Simulation Results

2025-10-14

Simulation Setup

This simulation is performed with $n = 100$ and $d = 10$, using the 2-d lattice as the underlying graph. $s = 2$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.1$. The attached results are for a 5-replication simulation. The true values of the parameter vector θ are

```
 [1]  0  0  0  0  0  0 -1  0 -1  0
```

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

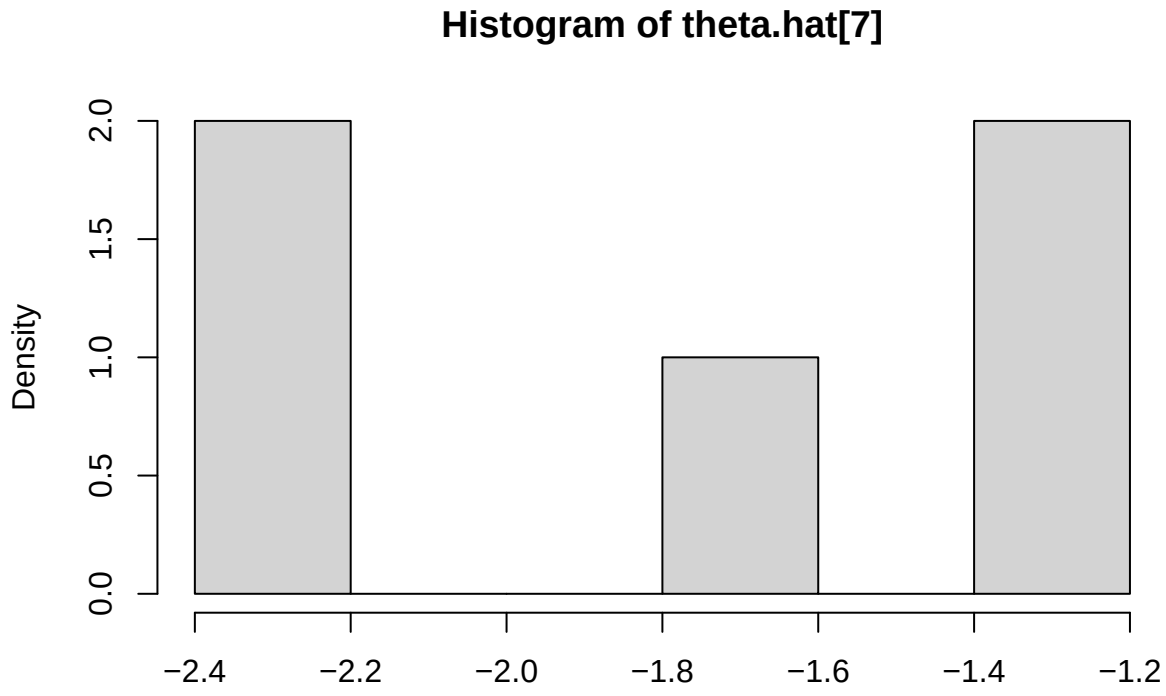
Results

Mean-squared error comparison ($\frac{1}{n.sim}\sum_{i=1}^{n.sim}\frac{1}{d}\|\hat{\theta}_i - \theta\|^2$)

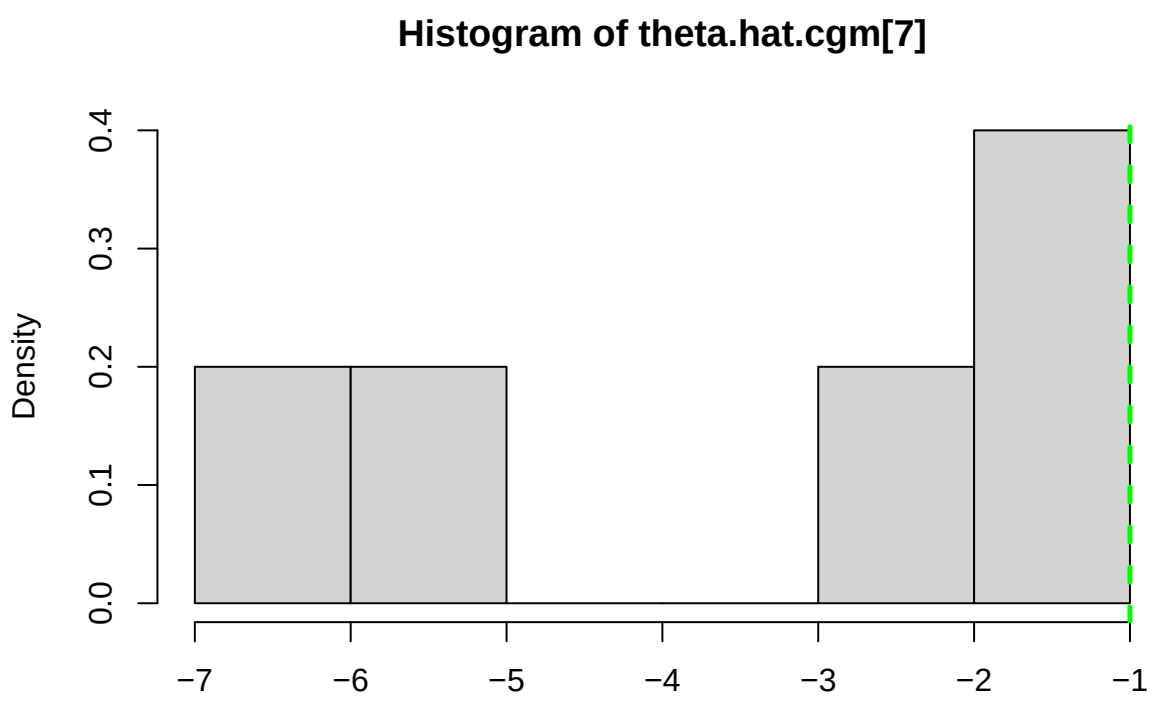
```
# A tibble: 1 x 2
  `MISLE (First-step) MSE` `MISLE MSE`
                     <dbl>       <dbl>
1                    0.310       0.716
```

```
# A tibble: 1 x 2
  `MISLE MSE` `CGM MSE`
        <dbl>     <dbl>
1       0.716     1853.
```

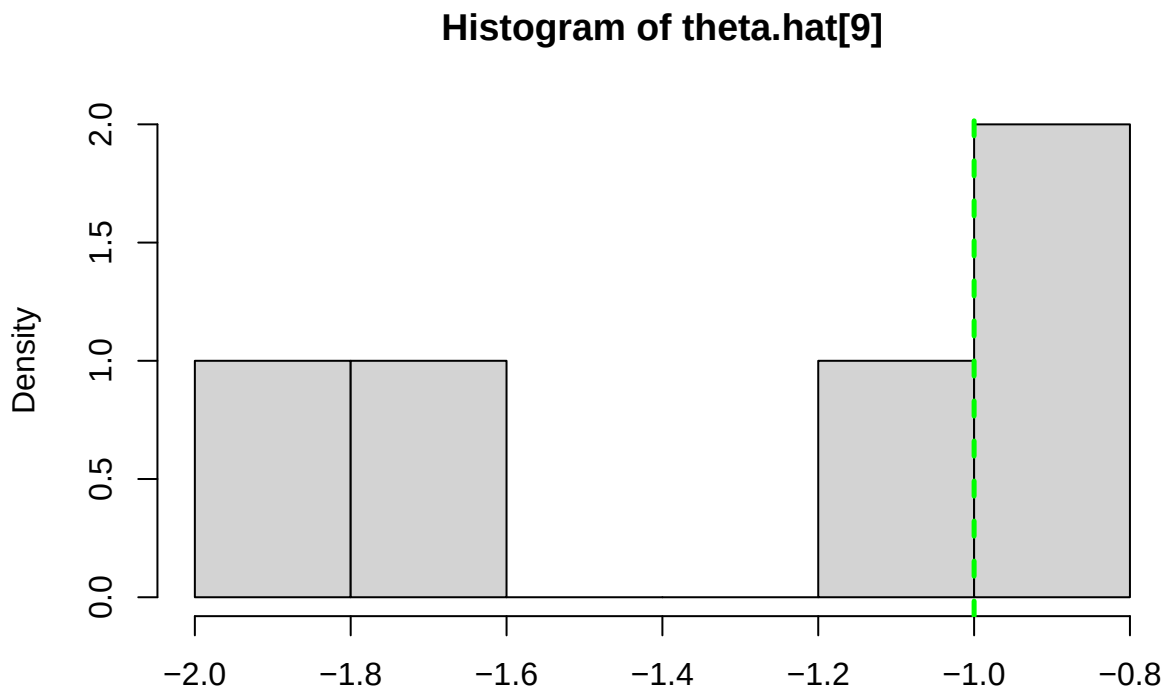
First Step Histograms



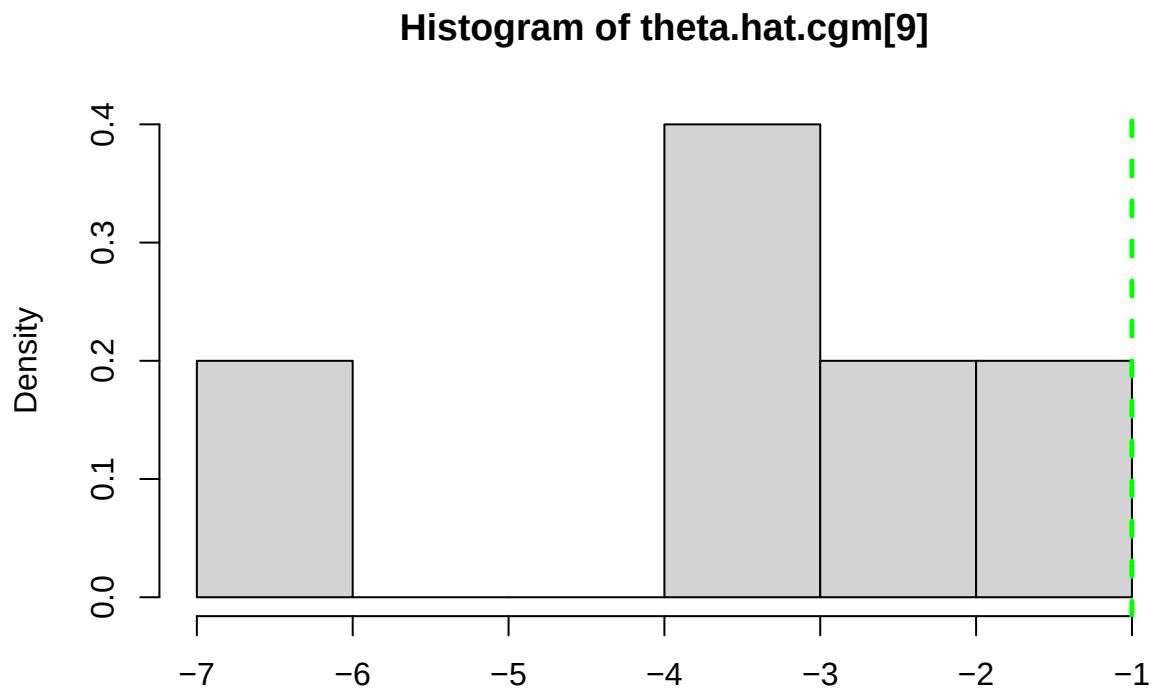

```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 -2.260  -2.201  -1.707  -1.758  -1.354  -1.268
[1] "95% CI based on bootstrap:"
      lower     upper
1 -2.253618 -1.276422
```



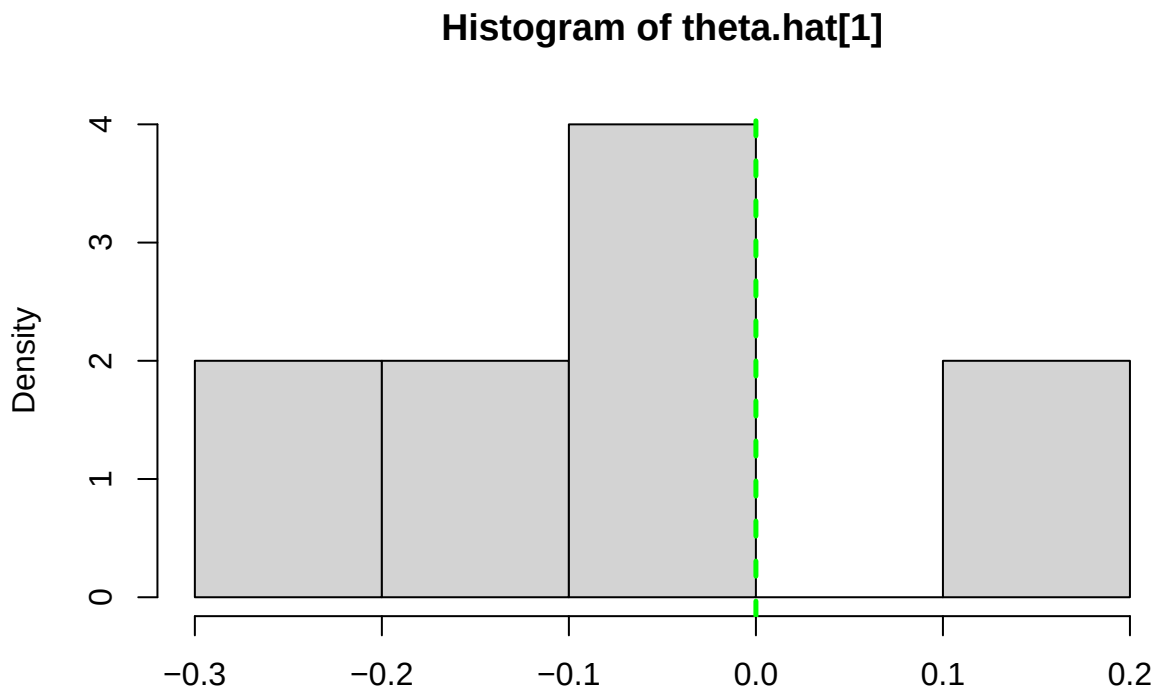

```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 -6.711  -5.281  -2.238  -3.544  -1.795  -1.694
[1] "95% CI based on bootstrap:"
  lower.cgm upper.cgm
1 -6.567691 -1.704274
```



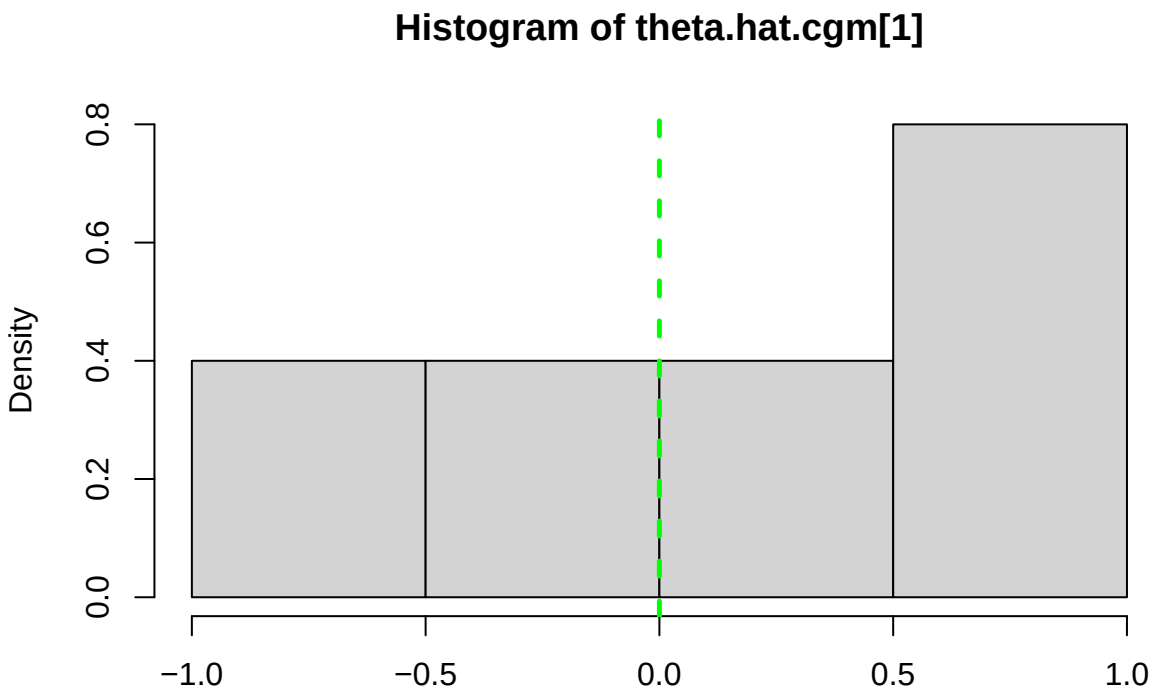

```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-1.9105 -1.6893 -1.0546 -1.2816 -0.9441 -0.8094
[1] "95% CI based on bootstrap:"
      lower      upper
1 -1.888354 -0.8229154
```



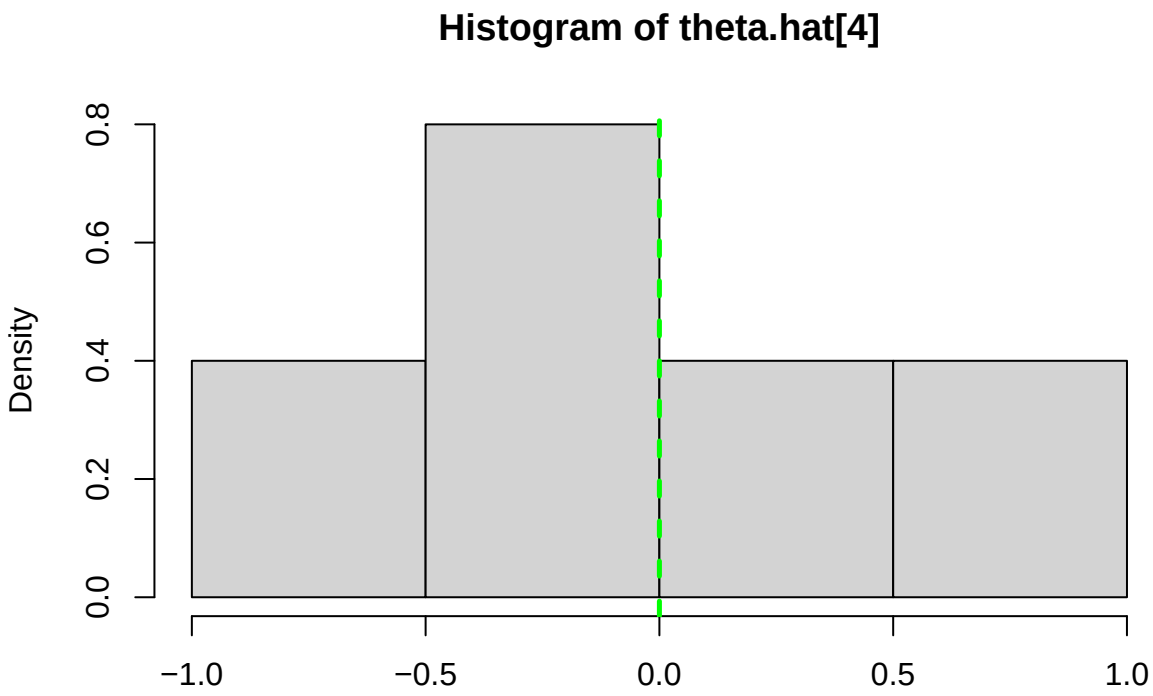

```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 -6.580  -3.880  -3.114  -3.562  -2.334  -1.902
[1] "95% CI based on bootstrap:"
  lower.cgm upper.cgm
1 -6.309669 -1.944949
```



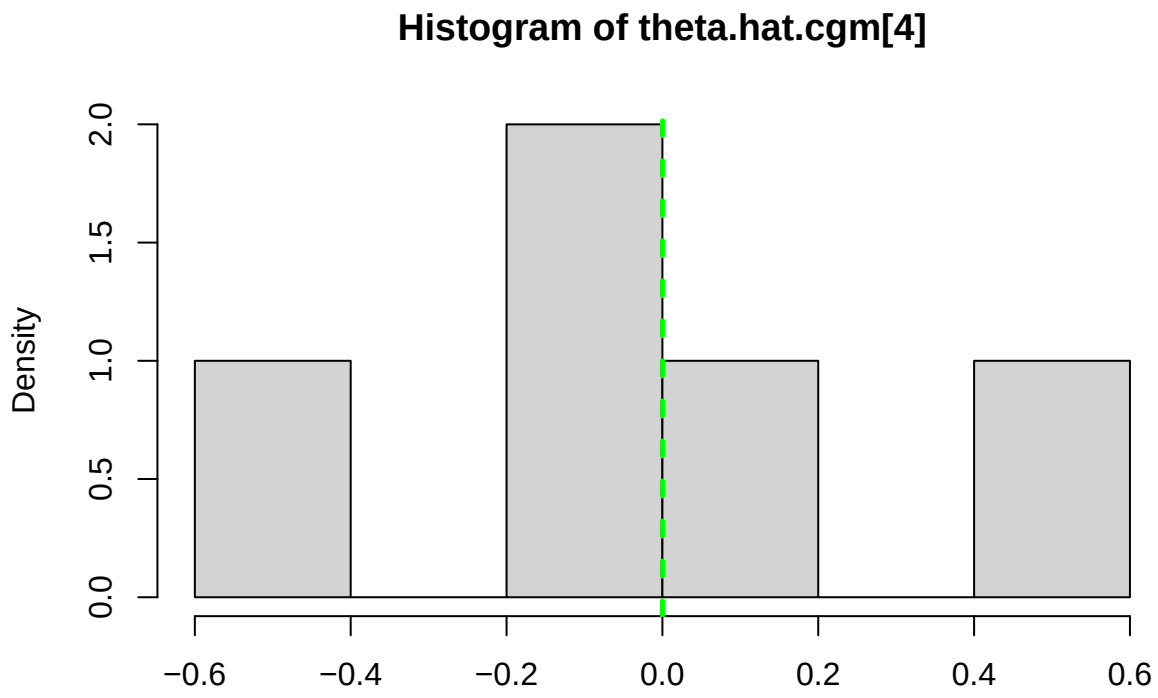

```
[1] "Summary statistics of bootstrap replicates:"
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-0.23588 -0.19950 -0.04604 -0.07854 -0.01451  0.10323
[1] "95% CI based on bootstrap:"
       lower      upper
1 -0.2322379 0.09145999
```




```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-0.9154 -0.1997  0.4120  0.1476  0.6216  0.8193
[1] "95% CI based on bootstrap:"
   lower.cgm upper.cgm
1 -0.8438653 0.7995425
```

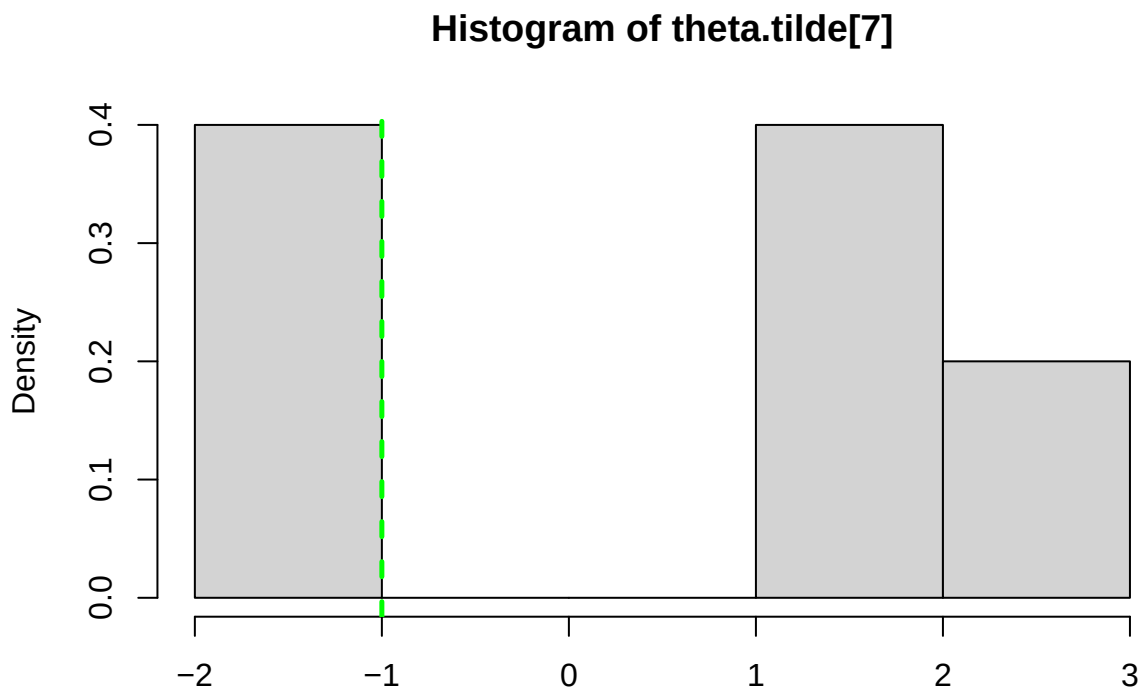



```
[1] "Summary statistics of bootstrap replicates:"
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-0.71441 -0.08266  0.00000  0.01784  0.20089  0.68539
[1] "95% CI based on bootstrap:"
       lower     upper
1 -0.6512336 0.6369397
```



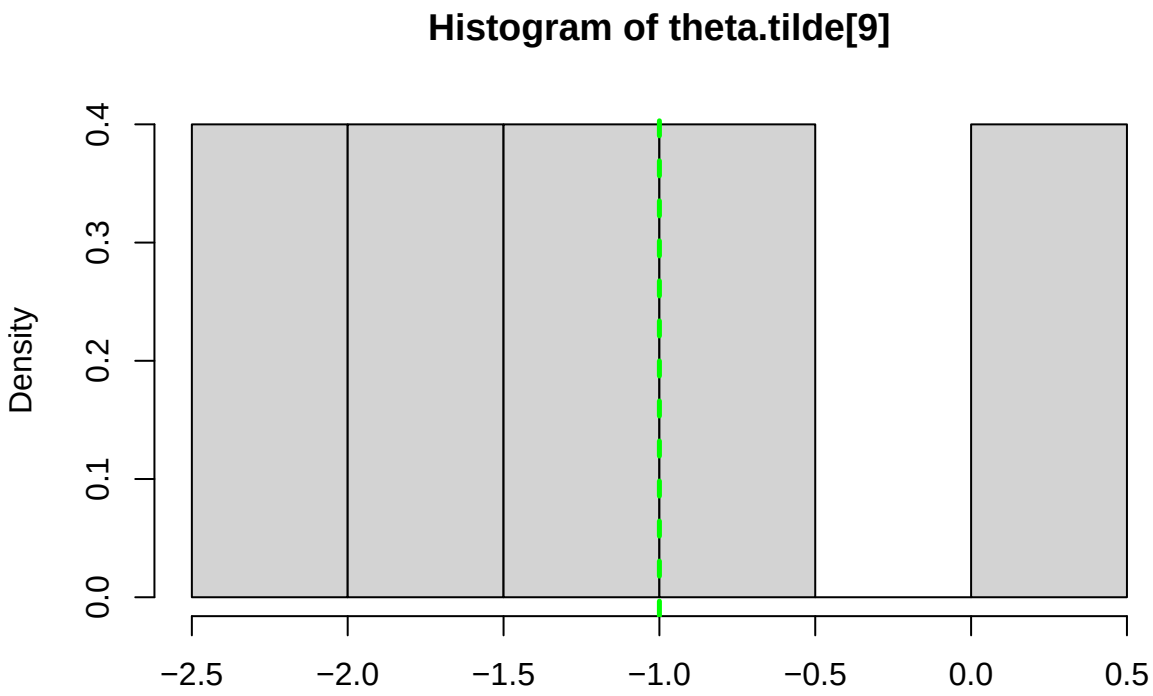

```
[1] "Summary statistics of bootstrap replicates:"
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-0.49463 -0.12265 -0.06961 -0.03418  0.01272  0.50330
[1] "95% CI based on bootstrap:"
   lower.cgm upper.cgm
1 -0.4574345 0.4542432
```

Statistics and 95% Confidence Intervals from per-Replicate Estimates

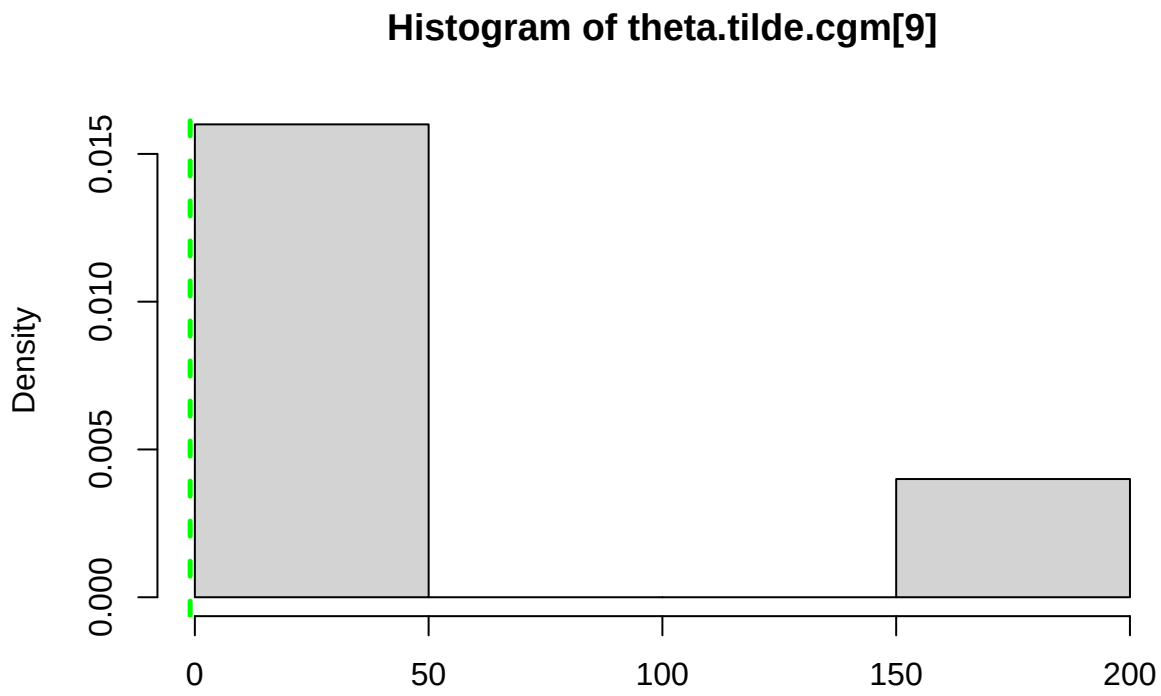



```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 -1.694  -1.070   1.401   0.518   1.868   2.085
[1] "95% CI based on bootstrap:"
     lower    upper
1 -1.63167 2.063578
```

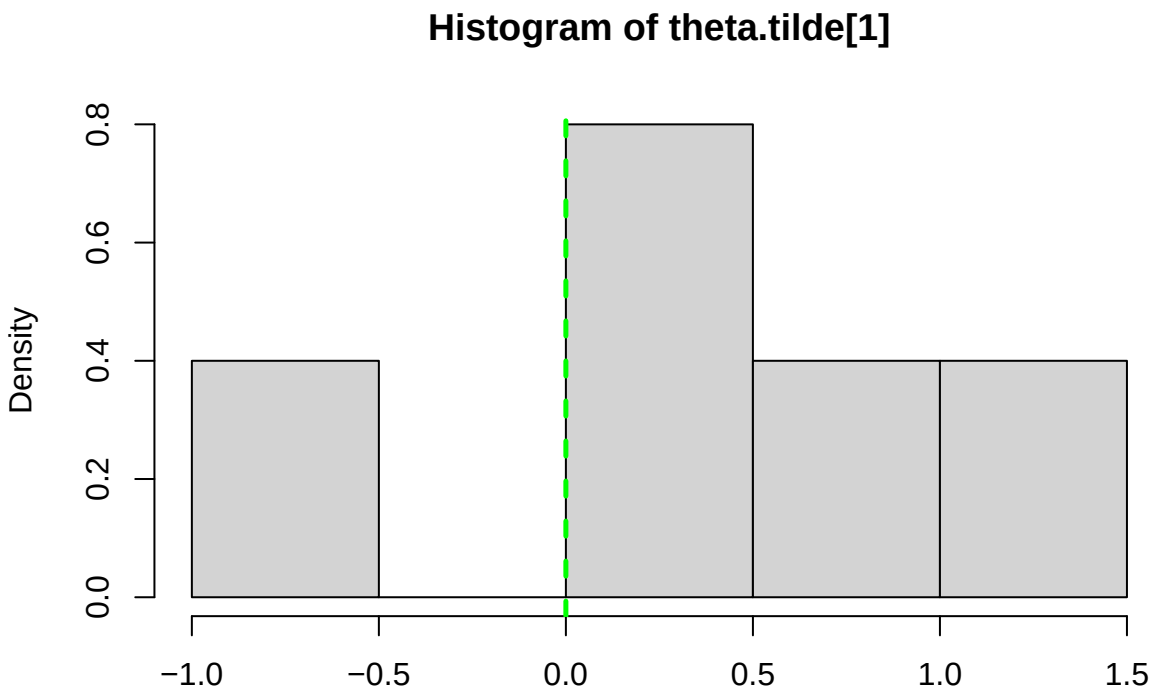

```
[1] "Summary statistics of bootstrap replicates:"
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
 -0.3724   1.1876   1.9399  59.2348  62.9124 230.5063
[1] "95% CI based on bootstrap:"
   lower.cgm upper.cgm
1 -0.2164006  213.7469
```



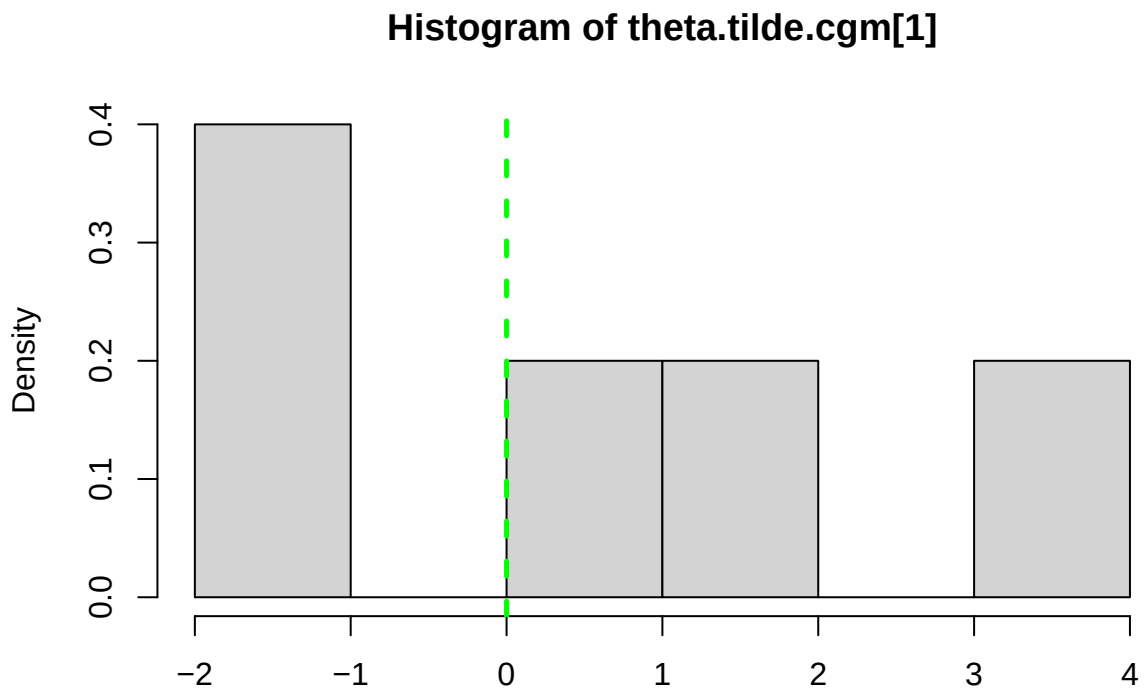

```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-2.1451 -1.7921 -1.1323 -1.1114 -0.6824  0.1949
[1] "95% CI based on bootstrap:"
      lower    upper
1 -2.109781 0.107138
```



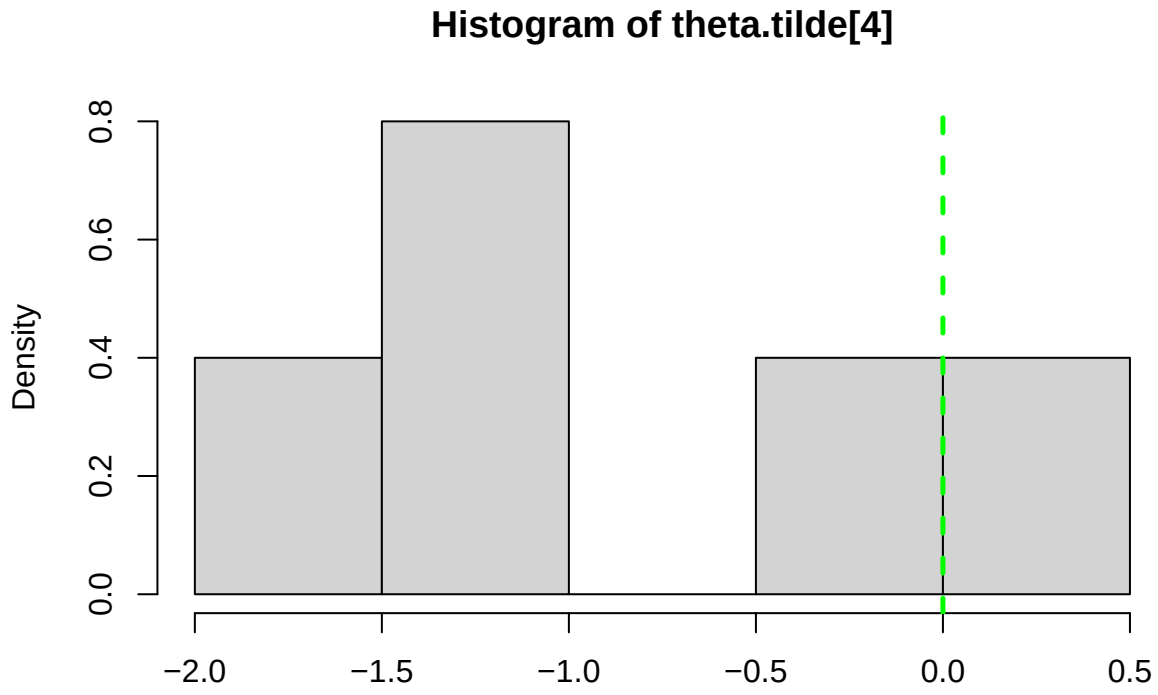

```
[1] "Summary statistics of bootstrap replicates:"
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
  0.4665   3.3571   9.8112  47.5187  44.0837 179.8751
[1] "95% CI based on bootstrap:"
  lower.cgm upper.cgm
1 0.7555556   166.296
```



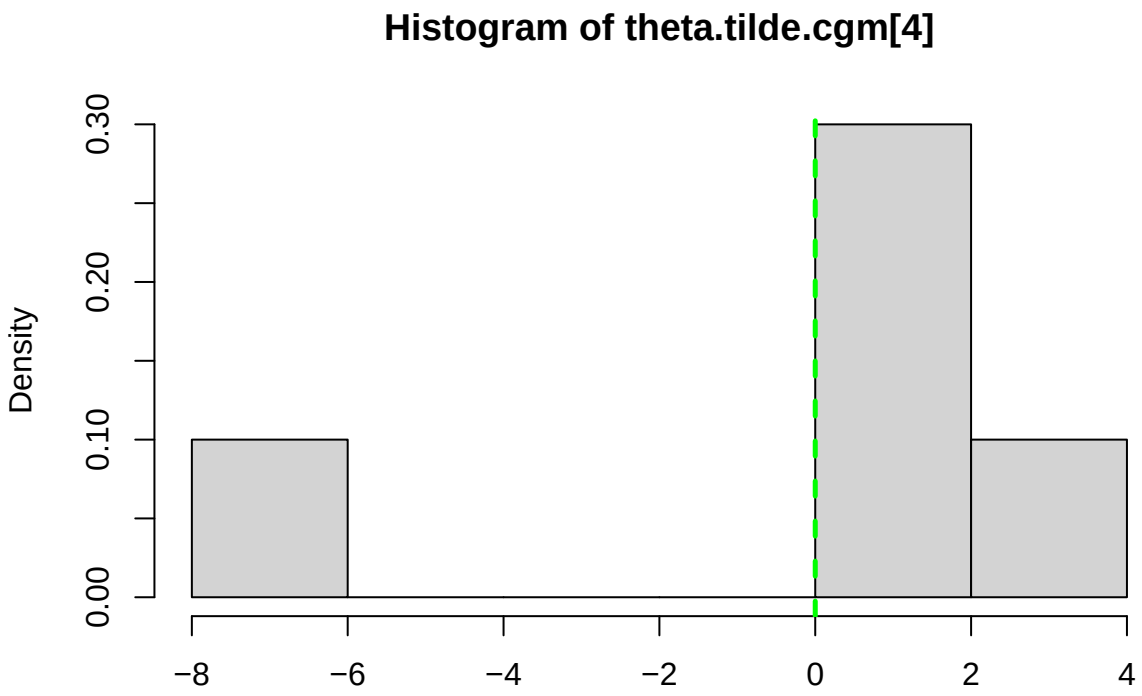

```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-0.5149  0.3024  0.3717  0.4534  0.8944  1.2136
[1] "95% CI based on bootstrap:"
       lower    upper
1 -0.4331814 1.181703
```




```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-1.5854 -1.1652  0.6127  0.5181  1.1089  3.6193
[1] "95% CI based on bootstrap:"
  lower.cgm upper.cgm
1 -1.543357  3.368292
```




```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-1.5161 -1.3396 -1.1860 -0.8237 -0.1995  0.1229
[1] "95% CI based on bootstrap:"
      lower      upper
1 -1.498489 0.09063924
```




```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-6.7767  0.1267  0.3991 -0.3306  1.9360  2.6618
[1] "95% CI based on bootstrap:"
  lower.cgm upper.cgm
1  -6.08639  2.589255
```

Statistics for Theoretical 95% Confidence Intervals

```
[1] Length of Confidence Intervals for theta[7]
[1] Coverage proportion: 0.2
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.7569  1.0638  1.8233  1.6675  1.9125  2.7809
[1] Length of Confidence Intervals for theta[7] (CGM Method)
[1] Coverage proportion: 0.6
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  5.467   6.744  13.310  78.145  94.403 270.799
[1] Length of Confidence Intervals for theta[9]
[1] Coverage proportion: 0.6
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.8254  1.1080  1.2291  1.4439  1.9188  2.1385
[1] Length of Confidence Intervals for theta[9] (CGM Method)
[1] Coverage proportion: 0.4
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  6.842  12.331  20.816  63.990  64.871 215.090
[1] Length of Confidence Intervals for theta[1]
[1] Coverage proportion: 0.4
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.7893  0.8272  0.8284  1.1685  1.0131  2.3845
[1] Length of Confidence Intervals for theta[1] (CGM Method)
[1] Coverage proportion: 1
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  2.196   3.864   4.456   5.536   5.872  11.292
[1] Length of Confidence Intervals for theta[4]
[1] Coverage proportion: 0.4
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.6802  0.9176  0.9840  1.1022  1.1911  1.7382
[1] Length of Confidence Intervals for theta[4] (CGM Method)
[1] Coverage proportion: 1
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  1.540   5.345   6.590   6.011   8.172   8.406
```