Simulation Results

2026-01-20

Simulation Setup

This simulation is performed with $n = 200$ and $d = 200$, using the 2-d lattice as the underlying graph. $s = 5$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.2$. The attached results are for a 10-replication simulation. The parameter vector θ has sparse components other than the following:

θ[7] = 0.447θ[9] = 0.447θ[112] = 0.447θ[160] = 0.447θ[199] = -0.447

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C}$ = { 7, 9, 70, 171} elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

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### Mean-squared error comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat\theta
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Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[7]	0.070	0.200
theta[9]	0.059	0.505
theta[70]	0.024	0.157
theta[171]	0.014	0.387
total	0.042	0.312

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[7]	0.182	0.076
theta[9]	0.178	0.035
theta[70]	0.000	0.001
theta[171]	0.000	0.000
total	0.090	0.028

Mean absolute deviation comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat

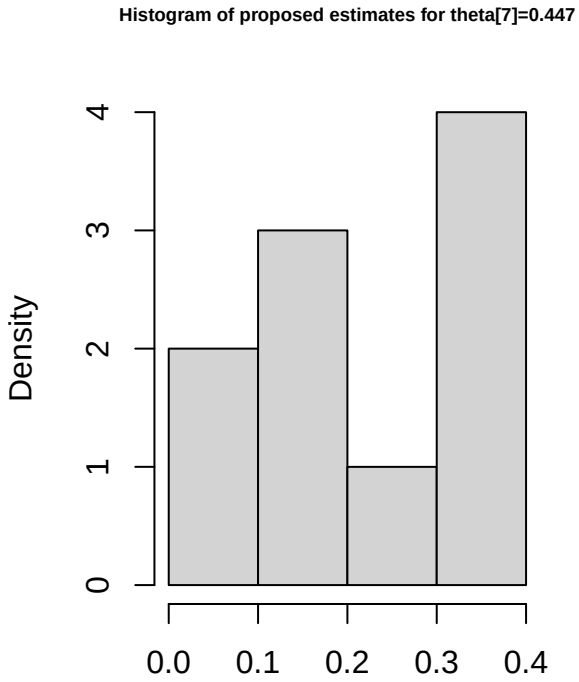
Table 3: Mean Absolute Deviation of Parameter Estimates

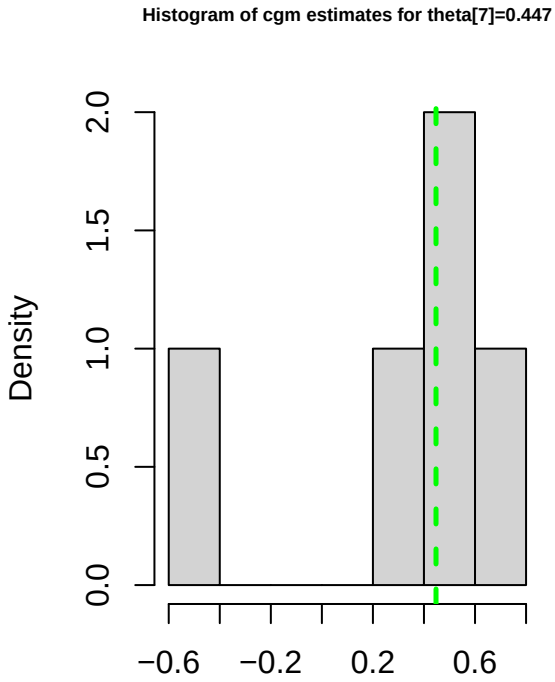
	proposed	cgm
theta[7]	0.234	0.288
theta[9]	0.216	0.410
theta[70]	0.145	0.292
theta[171]	0.098	0.386
total	0.173	0.344

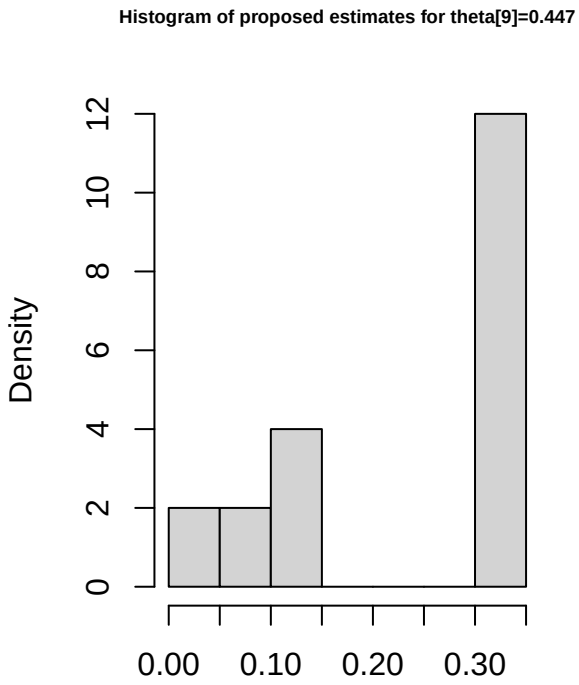
Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

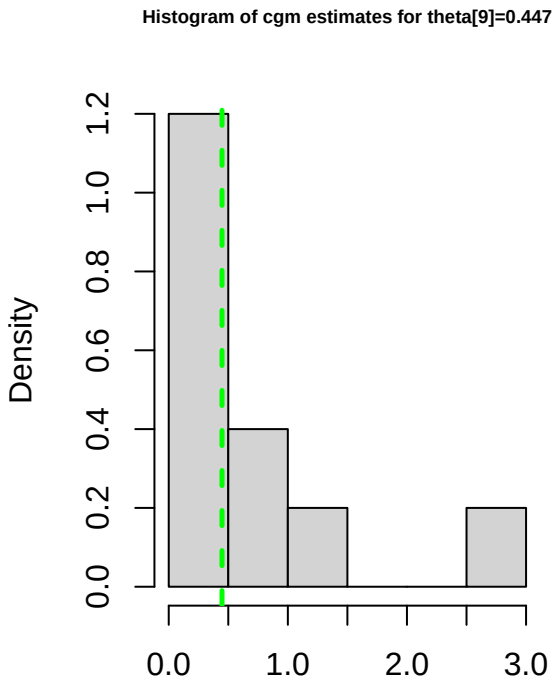
	proposed	cgm
theta[7]	0.423	0.236
theta[9]	0.417	0.149
theta[70]	0.000	0.010
theta[171]	0.000	0.000
total	0.210	0.099

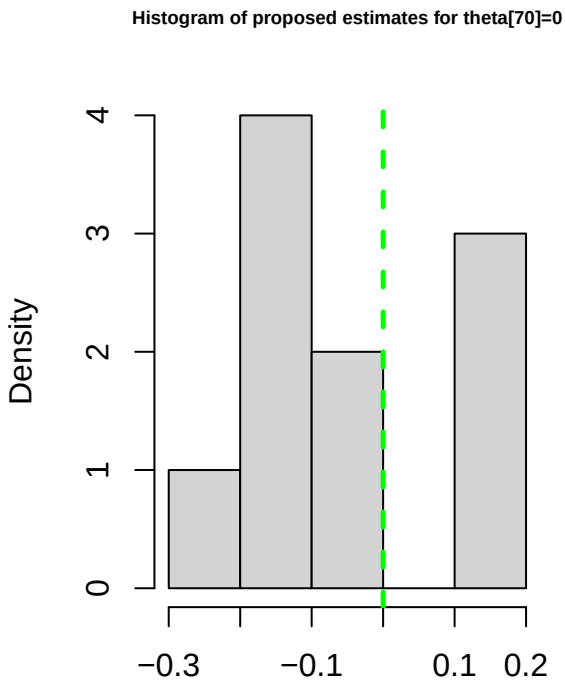
Boxplots

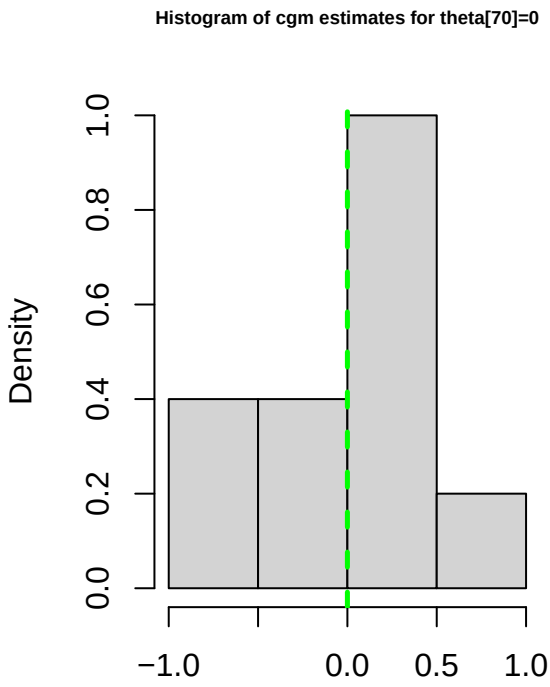

Histogram of proposed estimates for theta[7]=0.447
Density
0
1
2
3
4
0.0
0.1
0.2
0.3
0.4

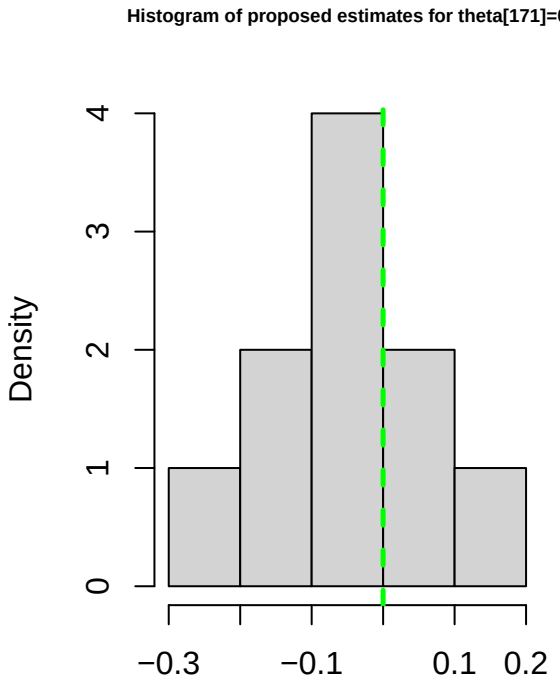

Histogram of cgm estimates for theta[7]=0.447
Density
0.0
0.5
1.0
1.5
2.0
−0.6
−0.2
0.2
0.6

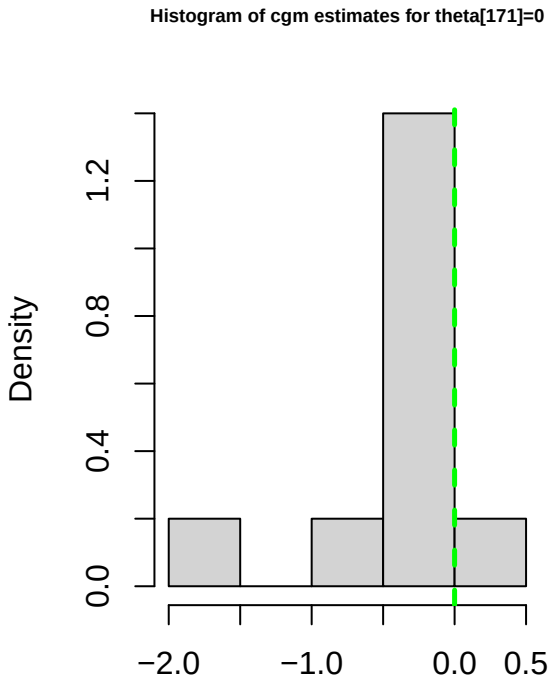

Histogram of proposed estimates for theta[9]=0.447
Density
0
2
4
6
8
10
12
0.00
0.10
0.20
0.30


Histogram of cgm estimates for theta[9]=0.447
Density
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0.0
1.0
2.0
3.0

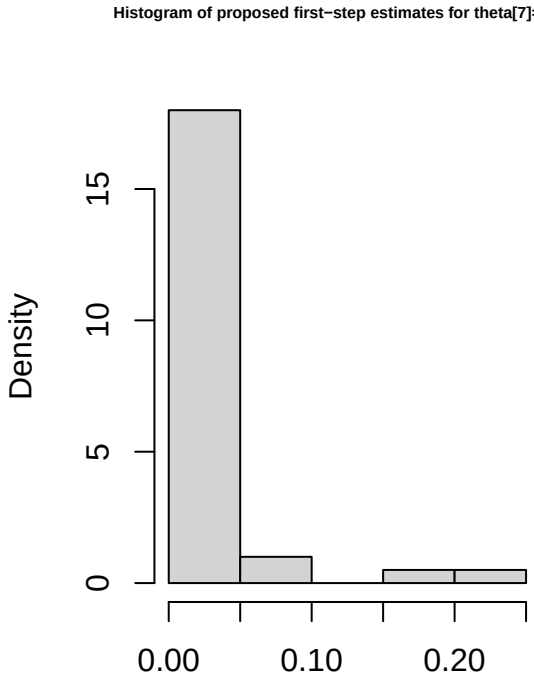

Histogram of proposed estimates for theta[70]=0
Density
0
1
2
3
4
−0.3
−0.1
0.1
0.2

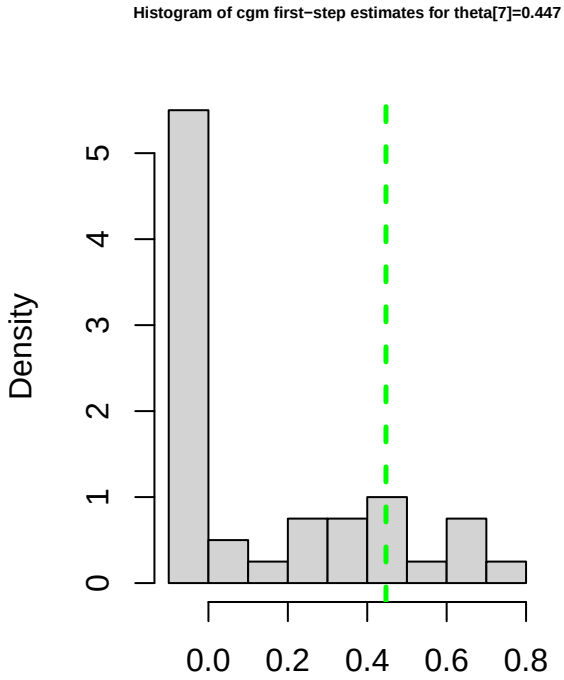

Histogram of cgm estimates for theta[70]=0
Density
0.0
0.2
0.4
0.6
0.8
1.0
−1.0
0.0
0.5
1.0

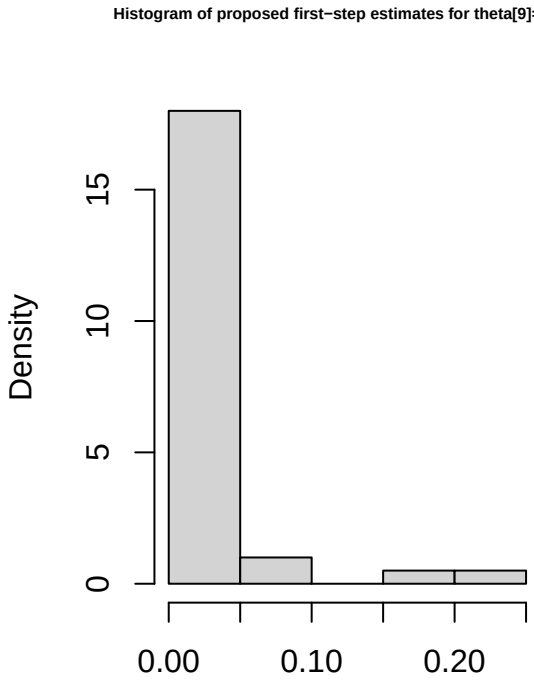

Histogram of proposed estimates for theta[171]=0
Density
0
1
2
3
4
−0.3
−0.1
0.1
0.2

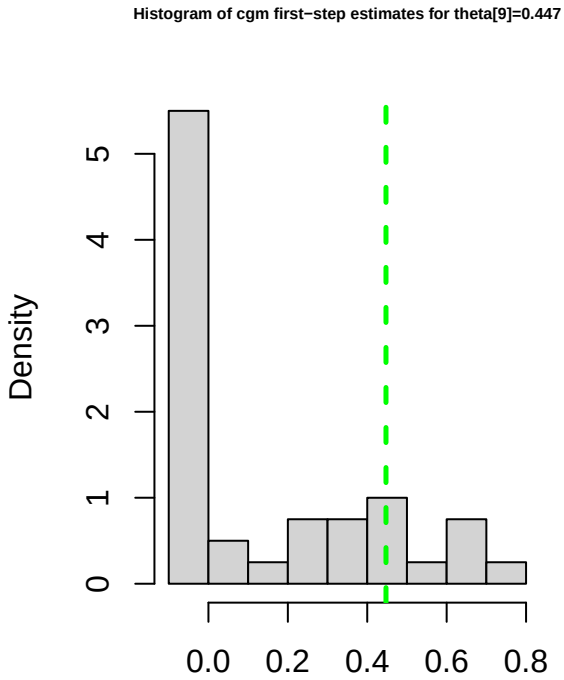

Histogram of cgm estimates for theta[171]=0
Density
0.0
0.4
0.8
1.2
−2.0
−1.0
0.0
0.5

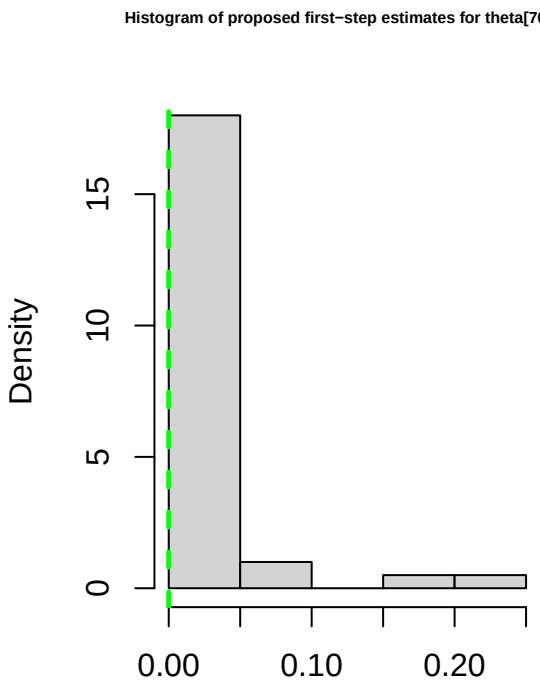
First Step Histograms

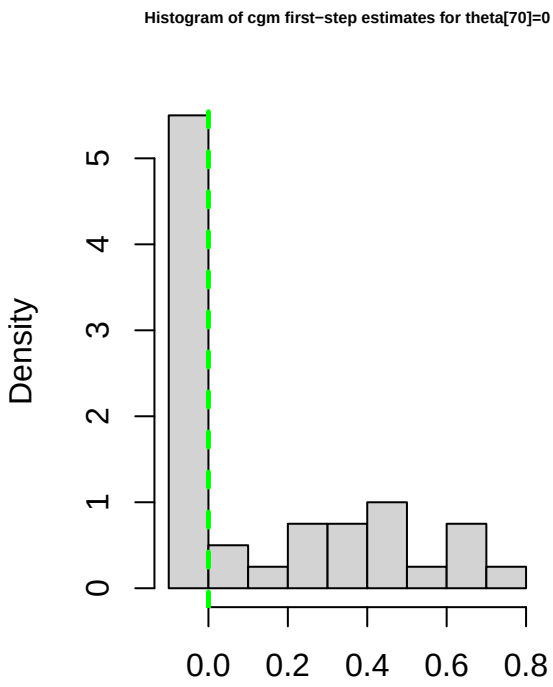


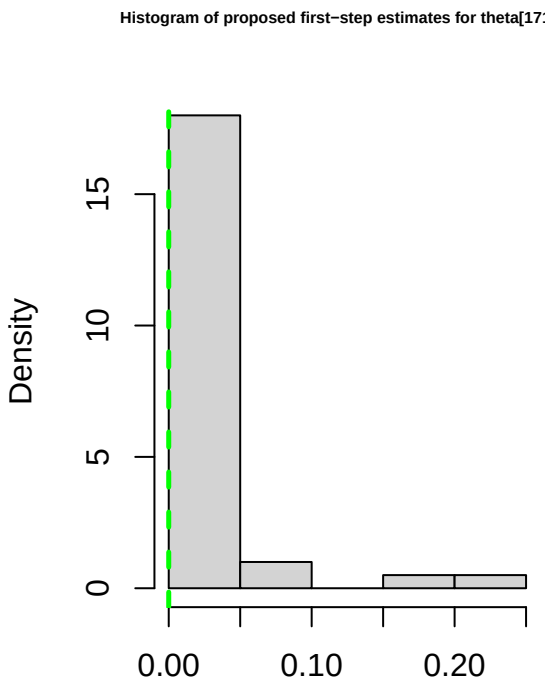


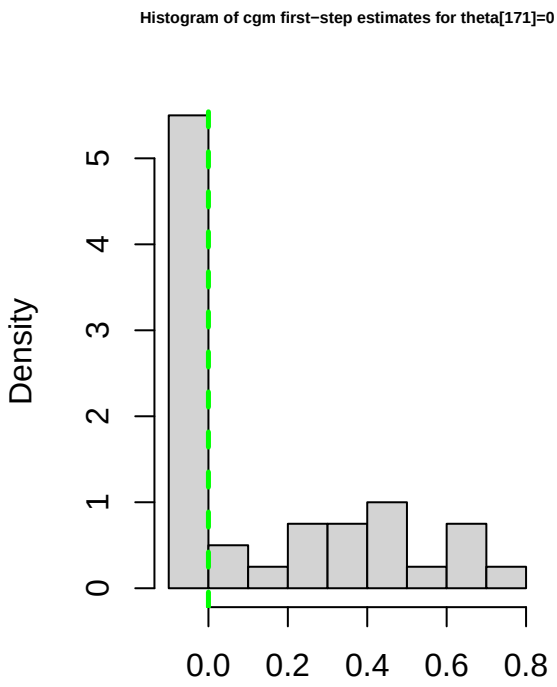





Histogram of proposed first−step estimates for theta[70]=0
Density
0
5
10
15
0.00
0.10
0.20


Histogram of cgm first−step estimates for theta[70]=0
Density
0
1
2
3
4
5
0.0
0.2
0.4
0.6
0.8


Histogram of proposed first−step estimates for theta[171]=0
Density
0
5
10
15
0.00
0.10
0.20


Histogram of cgm first−step estimates for theta[171]=0
Density
0
1
2
3
4
5
0.0
0.2
0.4
0.6
0.8

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 5: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[7]	0.005	0.238	0.360	0.022	0.354
theta[9]	0.045	0.308	0.340	0.055	0.337
theta[70]	-0.213	-0.084	0.194	-0.209	0.188
theta[171]	-0.233	-0.045	0.157	-0.216	0.137

Table 6: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[7]	-0.581	0.449	0.749	-0.546	0.725
theta[9]	0.220	0.479	2.506	0.222	2.205
theta[70]	-0.684	0.048	0.760	-0.662	0.613
theta[171]	-1.749	-0.201	0.113	-1.503	0.087

Statistics for Theoretical 95% Confidence Intervals

Table 7: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[7]	0.213	0.113	-0.009	0.435	0.5
theta[9]	0.231	0.110	0.015	0.447	0.6
theta[70]	-0.051	0.116	-0.279	0.178	1.0
theta[171]	-0.047	0.114	-0.271	0.178	0.9

Table 8: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[7]	0.305	0.207	-0.100	0.710	0.8
theta[9]	0.720	0.203	0.322	1.117	0.8
theta[70]	-0.059	0.229	-0.507	0.390	0.7
theta[171]	-0.363	0.186	-0.727	0.001	0.7