

Simulation Results

2025-10-14

Simulation Setup

This simulation is performed with $n = 400$ and $d = 10$, using the 2-d lattice as the underlying graph. $s = 2$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.1$. The attached results are for a 5-replication simulation. The true values of the parameter vector θ are

```
[1] 0 0 0 0 0 0 0 1 -1 0
```

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

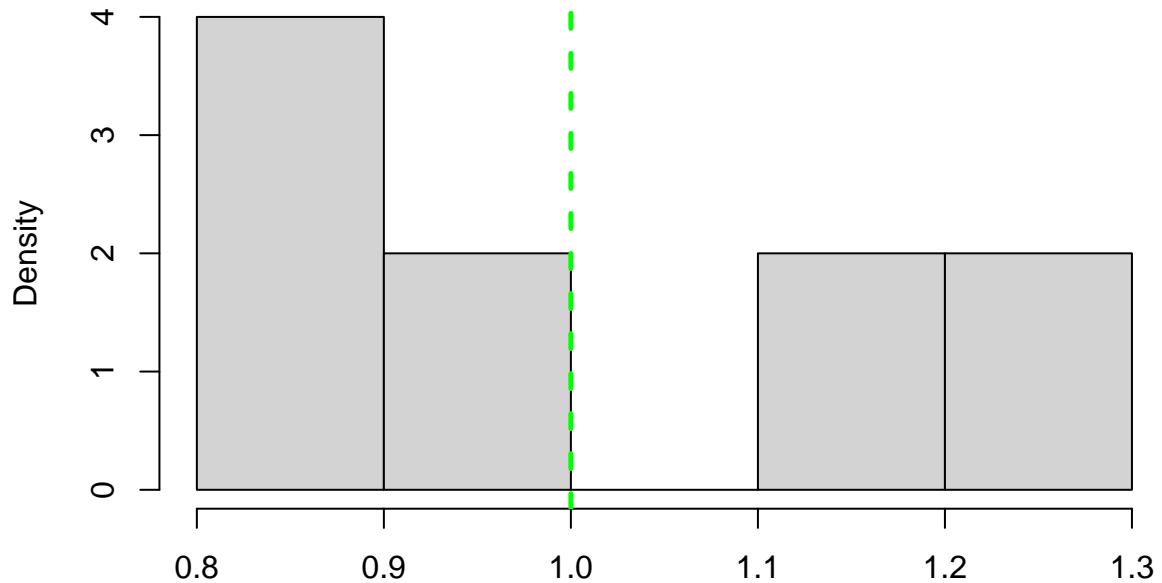
Mean-squared error comparison ($\frac{1}{n.sim} \sum_{i=1}^{n.sim} \frac{1}{d} \|\hat{\theta}_i - \theta\|^2$)

```
# A tibble: 1 x 2
`MISLE (First-step) MSE` `MISLE MSE`
<dbl> <dbl>
1 0.0187 0.0272

# A tibble: 1 x 2
`MISLE MSE` `CGM MSE`
<dbl> <dbl>
1 0.0272 2.61
```

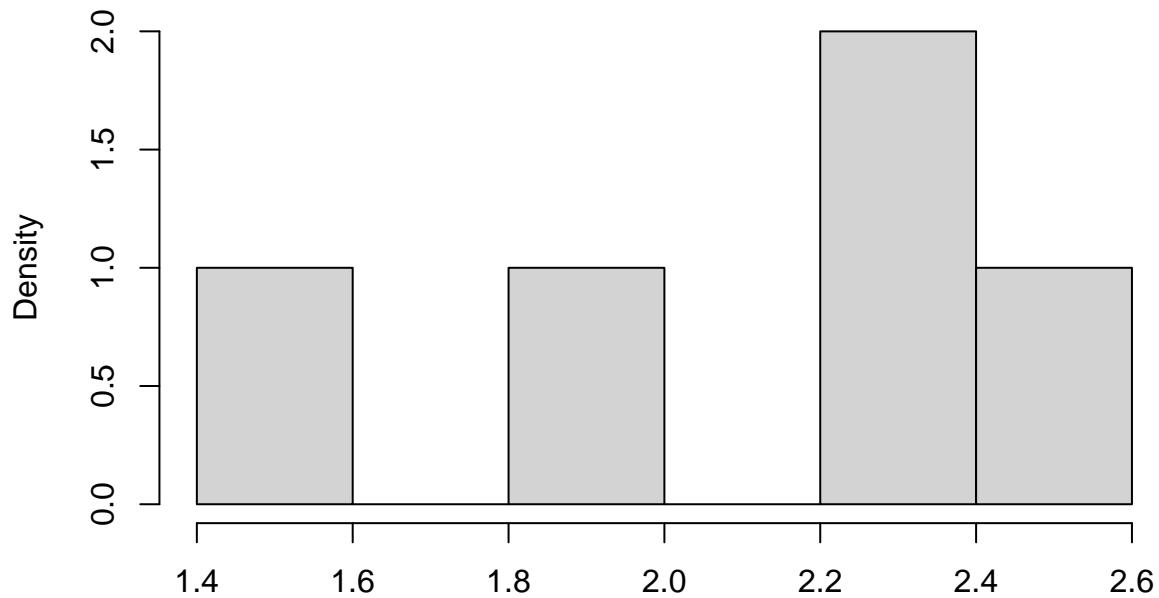
First Step Histograms

Histogram of theta.hat[8]



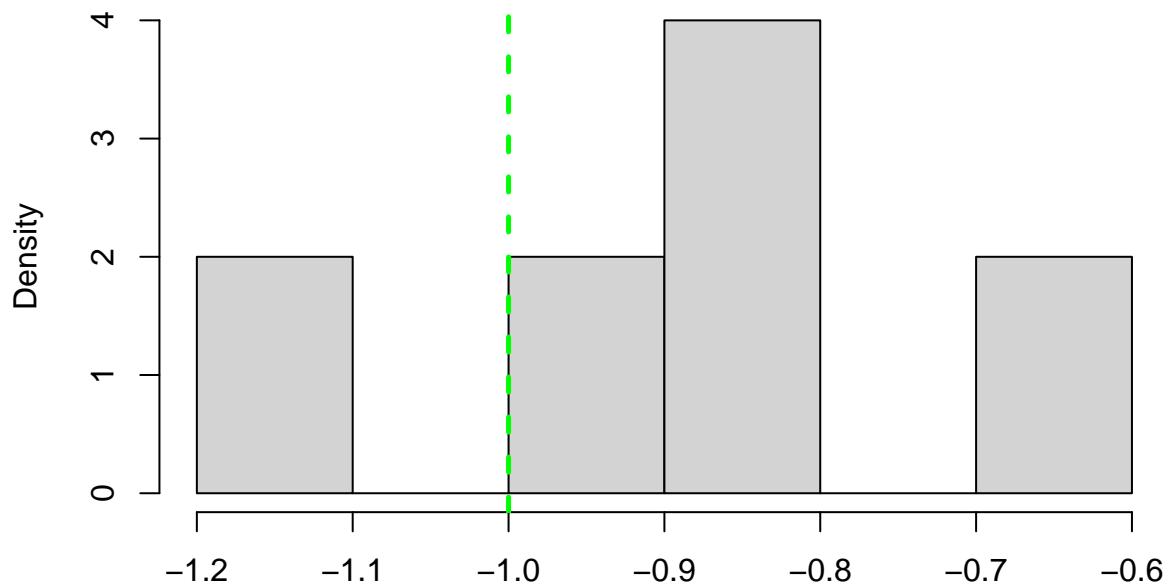
```
[1] "Summary statistics of bootstrap replicates:"  
Min. 1st Qu. Median Mean 3rd Qu. Max.  
0.8603 0.8830 0.9275 1.0136 1.1062 1.2909  
[1] "95% CI based on bootstrap:"  
lower upper  
1 0.8625514 1.272444
```

Histogram of theta.hat.cgm[8]



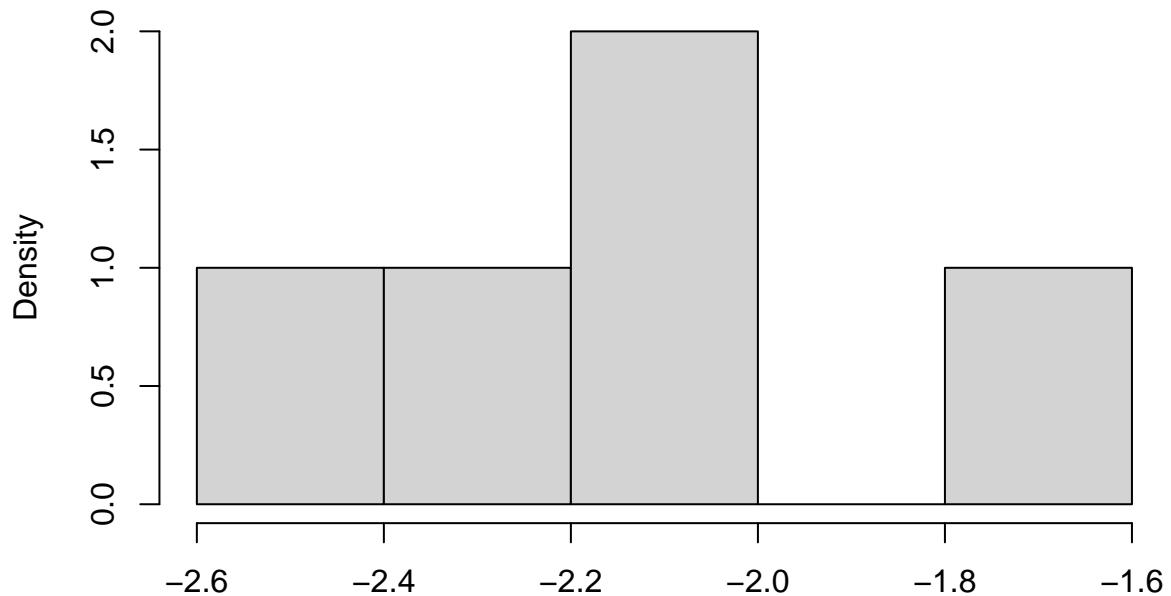
```
[1] "Summary statistics of bootstrap replicates:"  
Min. 1st Qu. Median Mean 3rd Qu. Max.  
1.476 1.881 2.228 2.099 2.379 2.531  
[1] "95% CI based on bootstrap:"  
lower.cgm upper.cgm  
1 1.516124 2.515654
```

Histogram of theta.hat[9]



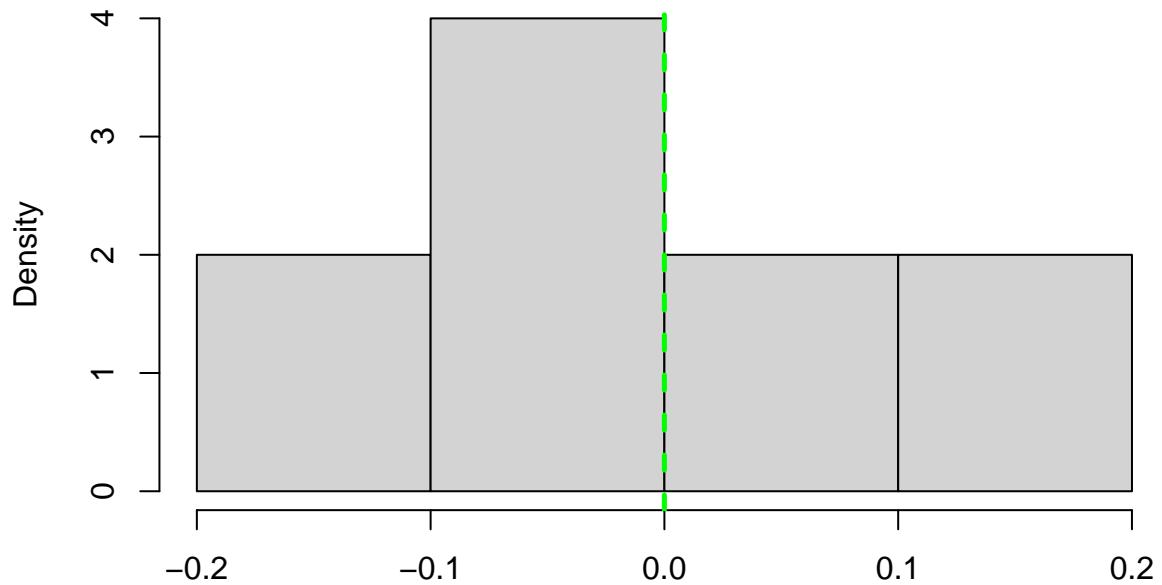
```
[1] "Summary statistics of bootstrap replicates:"  
Min. 1st Qu. Median Mean 3rd Qu. Max.  
-1.1681 -0.9963 -0.8534 -0.9063 -0.8151 -0.6985  
[1] "95% CI based on bootstrap:"  
lower upper  
1 -1.150889 -0.7101637
```

Histogram of theta.hat.cgm[9]



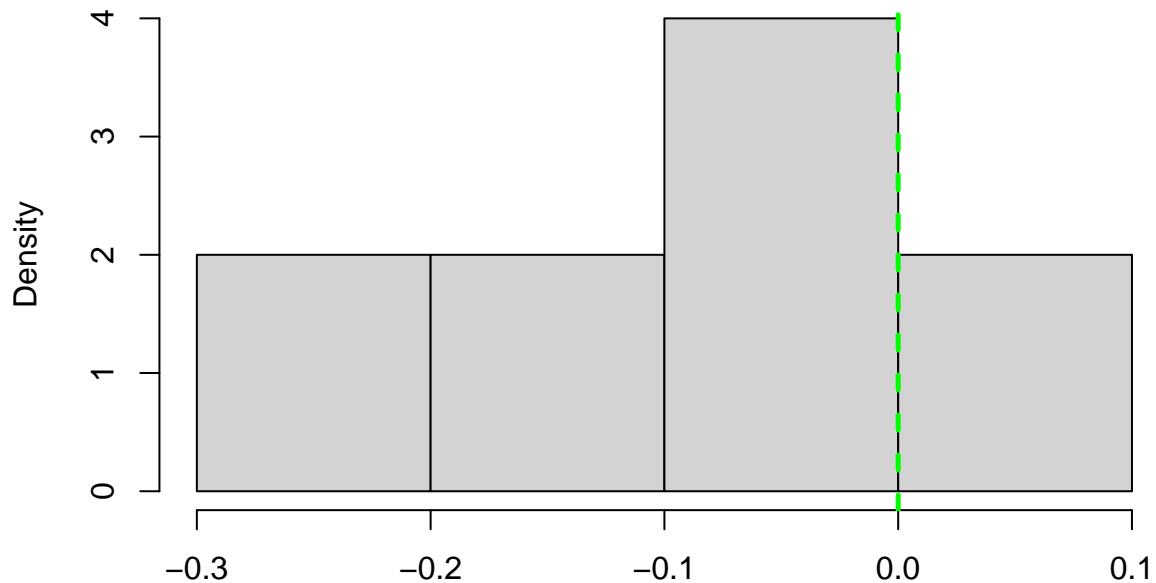
```
[1] "Summary statistics of bootstrap replicates:"  
   Min. 1st Qu. Median Mean 3rd Qu. Max.  
 -2.564 -2.381 -2.191 -2.206 -2.153 -1.740  
[1] "95% CI based on bootstrap:"  
lower.cgm upper.cgm  
1 -2.545526 -1.780897
```

Histogram of theta.hat[1]



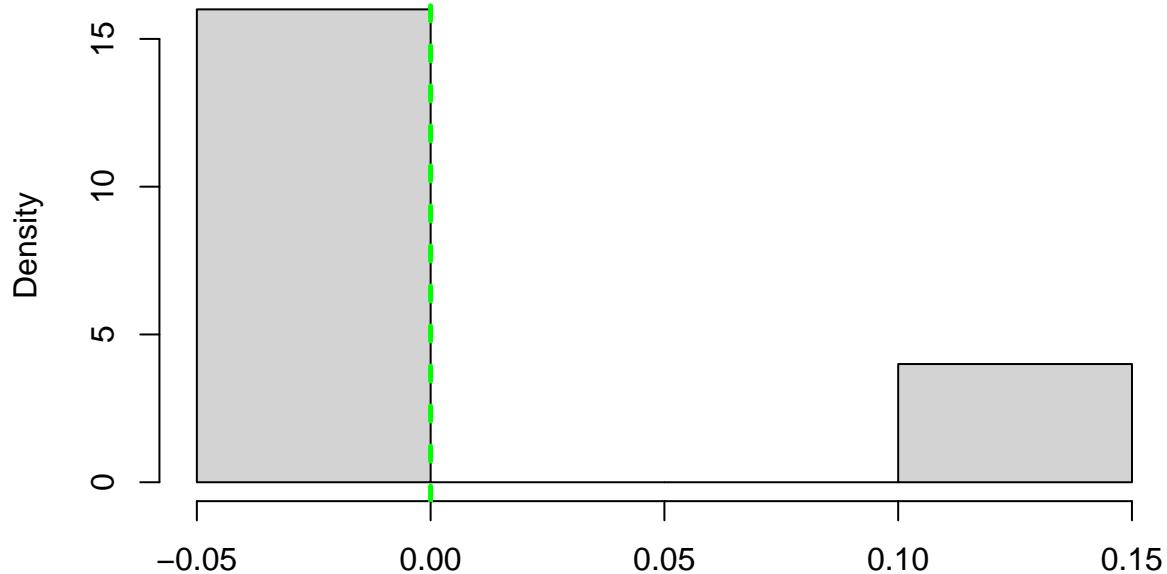
```
[1] "Summary statistics of bootstrap replicates:"  
Min. 1st Qu. Median Mean 3rd Qu. Max.  
-0.14009 -0.08891 -0.02909 -0.02276 0.00420 0.14010  
[1] "95% CI based on bootstrap:"  
lower upper  
1 -0.1349692 0.1265063
```

Histogram of theta.hat.cgm[1]



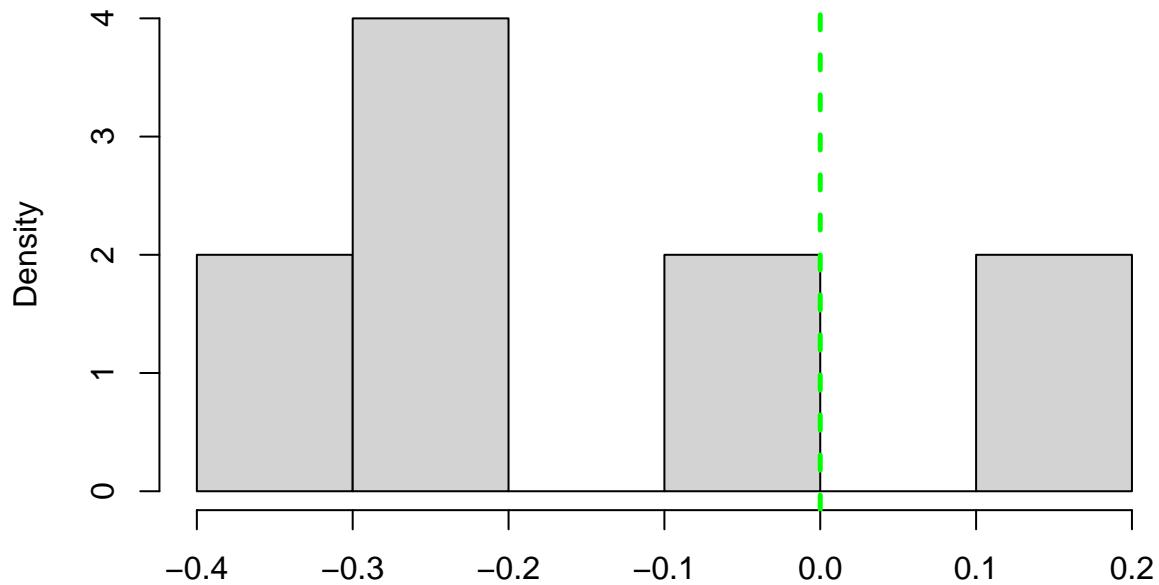
```
[1] "Summary statistics of bootstrap replicates:"  
Min. 1st Qu. Median Mean 3rd Qu. Max.  
-0.29922 -0.18235 -0.09957 -0.10136 0.00000 0.07432  
[1] "95% CI based on bootstrap:"  
lower.cgm upper.cgm  
1 -0.2875298 0.06688716
```

Histogram of theta.hat[4]



```
[1] "Summary statistics of bootstrap replicates:"  
     Min.   1st Qu.    Median      Mean   3rd Qu.      Max.  
-0.0472829 -0.0465674 -0.0147296 -0.0001757  0.0000000  0.1077016  
[1] "95% CI based on bootstrap:"  
     lower      upper  
1 -0.04721133  0.09693141
```

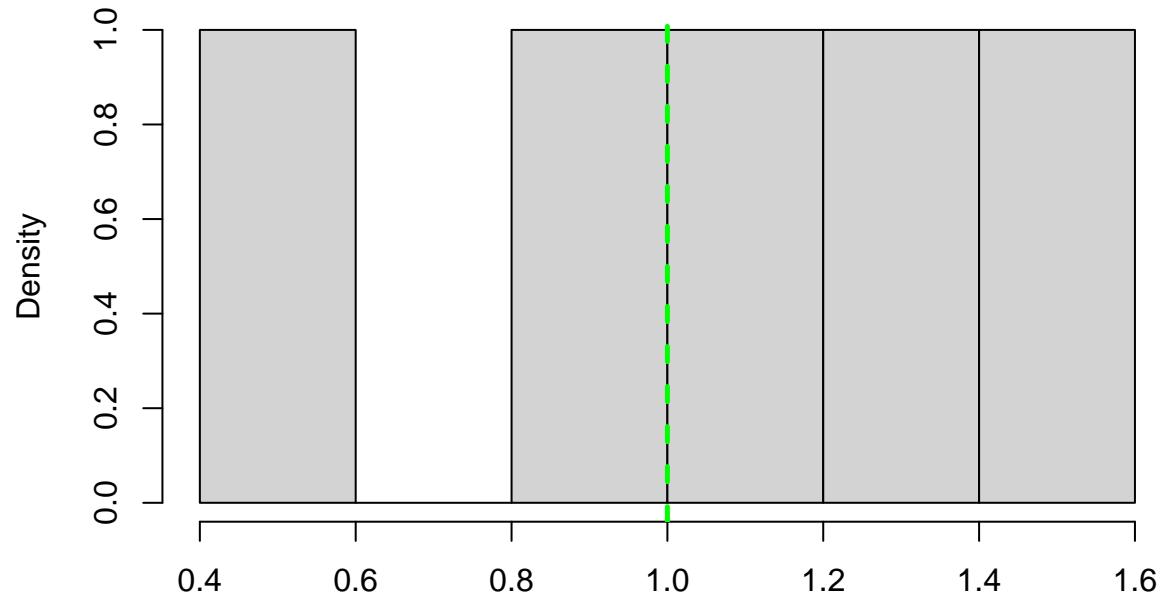
Histogram of theta.hat.cgm[4]



```
[1] "Summary statistics of bootstrap replicates:"  
Min. 1st Qu. Median Mean 3rd Qu. Max.  
-0.3941 -0.2787 -0.2552 -0.1637 0.0000 0.1094  
[1] "95% CI based on bootstrap:"  
lower.cgm upper.cgm  
1 -0.3825272 0.09841997
```

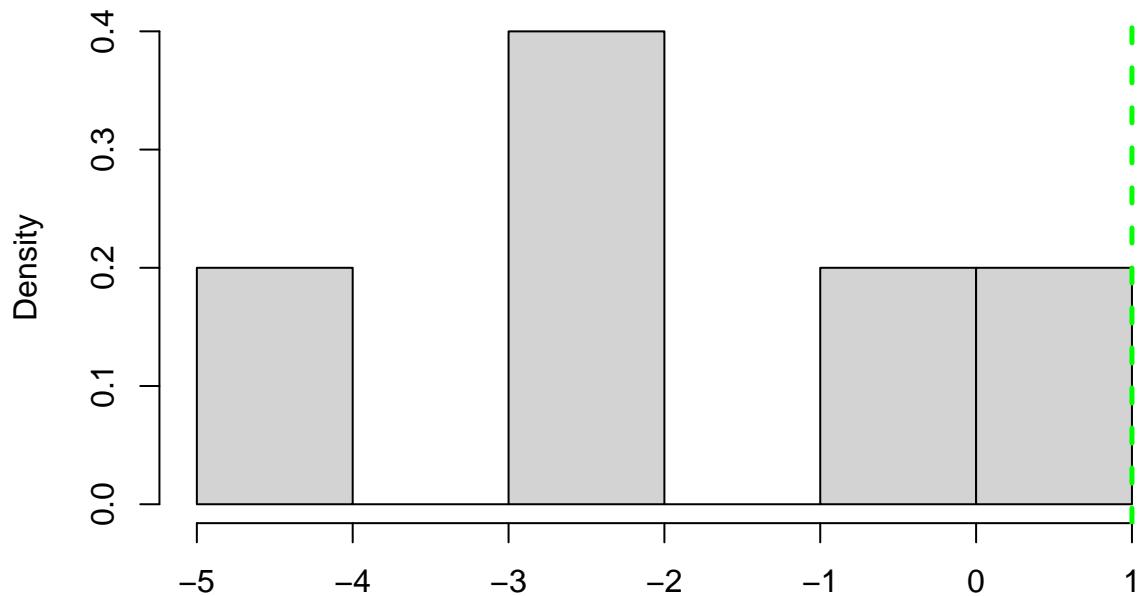
Statistics and 95% Confidence Intervals from per-Replicate Estimates

Histogram of $\theta_{\tilde{8}}$



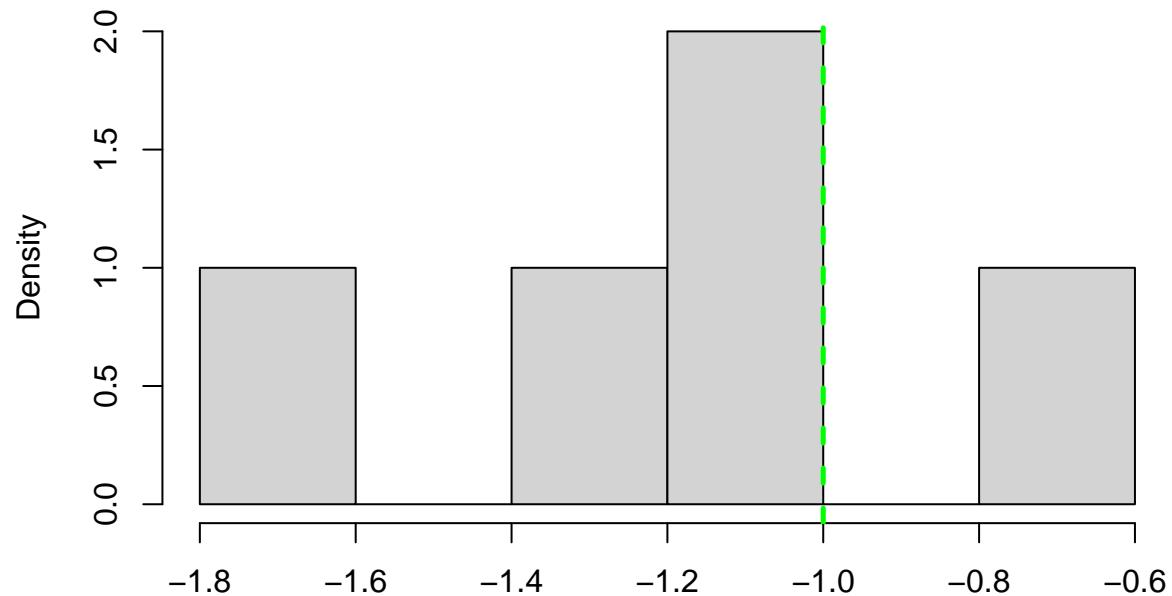
```
[1] "Summary statistics of bootstrap replicates:"  
Min. 1st Qu. Median Mean 3rd Qu. Max.  
0.5688 0.8843 1.0421 1.0747 1.3808 1.4974  
[1] "95% CI based on bootstrap:"  
lower upper  
1 0.6003282 1.485786
```

Histogram of theta.tilde.cgm[8]



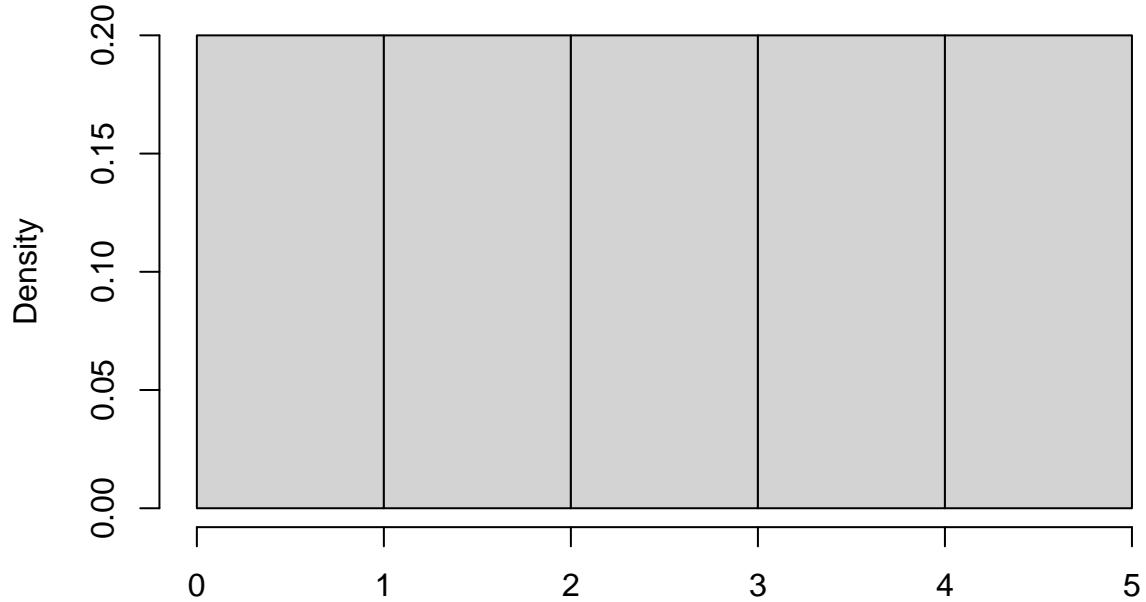
```
[1] "Summary statistics of bootstrap replicates:"  
Min. 1st Qu. Median Mean 3rd Qu. Max.  
-4.9666 -2.6021 -2.4412 -2.1121 -0.6569 0.1064  
[1] "95% CI based on bootstrap:"  
lower.cgm upper.cgm  
1 -4.730149 0.03004351
```

Histogram of theta.tilde[9]



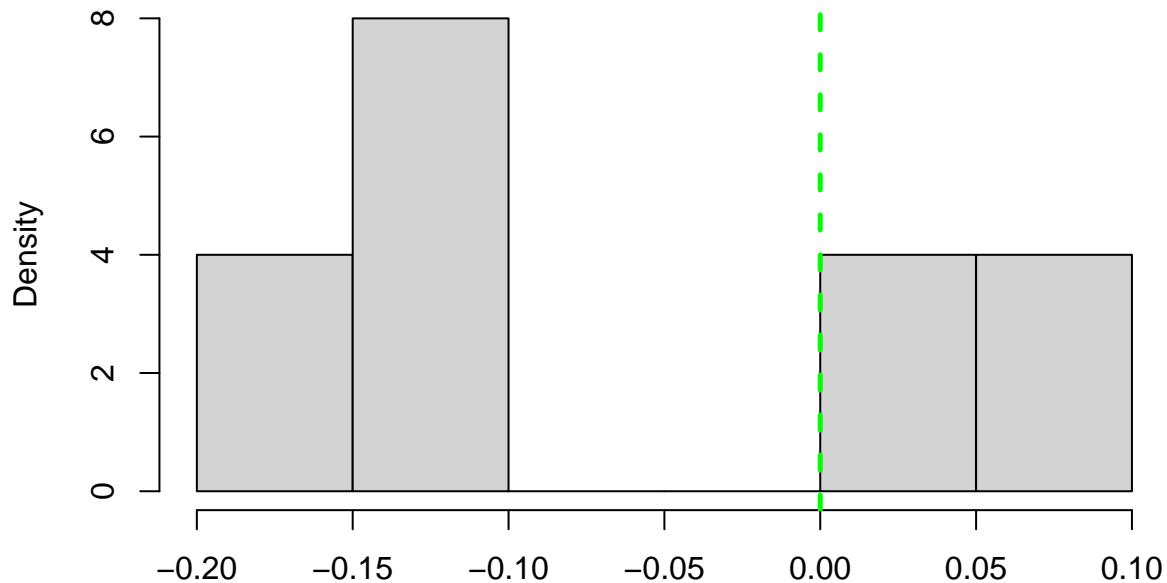
```
[1] "Summary statistics of bootstrap replicates:"  
Min. 1st Qu. Median Mean 3rd Qu. Max.  
-1.6409 -1.2229 -1.1138 -1.1359 -1.0933 -0.6088  
[1] "95% CI based on bootstrap:"  
lower upper  
1 -1.599121 -0.6572445
```

Histogram of theta.tilde.cgm[9]



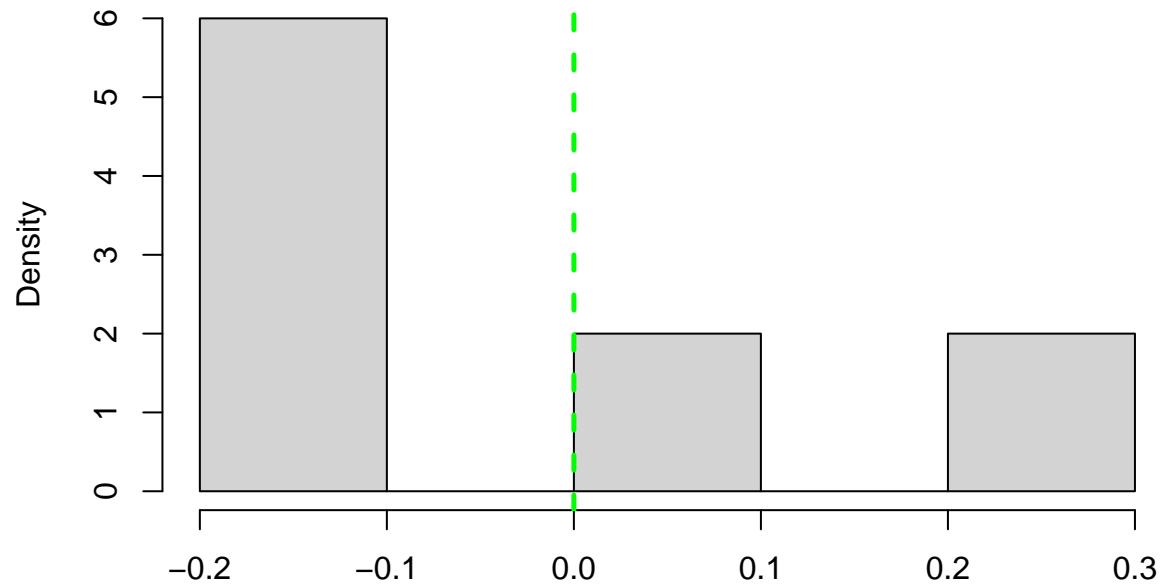
```
[1] "Summary statistics of bootstrap replicates:"  
    Min. 1st Qu. Median     Mean 3rd Qu.   Max.  
0.4224  1.0728  2.1920  2.2949  3.4078  4.3794  
[1] "95% CI based on bootstrap:"  
lower.cgm upper.cgm  
1 0.487402 4.282282
```

Histogram of theta.tilde[1]



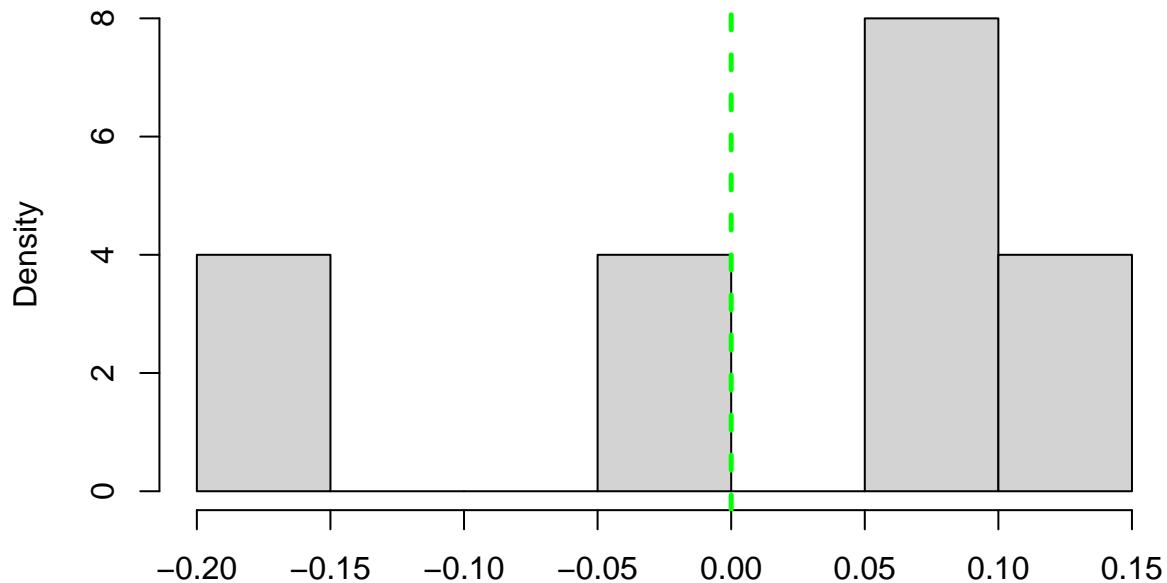
```
[1] "Summary statistics of bootstrap replicates:"  
Min. 1st Qu. Median Mean 3rd Qu. Max.  
-0.17824 -0.14891 -0.14088 -0.07520 0.03717 0.05488  
[1] "95% CI based on bootstrap:"  
lower upper  
1 -0.1753078 0.05310652
```

Histogram of theta.tilde.cgm[1]



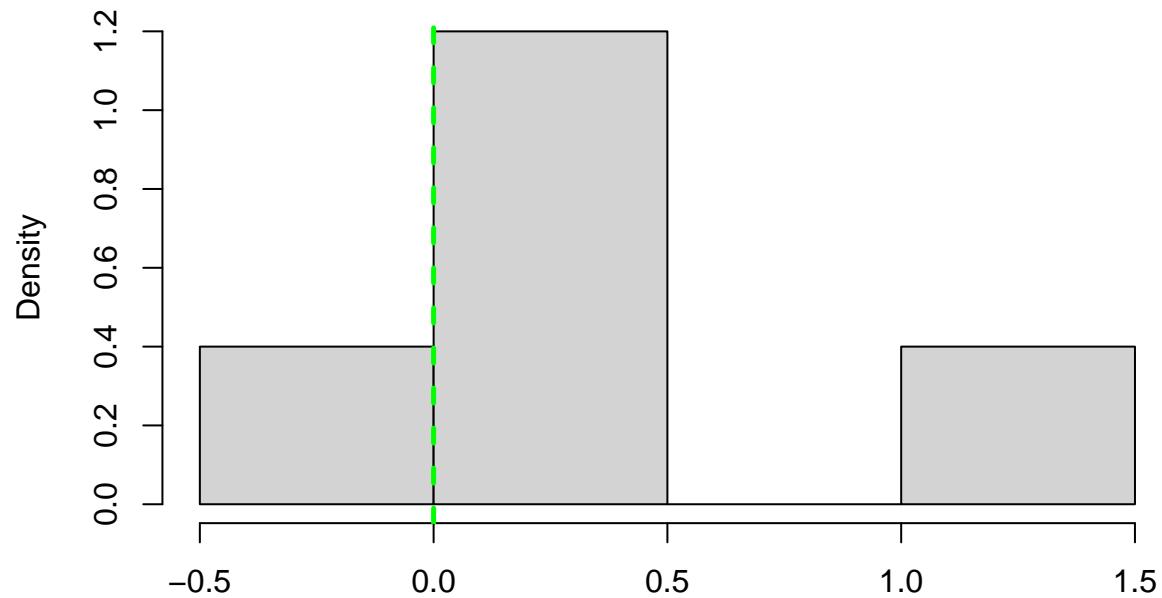
```
[1] "Summary statistics of bootstrap replicates:"  
Min. 1st Qu. Median Mean 3rd Qu. Max.  
-0.18009 -0.16501 -0.14664 -0.03553 0.09441 0.21970  
[1] "95% CI based on bootstrap:"  
lower.cgm upper.cgm  
1 -0.1785809 0.2071715
```

Histogram of theta.tilde[4]



```
[1] "Summary statistics of bootstrap replicates:"  
Min. 1st Qu. Median Mean 3rd Qu. Max.  
-0.16987 -0.02448 0.08017 0.01515 0.08157 0.10836  
[1] "95% CI based on bootstrap:"  
lower upper  
1 -0.1553294 0.1056838
```

Histogram of theta.tilde.cgm[4]



```
[1] "Summary statistics of bootstrap replicates:"  
   Min. 1st Qu. Median      Mean 3rd Qu.      Max.  
-0.10835  0.02857  0.10464  0.32851  0.38194  1.23575  
[1] "95% CI based on bootstrap:"  
    lower.cgm upper.cgm  
1 -0.09465751  1.15037
```

Statistics for Theoretical 95% Confidence Intervals

```
[1] Length of Confidence Intervals for theta[8]
[1] Coverage proportion: 0.6
    Min. 1st Qu. Median Mean 3rd Qu. Max.
    0.7022 0.7176 0.7392 0.8223 0.9382 1.0143
[1] Length of Confidence Intervals for theta[8] (CGM Method)
[1] Coverage proportion: 1
    Min. 1st Qu. Median Mean 3rd Qu. Max.
    6.305 6.534 7.603 8.285 10.415 10.566
[1] Length of Confidence Intervals for theta[9]
[1] Coverage proportion: 0.6
    Min. 1st Qu. Median Mean 3rd Qu. Max.
    0.6242 0.7352 0.7519 0.7931 0.8545 0.9998
[1] Length of Confidence Intervals for theta[9] (CGM Method)
[1] Coverage proportion: 1
    Min. 1st Qu. Median Mean 3rd Qu. Max.
    10.63 15.22 16.87 15.45 16.90 17.60
[1] Length of Confidence Intervals for theta[1]
[1] Coverage proportion: 1
    Min. 1st Qu. Median Mean 3rd Qu. Max.
    0.4939 0.5475 0.5571 0.5595 0.5770 0.6220
[1] Length of Confidence Intervals for theta[1] (CGM Method)
[1] Coverage proportion: 1
    Min. 1st Qu. Median Mean 3rd Qu. Max.
    3.620 4.349 4.467 4.481 4.633 5.337
[1] Length of Confidence Intervals for theta[4]
[1] Coverage proportion: 1
    Min. 1st Qu. Median Mean 3rd Qu. Max.
    0.5161 0.5165 0.5295 0.5727 0.6041 0.6972
[1] Length of Confidence Intervals for theta[4] (CGM Method)
[1] Coverage proportion: 1
    Min. 1st Qu. Median Mean 3rd Qu. Max.
    3.847 4.406 4.550 5.073 4.763 7.800
```