Simulation Results

2026-01-12

Simulation Setup

This simulation is performed with $n = 200$ and $d = 20$, using the 2-d lattice as the underlying graph. $s = 3$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.2$. The attached results are for a 10-replication simulation. The true values of the parameter vector θ are

```
0 0 0 0 0 -0.5773503 0 0 0 0 -0.5773503 0 0 -0.5773503 0 0 0 0 0 0 ,
```

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{$ 6, 11, 4, 8$\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison ($\frac{1}{n.sim}\sum_{i=1}^{n.sim}\frac{1}{|\mathcal{C}|}\|\hat{\theta}_{i,\mathcal{C}} - \theta_{\mathcal{C}}\|^2$)

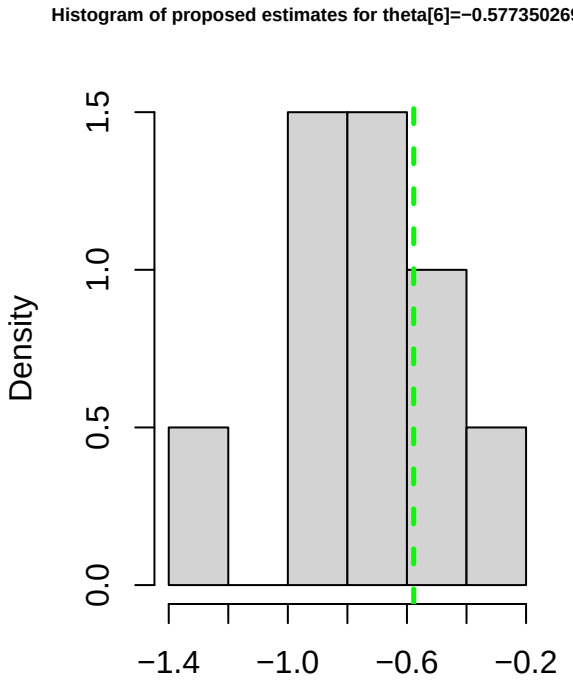
Table 1: Mean-Squared Error of Parameter Estimates

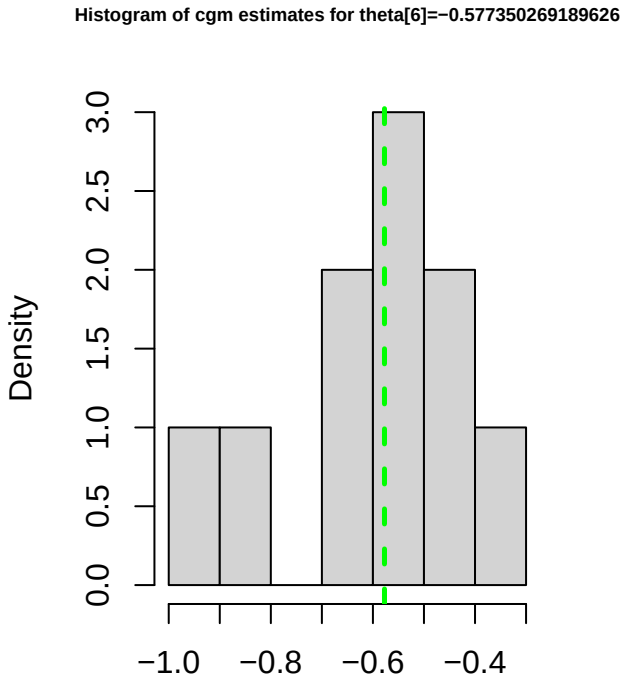
	proposed	cgm
theta[6]	0.108	0.034
theta[11]	0.880	0.109
theta[4]	0.091	0.037
theta[8]	0.132	0.018
total	0.303	0.049

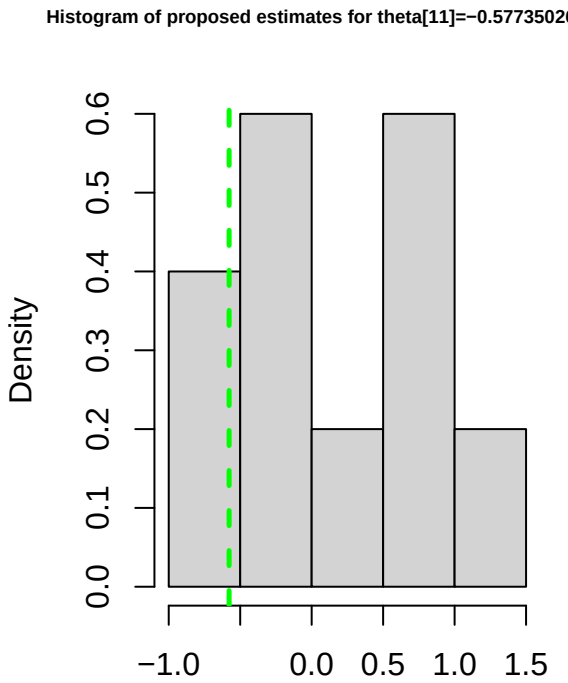
Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[6]	0.113	0.050
theta[11]	0.285	0.157
theta[4]	0.022	0.003
theta[8]	0.038	0.006
total	0.114	0.054

Boxplots


Histogram of proposed estimates for theta[6]=−0.5773502691896
Density
0.0
0.5
1.0
1.5
−1.4
−1.0
−0.6
−0.2


Histogram of cgm estimates for theta[6]=−0.577350269189626
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−1.0
−0.8
−0.6
−0.4


Histogram of proposed estimates for theta[11]=−0.577350269189
Density
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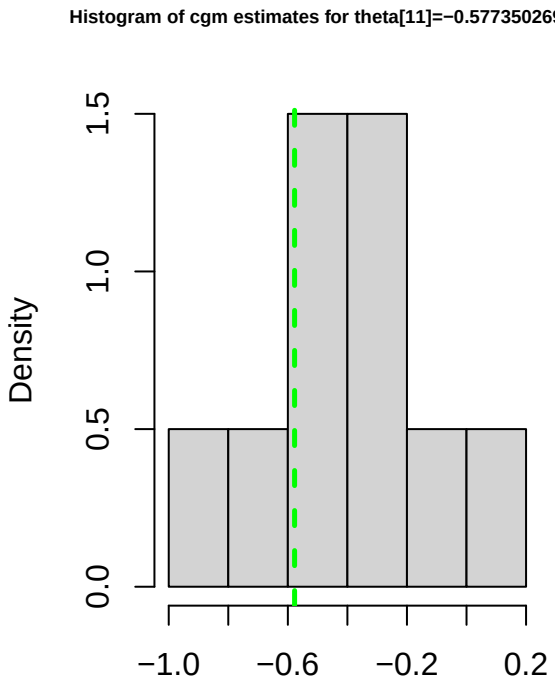
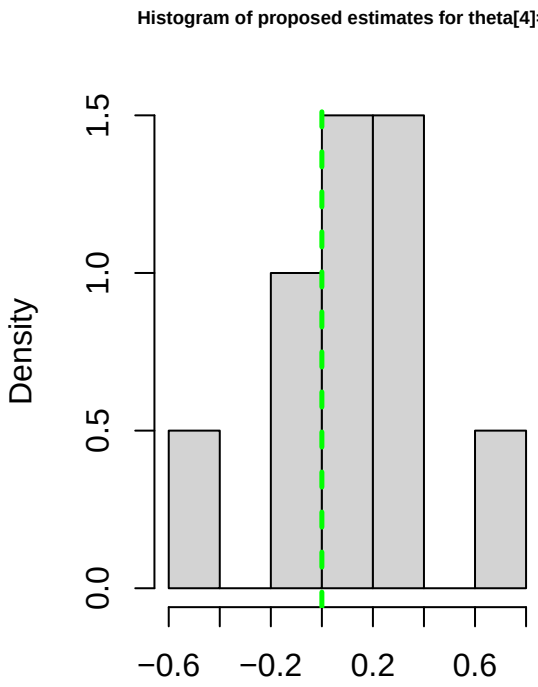
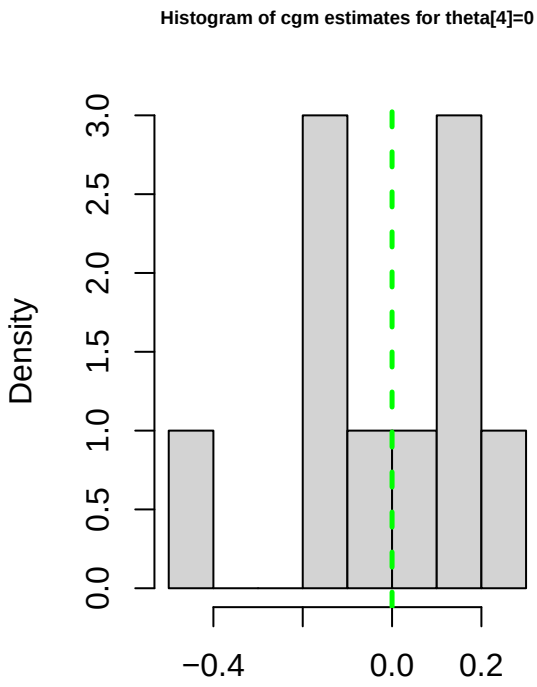
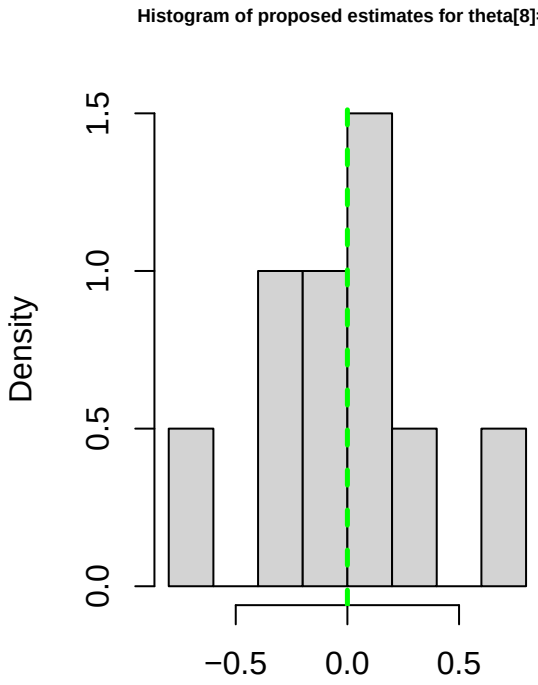
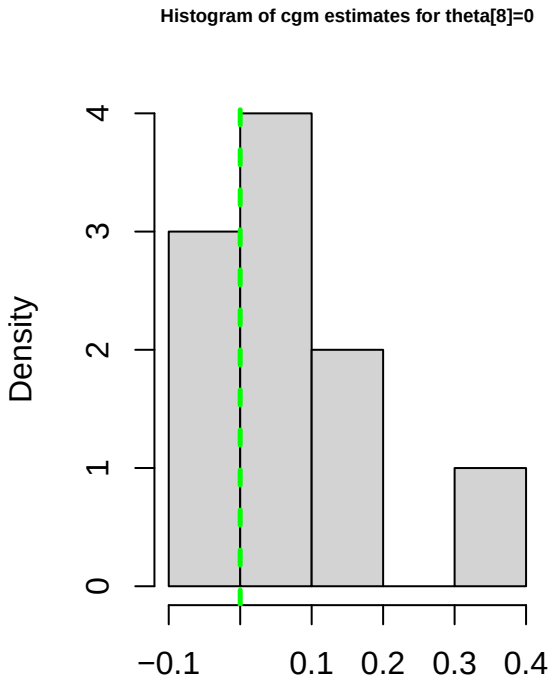
0.2
0.3
0.4
0.5
0.6
−1.0
0.0
0.5
1.0
1.5


Histogram of cgm estimates for theta[11]=−0.577350269189626
Density
0.0
0.5
1.0
1.5
−1.0
−0.6
−0.2
0.2

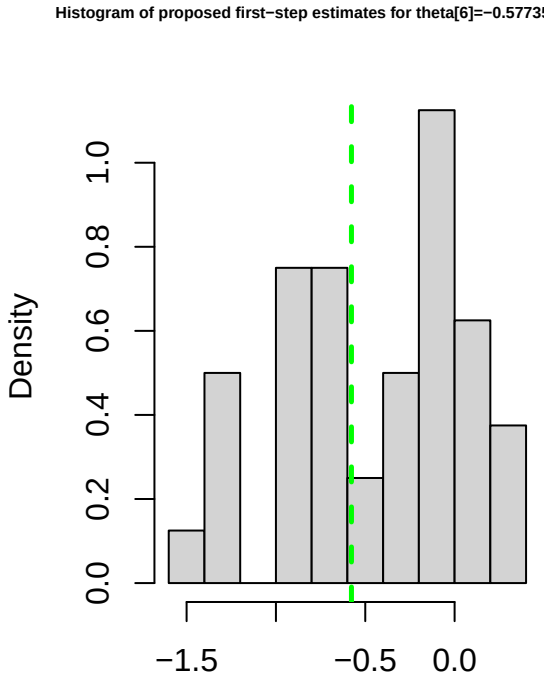

Histogram of proposed estimates for theta[4]=0
Density
0.0
0.5
1.0
1.5
−0.6
−0.2
0.2
0.6

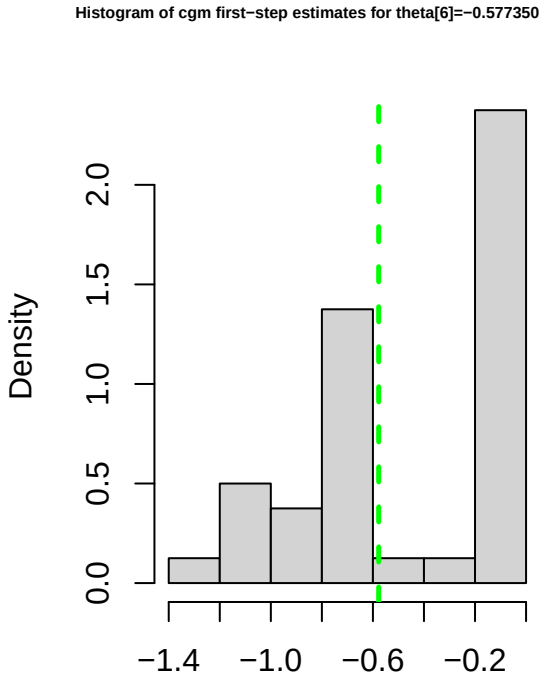

Histogram of cgm estimates for theta[4]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.4
0.0
0.2

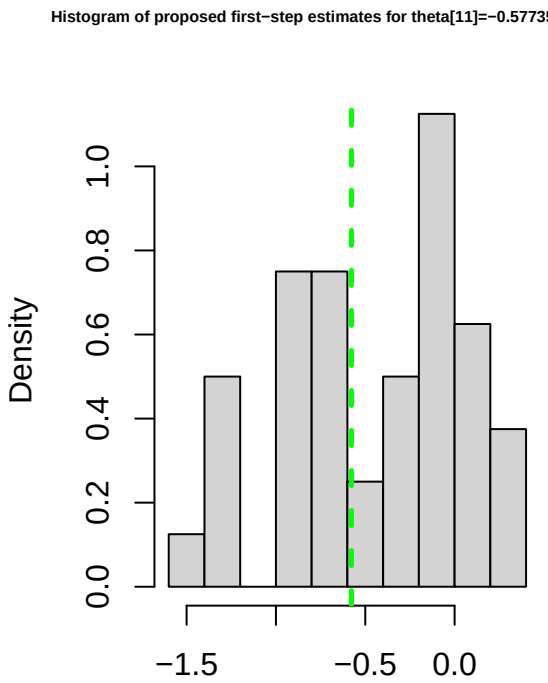

Histogram of proposed estimates for theta[8]=0
Density
0.0
0.5
1.0
1.5
−0.5
0.0
0.5

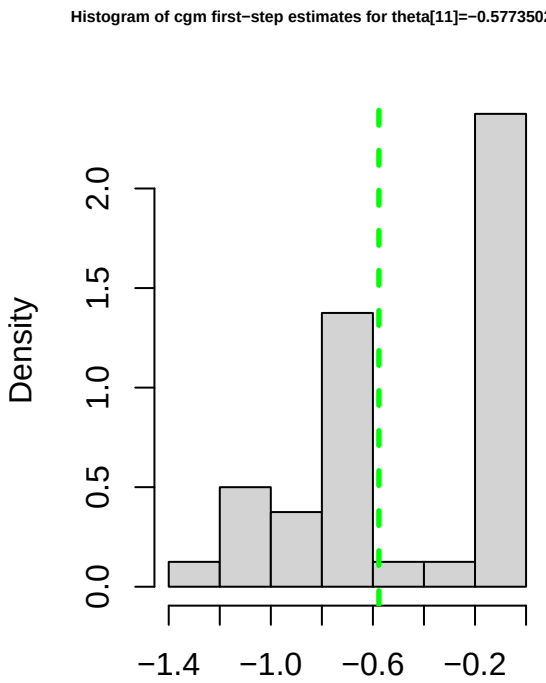

Histogram of cgm estimates for theta[8]=0
Density
0
1
2
3
4
−0.1
0.1
0.2
0.3
0.4

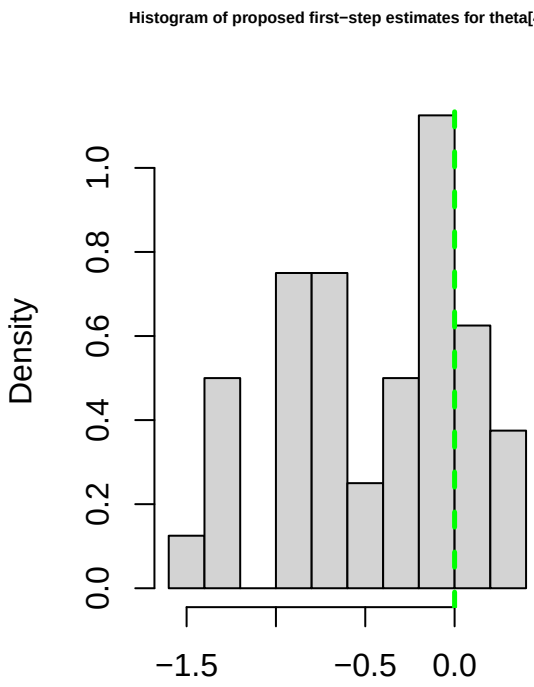
First Step Histograms

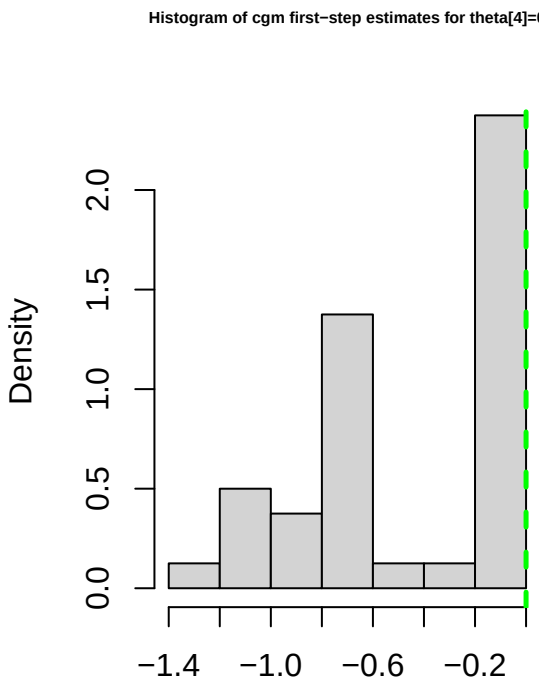


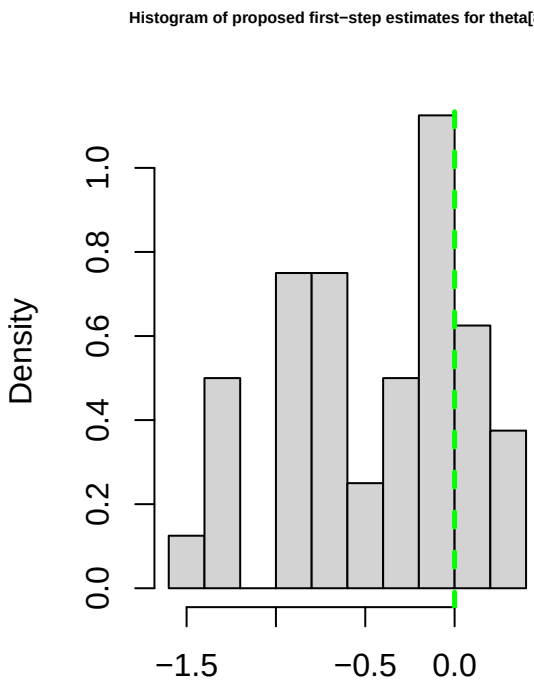


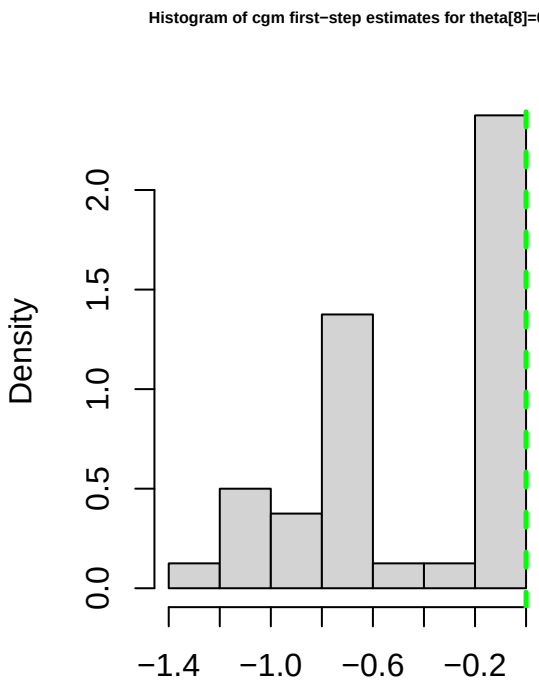





Histogram of proposed first−step estimates for theta[4]=0
Density
0.0
0.2
0.4
0.6
0.8
1.0
−1.5
−0.5
0.0


Histogram of cgm first−step estimates for theta[4]=0
Density
0.0
0.5
1.0
1.5
2.0
−1.4
−1.0
−0.6
−0.2


Histogram of proposed first−step estimates for theta[8]=0
Density
0.0
0.2
0.4
0.6
0.8
1.0
−1.5
−0.5
0.0


Histogram of cgm first−step estimates for theta[8]=0
Density
0.0
0.5
1.0
1.5
2.0
−1.4
−1.0
−0.6
−0.2

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 3: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[6]	-1.300	-0.782	-0.258	-1.218	-0.291
theta[11]	-0.850	-0.058	1.053	-0.790	0.996
theta[4]	-0.502	0.106	0.622	-0.421	0.546
theta[8]	-0.727	0.034	0.639	-0.644	0.560

Table 4: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[6]	-0.978	-0.586	-0.305	-0.944	-0.332
theta[11]	-0.804	-0.400	0.108	-0.797	0.067
theta[4]	-0.407	0.020	0.261	-0.351	0.241
theta[8]	-0.093	0.043	0.339	-0.083	0.296

Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[6]	-0.729	0.233	-1.186	-0.272	0.9
theta[11]	0.112	0.301	-0.477	0.701	0.6
theta[4]	0.082	0.231	-0.372	0.535	0.9
theta[8]	-0.016	0.218	-0.445	0.412	0.8

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[6]	-0.599	0.150	-0.894	-0.305	0.9
theta[11]	-0.388	0.174	-0.729	-0.047	0.7
theta[4]	-0.010	0.133	-0.271	0.252	0.9
theta[8]	0.066	0.133	-0.195	0.327	0.9