Simulation Results

2026-01-15

Simulation Setup

This simulation is performed with $n = 200$ and $d = 20$, using the 2-d lattice as the underlying graph. $s = 2$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0$. The attached results are for a 10-replication simulation. The true values of the parameter vector θ are

```
0 0 0 -0.7071068 0 0 0 0 0.7071068 0 0 0 0 0 0 0 0 0 0 0 ,
```

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{$ 4, 9, 5, 8$\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

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### Mean-squared error comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat\theta
```

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[4]	0.048	0.036
theta[9]	0.088	0.019
theta[5]	0.030	0.035
theta[8]	0.010	0.026
total	0.044	0.029

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[4]	0.171	0.140
theta[9]	0.212	0.072
theta[5]	0.000	0.004
theta[8]	0.001	0.007
total	0.096	0.056

Mean absolute deviation comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat

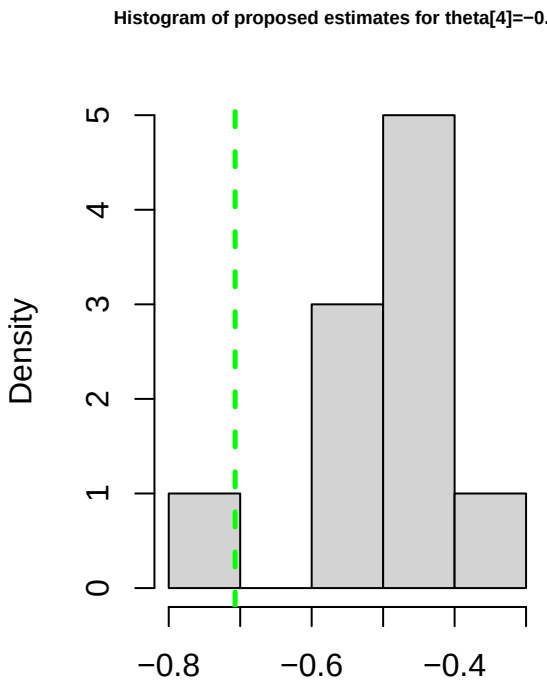
Table 3: Mean Absolute Deviation of Parameter Estimates

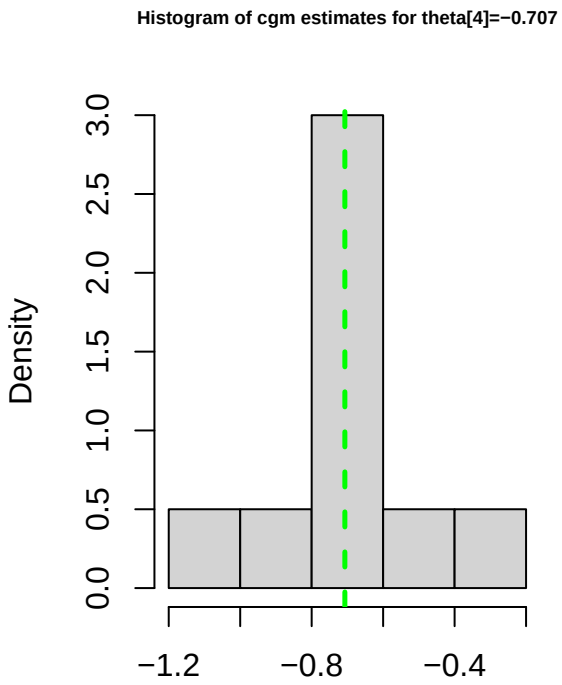
	proposed	cgm
theta[4]	0.203	0.145
theta[9]	0.278	0.122
theta[5]	0.153	0.162
theta[8]	0.078	0.118
total	0.178	0.137

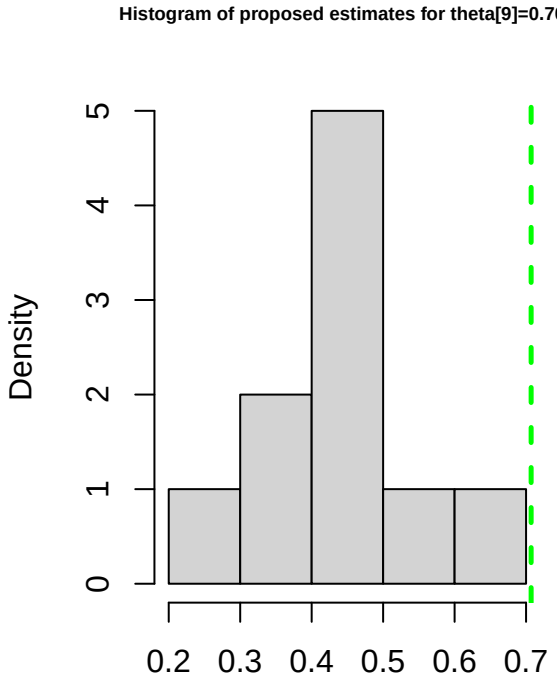
Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

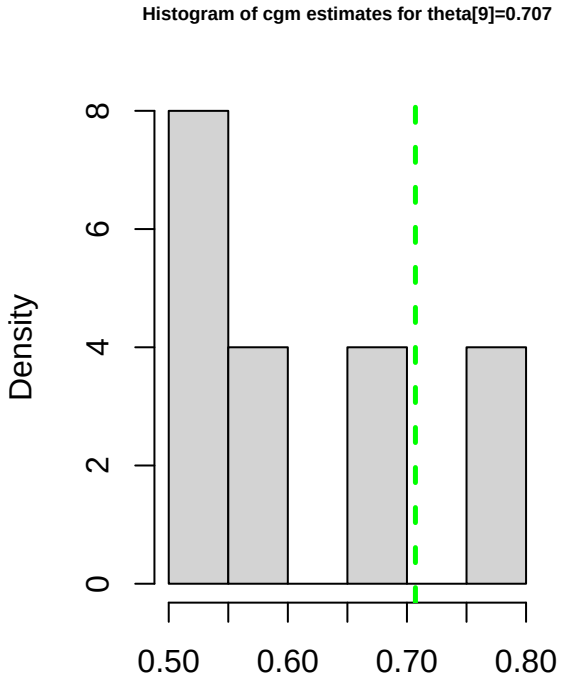
	proposed	cgm
theta[4]	0.407	0.325
theta[9]	0.436	0.246
theta[5]	0.002	0.041
theta[8]	0.017	0.052
total	0.216	0.166

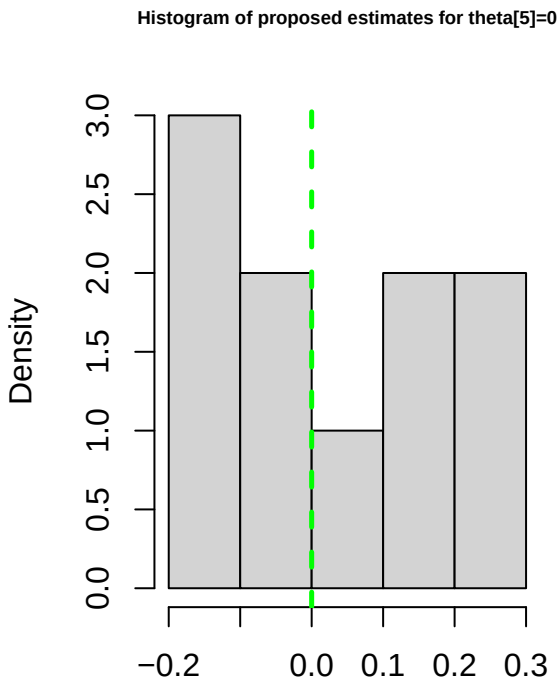
Boxplots

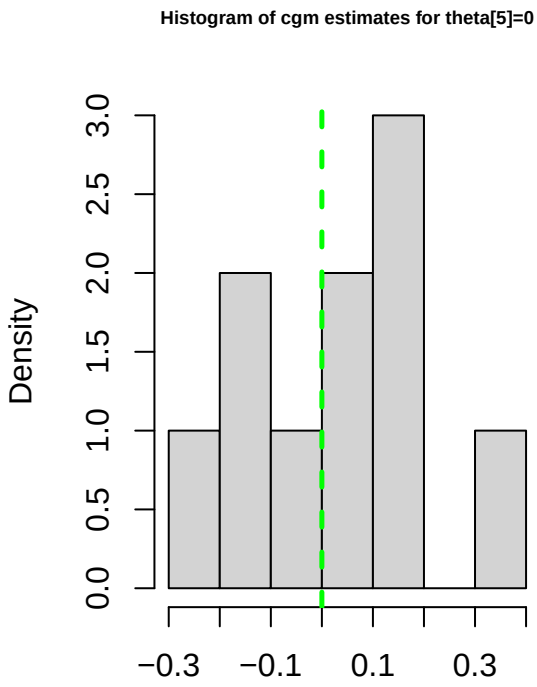

Histogram of proposed estimates for theta[4]=−0.707
Density
0
1
2
3
4
5
−0.8
−0.6
−0.4

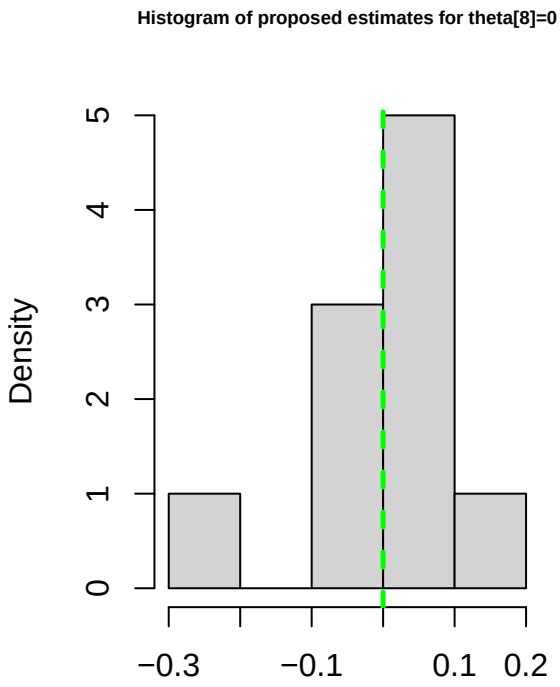

Histogram of cgm estimates for theta[4]=−0.707
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−1.2
−0.8
−0.4

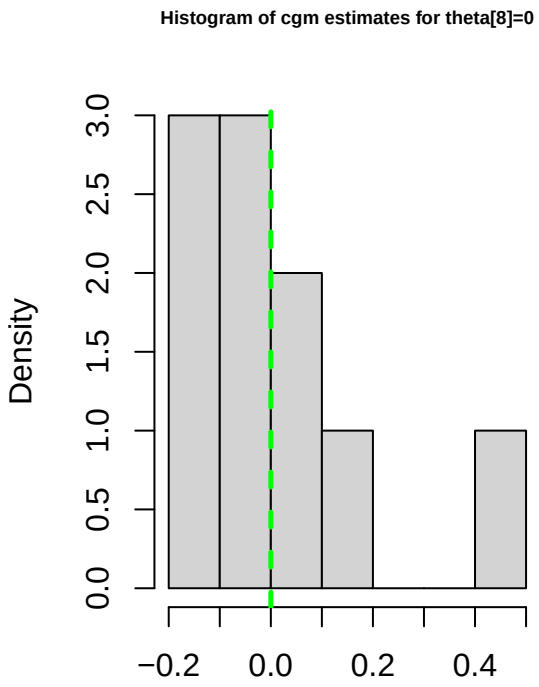

Histogram of proposed estimates for theta[9]=0.707
Density
0
1
2
3
4
5
0.2
0.3
0.4
0.5
0.6
0.7


Histogram of cgm estimates for theta[9]=0.707
Density
0
2
4
6
8
0.50
0.60
0.70
0.80

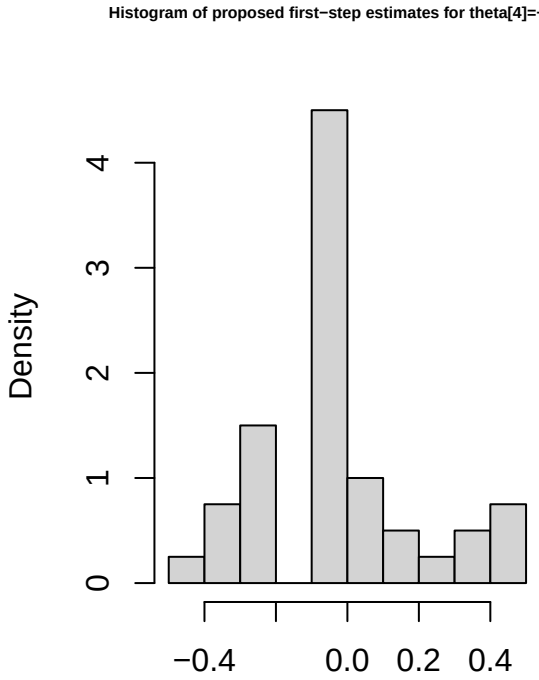

Histogram of proposed estimates for theta[5]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.2
0.0
0.1
0.2
0.3

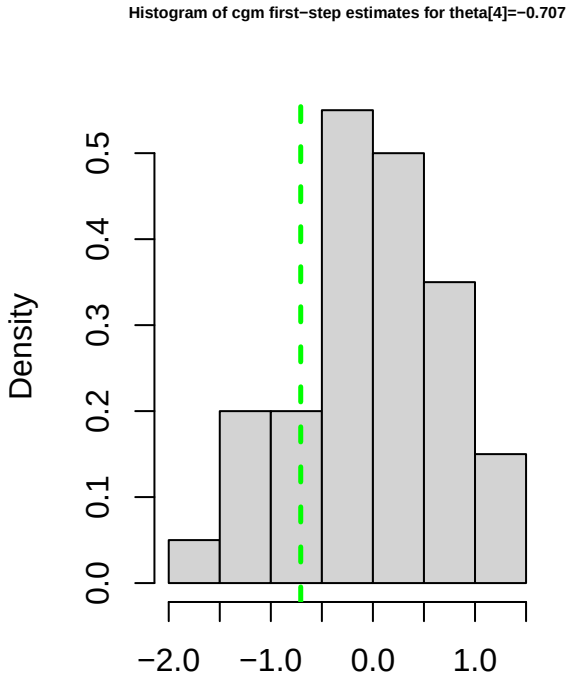

Histogram of cgm estimates for theta[5]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.3
−0.1
0.1
0.3

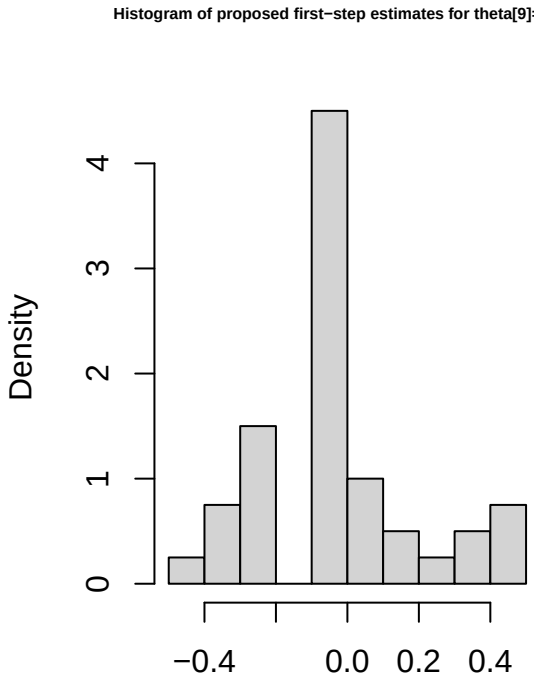

Histogram of proposed estimates for theta[8]=0
Density
0
1
2
3
4
5
−0.3
−0.1
0.1
0.2

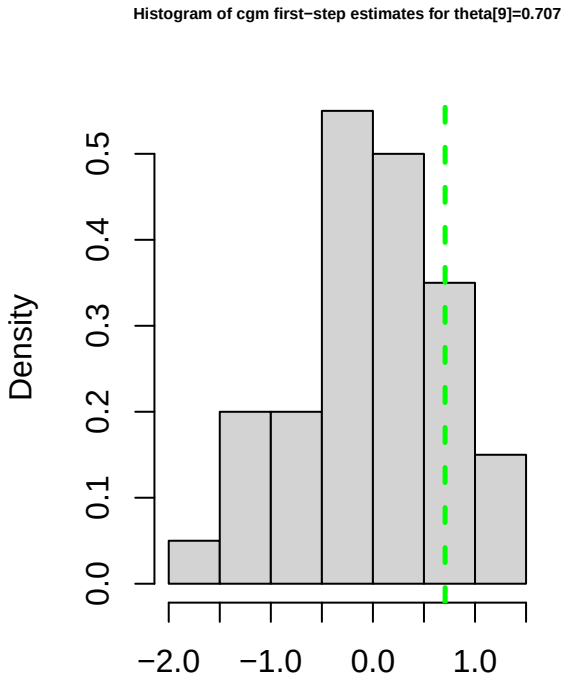

Histogram of cgm estimates for theta[8]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.2
0.0
0.2
0.4

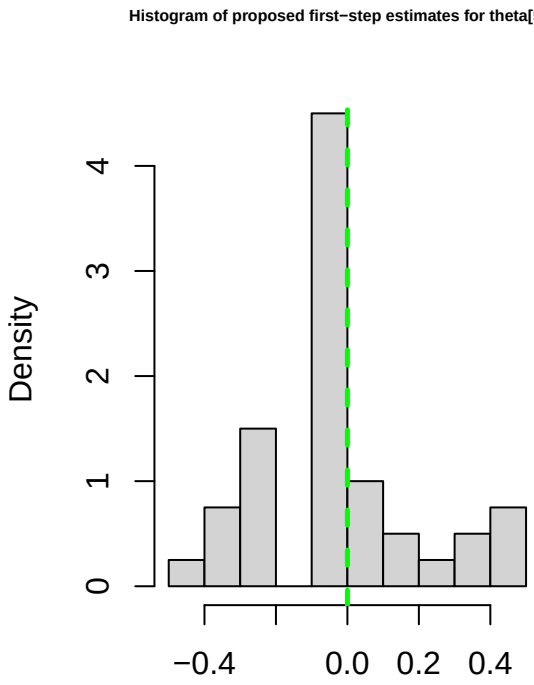
First Step Histograms

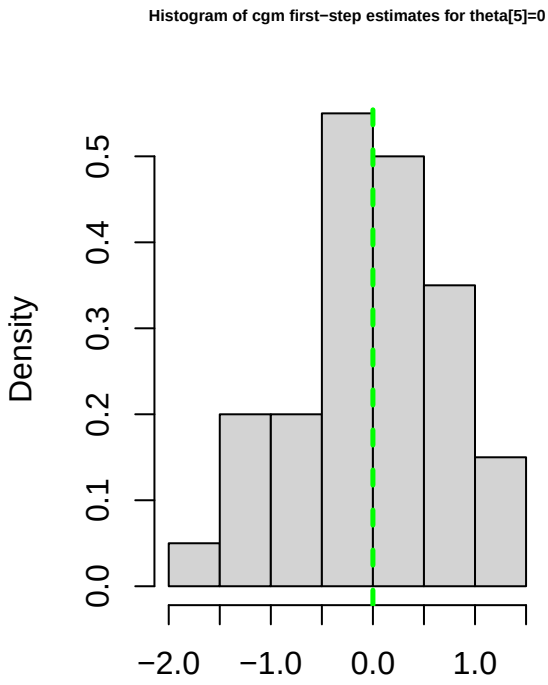


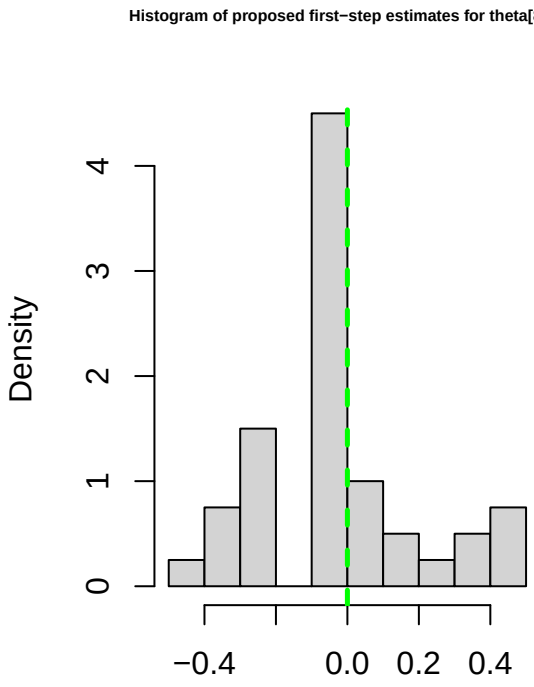


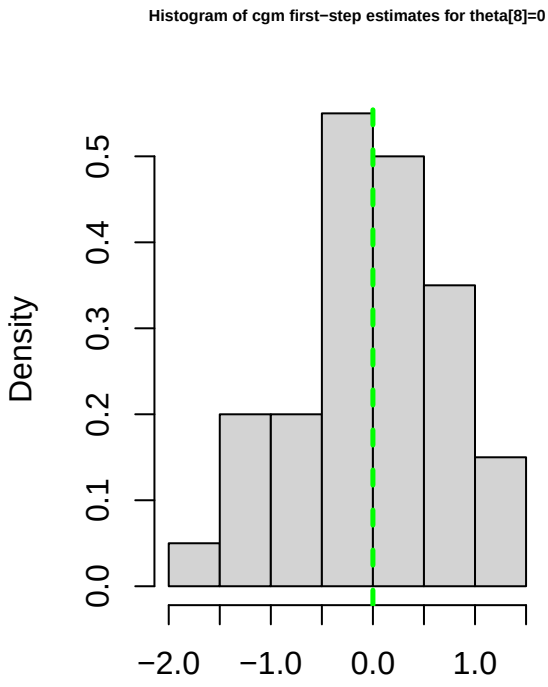





Histogram of proposed first−step estimates for theta[5]=0
Density
0
1
2
3
4
−0.4
0.0
0.2
0.4


Histogram of cgm first−step estimates for theta[5]=0
Density
0.0
0.1
0.2
0.3
0.4
0.5
−2.0
−1.0
0.0
1.0


Histogram of proposed first−step estimates for theta[8]=0
Density
0
1
2
3
4
−0.4
0.0
0.2
0.4


Histogram of cgm first−step estimates for theta[8]=0
Density
0.0
0.1
0.2
0.3
0.4
0.5
−2.0
−1.0
0.0
1.0

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 5: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[4]	-0.751	-0.483	-0.355	-0.713	-0.377
theta[9]	0.253	0.429	0.660	0.272	0.626
theta[5]	-0.195	0.018	0.289	-0.195	0.270
theta[8]	-0.225	0.017	0.157	-0.193	0.144

Table 6: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[4]	-1.088	-0.662	-0.378	-1.056	-0.415
theta[9]	0.513	0.563	0.795	0.515	0.791
theta[5]	-0.265	0.051	0.378	-0.246	0.332
theta[8]	-0.166	-0.034	0.406	-0.164	0.345

Statistics for Theoretical 95% Confidence Intervals

Table 7: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[4]	-0.513	0.134	-0.775	-0.250	0.9
theta[9]	0.429	0.125	0.183	0.675	0.3
theta[5]	0.032	0.124	-0.210	0.274	0.9
theta[8]	0.003	0.119	-0.231	0.236	1.0

Table 8: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[4]	-0.704	0.183	-1.064	-0.345	0.9
theta[9]	0.617	0.176	0.272	0.961	1.0
theta[5]	0.029	0.128	-0.221	0.280	0.9
theta[8]	-0.001	0.129	-0.253	0.252	0.9