Simulation Results

2026-01-13

Simulation Setup

This simulation is performed with $n = 500$ and $d = 10$, using the 2-d lattice as the underlying graph. $s = 2$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.2$. The attached results are for a 10-replication simulation. The true values of the parameter vector θ are

```
0 0 0 -3 0 0 0 0 -3 0 ,
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but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{$ 4, 9, 1, 5$\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison ($\frac{1}{n.sim}\sum_{i=1}^{n.sim}\frac{1}{|\mathcal{C}|}\|\hat{\theta}_{i,\mathcal{C}} - \theta_{\mathcal{C}}\|^2$)

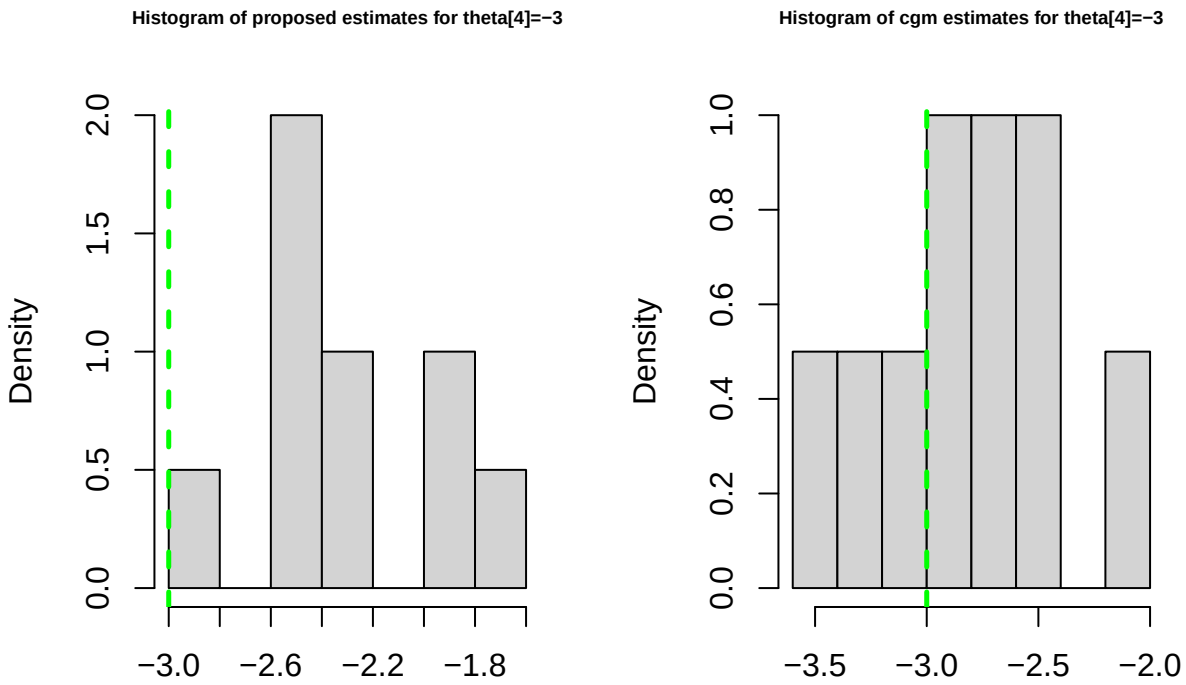
Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[4]	0.606	0.173
theta[9]	0.707	0.193
theta[1]	0.019	0.024
theta[5]	0.018	0.009
total	0.337	0.100

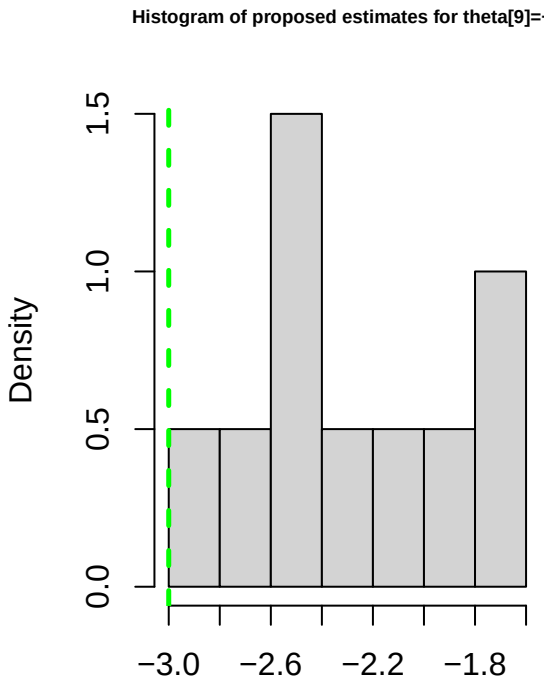
Table 2: Mean-Squared Error of First-Step Parameter Estimates

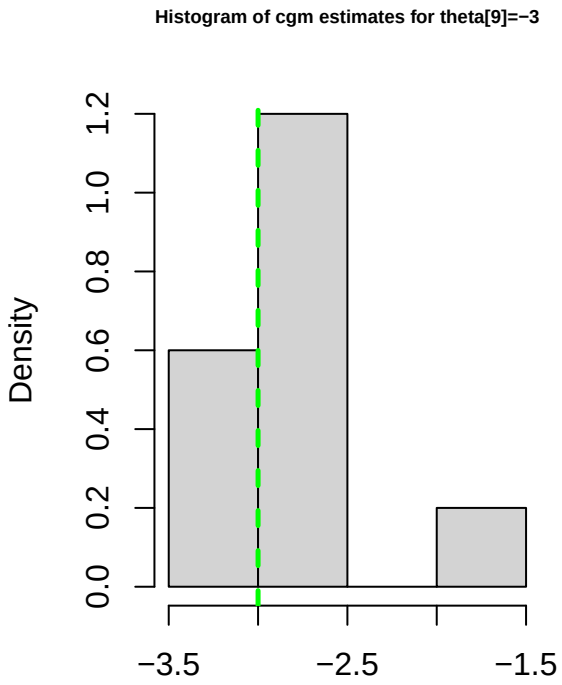
	proposed	cgm
theta[4]	1.569	0.248
theta[9]	1.538	0.230
theta[1]	0.002	0.003
theta[5]	0.005	0.002
total	0.779	0.121

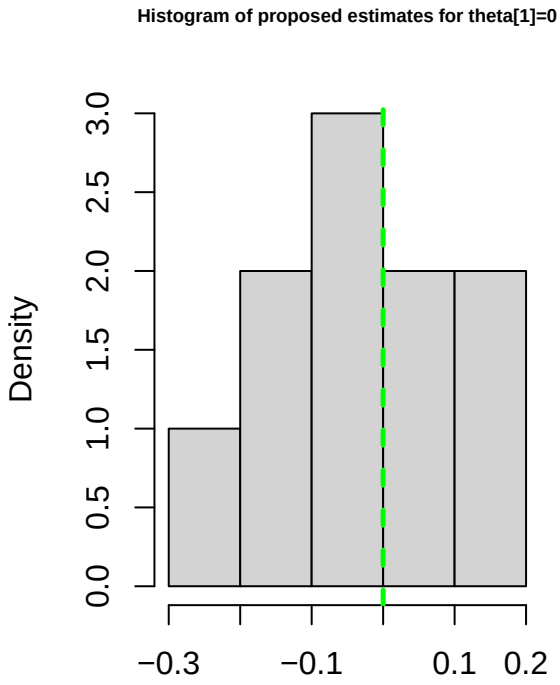
Boxplots

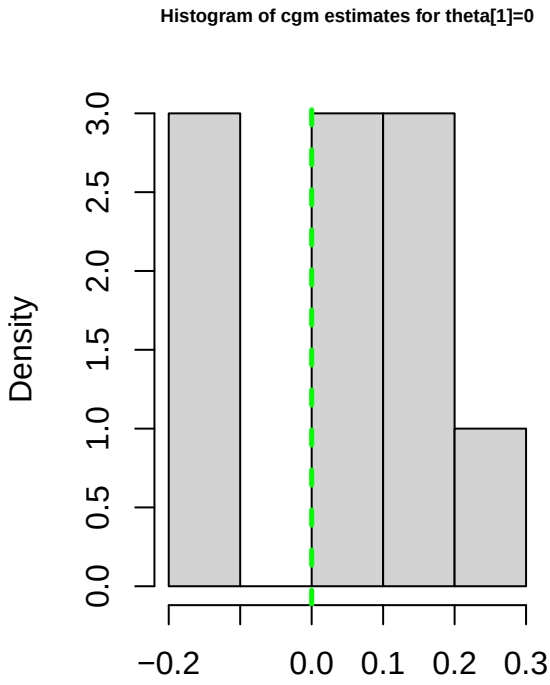

Histogram of proposed estimates for theta[4]=−3
Density
0.0
0.5
1.0
1.5
2.0
−3.0
−2.6
−2.2
−1.8

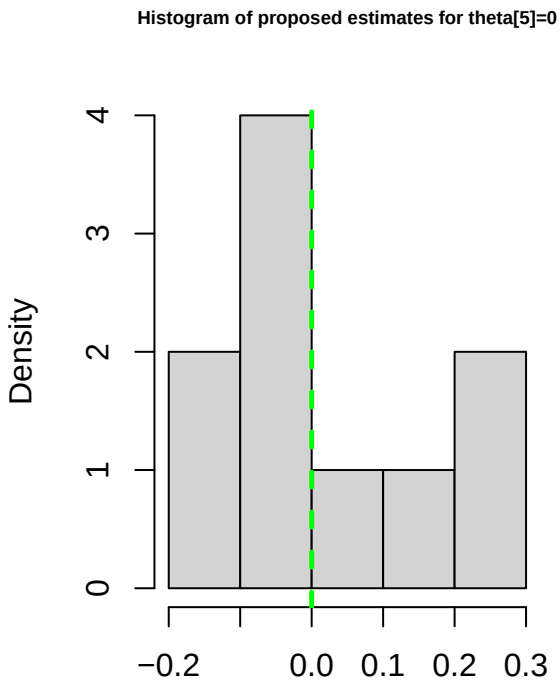
Histogram of cgm estimates for theta[4]=−3
Density
0.0
0.2
0.4
0.6
0.8
1.0
−3.5
−3.0
−2.5
−2.0

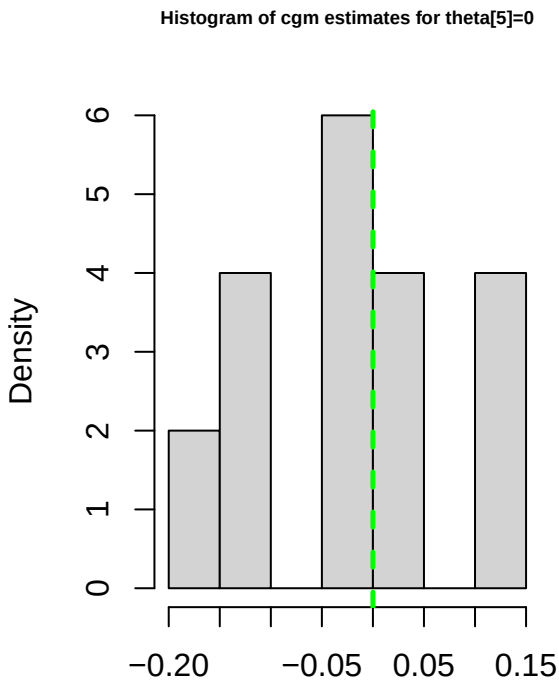

Histogram of proposed estimates for theta[9]=−3
Density
0.0
0.5
1.0
1.5
−3.0
−2.6
−2.2
−1.8


Histogram of cgm estimates for theta[9]=−3
Density
0.0
0.2
0.4
0.6
0.8
1.0
1.2
−3.5
−2.5
−1.5

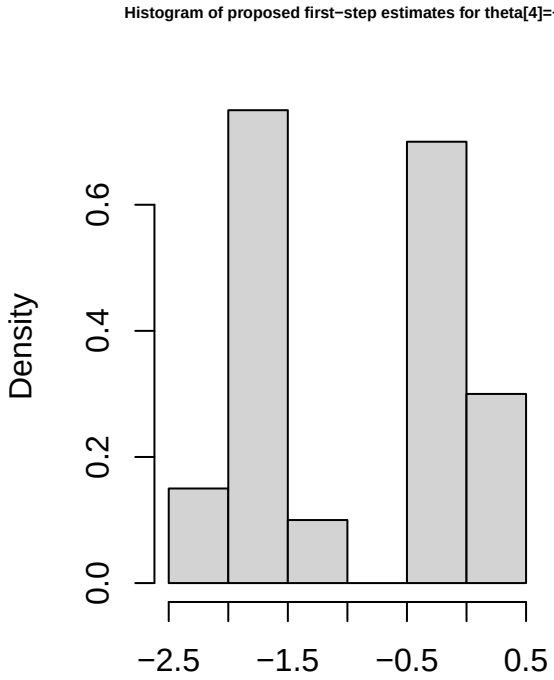

Histogram of proposed estimates for theta[1]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.3
−0.1
0.1
0.2

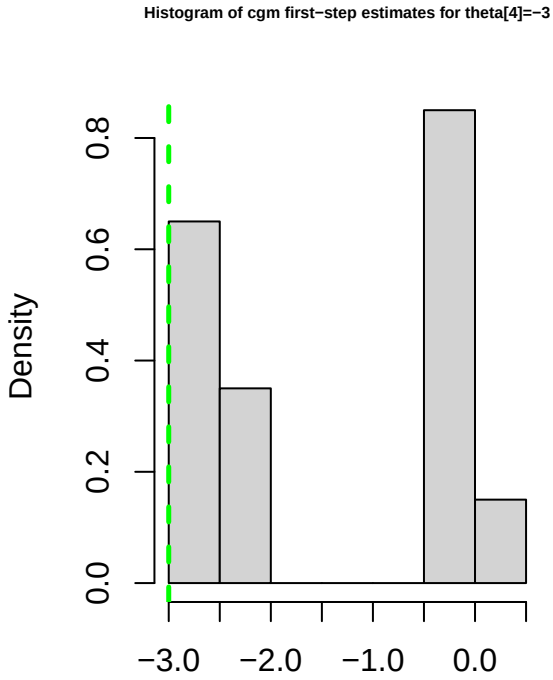

Histogram of cgm estimates for theta[1]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.2
0.0
0.1
0.2
0.3

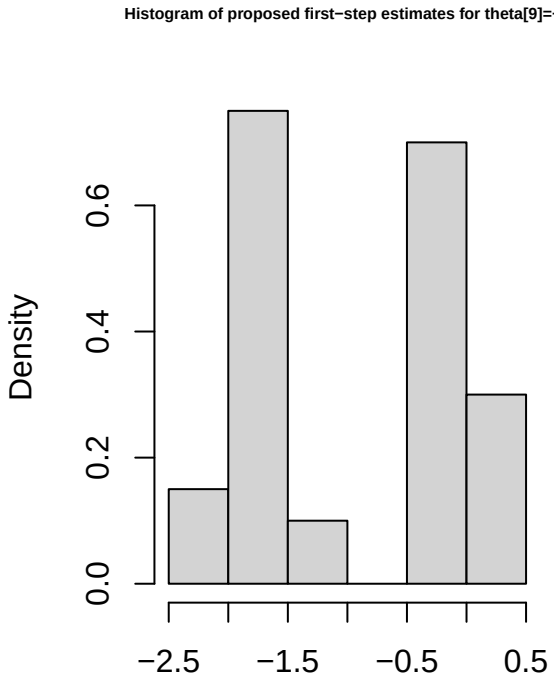

Histogram of proposed estimates for theta[5]=0
Density
0
1
2
3
4
−0.2
0.0
0.1
0.2
0.3

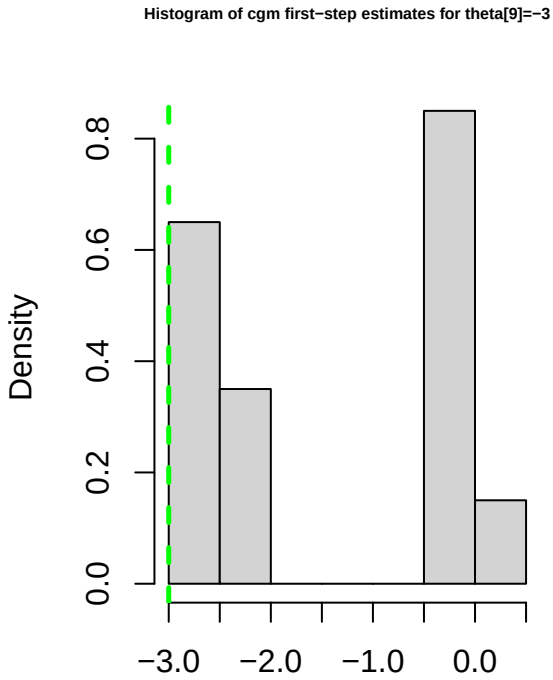

Histogram of cgm estimates for theta[5]=0
Density
0
1
2
3
4
5
6
−0.20
−0.05
0.05
0.15

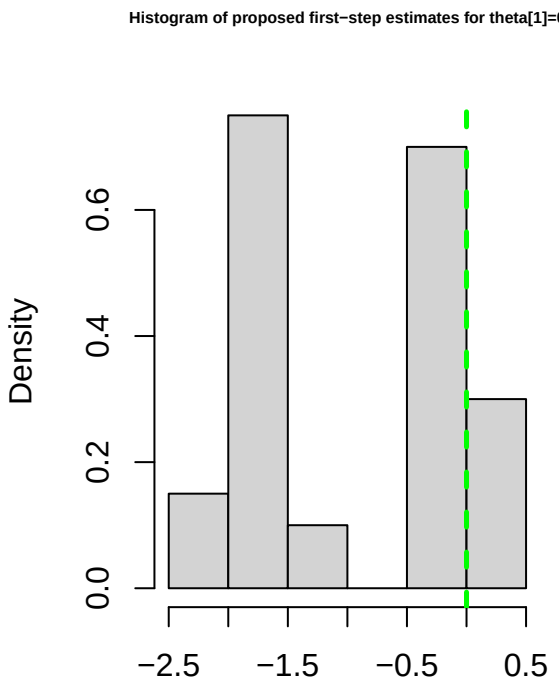
First Step Histograms

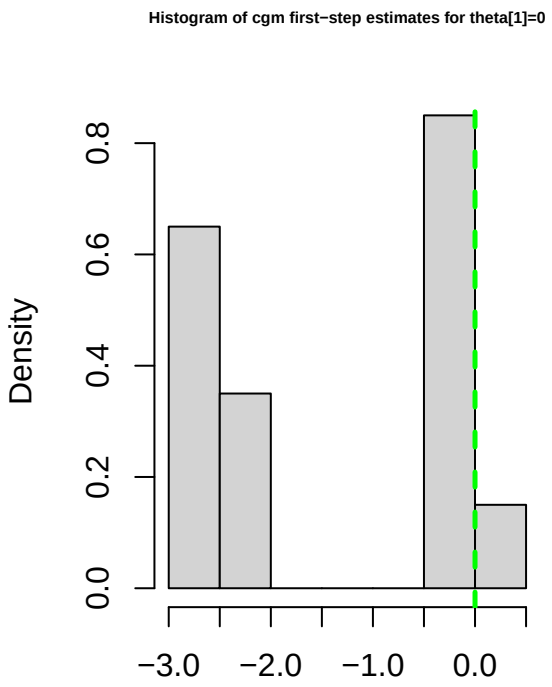


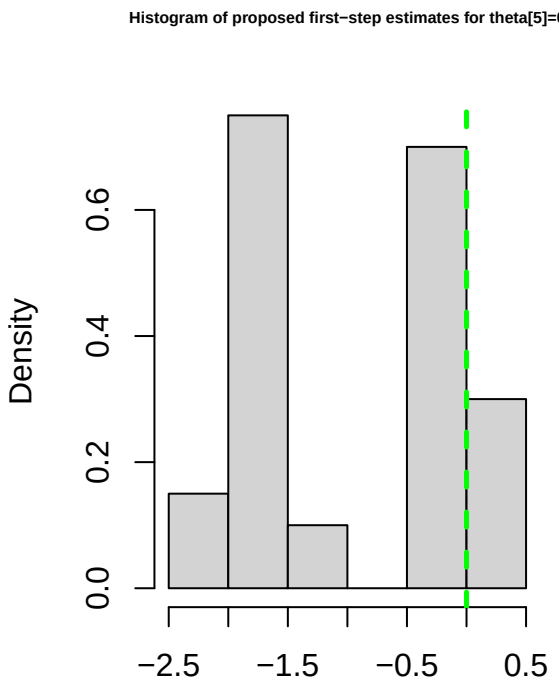


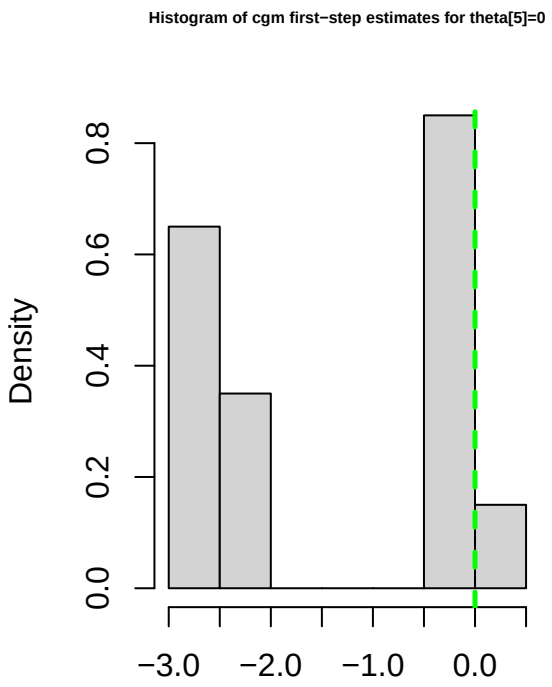





Histogram of proposed first−step estimates for theta[1]=0
Density
0.0
0.2
0.4
0.6
−2.5
−1.5
−0.5
0.5


Histogram of cgm first−step estimates for theta[1]=0
Density
0.0
0.2
0.4
0.6
0.8
−3.0
−2.0
−1.0
0.0


Histogram of proposed first−step estimates for theta[5]=0
Density
0.0
0.2
0.4
0.6
−2.5
−1.5
−0.5
0.5


Histogram of cgm first−step estimates for theta[5]=0
Density
0.0
0.2
0.4
0.6
0.8
−3.0
−2.0
−1.0
0.0

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 3: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[4]	-2.908	-2.415	-1.764	-2.815	-1.774
theta[9]	-2.820	-2.307	-1.681	-2.782	-1.708
theta[1]	-0.295	-0.046	0.153	-0.268	0.148
theta[5]	-0.154	-0.025	0.210	-0.147	0.209

Table 4: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[4]	-3.407	-2.794	-2.072	-3.362	-2.173
theta[9]	-3.354	-2.729	-1.916	-3.290	-2.070
theta[1]	-0.135	0.091	0.261	-0.135	0.244
theta[5]	-0.178	-0.025	0.108	-0.165	0.106

Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[4]	-2.300	0.404	-3.091	-1.509	0.6
theta[9]	-2.240	0.412	-3.047	-1.432	0.6
theta[1]	-0.043	0.169	-0.373	0.288	1.0
theta[5]	0.023	0.174	-0.318	0.365	1.0

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[4]	-2.799	0.456	-3.692	-1.905	0.9
theta[9]	-2.758	0.453	-3.646	-1.869	0.9
theta[1]	0.065	0.135	-0.201	0.330	1.0
theta[5]	-0.022	0.134	-0.286	0.241	1.0