Simulation Results

2026-01-26

Simulation Setup

This simulation is performed with $n = 200$ and $d = 400$, using the 2-d lattice as the underlying graph. $s = 5$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.4$. The attached results are for a 10-replication simulation. The parameter vector θ has sparse components other than the following:

Parameter.Index	Value
61	-0.447
62	-0.447
183	-0.447
263	0.447
269	-0.447

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{$ 61, 62, 329, 169$\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[61]	0.104	0.043
theta[62]	0.050	0.032
theta[329]	0.041	0.014
theta[169]	0.020	0.004
total	0.054	0.023

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[61]	0.149	0.125

	proposed	cgm
theta[62]	0.123	0.057
theta[329]	0.000	0.000
theta[169]	0.001	0.000
total	0.068	0.046

Mean absolute deviation comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat

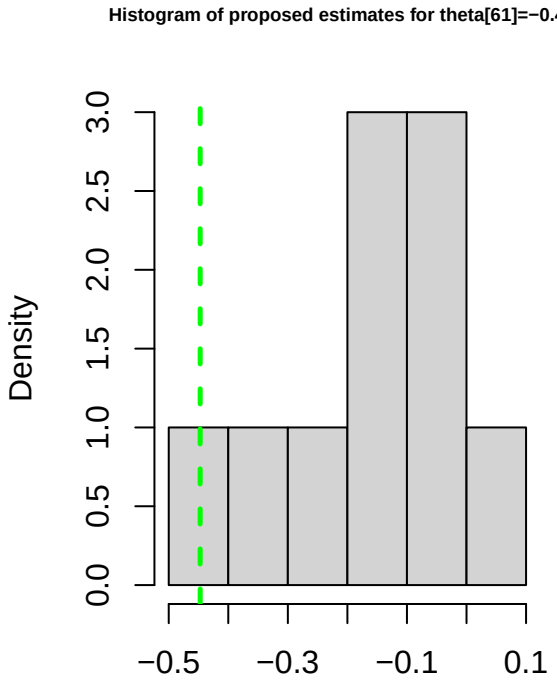
Table 3: Mean Absolute Deviation of Parameter Estimates

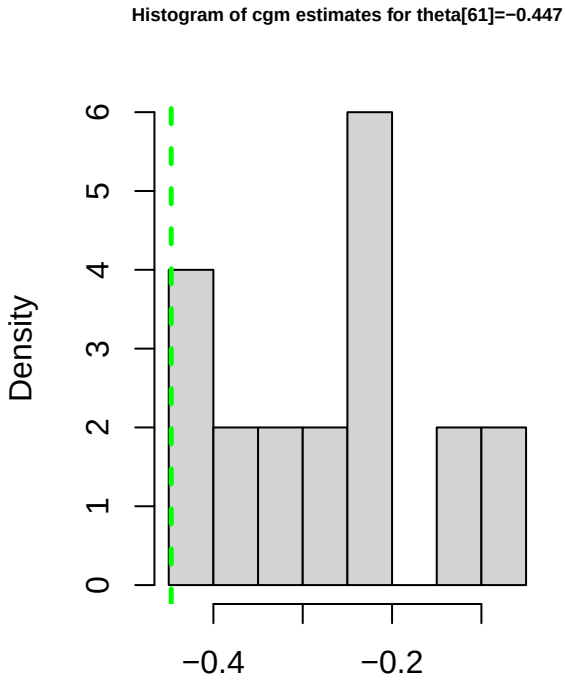
	proposed	cgm
theta[61]	0.286	0.179
theta[62]	0.187	0.144
theta[329]	0.153	0.086
theta[169]	0.123	0.048
total	0.187	0.114

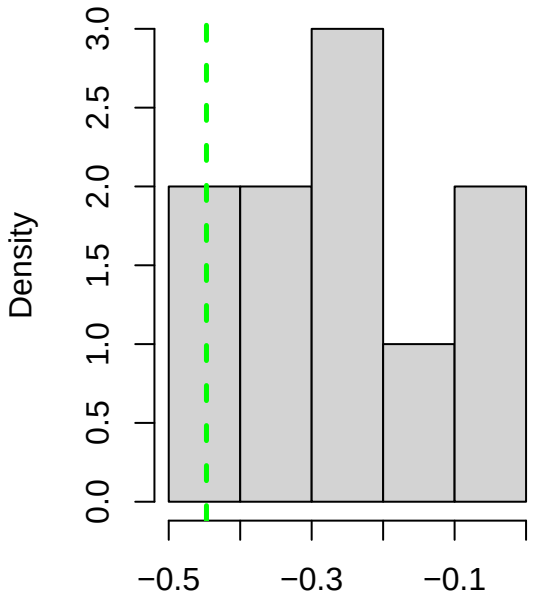
Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

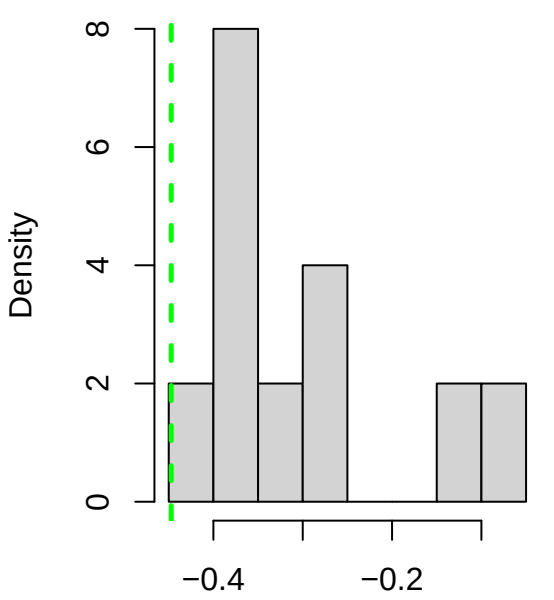
	proposed	cgm
theta[61]	0.377	0.333
theta[62]	0.336	0.204
theta[329]	0.000	0.000
theta[169]	0.009	0.000
total	0.181	0.134

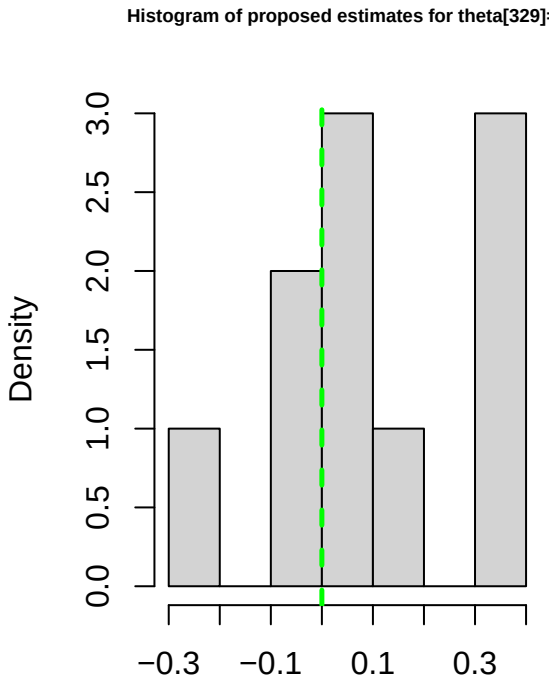
Boxplots

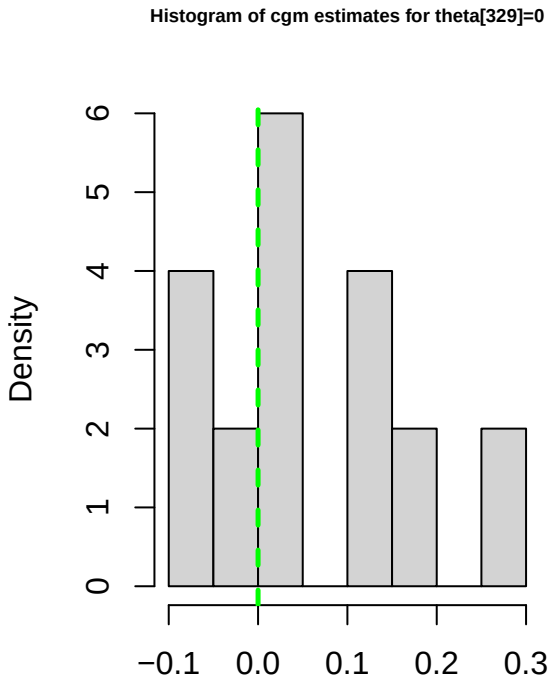

Histogram of proposed estimates for theta[61]=−0.447
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.5
−0.3
−0.1
0.1

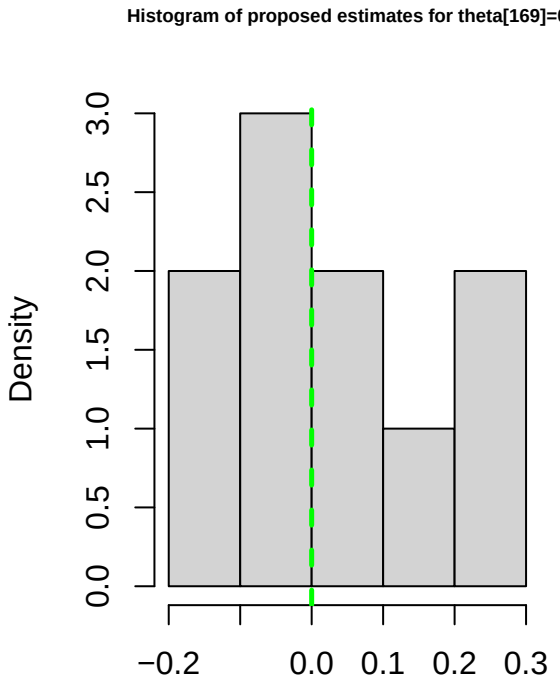

Histogram of cgm estimates for theta[61]=−0.447
Density
0
1
2
3
4
5
6
−0.4
−0.2

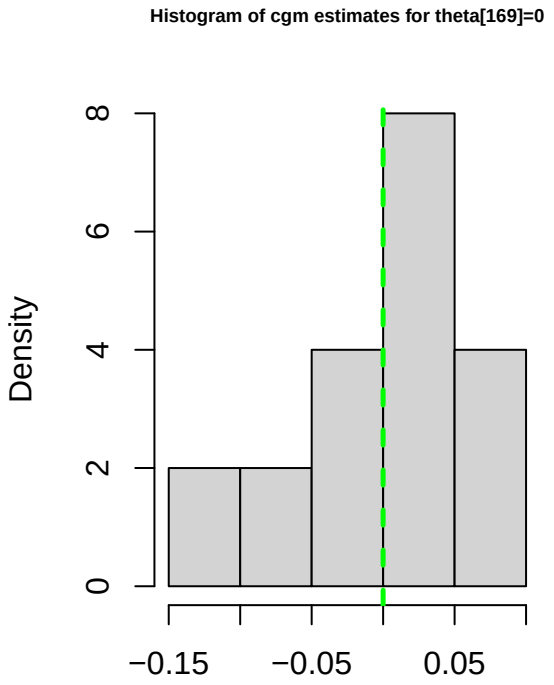

Histogram of proposed estimates for theta[62]=−0.447
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.5
−0.3
−0.1


Histogram of cgm estimates for theta[62]=−0.447
Density
0
2
4
6
8
−0.4
−0.2


Histogram of proposed estimates for theta[329]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.3
−0.1
0.1
0.3


Histogram of cgm estimates for theta[329]=0
Density
0
1
2
3
4
5
6
−0.1
0.0
0.1
0.2
0.3


Histogram of proposed estimates for theta[169]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.2
0.0
0.1
0.2
0.3


Histogram of cgm estimates for theta[169]=0
Density
0
2
4
6
8
−0.15
−0.05
0.05

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[61]	-0.162	0.110	-0.377	0.054	0.3
theta[62]	-0.271	0.121	-0.509	-0.033	0.7
theta[329]	0.092	0.118	-0.140	0.323	0.6
theta[169]	0.024	0.118	-0.208	0.255	0.9

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[61]	-0.268	0.094	-0.453	-0.083	0.5
theta[62]	-0.303	0.101	-0.500	-0.106	0.8
theta[329]	0.053	0.096	-0.135	0.240	0.9
theta[169]	-0.007	0.091	-0.184	0.171	1.0