Simulation Results

2026-01-26

Simulation Setup

This simulation is performed with $n = 200$ and $d = 200$, using the 2-d lattice as the underlying graph. $s = 5$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.2$. The attached results are for a 10-replication simulation. The parameter vector θ has sparse components other than the following:

Parameter.Index	Value
7	-0.447
72	0.447
81	0.447
90	0.447
138	-0.447

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{$ 7, 72, 69, 172$\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[7]	0.049	0.037
theta[72]	0.017	0.006
theta[69]	0.014	0.011
theta[172]	0.010	0.017
total	0.023	0.018

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[7]	0.137	0.064

	proposed	cgm
theta[72]	0.146	0.039
theta[69]	0.000	0.000
theta[172]	0.001	0.000
total	0.071	0.026

Mean absolute deviation comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat

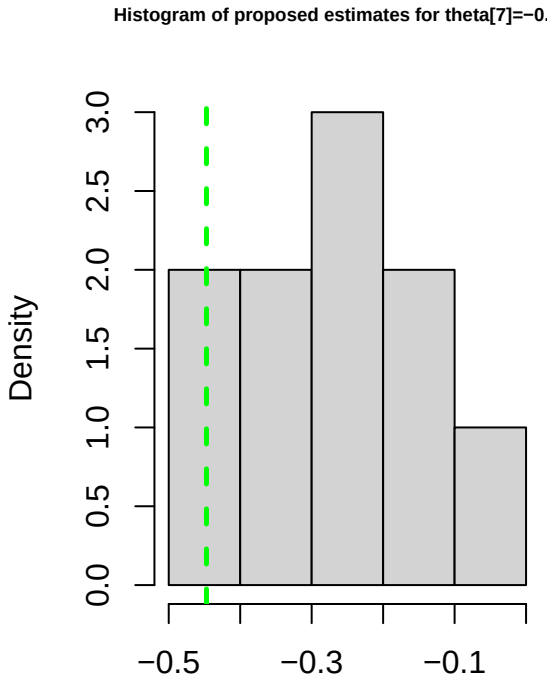
Table 3: Mean Absolute Deviation of Parameter Estimates

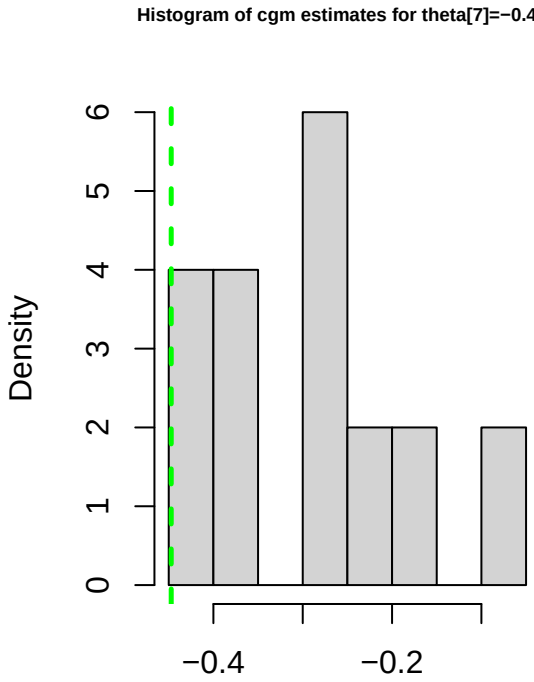
	proposed	cgm
theta[7]	0.186	0.165
theta[72]	0.105	0.064
theta[69]	0.088	0.087
theta[172]	0.089	0.107
total	0.117	0.106

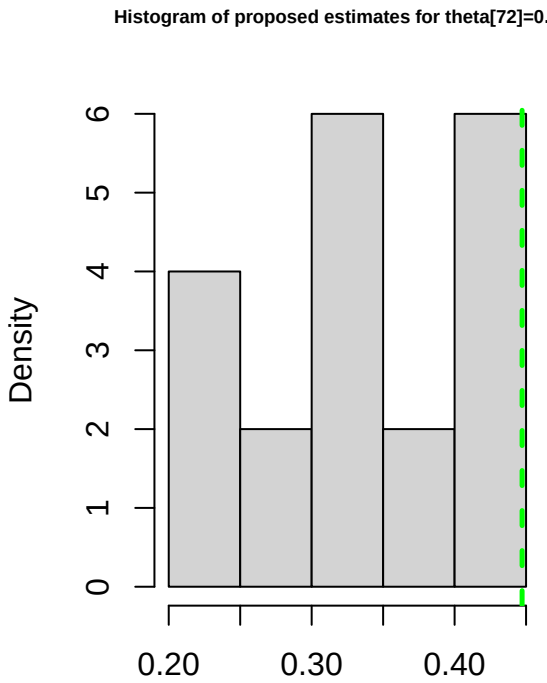
Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

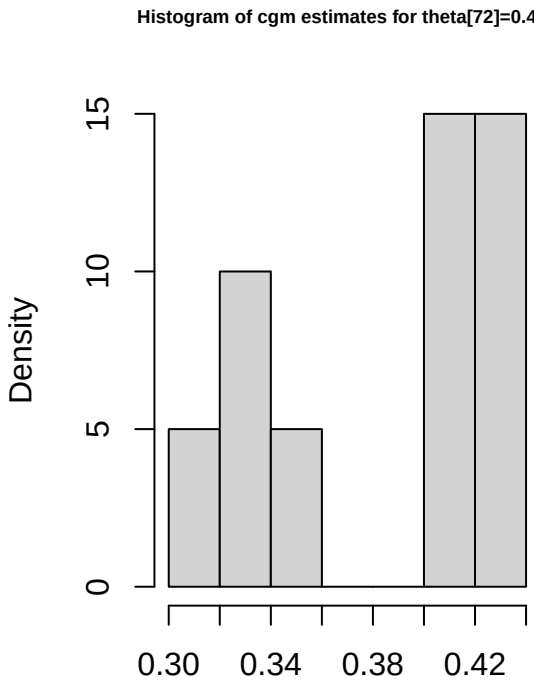
	proposed	cgm
theta[7]	0.357	0.217
theta[72]	0.371	0.175
theta[69]	0.000	0.000
theta[172]	0.007	0.000
total	0.184	0.098

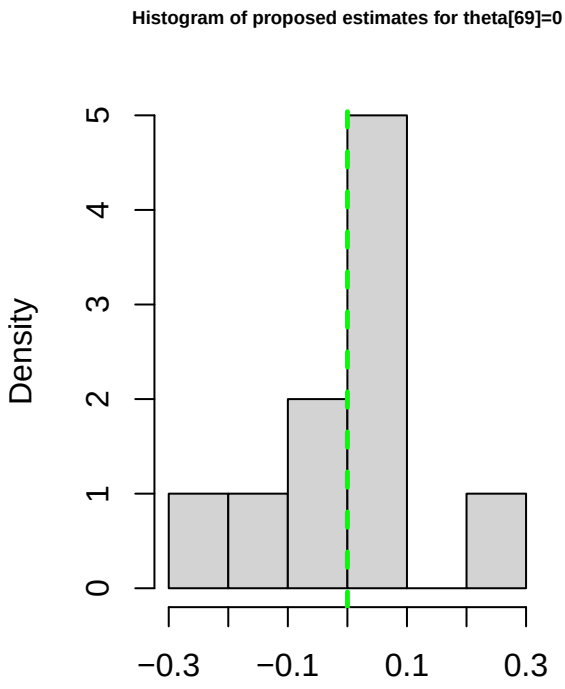
Boxplots

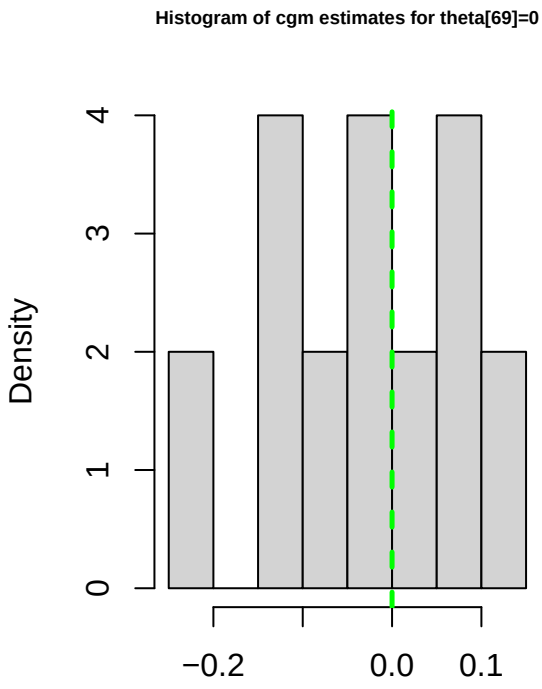

Histogram of proposed estimates for theta[7]=−0.447
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.5
−0.3
−0.1

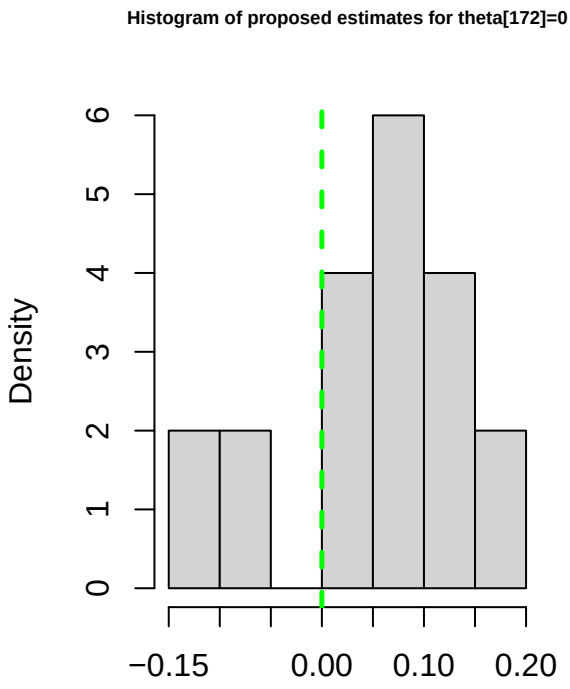

Histogram of cgm estimates for theta[7]=−0.447
Density
0
1
2
3
4
5
6
−0.4
−0.2

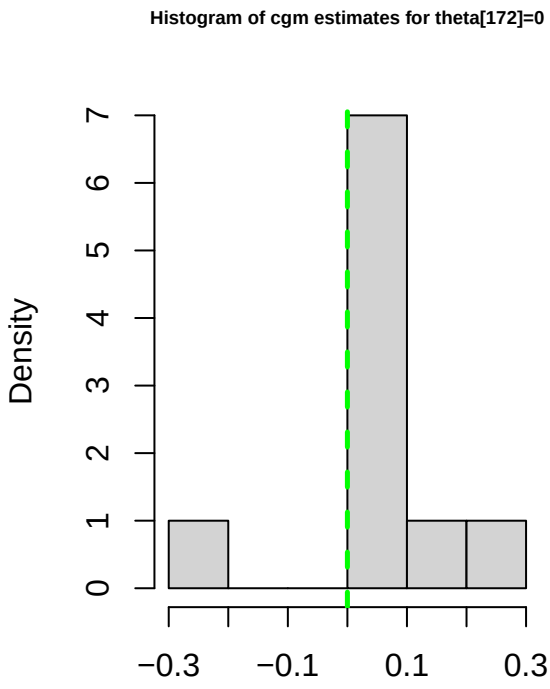

Histogram of proposed estimates for theta[72]=0.447
Density
0
1
2
3
4
5
6
0.20
0.30
0.40


Histogram of cgm estimates for theta[72]=0.447
Density
0
5
10
15
0.30
0.34
0.38
0.42


Histogram of proposed estimates for theta[69]=0
Density
0
1
2
3
4
5
−0.3
−0.1
0.1
0.3


Histogram of cgm estimates for theta[69]=0
Density
0
1
2
3
4
−0.2
0.0
0.1


Histogram of proposed estimates for theta[172]=0
Density
0
1
2
3
4
5
6
−0.15
0.00
0.10
0.20


Histogram of cgm estimates for theta[172]=0
Density
0
1
2
3
4
5
6
7
−0.3
−0.1
0.1
0.3

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[7]	-0.261	0.116	-0.489	-0.034	0.7
theta[72]	0.342	0.117	0.112	0.572	0.9
theta[69]	0.008	0.118	-0.223	0.239	0.9
theta[172]	0.052	0.109	-0.162	0.266	1.0

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[7]	-0.282	0.122	-0.520	-0.043	0.8
theta[72]	0.383	0.113	0.162	0.605	1.0
theta[69]	-0.033	0.121	-0.270	0.204	0.9
theta[172]	0.060	0.114	-0.164	0.284	1.0