Simulation Results

2026-01-16

Simulation Setup

This simulation is performed with $n = 200$ and $d = 50$, using the 2-d lattice as the underlying graph. $s = 5$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.2$. The attached results are for a 10-replication simulation. The true values of the parameter vector θ are

```
0 0 0 0 0 0.4472136 0 0 0 0 0 0 0 0 -0.4472136 0 0 0 -0.4472136 0 0 0 0.4472136 0 0 0 0 0 0 0 0 0 0 0 0
```

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{ 6, 15, 4, 44\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

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### Mean-squared error comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat\theta
```

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[6]	0.047	0.049
theta[15]	0.044	0.019
theta[4]	0.011	0.011
theta[44]	0.008	0.010
total	0.027	0.022

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[6]	0.074	0.060
theta[15]	0.128	0.036
theta[4]	0.000	0.003
theta[44]	0.001	0.005
total	0.051	0.026

Mean absolute deviation comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat

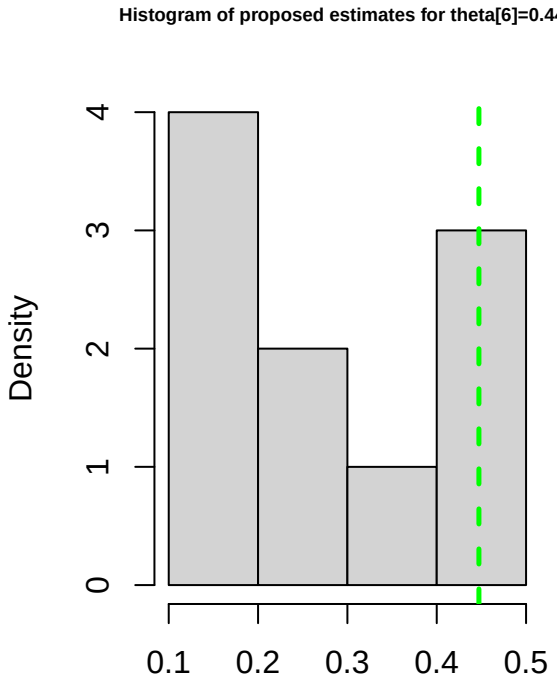
Table 3: Mean Absolute Deviation of Parameter Estimates

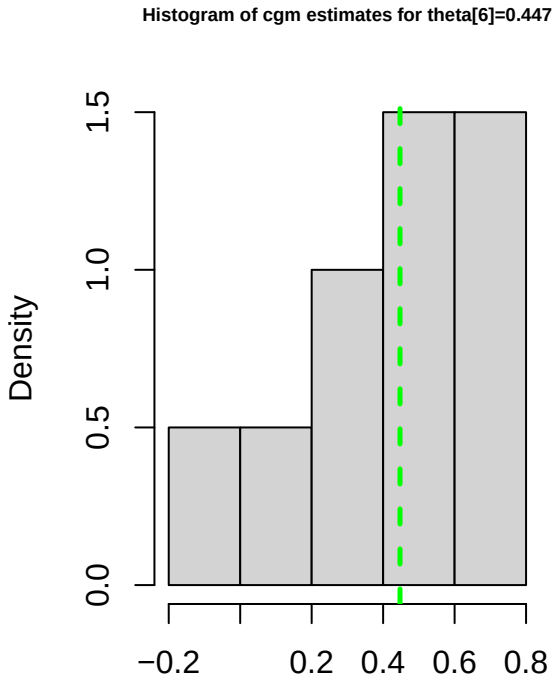
	proposed	cgm
theta[6]	0.182	0.178
theta[15]	0.171	0.117
theta[4]	0.080	0.086
theta[44]	0.069	0.085
total	0.126	0.117

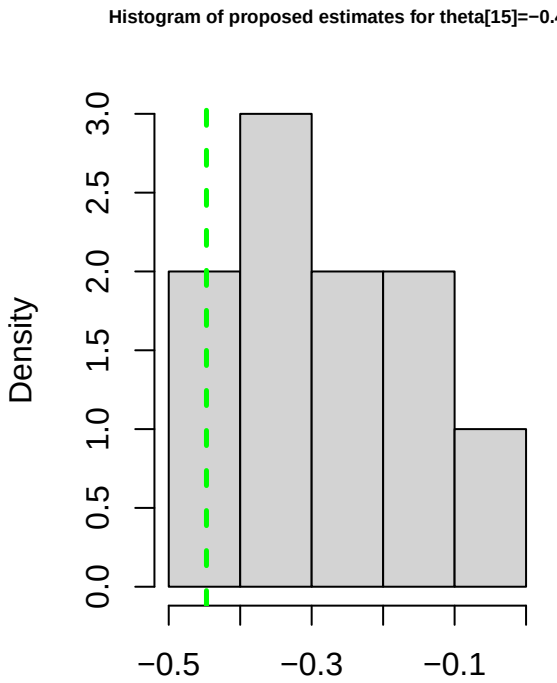
Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

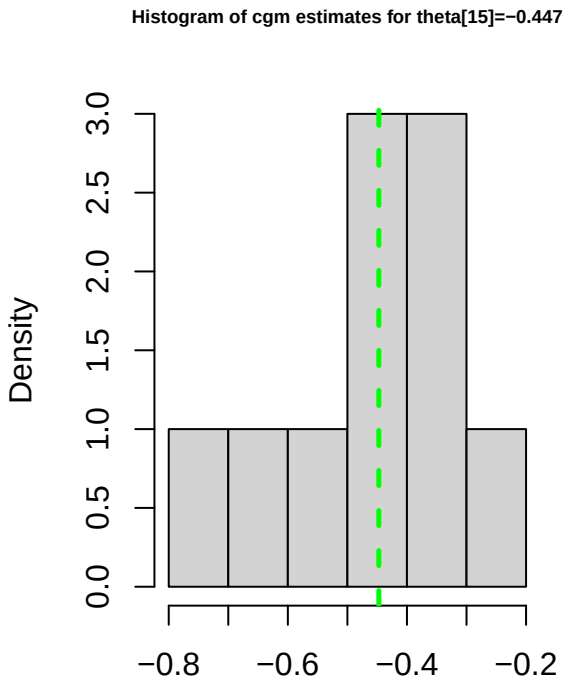
	proposed	cgm
theta[6]	0.252	0.197
theta[15]	0.336	0.139
theta[4]	0.003	0.018
theta[44]	0.008	0.031
total	0.150	0.096

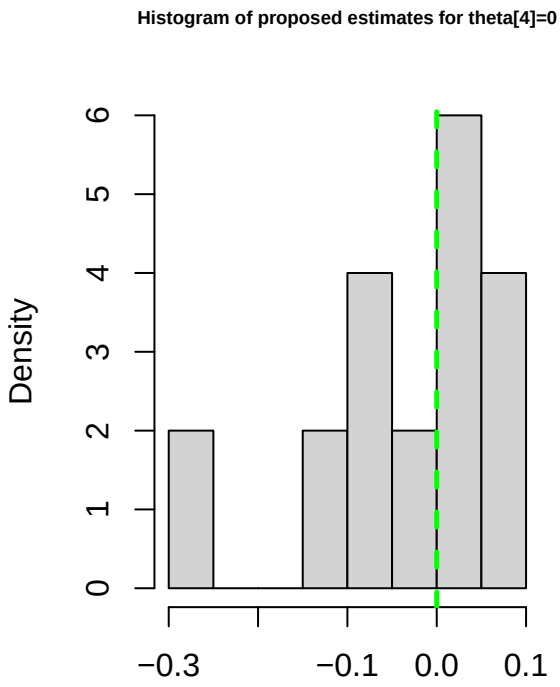
Boxplots

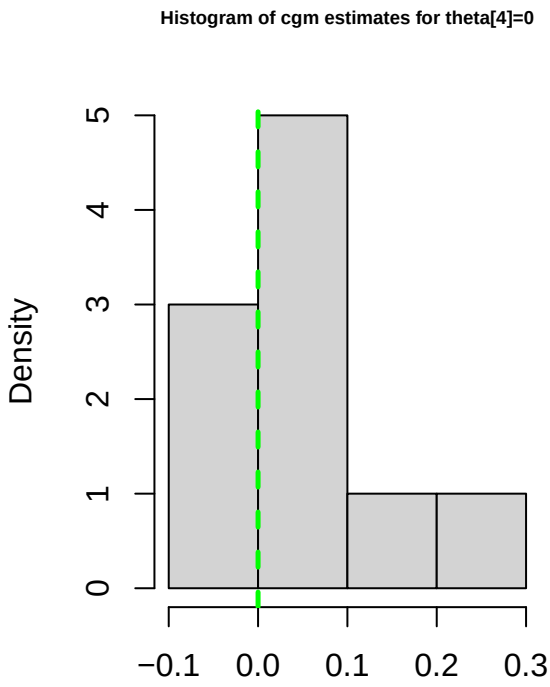

Histogram of proposed estimates for theta[6]=0.447
Density
0
1
2
3
4
0.1
0.2
0.3
0.4
0.5

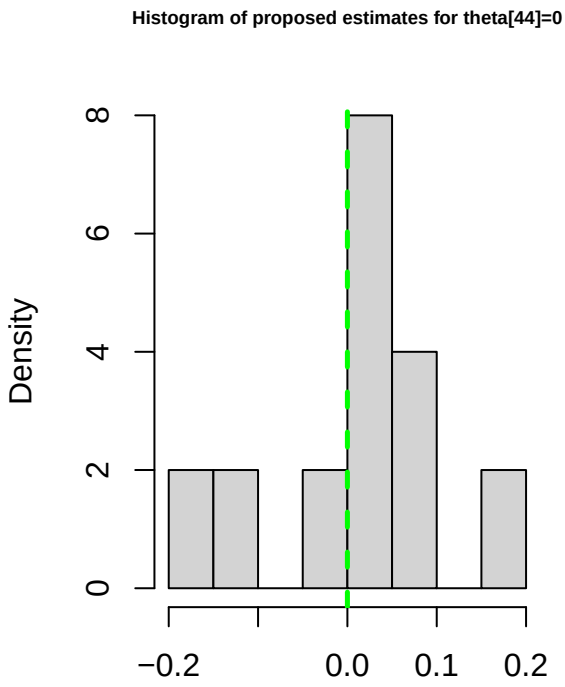

Histogram of cgm estimates for theta[6]=0.447
Density
0.0
0.5
1.0
1.5
−0.2
0.2
0.4
0.6
0.8

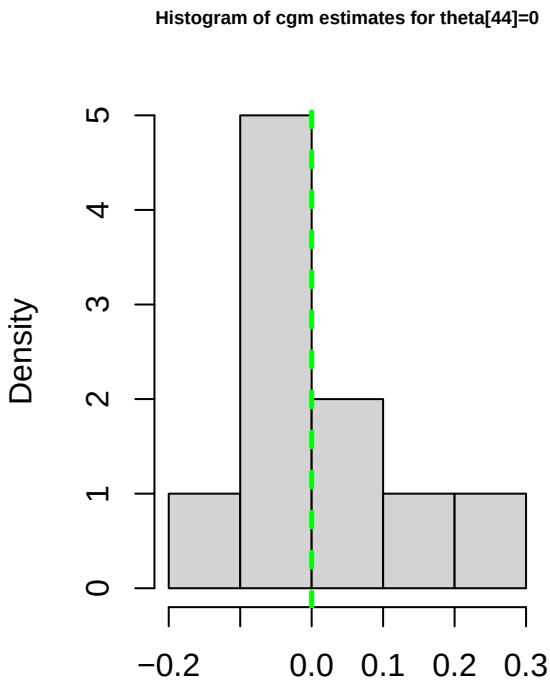

Histogram of proposed estimates for theta[15]=−0.447
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.5
−0.3
−0.1


Histogram of cgm estimates for theta[15]=−0.447
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.8
−0.6
−0.4
−0.2

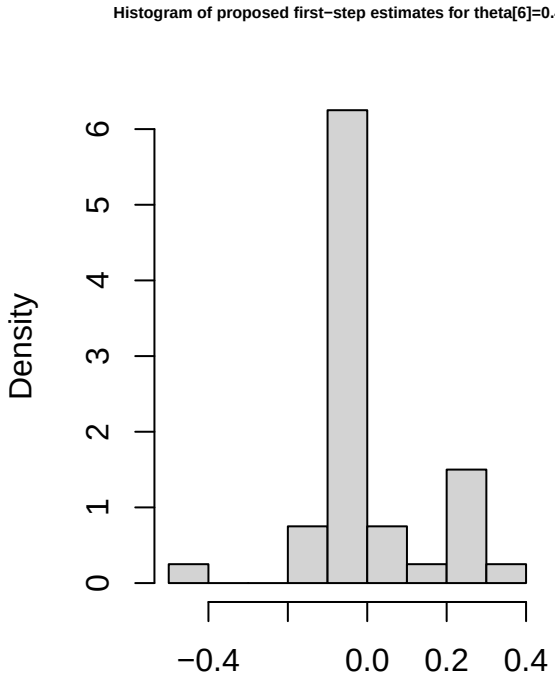

Histogram of proposed estimates for theta[4]=0
Density
0
1
2
3
4
5
6
−0.3
−0.1
0.0
0.1

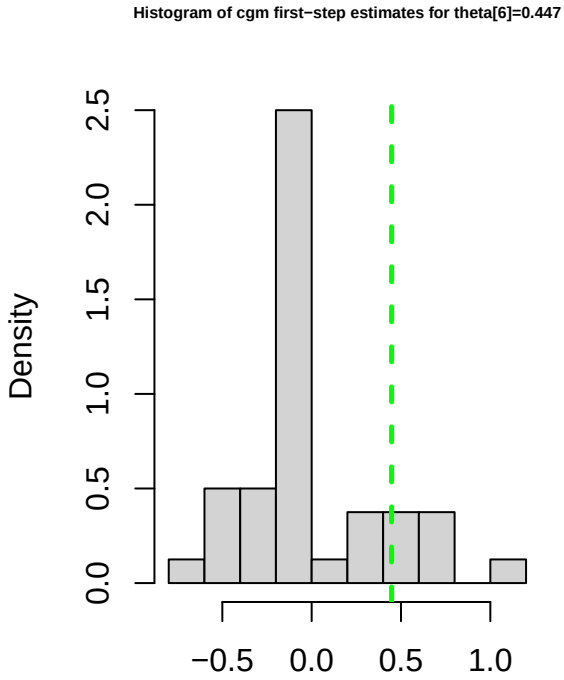

Histogram of cgm estimates for theta[4]=0
Density
0
1
2
3
4
5
−0.1
0.0
0.1
0.2
0.3

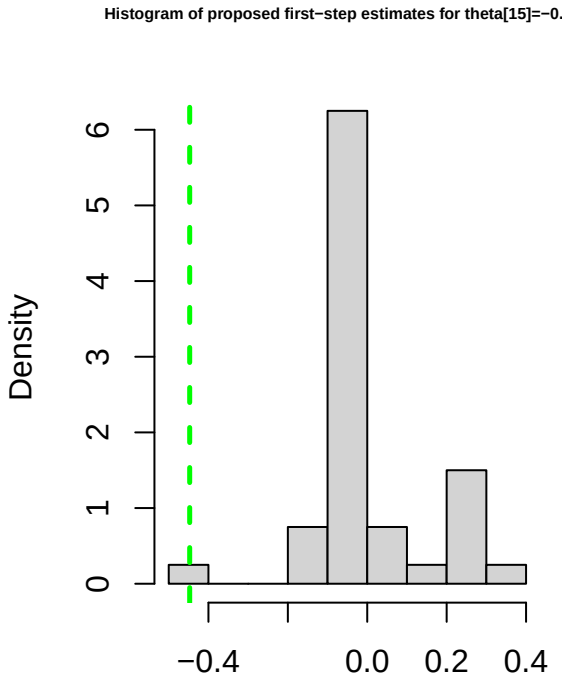

Histogram of proposed estimates for theta[44]=0
Density
0
2
4
6
8
−0.2
0.0
0.1
0.2

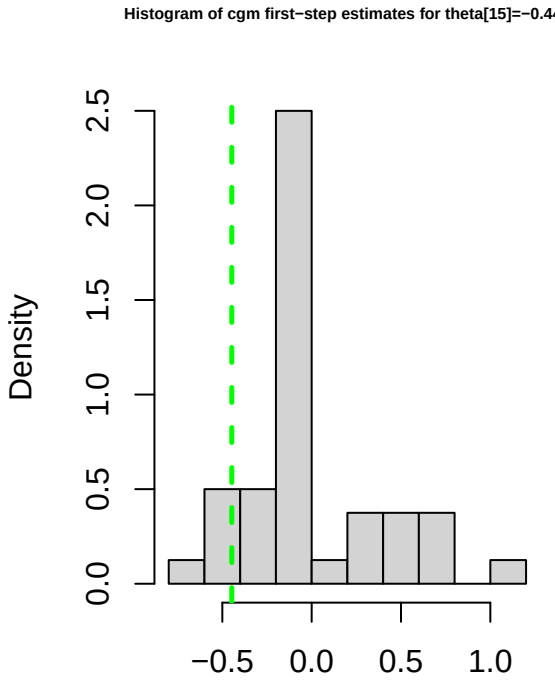

Histogram of cgm estimates for theta[44]=0
Density
0
1
2
3
4
5
−0.2
0.0
0.1
0.2
0.3

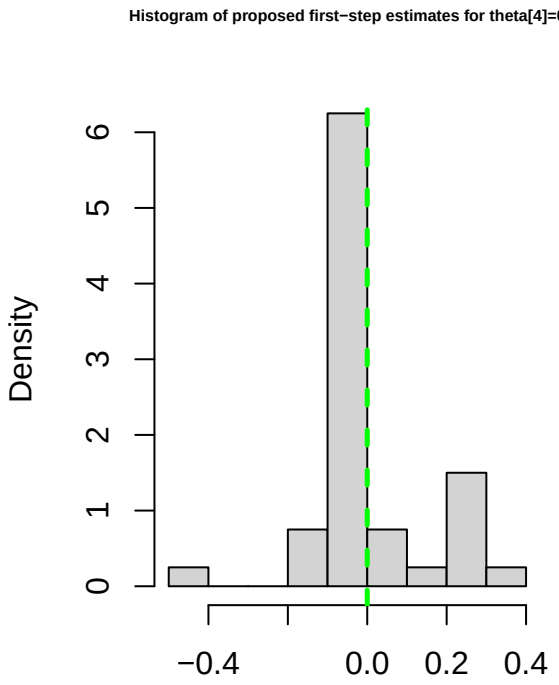
First Step Histograms

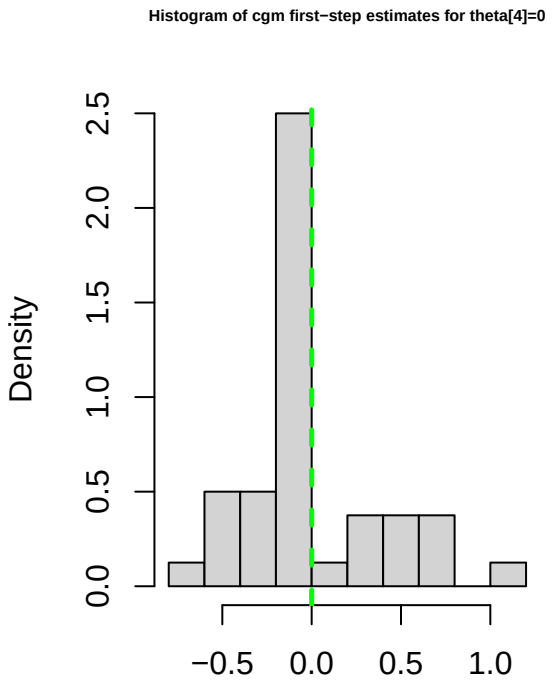


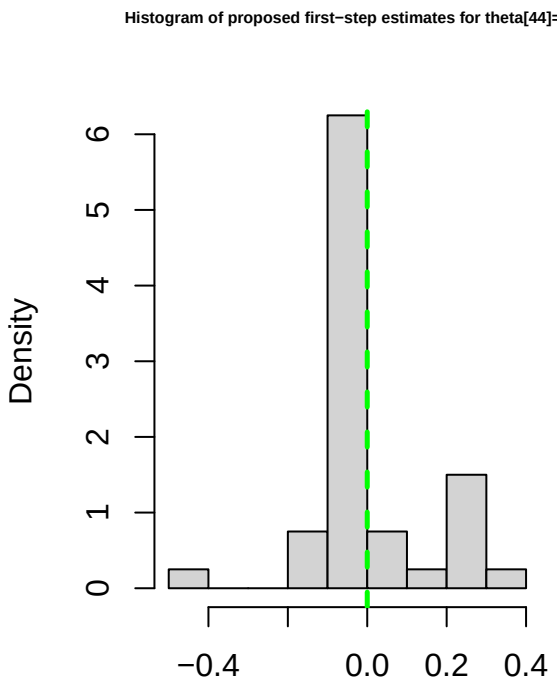


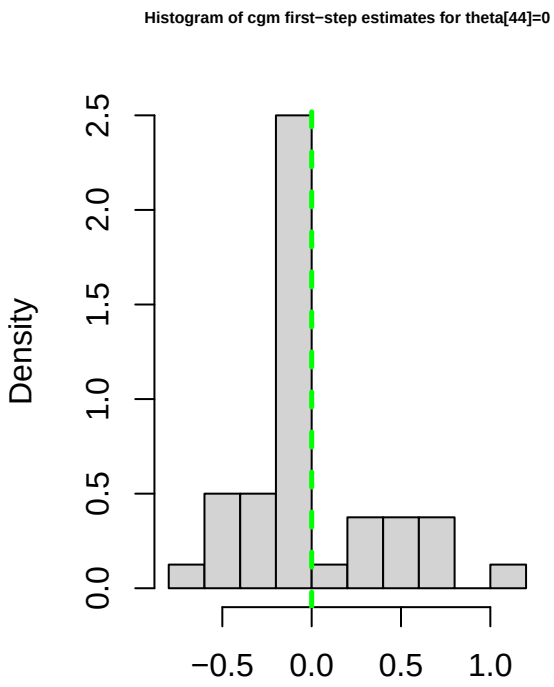





Histogram of proposed first−step estimates for theta[4]=0
Density
0
1
2
3
4
5
6
−0.4
0.0
0.2
0.4


Histogram of cgm first−step estimates for theta[4]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
−0.5
0.0
0.5
1.0


Histogram of proposed first−step estimates for theta[44]=0
Density
0
1
2
3
4
5
6
−0.4
0.0
0.2
0.4


Histogram of cgm first−step estimates for theta[44]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
−0.5
0.0
0.5
1.0

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 5: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[6]	0.122	0.220	0.495	0.127	0.484
theta[15]	-0.440	-0.282	-0.064	-0.438	-0.079
theta[4]	-0.253	0.008	0.064	-0.229	0.062
theta[44]	-0.150	0.029	0.159	-0.143	0.139

Table 6: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[6]	-0.004	0.465	0.733	0.041	0.722
theta[15]	-0.708	-0.416	-0.284	-0.687	-0.288
theta[4]	-0.098	0.040	0.255	-0.095	0.226
theta[44]	-0.149	-0.036	0.209	-0.134	0.191

Statistics for Theoretical 95% Confidence Intervals

Table 7: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[6]	0.274	0.125	0.029	0.520	0.6
theta[15]	-0.277	0.127	-0.525	-0.029	0.8
theta[4]	-0.036	0.119	-0.269	0.198	0.9
theta[44]	0.012	0.126	-0.236	0.259	1.0

Table 8: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[6]	0.449	0.131	0.192	0.706	0.7
theta[15]	-0.442	0.123	-0.684	-0.200	0.9
theta[4]	0.037	0.121	-0.200	0.273	0.9
theta[44]	0.001	0.120	-0.234	0.235	1.0