Simulation Results

2026-01-13

Simulation Setup

This simulation is performed with $n = 200$ and $d = 20$, using the 2-d lattice as the underlying graph. $s = 3$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.2$. The attached results are for a 10-replication simulation. The true values of the parameter vector θ are

```
0 0 0 0 0.5773503 0 0 0 0 0 0 0 0 0 0 -0.5773503 0 0 0 0.5773503 ,
```

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{$ 5, 16, 4, 8$\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison ($\frac{1}{n.sim}\sum_{i=1}^{n.sim}\frac{1}{|\mathcal{C}|}\|\hat{\theta}_{i,\mathcal{C}} - \theta_{\mathcal{C}}\|^2$)

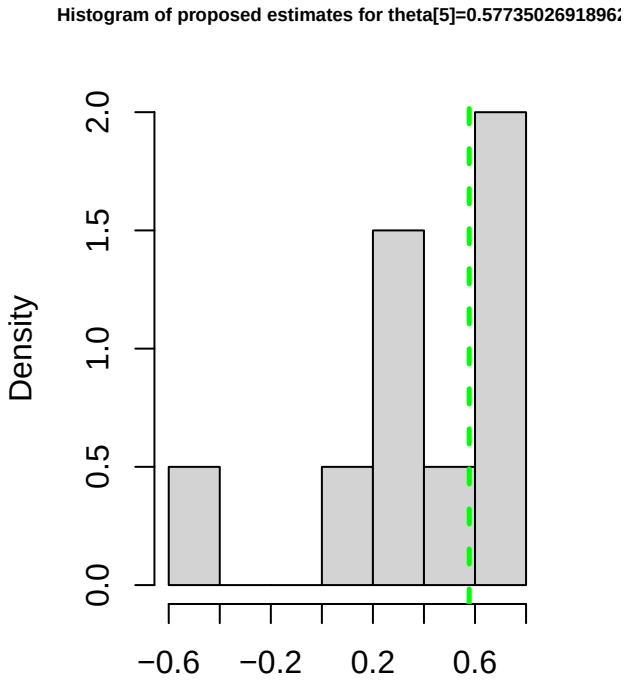
Table 1: Mean-Squared Error of Parameter Estimates

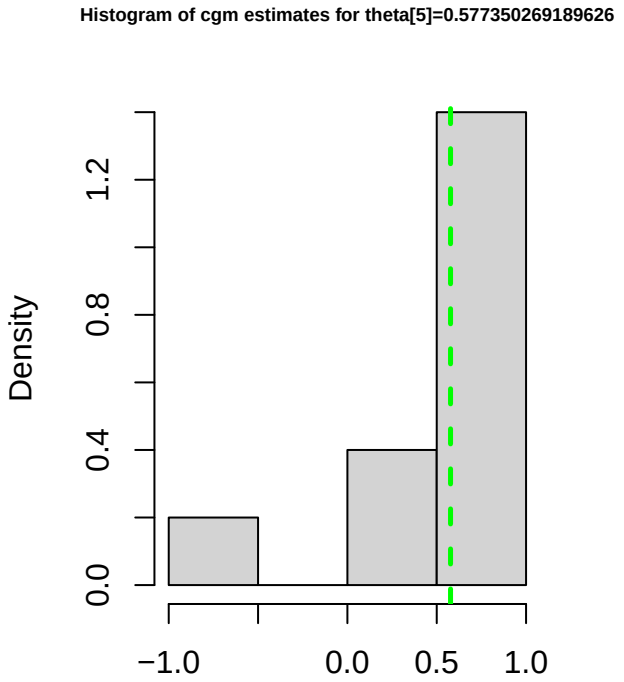
	proposed	cgm
theta[5]	0.184	0.191
theta[16]	0.073	0.067
theta[4]	0.117	0.024
theta[8]	0.143	0.069
total	0.129	0.088

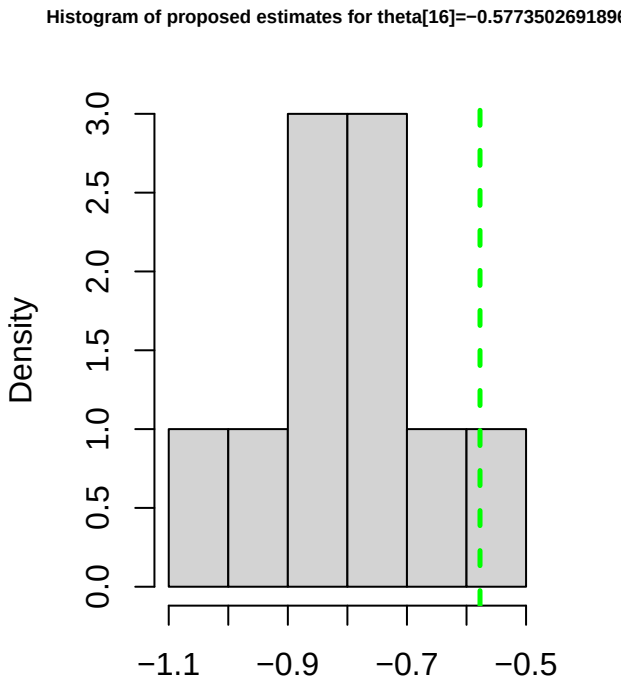
Table 2: Mean-Squared Error of First-Step Parameter Estimates

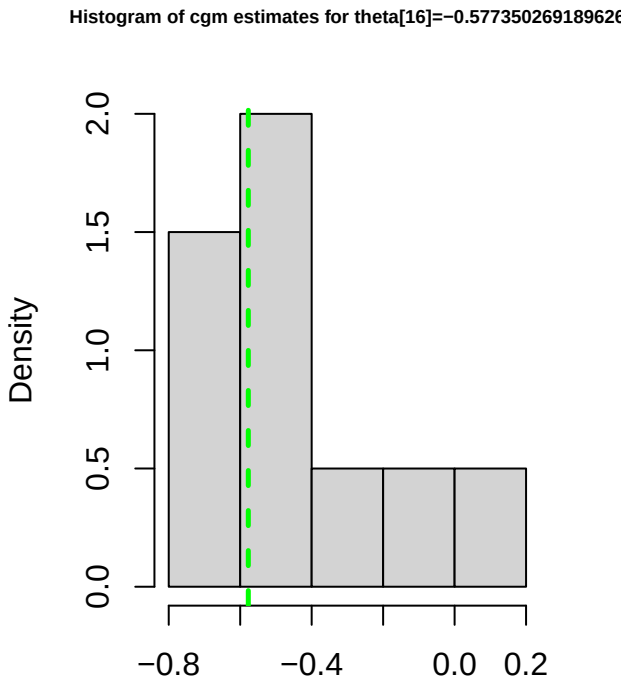
	proposed	cgm
theta[5]	0.162	0.116
theta[16]	0.105	0.113
theta[4]	0.066	0.003
theta[8]	0.097	0.011
total	0.108	0.061

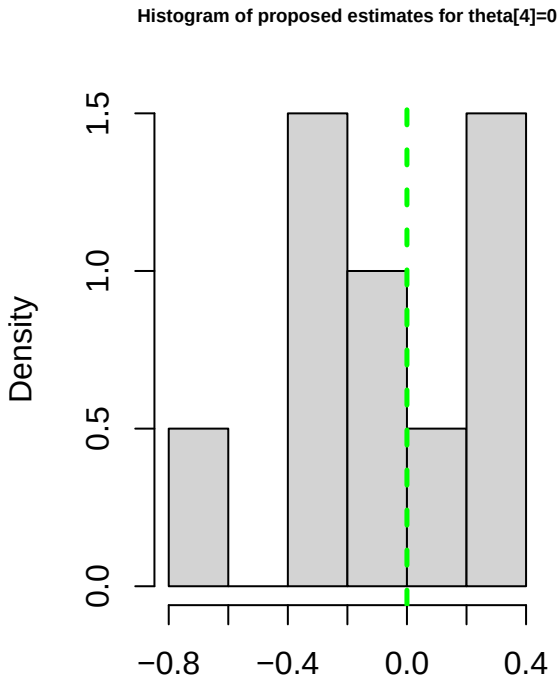
Boxplots

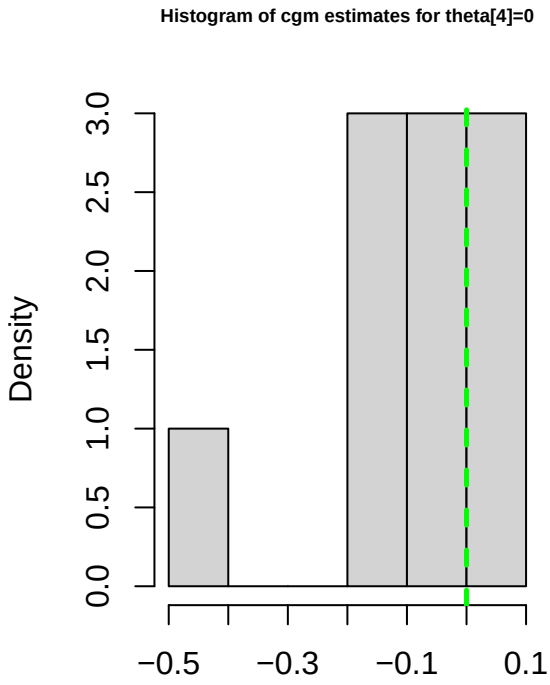

Histogram of proposed estimates for theta[5]=0.57735026918962
Density
0.0
0.5
1.0
1.5
2.0
−0.6
−0.2
0.2
0.6

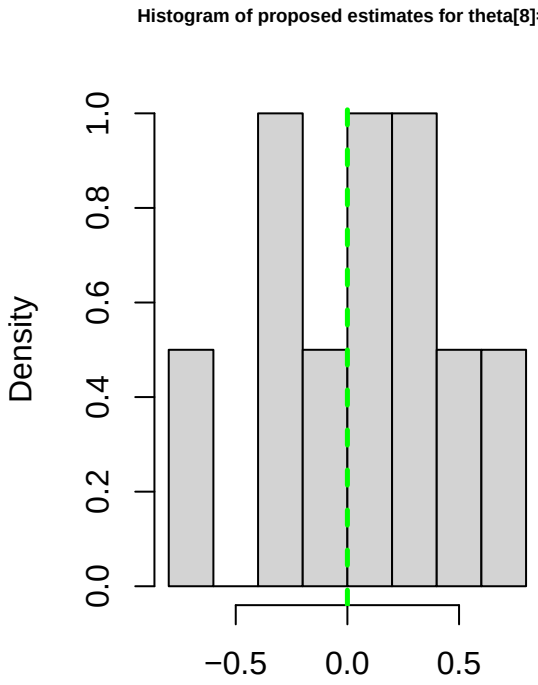

Histogram of cgm estimates for theta[5]=0.577350269189626
Density
0.0
0.4
0.8
1.2
−1.0
0.0
0.5
1.0

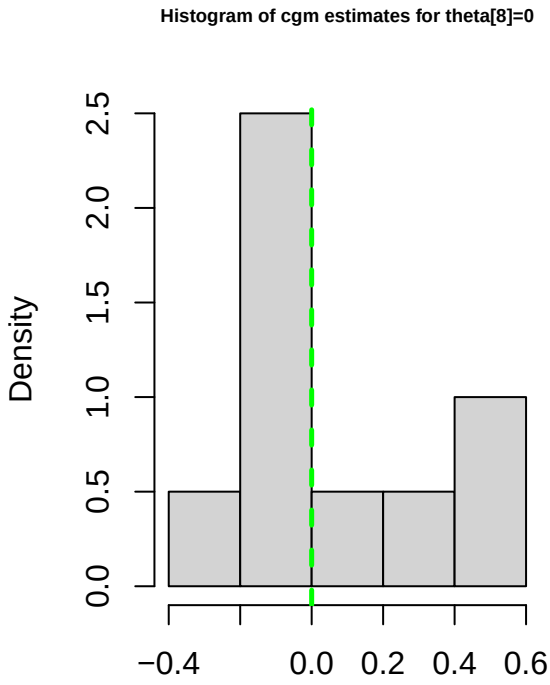

Histogram of proposed estimates for theta[16]=−0.577350269189
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−1.1
−0.9
−0.7
−0.5


Histogram of cgm estimates for theta[16]=−0.577350269189626
Density
0.0
0.5
1.0
1.5
2.0
−0.8
−0.4
0.0
0.2

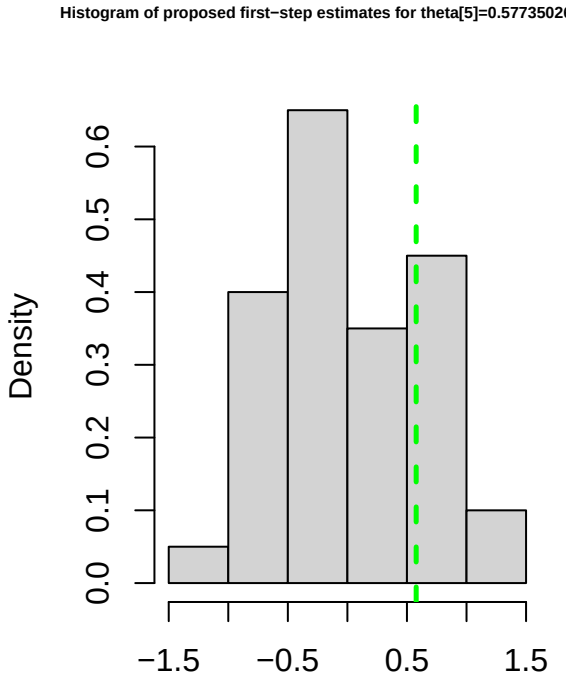

Histogram of proposed estimates for theta[4]=0
Density
0.0
0.5
1.0
1.5
−0.8
−0.4
0.0
0.4

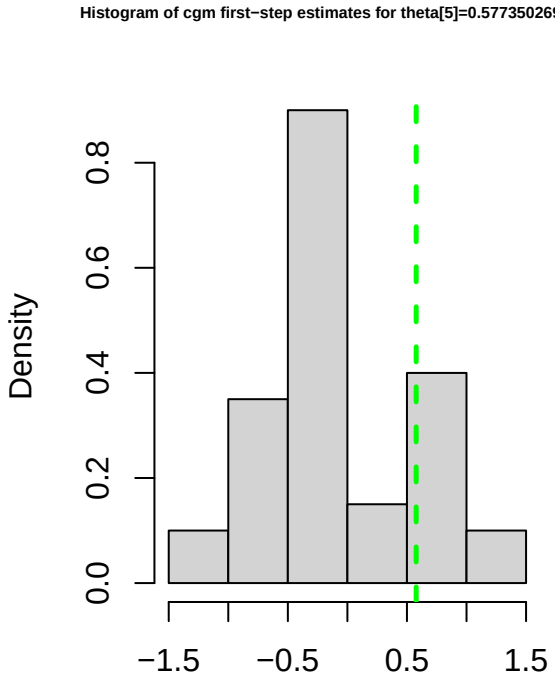

Histogram of cgm estimates for theta[4]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.5
−0.3
−0.1
0.1

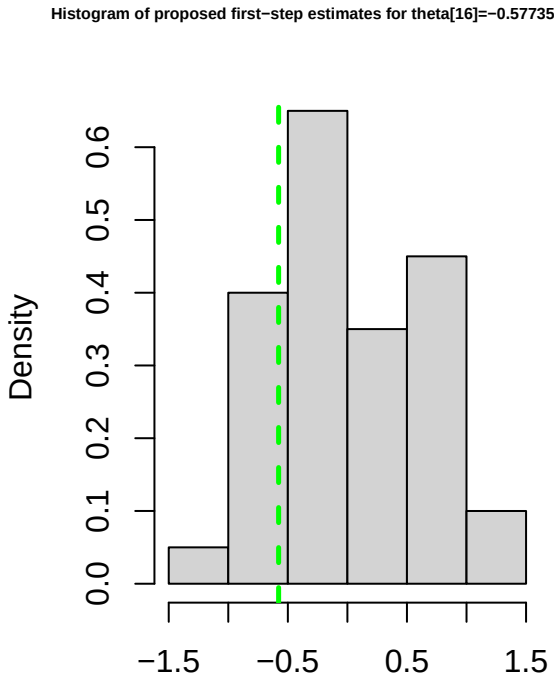

Histogram of proposed estimates for theta[8]=0
Density
0.0
0.2
0.4
0.6
0.8
1.0
−0.5
0.0
0.5

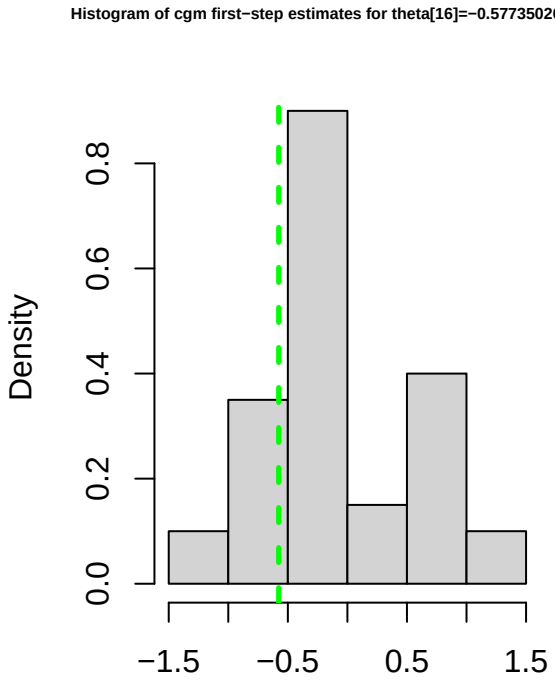

Histogram of cgm estimates for theta[8]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
−0.4
0.0
0.2
0.4
0.6

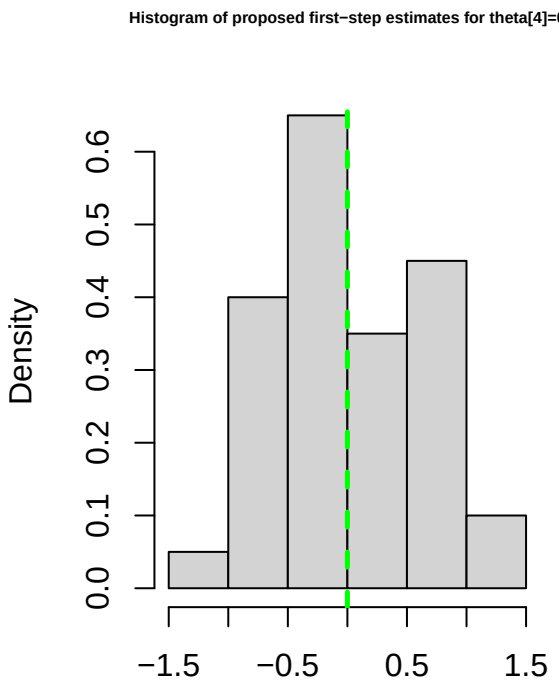
First Step Histograms

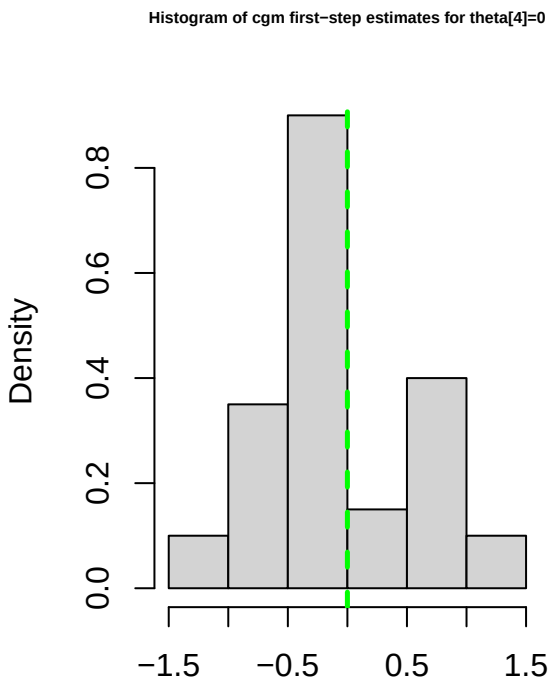


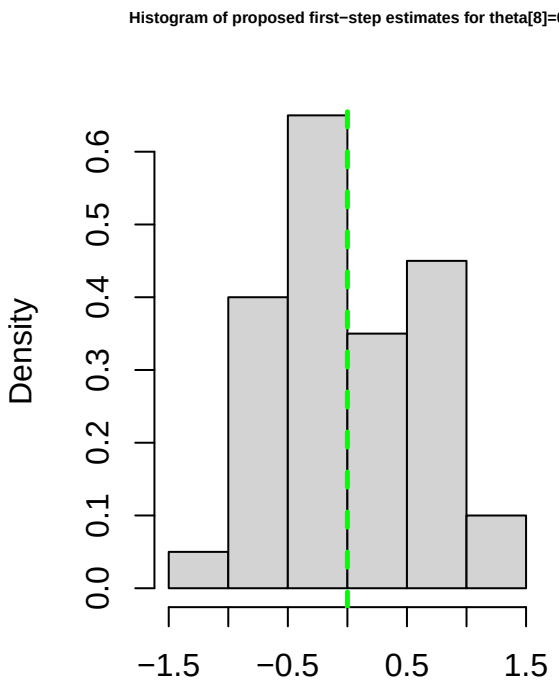


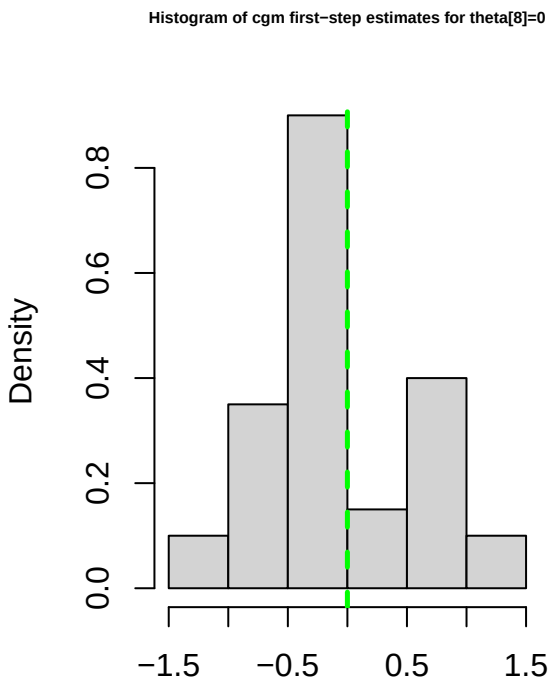





Histogram of proposed first−step estimates for theta[4]=0
Density
0.0
0.1
0.2
0.3
0.4
0.5
0.6
−1.5
−0.5
0.5
1.5


Histogram of cgm first−step estimates for theta[4]=0
Density
0.0
0.2
0.4
0.6
0.8
−1.5
−0.5
0.5
1.5


Histogram of proposed first−step estimates for theta[8]=0
Density
0.0
0.1
0.2
0.3
0.4
0.5
0.6
−1.5
−0.5
0.5
1.5


Histogram of cgm first−step estimates for theta[8]=0
Density
0.0
0.2
0.4
0.6
0.8
−1.5
−0.5
0.5
1.5

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 3: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[5]	-0.551	0.388	0.764	-0.422	0.755
theta[16]	-1.072	-0.804	-0.500	-1.042	-0.538
theta[4]	-0.763	-0.089	0.314	-0.675	0.310
theta[8]	-0.606	0.007	0.726	-0.548	0.657

Table 4: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[5]	-0.730	0.589	0.800	-0.494	0.792
theta[16]	-0.764	-0.533	0.023	-0.752	-0.014
theta[4]	-0.406	-0.070	0.097	-0.356	0.086
theta[8]	-0.307	-0.065	0.581	-0.276	0.541

Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[5]	0.358	0.234	-0.101	0.817	0.8
theta[16]	-0.802	0.214	-1.221	-0.383	0.9
theta[4]	-0.077	0.190	-0.449	0.296	0.8
theta[8]	0.041	0.198	-0.348	0.429	0.7

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[5]	0.469	0.163	0.150	0.789	0.9
theta[16]	-0.471	0.158	-0.781	-0.161	0.8
theta[4]	-0.077	0.126	-0.324	0.170	0.9
theta[8]	0.041	0.127	-0.208	0.290	0.7