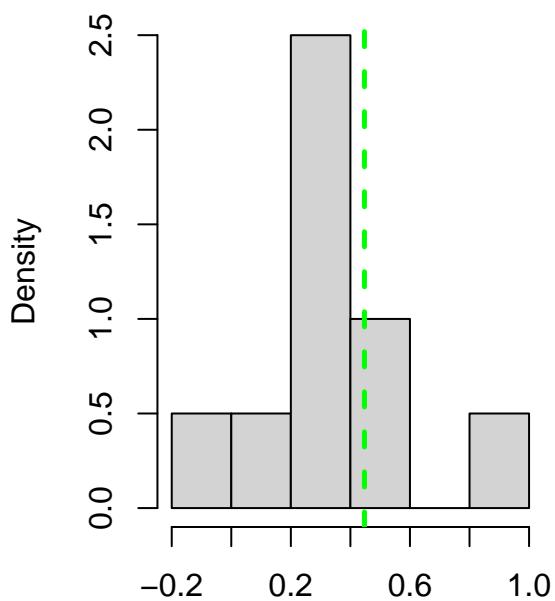
	proposed	cgm
theta[10]	0.391	0.153
theta[30]	0.348	0.150
theta[70]	0.002	0.013
theta[171]	0.003	0.002
total	0.186	0.079

Boxplots

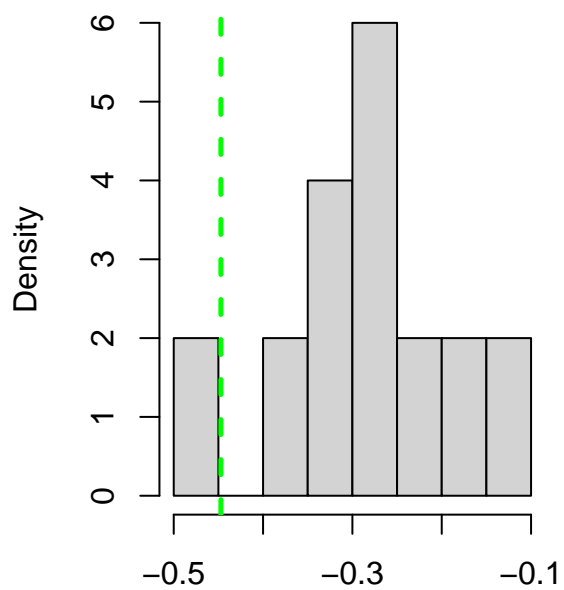
Histogram of proposed estimates for $\theta_{10}=0.447$



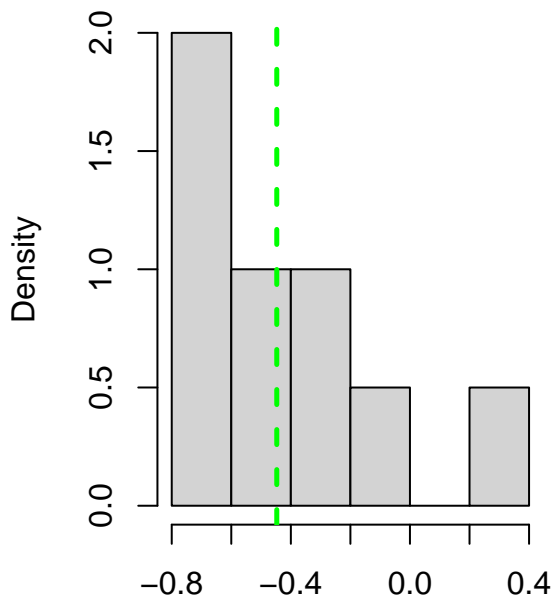
Histogram of cgm estimates for $\theta_{10}=0.447$



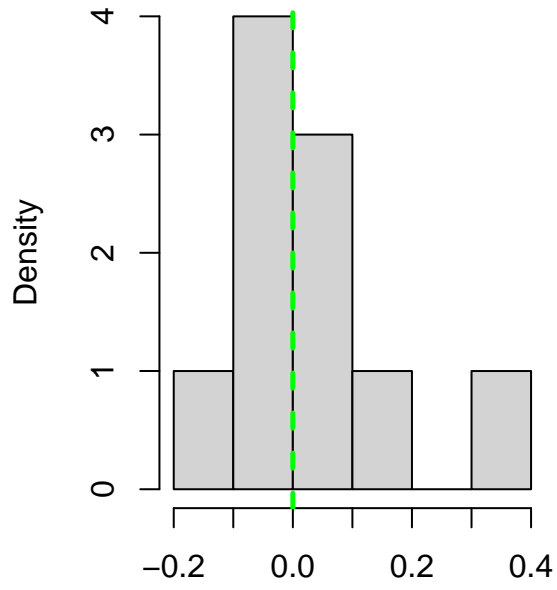
Histogram of proposed estimates for $\theta_{30}=-0.447$



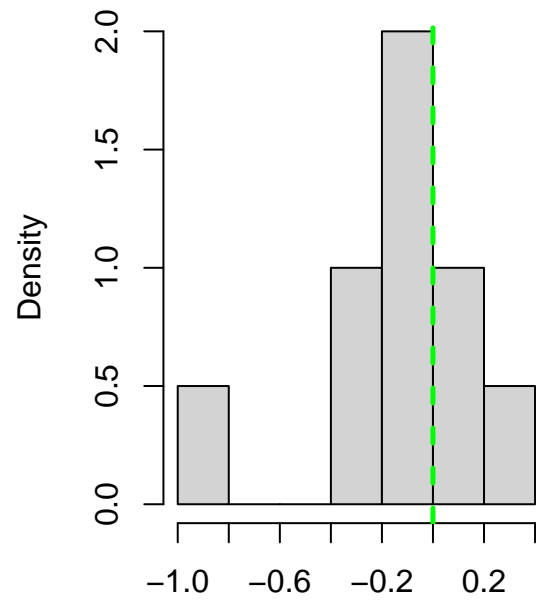
Histogram of cgm estimates for $\theta_{30}=-0.447$



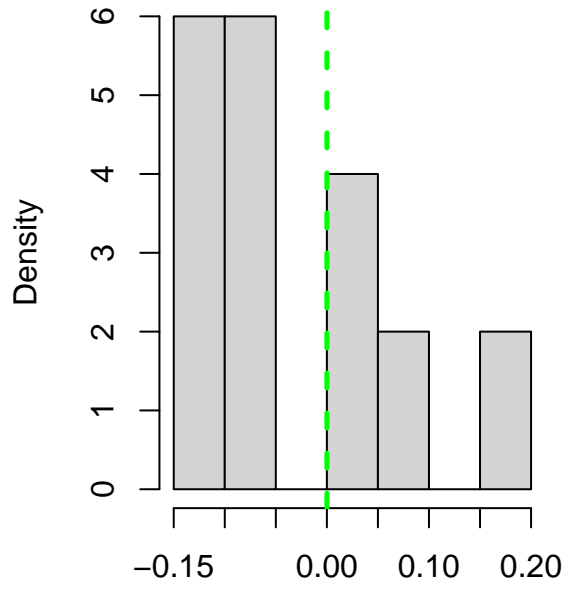
Histogram of proposed estimates for $\theta_{[70]}=0$



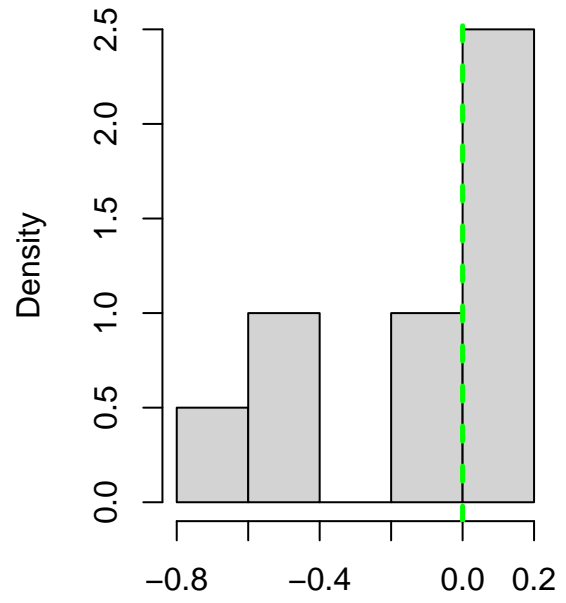
Histogram of cgm estimates for $\theta_{[70]}=0$



Histogram of proposed estimates for $\theta_{171}=0$



Histogram of cgm estimates for $\theta_{171}=0$



Statistics and 95% Confidence Intervals from per-Replicate Estimates

Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[10]	0.255	0.113	0.033	0.477	0.6
theta[30]	-0.291	0.109	-0.506	-0.077	0.8
theta[70]	0.026	0.109	-0.189	0.240	0.9
theta[171]	-0.028	0.116	-0.255	0.199	1.0

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[10]	0.346	0.212	-0.070	0.761	0.9
theta[30]	-0.395	0.165	-0.718	-0.071	0.8
theta[70]	-0.117	0.185	-0.480	0.245	0.9
theta[171]	-0.162	0.159	-0.474	0.150	0.8