

Simulation Results

2026-01-07

Simulation Setup

This simulation is performed with $n = 400$ and $d = 10$, using the 2-d lattice as the underlying graph. $s = 2$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.4$. The attached results are for a 10-replication simulation. The true values of the parameter vector θ are

0.7071068 0 0 0 0 0 -0.7071068 0 0 0 ,

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{1, 7, 2, 5\}$ elements of θ . Accordingly, **all statistics and visuals are indicative of performance only on the set \mathcal{C} .**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison ($\frac{1}{n_{sim}} \sum_{i=1}^{n_{sim}} \frac{1}{|\mathcal{C}|} \|\hat{\theta}_{i,C} - \theta_C\|^2$)

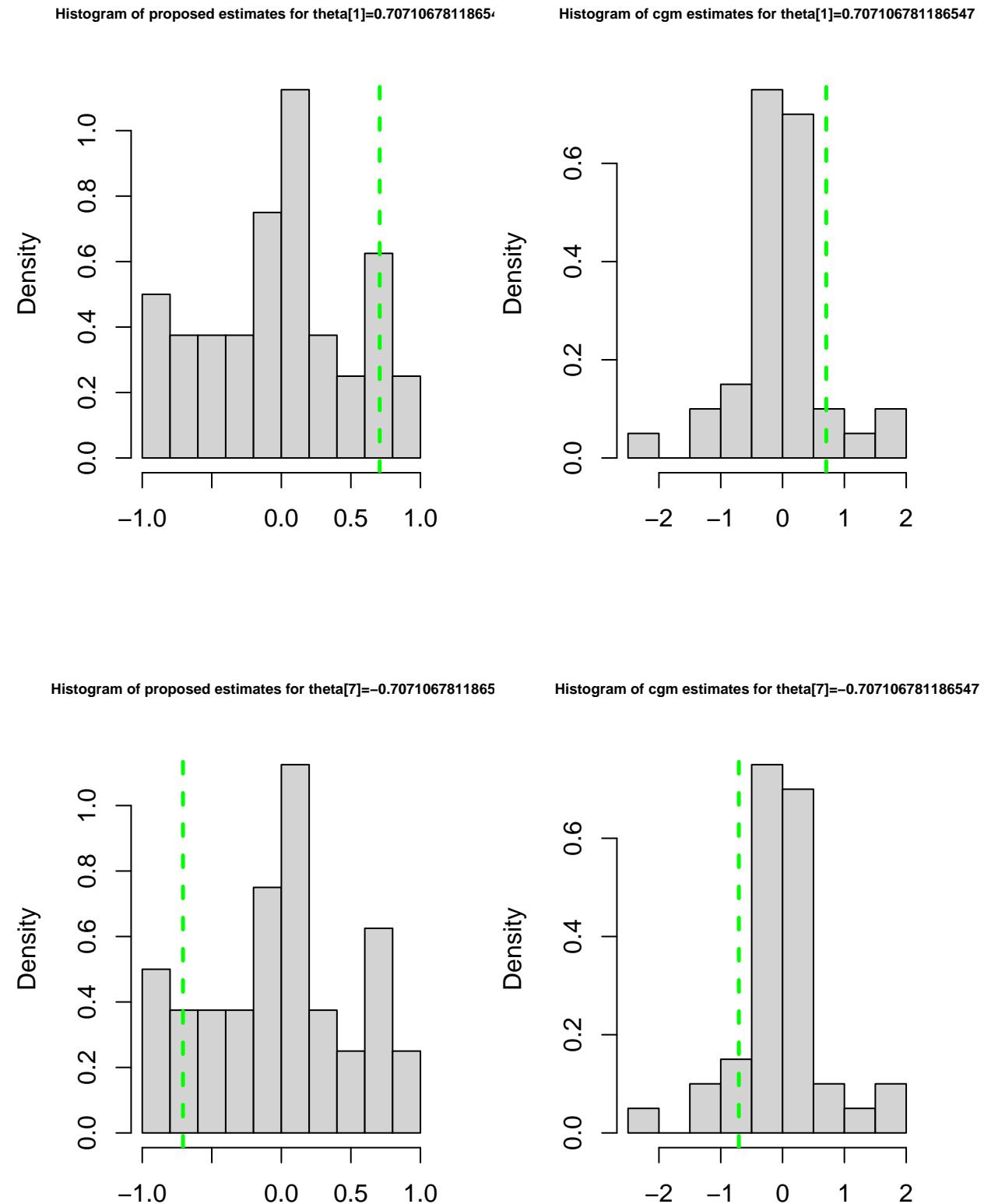
Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[1]	0.025	2.331
theta[7]	0.028	2.475
theta[2]	0.039	0.054
theta[5]	0.020	0.015
total	0.028	1.219

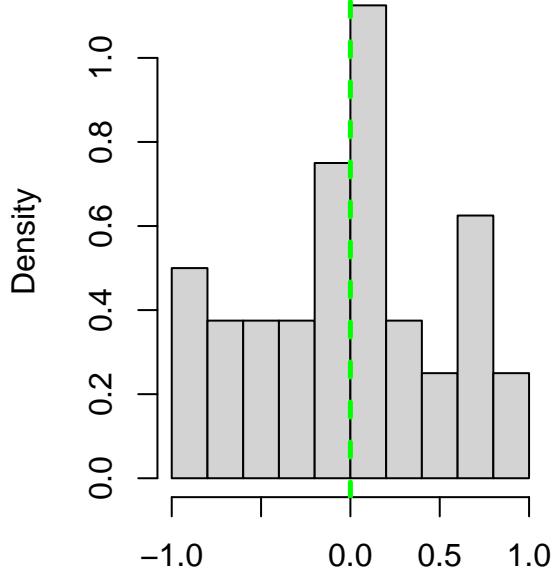
Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[1]	0.022	0.016
theta[7]	0.008	0.024
theta[2]	0.011	0.012
theta[5]	0.010	0.007
total	0.013	0.015

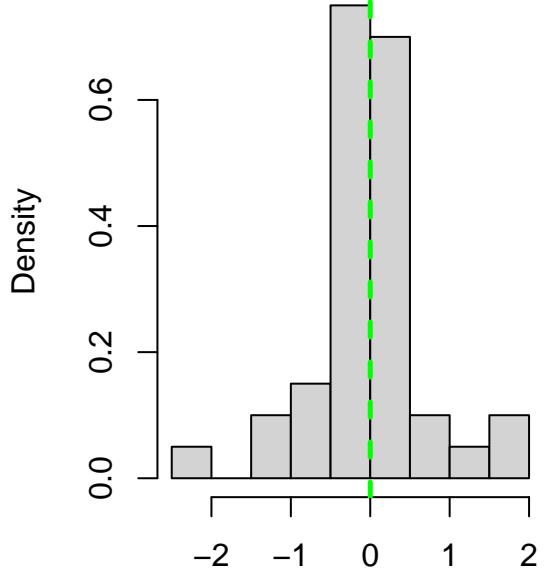
Boxplots



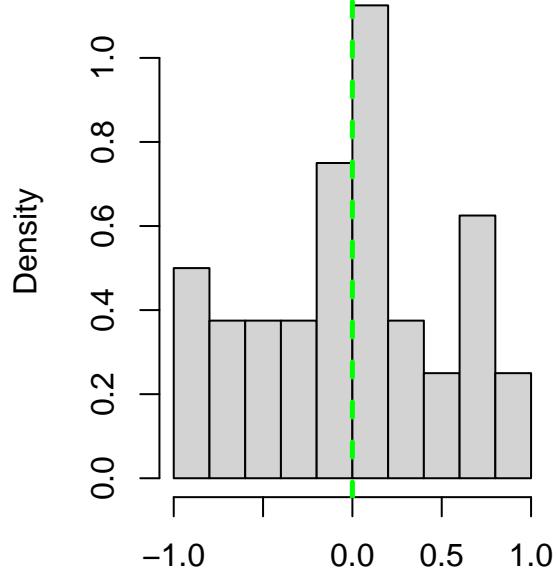
Histogram of proposed estimates for $\theta[2]=0$



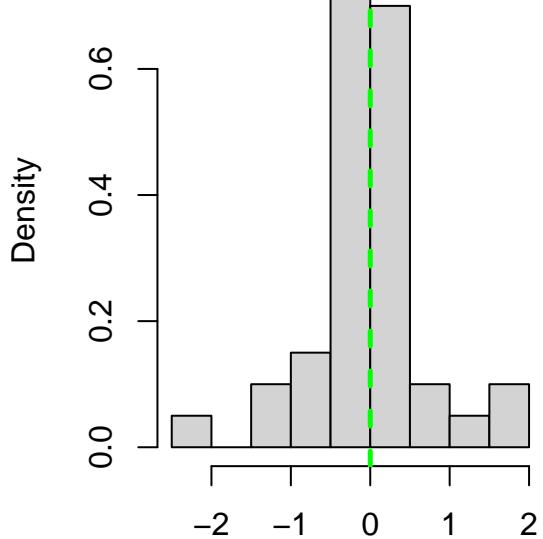
Histogram of cgm estimates for $\theta[2]=0$



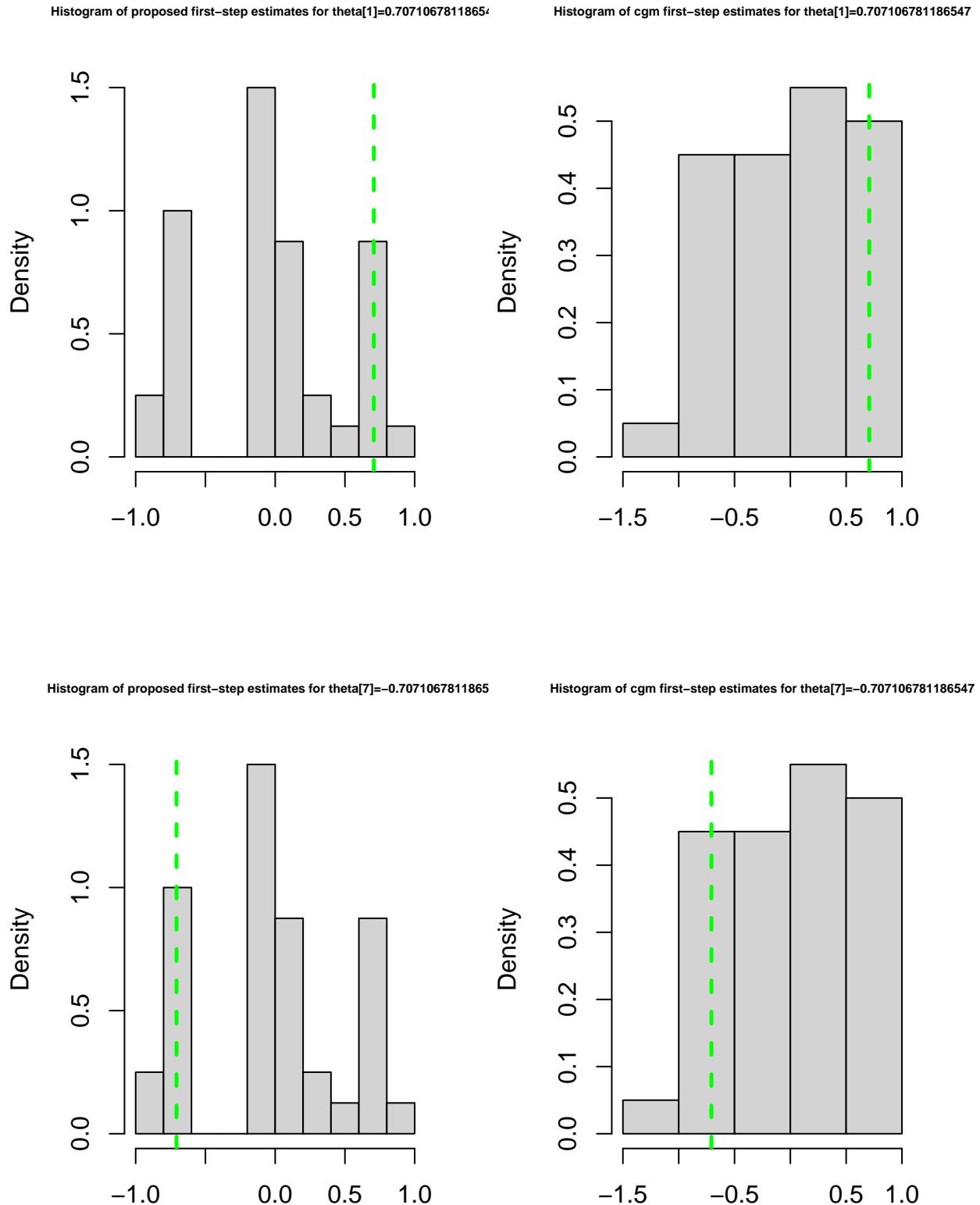
Histogram of proposed estimates for $\theta[5]=0$



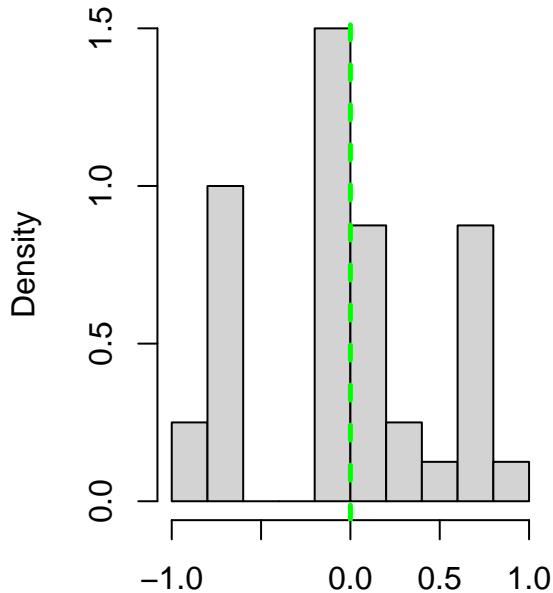
Histogram of cgm estimates for $\theta[5]=0$



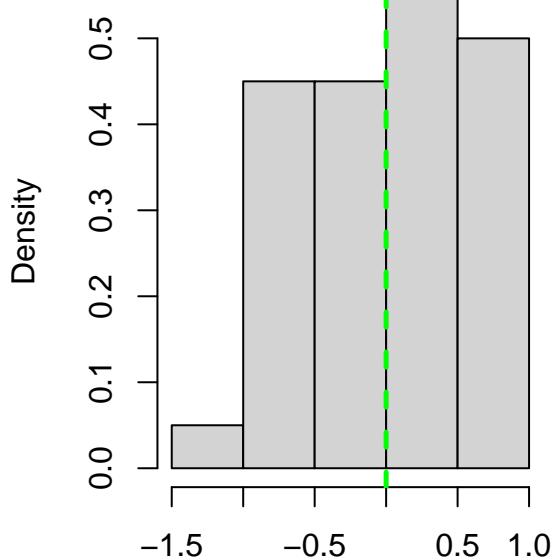
First Step Histograms



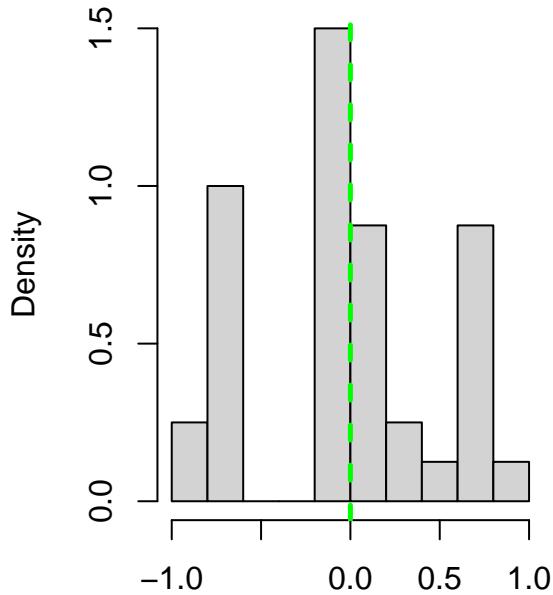
Histogram of proposed first-step estimates for $\theta[2]=0$



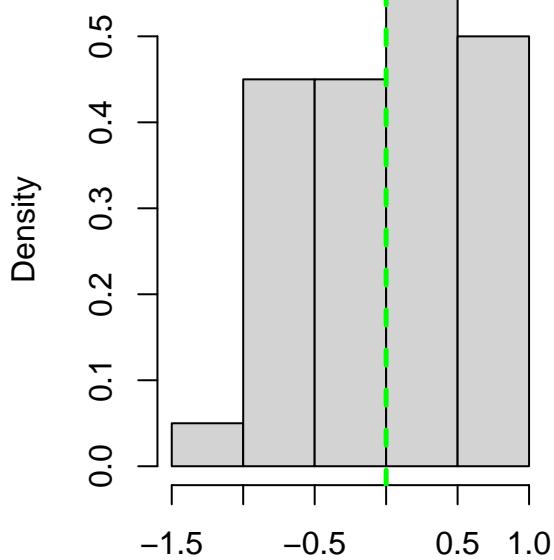
Histogram of cgm first-step estimates for $\theta[2]=0$



Histogram of proposed first-step estimates for $\theta[5]=0$



Histogram of cgm first-step estimates for $\theta[5]=0$



Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 3: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[1]	0.371	0.671	0.927	0.401	0.904
theta[7]	-0.912	-0.769	-0.372	-0.905	-0.409
theta[2]	-0.406	0.001	0.239	-0.372	0.229
theta[5]	-0.278	0.014	0.242	-0.245	0.228

Table 4: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[1]	-2.077	-0.537	0.098	-1.940	0.087
theta[7]	-0.074	0.628	1.926	-0.064	1.861
theta[2]	-0.580	-0.011	0.248	-0.503	0.231
theta[5]	-0.285	-0.009	0.127	-0.247	0.121

Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[1]	0.663	0.164	0.341	0.985	0.9
theta[7]	-0.718	0.170	-1.051	-0.385	1.0
theta[2]	-0.008	0.134	-0.271	0.254	0.7
theta[5]	0.015	0.131	-0.241	0.271	0.9

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[1]	-0.656	0.132	-0.916	-0.396	0.0
theta[7]	0.699	0.139	0.427	0.971	0.0
theta[2]	-0.039	0.063	-0.162	0.085	0.4
theta[5]	-0.029	0.061	-0.149	0.091	0.8