

# Simulation Results

2026-01-07

## Simulation Setup

This simulation is performed with  $n = 200$  and  $d = 10$ , using the 2-d lattice as the underlying graph.  $s = 2$  parameters are set to be nonzero, and the beta parameter is chosen to be  $\beta = 0$ . The attached results are for a 10-replication simulation. The true values of the parameter vector  $\theta$  are

0 0 0 -0.7071068 0 0 0 0.7071068 0 0 ,

but for brevity, our simulation only estimates the indices of  $\theta$  in  $\mathcal{C} = \{4, 8, 1, 5\}$  elements of  $\theta$ . Accordingly, **all statistics and visuals are indicative of performance only on the set  $\mathcal{C}$ .**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

## Results

**Mean-squared error comparison** ( $\frac{1}{n.sim} \sum_{i=1}^{n.sim} \frac{1}{|\mathcal{C}|} \|\hat{\theta}_{i,\mathcal{C}} - \theta_{\mathcal{C}}\|^2$ )

Table 1: Mean-Squared Error of Parameter Estimates

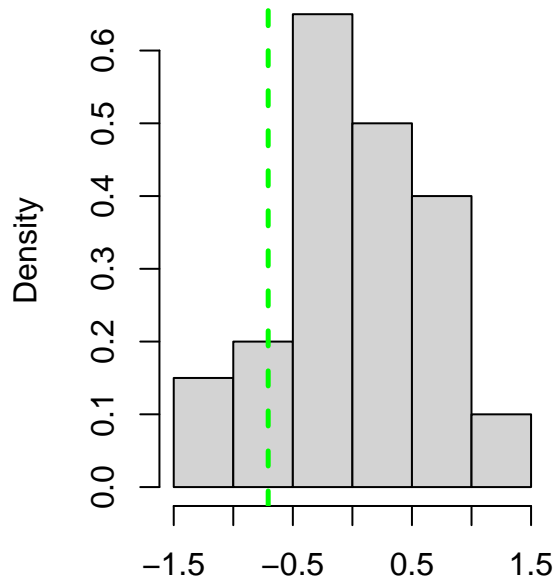
	proposed	cgm
theta[4]	0.372	10.155
theta[8]	0.112	8.670
theta[1]	0.065	0.162
theta[5]	0.055	0.530
total	0.151	4.879

Table 2: Mean-Squared Error of First-Step Parameter Estimates

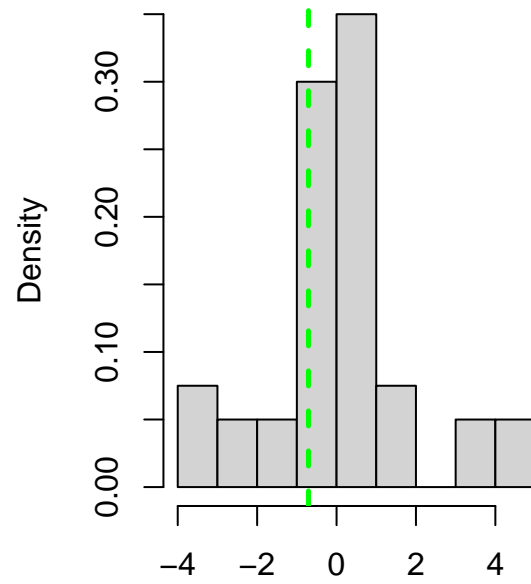
	proposed	cgm
theta[4]	0.193	0.174
theta[8]	0.114	0.094
theta[1]	0.019	0.017
theta[5]	0.032	0.064
total	0.090	0.087

## Boxplots

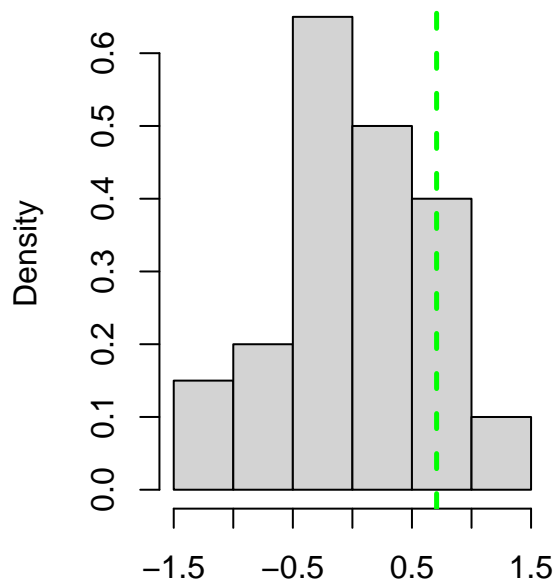
Histogram of proposed estimates for  $\theta_4 = -0.7071067811865$



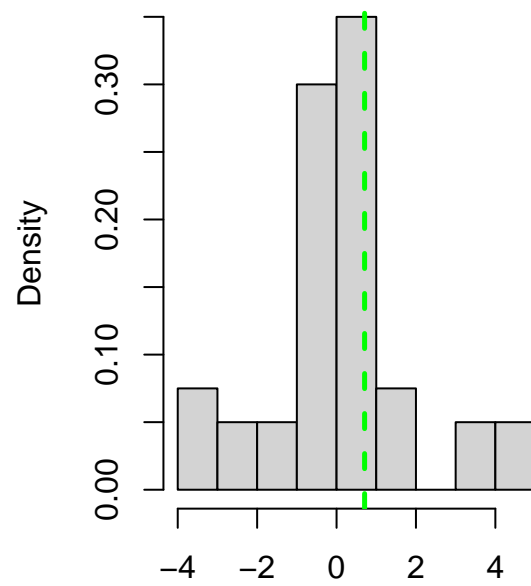
Histogram of cgm estimates for  $\theta_4 = -0.707106781186547$

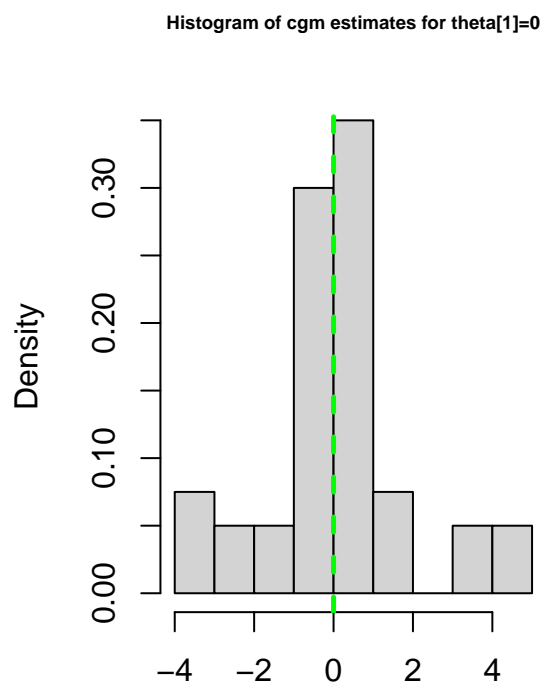
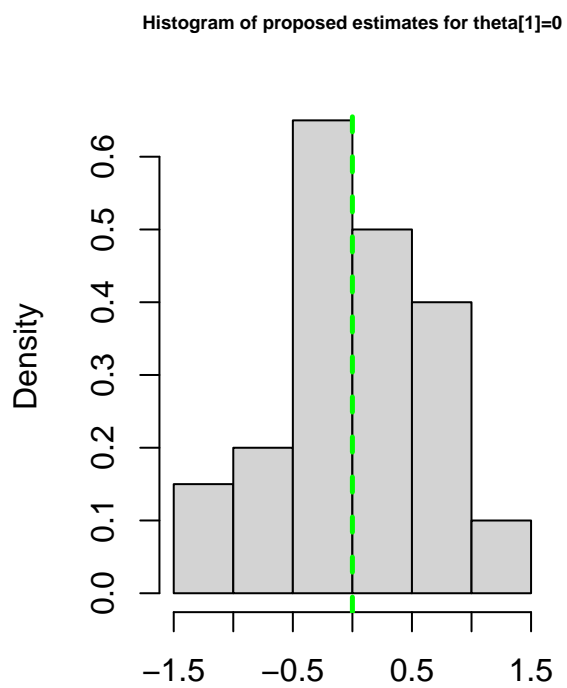


Histogram of proposed estimates for  $\theta_8 = 0.707106781186547$

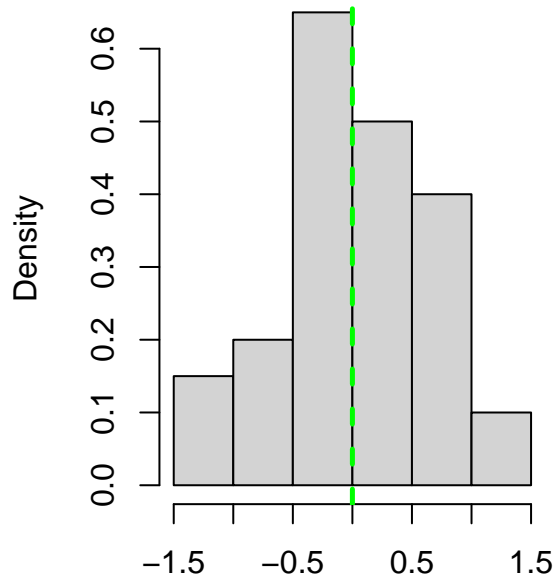


Histogram of cgm estimates for  $\theta_8 = 0.707106781186547$

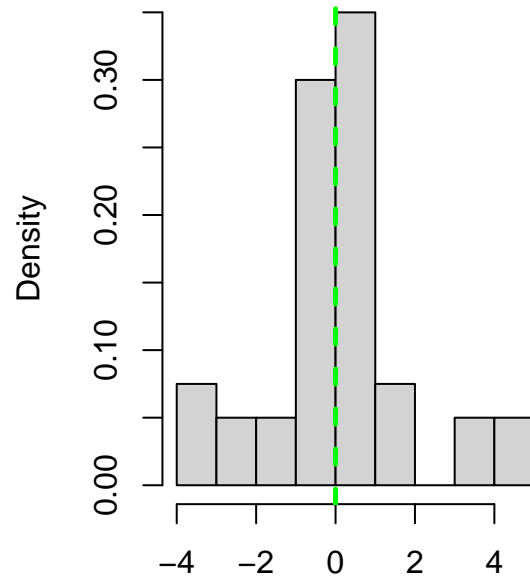




Histogram of proposed estimates for  $\theta[5]=0$

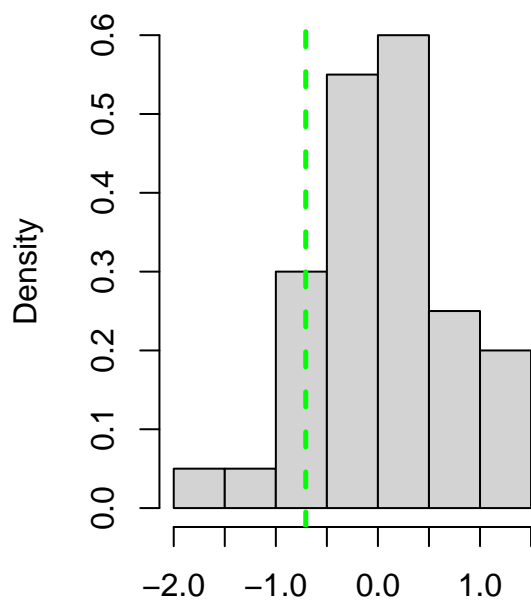


Histogram of cgm estimates for  $\theta[5]=0$

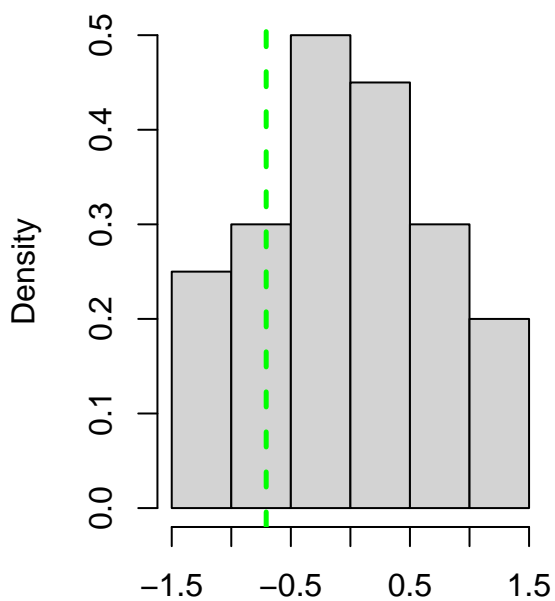


## First Step Histograms

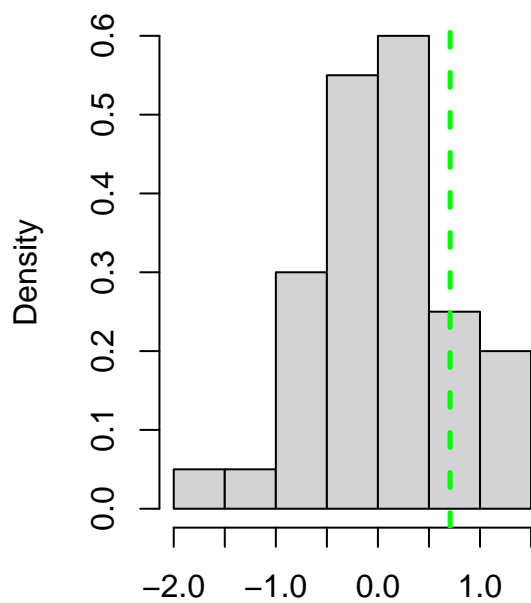
Histogram of proposed first-step estimates for  $\theta_4 = -0.7071067811865$



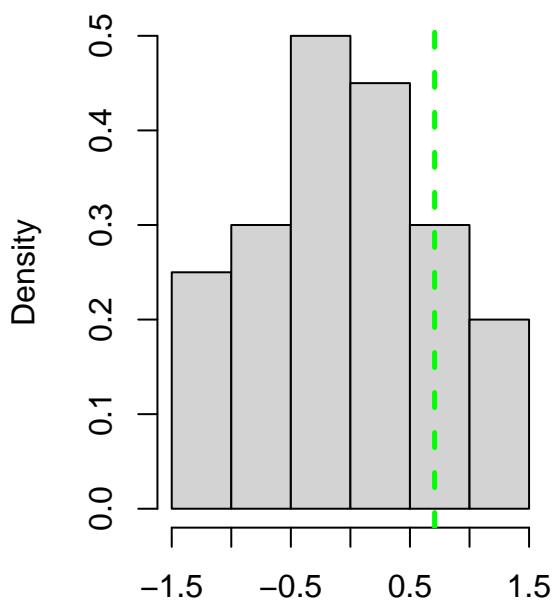
Histogram of cgm first-step estimates for  $\theta_4 = -0.707106781186547$



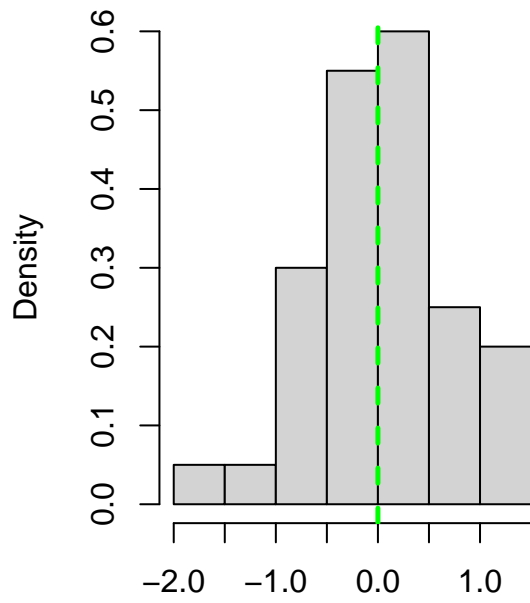
Histogram of proposed first-step estimates for  $\theta_8 = 0.707106781186547$



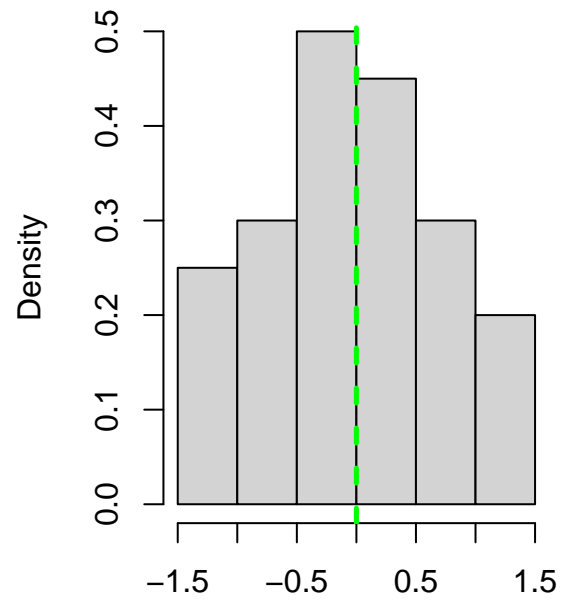
Histogram of cgm first-step estimates for  $\theta_8 = 0.707106781186547$



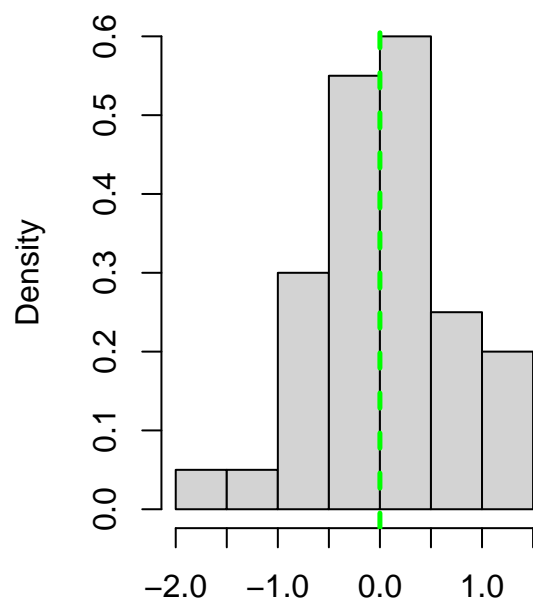
Histogram of proposed first-step estimates for  $\theta[1]=0$



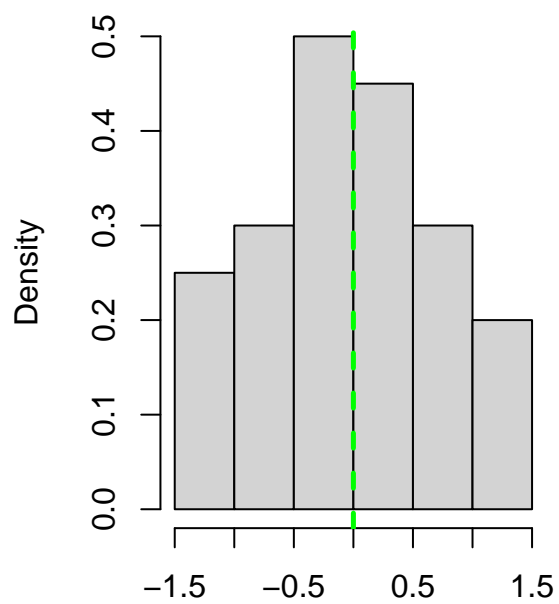
Histogram of cgm first-step estimates for  $\theta[1]=0$



Histogram of proposed first-step estimates for  $\theta[5]=0$



Histogram of cgm first-step estimates for  $\theta[5]=0$



## Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 3: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[4]	-1.054	-0.811	1.061	-1.048	0.744
theta[8]	-0.194	0.745	1.057	-0.040	1.023
theta[1]	-0.487	-0.090	0.318	-0.461	0.283
theta[5]	-0.348	0.050	0.536	-0.300	0.474

Table 4: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[4]	0.702	1.389	4.356	0.718	4.280
theta[8]	-3.819	-1.639	-0.422	-3.778	-0.462
theta[1]	-0.569	0.064	0.748	-0.529	0.684
theta[5]	-1.720	0.043	1.087	-1.516	0.937

## Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[4]	-0.587	0.199	-0.977	-0.197	0.7
theta[8]	0.651	0.183	0.293	1.009	0.9
theta[1]	-0.073	0.157	-0.380	0.234	0.7
theta[5]	0.053	0.145	-0.230	0.337	0.8

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[4]	2.129	0.274	1.593	2.665	0.0
theta[8]	-1.943	0.264	-2.460	-1.426	0.0
theta[1]	0.104	0.106	-0.104	0.313	0.4
theta[5]	-0.112	0.119	-0.346	0.122	0.2