

# Simulation Results

2026-01-21

## Simulation Setup

This simulation is performed with  $n = 400$  and  $d = 200$ , using the 2-d lattice as the underlying graph.  $s = 20$  parameters are set to be nonzero, and the beta parameter is chosen to be  $\beta = 0.4$ . The attached results are for a 10-replication simulation. The parameter vector  $\theta$  has sparse components other than the following:

Parameter.Index	Value
2	0.224
23	0.224
38	-0.224
39	0.224
51	-0.224
53	0.224
57	-0.224
60	0.224
81	-0.224
82	-0.224
83	0.224
89	-0.224
120	-0.224
123	-0.224
125	0.224
129	-0.224
162	0.224
174	0.224
177	-0.224
187	0.224

but for brevity, our simulation only estimates the indices of  $\theta$  in  $\mathcal{C} = \{ 2, 23, 76, 186 \}$  elements of  $\theta$ . Accordingly, **all statistics and visuals are indicative of performance only on the set  $\mathcal{C}$ .**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

## Results

### Mean-squared error comparison

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[2]	0.021	0.017
theta[23]	0.009	0.025
theta[76]	0.008	0.008
theta[186]	0.007	0.014
total	0.011	0.016

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[2]	0.044	0.019
theta[23]	0.045	0.018
theta[76]	0.000	0.008
theta[186]	0.000	0.004
total	0.022	0.012

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### Mean absolute deviation comparison $(\frac{1}{n.sim} \sum_{i=1}^{n.sim} \frac{1}{|\mathcal{C}|} |\hat{C}
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Table 3: Mean Absolute Deviation of Parameter Estimates

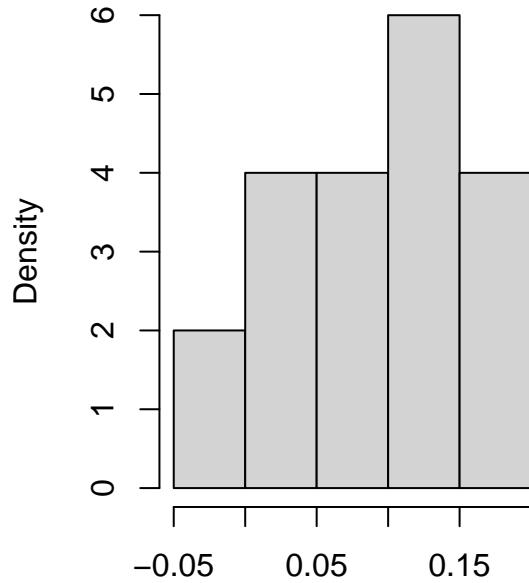
	proposed	cgm
theta[2]	0.129	0.096
theta[23]	0.063	0.108
theta[76]	0.076	0.075
theta[186]	0.062	0.098
total	0.083	0.094

Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

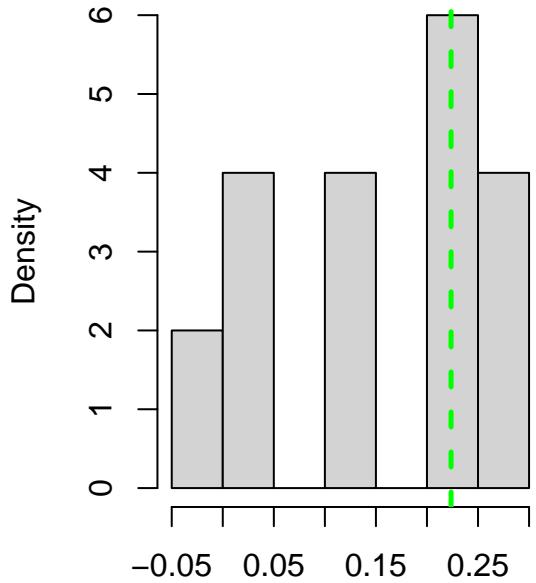
	proposed	cgm
theta[2]	0.208	0.120
theta[23]	0.209	0.107
theta[76]	0.000	0.028
theta[186]	0.000	0.027
total	0.104	0.071

## Boxplots

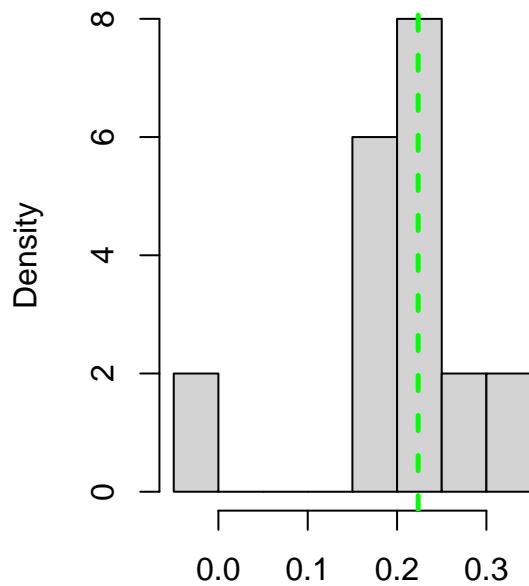
Histogram of proposed estimates for  $\theta[2]=0.224$



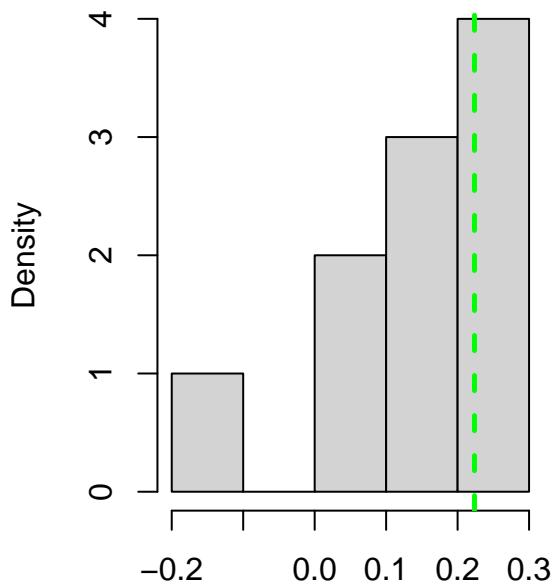
Histogram of cgm estimates for  $\theta[2]=0.224$



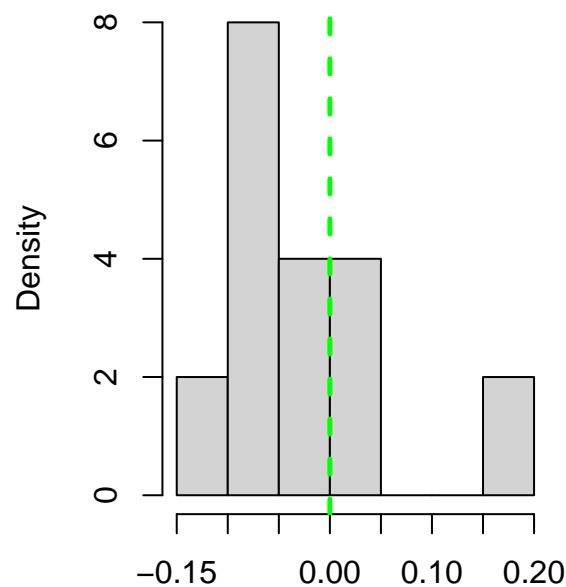
Histogram of proposed estimates for  $\theta[23]=0.224$



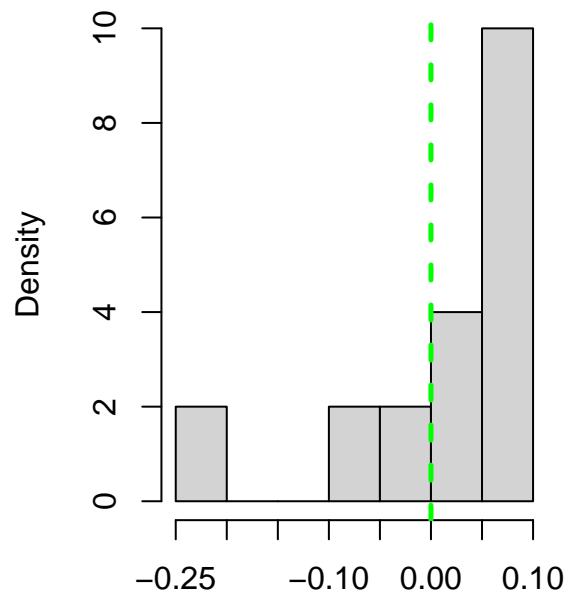
Histogram of cgm estimates for  $\theta[23]=0.224$



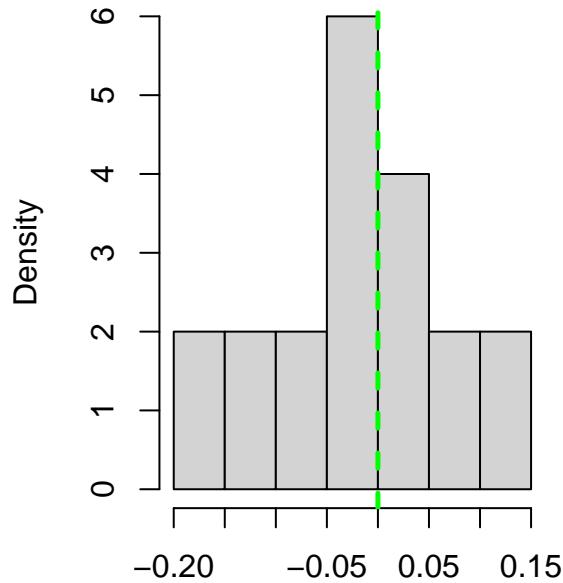
Histogram of proposed estimates for theta[76]=0



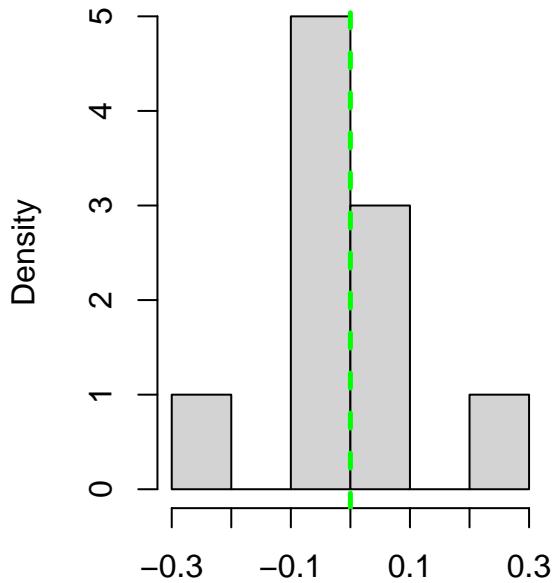
Histogram of cgm estimates for theta[76]=0



Histogram of proposed estimates for theta[186]=0



Histogram of cgm estimates for theta[186]=0



## Statistics and 95% Confidence Intervals from per-Replicate Estimates

### Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[2]	0.094	0.094	-0.090	0.278	0.7
theta[23]	0.197	0.108	-0.014	0.409	0.9
theta[76]	-0.024	0.103	-0.225	0.177	1.0
theta[186]	-0.021	0.103	-0.222	0.180	1.0

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[2]	0.147	0.084	-0.018	0.312	0.7
theta[23]	0.133	0.090	-0.043	0.309	0.9
theta[76]	0.016	0.084	-0.148	0.180	0.9
theta[186]	0.003	0.087	-0.167	0.173	0.8