Simulation Results

2026-01-20

Simulation Setup

This simulation is performed with $n = 200$ and $d = 200$, using the 2-d lattice as the underlying graph. $s = 5$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.2$. The attached results are for a 10-replication simulation. The parameter vector θ has sparse components other than the following:

Parameter.Index	Value
17	-0.447
64	0.447
116	0.447
169	0.447
171	0.447

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{$ 17, 64, 70, 172$\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[17]	0.062	0.595
theta[64]	0.037	0.095
theta[70]	0.010	0.046
theta[172]	0.015	0.079
total	0.031	0.204

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[17]	0.192	0.049

	proposed	cgm
theta[64]	0.097	0.054
theta[70]	0.000	0.000
theta[172]	0.000	0.005
total	0.072	0.027

Mean absolute deviation comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat

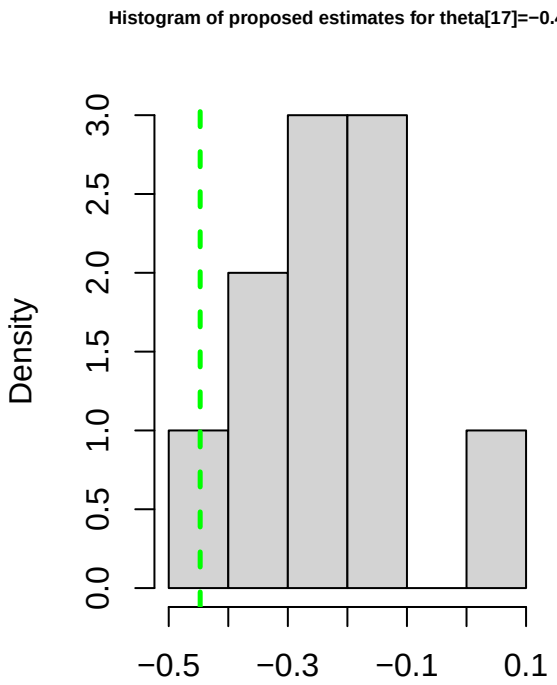
Table 3: Mean Absolute Deviation of Parameter Estimates

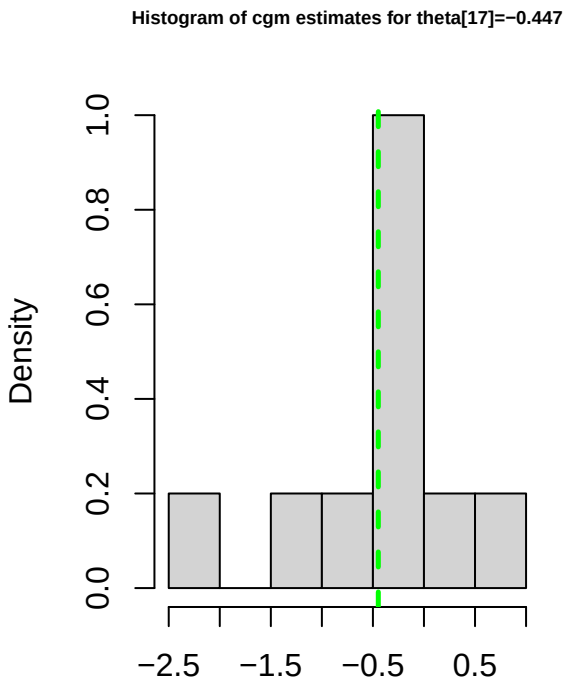
	proposed	cgm
theta[17]	0.208	0.580
theta[64]	0.171	0.230
theta[70]	0.086	0.177
theta[172]	0.094	0.225
total	0.140	0.303

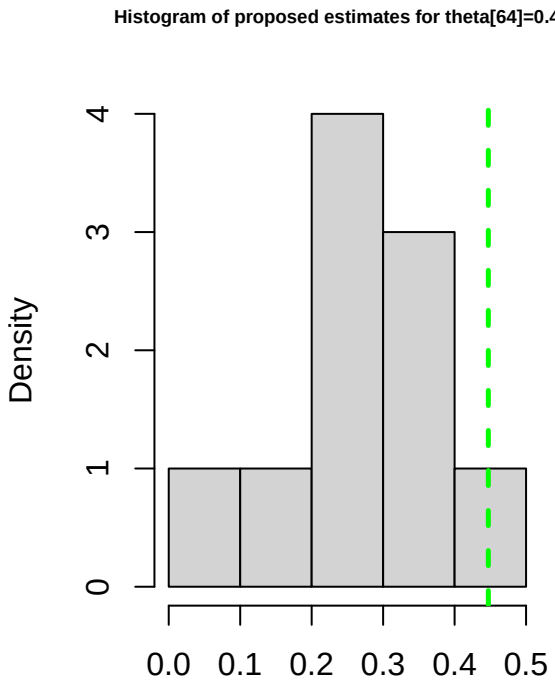
Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

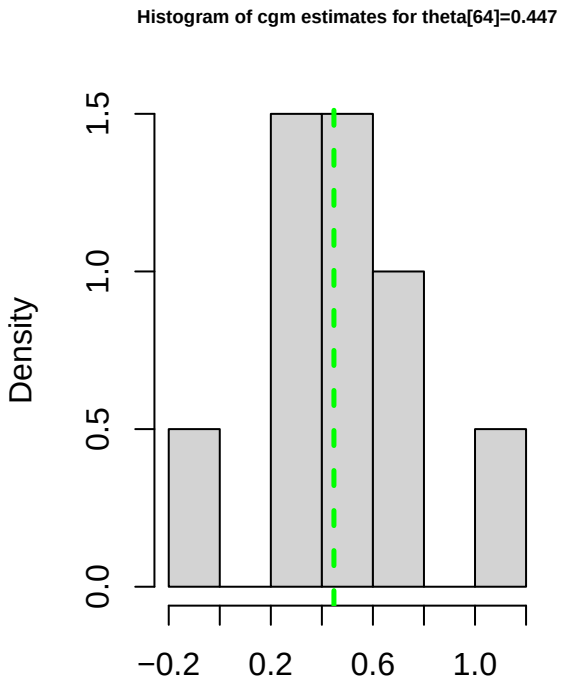
	proposed	cgm
theta[17]	0.438	0.166
theta[64]	0.288	0.192
theta[70]	0.000	0.000
theta[172]	0.000	0.028
total	0.182	0.096

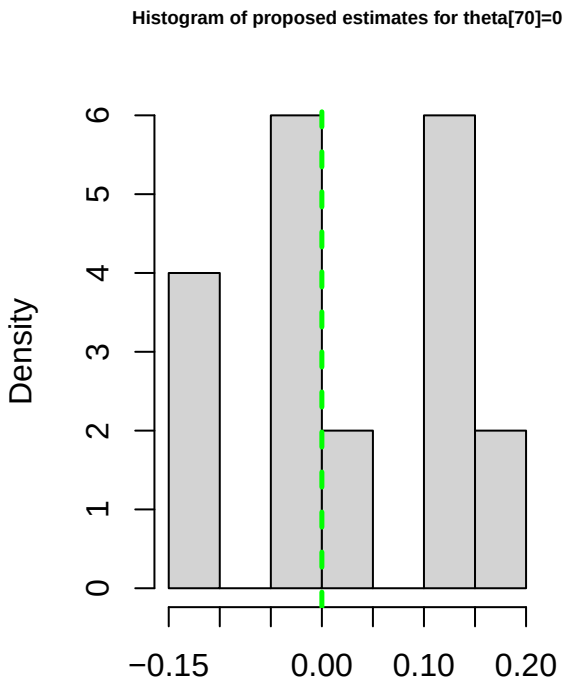
Boxplots

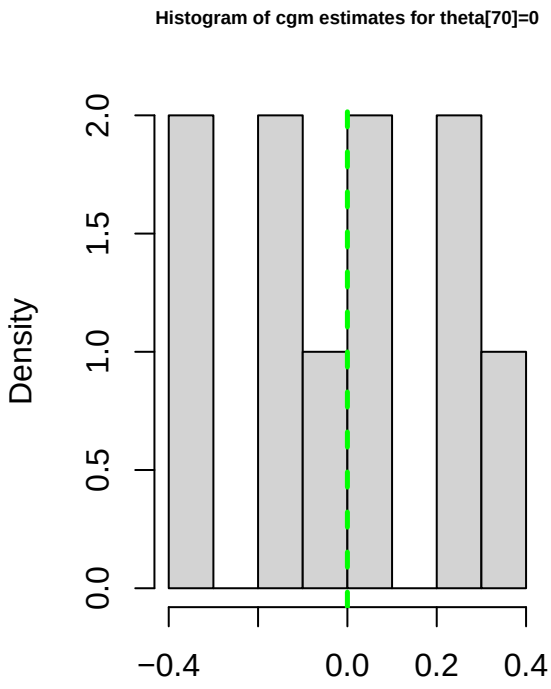

Histogram of proposed estimates for theta[17]=−0.447
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.5
−0.3
−0.1
0.1

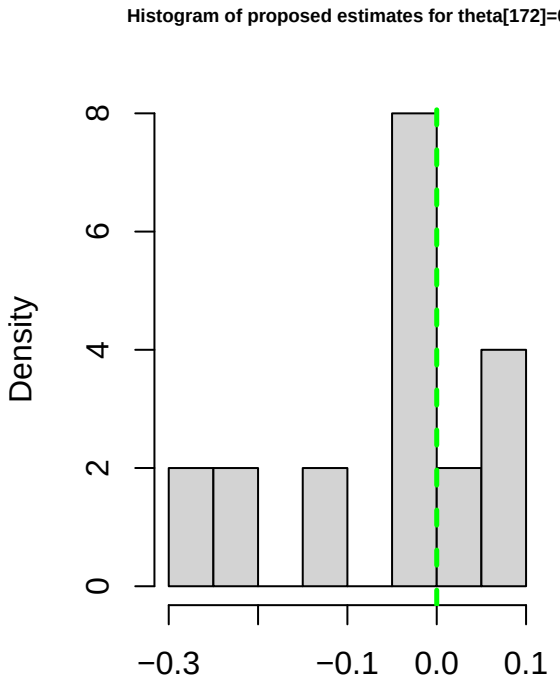

Histogram of cgm estimates for theta[17]=−0.447
Density
0.0
0.2
0.4
0.6
0.8
1.0
−2.5
−1.5
−0.5
0.5

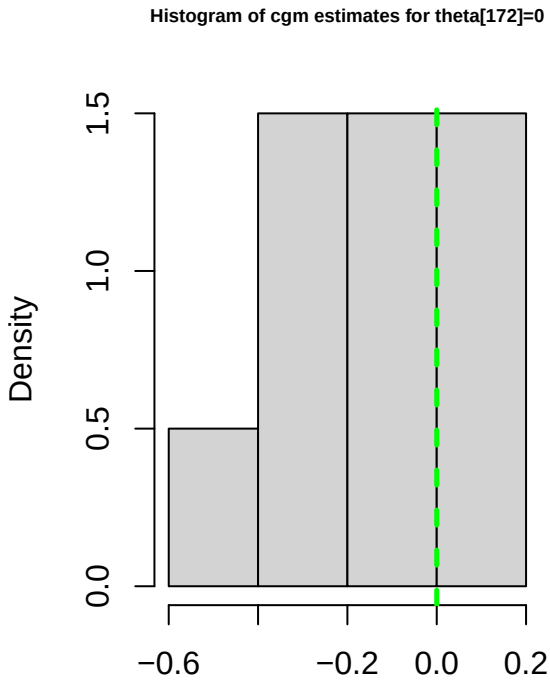

Histogram of proposed estimates for theta[64]=0.447
Density
0
1
2
3
4
0.0
0.1
0.2
0.3
0.4
0.5


Histogram of cgm estimates for theta[64]=0.447
Density
0.0
0.5
1.0
1.5
−0.2
0.2
0.6
1.0


Histogram of proposed estimates for theta[70]=0
Density
0
1
2
3
4
5
6
−0.15
0.00
0.10
0.20


Histogram of cgm estimates for theta[70]=0
Density
0.0
0.5
1.0
1.5
2.0
−0.4
0.0
0.2
0.4


Histogram of proposed estimates for theta[172]=0
Density
0
2
4
6
8
−0.3
−0.1
0.0
0.1


Histogram of cgm estimates for theta[172]=0
Density
0.0
0.5
1.0
1.5
−0.6
−0.2
0.0
0.2

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[17]	-0.239	0.115	-0.463	-0.014	0.6
theta[64]	0.285	0.117	0.055	0.514	0.8
theta[70]	0.016	0.116	-0.211	0.243	1.0
theta[172]	-0.054	0.115	-0.280	0.172	0.9

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[17]	-0.436	0.219	-0.865	-0.007	0.5
theta[64]	0.511	0.248	0.025	0.996	0.8
theta[70]	-0.004	0.181	-0.358	0.350	0.8
theta[172]	-0.146	0.178	-0.495	0.202	0.8