

# Simulation Results

2026-01-21

## Simulation Setup

This simulation is performed with  $n = 200$  and  $d = 200$ , using the 2-d lattice as the underlying graph.  $s = 5$  parameters are set to be nonzero, and the beta parameter is chosen to be  $\beta = 0.4$ . The attached results are for a 10-replication simulation. The parameter vector  $\theta$  has sparse components other than the following:

Parameter.Index	Value
15	-0.447
95	-0.447
98	0.447
174	-0.447
197	0.447

but for brevity, our simulation only estimates the indices of  $\theta$  in  $\mathcal{C} = \{15, 95, 69, 170\}$  elements of  $\theta$ . Accordingly, **all statistics and visuals are indicative of performance only on the set  $\mathcal{C}$ .**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

## Results

### Mean-squared error comparison

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[15]	0.045	1.083
theta[95]	0.046	0.370
theta[69]	0.032	0.166
theta[170]	0.013	0.165
total	0.034	0.446

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[15]	0.167	0.053

	proposed	cgm
theta[95]	0.187	0.039
theta[69]	0.007	0.004
theta[170]	0.000	0.000
total	0.090	0.024

```
### Mean absolute deviation comparison $(\frac{1}{n.sim} \sum_{i=1}^{n.sim} \frac{1}{|\mathcal{C}|} |\hat{C}
```

Table 3: Mean Absolute Deviation of Parameter Estimates

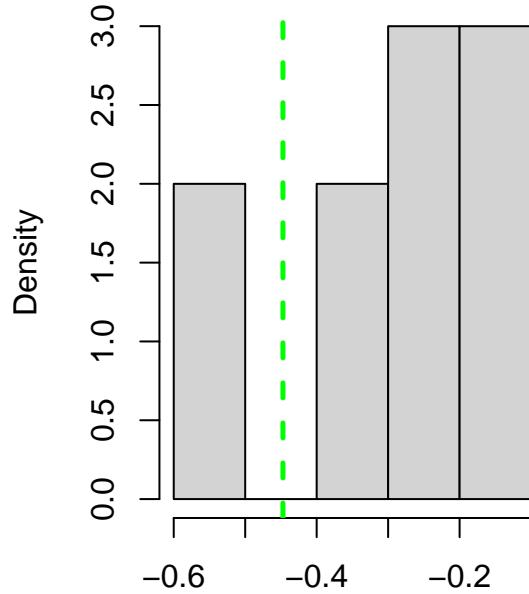
	proposed	cgm
theta[15]	0.193	0.645
theta[95]	0.184	0.389
theta[69]	0.146	0.318
theta[170]	0.087	0.323
total	0.152	0.419

Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

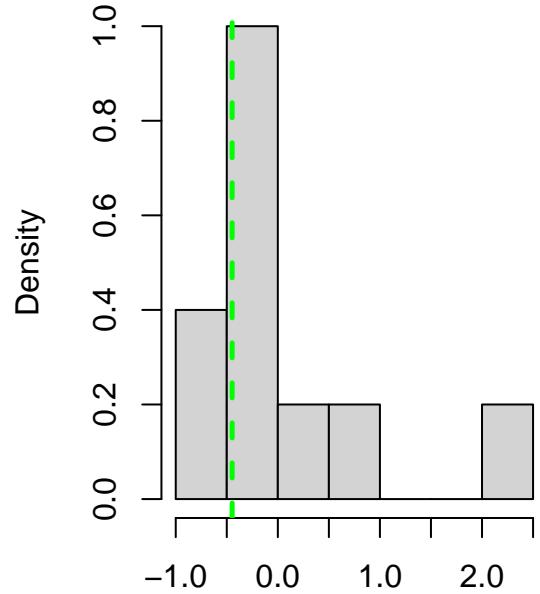
	proposed	cgm
theta[15]	0.406	0.205
theta[95]	0.431	0.160
theta[69]	0.027	0.031
theta[170]	0.003	0.000
total	0.217	0.099

## Boxplots

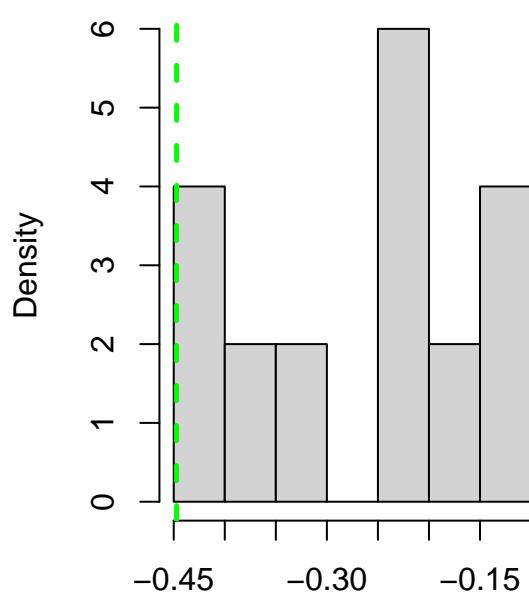
Histogram of proposed estimates for  $\theta[15] = -0.447$



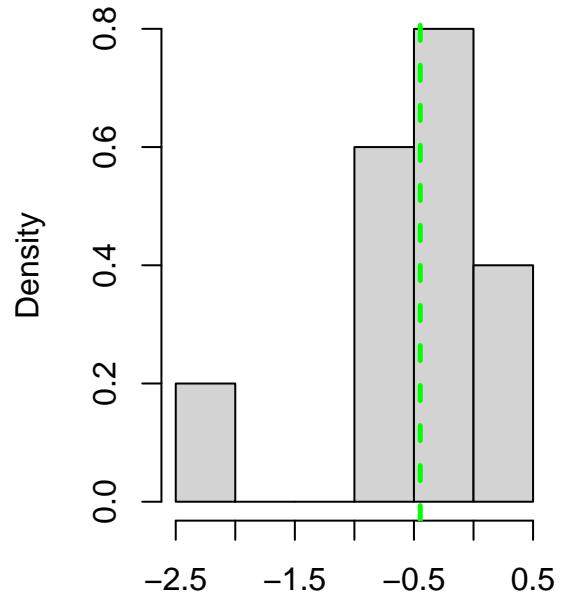
Histogram of cgm estimates for  $\theta[15] = -0.447$



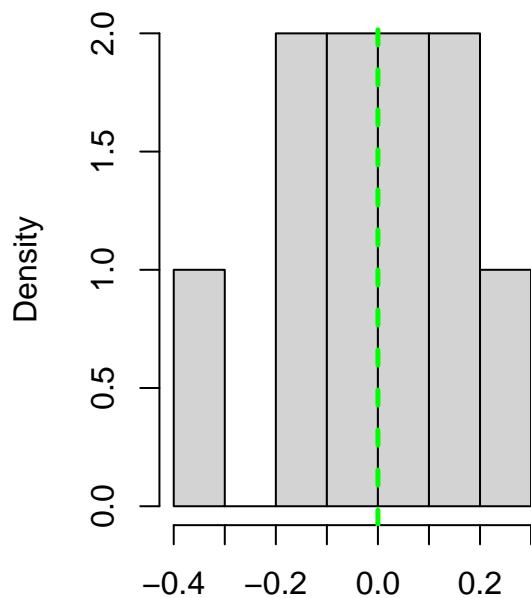
Histogram of proposed estimates for  $\theta[95] = -0.447$



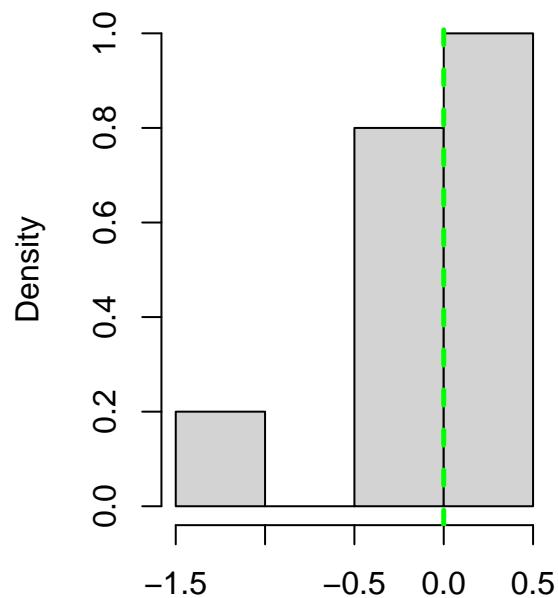
Histogram of cgm estimates for  $\theta[95] = -0.447$



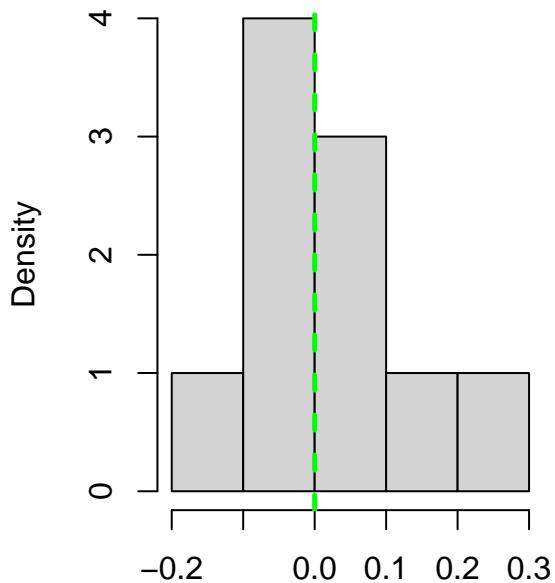
Histogram of proposed estimates for theta[69]=0



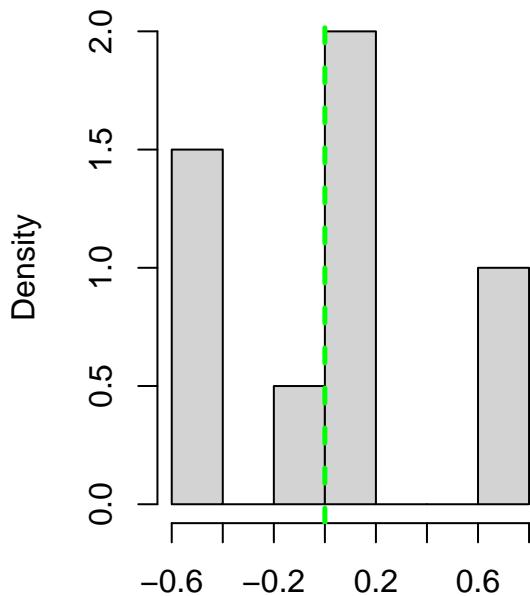
Histogram of cgm estimates for theta[69]=0



Histogram of proposed estimates for theta[170]=0



Histogram of cgm estimates for theta[170]=0



## Statistics and 95% Confidence Intervals from per-Replicate Estimates

### Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[15]	-0.300	0.124	-0.542	-0.058	0.6
theta[95]	-0.263	0.117	-0.493	-0.034	0.5
theta[69]	0.006	0.129	-0.246	0.258	0.7
theta[170]	0.008	0.119	-0.226	0.241	0.9

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[15]	0.069	0.213	-0.348	0.485	0.6
theta[95]	-0.518	0.176	-0.863	-0.173	0.6
theta[69]	-0.087	0.205	-0.490	0.316	0.9
theta[170]	-0.005	0.212	-0.421	0.412	0.6