

# Simulation Results

2026-01-15

## Simulation Setup

This simulation is performed with  $n = 200$  and  $d = 20$ , using the 2-d lattice as the underlying graph.  $s = 2$  parameters are set to be nonzero, and the beta parameter is chosen to be  $\beta = 0.2$ . The attached results are for a 10-replication simulation. The true values of the parameter vector  $\theta$  are

0 0 0 0 0 0 -0.7071068 0 0 0 0 0 0.7071068 0 0 0 0 0 0 0 ,

but for brevity, our simulation only estimates the indices of  $\theta$  in  $\mathcal{C} = \{7, 13, 4, 8\}$  elements of  $\theta$ . Accordingly, **all statistics and visuals are indicative of performance only on the set  $\mathcal{C}$ .**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

## Results

### Mean-squared error comparison  $\frac{1}{n.\text{sim}} \sum_{i=1}^{n.\text{sim}} \frac{1}{|\mathcal{C}|} \|\hat{\theta}_i - \theta\|^2$

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[7]	0.036	0.087
theta[13]	0.054	0.085
theta[4]	0.006	0.030
theta[8]	0.008	0.017
total	0.026	0.055

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[7]	0.123	0.132
theta[13]	0.158	0.136
theta[4]	0.000	0.004
theta[8]	0.000	0.003
total	0.070	0.069

### Mean absolute deviation comparison  $\frac{1}{n.\text{sim}} \sum_{i=1}^{n.\text{sim}} \frac{1}{|\mathcal{C}|} \|\hat{\theta}_i - \theta\|$

Table 3: Mean Absolute Deviation of Parameter Estimates

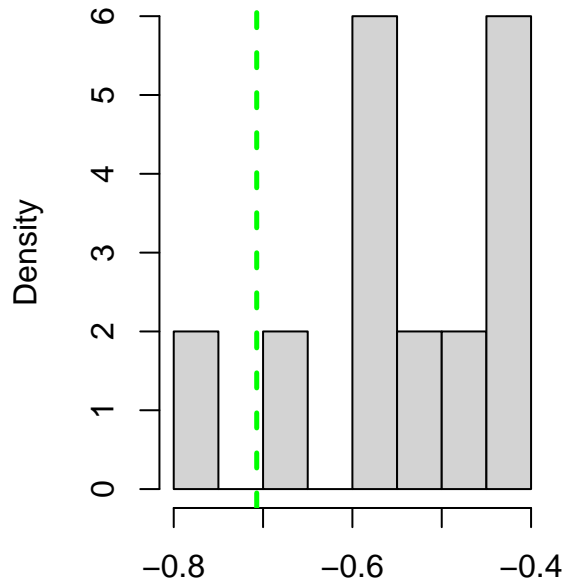
	proposed	cgm
theta[7]	0.166	0.217
theta[13]	0.204	0.202
theta[4]	0.054	0.145
theta[8]	0.079	0.114
total	0.126	0.169

Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

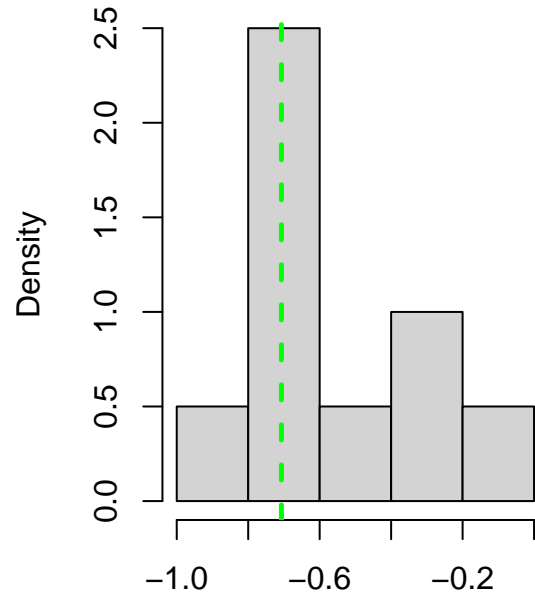
	proposed	cgm
theta[7]	0.334	0.306
theta[13]	0.386	0.347
theta[4]	0.003	0.034
theta[8]	0.000	0.021
total	0.181	0.177

## Boxplots

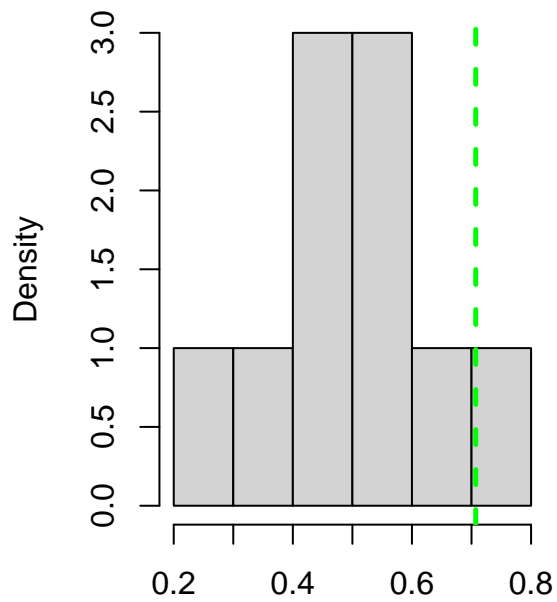
Histogram of proposed estimates for  $\theta[7] = -0.707$



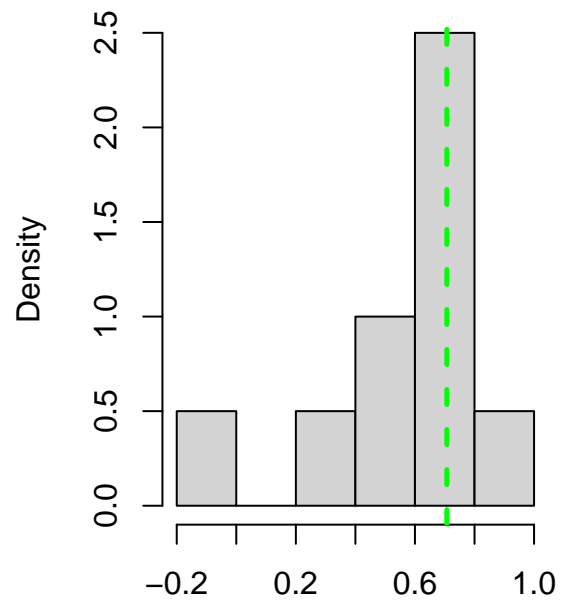
Histogram of cgm estimates for  $\theta[7] = -0.707$

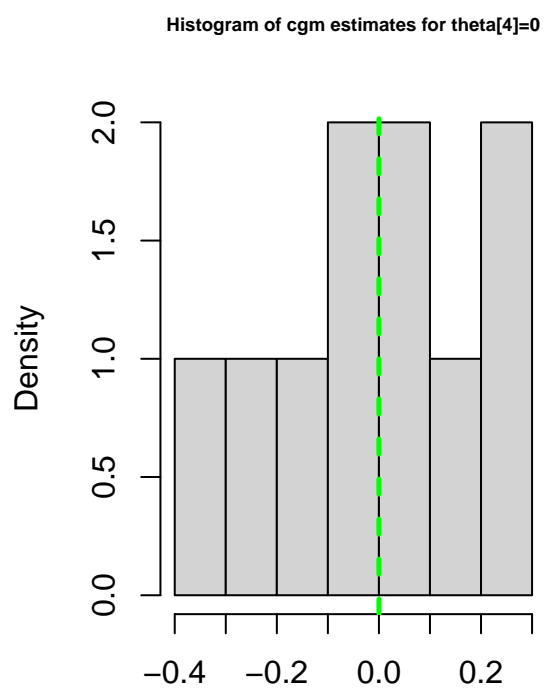
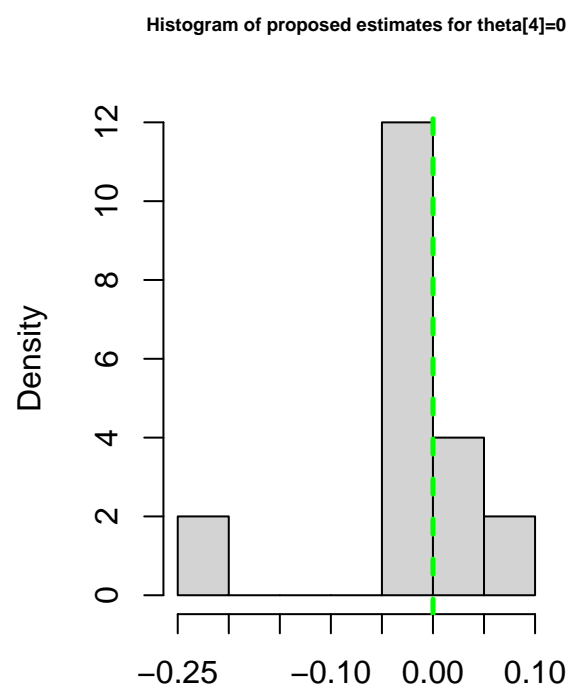


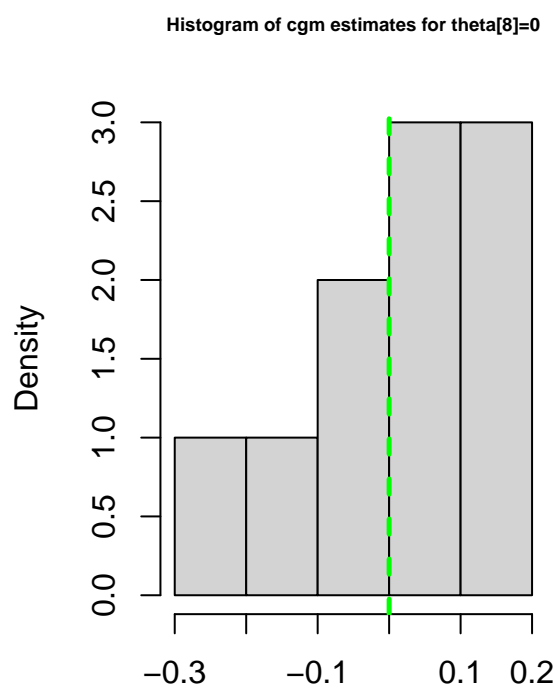
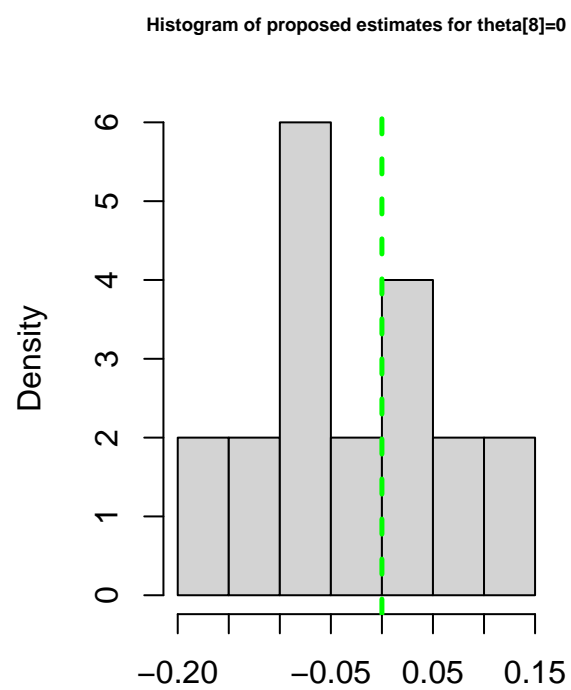
Histogram of proposed estimates for  $\theta[13] = 0.707$



Histogram of cgm estimates for  $\theta[13] = 0.707$

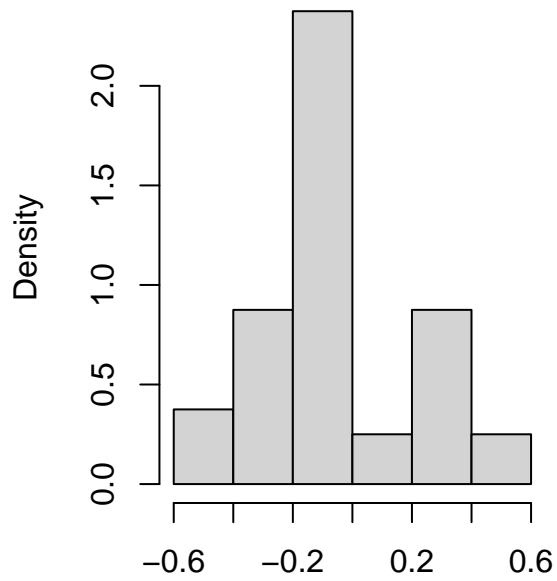




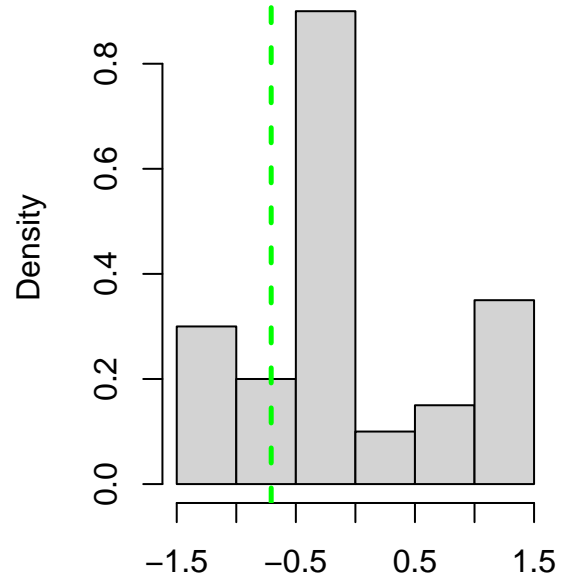


## First Step Histograms

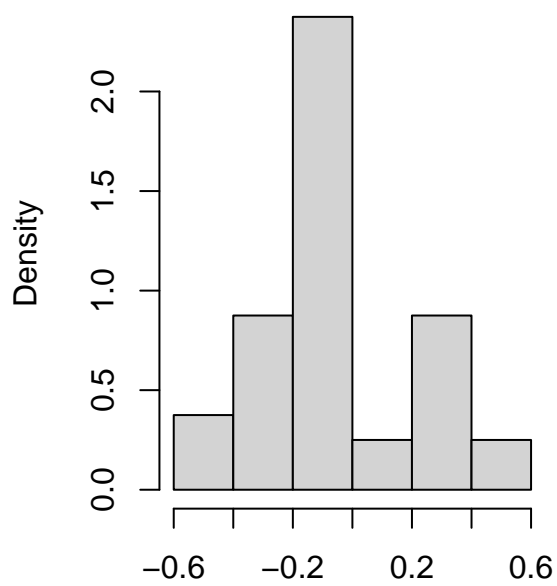
Histogram of proposed first-step estimates for  $\theta_7 = -0.707$



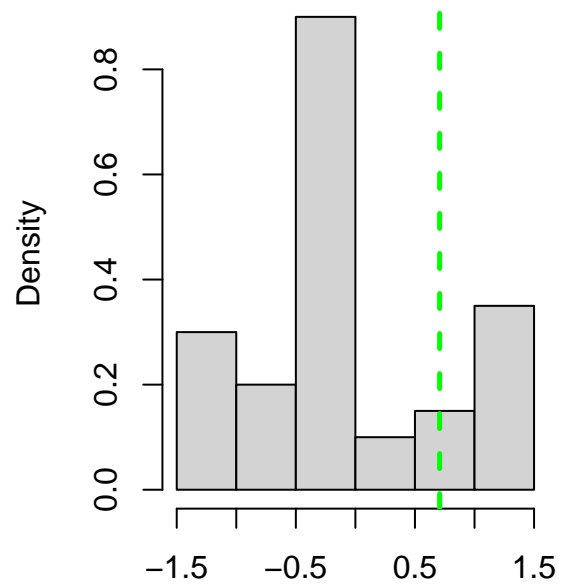
Histogram of cgm first-step estimates for  $\theta_7 = -0.707$



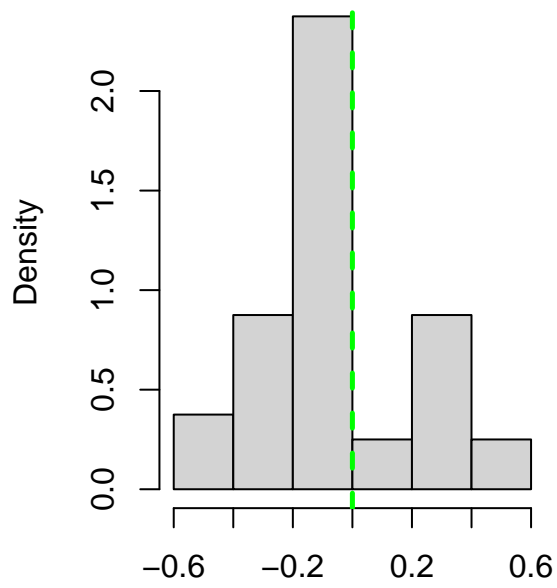
Histogram of proposed first-step estimates for  $\theta_{13} = 0.707$



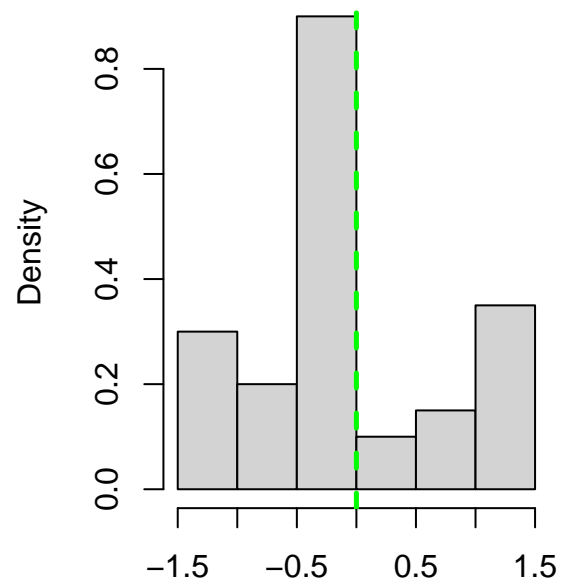
Histogram of cgm first-step estimates for  $\theta_{13} = 0.707$



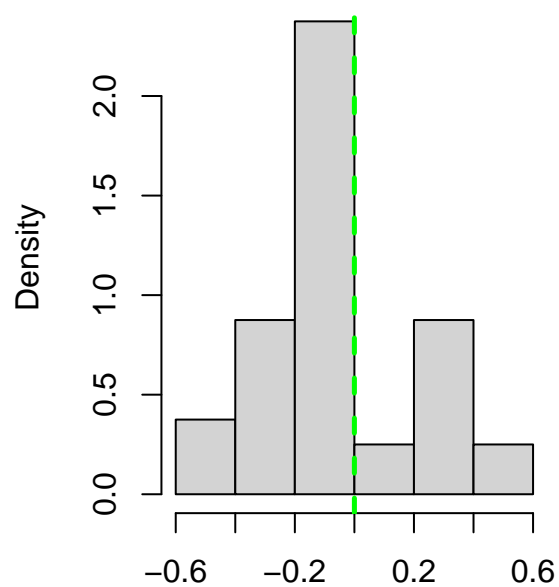
Histogram of proposed first-step estimates for  $\theta_4=0$



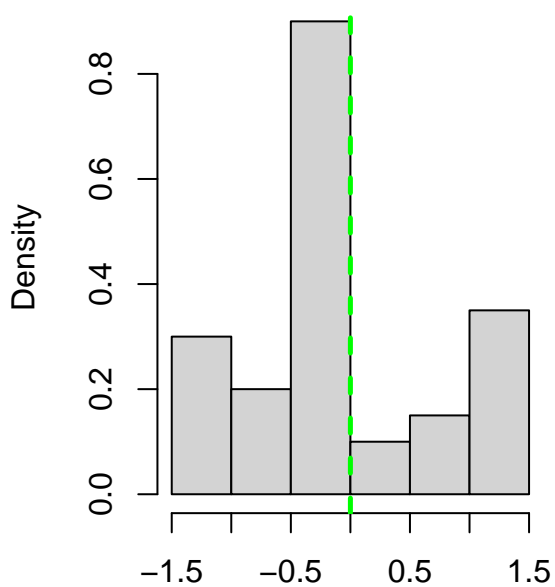
Histogram of cgm first-step estimates for  $\theta_4=0$



Histogram of proposed first-step estimates for  $\theta[8]=0$



Histogram of cgm first-step estimates for  $\theta[8]=0$





## Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 5: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[7]	-0.757	-0.542	-0.417	-0.744	-0.422
theta[13]	0.296	0.510	0.713	0.316	0.692
theta[4]	-0.220	-0.028	0.059	-0.182	0.051
theta[8]	-0.165	-0.048	0.106	-0.158	0.101

Table 6: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[7]	-0.988	-0.664	-0.016	-0.945	-0.082
theta[13]	-0.027	0.639	0.867	0.050	0.846
theta[4]	-0.310	-0.003	0.268	-0.287	0.261
theta[8]	-0.220	0.044	0.167	-0.209	0.167

## Statistics for Theoretical 95% Confidence Intervals

Table 7: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[7]	-0.552	0.136	-0.819	-0.285	0.8
theta[13]	0.504	0.132	0.246	0.762	0.7
theta[4]	-0.033	0.123	-0.273	0.208	1.0
theta[8]	-0.032	0.124	-0.276	0.211	1.0

Table 8: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[7]	-0.593	0.183	-0.952	-0.233	0.8
theta[13]	0.550	0.189	0.179	0.921	0.8
theta[4]	-0.003	0.128	-0.254	0.248	0.8
theta[8]	0.021	0.124	-0.222	0.264	1.0