

# Simulation Results

2026-01-20

## Simulation Setup

This simulation is performed with  $n = 400$  and  $d = 200$ , using the 2-d lattice as the underlying graph.  $s = 20$  parameters are set to be nonzero, and the beta parameter is chosen to be  $\beta = 0$ . The attached results are for a 10-replication simulation. The parameter vector  $\theta$  has sparse components other than the following:

Parameter.Index	Value
3	0.224
9	0.224
10	-0.224
24	-0.224
38	0.224
43	0.224
63	-0.224
71	-0.224
88	0.224
92	0.224
99	0.224
102	-0.224
103	-0.224
107	0.224
125	0.224
136	0.224
138	-0.224
189	0.224
197	-0.224
200	0.224

but for brevity, our simulation only estimates the indices of  $\theta$  in  $\mathcal{C} = \{ 3, 9, 76, 184 \}$  elements of  $\theta$ . Accordingly, **all statistics and visuals are indicative of performance only on the set  $\mathcal{C}$ .**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

## Results

### Mean-squared error comparison

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[3]	0.014	0.004
theta[9]	0.017	0.022
theta[76]	0.008	0.026
theta[184]	0.005	0.013
total	0.011	0.016

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[3]	0.043	0.018
theta[9]	0.043	0.020
theta[76]	0.003	0.003
theta[184]	0.000	0.001
total	0.022	0.011

```
### Mean absolute deviation comparison $(\frac{1}{n.sim} \sum_{i=1}^{n.sim} \frac{1}{|\mathcal{C}|} |\hat{C}
```

Table 3: Mean Absolute Deviation of Parameter Estimates

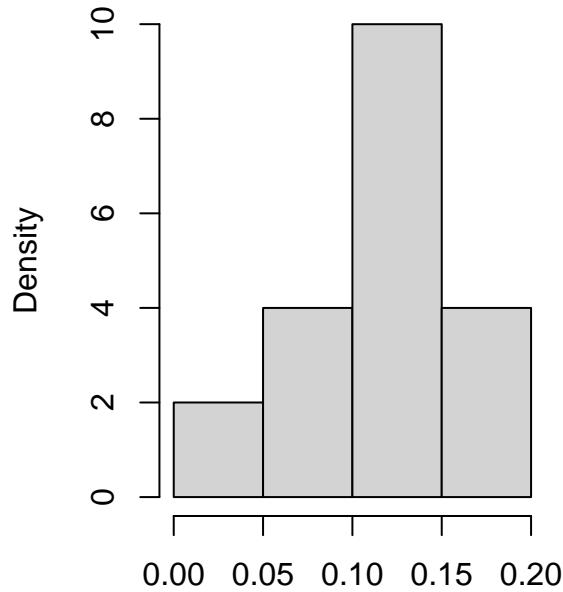
	proposed	cgm
theta[3]	0.109	0.052
theta[9]	0.103	0.123
theta[76]	0.087	0.125
theta[184]	0.058	0.088
total	0.089	0.097

Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

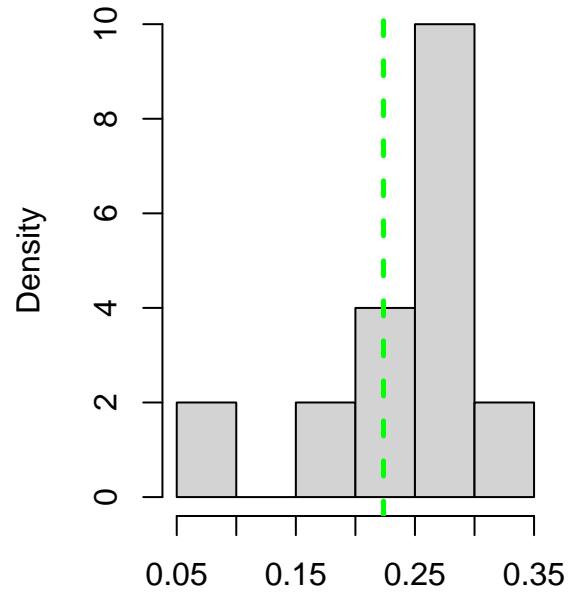
	proposed	cgm
theta[3]	0.206	0.110
theta[9]	0.204	0.124
theta[76]	0.024	0.025
theta[184]	0.002	0.009
total	0.109	0.067

## Boxplots

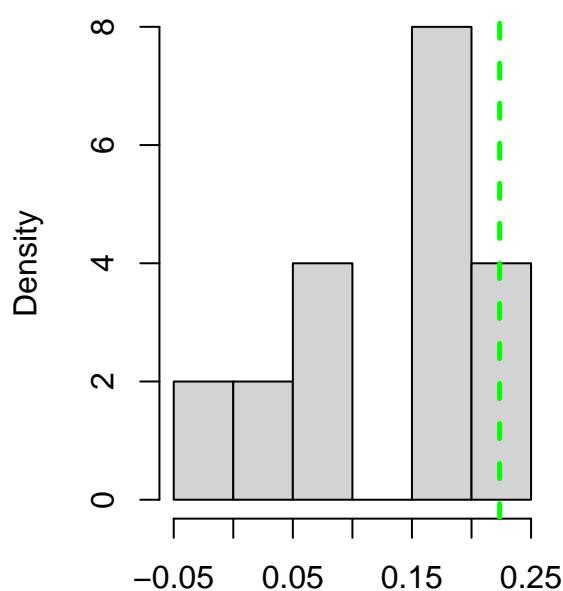
Histogram of proposed estimates for  $\theta[3]=0.224$



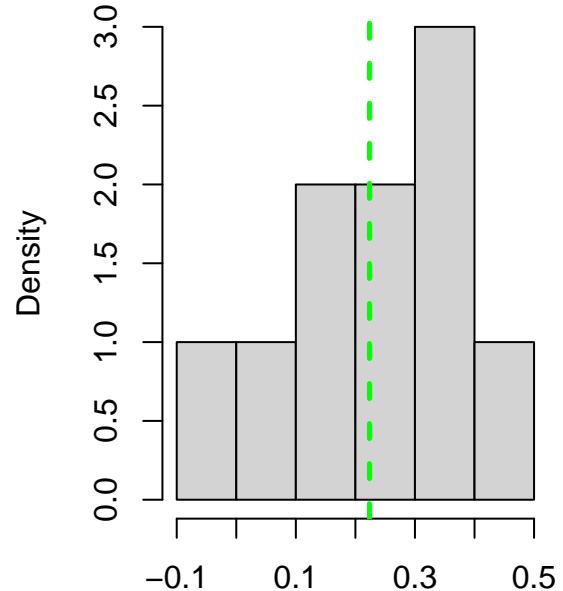
Histogram of cgm estimates for  $\theta[3]=0.224$



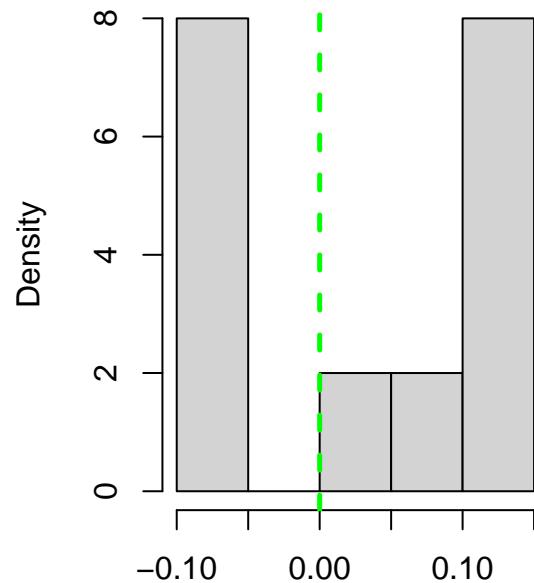
Histogram of proposed estimates for  $\theta[9]=0.224$



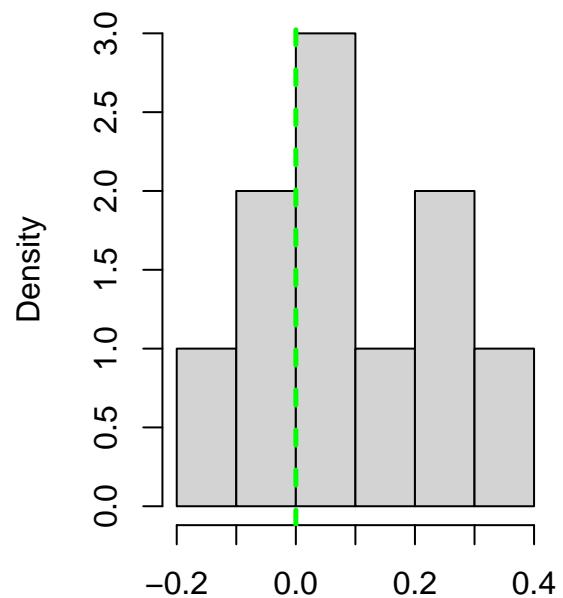
Histogram of cgm estimates for  $\theta[9]=0.224$



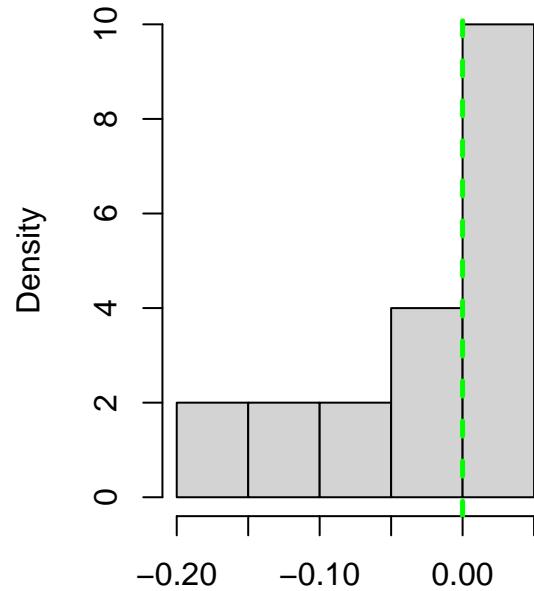
Histogram of proposed estimates for theta[76]=0



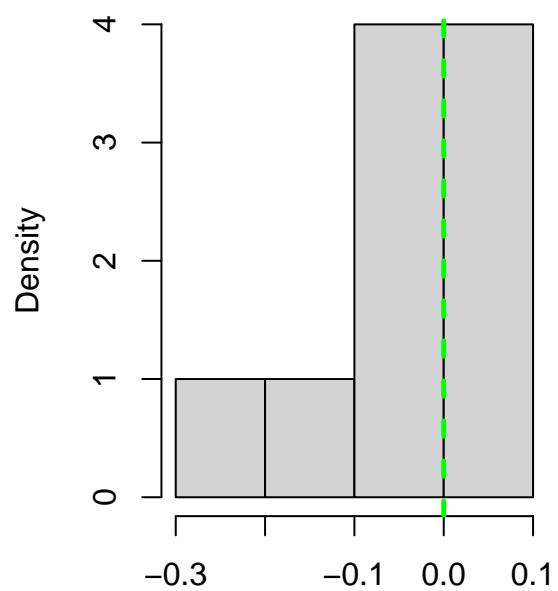
Histogram of cgm estimates for theta[76]=0



Histogram of proposed estimates for theta[184]=0



Histogram of cgm estimates for theta[184]=0



## Statistics and 95% Confidence Intervals from per-Replicate Estimates

### Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[3]	0.115	0.091	-0.065	0.294	0.9
theta[9]	0.121	0.091	-0.057	0.300	0.8
theta[76]	0.022	0.092	-0.159	0.203	1.0
theta[184]	-0.024	0.089	-0.199	0.150	0.9

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[3]	0.242	0.082	0.081	0.404	1.0
theta[9]	0.234	0.090	0.058	0.410	0.7
theta[76]	0.081	0.085	-0.085	0.248	0.6
theta[184]	-0.037	0.089	-0.211	0.137	0.9