Simulation Results

2026-01-21

Simulation Setup

This simulation is performed with $n = 200$ and $d = 400$, using the 2-d lattice as the underlying graph. $s = 5$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0$. The attached results are for a 10-replication simulation. The parameter vector θ has sparse components other than the following:

Parameter.Index	Value
28	-0.447
36	0.447
47	0.447
61	0.447
117	0.447

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{$ 28, 36, 329, 172$\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[28]	0.075	0.077
theta[36]	0.058	0.090
theta[329]	0.019	0.065
theta[172]	0.012	0.048
total	0.041	0.070

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[28]	0.132	0.087

	proposed	cgm
theta[36]	0.100	0.029
theta[329]	0.005	0.000
theta[172]	0.000	0.004
total	0.059	0.030

Mean absolute deviation comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat

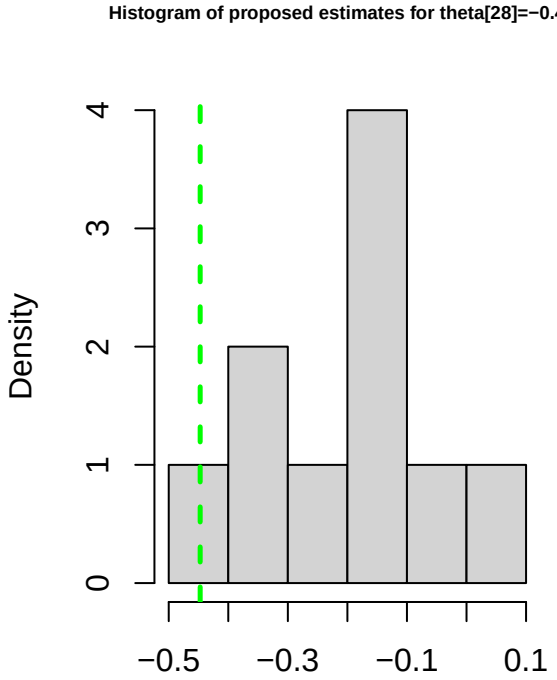
Table 3: Mean Absolute Deviation of Parameter Estimates

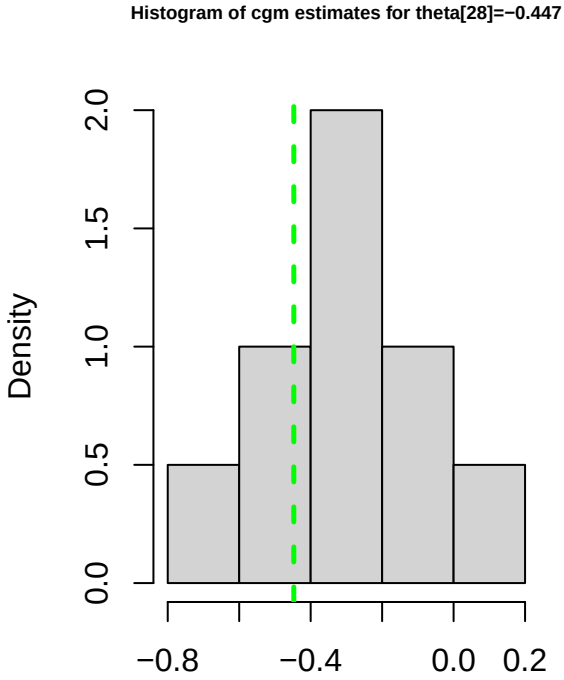
	proposed	cgm
theta[28]	0.236	0.226
theta[36]	0.212	0.228
theta[329]	0.111	0.194
theta[172]	0.087	0.172
total	0.162	0.205

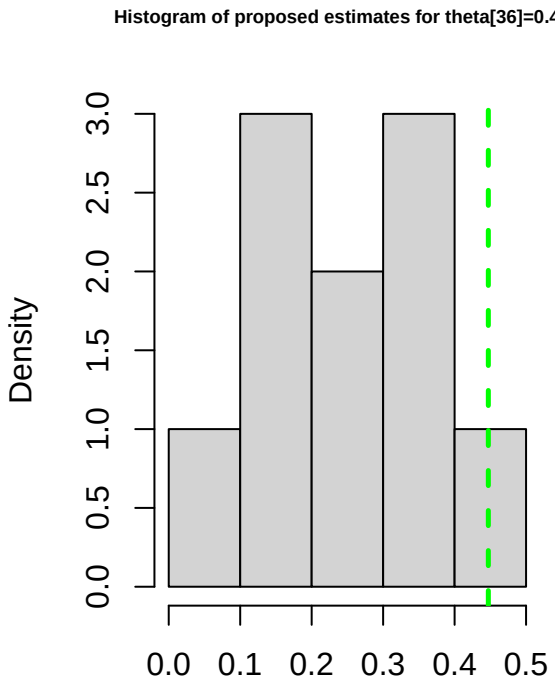
Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

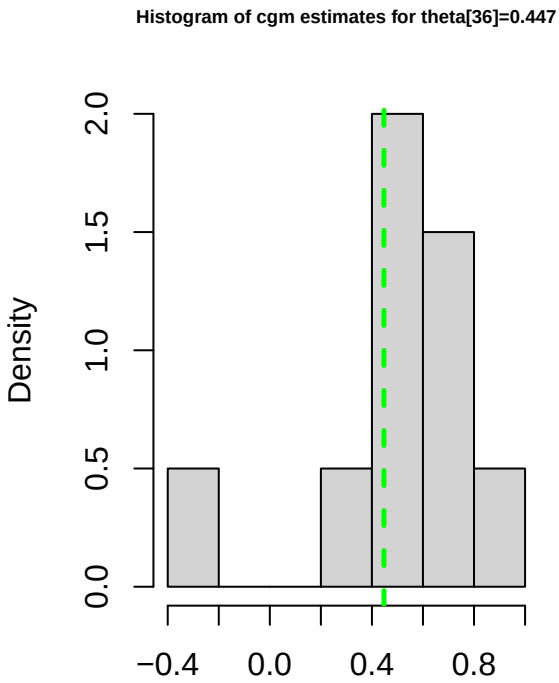
	proposed	cgm
theta[28]	0.356	0.249
theta[36]	0.299	0.135
theta[329]	0.022	0.000
theta[172]	0.000	0.020
total	0.169	0.101

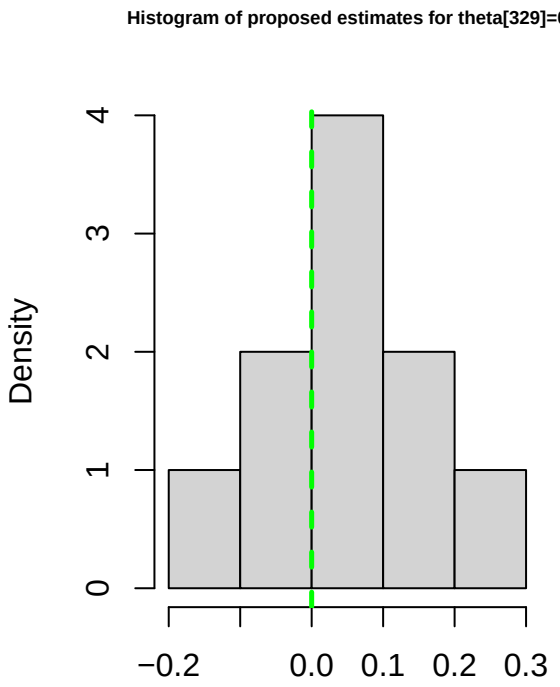
Boxplots

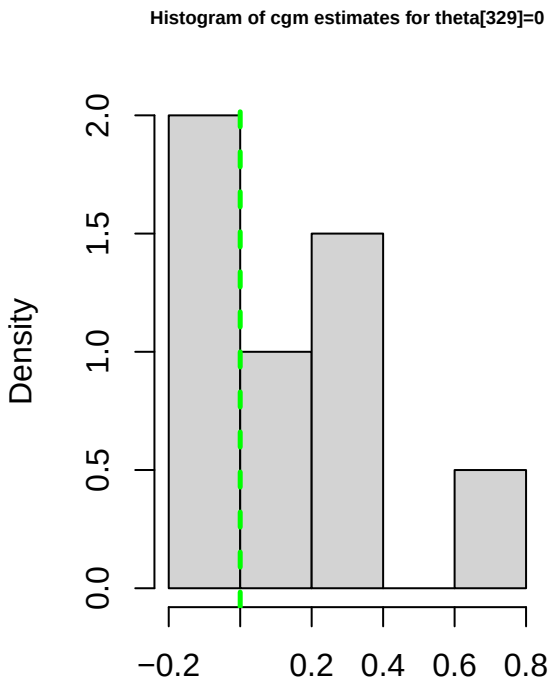

Histogram of proposed estimates for theta[28]=−0.447
Density
0
1
2
3
4
−0.5
−0.3
−0.1
0.1

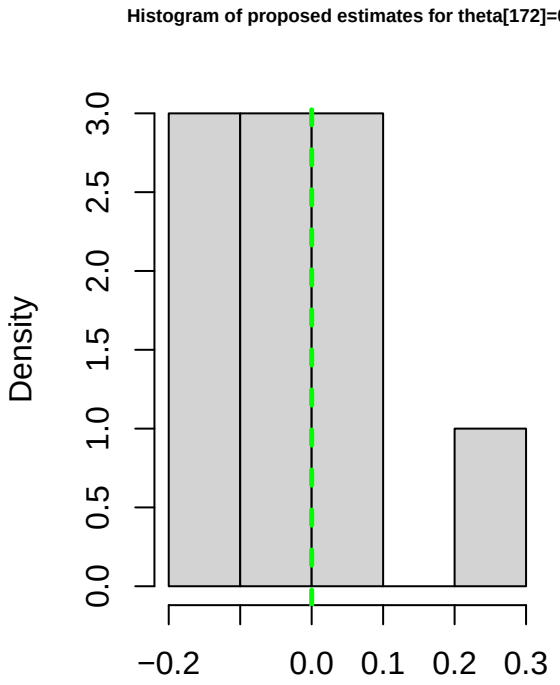

Histogram of cgm estimates for theta[28]=−0.447
Density
0.0
0.5
1.0
1.5
2.0
−0.8
−0.4
0.0
0.2

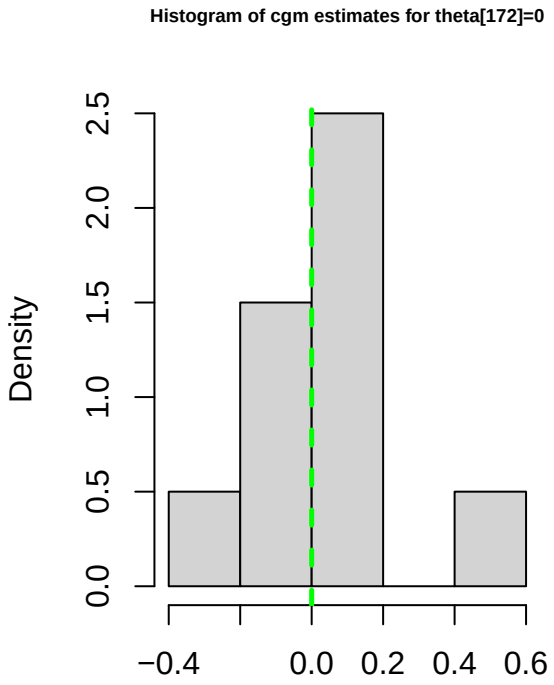

Histogram of proposed estimates for theta[36]=0.447
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0.0
0.1
0.2
0.3
0.4
0.5


Histogram of cgm estimates for theta[36]=0.447
Density
0.0
0.5
1.0
1.5
2.0
−0.4
0.0
0.4
0.8


Histogram of proposed estimates for theta[329]=0
Density
0
1
2
3
4
−0.2
0.0
0.1
0.2
0.3


Histogram of cgm estimates for theta[329]=0
Density
0.0
0.5
1.0
1.5
2.0
−0.2
0.2
0.4
0.6
0.8


Histogram of proposed estimates for theta[172]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.2
0.0
0.1
0.2
0.3


Histogram of cgm estimates for theta[172]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
−0.4
0.0
0.2
0.4
0.6

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[28]	-0.213	0.106	-0.422	-0.005	0.4
theta[36]	0.235	0.106	0.027	0.443	0.6
theta[329]	0.063	0.108	-0.149	0.276	0.9
theta[172]	-0.013	0.107	-0.223	0.197	0.9

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[28]	-0.309	0.131	-0.566	-0.053	0.6
theta[36]	0.478	0.129	0.226	0.731	0.7
theta[329]	0.105	0.118	-0.126	0.336	0.7
theta[172]	0.072	0.129	-0.180	0.324	0.9