Simulation Results

2026-01-13

Simulation Setup

This simulation is performed with $n = 500$ and $d = 10$, using the 2-d lattice as the underlying graph. $s = 2$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.2$. The attached results are for a 10-replication simulation. The true values of the parameter vector θ are

```
0 0.7071068 0 -0.7071068 0 0 0 0 0 0 ,
```

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{$ 2, 4, 1, 6$\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison ($\frac{1}{n.sim}\sum_{i=1}^{n.sim}\frac{1}{|\mathcal{C}|}\|\hat{\theta}_{i,\mathcal{C}} - \theta_{\mathcal{C}}\|^2$)

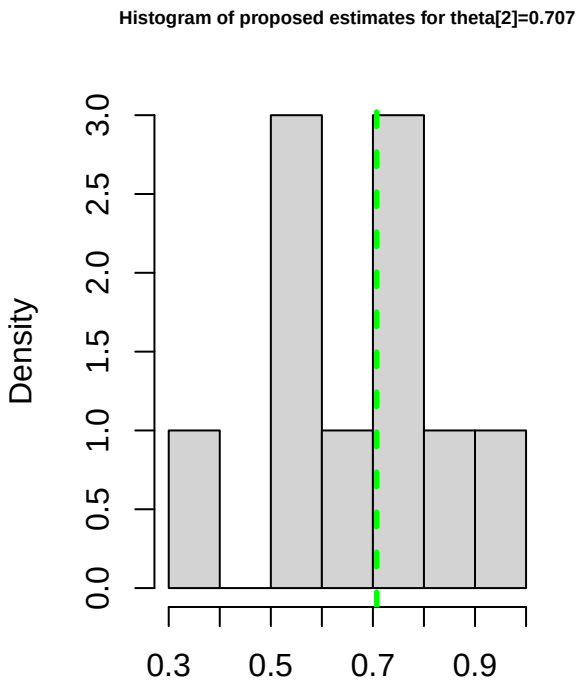
Table 1: Mean-Squared Error of Parameter Estimates

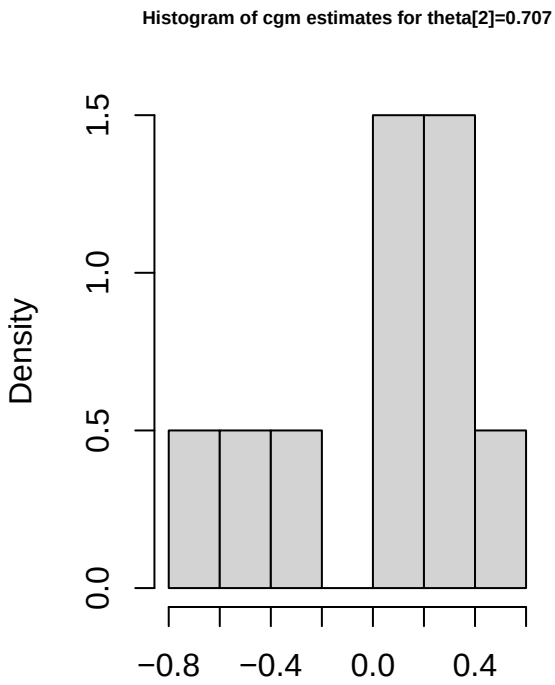
	proposed	cgm
theta[2]	0.027	0.618
theta[4]	0.012	0.573
theta[1]	0.009	0.015
theta[6]	0.016	0.014
total	0.016	0.305

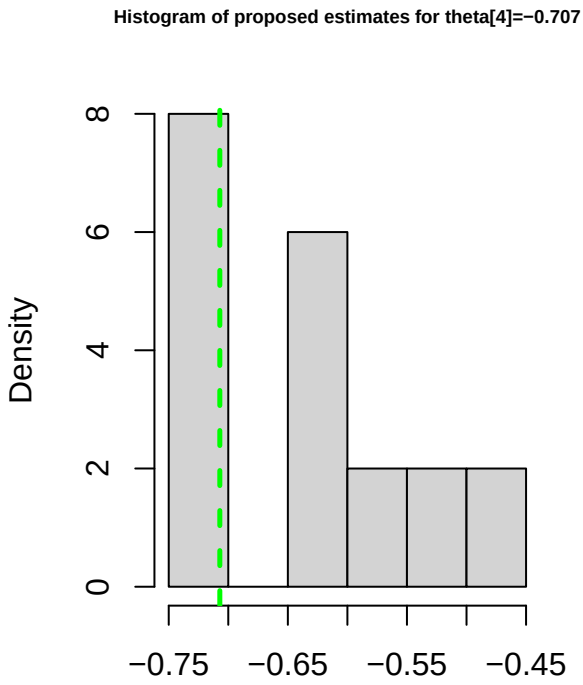
Table 2: Mean-Squared Error of First-Step Parameter Estimates

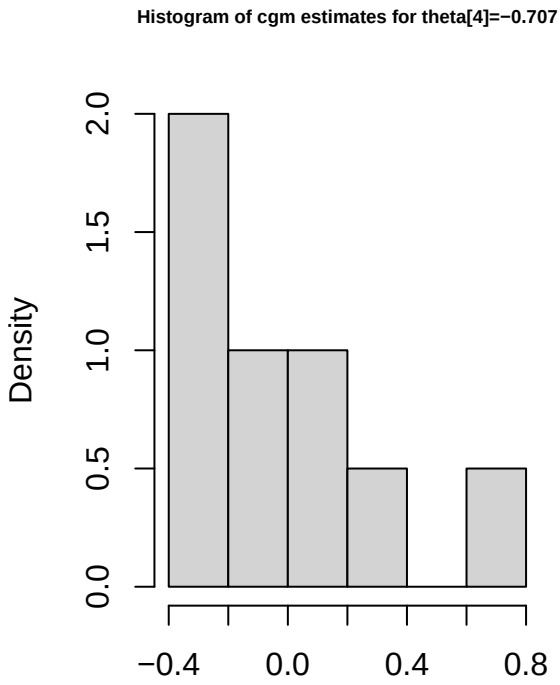
	proposed	cgm
theta[2]	0.018	0.244
theta[4]	0.020	0.253
theta[1]	0.009	0.000
theta[6]	0.002	0.004
total	0.012	0.125

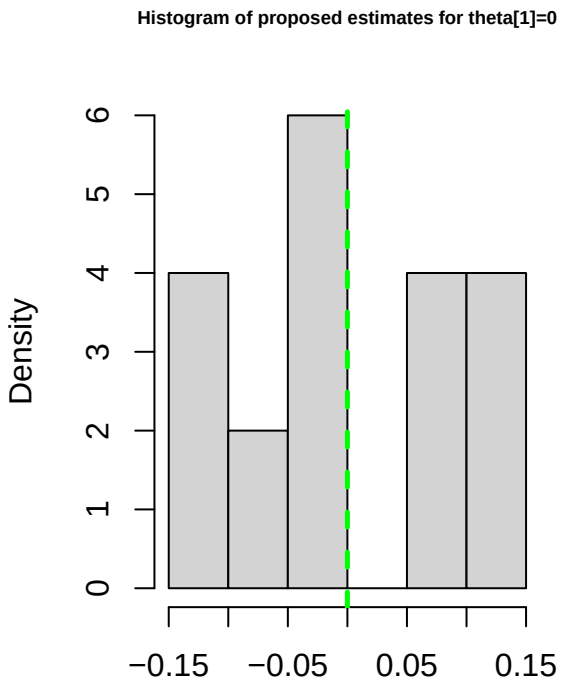
Boxplots

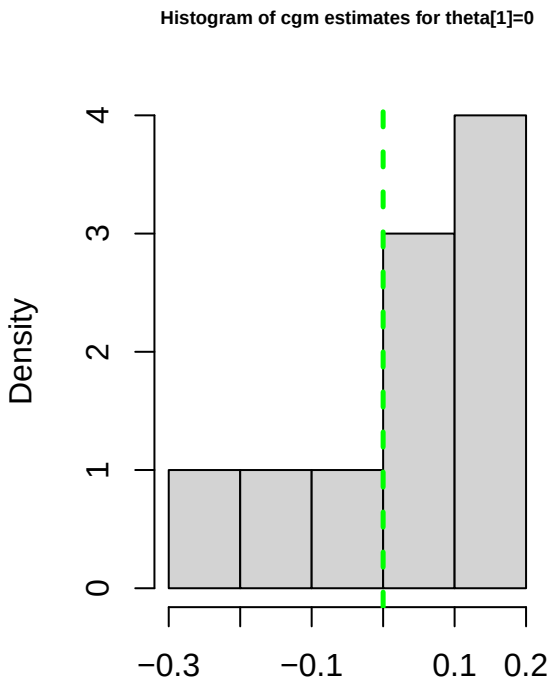

Histogram of proposed estimates for theta[2]=0.707
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0.3
0.5
0.7
0.9

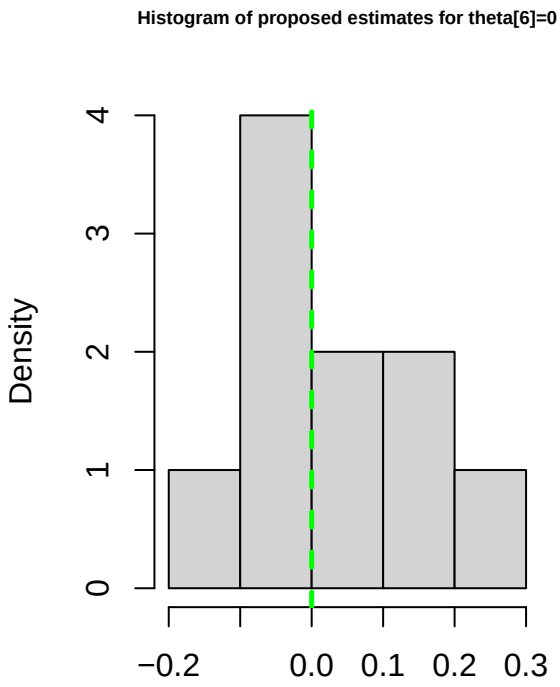

Histogram of cgm estimates for theta[2]=0.707
Density
0.0
0.5
1.0
1.5
−0.8
−0.4
0.0
0.4

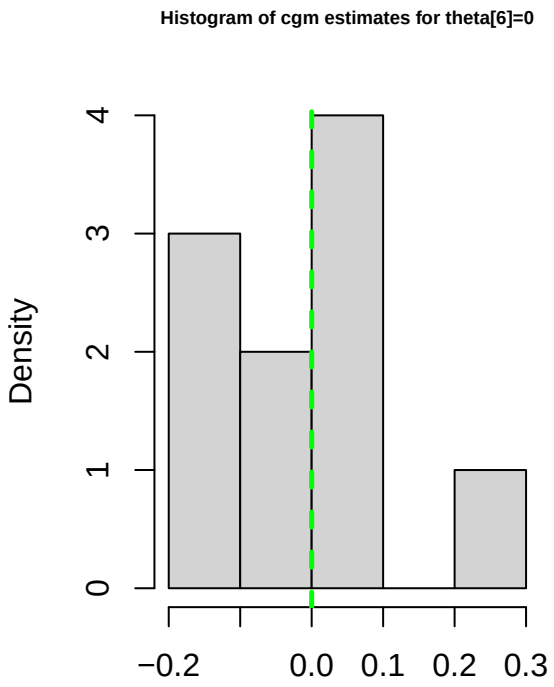

Histogram of proposed estimates for theta[4]=−0.707
Density
0
2
4
6
8
−0.75
−0.65
−0.55
−0.45


Histogram of cgm estimates for theta[4]=−0.707
Density
0.0
0.5
1.0
1.5
2.0
−0.4
0.0
0.4
0.8

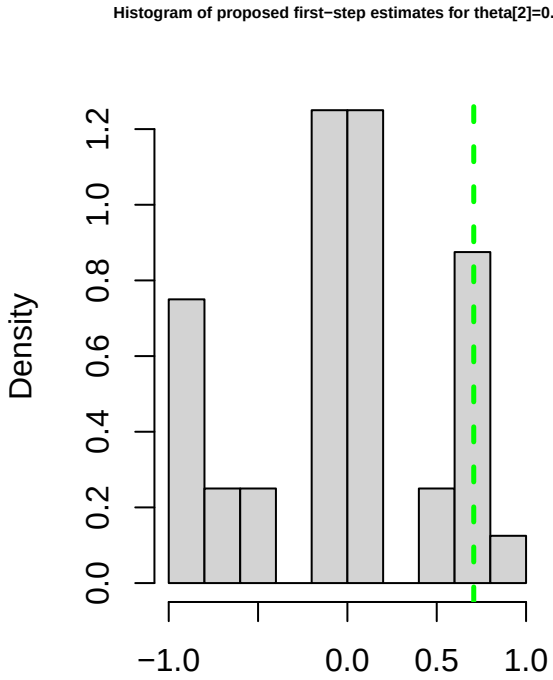

Histogram of proposed estimates for theta[1]=0
Density
0
1
2
3
4
5
6
−0.15
−0.05
0.05
0.15

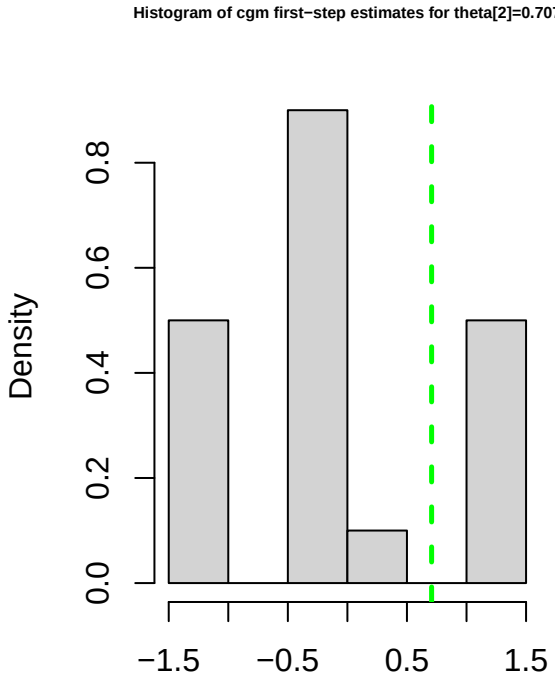

Histogram of cgm estimates for theta[1]=0
Density
0
1
2
3
4
−0.3
−0.1
0.1
0.2

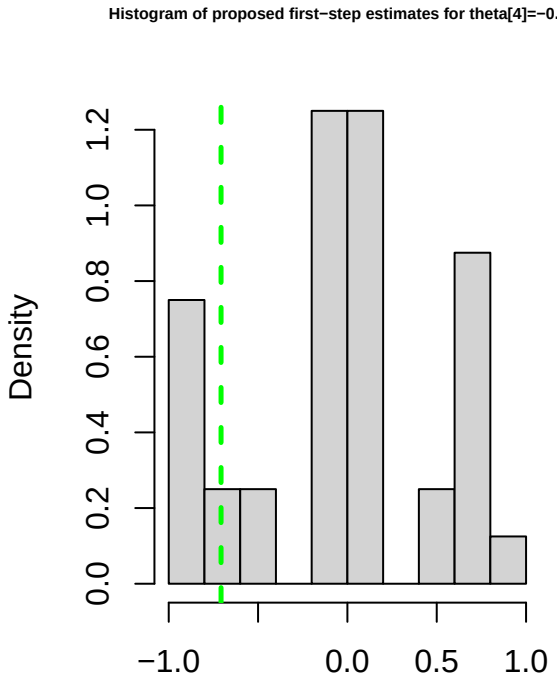

Histogram of proposed estimates for theta[6]=0
Density
0
1
2
3
4
−0.2
0.0
0.1
0.2
0.3

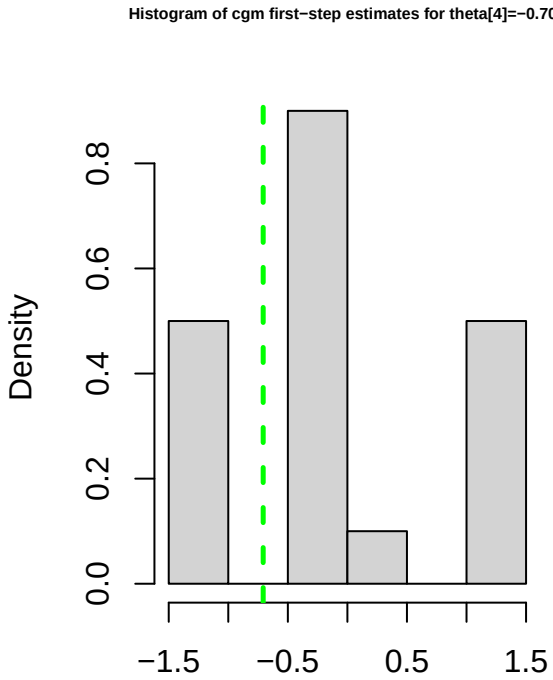

Histogram of cgm estimates for theta[6]=0
Density
0
1
2
3
4
−0.2
0.0
0.1
0.2
0.3

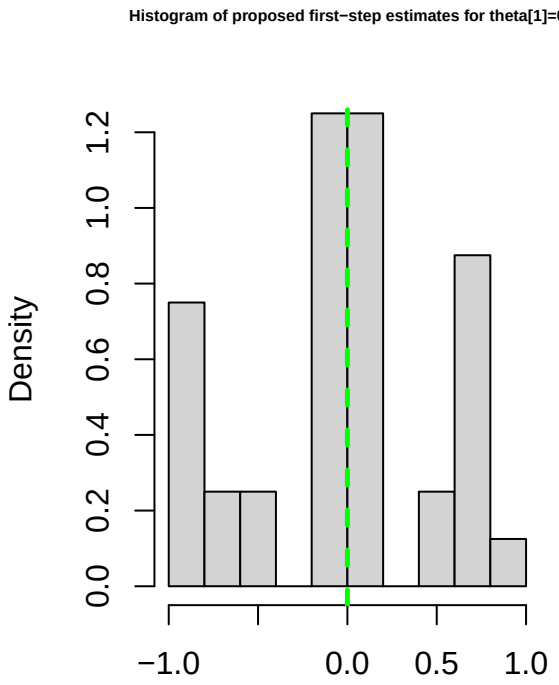
First Step Histograms

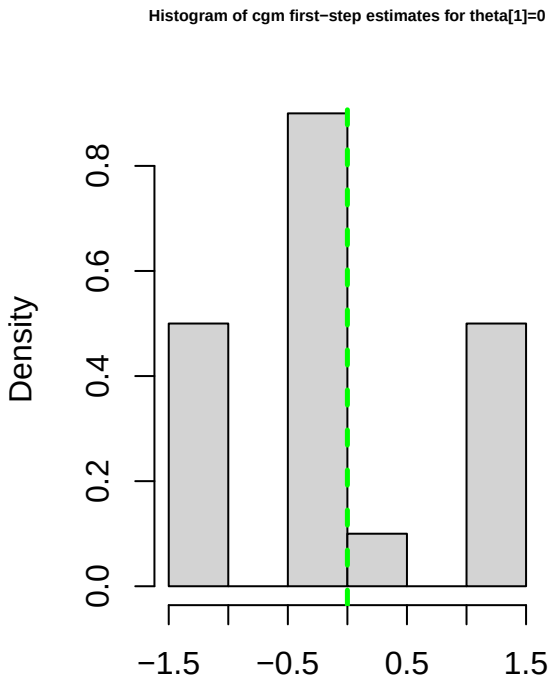


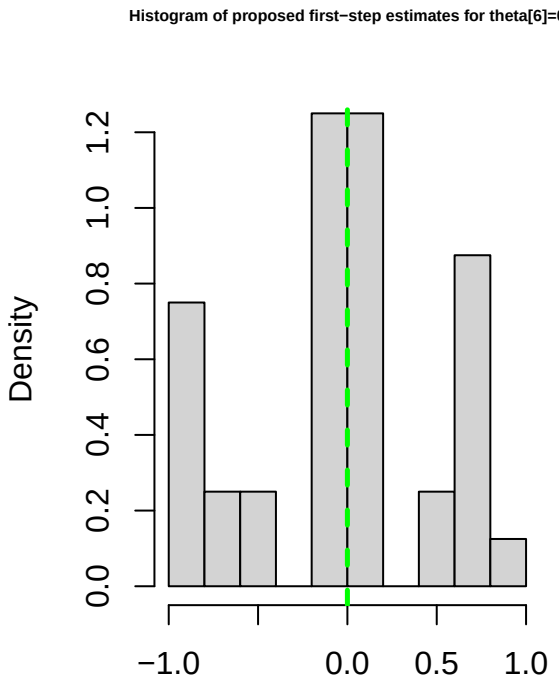


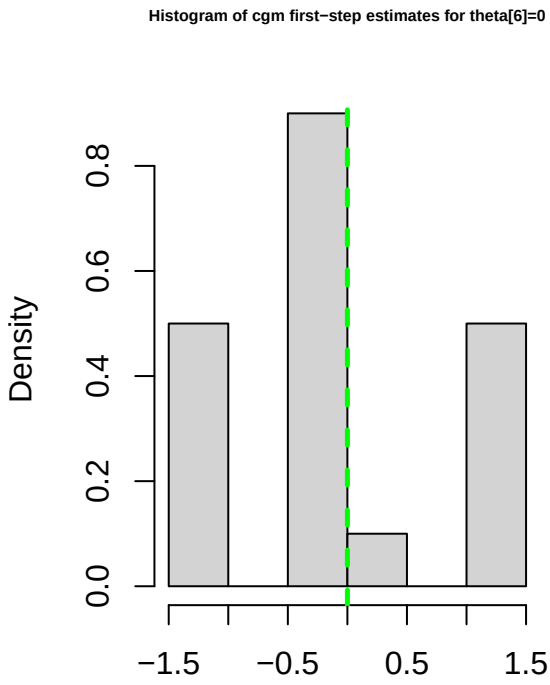





Histogram of proposed first−step estimates for theta[1]=0
Density
0.0
0.2
0.4
0.6
0.8
1.0
1.2
−1.0
0.0
0.5
1.0


Histogram of cgm first−step estimates for theta[1]=0
Density
0.0
0.2
0.4
0.6
0.8
−1.5
−0.5
0.5
1.5


Histogram of proposed first−step estimates for theta[6]=0
Density
0.0
0.2
0.4
0.6
0.8
1.0
1.2
−1.0
0.0
0.5
1.0


Histogram of cgm first−step estimates for theta[6]=0
Density
0.0
0.2
0.4
0.6
0.8
−1.5
−0.5
0.5
1.5

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 3: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[2]	0.351	0.678	0.908	0.386	0.885
theta[4]	-0.728	-0.627	-0.499	-0.728	-0.505
theta[1]	-0.134	-0.019	0.143	-0.130	0.135
theta[6]	-0.172	0.001	0.286	-0.148	0.254

Table 4: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[2]	-0.640	0.124	0.416	-0.599	0.390
theta[4]	-0.394	-0.082	0.695	-0.381	0.628
theta[1]	-0.206	0.084	0.178	-0.183	0.168
theta[6]	-0.183	-0.001	0.232	-0.179	0.196

Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[2]	0.651	0.147	0.363	0.939	0.9
theta[4]	-0.635	0.154	-0.937	-0.333	1.0
theta[1]	-0.001	0.115	-0.227	0.225	1.0
theta[6]	0.028	0.109	-0.187	0.242	0.9

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[2]	0.002	0.173	-0.337	0.341	0.1
theta[4]	-0.028	0.170	-0.361	0.305	0.0
theta[1]	0.036	0.097	-0.153	0.226	0.8
theta[6]	-0.008	0.097	-0.199	0.182	0.8