

# Simulation Results

2026-01-26

## Simulation Setup

This simulation is performed with  $n = 400$  and  $d = 200$ , using the 2-d lattice as the underlying graph.  $s = 5$  parameters are set to be nonzero, and the beta parameter is chosen to be  $\beta = 0$ . The attached results are for a 10-replication simulation. The parameter vector  $\theta$  has sparse components other than the following:

Parameter.Index	Value
42	-0.447
59	0.447
121	-0.447
136	0.447
162	-0.447

but for brevity, our simulation only estimates the indices of  $\theta$  in  $\mathcal{C} = \{42, 59, 70, 172\}$  elements of  $\theta$ . Accordingly, **all statistics and visuals are indicative of performance only on the set  $\mathcal{C}$ .**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

## Results

### Mean-squared error comparison

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[42]	0.054	0.020
theta[59]	0.048	0.024
theta[70]	0.005	0.005
theta[172]	0.021	0.006
total	0.032	0.014

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[42]	0.130	0.078

	proposed	cgm
theta[59]	0.144	0.069
theta[70]	0.000	0.000
theta[172]	0.000	0.000
total	0.068	0.037

### Mean absolute deviation comparison  $\frac{1}{n.\text{sim}} \sum_{i=1}^{n.\text{sim}} \frac{1}{|\mathcal{C}|} \|\hat{\theta}_i - \theta\|$

Table 3: Mean Absolute Deviation of Parameter Estimates

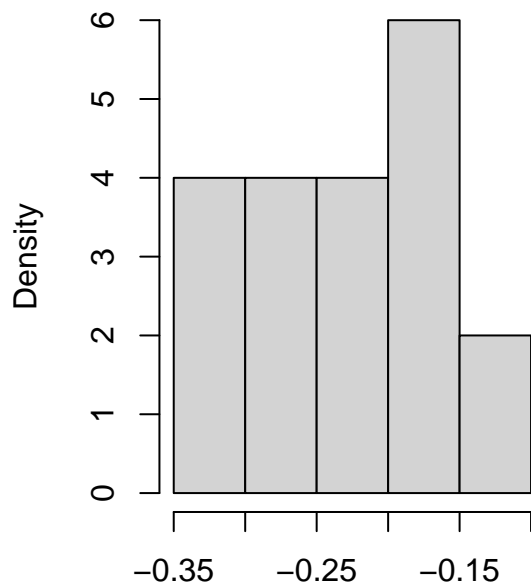
	proposed	cgm
theta[42]	0.223	0.133
theta[59]	0.207	0.148
theta[70]	0.059	0.050
theta[172]	0.109	0.064
total	0.150	0.099

Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

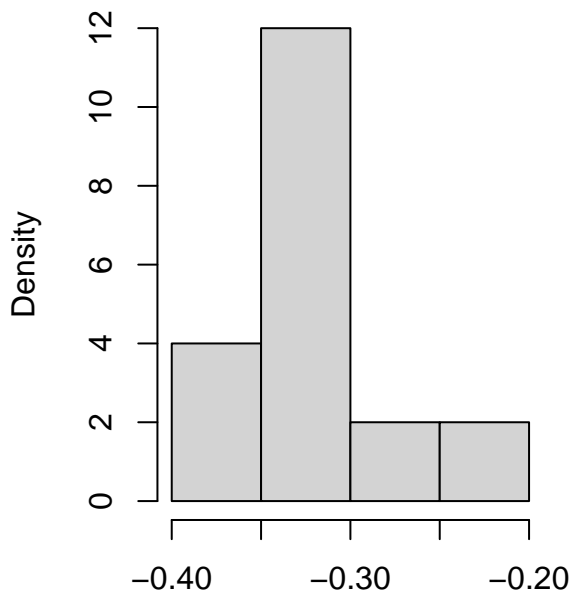
	proposed	cgm
theta[42]	0.349	0.258
theta[59]	0.376	0.233
theta[70]	0.005	0.000
theta[172]	0.000	0.000
total	0.183	0.123

## Boxplots

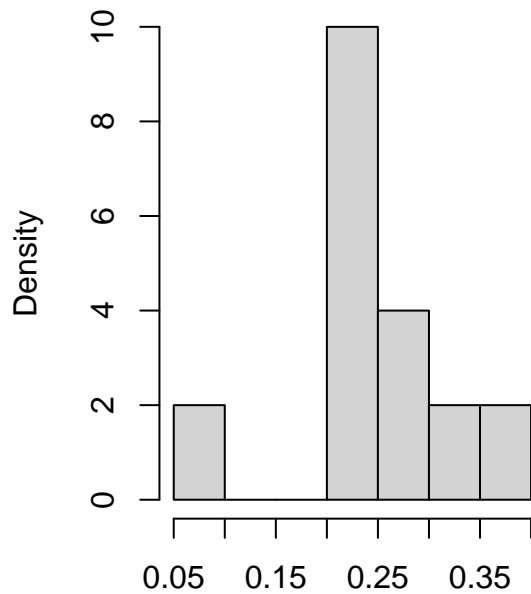
Histogram of proposed estimates for  $\theta_{[42]} = -0.447$



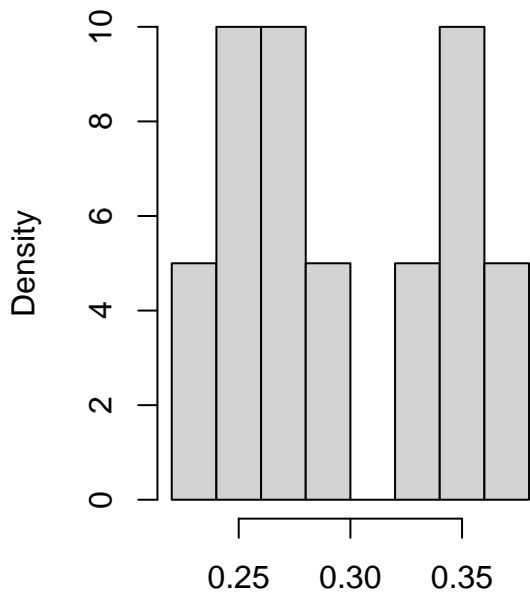
Histogram of cgm estimates for  $\theta_{[42]} = -0.447$



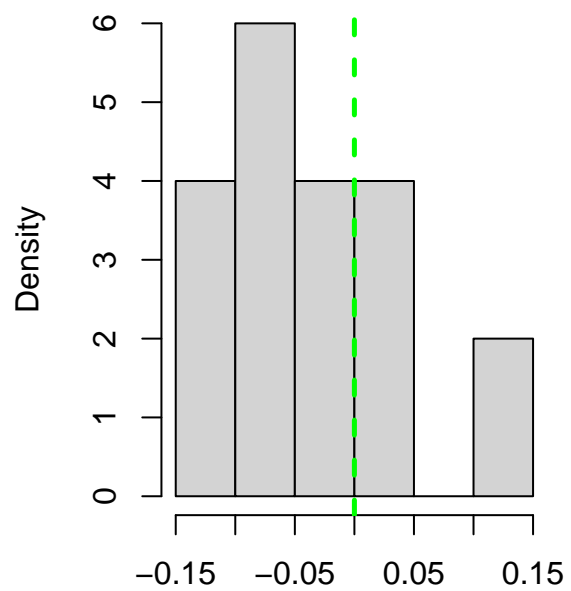
Histogram of proposed estimates for  $\theta_{[59]} = 0.447$



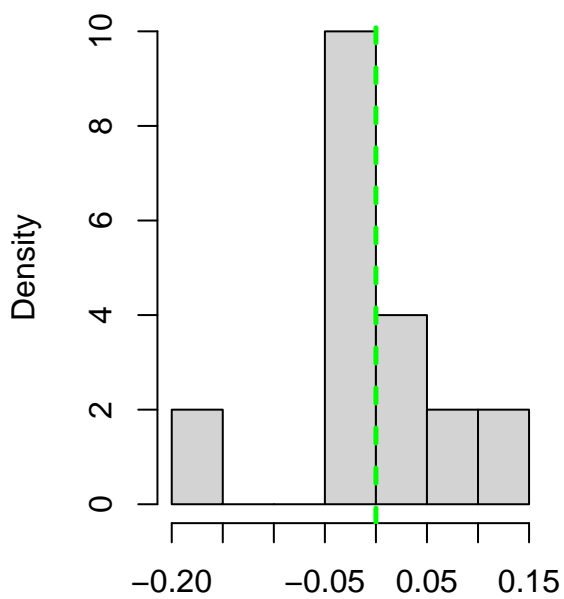
Histogram of cgm estimates for  $\theta_{[59]} = 0.447$



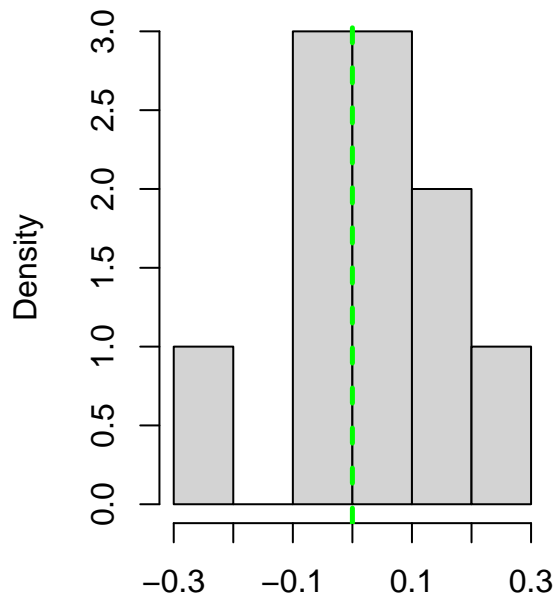
Histogram of proposed estimates for  $\theta_{70}=0$



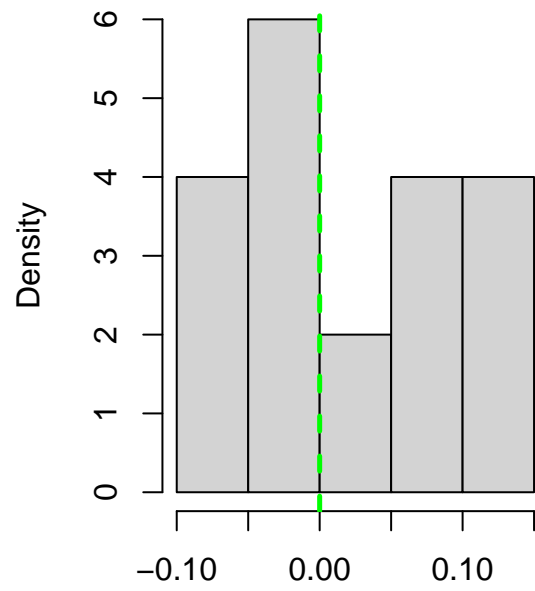
Histogram of cgm estimates for  $\theta_{70}=0$



Histogram of proposed estimates for  $\theta_{172}=0$



Histogram of cgm estimates for  $\theta_{172}=0$



## Statistics and 95% Confidence Intervals from per-Replicate Estimates

### Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[42]	-0.224	0.088	-0.397	-0.051	0.2
theta[59]	0.240	0.090	0.063	0.417	0.2
theta[70]	-0.028	0.096	-0.217	0.161	1.0
theta[172]	0.025	0.108	-0.187	0.238	0.8

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[42]	-0.315	0.072	-0.455	-0.174	0.8
theta[59]	0.299	0.070	0.161	0.437	0.4
theta[70]	-0.007	0.067	-0.139	0.124	0.9
theta[172]	0.020	0.072	-0.120	0.160	0.9