Simulation Results

2026-01-12

Simulation Setup

This simulation is performed with $n = 400$ and $d = 40$, using the 2-d lattice as the underlying graph. $s = 10$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.4$. The attached results are for a 10-replication simulation. The true values of the parameter vector θ are

```
0 0 0 0 0 0 0 0 0 0 0.3162278 0 -0.3162278 0 0 0 -0.3162278 0 -0.3162278 0 0 0 0 0 0 0.3162278 0 0 0 0
```

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{\ 11,\ 13,\ 30,\ 4\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison ($\frac{1}{n.sim}\sum_{i=1}^{n.sim}\frac{1}{|\mathcal{C}|}\|\hat{\theta}_{i,\mathcal{C}} - \theta_{\mathcal{C}}\|^2$)

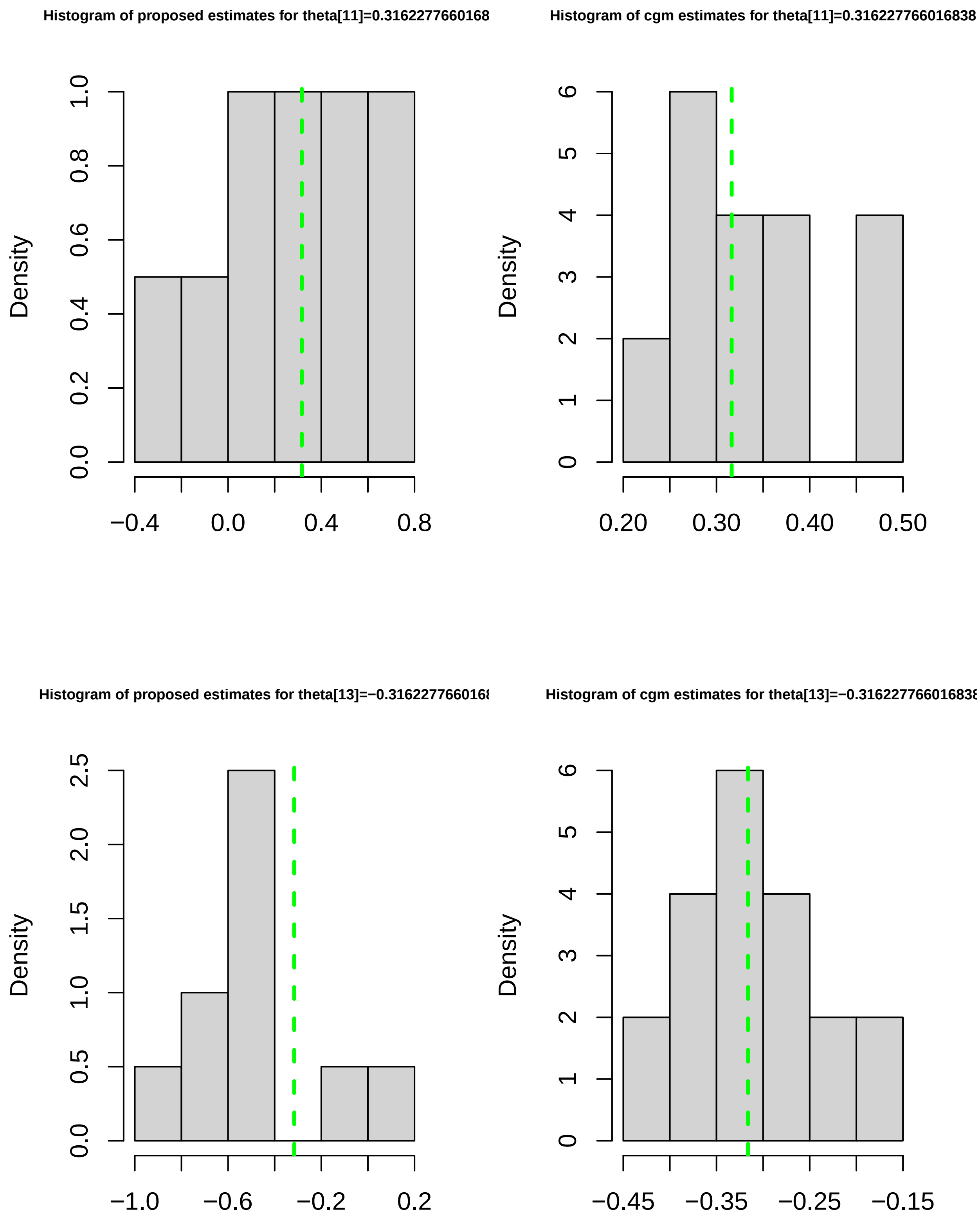
Table 1: Mean-Squared Error of Parameter Estimates

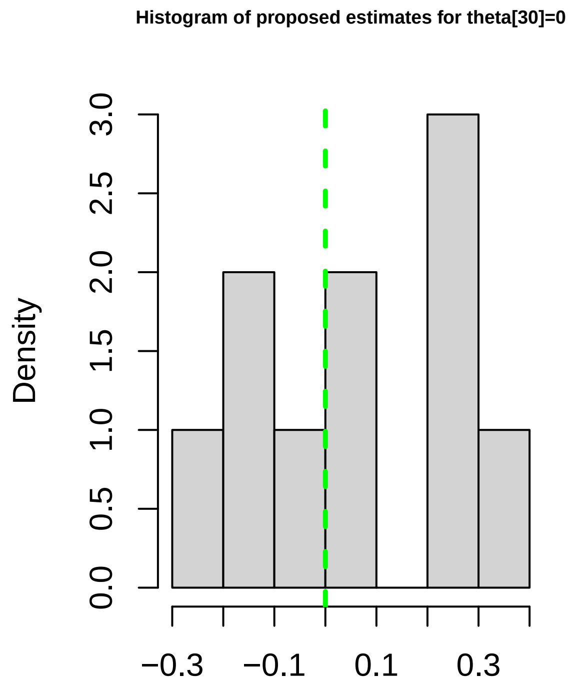
	proposed	cgm
theta[11]	0.089	0.007
theta[13]	0.099	0.006
theta[30]	0.045	0.008
theta[4]	0.045	0.002
total	0.069	0.006

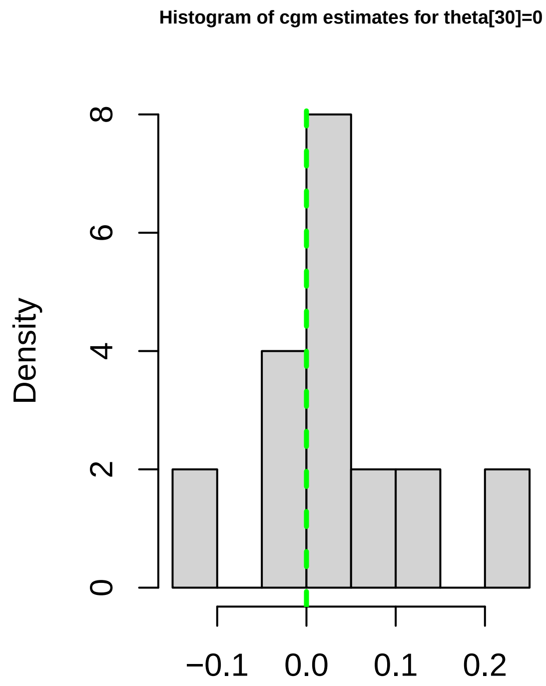
Table 2: Mean-Squared Error of First-Step Parameter Estimates

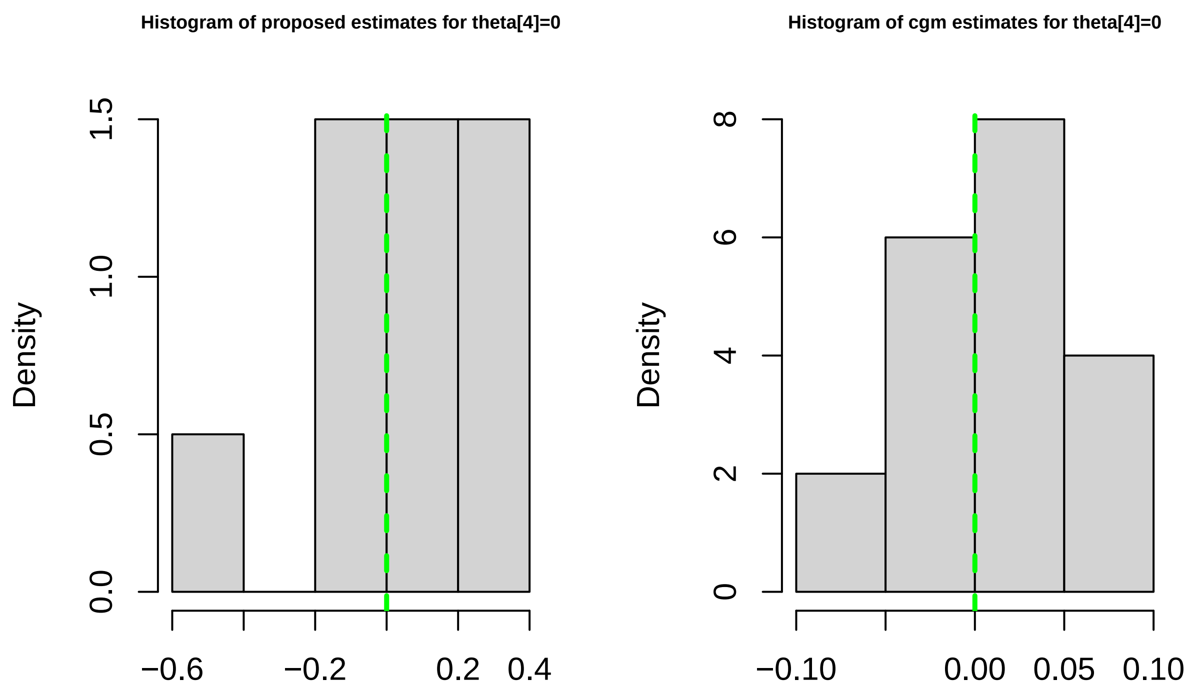
	proposed	cgm
theta[11]	0.050	0.013
theta[13]	0.043	0.016
theta[30]	0.009	0.006
theta[4]	0.015	0.003
total	0.029	0.009

Boxplots

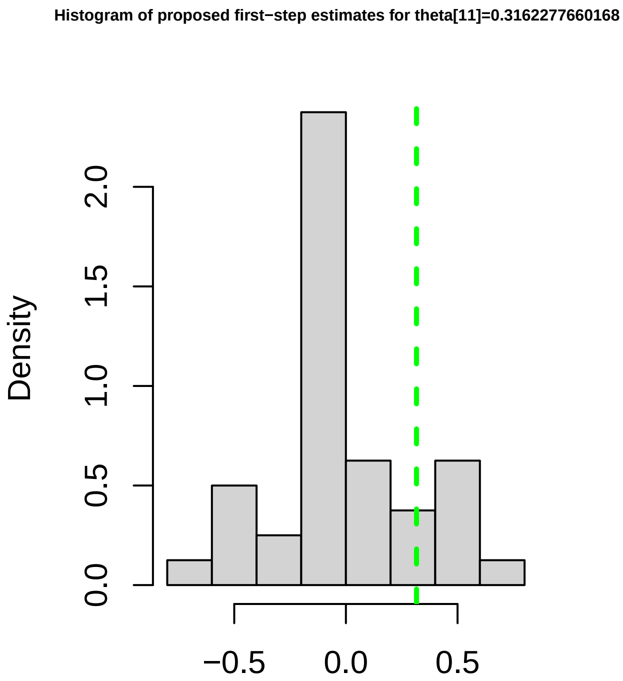

Histogram of proposed estimates for theta[11]=0.3162277660168
Density
1.0
0.8
0.6
0.4
0.2
0.0
−0.4
0.0
0.4
0.8
Histogram of cgm estimates for theta[11]=0.316227766016838
Density
6
5
4
3
2
1
0
0.20
0.30
0.40
0.50
Histogram of proposed estimates for theta[13]=−0.316227766016
Density
2.5
2.0
1.5
1.0
0.5
0.0
−1.0
−0.6
−0.2
0.2
Histogram of cgm estimates for theta[13]=−0.316227766016838
Density
6
5
4
3
2
1
0
−0.45
−0.35
−0.25
−0.15

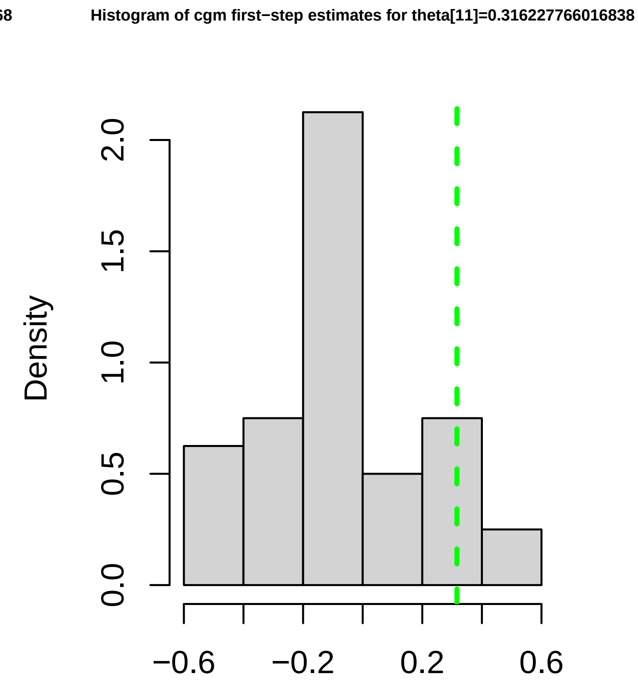

Histogram of proposed estimates for theta[30]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.3
−0.1
0.1
0.3

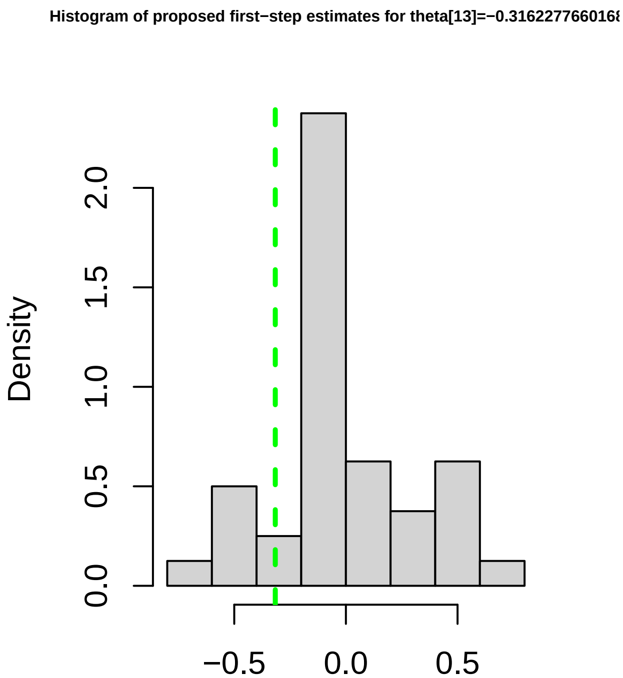

Histogram of cgm estimates for theta[30]=0
Density
0
2
4
6
8
−0.1
0.0
0.1
0.2

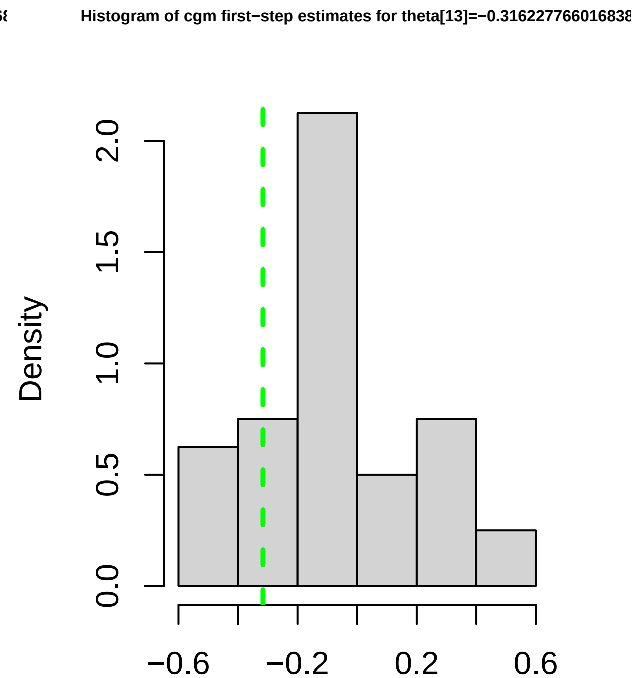

Histogram of proposed estimates for theta[4]=0
Density
0.0
0.5
1.0
1.5
−0.6
−0.2
0.2
0.4
Histogram of cgm estimates for theta[4]=0
Density
0
2
4
6
8
−0.10
0.00
0.05
0.10

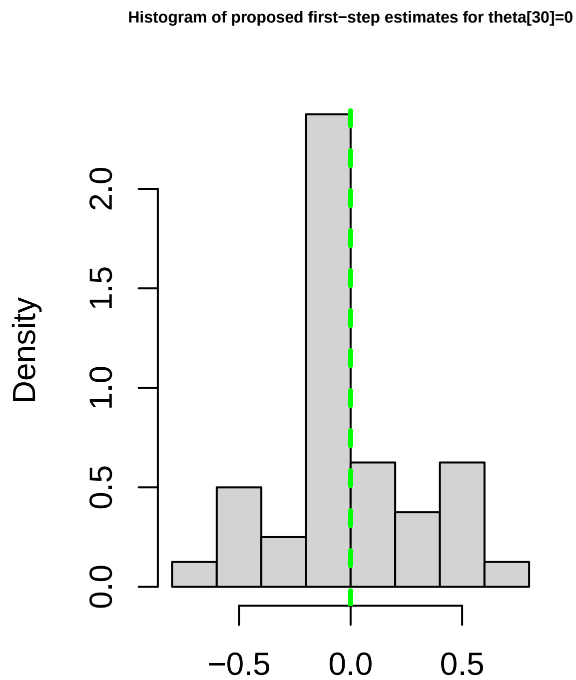
First Step Histograms

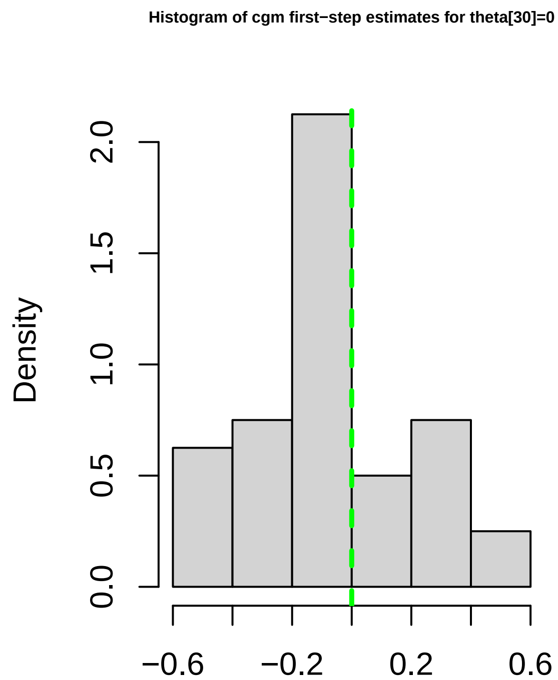


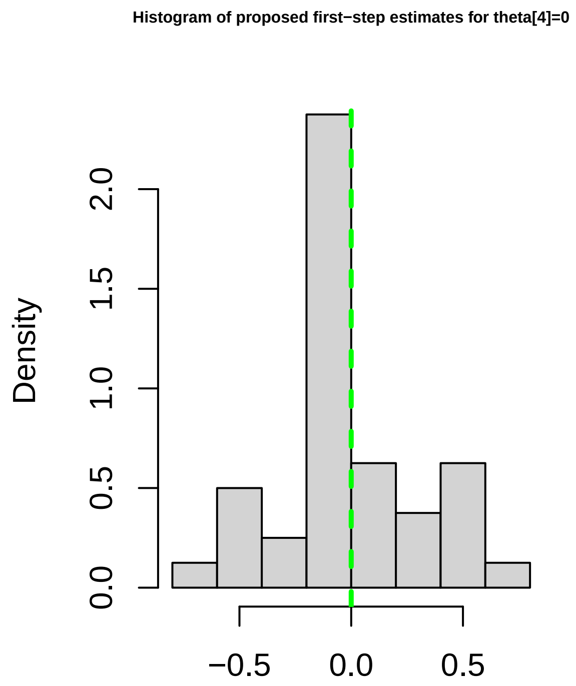


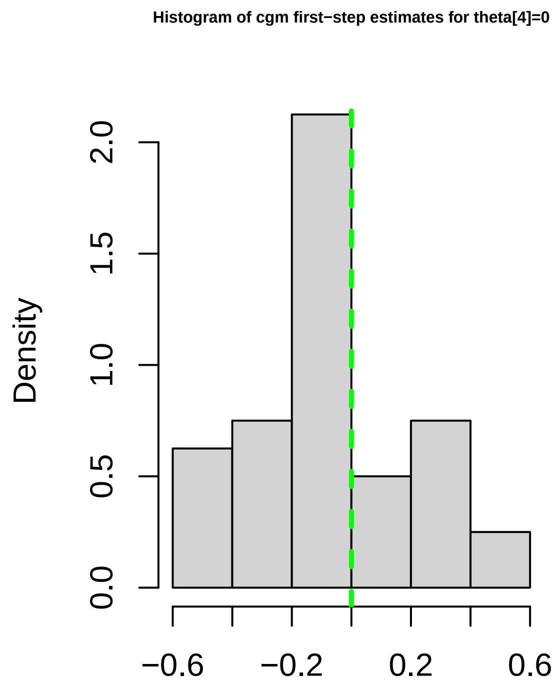





Histogram of proposed first−step estimates for theta[30]=0
Density
0.0
0.5
1.0
1.5
2.0
−0.5
0.0
0.5


Histogram of cgm first−step estimates for theta[30]=0
Density
0.0
0.5
1.0
1.5
2.0
−0.6
−0.2
0.2
0.6


Histogram of proposed first−step estimates for theta[4]=0
Density
0.0
0.5
1.0
1.5
2.0
−0.5
0.0
0.5


Histogram of cgm first−step estimates for theta[4]=0
Density
0.0
0.5
1.0
1.5
2.0
−0.6
−0.2
0.2
0.6

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 3: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[11]	-0.243	0.304	0.656	-0.207	0.654
theta[13]	-0.856	-0.465	0.062	-0.837	0.020
theta[30]	-0.256	0.014	0.330	-0.236	0.322
theta[4]	-0.432	0.093	0.273	-0.360	0.272

Table 4: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[11]	0.234	0.336	0.472	0.239	0.471
theta[13]	-0.403	-0.320	-0.168	-0.402	-0.176
theta[30]	-0.113	0.022	0.220	-0.095	0.194
theta[4]	-0.066	0.016	0.078	-0.059	0.073

Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[11]	0.285	0.141	0.008	0.562	0.5
theta[13]	-0.480	0.137	-0.748	-0.212	0.6
theta[30]	0.055	0.135	-0.209	0.318	0.6
theta[4]	0.046	0.133	-0.216	0.307	0.8

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[11]	0.344	0.087	0.174	0.514	0.9
theta[13]	-0.308	0.089	-0.482	-0.134	1.0
theta[30]	0.034	0.086	-0.135	0.202	0.9
theta[4]	0.011	0.087	-0.158	0.181	1.0