Simulation Results

2026-01-26

Simulation Setup

This simulation is performed with $n = 400$ and $d = 200$, using the 2-d lattice as the underlying graph. $s = 5$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.4$. The attached results are for a 10-replication simulation. The parameter vector θ has sparse components other than the following:

Parameter.Index	Value
11	-0.447
79	-0.447
89	-0.447
99	-0.447
131	0.447

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{$ 11, 79, 69, 172$\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[11]	0.033	0.025
theta[79]	0.043	0.016
theta[69]	0.022	0.004
theta[172]	0.014	0.003
total	0.028	0.012

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[11]	0.110	0.064

	proposed	cgm
theta[79]	0.109	0.064
theta[69]	0.000	0.000
theta[172]	0.000	0.000
total	0.055	0.032

Mean absolute deviation comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat

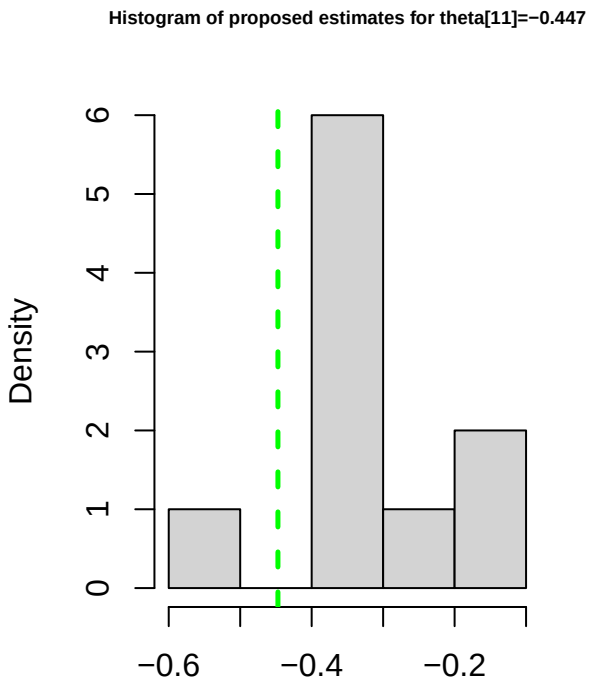
Table 3: Mean Absolute Deviation of Parameter Estimates

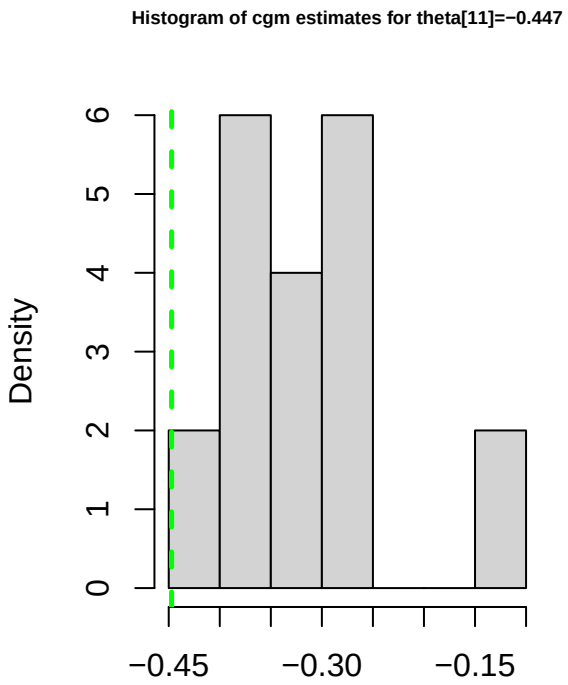
	proposed	cgm
theta[11]	0.164	0.135
theta[79]	0.191	0.116
theta[69]	0.140	0.055
theta[172]	0.090	0.048
total	0.146	0.088

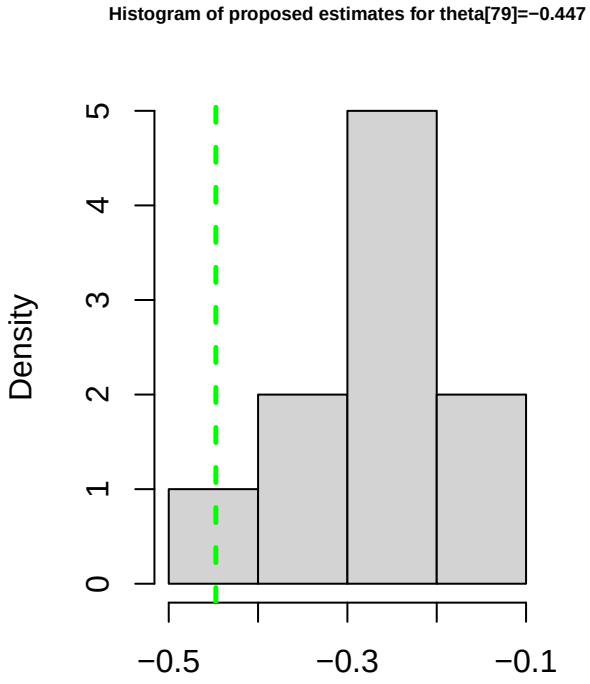
Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

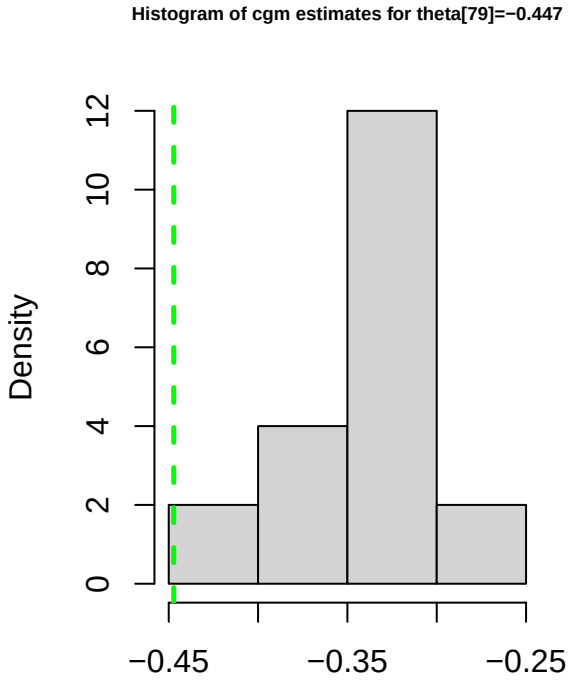
	proposed	cgm
theta[11]	0.319	0.224
theta[79]	0.321	0.246
theta[69]	0.000	0.000
theta[172]	0.000	0.000
total	0.160	0.118

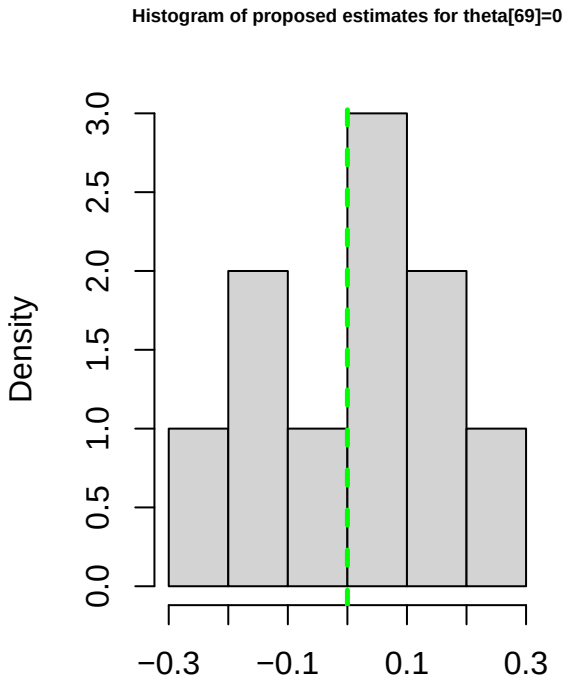
Boxplots

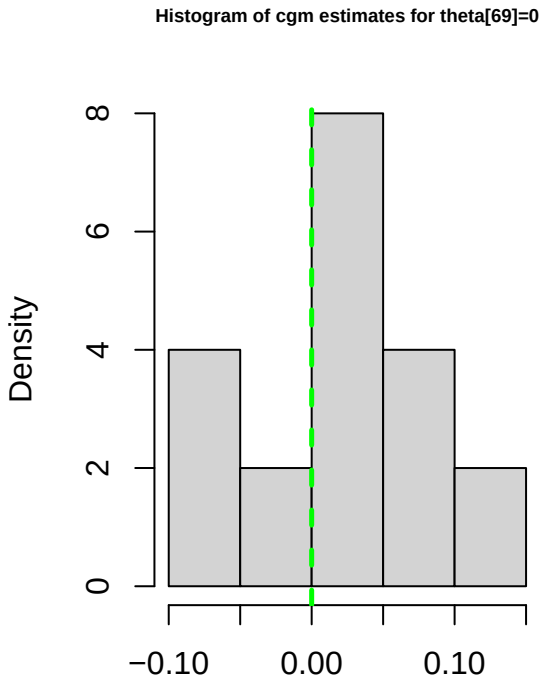

Histogram of proposed estimates for theta[11]=−0.447
Density
0
1
2
3
4
5
6
−0.6
−0.4
−0.2

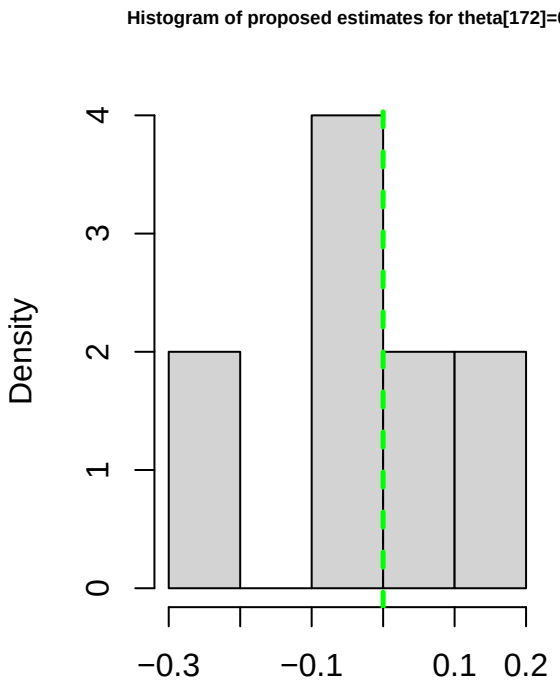

Histogram of cgm estimates for theta[11]=−0.447
Density
0
1
2
3
4
5
6
−0.45
−0.30
−0.15

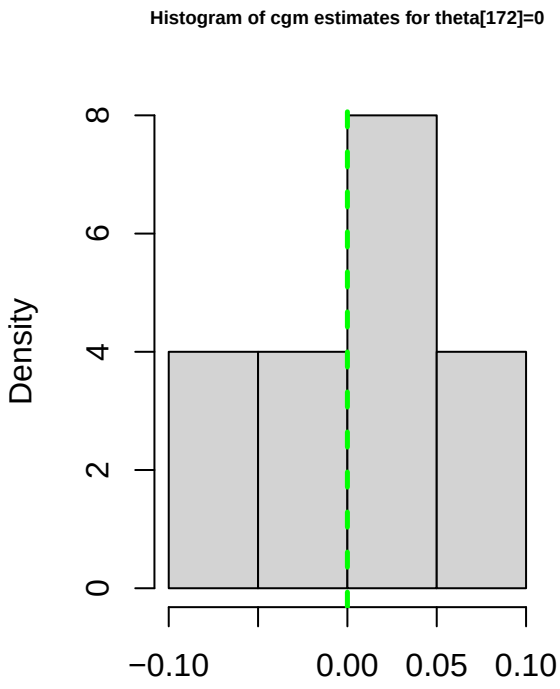

Histogram of proposed estimates for theta[79]=−0.447
Density
0
1
2
3
4
5
−0.5
−0.3
−0.1


Histogram of cgm estimates for theta[79]=−0.447
Density
0
2
4
6
8
10
12
−0.45
−0.35
−0.25


Histogram of proposed estimates for theta[69]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.3
−0.1
0.1
0.3


Histogram of cgm estimates for theta[69]=0
Density
0
2
4
6
8
−0.10
0.00
0.10


Histogram of proposed estimates for theta[172]=0
Density
0
1
2
3
4
−0.3
−0.1
0.1
0.2


Histogram of cgm estimates for theta[172]=0
Density
0
2
4
6
8
−0.10
0.00
0.05
0.10

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[11]	-0.301	0.098	-0.493	-0.109	0.7
theta[79]	-0.265	0.107	-0.476	-0.055	0.6
theta[69]	0.012	0.117	-0.219	0.242	0.9
theta[172]	-0.015	0.106	-0.223	0.193	0.8

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[11]	-0.312	0.073	-0.455	-0.169	0.5
theta[79]	-0.331	0.069	-0.466	-0.197	0.6
theta[69]	0.016	0.068	-0.117	0.149	1.0
theta[172]	-0.001	0.069	-0.137	0.135	1.0