

# Simulation Results

2026-01-16

## Simulation Setup

This simulation is performed with  $n = 200$  and  $d = 100$ , using the 2-d lattice as the underlying graph.  $s = 5$  parameters are set to be nonzero, and the beta parameter is chosen to be  $\beta = 0$ . The attached results are for a 10-replication simulation. The true values of the parameter vector  $\theta$  are

0 0 0 0 0 0.4472136 0 0 0 0 0 0 0 0 0 0 0 0 0 0.4472136 0 0 0 0.4472136 0 0 0 0 0 0 0 0 0 0 0 0

but for brevity, our simulation only estimates the indices of  $\theta$  in  $\mathcal{C} = \{6, 24, 71, 42\}$  elements of  $\theta$ . Accordingly, **all statistics and visuals are indicative of performance only on the set  $\mathcal{C}$ .**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

## Results

### Mean-squared error comparison  $\frac{1}{n_{\text{sim}}} \sum_{i=1}^{n_{\text{sim}}} \frac{1}{|\mathcal{C}|} \|\hat{\theta}_i - \theta\|^2$

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[6]	0.059	0.027
theta[24]	0.033	0.054
theta[71]	0.012	0.017
theta[42]	0.020	0.025
total	0.031	0.031

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[6]	0.157	0.036
theta[24]	0.141	0.039
theta[71]	0.000	0.000
theta[42]	0.000	0.010
total	0.075	0.021

### Mean absolute deviation comparison  $\frac{1}{n.\text{sim}} \sum_{i=1}^{n.\text{sim}} \frac{1}{|\mathcal{C}|} \|\hat{\theta}_i - \theta\|$

Table 3: Mean Absolute Deviation of Parameter Estimates

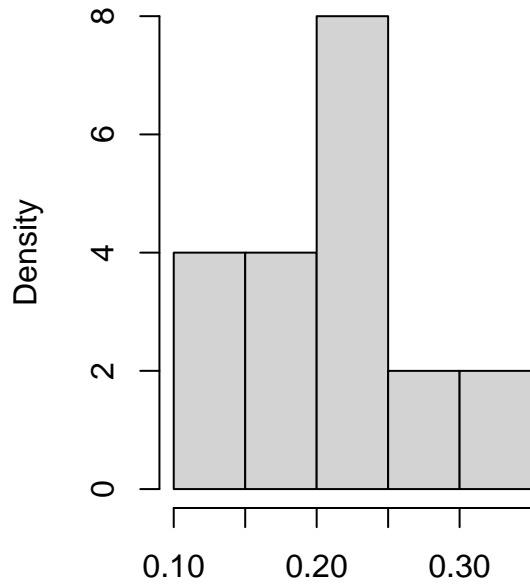
	proposed	cgm
theta[6]	0.236	0.104
theta[24]	0.141	0.182
theta[71]	0.084	0.111
theta[42]	0.113	0.133
total	0.144	0.132

Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

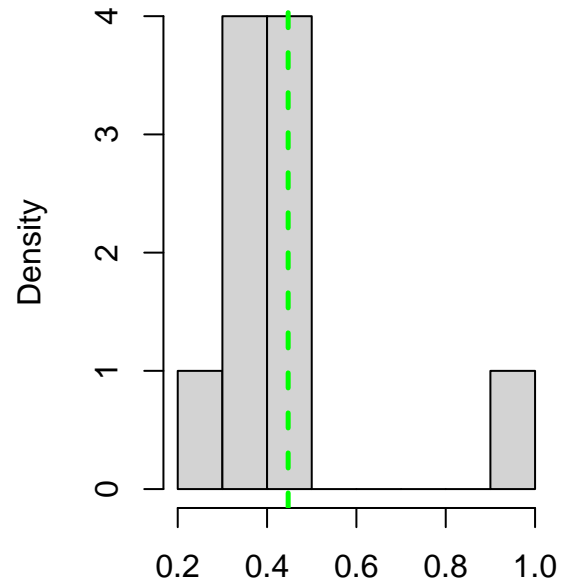
	proposed	cgm
theta[6]	0.389	0.168
theta[24]	0.363	0.159
theta[71]	0.000	0.000
theta[42]	0.007	0.042
total	0.190	0.092

## Boxplots

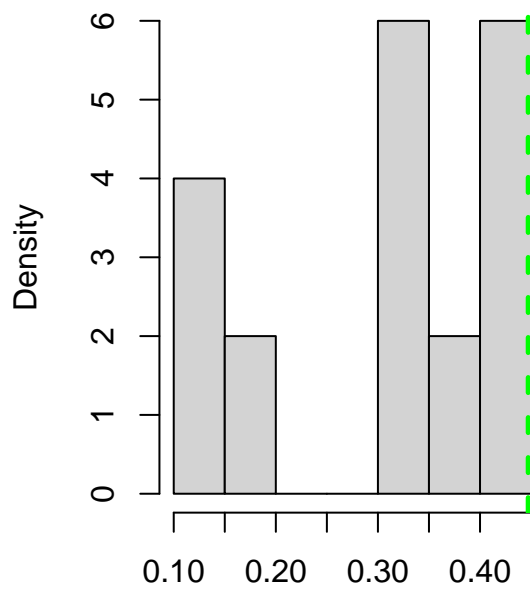
Histogram of proposed estimates for  $\theta[6]=0.447$



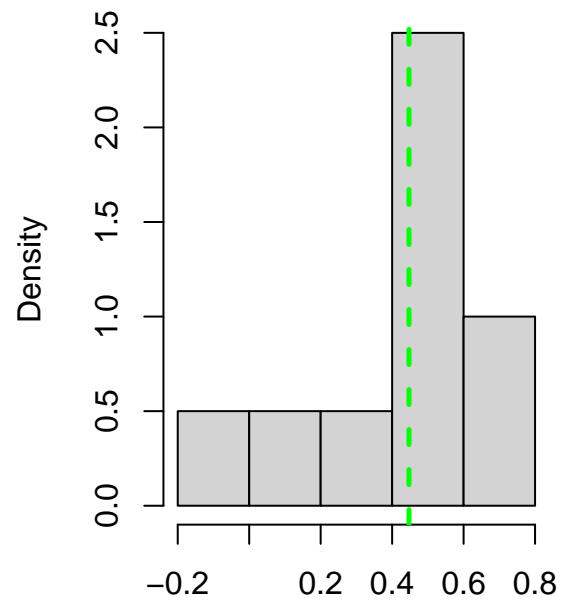
Histogram of cgm estimates for  $\theta[6]=0.447$



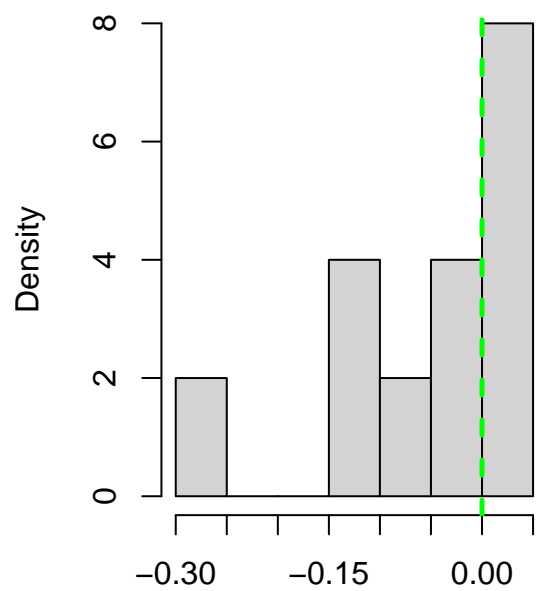
Histogram of proposed estimates for  $\theta[24]=0.447$



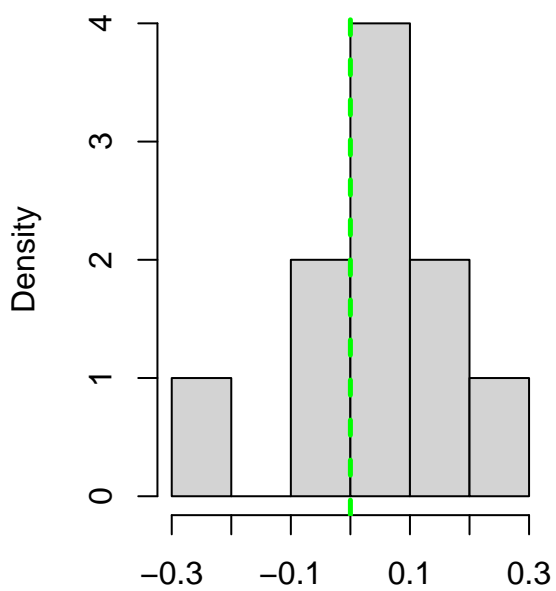
Histogram of cgm estimates for  $\theta[24]=0.447$



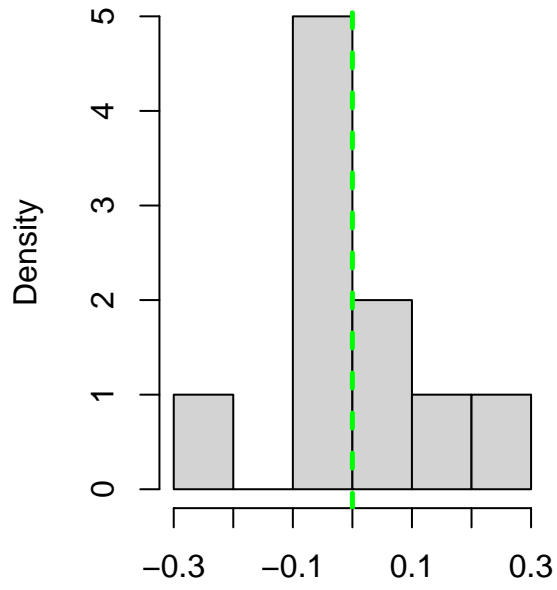
Histogram of proposed estimates for  $\theta_{[71]}=0$



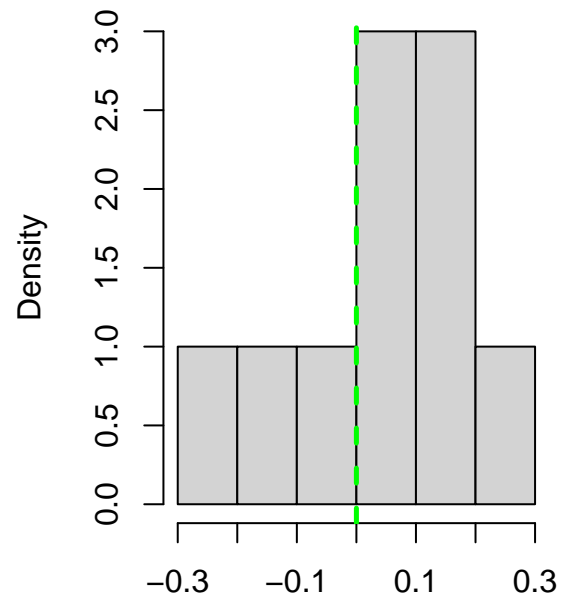
Histogram of cgm estimates for  $\theta_{[71]}=0$



Histogram of proposed estimates for  $\theta_{42}=0$

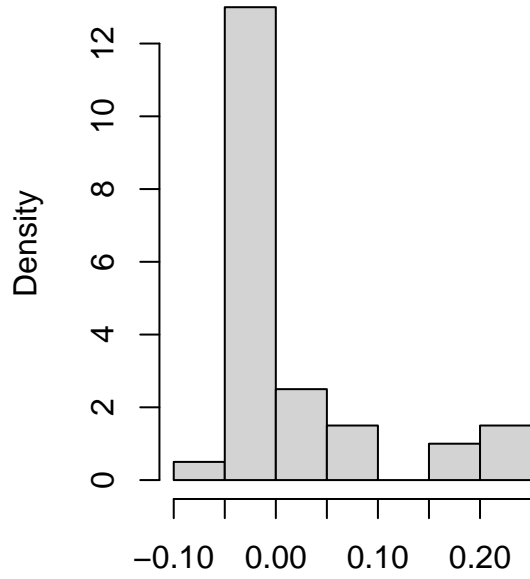


Histogram of cgm estimates for  $\theta_{42}=0$

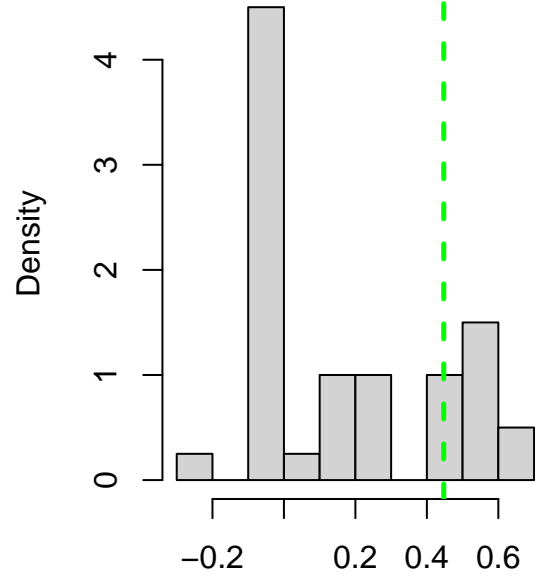


## First Step Histograms

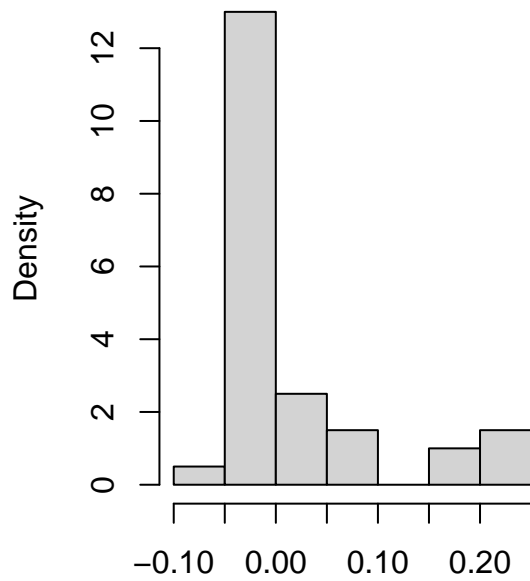
Histogram of proposed first-step estimates for  $\theta[6]=0.447$



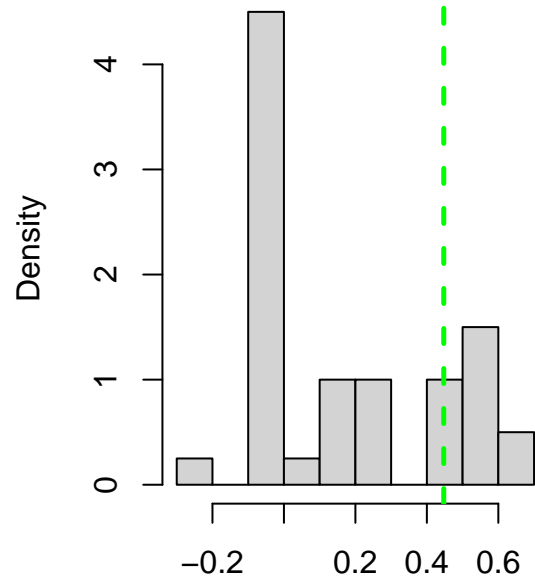
Histogram of cgm first-step estimates for  $\theta[6]=0.447$



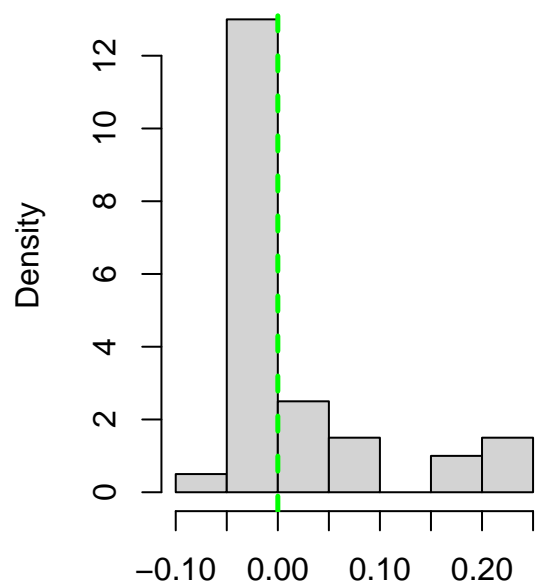
Histogram of proposed first-step estimates for  $\theta[24]=0.447$



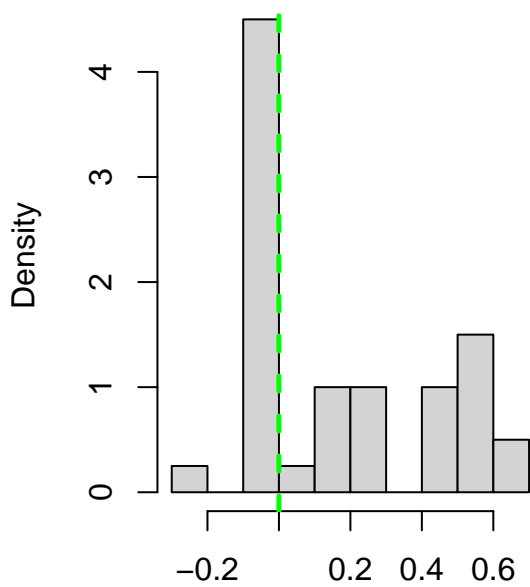
Histogram of cgm first-step estimates for  $\theta[24]=0.447$



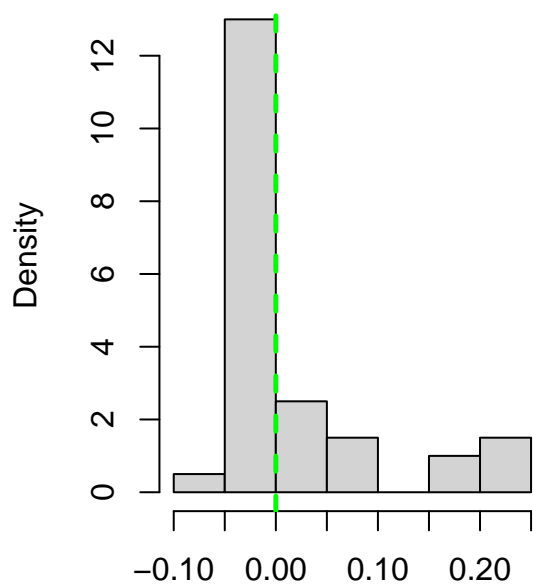
Histogram of proposed first-step estimates for  $\theta_{71}=0$



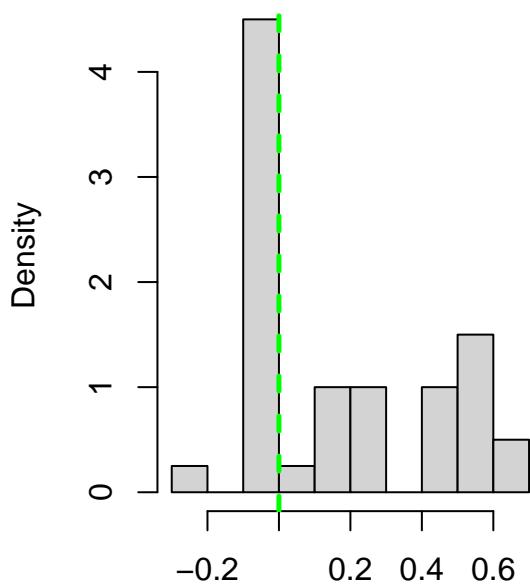
Histogram of cgm first-step estimates for  $\theta_{71}=0$



Histogram of proposed first-step estimates for  $\theta_{[42]}=0$



Histogram of cgm first-step estimates for  $\theta_{[42]}=0$





## Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 5: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[6]	0.126	0.216	0.316	0.128	0.310
theta[24]	0.131	0.334	0.424	0.132	0.423
theta[71]	-0.261	-0.047	0.047	-0.234	0.046
theta[42]	-0.259	-0.035	0.259	-0.220	0.240

Table 6: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[6]	0.295	0.397	0.906	0.300	0.807
theta[24]	-0.065	0.502	0.783	-0.008	0.752
theta[71]	-0.224	0.053	0.230	-0.192	0.217
theta[42]	-0.282	0.084	0.262	-0.244	0.240

## Statistics for Theoretical 95% Confidence Intervals

Table 7: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[6]	0.212	0.123	-0.030	0.453	0.5
theta[24]	0.306	0.127	0.058	0.554	0.7
theta[71]	-0.056	0.119	-0.290	0.178	0.9
theta[42]	0.003	0.120	-0.233	0.238	0.8

Table 8: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[6]	0.439	0.120	0.203	0.674	0.9
theta[24]	0.435	0.133	0.174	0.696	0.7
theta[71]	0.034	0.121	-0.202	0.271	1.0
theta[42]	0.049	0.122	-0.190	0.288	0.8