

# Simulation Results

2025-10-14

## Simulation Setup

This simulation is performed with  $n = 200$  and  $d = 10$ , using the 2-d lattice as the underlying graph.  $s = 2$  parameters are set to be nonzero, and the beta parameter is chosen to be  $\beta = 0.1$ . The attached results are for a 5-replication simulation. The true values of the parameter vector  $\theta$  are

0 1 0 0 0 0 0 0 0 -1 ,

but for brevity, our simulation only estimates the indices of  $\theta$  in  $\mathcal{C} = \{2, 10, 1, 5\}$  elements of  $\theta$ . Accordingly, **all statistics and visuals are indicative of performance only on the set  $\mathcal{C}$ .**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

## Results

**Mean-squared error comparison** ( $\frac{1}{n_{sim}} \sum_{i=1}^{n_{sim}} \frac{1}{|\mathcal{C}|} \|\hat{\theta}_{i,C} - \theta_C\|^2$ )

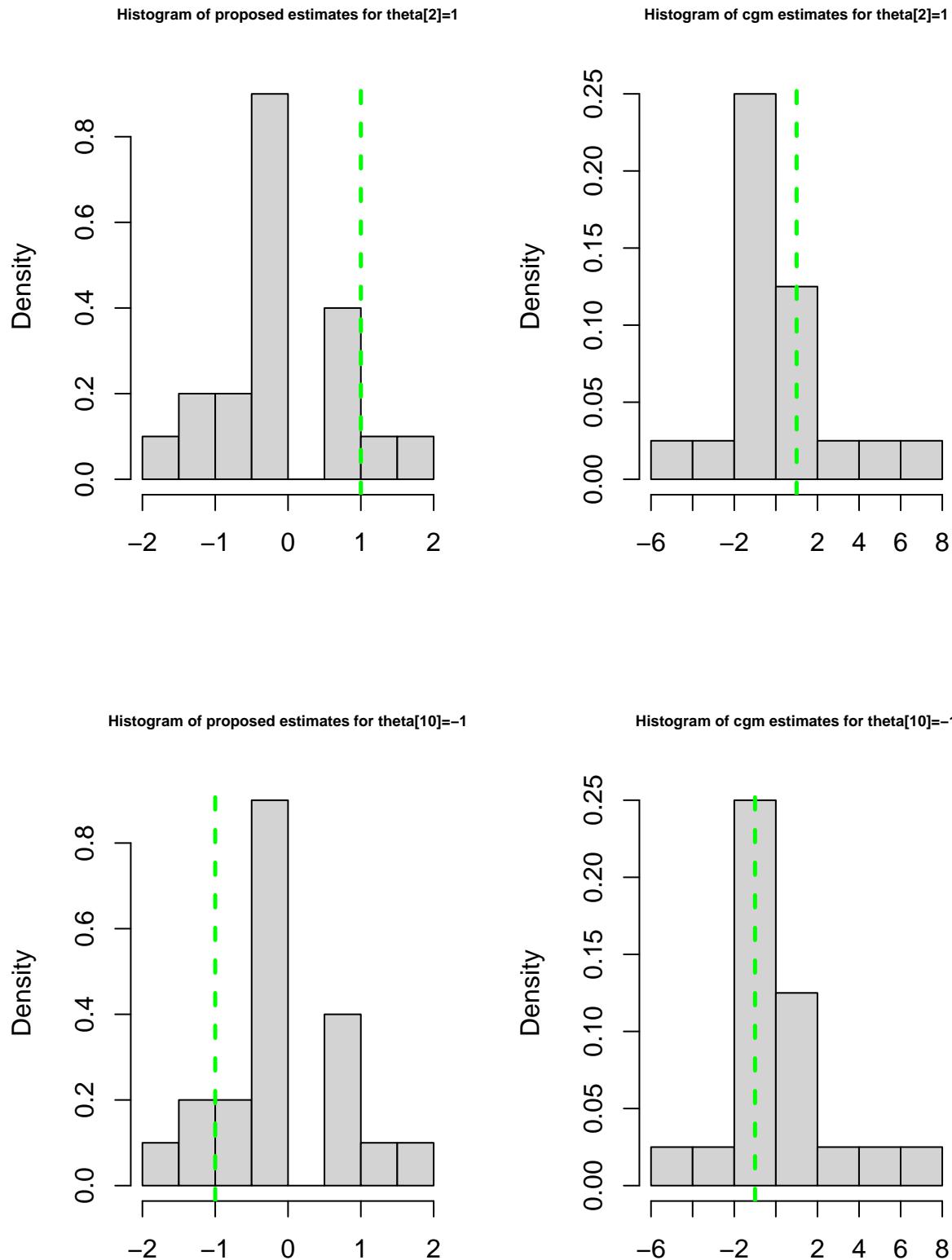
Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[2]	0.161	11.259
theta[10]	0.131	21.325
theta[1]	0.070	0.970
theta[5]	0.096	0.917
total	0.115	8.618

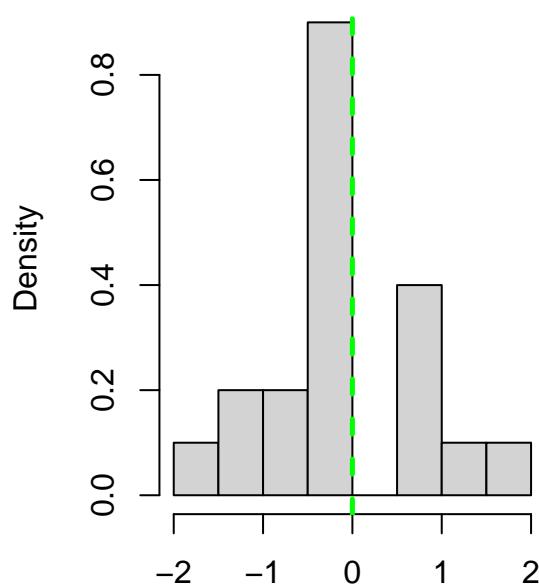
Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[2]	0.232	1.028
theta[10]	0.149	1.816
theta[1]	0.051	0.098
theta[5]	0.030	0.114
total	0.115	0.764

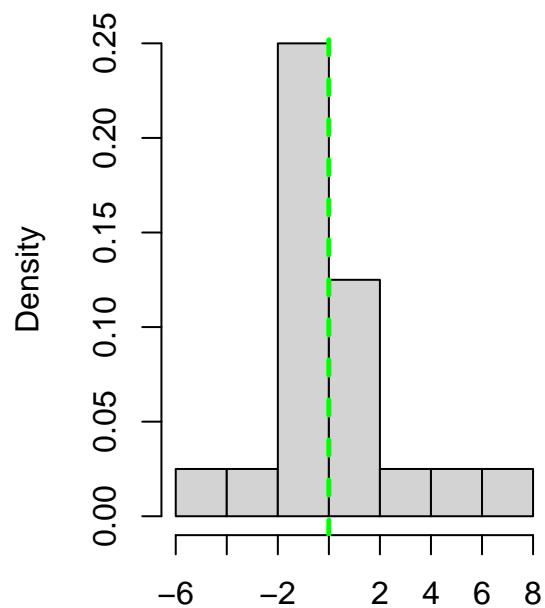
## Boxplots



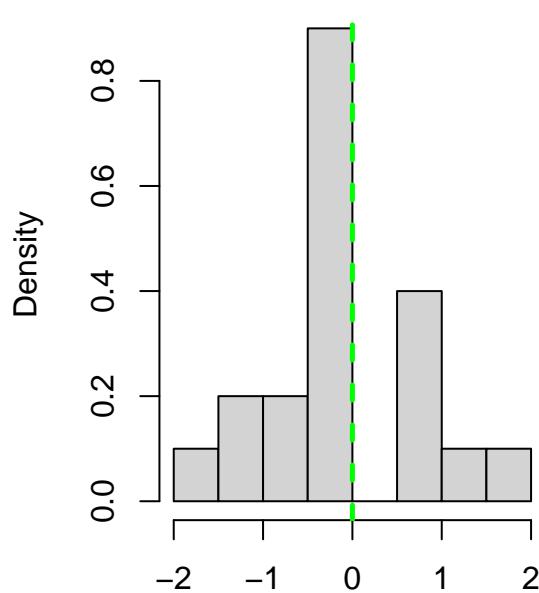
Histogram of proposed estimates for  $\theta[1]=0$



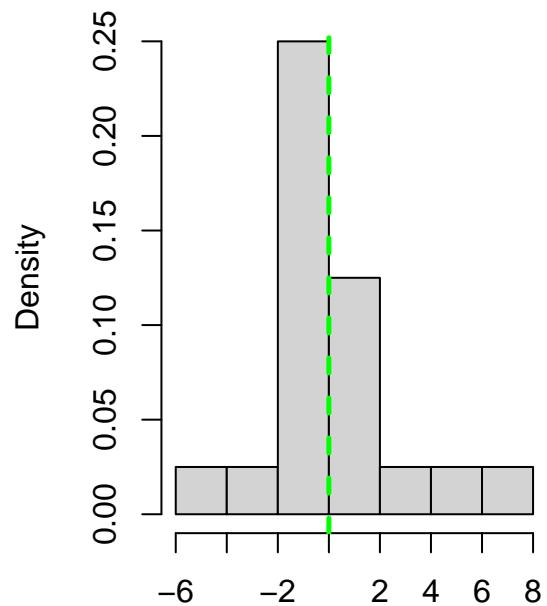
Histogram of cgm estimates for  $\theta[1]=0$



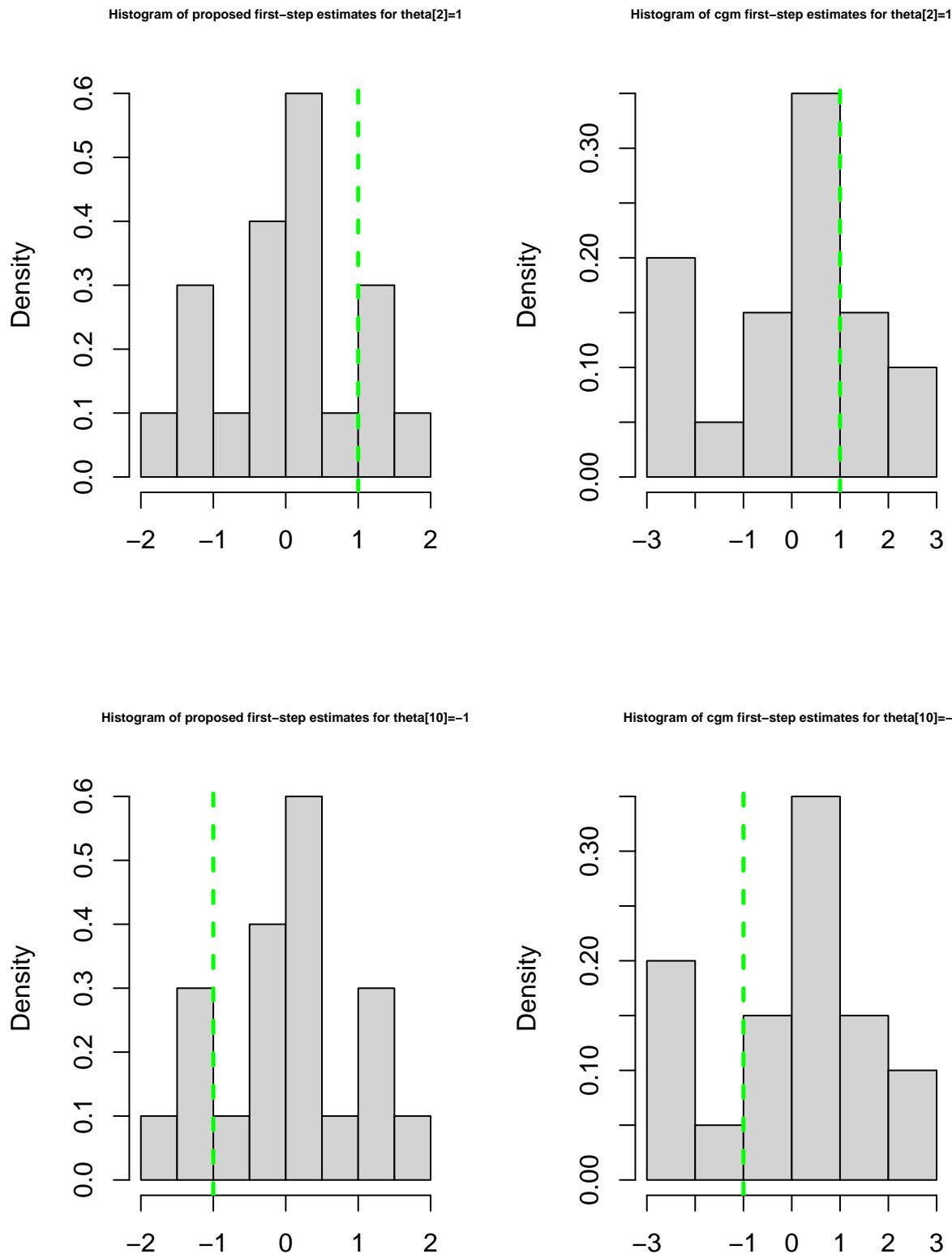
Histogram of proposed estimates for theta[5]=0



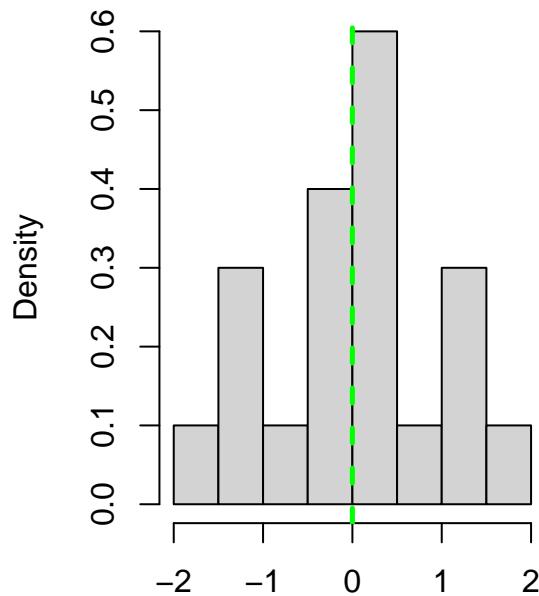
Histogram of cgm estimates for theta[5]=0



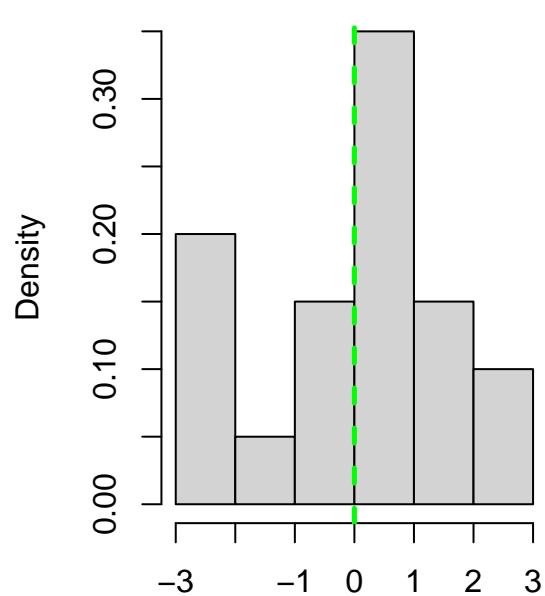
## First Step Histograms



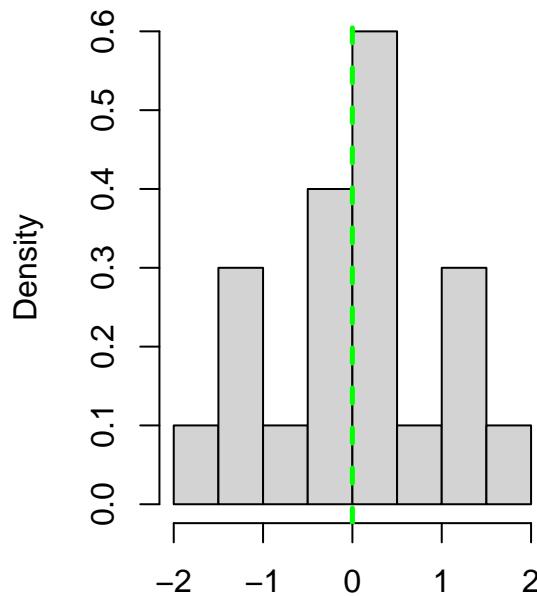
Histogram of proposed first-step estimates for  $\theta[1]=0$



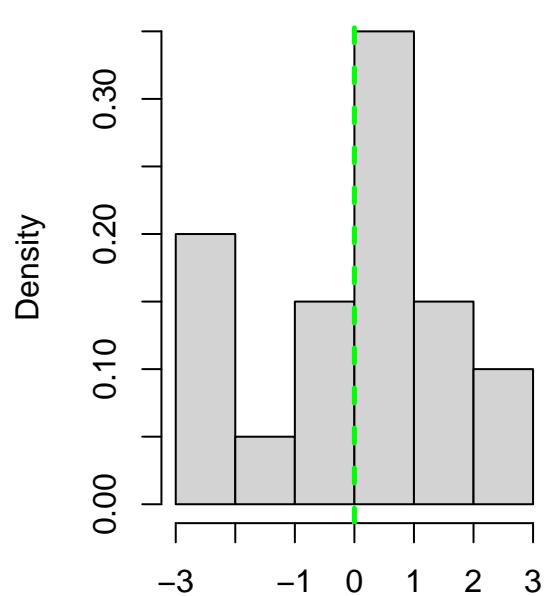
Histogram of cgm first-step estimates for  $\theta[1]=0$



Histogram of proposed first-step estimates for  $\theta[5]=0$



Histogram of cgm first-step estimates for  $\theta[5]=0$



### Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 3: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[2]	0.759	0.923	1.770	0.767	1.728
theta[10]	-1.562	-1.117	-0.444	-1.518	-0.499
theta[1]	-0.566	-0.105	-0.025	-0.521	-0.031
theta[5]	-0.185	-0.054	0.646	-0.182	0.576

Table 4: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[2]	-4.921	-1.098	0.741	-4.722	0.653
theta[10]	-0.892	2.486	6.384	-0.762	6.261
theta[1]	-1.470	-0.594	1.152	-1.422	1.021
theta[5]	-1.749	-0.522	0.043	-1.686	0.041

### Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[2]	1.128	0.263	0.612	1.644	1.0
theta[10]	-1.049	0.271	-1.579	-0.518	1.0
theta[1]	-0.178	0.180	-0.532	0.176	0.8
theta[5]	0.040	0.178	-0.309	0.388	0.8

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[2]	-1.671	5.569	-12.585	9.243	1
theta[10]	2.709	6.436	-9.906	15.324	1
theta[1]	-0.412	2.431	-5.176	4.353	1
theta[5]	-0.666	2.526	-5.618	4.286	1