# Simulation Results

2026-01-26

## Simulation Setup

This simulation is performed with $n = 200$ and $d = 400$, using the 2-d lattice as the underlying graph. $s = 5$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.2$. The attached results are for a 10-replication simulation. The parameter vector $\theta$ has sparse components other than the following:

| Parameter.Index | Value |
|---|---|
| 28 | -0.447 |
| 237 | -0.447 |
| 250 | 0.447 |
| 282 | -0.447 |
| 399 | -0.447 |

but for brevity, our simulation only estimates the indices of $\theta$ in $\mathcal{C} = \{$ 28, 237, 328, 168$\}$ elements of $\theta$. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

## Results

### Mean-squared error comparison

Table 1: Mean-Squared Error of Parameter Estimates

| | proposed | cgm |
|---|---|---|
| theta[28] | 0.047 | 0.013 |
| theta[237] | 0.039 | 0.020 |
| theta[328] | 0.012 | 0.025 |
| theta[168] | 0.011 | 0.006 |
| total | 0.027 | 0.016 |

Table 2: Mean-Squared Error of First-Step Parameter Estimates

| | proposed | cgm |
|---|---|---|
| theta[28] | 0.142 | 0.131 |

| | proposed | cgm |
|---|---|---|
| theta[237] | 0.154 | 0.077 |
| theta[328] | 0.000 | 0.000 |
| theta[168] | 0.000 | 0.000 |
| total | 0.074 | 0.052 |

### Mean absolute deviation comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat

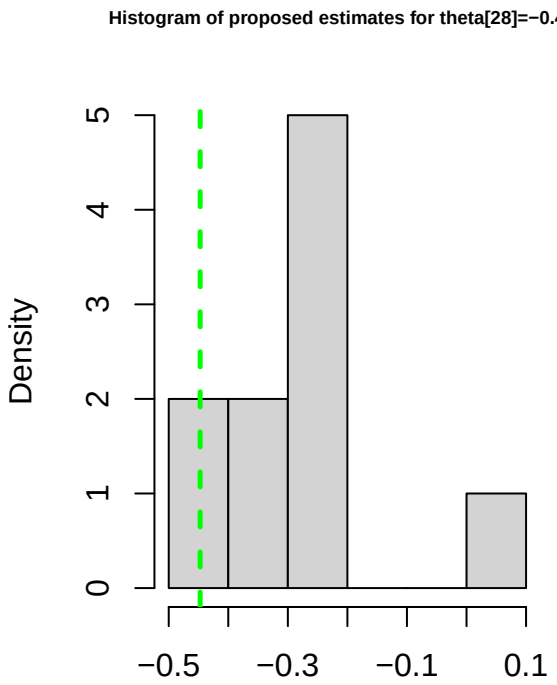
Table 3: Mean Absolute Deviation of Parameter Estimates

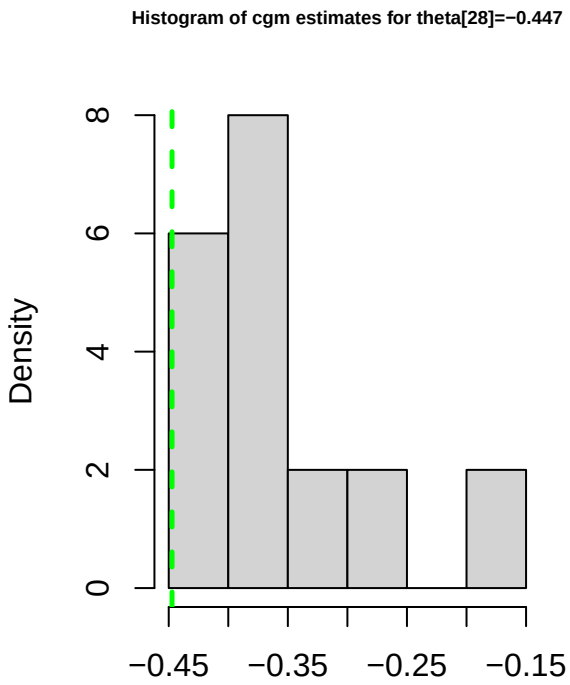
| | proposed | cgm |
|---|---|---|
| theta[28] | 0.170 | 0.085 |
| theta[237] | 0.187 | 0.128 |
| theta[328] | 0.086 | 0.128 |
| theta[168] | 0.086 | 0.065 |
| total | 0.132 | 0.102 |

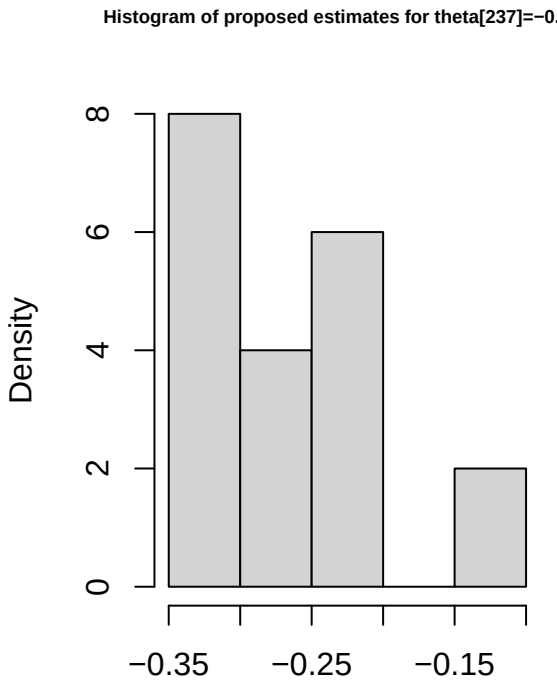
Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

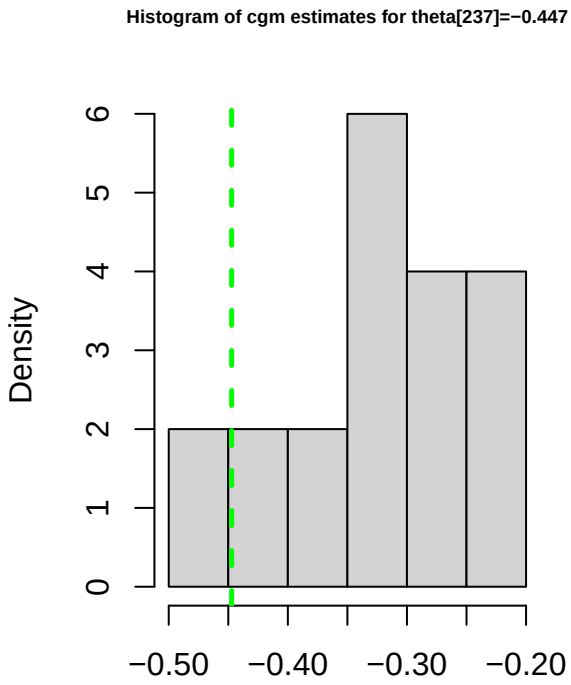
| | proposed | cgm |
|---|---|---|
| theta[28] | 0.360 | 0.333 |
| theta[237] | 0.383 | 0.229 |
| theta[328] | 0.000 | 0.000 |
| theta[168] | 0.000 | 0.000 |
| total | 0.186 | 0.140 |

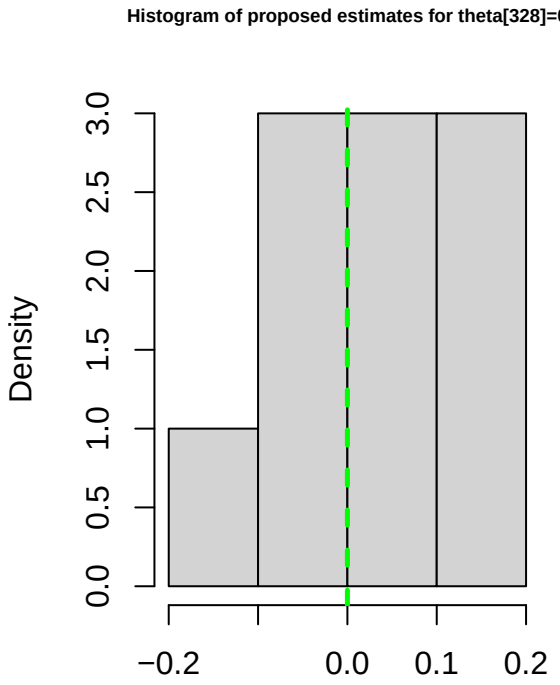
## Boxplots

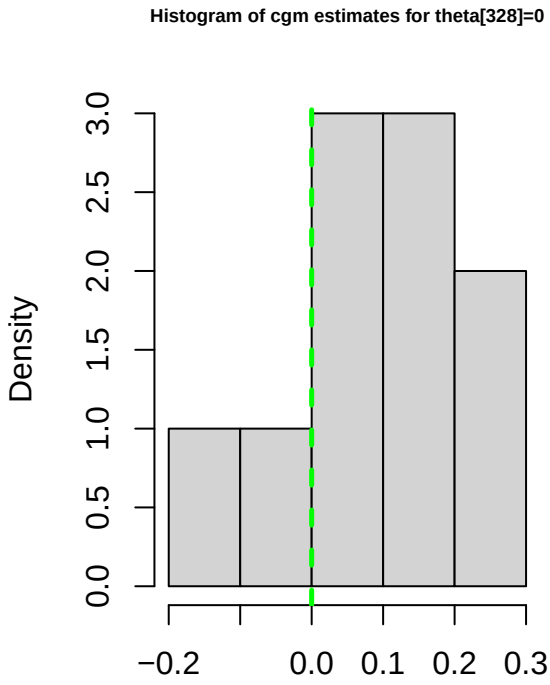

Histogram of proposed estimates for theta[28]=−0.447
Density
0
1
2
3
4
5
−0.5
−0.3
−0.1
0.1

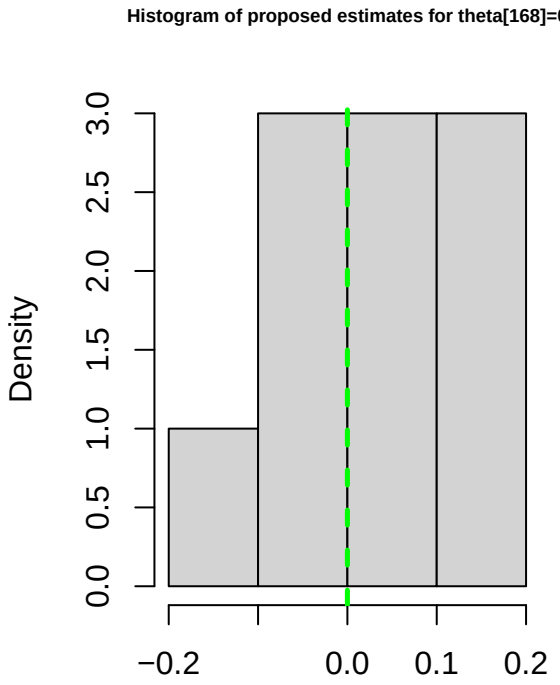

Histogram of cgm estimates for theta[28]=−0.447
Density
0
2
4
6
8
−0.45
−0.35
−0.25
−0.15

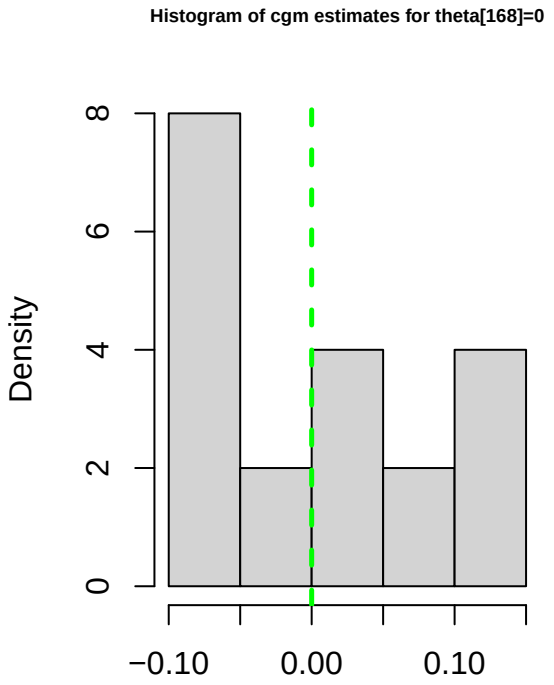

Histogram of proposed estimates for theta[237]=−0.447
Density
0
2
4
6
8
−0.35
−0.25
−0.15


Histogram of cgm estimates for theta[237]=−0.447
Density
0
1
2
3
4
5
6
−0.50
−0.40
−0.30
−0.20


Histogram of proposed estimates for theta[328]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.2
0.0
0.1
0.2


Histogram of cgm estimates for theta[328]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.2
0.0
0.1
0.2
0.3


Histogram of proposed estimates for theta[168]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.2
0.0
0.1
0.2


Histogram of cgm estimates for theta[168]=0
Density
0
2
4
6
8
−0.10
0.00
0.10

**Statistics and 95% Confidence Intervals from per-Replicate Estimates**

**Statistics for Theoretical 95% Confidence Intervals**

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

| | Estimate | SE | lower.CI | upper.CI | cvg |
|---|---|---|---|---|---|
| theta[28] | -0.282 | 0.108 | -0.494 | -0.070 | 0.7 |
| theta[237] | -0.260 | 0.118 | -0.490 | -0.030 | 0.6 |
| theta[328] | 0.025 | 0.103 | -0.177 | 0.228 | 0.9 |
| theta[168] | 0.023 | 0.107 | -0.187 | 0.232 | 1.0 |

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

| | Estimate | SE | lower.CI | upper.CI | cvg |
|---|---|---|---|---|---|
| theta[28] | -0.362 | 0.094 | -0.546 | -0.178 | 0.8 |
| theta[237] | -0.328 | 0.098 | -0.521 | -0.135 | 0.8 |
| theta[328] | 0.102 | 0.093 | -0.082 | 0.285 | 0.7 |
| theta[168] | 0.000 | 0.092 | -0.180 | 0.180 | 1.0 |