Simulation Results

2026-01-21

Simulation Setup

This simulation is performed with $n = 200$ and $d = 100$, using the 2-d lattice as the underlying graph. $s = 5$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.4$. The attached results are for a 10-replication simulation. The parameter vector θ has sparse components other than the following:

Parameter.Index	Value
13	-0.447
40	0.447
56	0.447
76	-0.447
89	0.447

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{$ 13, 40, 71, 41$\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[13]	0.157	0.022
theta[40]	0.051	0.020
theta[71]	0.009	0.010
theta[41]	0.014	0.007
total	0.058	0.015

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[13]	0.133	0.068

	proposed	cgm
theta[40]	0.130	0.058
theta[71]	0.000	0.000
theta[41]	0.000	0.000
total	0.066	0.031

Mean absolute deviation comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat

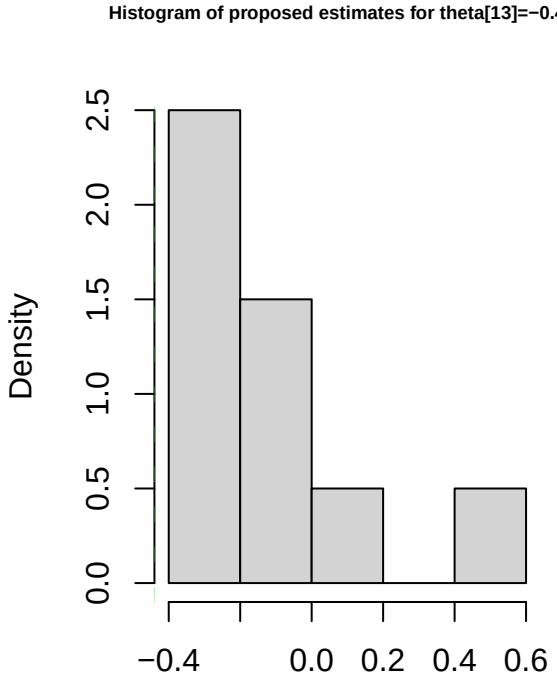
Table 3: Mean Absolute Deviation of Parameter Estimates

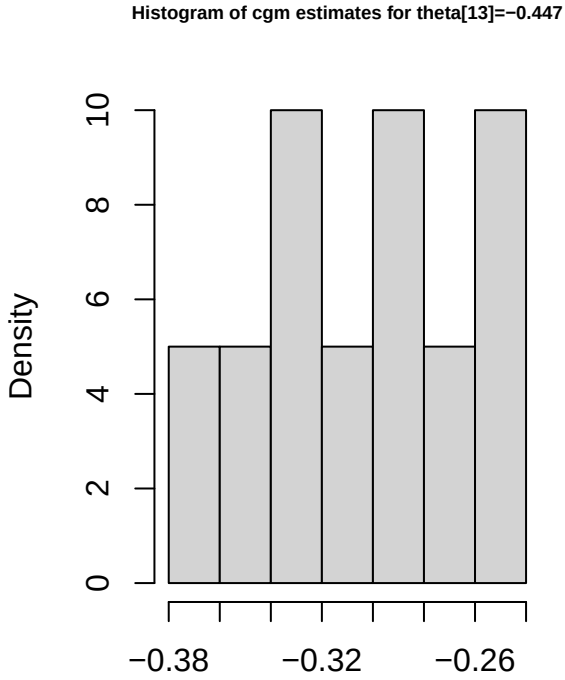
	proposed	cgm
theta[13]	0.323	0.144
theta[40]	0.180	0.108
theta[71]	0.083	0.088
theta[41]	0.099	0.072
total	0.171	0.103

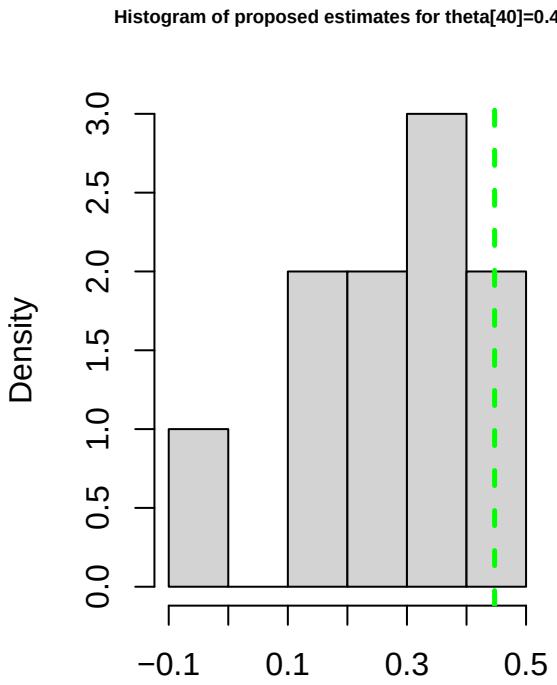
Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

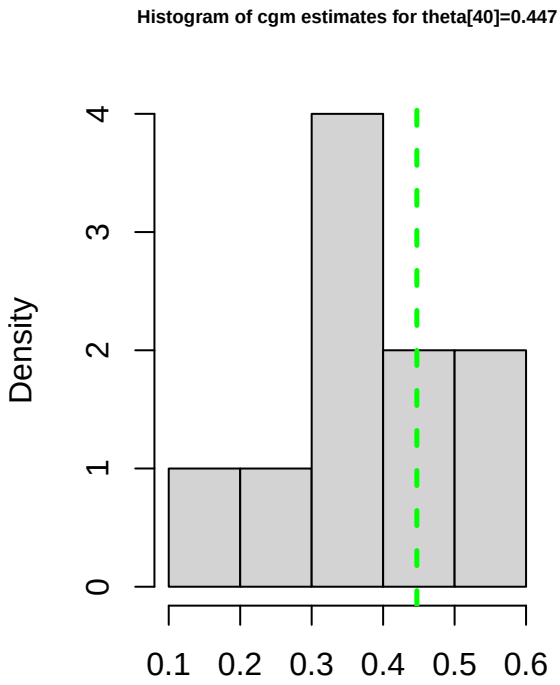
	proposed	cgm
theta[13]	0.351	0.249
theta[40]	0.336	0.203
theta[71]	0.004	0.000
theta[41]	0.000	0.000
total	0.173	0.113

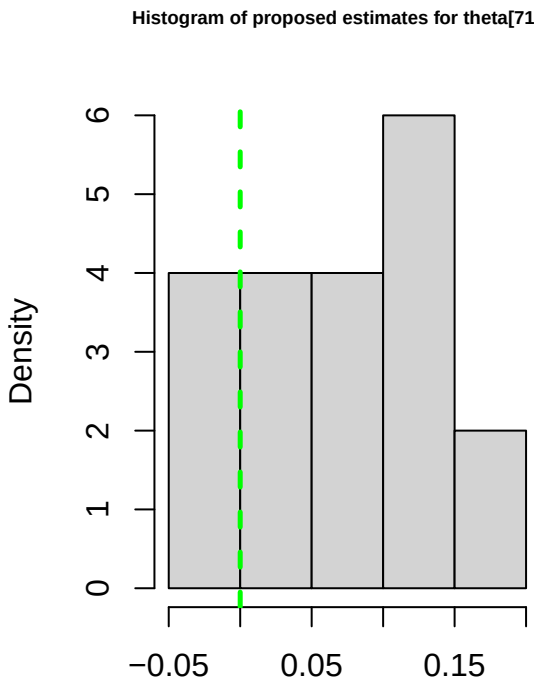
Boxplots

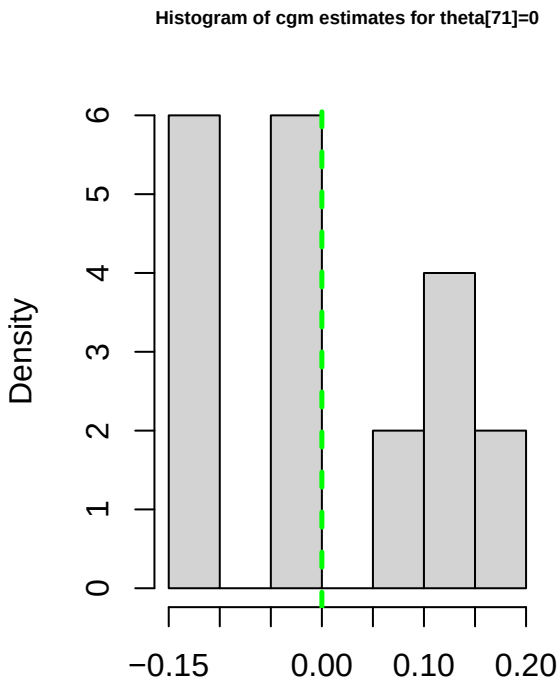

Histogram of proposed estimates for theta[13]=−0.447
Density
0.0
0.5
1.0
1.5
2.0
2.5
−0.4
0.0
0.2
0.4
0.6

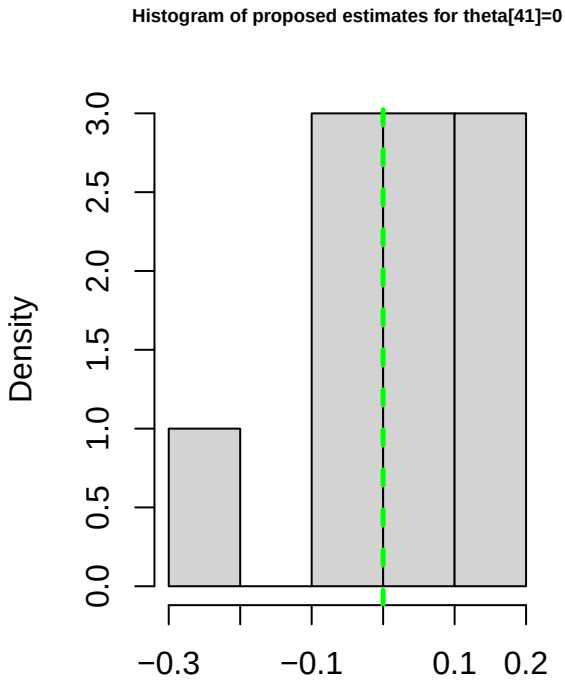

Histogram of cgm estimates for theta[13]=−0.447
Density
0
2
4
6
8
10
−0.38
−0.32
−0.26

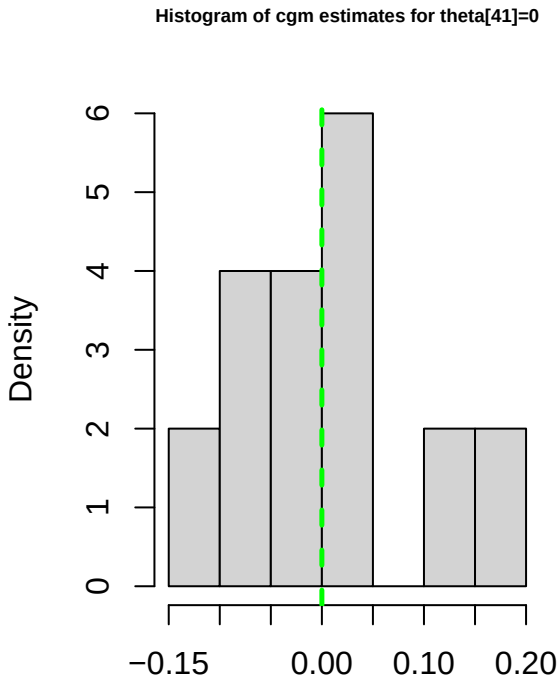

Histogram of proposed estimates for theta[40]=0.447
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.1
0.1
0.3
0.5


Histogram of cgm estimates for theta[40]=0.447
Density
0
1
2
3
4
0.1
0.2
0.3
0.4
0.5
0.6


Histogram of proposed estimates for theta[71]=0
Density
0
1
2
3
4
5
6
−0.05
0.05
0.15


Histogram of cgm estimates for theta[71]=0
Density
0
1
2
3
4
5
6
−0.15
0.00
0.10
0.20


Histogram of proposed estimates for theta[41]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.3
−0.1
0.1
0.2


Histogram of cgm estimates for theta[41]=0
Density
0
1
2
3
4
5
6
−0.15
0.00
0.10
0.20

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[13]	-0.124	0.136	-0.390	0.142	0.5
theta[40]	0.277	0.141	0.001	0.553	0.7
theta[71]	0.076	0.135	-0.188	0.340	1.0
theta[41]	0.003	0.140	-0.272	0.277	0.9

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[13]	-0.303	0.095	-0.491	-0.116	0.7
theta[40]	0.374	0.099	0.180	0.568	0.8
theta[71]	0.007	0.092	-0.173	0.188	1.0
theta[41]	-0.002	0.099	-0.195	0.191	1.0