Simulation Results

2026-01-16

Simulation Setup

This simulation is performed with $n = 200$ and $d = 50$, using the 2-d lattice as the underlying graph. $s = 5$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0$. The attached results are for a 10-replication simulation. The true values of the parameter vector θ are

```
0 -0.4472136 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -0.4472136 0 -0.4472136 0 -0.4472136 0 0 0
```

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{\ 2, 27, 5, 44\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

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### Mean-squared error comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat\theta
```

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[2]	0.054	0.030
theta[27]	0.036	0.071
theta[5]	0.006	0.012
theta[44]	0.015	0.016
total	0.028	0.032

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[2]	0.148	0.049
theta[27]	0.121	0.028
theta[5]	0.000	0.006
theta[44]	0.001	0.003
total	0.067	0.022

Mean absolute deviation comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat

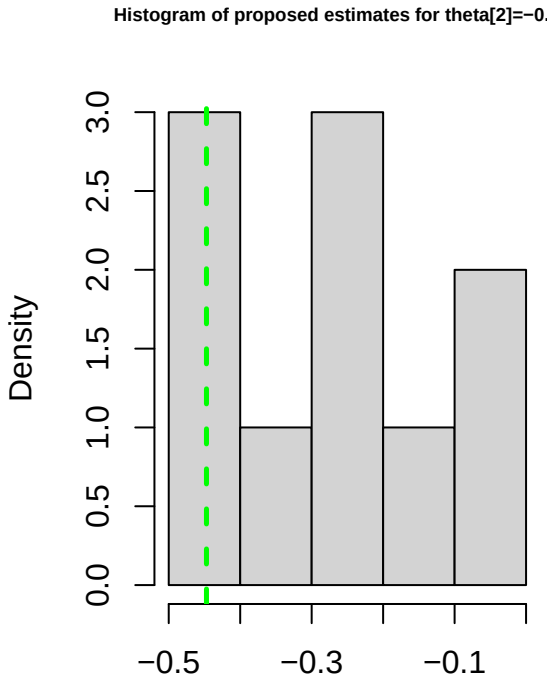
Table 3: Mean Absolute Deviation of Parameter Estimates

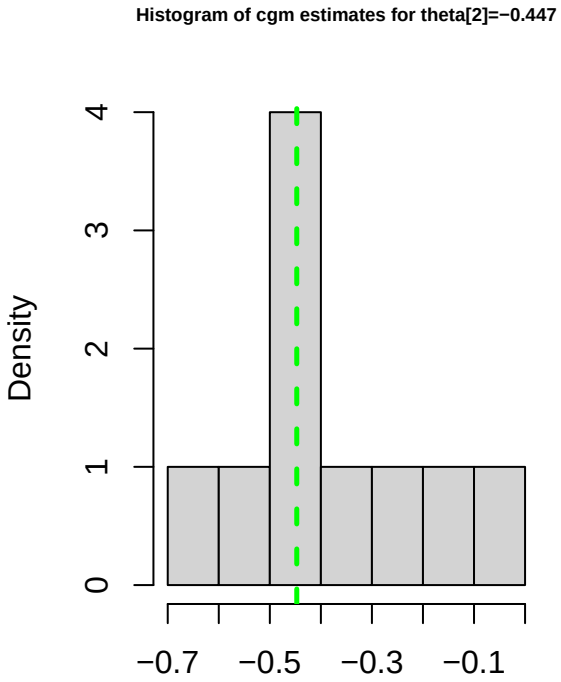
	proposed	cgm
theta[2]	0.188	0.131
theta[27]	0.171	0.206
theta[5]	0.057	0.096
theta[44]	0.107	0.106
total	0.131	0.135

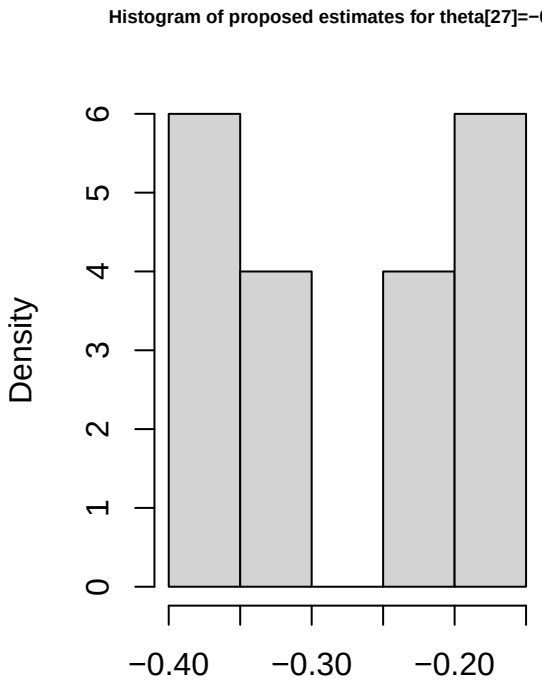
Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

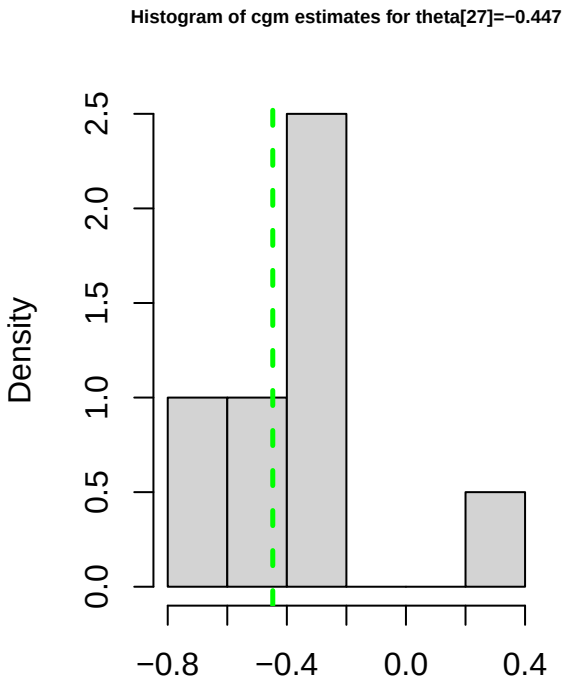
	proposed	cgm
theta[2]	0.378	0.193
theta[27]	0.325	0.137
theta[5]	0.000	0.025
theta[44]	0.009	0.018
total	0.178	0.093

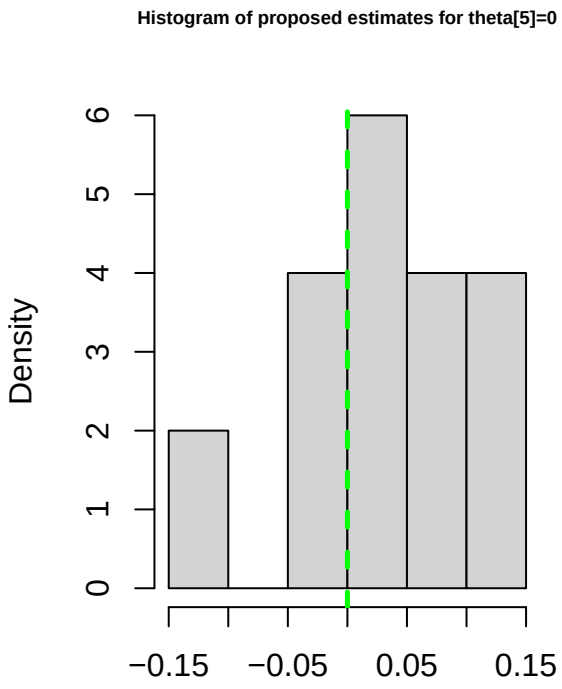
Boxplots

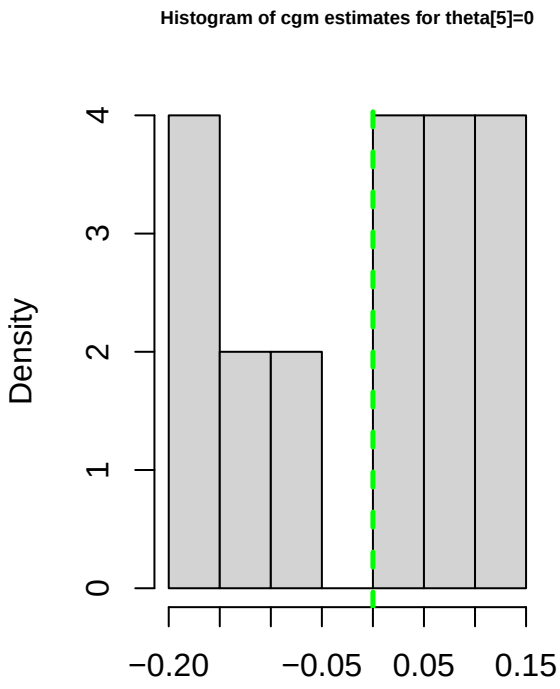

Histogram of proposed estimates for theta[2]=−0.447
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.5
−0.3
−0.1

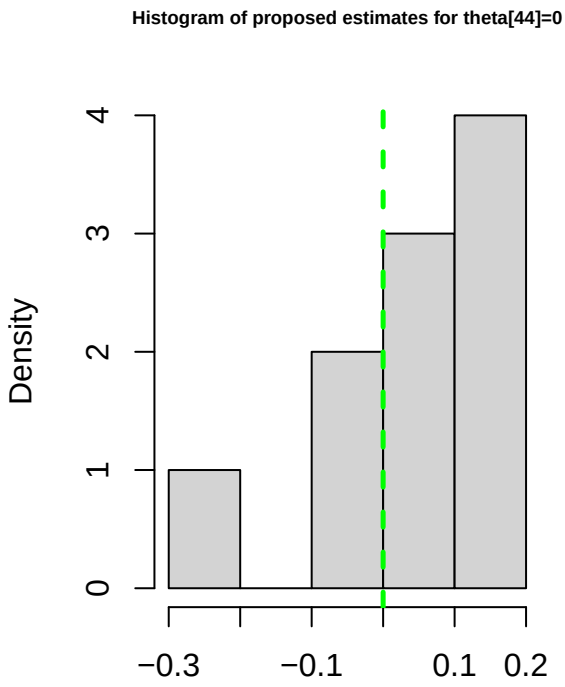

Histogram of cgm estimates for theta[2]=−0.447
Density
0
1
2
3
4
−0.7
−0.5
−0.3
−0.1

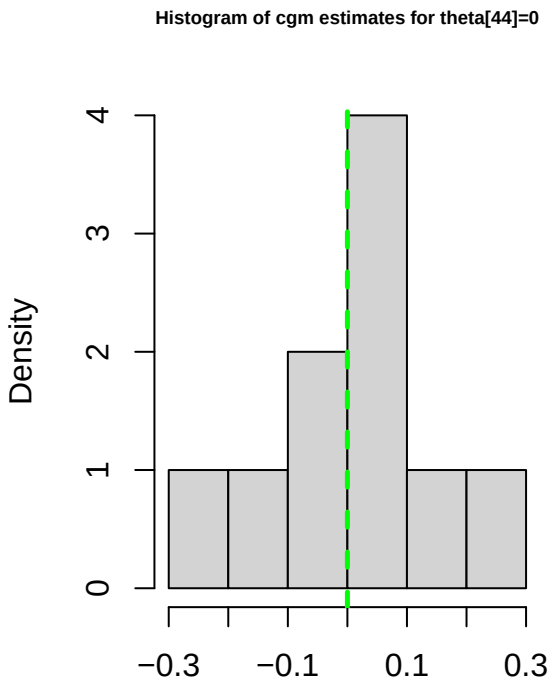

Histogram of proposed estimates for theta[27]=−0.447
Density
0
1
2
3
4
5
6
−0.40
−0.30
−0.20


Histogram of cgm estimates for theta[27]=−0.447
Density
0.0
0.5
1.0
1.5
2.0
2.5
−0.8
−0.4
0.0
0.4

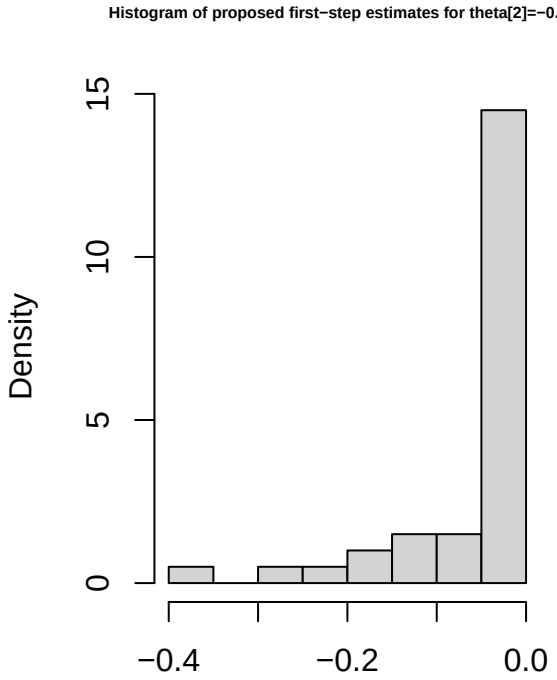

Histogram of proposed estimates for theta[5]=0
Density
0
1
2
3
4
5
6
−0.15
−0.05
0.05
0.15

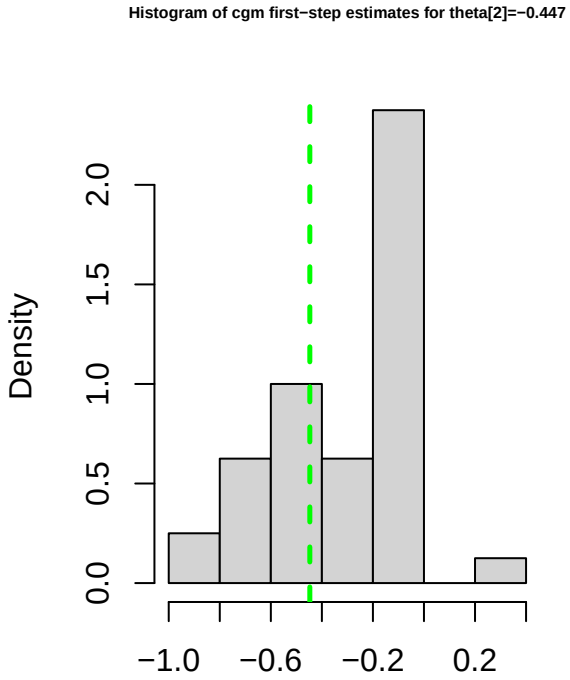

Histogram of cgm estimates for theta[5]=0
Density
0
1
2
3
4
−0.20
−0.05
0.05
0.15

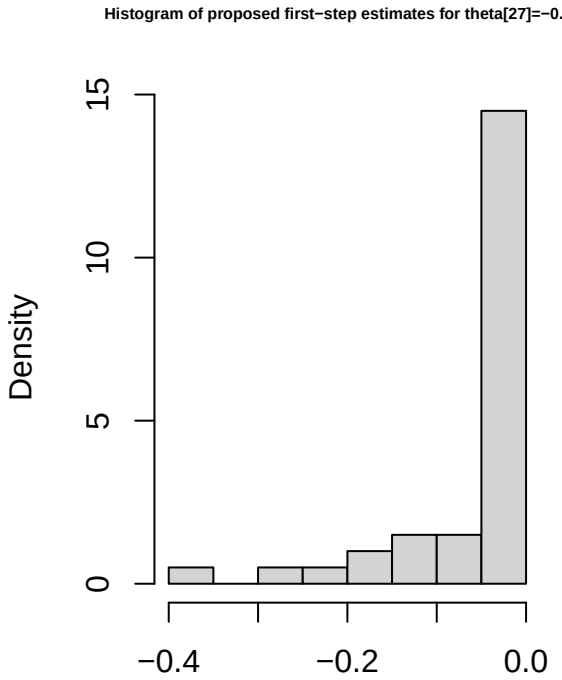

Histogram of proposed estimates for theta[44]=0
Density
0
1
2
3
4
−0.3
−0.1
0.1
0.2

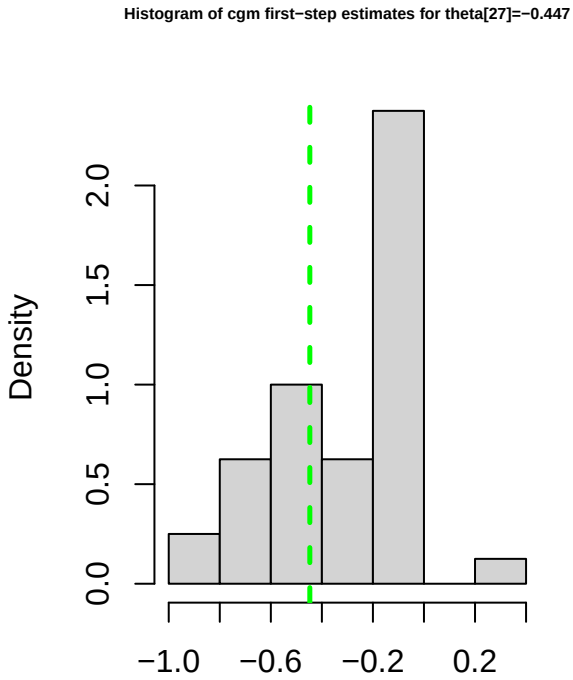

Histogram of cgm estimates for theta[44]=0
Density
0
1
2
3
4
−0.3
−0.1
0.1
0.3

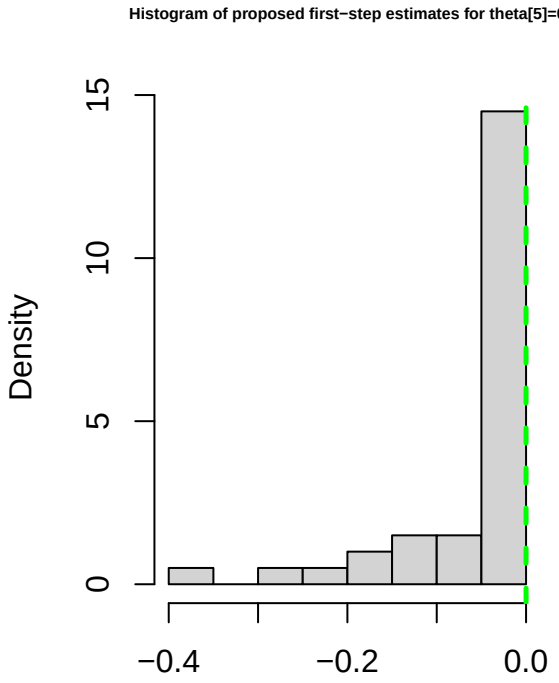
First Step Histograms

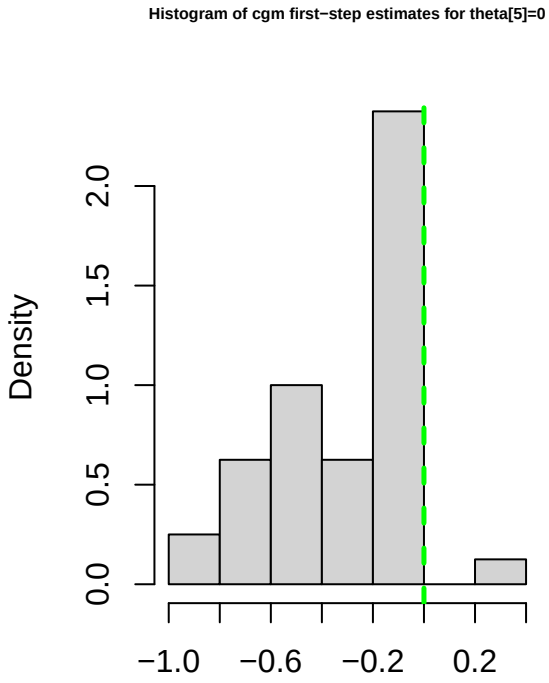


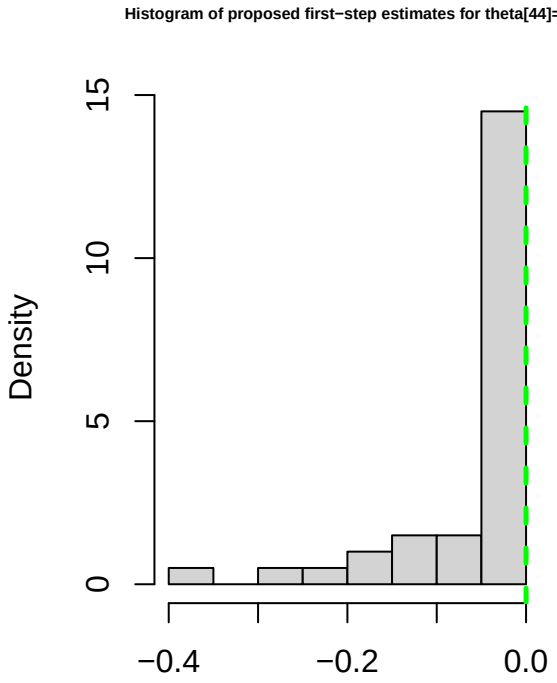


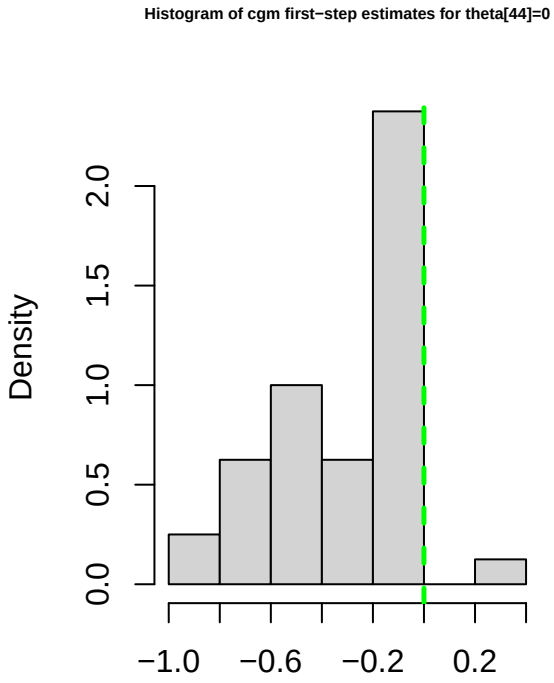





Histogram of proposed first−step estimates for theta[5]=0
Density
0
5
10
15
−0.4
−0.2
0.0


Histogram of cgm first−step estimates for theta[5]=0
Density
0.0
0.5
1.0
1.5
2.0
−1.0
−0.6
−0.2
0.2


Histogram of proposed first−step estimates for theta[44]=0
Density
0
5
10
15
−0.4
−0.2
0.0


Histogram of cgm first−step estimates for theta[44]=0
Density
0.0
0.5
1.0
1.5
2.0
−1.0
−0.6
−0.2
0.2

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 5: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[2]	-0.460	-0.236	-0.038	-0.447	-0.052
theta[27]	-0.391	-0.283	-0.155	-0.387	-0.162
theta[5]	-0.120	0.018	0.143	-0.102	0.136
theta[44]	-0.225	0.068	0.153	-0.196	0.153

Table 6: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[2]	-0.613	-0.405	-0.091	-0.595	-0.104
theta[27]	-0.705	-0.306	0.226	-0.694	0.120
theta[5]	-0.174	0.013	0.136	-0.172	0.128
theta[44]	-0.218	0.039	0.245	-0.194	0.217

Statistics for Theoretical 95% Confidence Intervals

Table 7: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[2]	-0.261	0.116	-0.489	-0.033	0.5
theta[27]	-0.276	0.121	-0.513	-0.039	0.6
theta[5]	0.022	0.119	-0.212	0.256	1.0
theta[44]	0.034	0.114	-0.189	0.257	0.9

Table 8: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[2]	-0.367	0.134	-0.629	-0.105	0.8
theta[27]	-0.347	0.134	-0.610	-0.085	0.9
theta[5]	-0.009	0.119	-0.243	0.226	1.0
theta[44]	0.010	0.120	-0.225	0.246	0.9