Simulation Results

2026-01-19

Simulation Setup

This simulation is performed with $n = 200$ and $d = 100$, using the 2-d lattice as the underlying graph. $s = 5$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.4$. The attached results are for a 10-replication simulation. The true values of the parameter vector θ are

```
0 0 0 0 0 -0.4472136 0 0 0 0 0 0 0 0 0.4472136 0 0 0 0 0 0 0 0 0 0 -0.4472136 0 0 0 0 0 0 0 0 0 0 0
```

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{$ 6, 15, 72, 42$\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

```
### Mean-squared error comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat\theta
```

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[6]	0.056	0.013
theta[15]	0.015	0.060
theta[72]	0.007	0.017
theta[42]	0.028	0.022
total	0.026	0.028

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[6]	0.162	0.021
theta[15]	0.098	0.054
theta[72]	0.000	0.004
theta[42]	0.000	0.003
total	0.065	0.021

Mean absolute deviation comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat

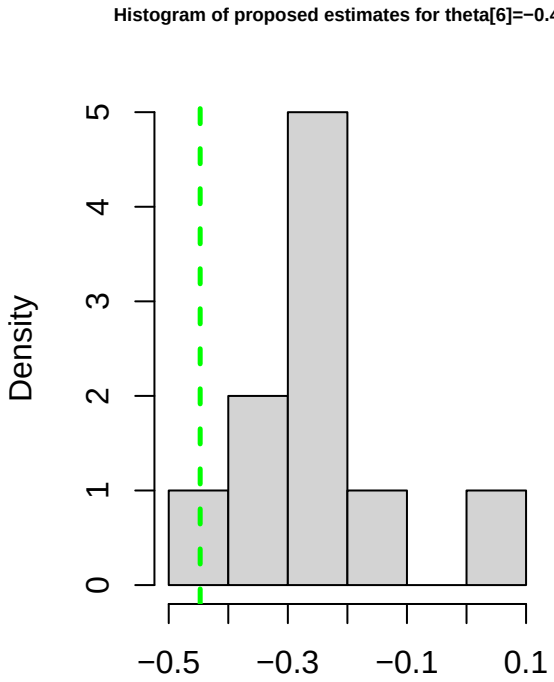
Table 3: Mean Absolute Deviation of Parameter Estimates

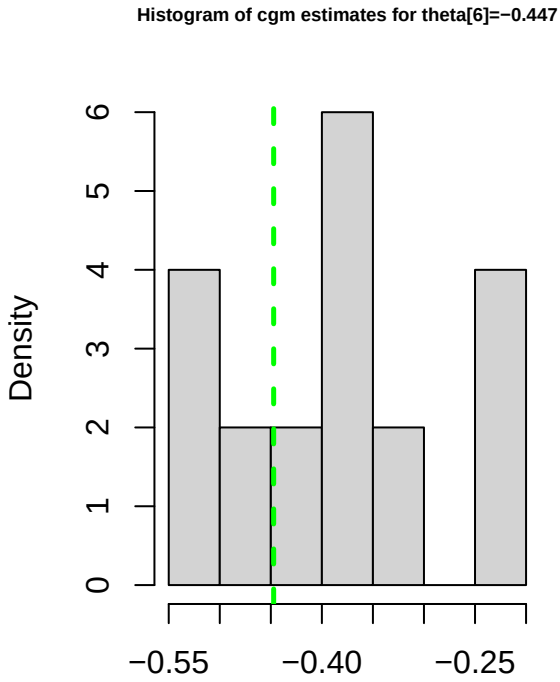
	proposed	cgm
theta[6]	0.201	0.099
theta[15]	0.118	0.190
theta[72]	0.058	0.101
theta[42]	0.142	0.121
total	0.130	0.128

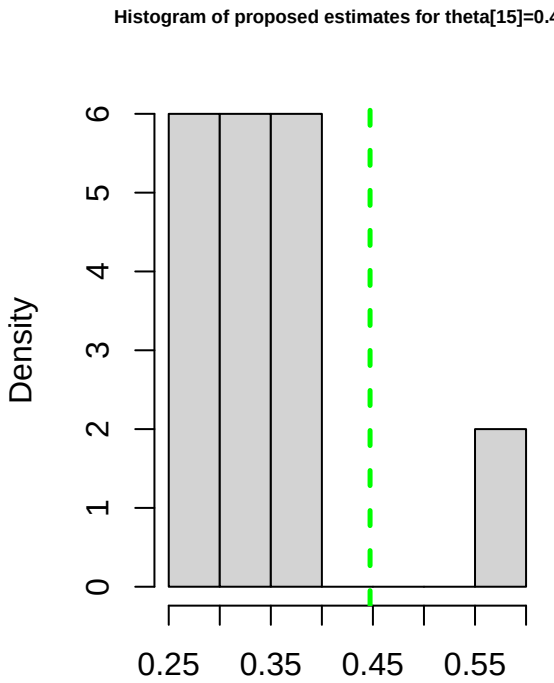
Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

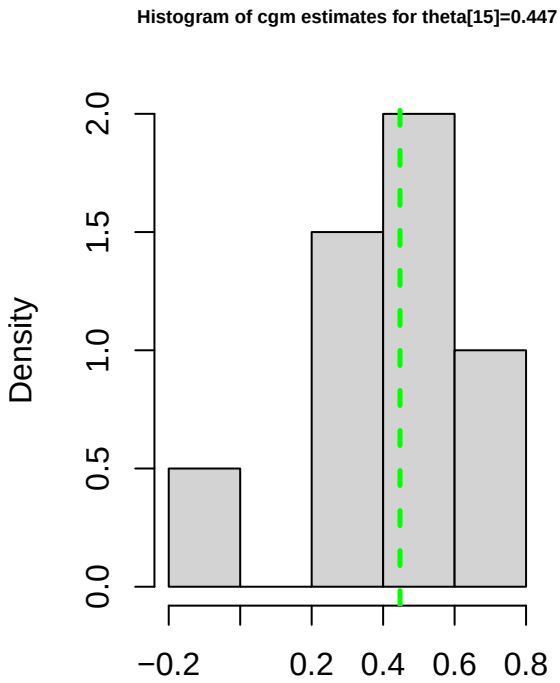
	proposed	cgm
theta[6]	0.401	0.114
theta[15]	0.283	0.189
theta[72]	0.000	0.027
theta[42]	0.006	0.025
total	0.172	0.089

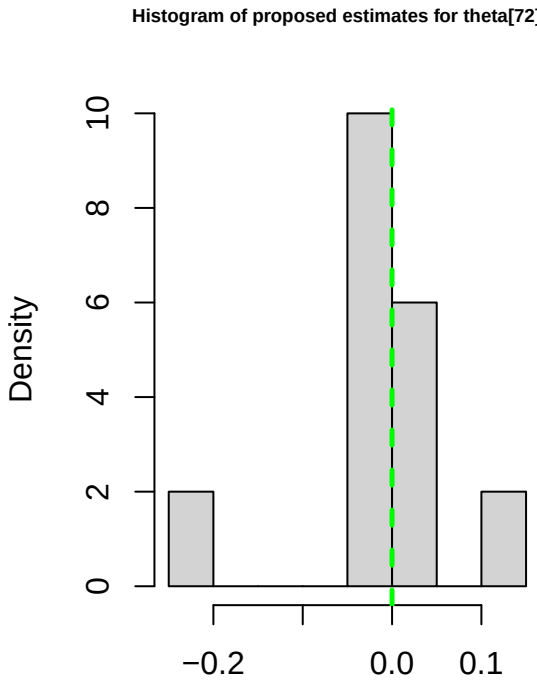
Boxplots

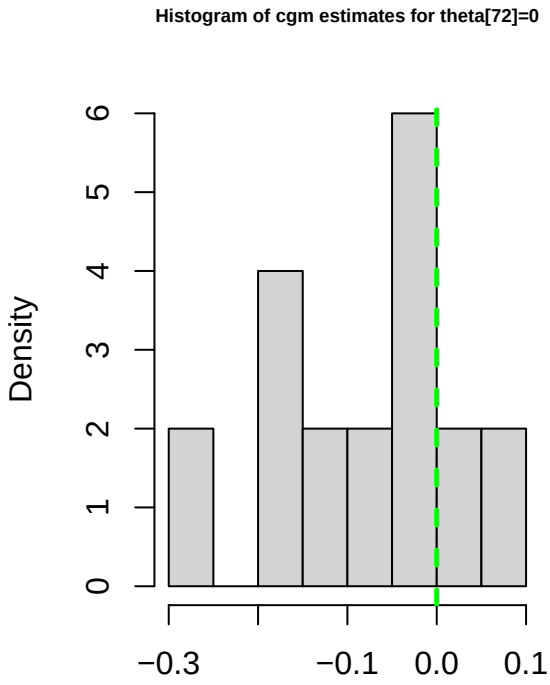

Histogram of proposed estimates for theta[6]=−0.447
Density
0
1
2
3
4
5
−0.5
−0.3
−0.1
0.1

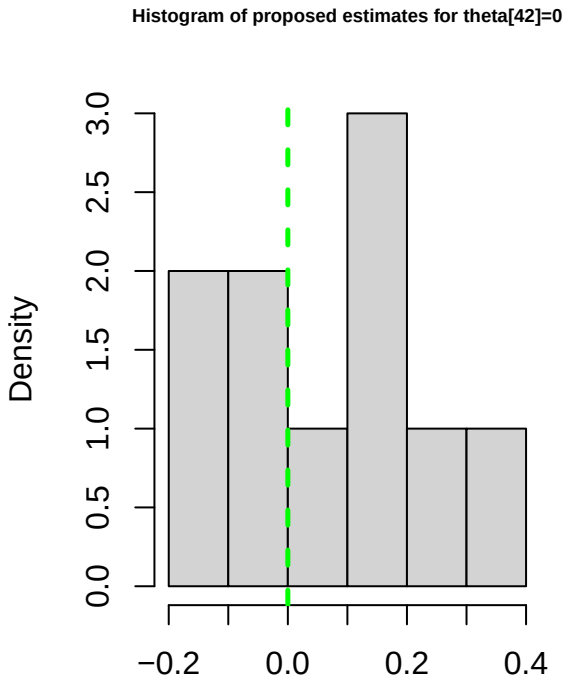

Histogram of cgm estimates for theta[6]=−0.447
Density
0
1
2
3
4
5
6
−0.55
−0.40
−0.25

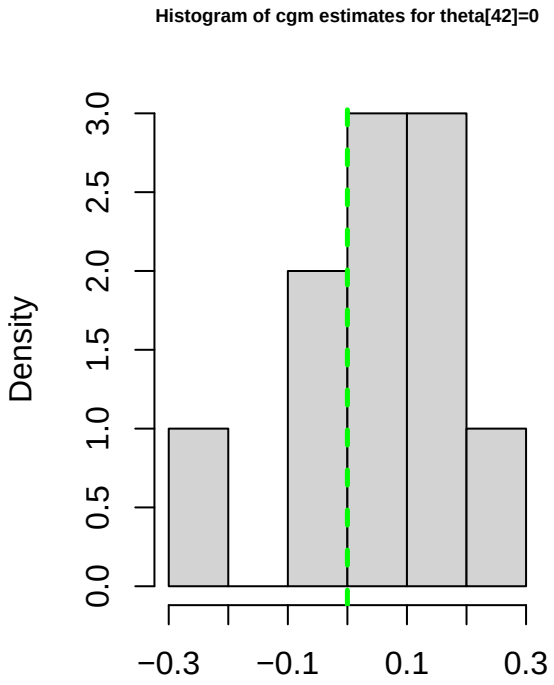

Histogram of proposed estimates for theta[15]=0.447
Density
0
1
2
3
4
5
6
0.25
0.35
0.45
0.55


Histogram of cgm estimates for theta[15]=0.447
Density
0.0
0.5
1.0
1.5
2.0
−0.2
0.2
0.4
0.6
0.8

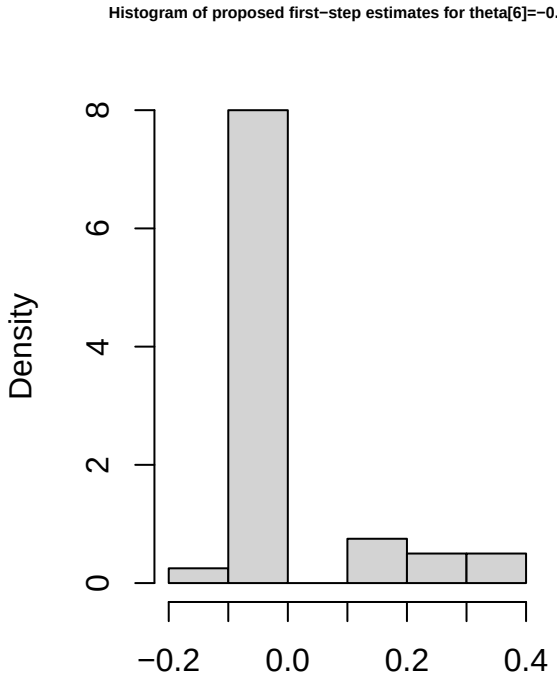

Histogram of proposed estimates for theta[72]=0
Density
0
2
4
6
8
10
−0.2
0.0
0.1

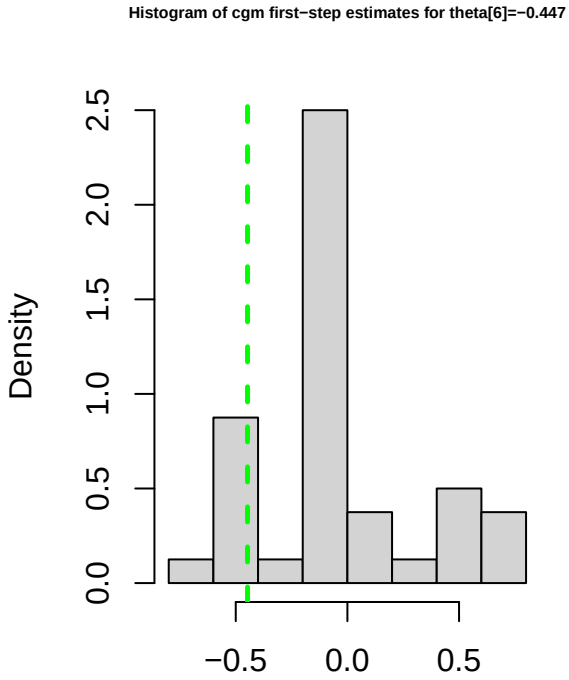

Histogram of cgm estimates for theta[72]=0
Density
0
1
2
3
4
5
6
−0.3
−0.1
0.0
0.1

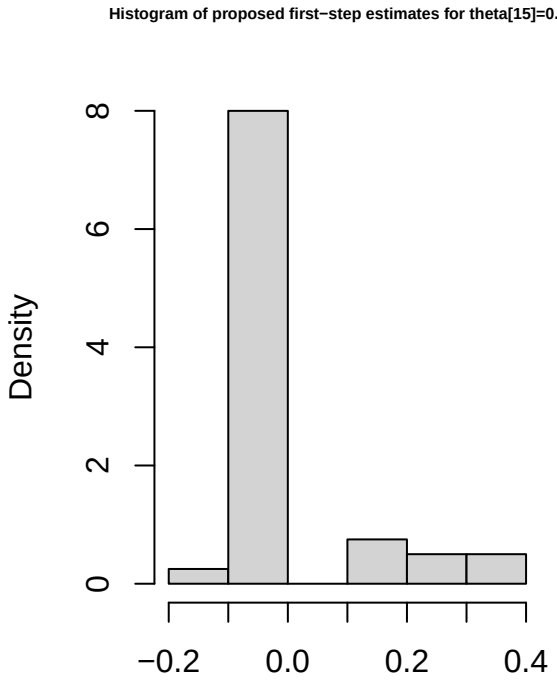

Histogram of proposed estimates for theta[42]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.2
0.0
0.2
0.4

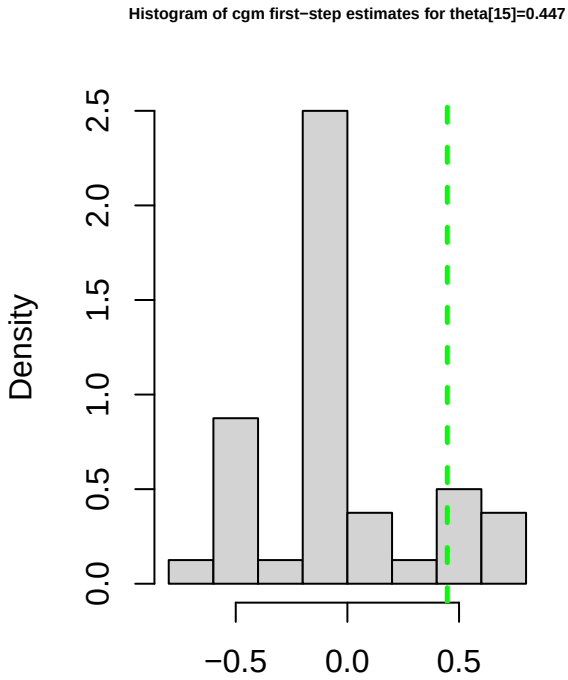

Histogram of cgm estimates for theta[42]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.3
−0.1
0.1
0.3

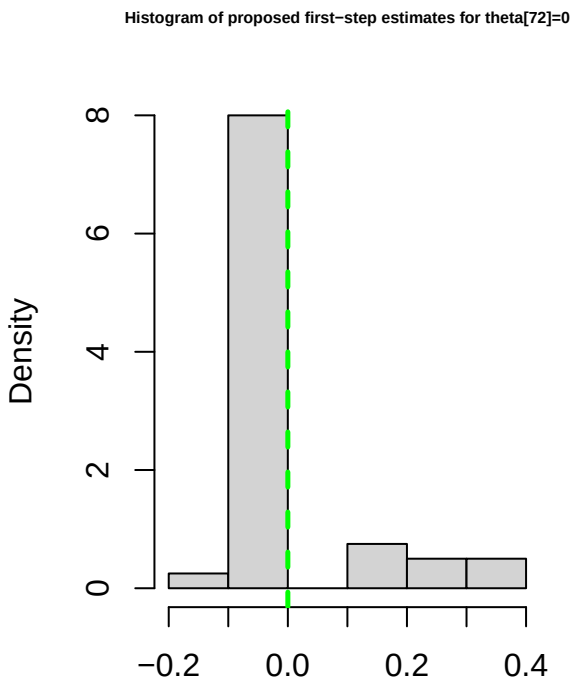
First Step Histograms

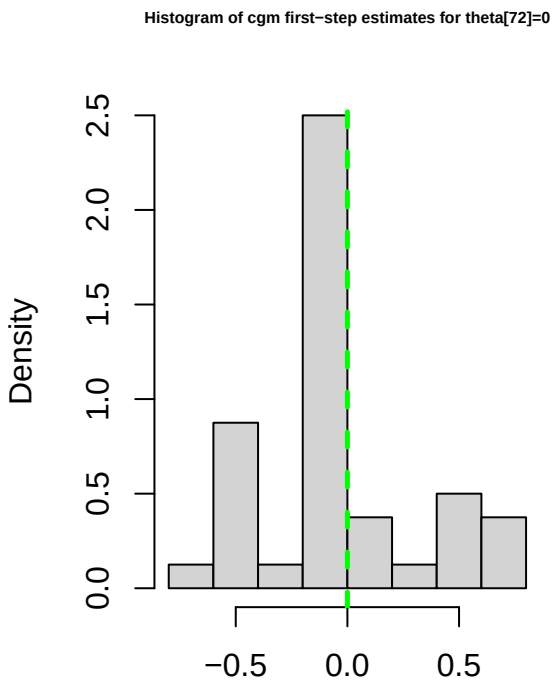


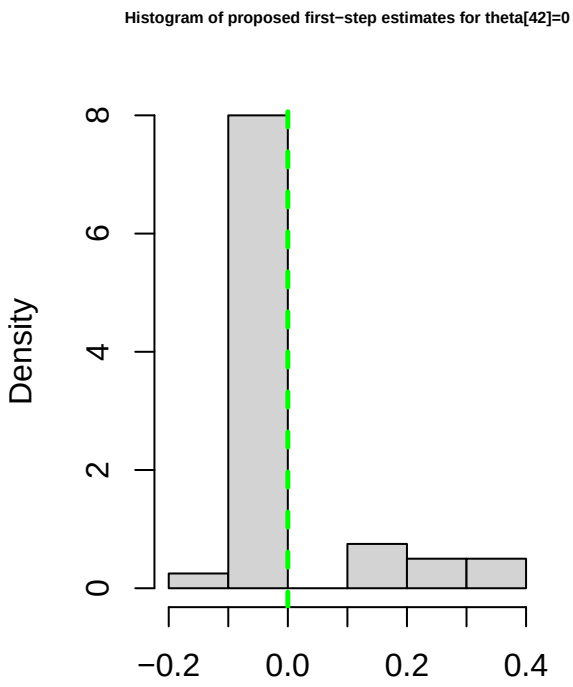


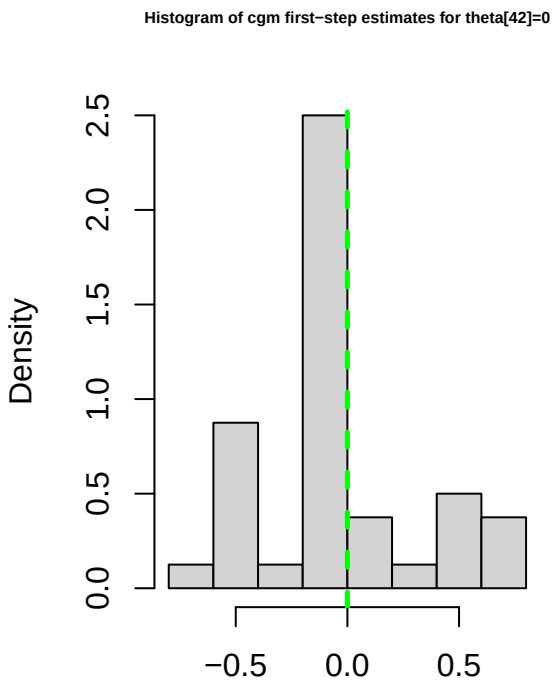





Histogram of proposed first−step estimates for theta[72]=0
Density
0
2
4
6
8
−0.2
0.0
0.2
0.4


Histogram of cgm first−step estimates for theta[72]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
−0.5
0.0
0.5


Histogram of proposed first−step estimates for theta[42]=0
Density
0
2
4
6
8
−0.2
0.0
0.2
0.4


Histogram of cgm first−step estimates for theta[42]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
−0.5
0.0
0.5

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 5: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[6]	-0.401	-0.277	0.060	-0.393	0.021
theta[15]	0.285	0.334	0.554	0.285	0.518
theta[72]	-0.216	-0.031	0.111	-0.177	0.091
theta[42]	-0.146	0.106	0.310	-0.140	0.304

Table 6: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[6]	-0.530	-0.382	-0.221	-0.529	-0.226
theta[15]	-0.176	0.537	0.718	-0.072	0.695
theta[72]	-0.266	-0.070	0.052	-0.249	0.050
theta[42]	-0.224	0.048	0.278	-0.194	0.255

Statistics for Theoretical 95% Confidence Intervals

Table 7: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[6]	-0.247	0.128	-0.497	0.004	0.8
theta[15]	0.351	0.134	0.089	0.613	1.0
theta[72]	-0.024	0.134	-0.287	0.239	0.9
theta[42]	0.076	0.136	-0.190	0.343	1.0

Table 8: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[6]	-0.391	0.125	-0.636	-0.145	1.0
theta[15]	0.429	0.142	0.151	0.708	0.8
theta[72]	-0.082	0.125	-0.328	0.164	1.0
theta[42]	0.046	0.125	-0.199	0.291	1.0