Simulation Results

2026-01-08

Simulation Setup

This simulation is performed with $n = 200$ and $d = 20$, using the 2-d lattice as the underlying graph. $s = 2$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0$. The attached results are for a 10-replication simulation. The true values of the parameter vector θ are

```
0 0 0 -0.7071068 0 0 0 0 0 0 0 0 0 0 0.7071068 0 0 0 0 ,
```

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{$ 4, 16, 5, 8$\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison ($\frac{1}{n.sim}\sum_{i=1}^{n.sim}\frac{1}{|\mathcal{C}|}\|\hat{\theta}_{i,\mathcal{C}} - \theta_{\mathcal{C}}\|^2$)

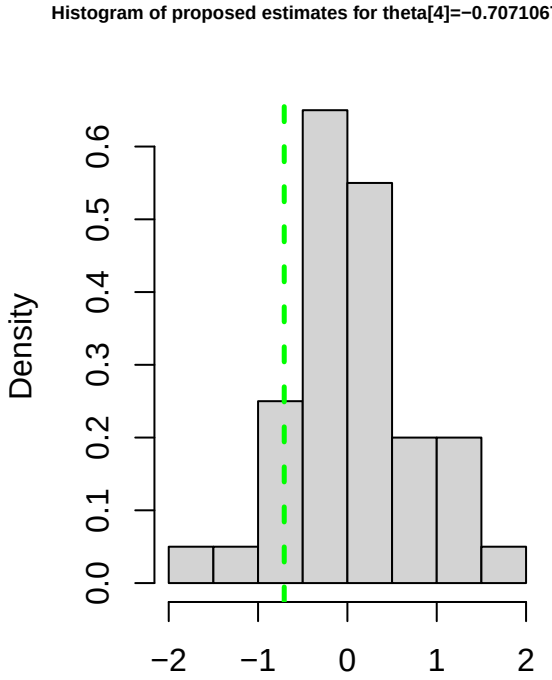
Table 1: Mean-Squared Error of Parameter Estimates

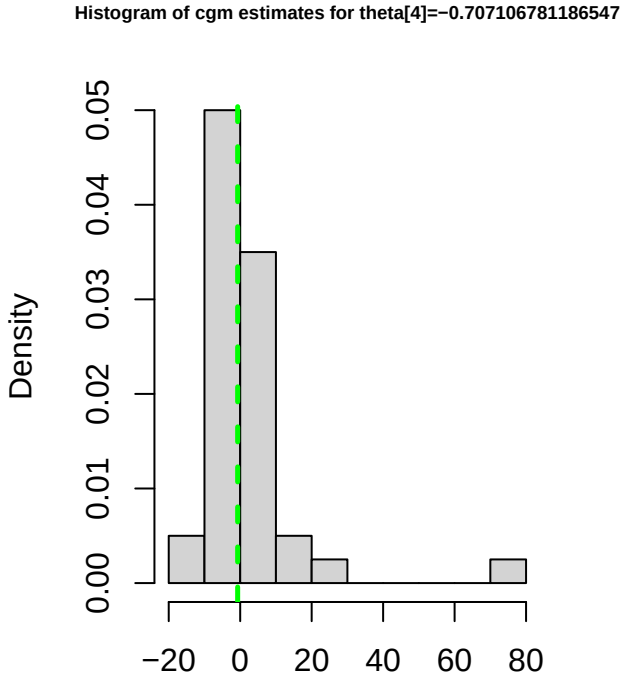
	proposed	cgm
theta[4]	0.228	693.351
theta[16]	0.353	63.410
theta[5]	0.108	0.315
theta[8]	0.132	1.439
total	0.206	189.629

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[4]	0.256	3.748
theta[16]	0.179	2.364
theta[5]	0.107	0.087
theta[8]	0.022	0.240
total	0.141	1.610

Boxplots


Histogram of proposed estimates for theta[4]=−0.7071067811865
Density
0.0
0.1
0.2
0.3
0.4
0.5
0.6
−2
−1
0
1
2


Histogram of cgm estimates for theta[4]=−0.707106781186547
Density
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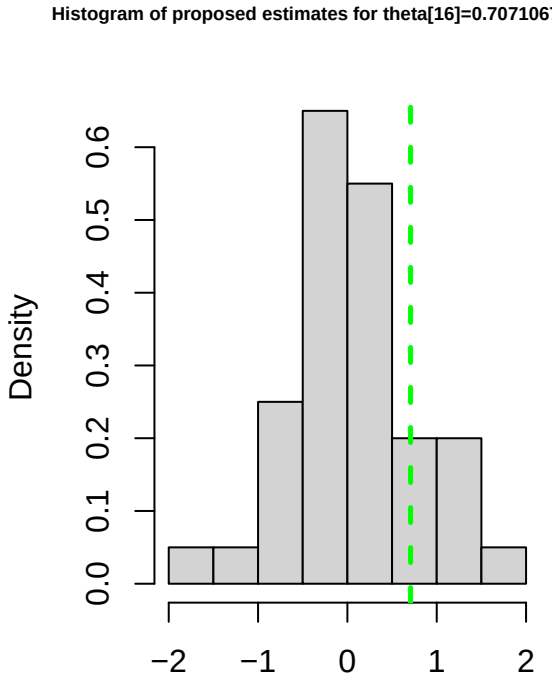
0.02
0.03
0.04
0.05
−20
0
20
40
60
80


Histogram of proposed estimates for theta[16]=0.7071067811865
Density
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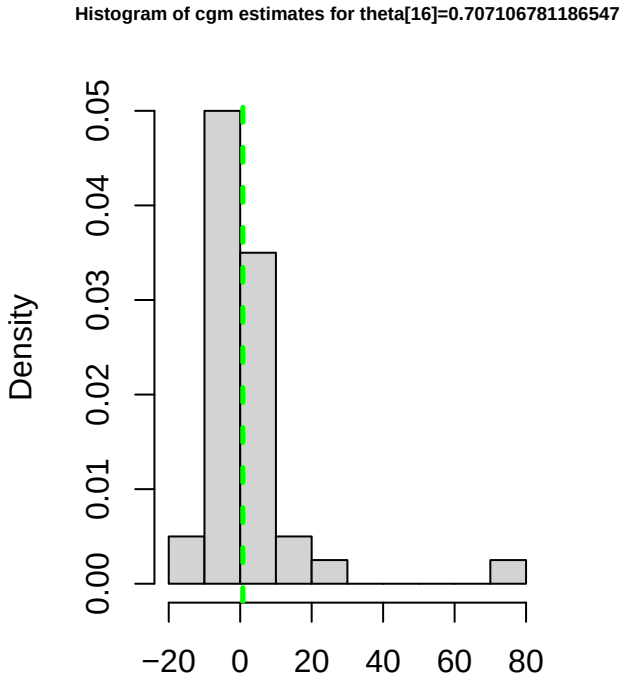
0.2
0.3
0.4
0.5
0.6
−2
−1
0
1
2


Histogram of cgm estimates for theta[16]=0.707106781186547
Density
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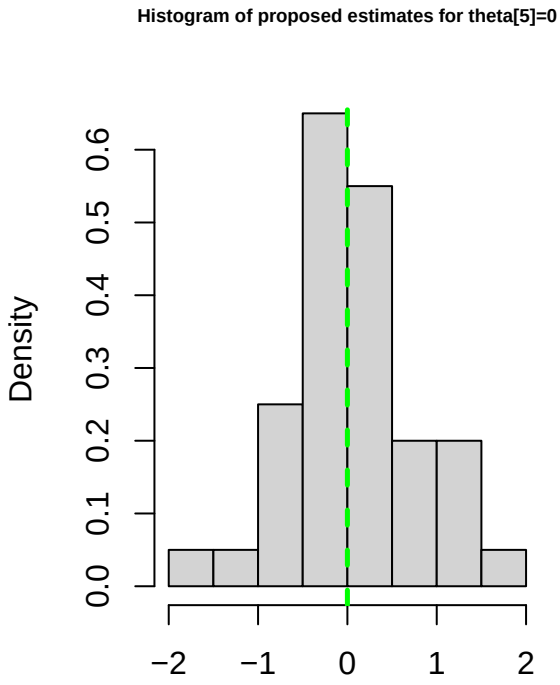
0.01
0.02
0.03
0.04
0.05
−20
0
20
40
60
80


Histogram of proposed estimates for theta[5]=0
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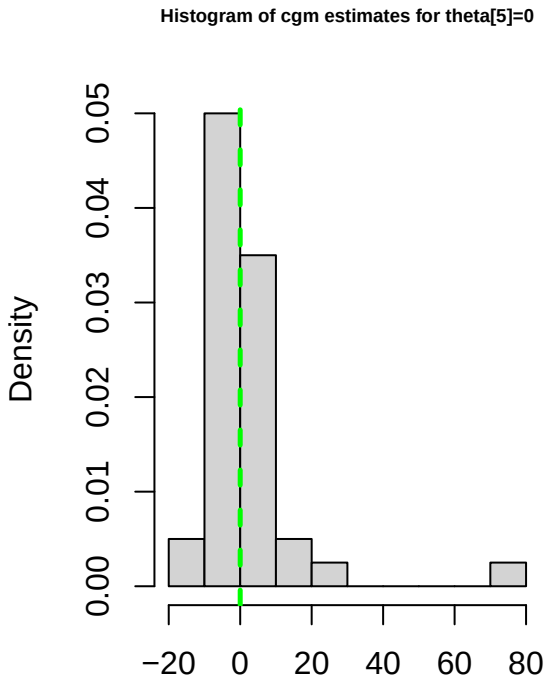
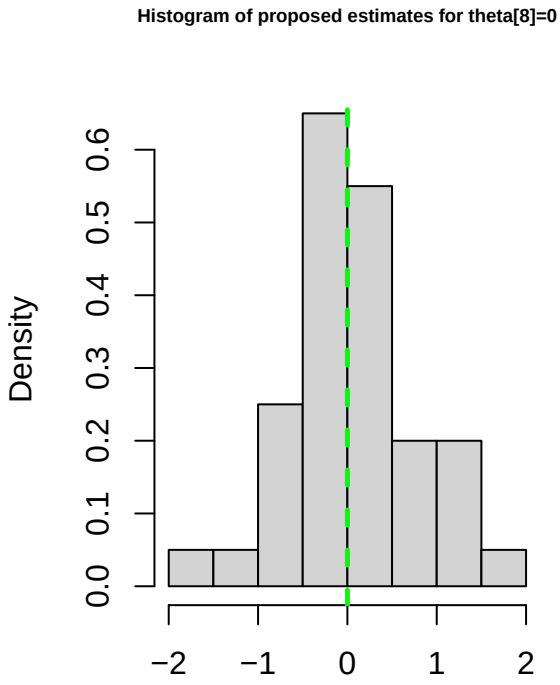
0.2
0.3
0.4
0.5
0.6
−2
−1
0
1
2


Histogram of cgm estimates for theta[5]=0
Density
0.00
0.01
0.02
0.03
0.04
0.05
−20
0
20
40
60
80


Histogram of proposed estimates for theta[8]=0
Density
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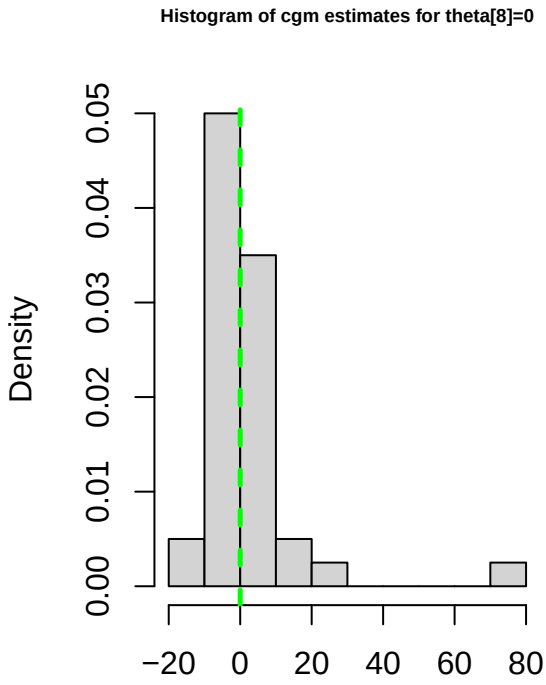
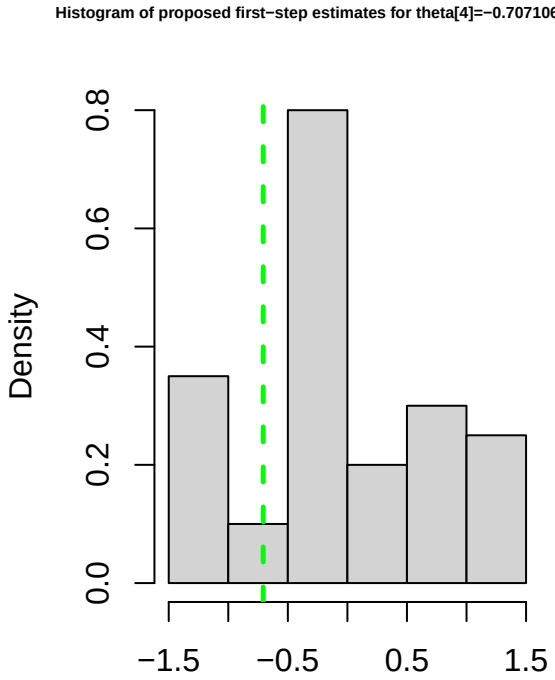
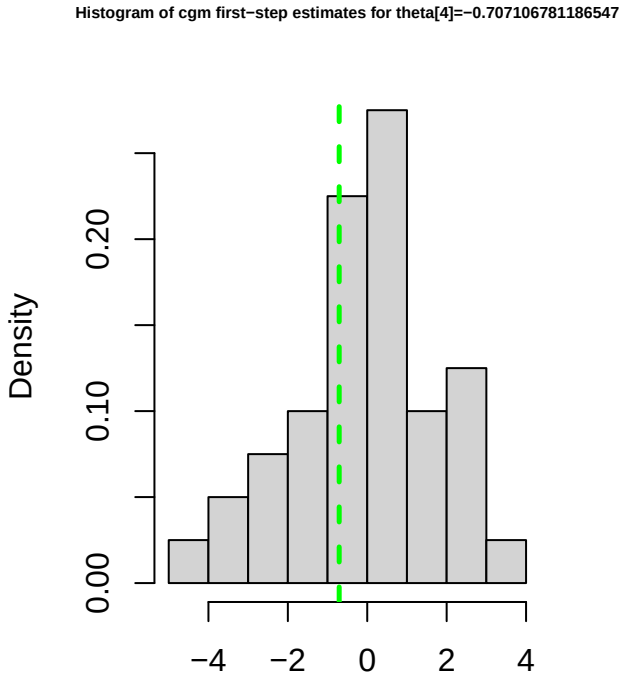
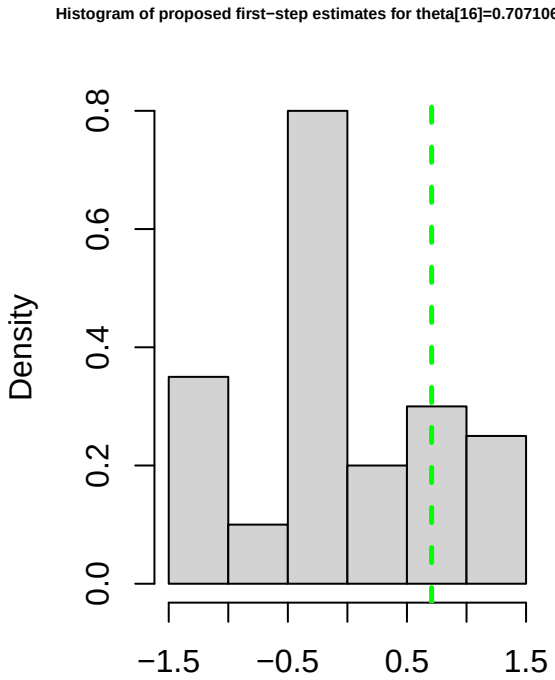
0.1
0.2
0.3
0.4
0.5
0.6
−2
−1
0
1
2

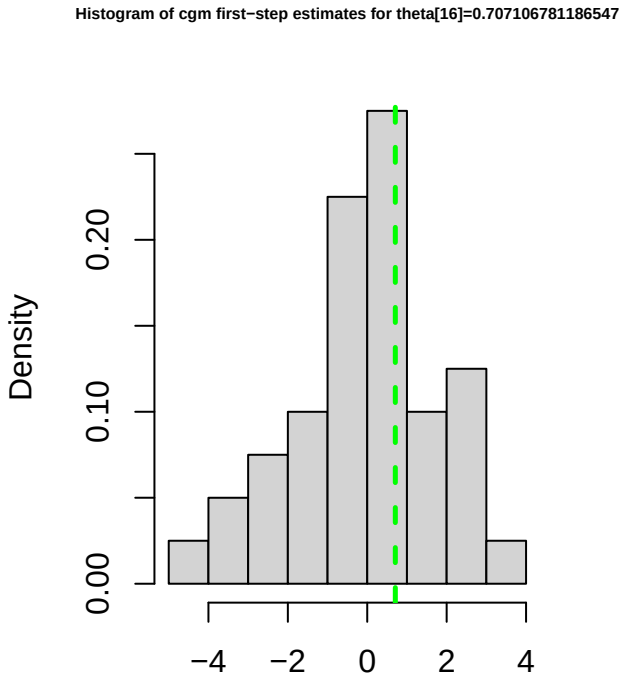

Histogram of cgm estimates for theta[8]=0
Density
0.00
0.01
0.02
0.03
0.04
0.05
−20
0
20
40
60
80

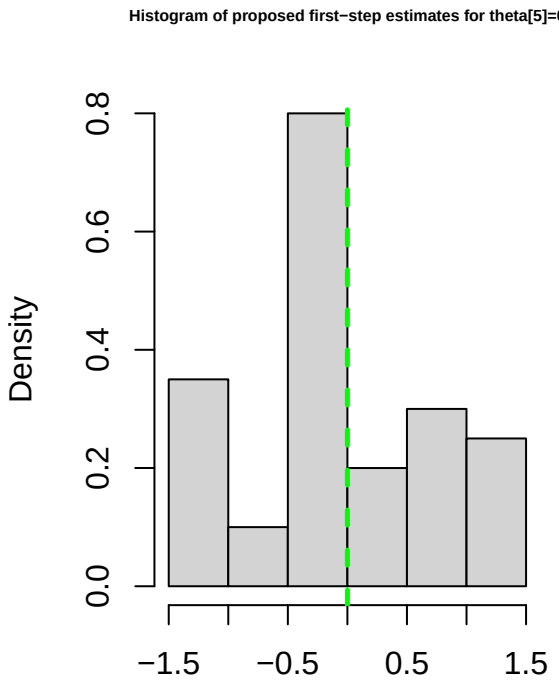
First Step Histograms

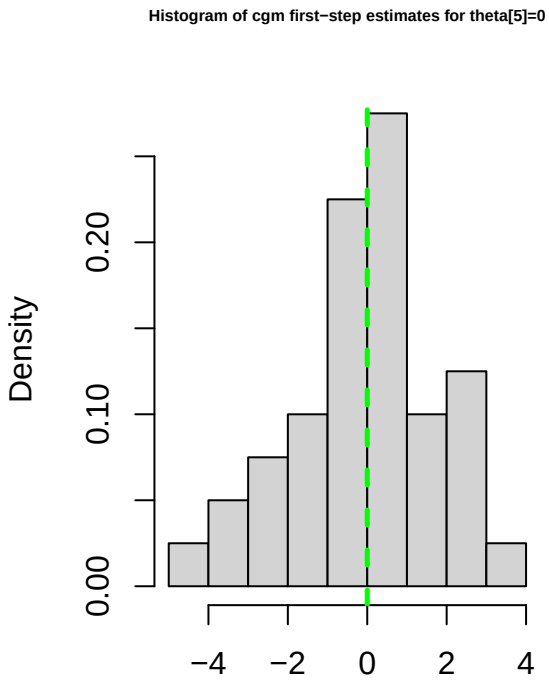


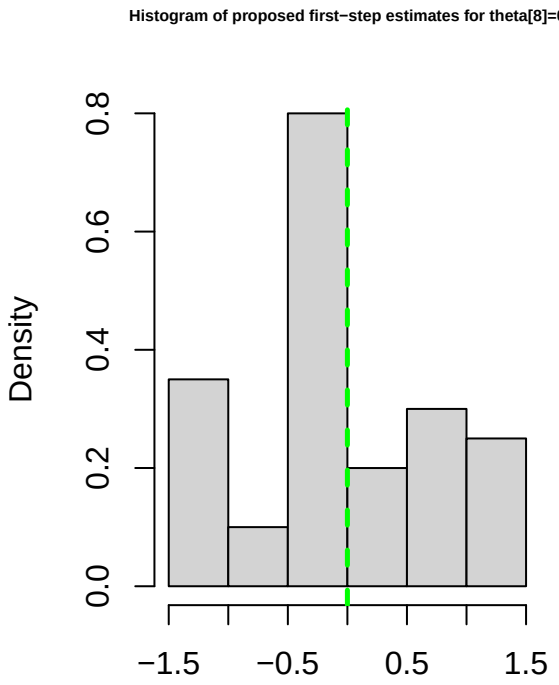


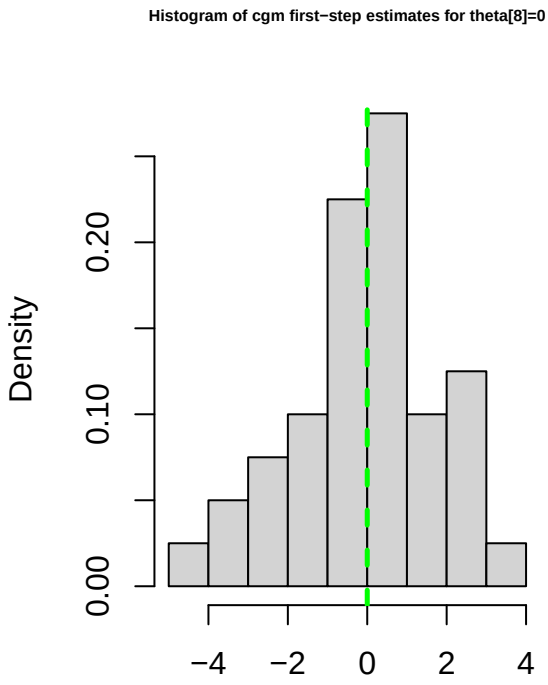





Histogram of proposed first−step estimates for theta[5]=0
Density
0.0
0.2
0.4
0.6
0.8
−1.5
−0.5
0.5
1.5


Histogram of cgm first−step estimates for theta[5]=0
Density
0.00
0.10
0.20
−4
−2
0
2
4


Histogram of proposed first−step estimates for theta[8]=0
Density
0.0
0.2
0.4
0.6
0.8
−1.5
−0.5
0.5
1.5


Histogram of cgm first−step estimates for theta[8]=0
Density
0.00
0.10
0.20
−4
−2
0
2
4

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 3: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[4]	-1.563	-0.497	0.086	-1.483	0.051
theta[16]	-0.403	0.996	1.795	-0.188	1.688
theta[5]	-0.815	0.094	0.316	-0.712	0.312
theta[8]	-0.874	-0.010	0.270	-0.814	0.247

Table 4: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[4]	-1.079	3.000	73.738	-0.991	63.865
theta[16]	-15.690	-2.757	0.738	-14.777	0.688
theta[5]	-1.115	0.051	0.928	-0.941	0.898
theta[8]	-3.240	-0.177	0.692	-2.815	0.676

Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[4]	-0.624	0.277	-1.166	-0.082	0.7
theta[16]	0.914	0.263	0.399	1.430	0.6
theta[5]	-0.018	0.212	-0.433	0.397	0.9
theta[8]	-0.131	0.200	-0.524	0.261	0.8

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[4]	13.449	0.470	12.528	14.371	0.3
theta[16]	-5.133	0.356	-5.830	-4.436	0.2
theta[5]	0.074	0.218	-0.354	0.502	0.6
theta[8]	-0.429	0.197	-0.815	-0.044	0.3