

## Simulation Results

2026-01-13

## Simulation Setup

This simulation is performed with  $n = 200$  and  $d = 100$ , using the 2-d lattice as the underlying graph.  $s = 10$  parameters are set to be nonzero, and the beta parameter is chosen to be  $\beta = 0.4$ . The attached results are for a 10-replication simulation. The true values of the parameter vector  $\theta$  are

but for brevity, our simulation only estimates the indices of  $\theta$  in  $\mathcal{C} = \{1, 11, 76, 45\}$  elements of  $\theta$ . Accordingly, all statistics and visuals are indicative of performance only on the set  $\mathcal{C}$ .

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

## Results

Mean-squared error comparison ( $\frac{1}{n.sim} \sum_{i=1}^{n.sim} \frac{1}{|C|} \|\hat{\theta}_{i,C} - \theta_C\|^2$ )

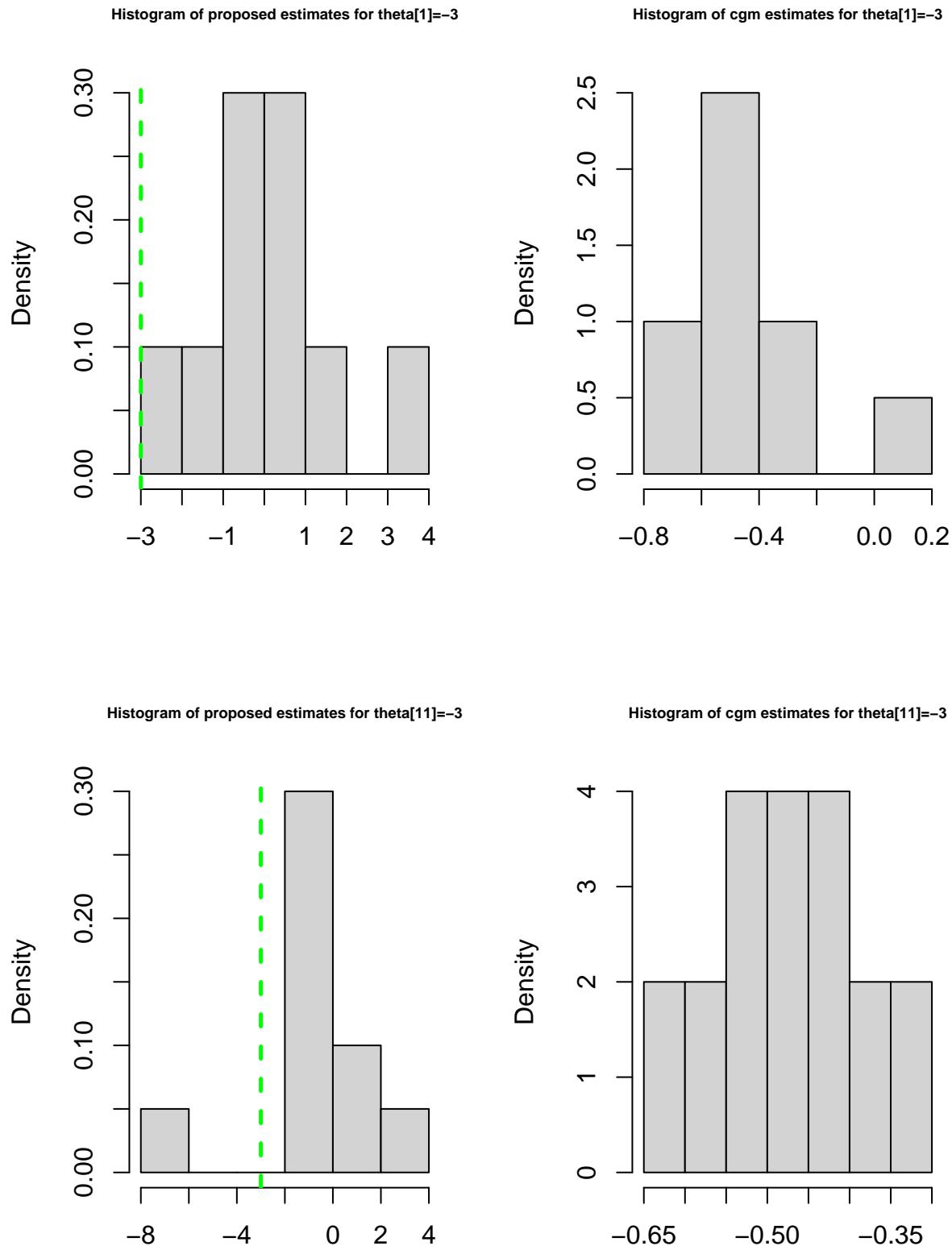
Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[1]	11.590	6.573
theta[11]	10.225	6.423
theta[76]	6.710	0.004
theta[45]	7.034	0.011
total	8.890	3.253

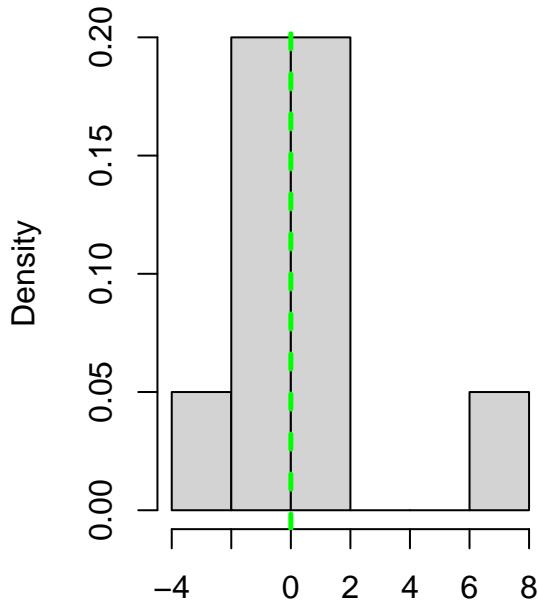
Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[1]	7.595	7.113
theta[11]	7.396	7.131
theta[76]	0.004	0.001
theta[45]	0.007	0.001
total	3.750	3.562

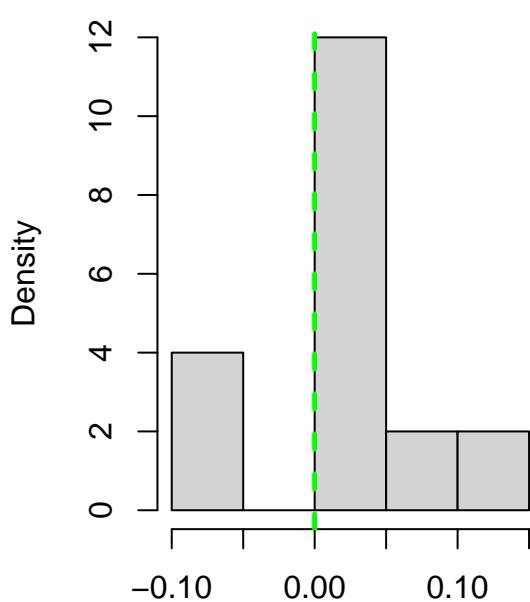
## Boxplots



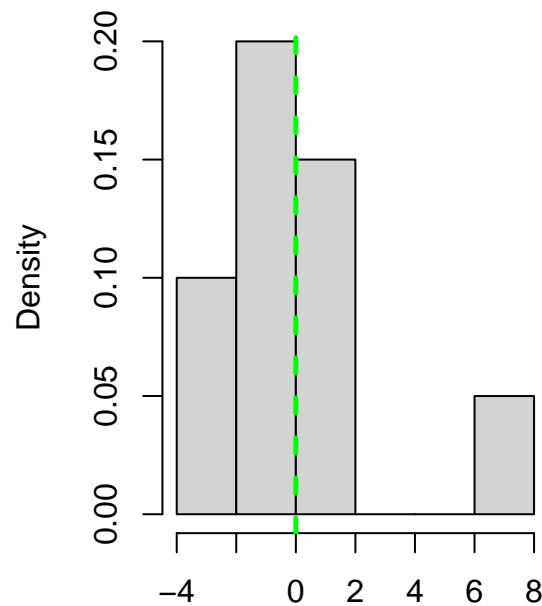
Histogram of proposed estimates for theta[76]=0



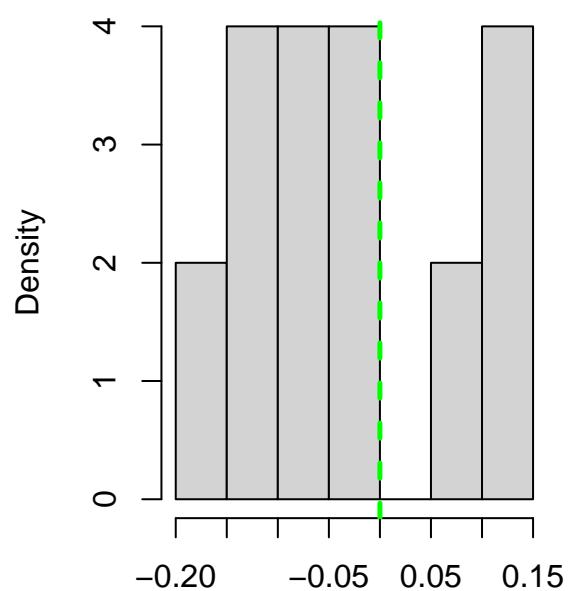
Histogram of cgm estimates for theta[76]=0



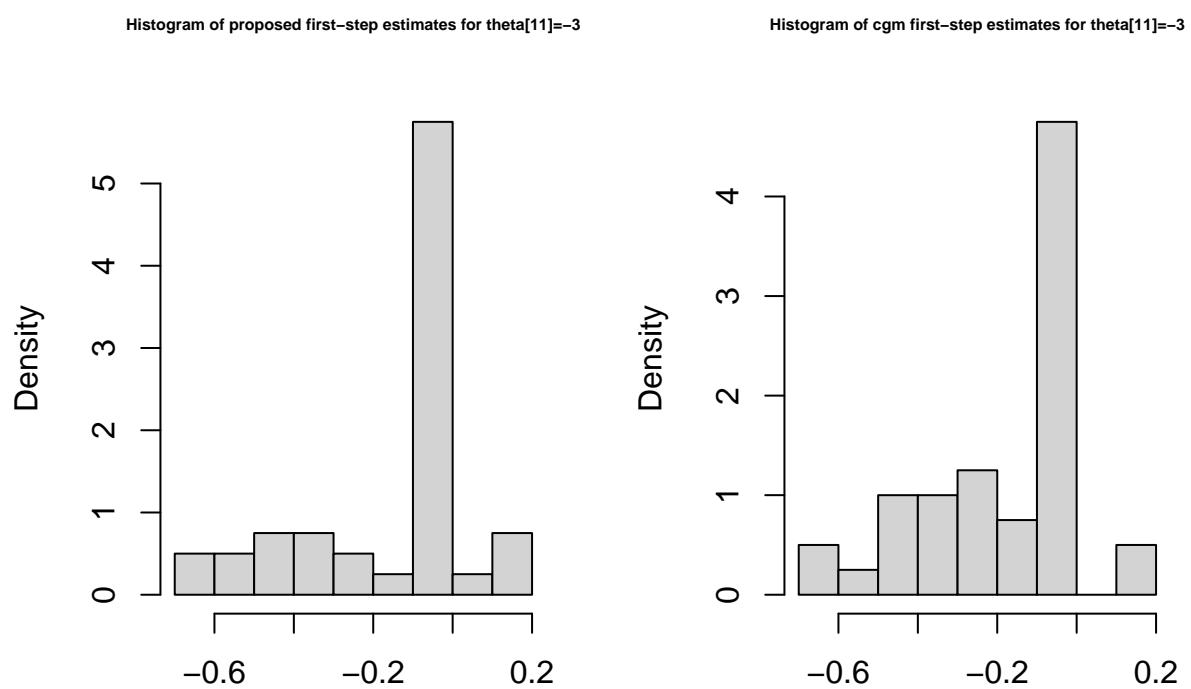
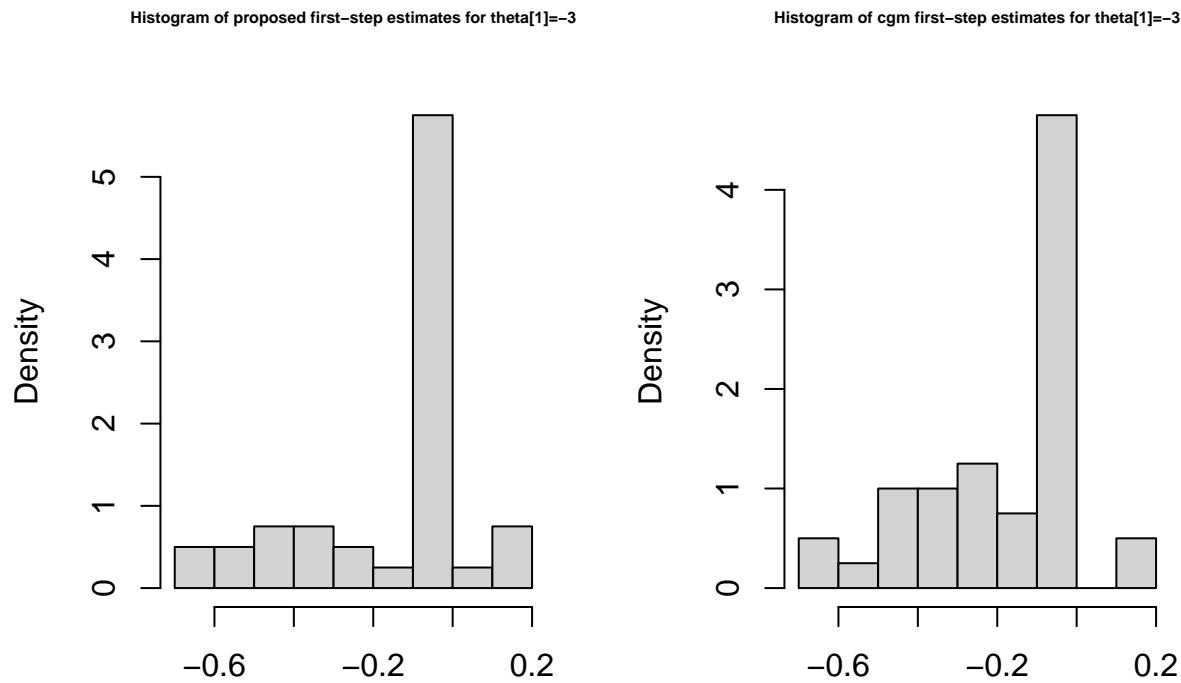
Histogram of proposed estimates for theta[45]=0



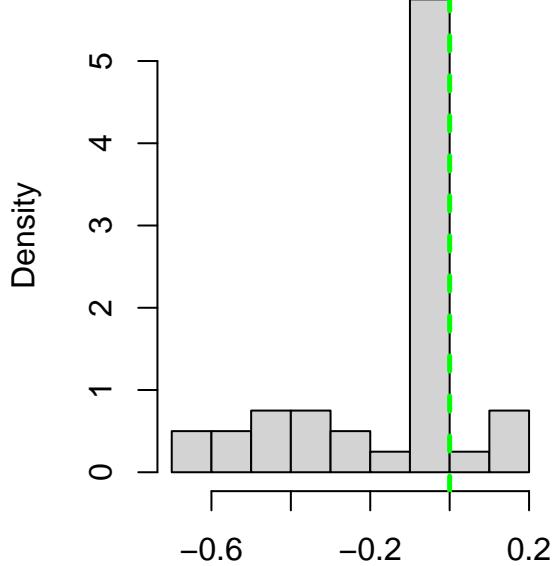
Histogram of cgm estimates for theta[45]=0



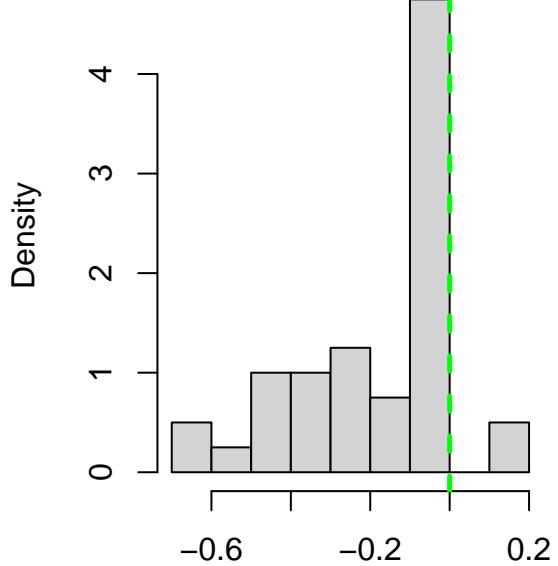
## First Step Histograms



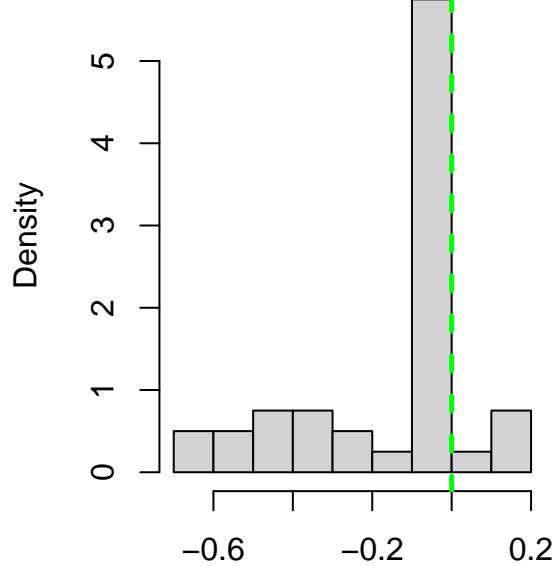
Histogram of proposed first-step estimates for theta[76]=0



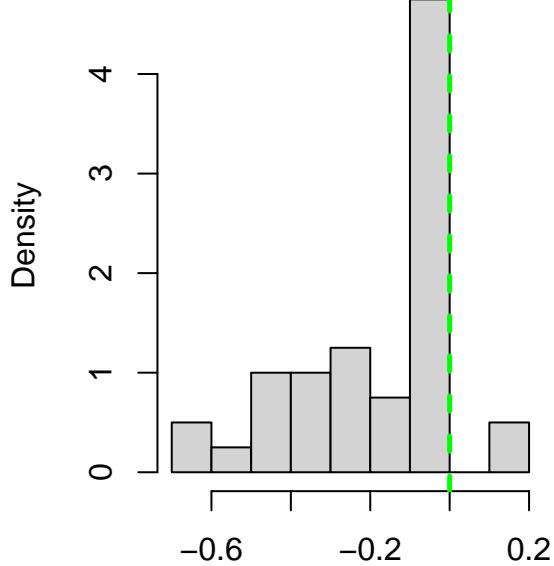
Histogram of cgm first-step estimates for theta[76]=0



Histogram of proposed first-step estimates for  $\theta[45]=0$



Histogram of cgm first-step estimates for  $\theta[45]=0$



### Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 3: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[1]	-2.634	-0.117	3.896	-2.354	3.263
theta[11]	-6.350	-0.220	2.521	-5.142	2.011
theta[76]	-3.601	0.016	7.252	-2.975	5.803
theta[45]	-3.418	-0.561	6.765	-3.309	5.442

Table 4: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[1]	-0.651	-0.504	0.054	-0.644	-0.024
theta[11]	-0.620	-0.467	-0.314	-0.607	-0.326
theta[76]	-0.098	0.014	0.109	-0.096	0.103
theta[45]	-0.187	-0.032	0.139	-0.173	0.132

### Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[1]	0.004	0.313	-0.609	0.617	0.1
theta[11]	-0.600	0.315	-1.218	0.017	0.0
theta[76]	0.361	0.594	-0.804	1.526	0.6
theta[45]	-0.171	0.594	-1.336	0.993	0.4

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[1]	-0.444	0.147	-0.732	-0.156	0
theta[11]	-0.467	0.132	-0.725	-0.209	0
theta[76]	0.011	0.135	-0.253	0.275	1
theta[45]	-0.026	0.134	-0.289	0.237	1