

Simulation Results

2026-01-16

Simulation Setup

This simulation is performed with $n = 200$ and $d = 20$, using the 2-d lattice as the underlying graph. $s = 5$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.4$. The attached results are for a 10-replication simulation. The true values of the parameter vector θ are

0 0 0 -0.4472136 0.4472136 0 0.4472136 0 0 0 0 0 0 0 0.4472136 0 0 -0.4472136 0 ,

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{4, 5, 12, 6\}$ elements of θ . Accordingly, **all statistics and visuals are indicative of performance only on the set \mathcal{C} .**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison $\frac{1}{n_{\text{sim}}} \sum_{i=1}^{n_{\text{sim}}} \frac{1}{|\mathcal{C}|} \|\hat{\theta}_i - \theta\|^2$

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
$\theta[4]$	0.048	0.017
$\theta[5]$	0.037	0.031
$\theta[12]$	0.011	0.033
$\theta[6]$	0.009	0.017
total	0.026	0.024

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
$\theta[4]$	0.123	0.022
$\theta[5]$	0.110	0.023
$\theta[12]$	0.000	0.013
$\theta[6]$	0.000	0.019
total	0.058	0.019

Mean absolute deviation comparison $\frac{1}{n.\text{sim}} \sum_{i=1}^{n.\text{sim}} \frac{1}{|\mathcal{C}|} \|\hat{\theta}_i - \theta\|$

Table 3: Mean Absolute Deviation of Parameter Estimates

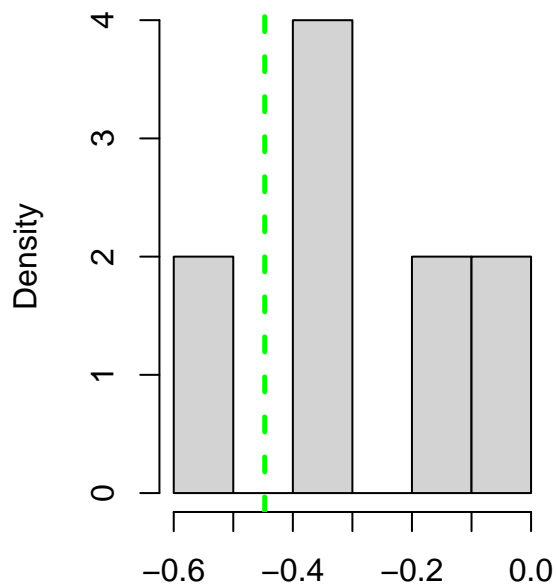
	proposed	cgm
theta[4]	0.185	0.110
theta[5]	0.177	0.149
theta[12]	0.075	0.145
theta[6]	0.076	0.096
total	0.128	0.125

Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

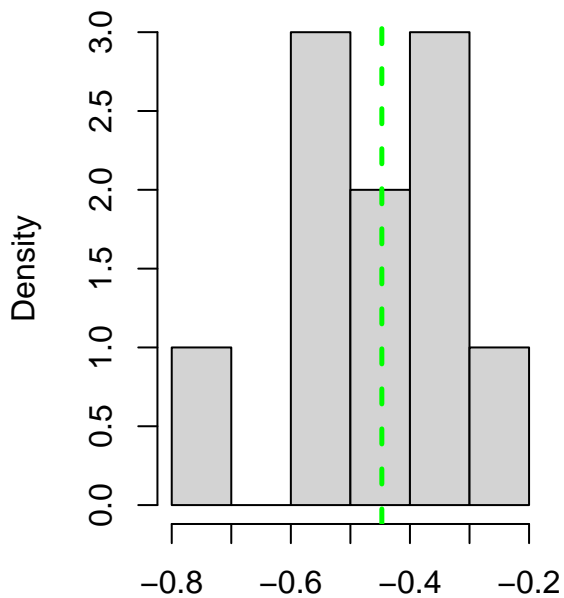
	proposed	cgm
theta[4]	0.343	0.125
theta[5]	0.315	0.133
theta[12]	0.000	0.054
theta[6]	0.003	0.068
total	0.165	0.095

Boxplots

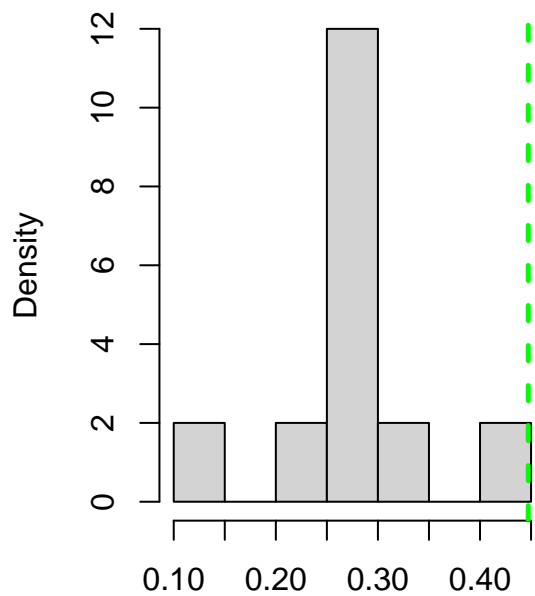
Histogram of proposed estimates for $\theta[4]=-0.447$



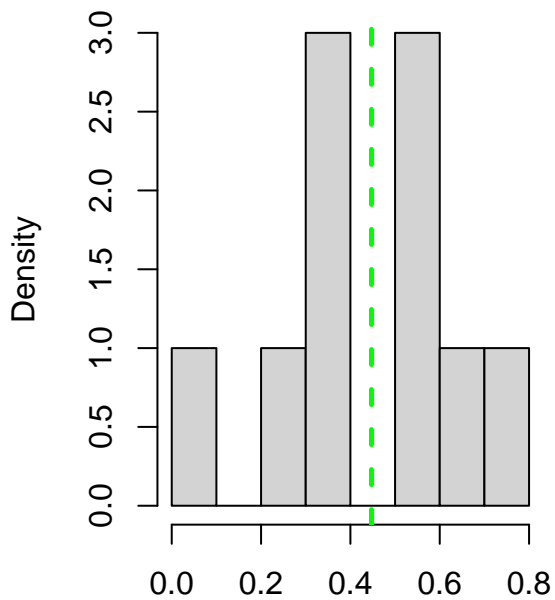
Histogram of cgm estimates for $\theta[4]=-0.447$



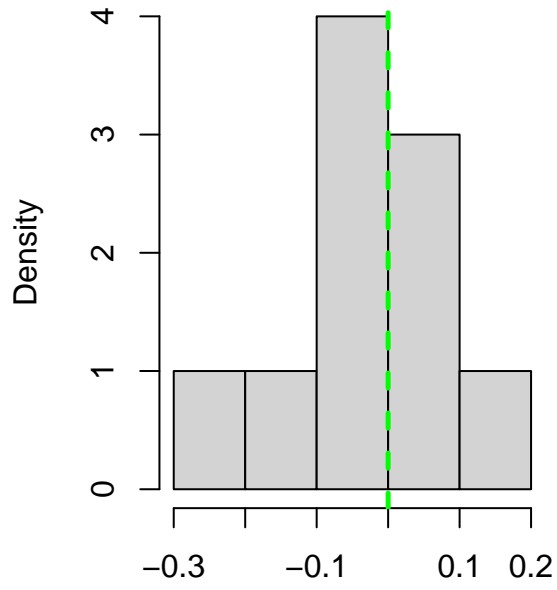
Histogram of proposed estimates for $\theta[5]=0.447$



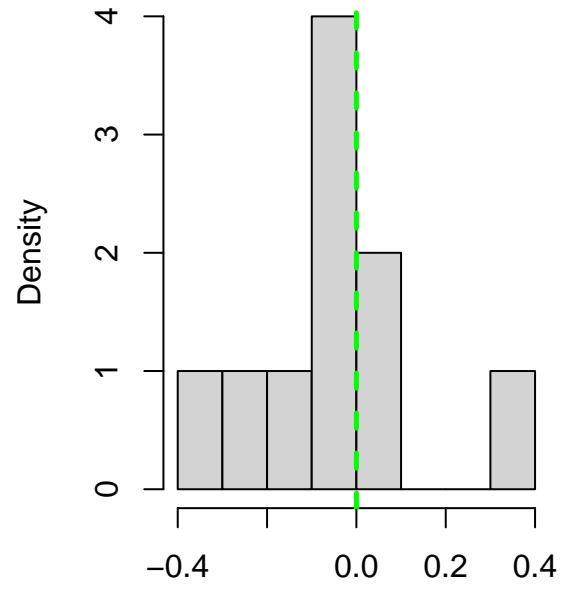
Histogram of cgm estimates for $\theta[5]=0.447$

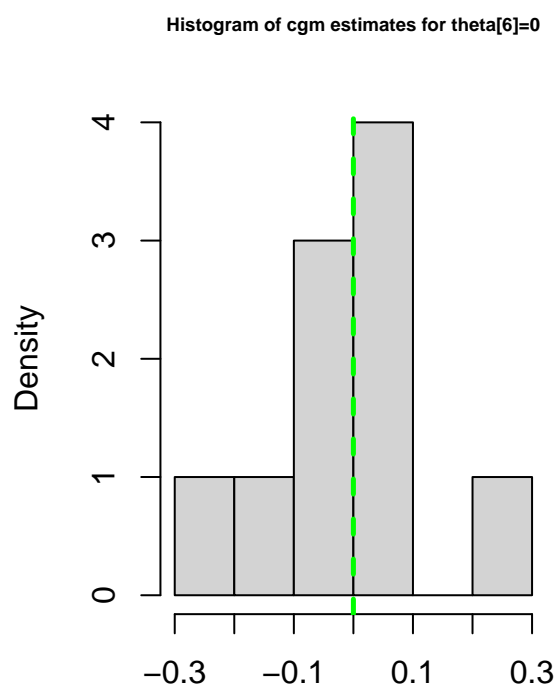
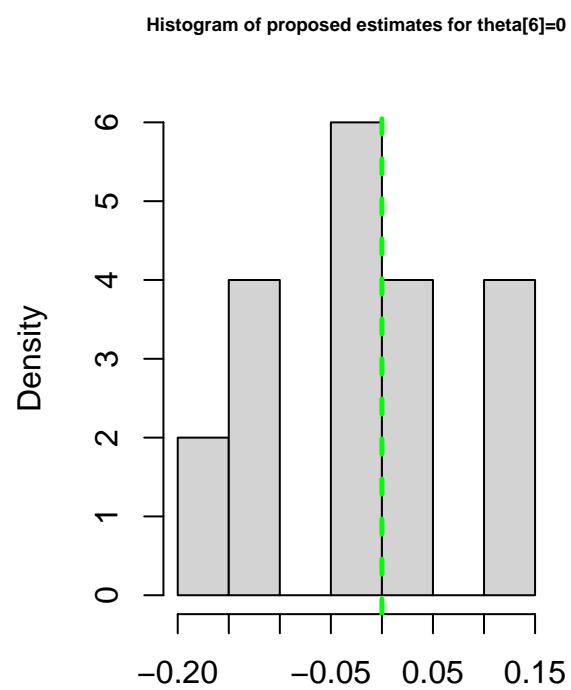


Histogram of proposed estimates for $\theta_{12}=0$



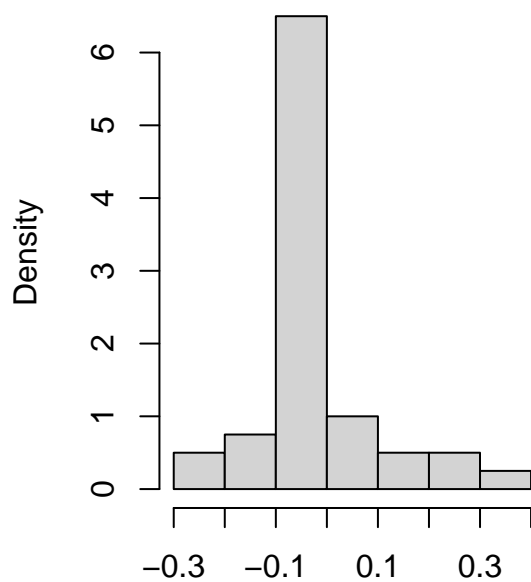
Histogram of cgm estimates for $\theta_{12}=0$



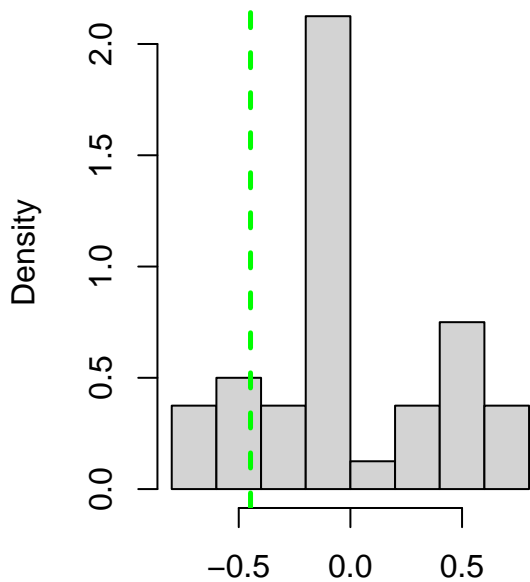


First Step Histograms

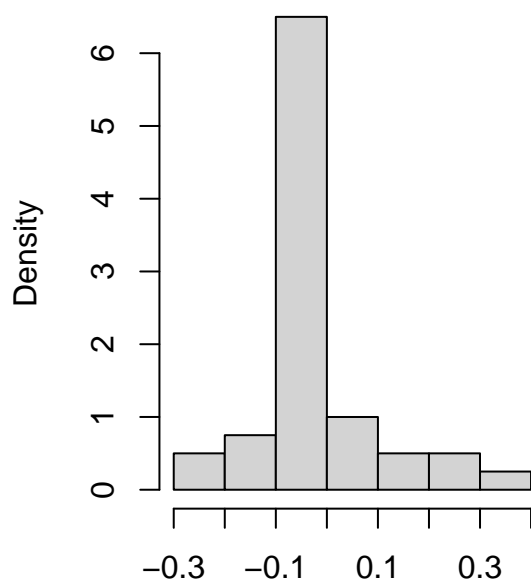
Histogram of proposed first-step estimates for $\theta_4 = -0.447$



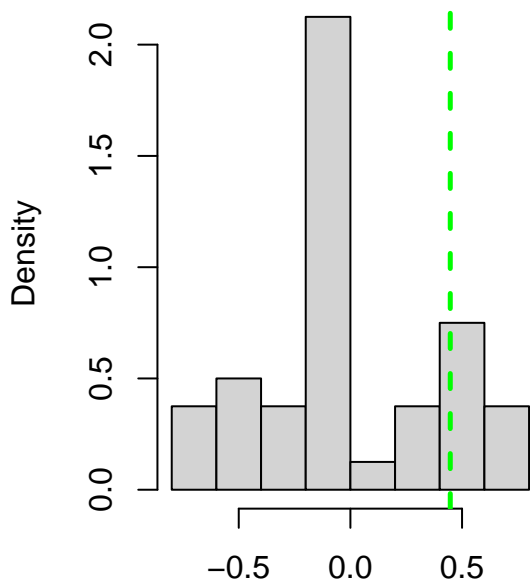
Histogram of cgm first-step estimates for $\theta_4 = -0.447$



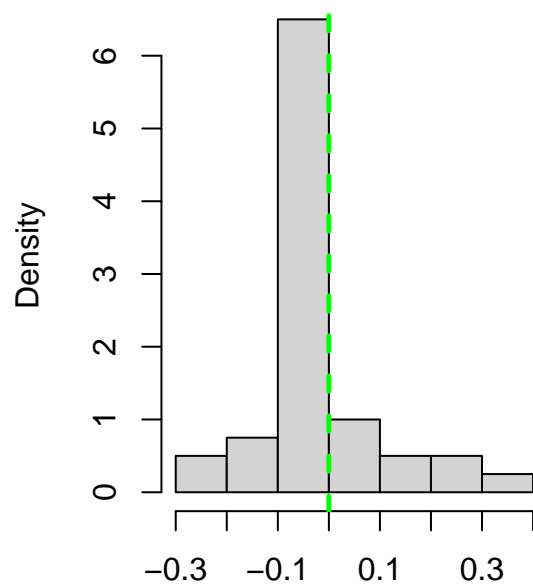
Histogram of proposed first-step estimates for $\theta_5 = 0.447$



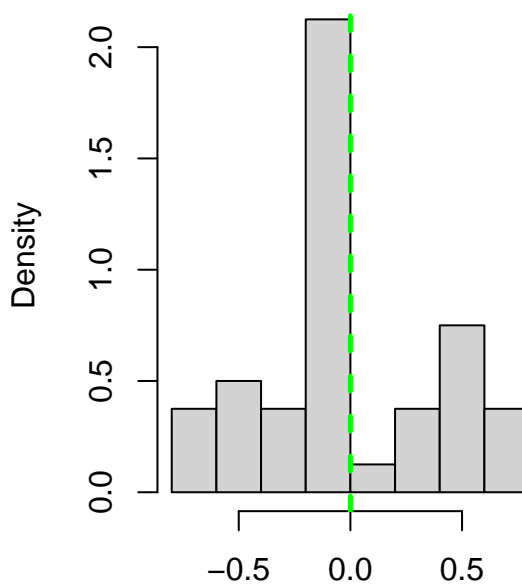
Histogram of cgm first-step estimates for $\theta_5 = 0.447$



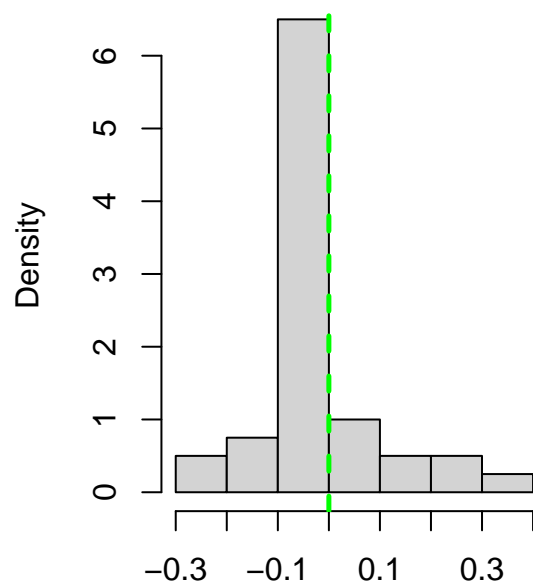
Histogram of proposed first-step estimates for $\theta_{12}=0$



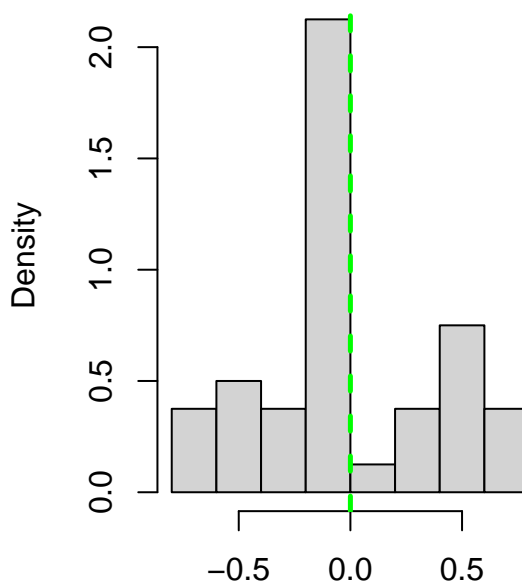
Histogram of cgm first-step estimates for $\theta_{12}=0$



Histogram of proposed first-step estimates for $\theta_6=0$



Histogram of cgm first-step estimates for $\theta_6=0$



Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 5: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[4]	-0.545	-0.338	-0.063	-0.538	-0.071
theta[5]	0.105	0.275	0.411	0.129	0.394
theta[12]	-0.265	-0.013	0.101	-0.233	0.095
theta[6]	-0.150	-0.005	0.122	-0.150	0.121

Table 6: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[4]	-0.708	-0.434	-0.258	-0.680	-0.270
theta[5]	0.091	0.451	0.717	0.136	0.698
theta[12]	-0.306	-0.072	0.333	-0.296	0.279
theta[6]	-0.277	-0.003	0.223	-0.245	0.189

Statistics for Theoretical 95% Confidence Intervals

Table 7: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[4]	-0.296	0.127	-0.545	-0.046	0.6
theta[5]	0.270	0.119	0.036	0.504	0.9
theta[12]	-0.036	0.123	-0.276	0.205	0.9
theta[6]	-0.011	0.122	-0.251	0.229	1.0

Table 8: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[4]	-0.450	0.126	-0.696	-0.204	1.0
theta[5]	0.439	0.124	0.197	0.682	0.8
theta[12]	-0.053	0.119	-0.285	0.180	0.7
theta[6]	-0.021	0.115	-0.247	0.204	0.8