Simulation Results

2026-01-15

Simulation Setup

This simulation is performed with $n = 200$ and $d = 20$, using the 2-d lattice as the underlying graph. $s = 2$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.4$. The attached results are for a 10-replication simulation. The true values of the parameter vector θ are

```
0.7071068 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -0.7071068 0 0 0 0 ,
```

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{\ 1, 16, 5, 8\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

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### Mean-squared error comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat\theta
```

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[1]	0.052	0.042
theta[16]	0.039	0.032
theta[5]	0.004	0.026
theta[8]	0.020	0.015
total	0.029	0.029

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[1]	0.130	0.113
theta[16]	0.163	0.086
theta[5]	0.000	0.002
theta[8]	0.002	0.005
total	0.074	0.051

Mean absolute deviation comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat

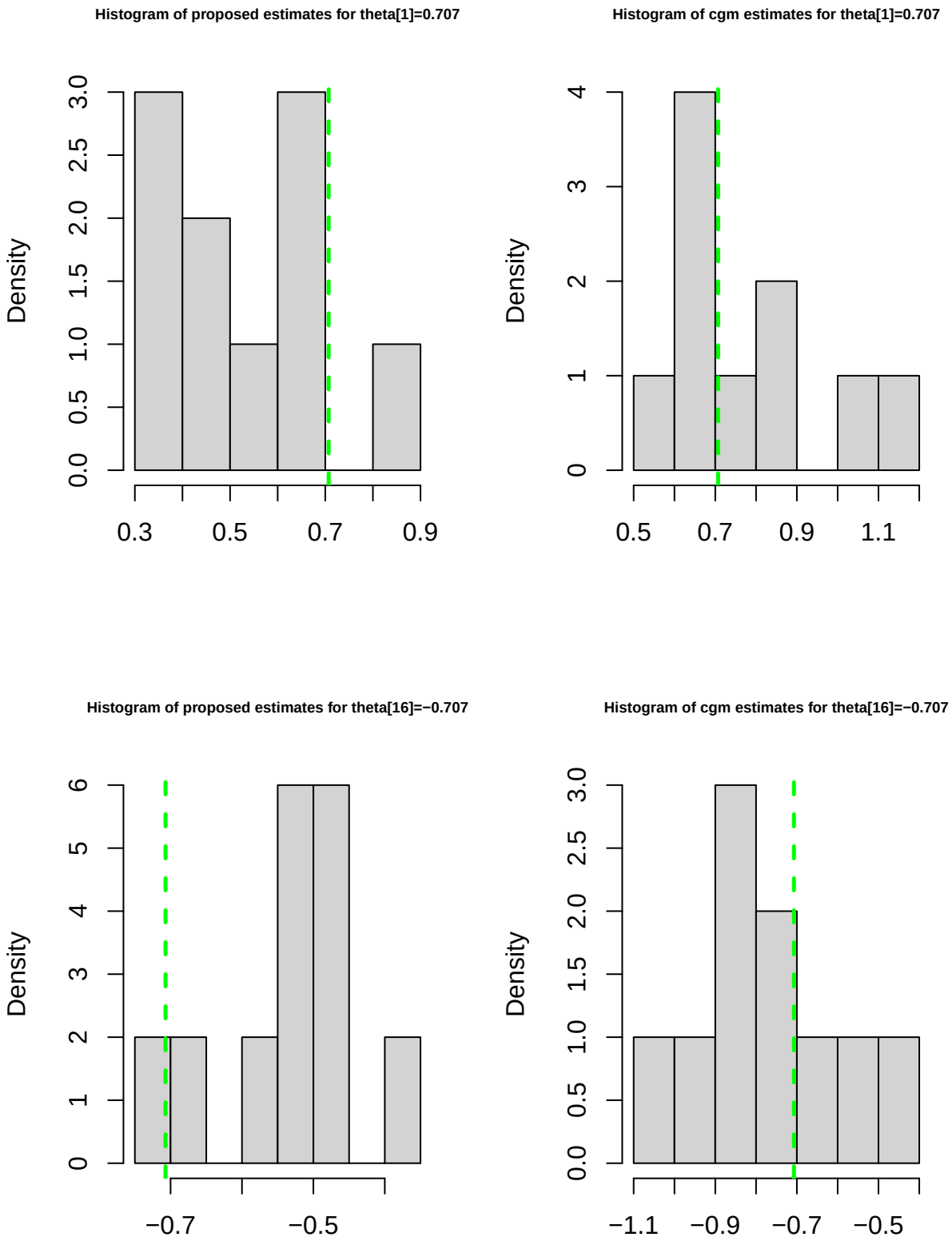
Table 3: Mean Absolute Deviation of Parameter Estimates

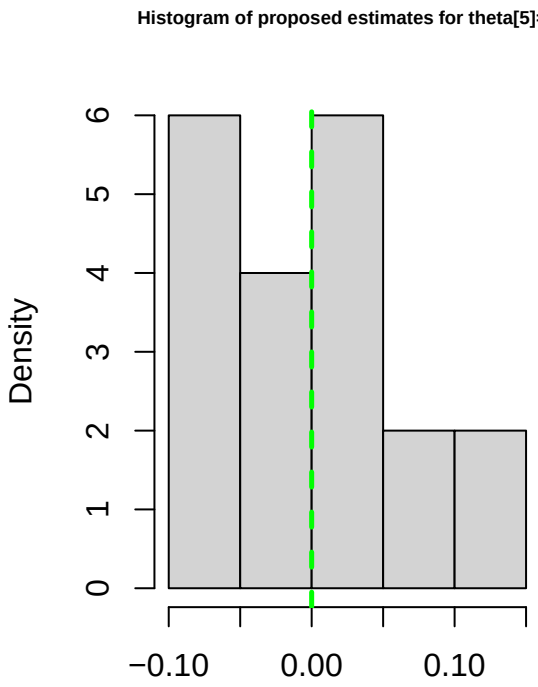
	proposed	cgm
theta[1]	0.188	0.156
theta[16]	0.175	0.151
theta[5]	0.056	0.122
theta[8]	0.092	0.091
total	0.128	0.130

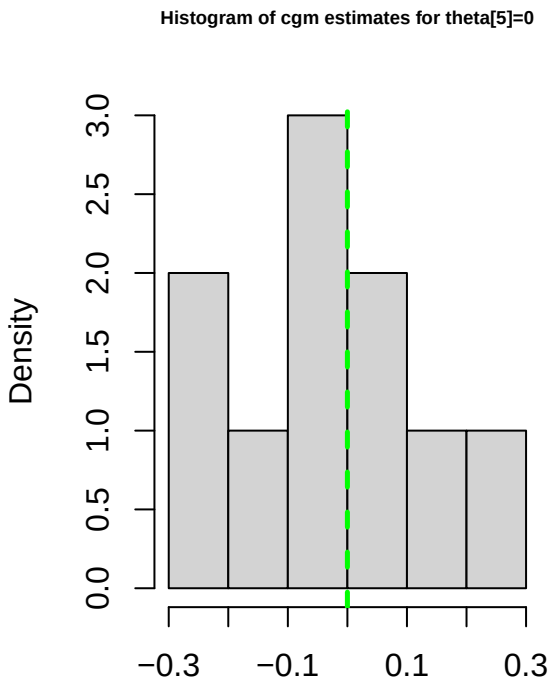
Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

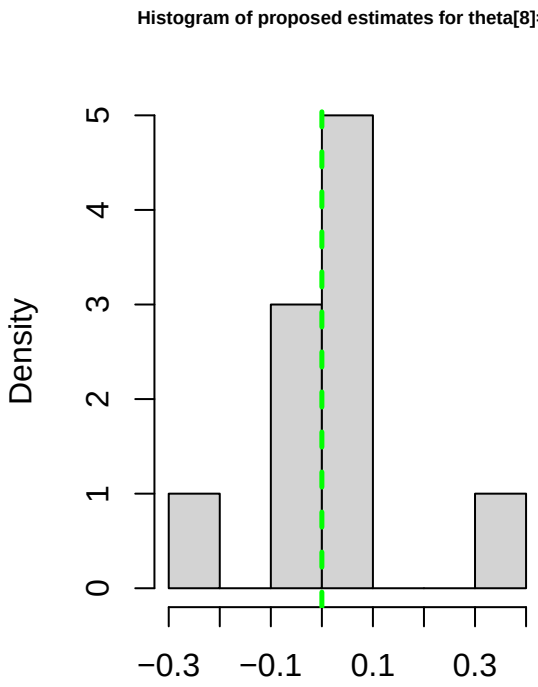
	proposed	cgm
theta[1]	0.337	0.268
theta[16]	0.374	0.247
theta[5]	0.000	0.020
theta[8]	0.016	0.042
total	0.182	0.145

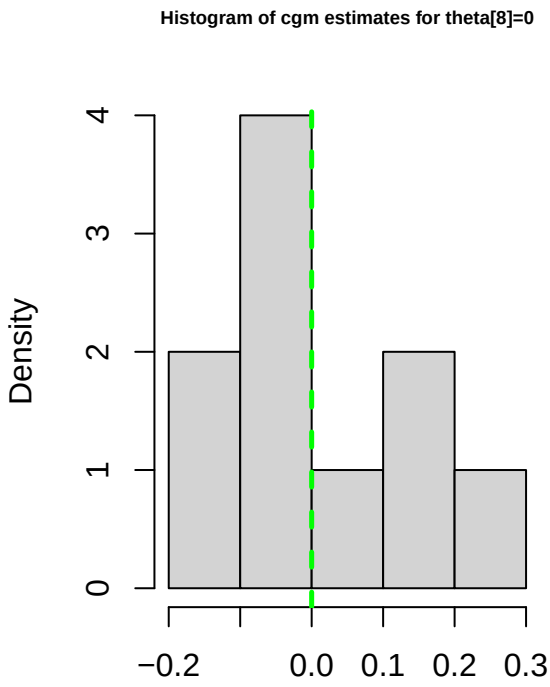
Boxplots


Histogram of proposed estimates for theta[1]=0.707
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0.3
0.5
0.7
0.9
Histogram of cgm estimates for theta[1]=0.707
Density
0
1
2
3
4
0.5
0.7
0.9
1.1
Histogram of proposed estimates for theta[16]=−0.707
Density
0
1
2
3
4
5
6
−0.7
−0.5
Histogram of cgm estimates for theta[16]=−0.707
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−1.1
−0.9
−0.7
−0.5

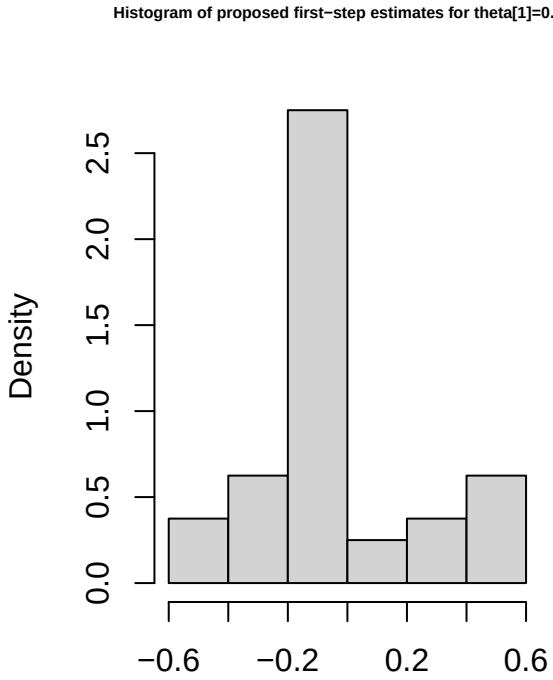

Histogram of proposed estimates for theta[5]=0
Density
0
1
2
3
4
5
6
−0.10
0.00
0.10

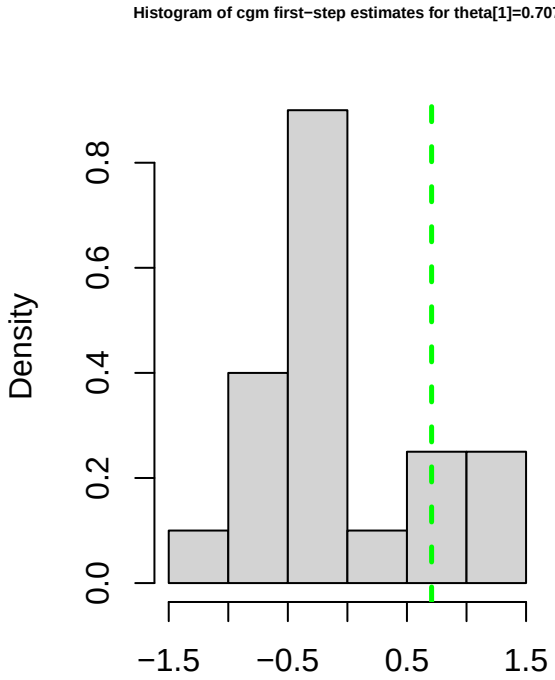

Histogram of cgm estimates for theta[5]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.3
−0.1
0.1
0.3

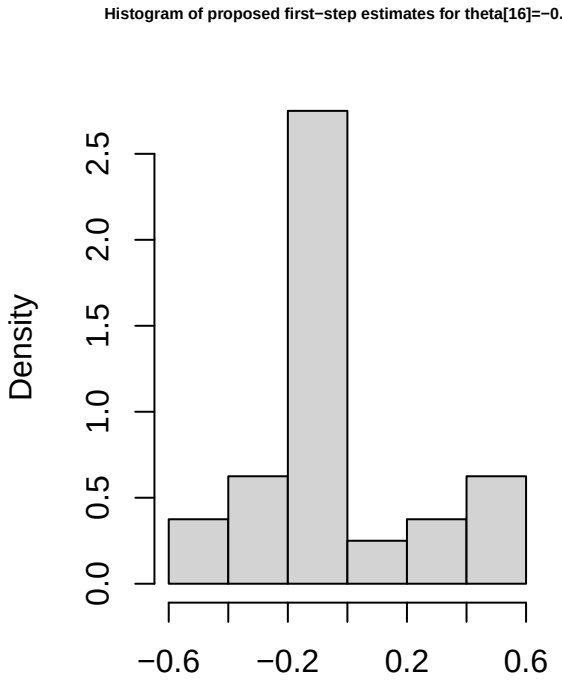

Histogram of proposed estimates for theta[8]=0
Density
0
1
2
3
4
5
−0.3
−0.1
0.1
0.3

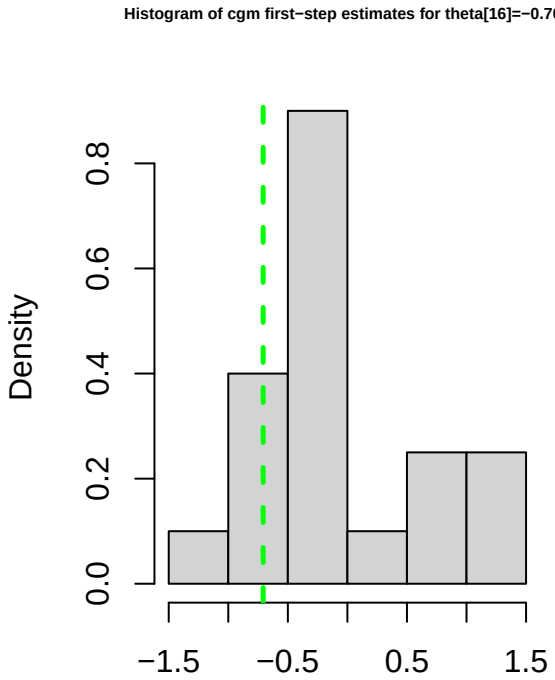

Histogram of cgm estimates for theta[8]=0
Density
0
1
2
3
4
−0.2
0.0
0.1
0.2
0.3

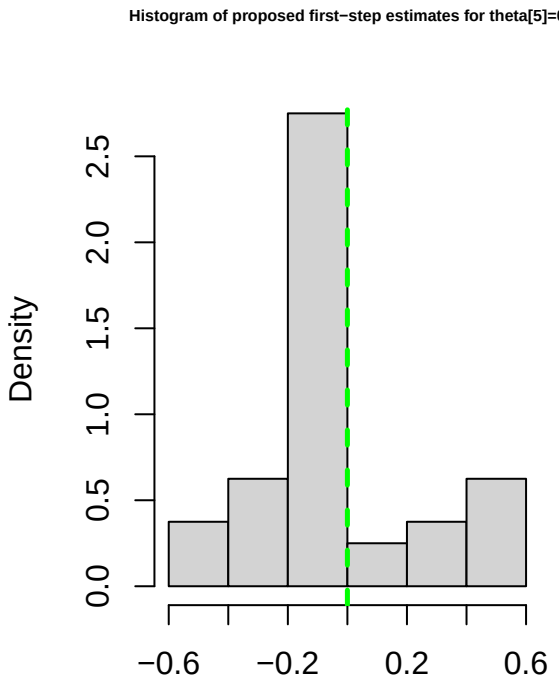
First Step Histograms

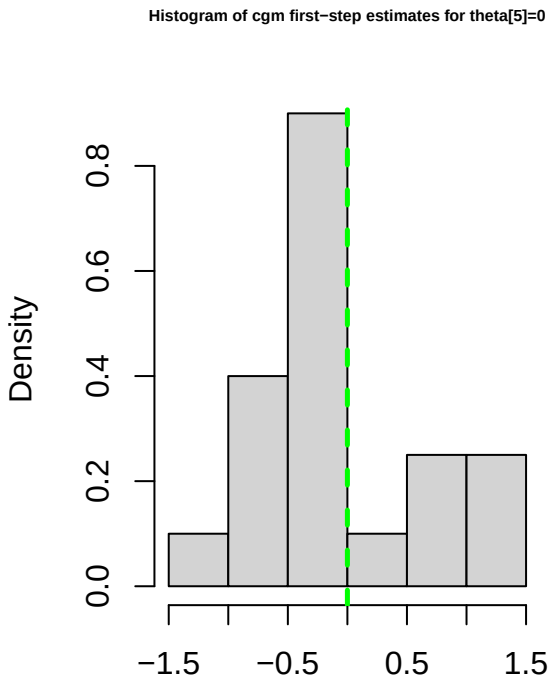


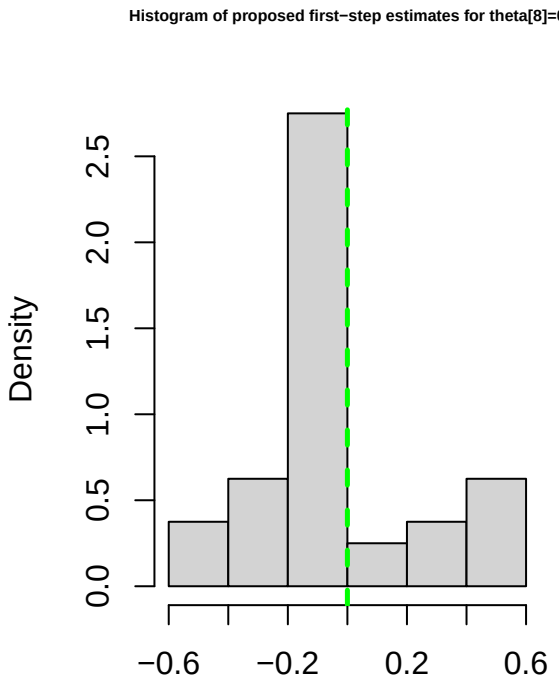


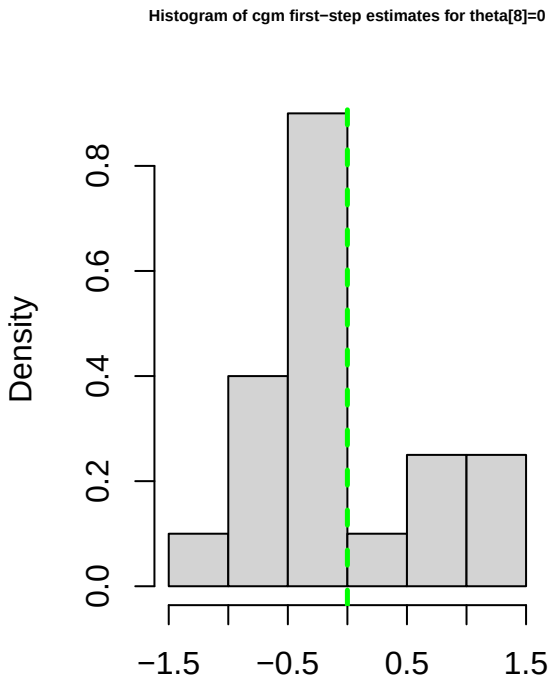





Histogram of proposed first−step estimates for theta[5]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
−0.6
−0.2
0.2
0.6


Histogram of cgm first−step estimates for theta[5]=0
Density
0.0
0.2
0.4
0.6
0.8
−1.5
−0.5
0.5
1.5


Histogram of proposed first−step estimates for theta[8]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
−0.6
−0.2
0.2
0.6


Histogram of cgm first−step estimates for theta[8]=0
Density
0.0
0.2
0.4
0.6
0.8
−1.5
−0.5
0.5
1.5

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 5: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[1]	0.327	0.516	0.801	0.339	0.776
theta[16]	-0.723	-0.514	-0.380	-0.711	-0.400
theta[5]	-0.068	-0.006	0.123	-0.064	0.112
theta[8]	-0.211	0.014	0.364	-0.175	0.304

Table 6: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[1]	0.514	0.714	1.167	0.541	1.138
theta[16]	-1.000	-0.792	-0.405	-0.988	-0.427
theta[5]	-0.267	-0.010	0.242	-0.266	0.229
theta[8]	-0.144	-0.006	0.299	-0.134	0.260

Statistics for Theoretical 95% Confidence Intervals

Table 7: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[1]	0.538	0.136	0.271	0.804	0.6
theta[16]	-0.535	0.136	-0.801	-0.269	0.9
theta[5]	0.008	0.126	-0.239	0.255	1.0
theta[8]	0.028	0.126	-0.219	0.276	0.9

Table 8: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[1]	0.778	0.176	0.434	1.122	0.9
theta[16]	-0.750	0.175	-1.092	-0.408	1.0
theta[5]	-0.025	0.130	-0.280	0.229	0.8
theta[8]	0.020	0.125	-0.225	0.264	1.0