

# Simulation Results

2026-01-20

## Simulation Setup

This simulation is performed with  $n = 400$  and  $d = 200$ , using the 2-d lattice as the underlying graph.  $s = 20$  parameters are set to be nonzero, and the beta parameter is chosen to be  $\beta = 0.2$ . The attached results are for a 10-replication simulation. The parameter vector  $\theta$  has sparse components other than the following:

Parameter.Index	Value
4	-0.224
8	0.224
13	-0.224
25	0.224
28	0.224
32	0.224
46	0.224
50	-0.224
53	-0.224
55	-0.224
58	-0.224
77	0.224
87	0.224
118	-0.224
127	0.224
129	0.224
146	0.224
167	0.224
180	0.224
195	-0.224

but for brevity, our simulation only estimates the indices of  $\theta$  in  $\mathcal{C} = \{4, 8, 80, 186\}$  elements of  $\theta$ . Accordingly, **all statistics and visuals are indicative of performance only on the set  $\mathcal{C}$ .**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

## Results

### Mean-squared error comparison

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[4]	0.015	0.015
theta[8]	0.021	0.008
theta[80]	0.006	0.007
theta[186]	0.007	0.008
total	0.012	0.010

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[4]	0.041	0.008
theta[8]	0.047	0.019
theta[80]	0.000	0.001
theta[186]	0.000	0.000
total	0.022	0.007

### Mean absolute deviation comparison  $\frac{1}{n.\text{sim}} \sum_{i=1}^{n.\text{sim}} \frac{1}{|\mathcal{C}|} \|\hat{\theta}_i - \theta\|$

Table 3: Mean Absolute Deviation of Parameter Estimates

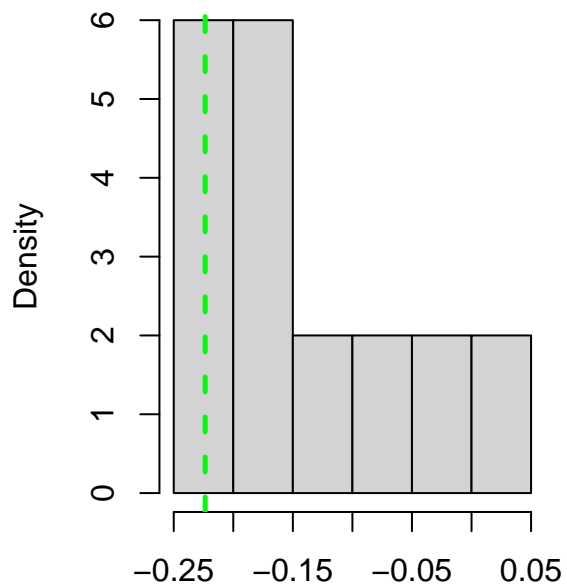
	proposed	cgm
theta[4]	0.091	0.106
theta[8]	0.125	0.081
theta[80]	0.064	0.070
theta[186]	0.060	0.068
total	0.085	0.081

Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

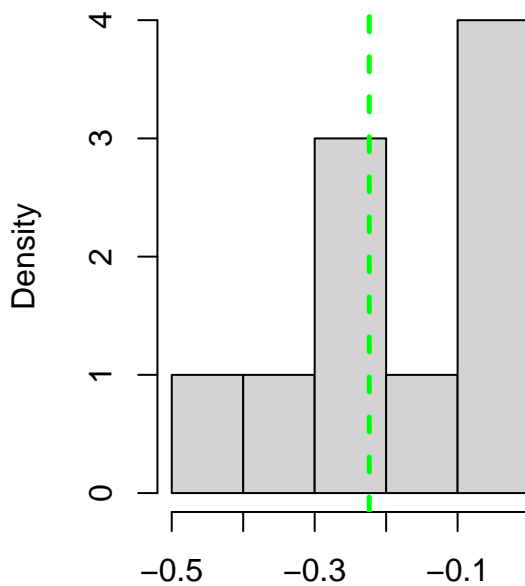
	proposed	cgm
theta[4]	0.200	0.062
theta[8]	0.216	0.120
theta[80]	0.000	0.014
theta[186]	0.001	0.003
total	0.104	0.049

## Boxplots

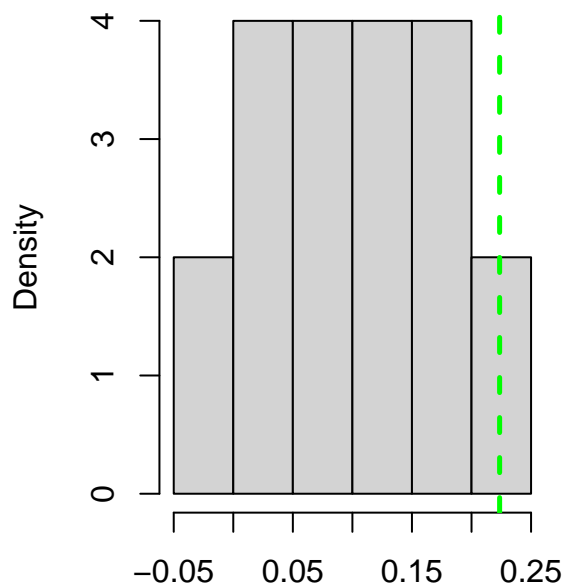
Histogram of proposed estimates for  $\theta[4]=-0.224$



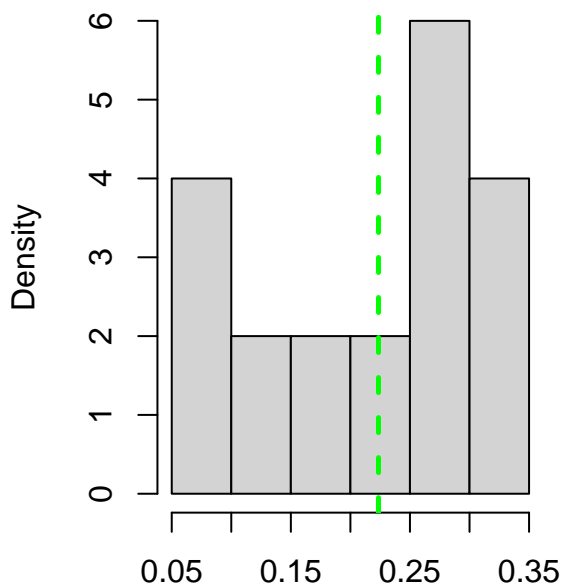
Histogram of cgm estimates for  $\theta[4]=-0.224$



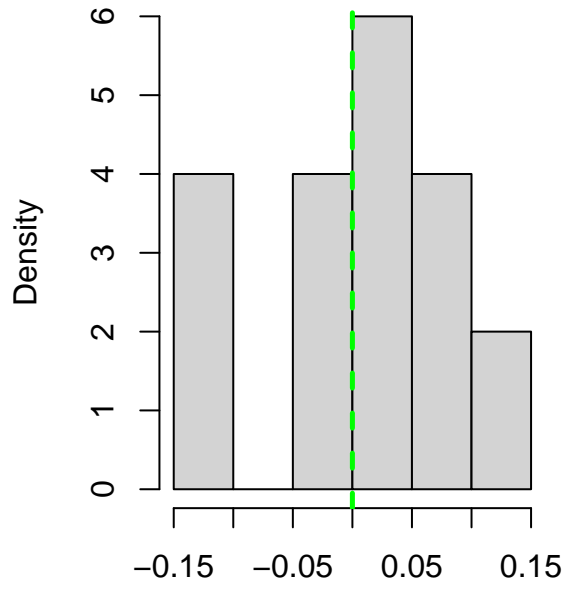
Histogram of proposed estimates for  $\theta[8]=0.224$



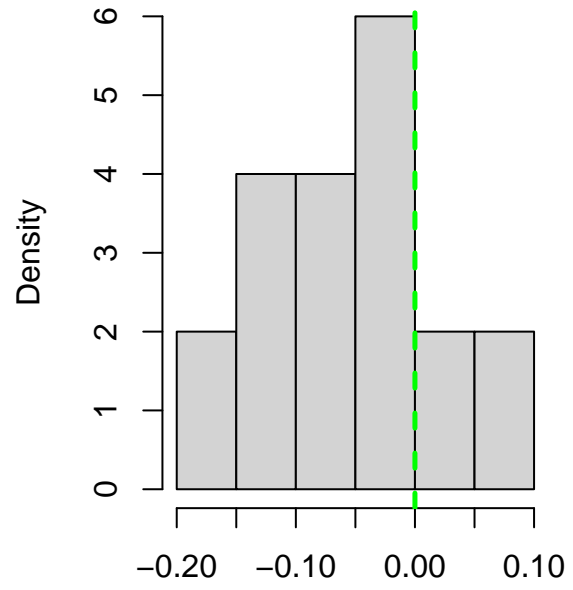
Histogram of cgm estimates for  $\theta[8]=0.224$



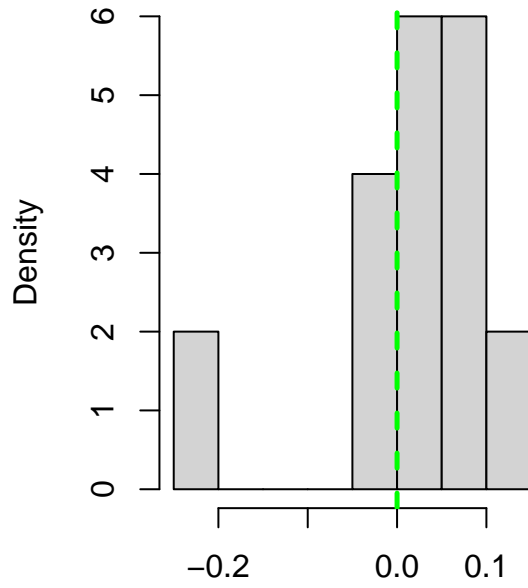
Histogram of proposed estimates for  $\theta_{[80]}=0$



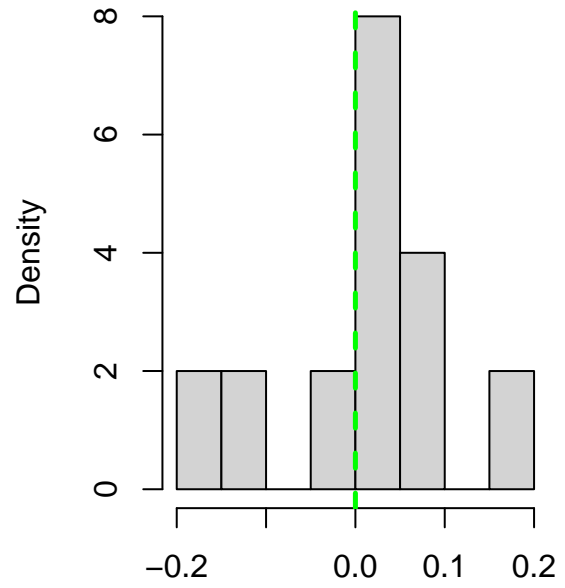
Histogram of cgm estimates for  $\theta_{[80]}=0$



Histogram of proposed estimates for  $\theta_{186}=0$



Histogram of cgm estimates for  $\theta_{186}=0$



## Statistics and 95% Confidence Intervals from per-Replicate Estimates

### Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[4]	-0.135	0.099	-0.329	0.060	0.8
theta[8]	0.099	0.095	-0.087	0.285	0.7
theta[80]	0.012	0.093	-0.171	0.194	1.0
theta[186]	0.017	0.091	-0.162	0.195	0.9

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[4]	-0.189	0.087	-0.359	-0.018	1.0
theta[8]	0.219	0.089	0.045	0.394	1.0
theta[80]	-0.052	0.084	-0.216	0.112	1.0
theta[186]	0.010	0.086	-0.157	0.178	0.9