Simulation Results

2026-01-12

Simulation Setup

This simulation is performed with $n = 100$ and $d = 40$, using the 2-d lattice as the underlying graph. $s = 5$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.4$. The attached results are for a 10-replication simulation. The true values of the parameter vector θ are

```
0.4472136 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.4472136 0 0 0 -0.4472136 0 -0.4472136 0 -0
```

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{ 1, 27, 5, 2\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison ($\frac{1}{n.sim}\sum_{i=1}^{n.sim}\frac{1}{|\mathcal{C}|}\|\hat{\theta}_{i,\mathcal{C}} - \theta_{\mathcal{C}}\|^2$)

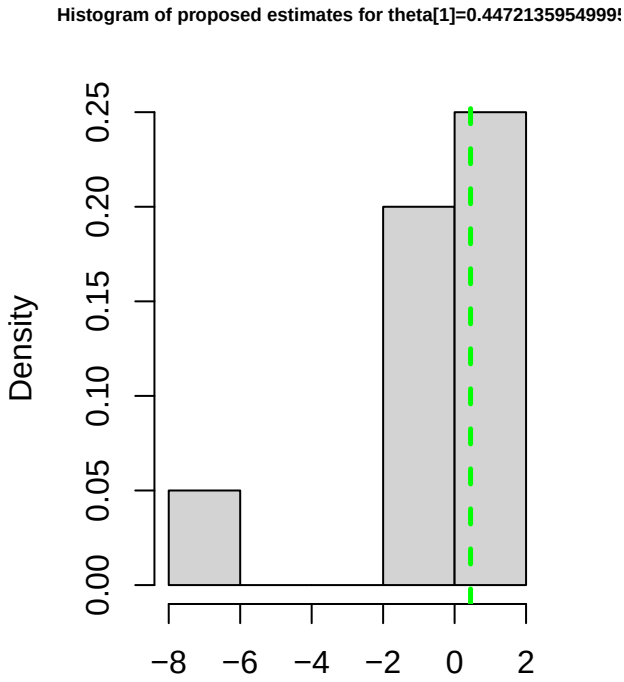
Table 1: Mean-Squared Error of Parameter Estimates

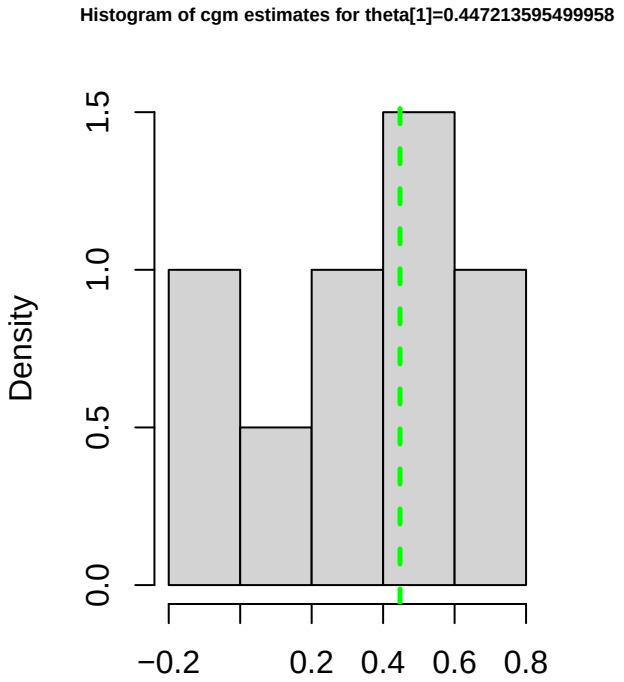
	proposed	cgm
theta[1]	5.565	0.096
theta[27]	1.494	0.132
theta[5]	0.658	0.051
theta[2]	7.230	0.051
total	3.737	0.083

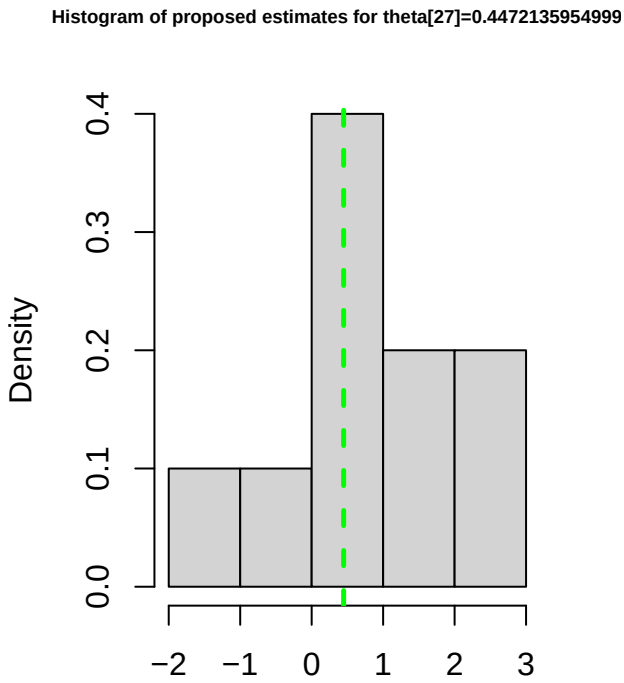
Table 2: Mean-Squared Error of First-Step Parameter Estimates

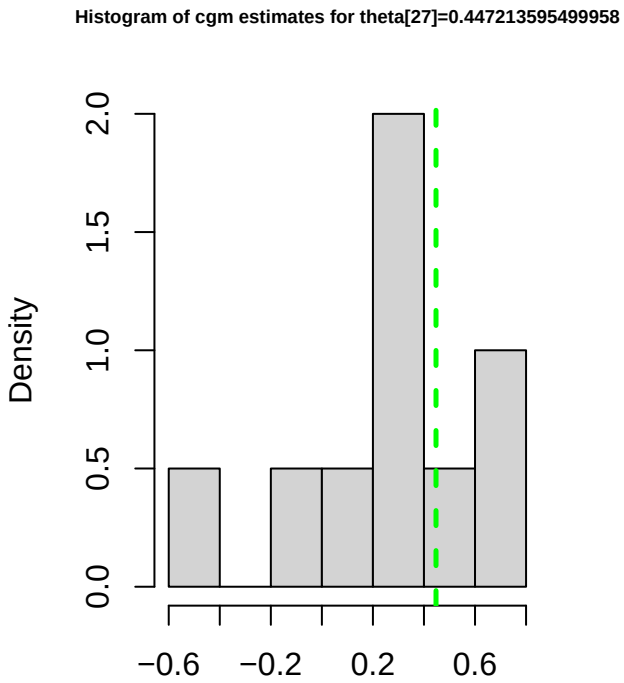
	proposed	cgm
theta[1]	0.280	0.058
theta[27]	0.128	0.113
theta[5]	0.007	0.002
theta[2]	0.000	0.002
total	0.104	0.044

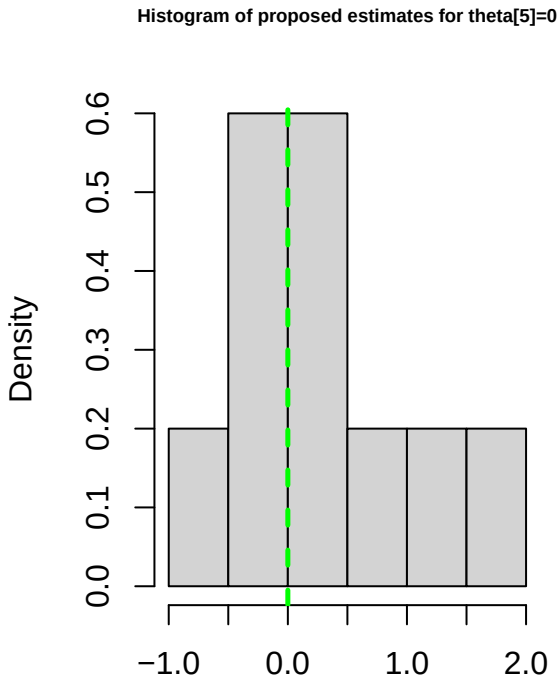
Boxplots

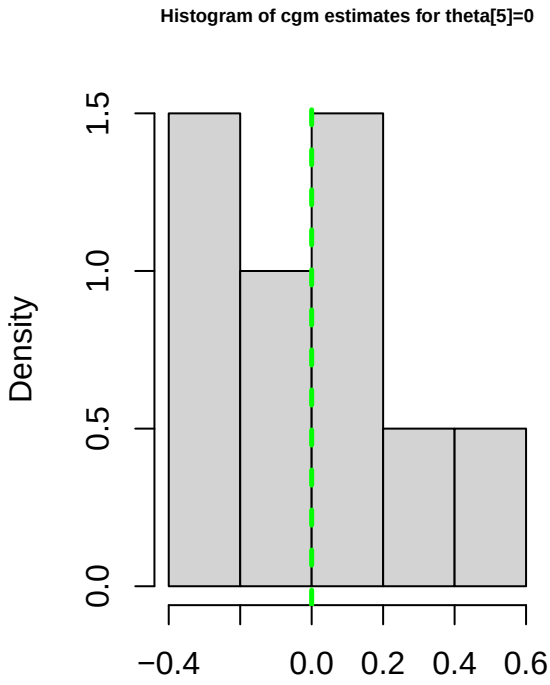

Histogram of proposed estimates for theta[1]=0.44721359549995
Density
0.00
0.05
0.10
0.15
0.20
0.25
−8
−6
−4
−2
0
2

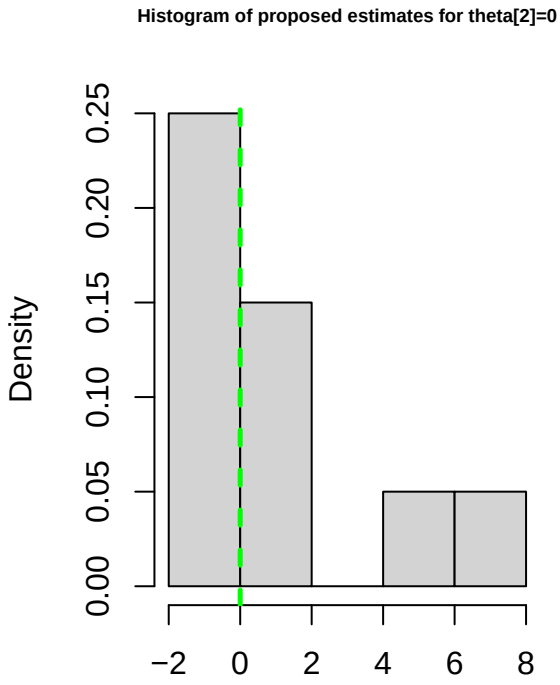

Histogram of cgm estimates for theta[1]=0.447213595499958
Density
0.0
0.5
1.0
1.5
−0.2
0.2
0.4
0.6
0.8

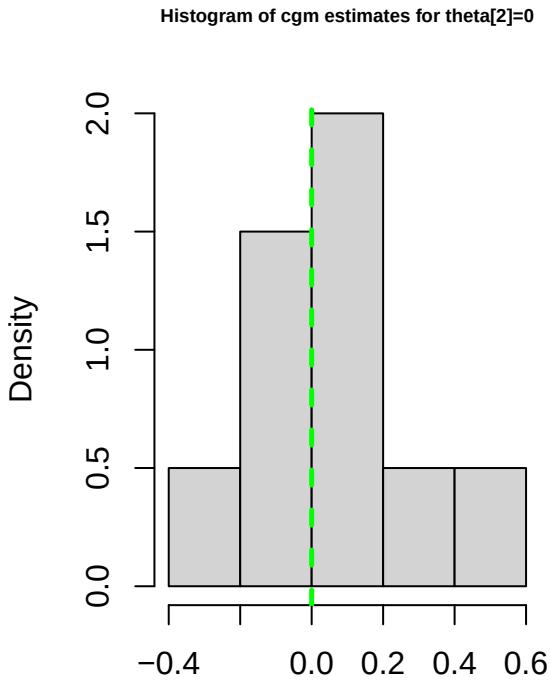

Histogram of proposed estimates for theta[27]=0.4472135954999
Density
0.0
0.1
0.2
0.3
0.4
−2
−1
0
1
2
3


Histogram of cgm estimates for theta[27]=0.447213595499958
Density
0.0
0.5
1.0
1.5
2.0
−0.6
−0.2
0.2
0.6

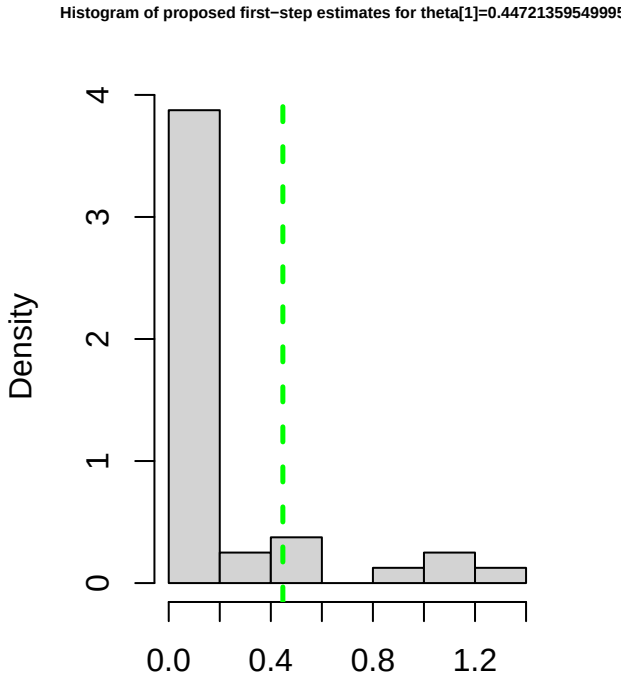

Histogram of proposed estimates for theta[5]=0
Density
0.0
0.1
0.2
0.3
0.4
0.5
0.6
−1.0
0.0
1.0
2.0

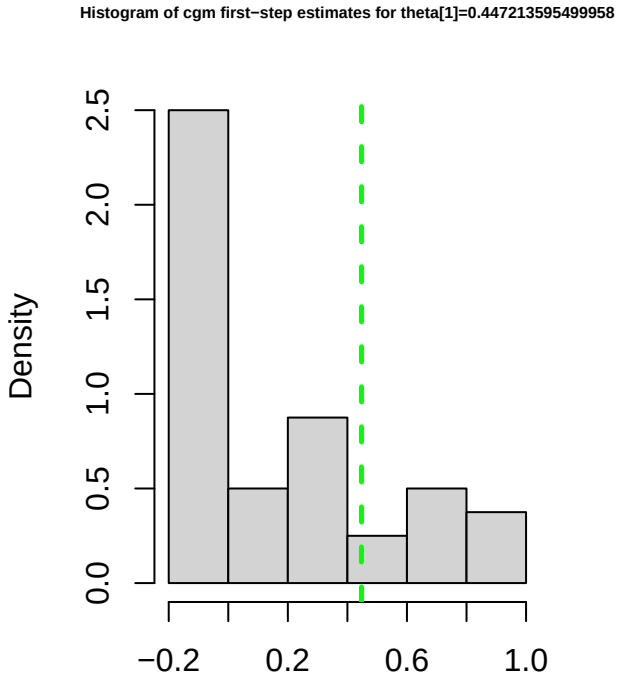

Histogram of cgm estimates for theta[5]=0
Density
0.0
0.5
1.0
1.5
−0.4
0.0
0.2
0.4
0.6

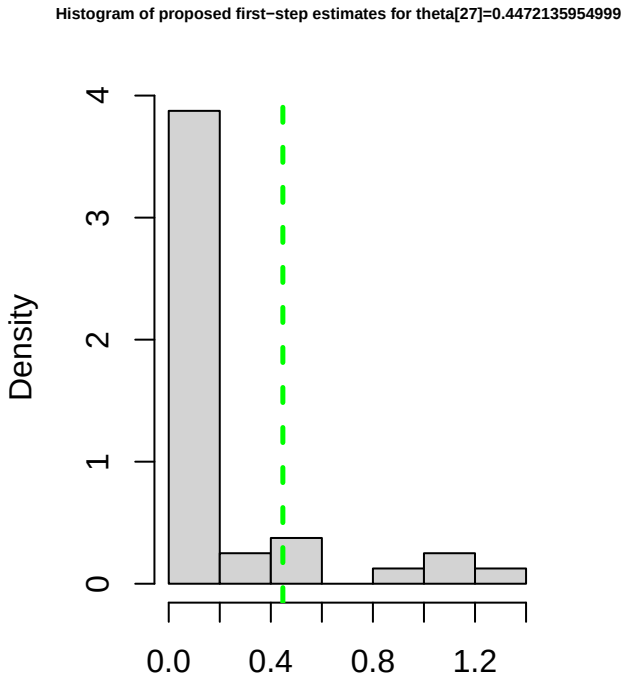

Histogram of proposed estimates for theta[2]=0
Density
0.00
0.05
0.10
0.15
0.20
0.25
−2
0
2
4
6
8

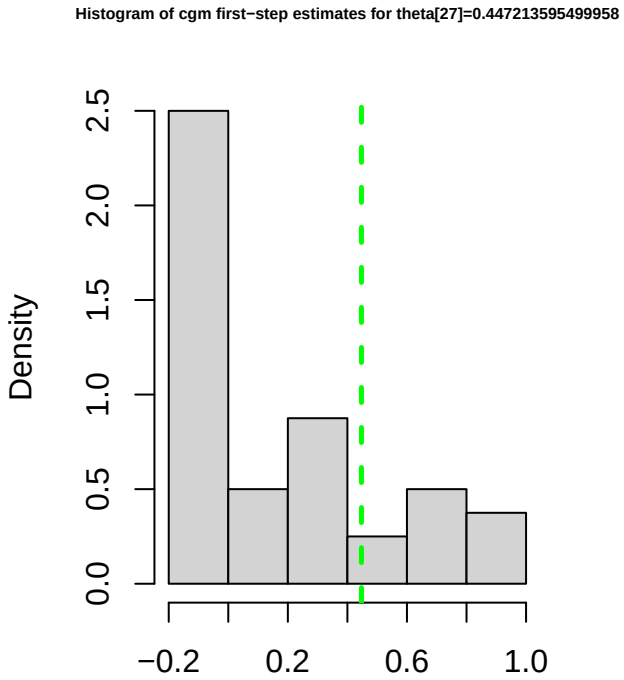

Histogram of cgm estimates for theta[2]=0
Density
0.0
0.5
1.0
1.5
2.0
−0.4
0.0
0.2
0.4
0.6

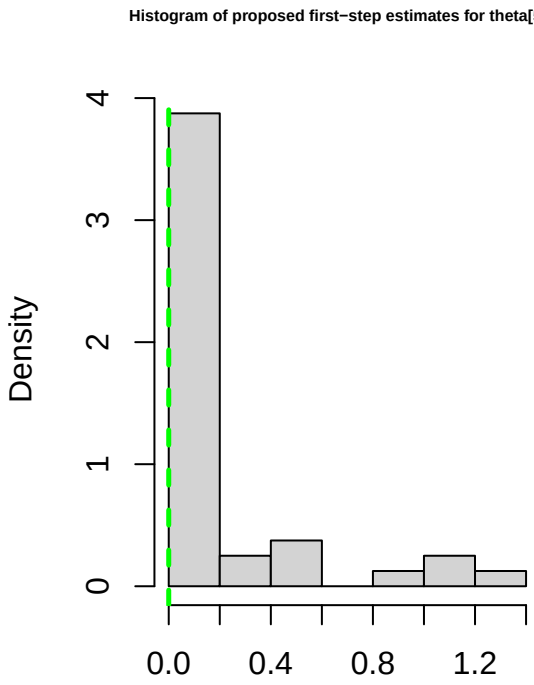
First Step Histograms

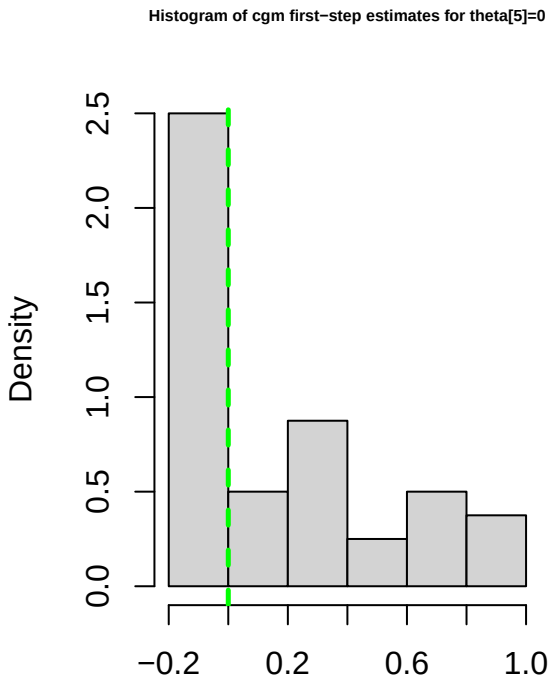


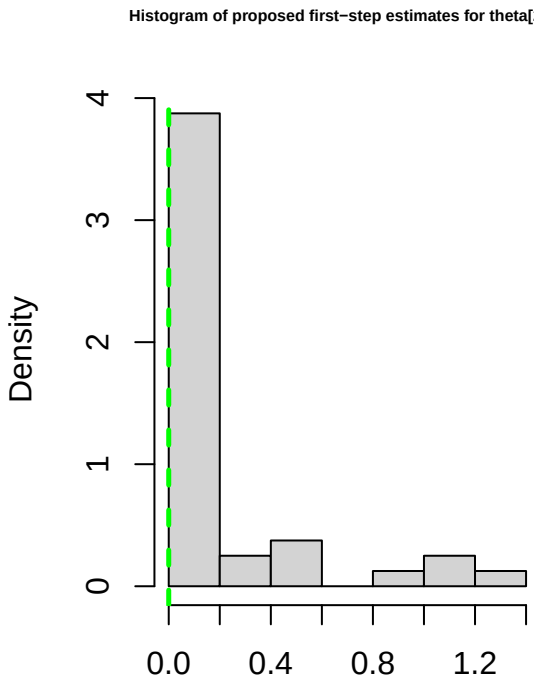


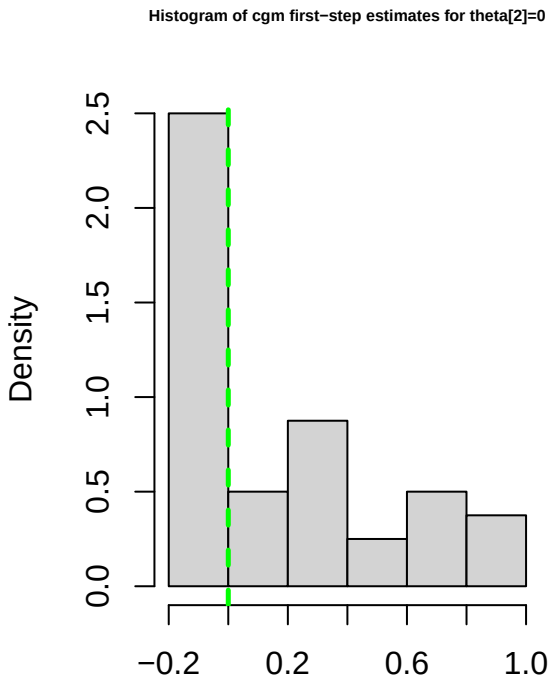





Histogram of proposed first−step estimates for theta[5]=0
Density
0
1
2
3
4
0.0
0.4
0.8
1.2


Histogram of cgm first−step estimates for theta[5]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
−0.2
0.2
0.6
1.0


Histogram of proposed first−step estimates for theta[2]=0
Density
0
1
2
3
4
0.0
0.4
0.8
1.2


Histogram of cgm first−step estimates for theta[2]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
−0.2
0.2
0.6
1.0

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 3: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[1]	-6.529	0.016	1.395	-5.440	1.270
theta[27]	-1.106	0.843	2.903	-0.921	2.747
theta[5]	-0.875	0.167	1.575	-0.764	1.535
theta[2]	-1.002	0.087	6.132	-0.934	6.020

Table 4: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[1]	-0.197	0.364	0.750	-0.164	0.742
theta[27]	-0.439	0.311	0.686	-0.353	0.683
theta[5]	-0.373	-0.008	0.403	-0.343	0.372
theta[2]	-0.253	0.027	0.536	-0.228	0.487

Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[1]	-0.555	0.319	-1.179	0.070	0.5
theta[27]	0.868	0.271	0.337	1.399	0.4
theta[5]	0.302	0.268	-0.223	0.827	0.5
theta[2]	1.153	0.290	0.585	1.721	0.5

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[1]	0.336	0.170	0.003	0.668	0.8
theta[27]	0.265	0.174	-0.076	0.606	0.8
theta[5]	-0.013	0.156	-0.319	0.294	0.7
theta[2]	0.061	0.163	-0.260	0.381	0.9