

# Simulation Results

2026-01-20

## Simulation Setup

This simulation is performed with  $n = 200$  and  $d = 100$ , using the 2-d lattice as the underlying graph.  $s = 5$  parameters are set to be nonzero, and the beta parameter is chosen to be  $\beta = 0.2$ . The attached results are for a 10-replication simulation. The parameter vector  $\theta$  has sparse components other than the following:

Parameter.Index	Value
20	-0.447
51	-0.447
68	0.447
76	0.447
85	0.447

but for brevity, our simulation only estimates the indices of  $\theta$  in  $\mathcal{C} = \{20, 51, 71, 40\}$  elements of  $\theta$ . Accordingly, **all statistics and visuals are indicative of performance only on the set  $\mathcal{C}$ .**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

## Results

### Mean-squared error comparison

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[20]	0.040	0.050
theta[51]	0.071	0.035
theta[71]	0.015	0.028
theta[40]	0.020	0.041
total	0.036	0.038

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[20]	0.131	0.049

	proposed	cgm
theta[51]	0.117	0.035
theta[71]	0.000	0.005
theta[40]	0.000	0.003
total	0.062	0.023

### Mean absolute deviation comparison  $\frac{1}{n.\text{sim}} \sum_{i=1}^{n.\text{sim}} \frac{1}{|\mathcal{C}|} \|\hat{\theta}_i - \theta\|$

Table 3: Mean Absolute Deviation of Parameter Estimates

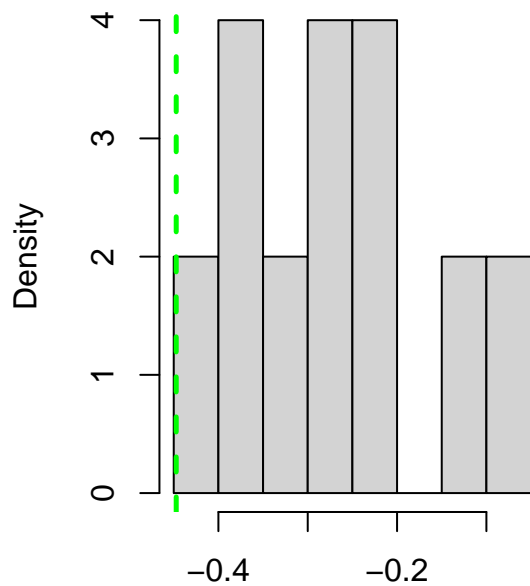
	proposed	cgm
theta[20]	0.168	0.180
theta[51]	0.233	0.169
theta[71]	0.108	0.126
theta[40]	0.087	0.170
total	0.149	0.161

Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

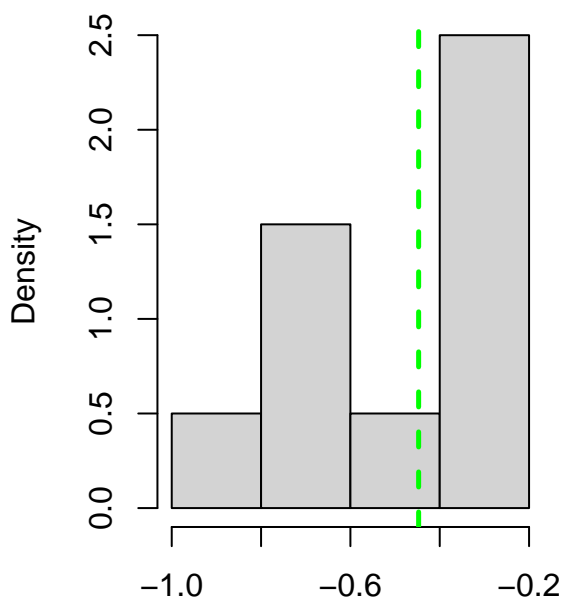
	proposed	cgm
theta[20]	0.352	0.189
theta[51]	0.317	0.151
theta[71]	0.006	0.033
theta[40]	0.005	0.023
total	0.170	0.099

## Boxplots

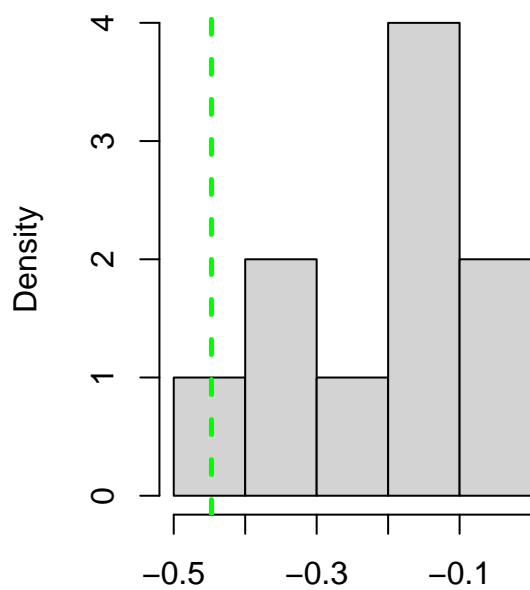
Histogram of proposed estimates for  $\theta_{[20]} = -0.447$



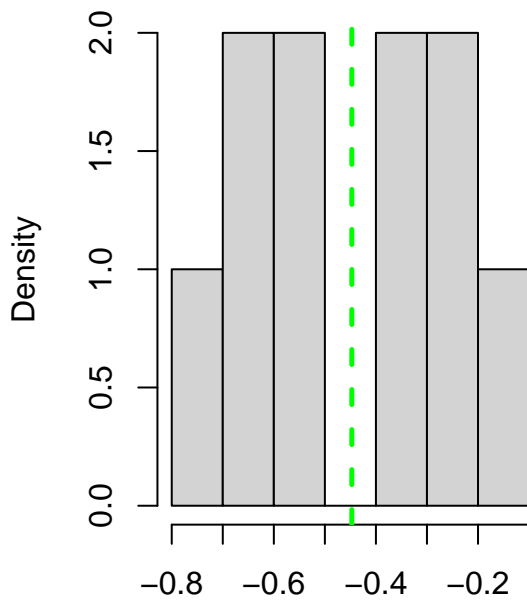
Histogram of cgm estimates for  $\theta_{[20]} = -0.447$



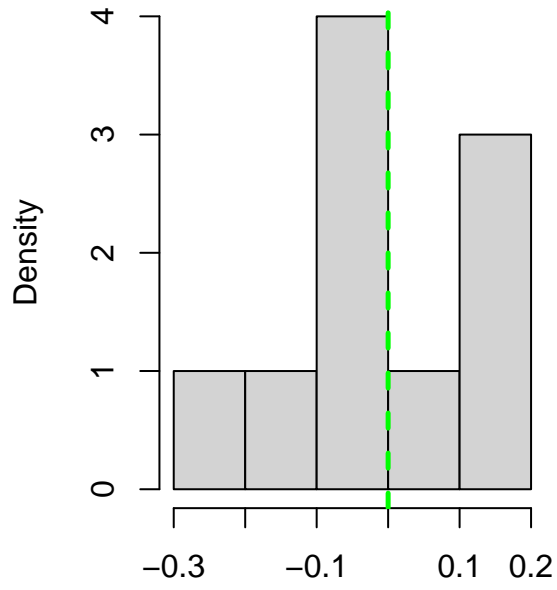
Histogram of proposed estimates for  $\theta_{[51]} = -0.447$



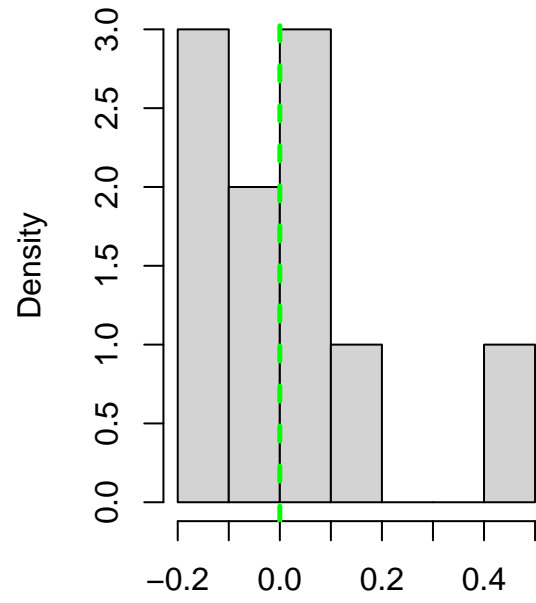
Histogram of cgm estimates for  $\theta_{[51]} = -0.447$



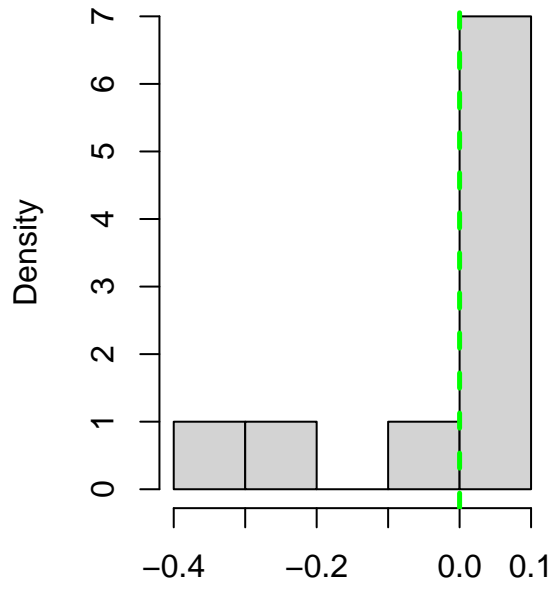
Histogram of proposed estimates for  $\theta_{71}=0$



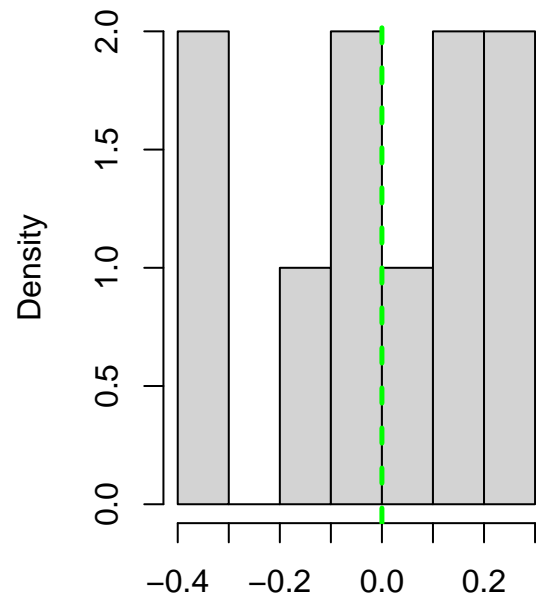
Histogram of cgm estimates for  $\theta_{71}=0$



Histogram of proposed estimates for  $\theta_{40}=0$



Histogram of cgm estimates for  $\theta_{40}=0$



## Statistics and 95% Confidence Intervals from per-Replicate Estimates

### Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[20]	-0.279	0.142	-0.557	-0.001	0.9
theta[51]	-0.218	0.131	-0.474	0.038	0.5
theta[71]	-0.019	0.131	-0.276	0.238	1.0
theta[40]	-0.040	0.129	-0.292	0.212	0.9

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[20]	-0.489	0.129	-0.743	-0.235	0.9
theta[51]	-0.452	0.132	-0.711	-0.192	0.9
theta[71]	0.013	0.133	-0.249	0.274	0.9
theta[40]	0.002	0.141	-0.273	0.278	0.8