Simulation Results

2026-01-09

Simulation Setup

This simulation is performed with $n = 200$ and $d = 10$, using the 2-d lattice as the underlying graph. $s = 2$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.2$. The attached results are for a 10-replication simulation. The true values of the parameter vector θ are

```
0 0 0 0 0.7071068 0 0 0 0 -0.7071068 ,
```

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{ 5, 10, 1, 4\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison ($\frac{1}{n.sim}\sum_{i=1}^{n.sim}\frac{1}{|\mathcal{C}|}\|\hat{\theta}_{i,\mathcal{C}} - \theta_{\mathcal{C}}\|^2$)

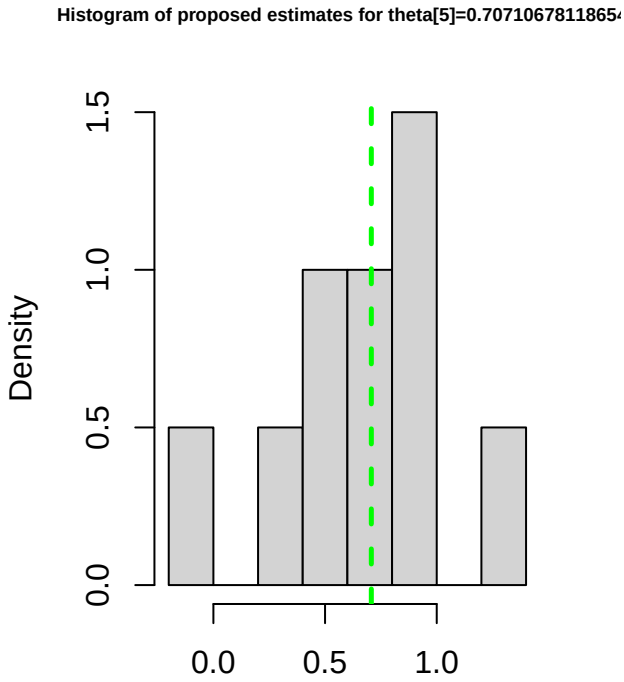
Table 1: Mean-Squared Error of Parameter Estimates

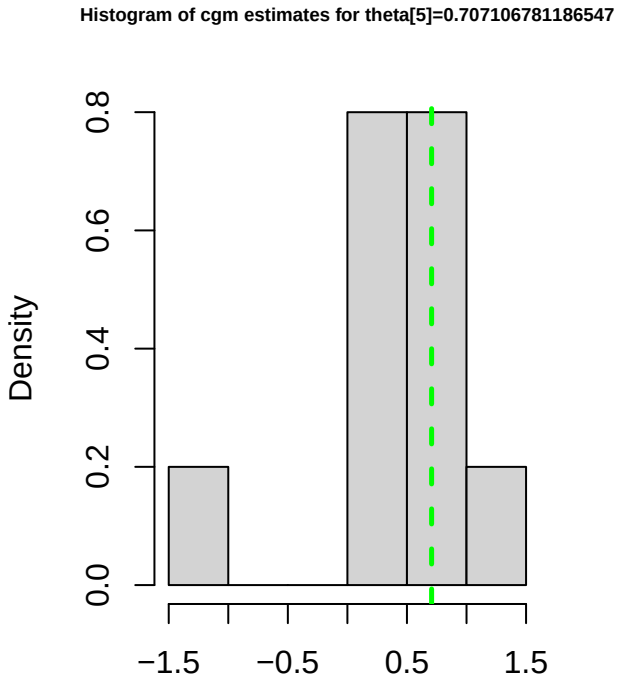
	proposed	cgm
theta[5]	0.136	0.473
theta[10]	0.073	0.320
theta[1]	0.030	0.027
theta[4]	0.058	0.054
total	0.074	0.219

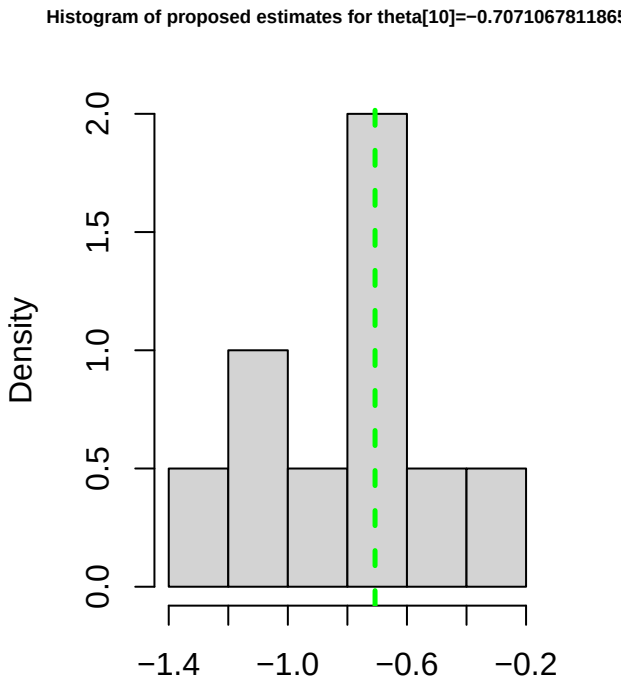
Table 2: Mean-Squared Error of First-Step Parameter Estimates

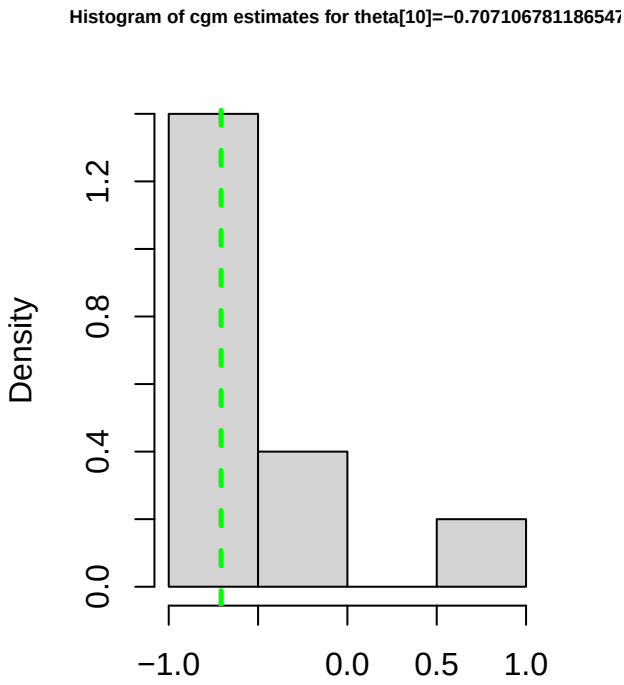
	proposed	cgm
theta[5]	0.203	0.128
theta[10]	0.131	0.130
theta[1]	0.026	0.010
theta[4]	0.023	0.005
total	0.096	0.068

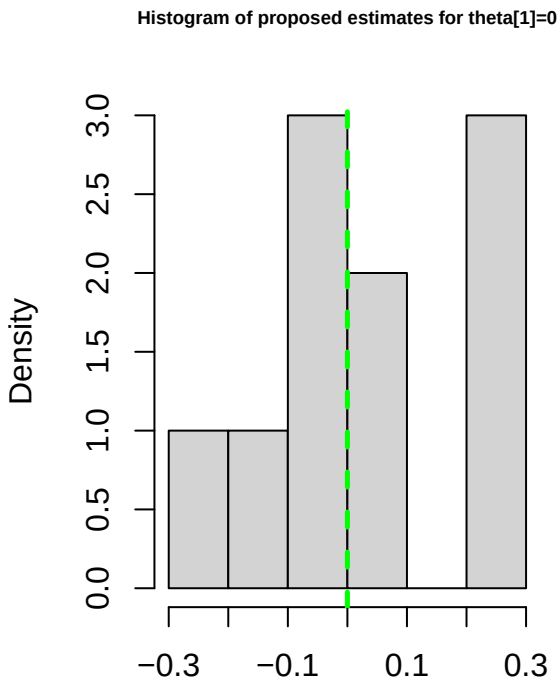
Boxplots

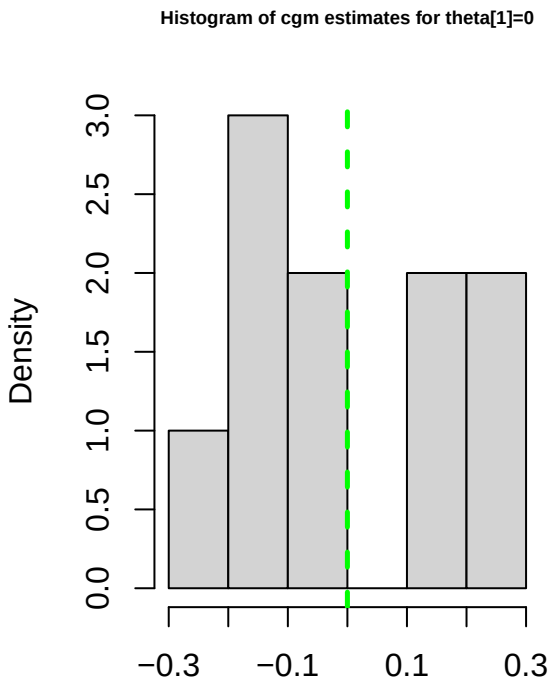

Histogram of proposed estimates for theta[5]=0.70710678118654
Density
0.0
0.5
1.0
1.5
0.0
0.5
1.0

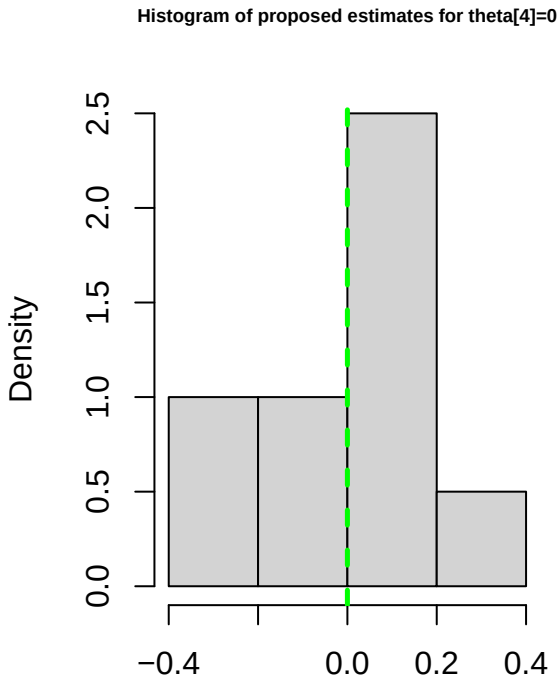

Histogram of cgm estimates for theta[5]=0.707106781186547
Density
0.0
0.2
0.4
0.6
0.8
−1.5
−0.5
0.5
1.5

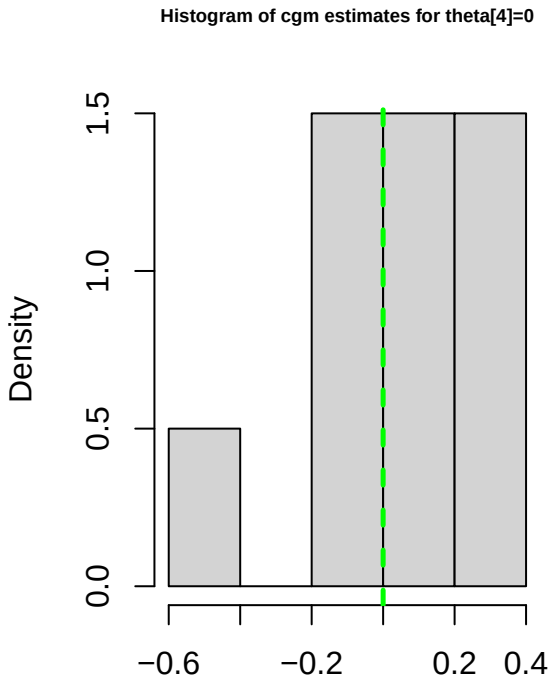

Histogram of proposed estimates for theta[10]=−0.707106781186
Density
0.0
0.5
1.0
1.5
2.0
−1.4
−1.0
−0.6
−0.2


Histogram of cgm estimates for theta[10]=−0.707106781186547
Density
0.0
0.4
0.8
1.2
−1.0
0.0
0.5
1.0

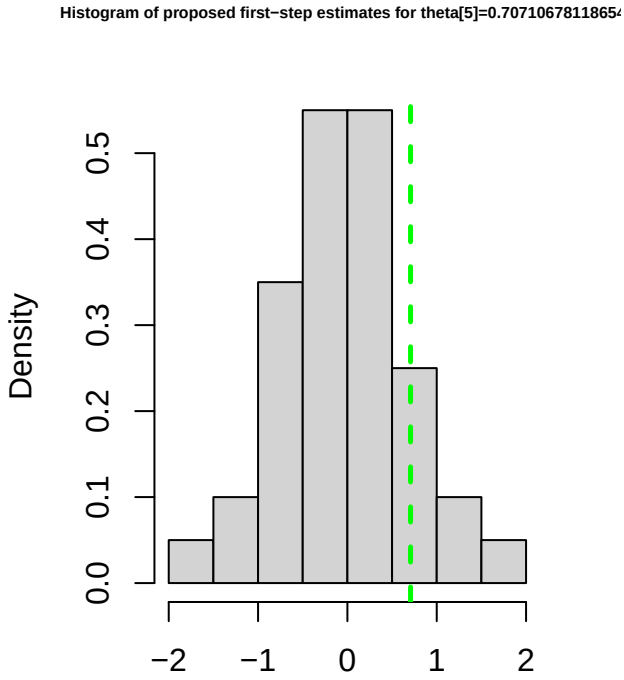

Histogram of proposed estimates for theta[1]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.3
−0.1
0.1
0.3

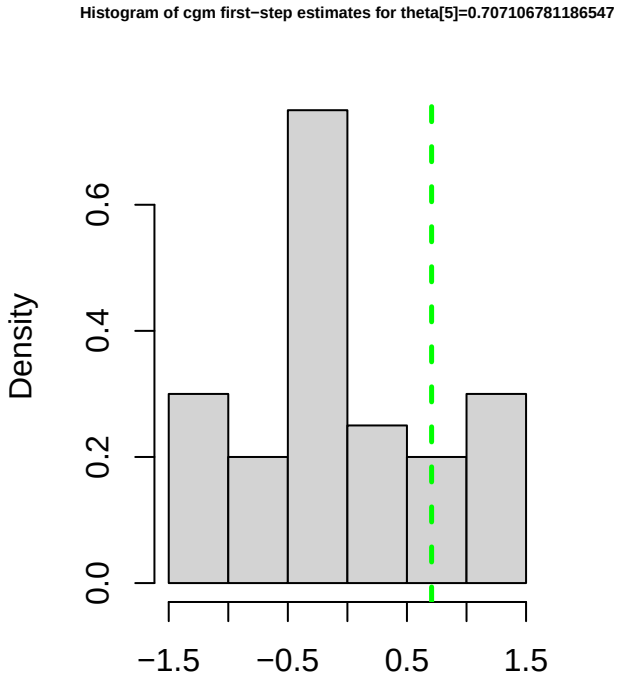

Histogram of cgm estimates for theta[1]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.3
−0.1
0.1
0.3

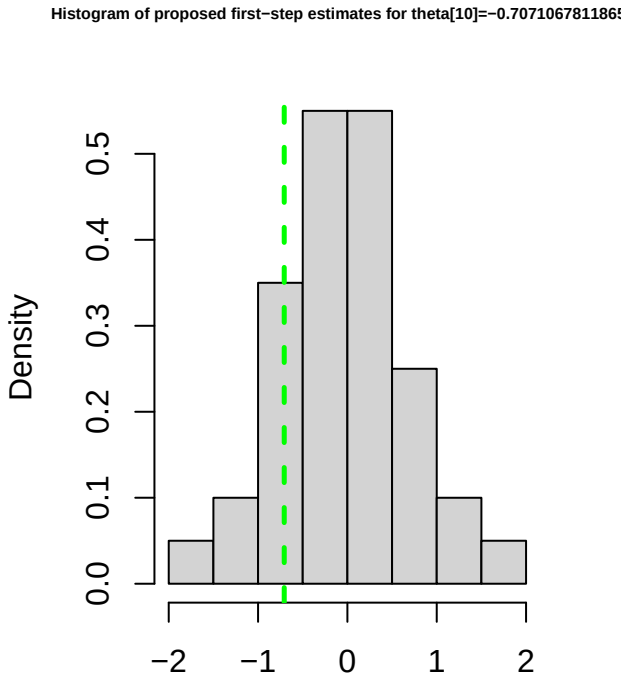

Histogram of proposed estimates for theta[4]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
−0.4
0.0
0.2
0.4

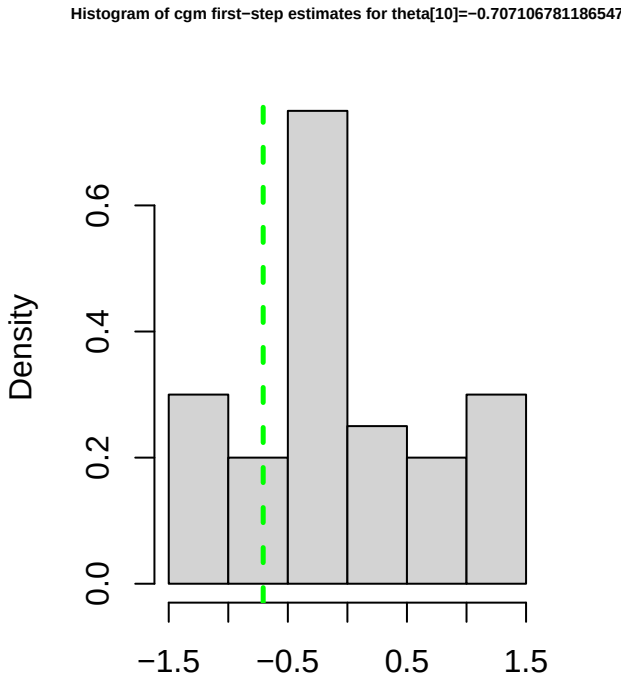

Histogram of cgm estimates for theta[4]=0
Density
0.0
0.5
1.0
1.5
−0.6
−0.2
0.2
0.4

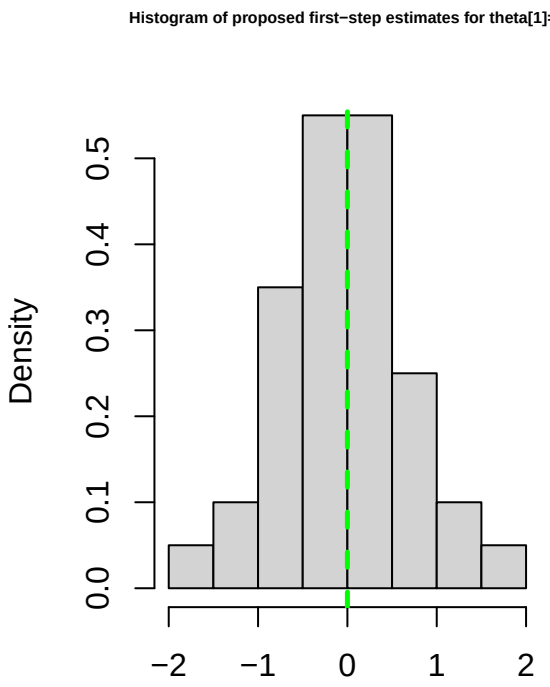
First Step Histograms

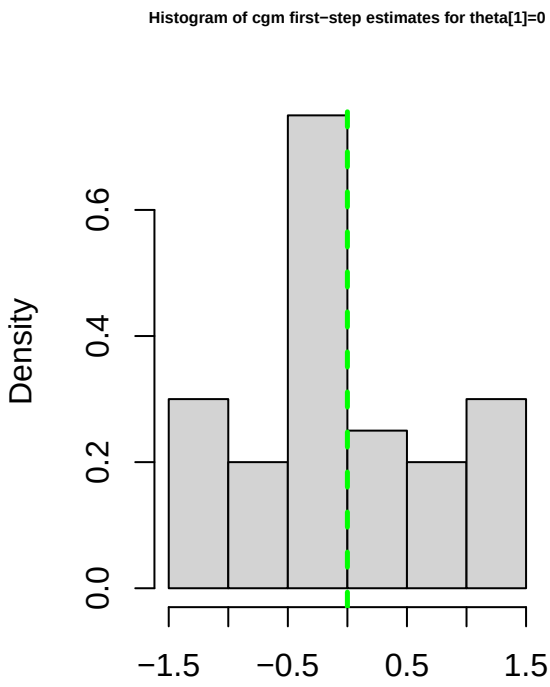


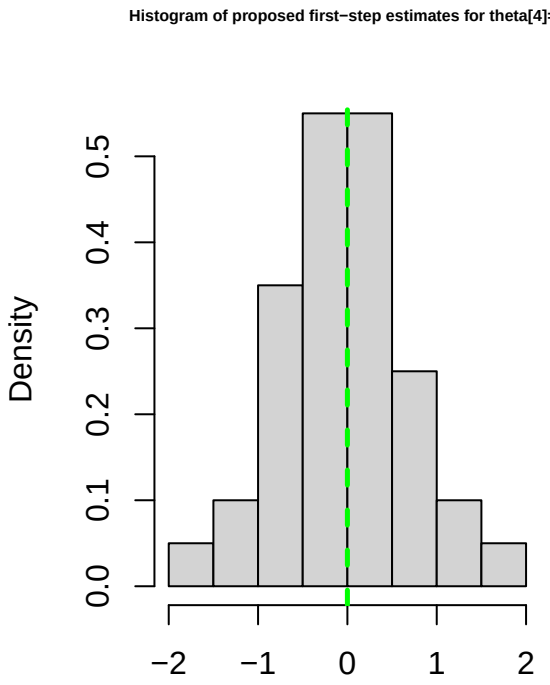


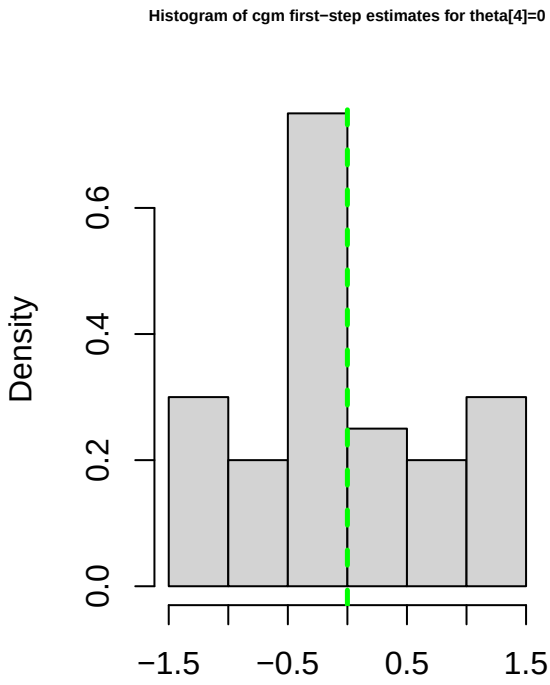





Histogram of proposed first−step estimates for theta[1]=0
Density
0.0
0.1
0.2
0.3
0.4
0.5
−2
−1
0
1
2


Histogram of cgm first−step estimates for theta[1]=0
Density
0.0
0.2
0.4
0.6
−1.5
−0.5
0.5
1.5


Histogram of proposed first−step estimates for theta[4]=0
Density
0.0
0.1
0.2
0.3
0.4
0.5
−2
−1
0
1
2


Histogram of cgm first−step estimates for theta[4]=0
Density
0.0
0.2
0.4
0.6
−1.5
−0.5
0.5
1.5

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 3: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[5]	-0.135	0.668	1.209	-0.035	1.162
theta[10]	-1.213	-0.761	-0.318	-1.191	-0.375
theta[1]	-0.280	0.003	0.275	-0.241	0.270
theta[4]	-0.366	0.153	0.356	-0.364	0.321

Table 4: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[5]	-1.339	0.507	1.035	-0.966	0.953
theta[10]	-0.897	-0.583	0.990	-0.863	0.700
theta[1]	-0.245	-0.035	0.236	-0.227	0.229
theta[4]	-0.446	0.079	0.347	-0.386	0.334

Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[5]	0.667	0.179	0.317	1.018	0.8
theta[10]	-0.798	0.177	-1.144	-0.451	0.7
theta[1]	0.027	0.149	-0.265	0.320	0.9
theta[4]	0.029	0.152	-0.269	0.327	0.7

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[5]	0.374	0.238	-0.092	0.840	0.9
theta[10]	-0.450	0.234	-0.908	0.009	0.9
theta[1]	-0.001	0.149	-0.292	0.290	1.0
theta[4]	0.028	0.151	-0.269	0.325	0.8