

# Simulation Results

2026-01-26

## Simulation Setup

This simulation is performed with  $n = 200$  and  $d = 200$ , using the 2-d lattice as the underlying graph.  $s = 5$  parameters are set to be nonzero, and the beta parameter is chosen to be  $\beta = 0$ . The attached results are for a 10-replication simulation. The parameter vector  $\theta$  has sparse components other than the following:

Parameter.Index	Value
59	-0.447
60	-0.447
76	0.447
78	0.447
134	0.447

but for brevity, our simulation only estimates the indices of  $\theta$  in  $\mathcal{C} = \{ 59, 60, 70, 172 \}$  elements of  $\theta$ . Accordingly, **all statistics and visuals are indicative of performance only on the set  $\mathcal{C}$ .**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

## Results

### Mean-squared error comparison

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[59]	0.042	0.012
theta[60]	0.036	0.036
theta[70]	0.011	0.006
theta[172]	0.017	0.008
total	0.027	0.016

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[59]	0.133	0.064

	proposed	cgm
theta[60]	0.143	0.100
theta[70]	0.000	0.000
theta[172]	0.001	0.000
total	0.070	0.041

### Mean absolute deviation comparison  $\frac{1}{n_{\text{sim}}} \sum_{i=1}^{n_{\text{sim}}} \frac{1}{|\mathcal{C}|} \|\hat{\theta}_i - \theta\|$

Table 3: Mean Absolute Deviation of Parameter Estimates

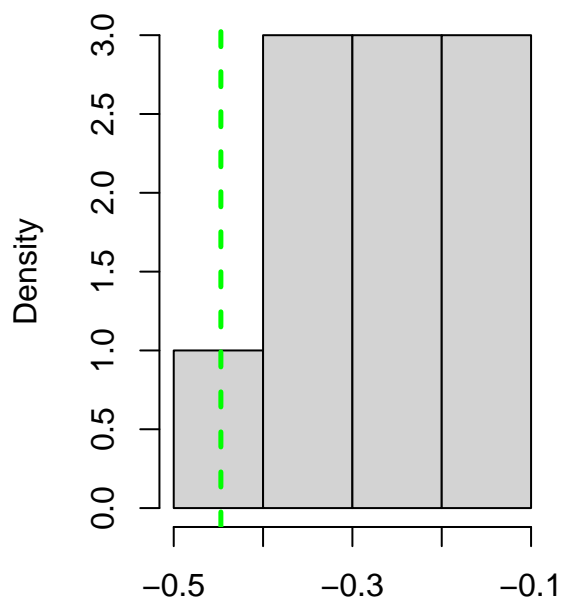
	proposed	cgm
theta[59]	0.187	0.090
theta[60]	0.170	0.172
theta[70]	0.072	0.058
theta[172]	0.101	0.070
total	0.133	0.098

Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

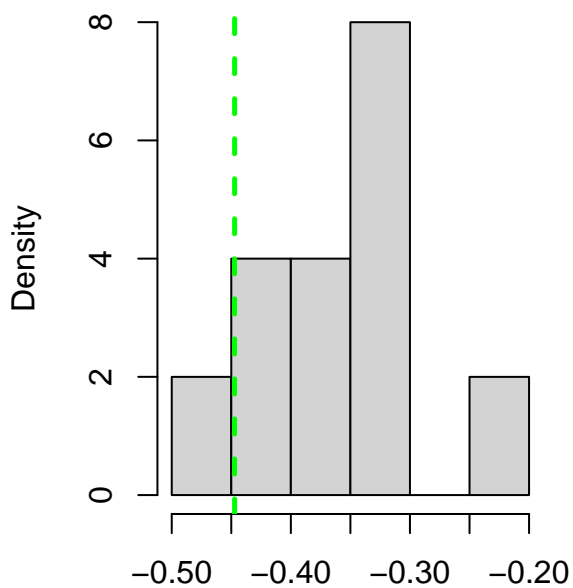
	proposed	cgm
theta[59]	0.357	0.236
theta[60]	0.362	0.295
theta[70]	0.000	0.000
theta[172]	0.014	0.000
total	0.183	0.133

## Boxplots

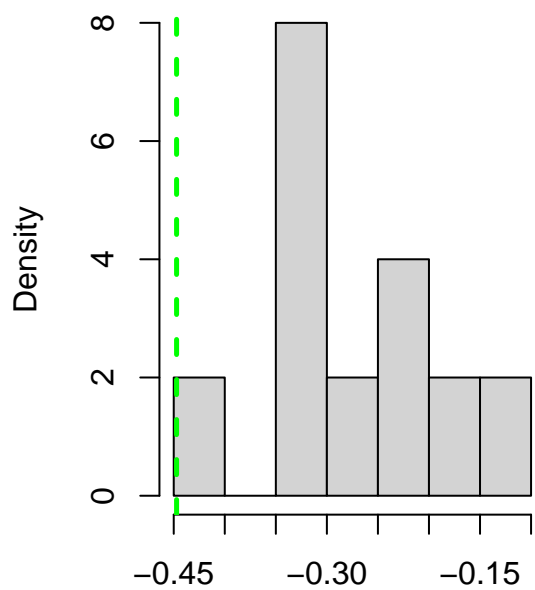
Histogram of proposed estimates for  $\theta[59] = -0.447$



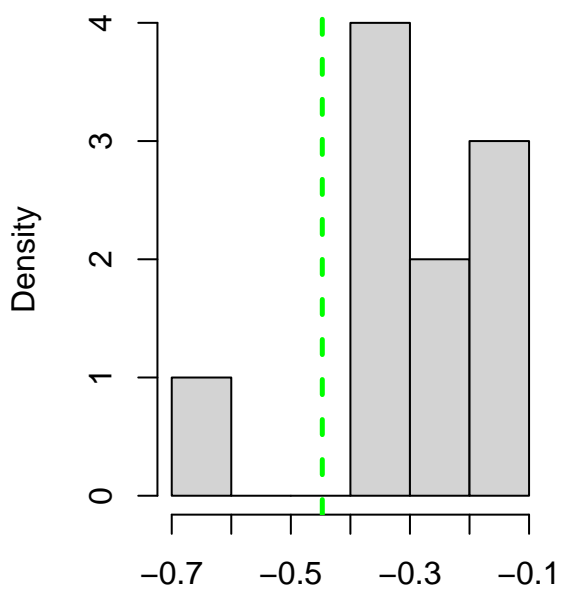
Histogram of cgm estimates for  $\theta[59] = -0.447$



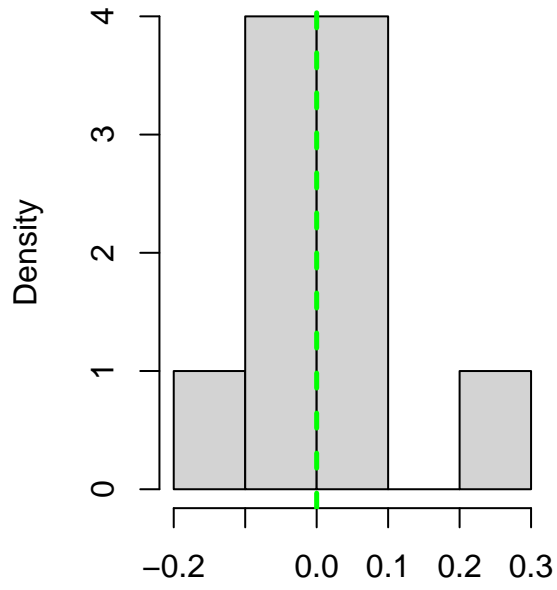
Histogram of proposed estimates for  $\theta[60] = -0.447$



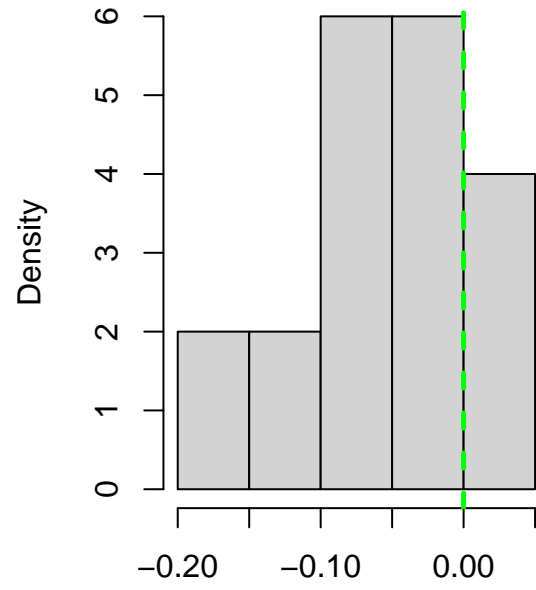
Histogram of cgm estimates for  $\theta[60] = -0.447$



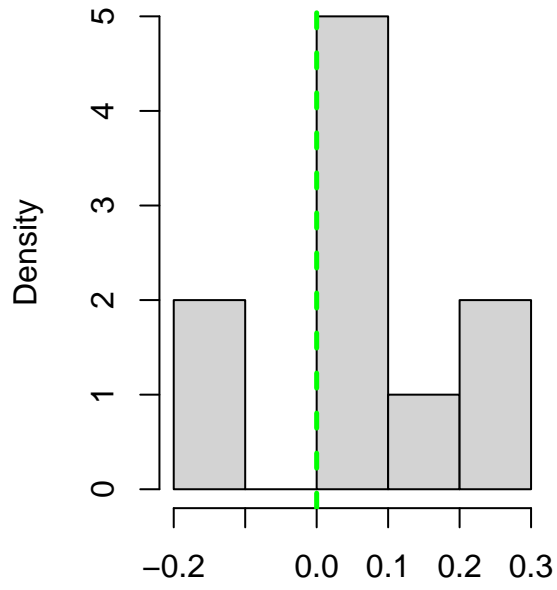
Histogram of proposed estimates for  $\theta_{[70]}=0$



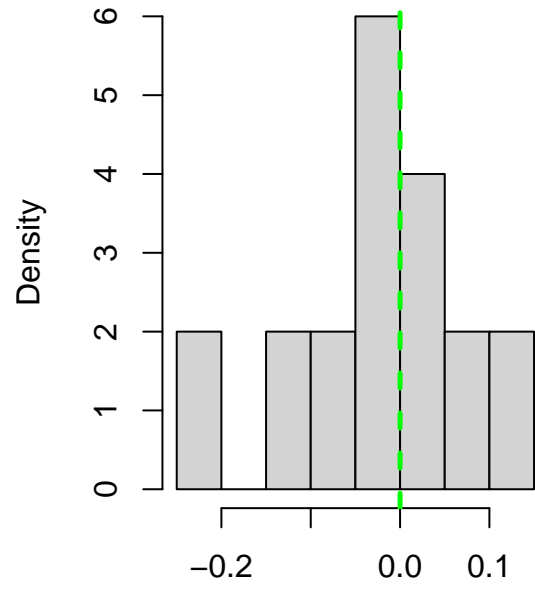
Histogram of cgm estimates for  $\theta_{[70]}=0$



Histogram of proposed estimates for  $\theta_{172}=0$



Histogram of cgm estimates for  $\theta_{172}=0$



## Statistics and 95% Confidence Intervals from per-Replicate Estimates

### Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[59]	-0.269	0.116	-0.498	-0.041	0.7
theta[60]	-0.277	0.114	-0.500	-0.054	0.8
theta[70]	0.015	0.117	-0.214	0.245	1.0
theta[172]	0.049	0.120	-0.186	0.283	0.9

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[59]	-0.363	0.107	-0.573	-0.154	1.0
theta[60]	-0.306	0.111	-0.523	-0.089	0.6
theta[70]	-0.046	0.105	-0.252	0.159	1.0
theta[172]	-0.019	0.111	-0.237	0.199	1.0