Simulation Results

2026-01-14

Simulation Setup

This simulation is performed with $n = 200$ and $d = 20$, using the 2-d lattice as the underlying graph. $s = 2$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.2$. The attached results are for a 10-replication simulation. The true values of the parameter vector θ are

```
0 0 -0.7071068 0 0 0 -0.7071068 0 0 0 0 0 0 0 0 0 0 0 0 0 ,
```

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{ 3, 7, 5, 9\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat\thet$

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[3]	0.216	0.080
theta[7]	0.080	0.290
theta[5]	0.462	0.035
theta[9]	0.050	0.032
total	0.202	0.109

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[3]	0.163	0.152
theta[7]	0.110	0.191
theta[5]	0.074	0.001
theta[9]	0.024	0.000
total	0.093	0.086

Mean absolute deviation comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat$

Table 3: Mean Absolute Deviation of Parameter Estimates

	proposed	cgm
theta[3]	0.328	0.250
theta[7]	0.232	0.438
theta[5]	0.472	0.164
theta[9]	0.178	0.148
total	0.302	0.250

Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

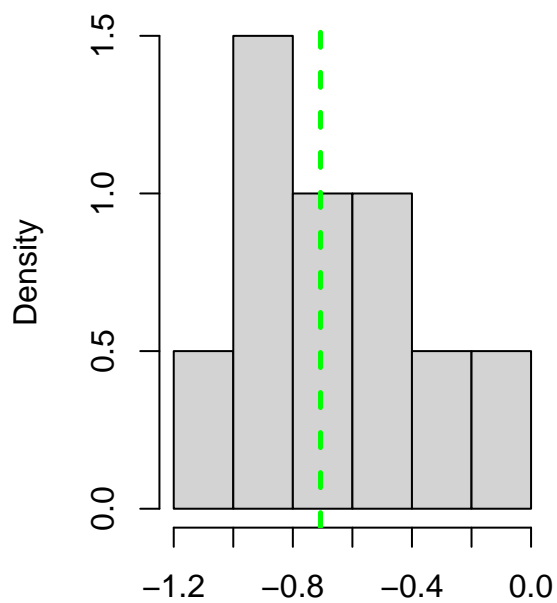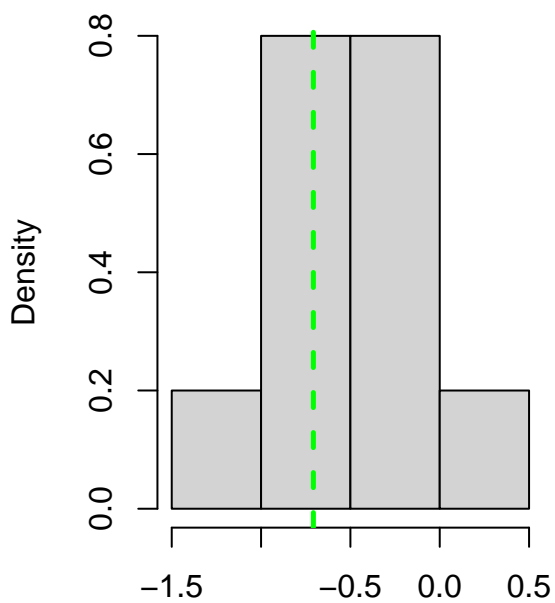|	proposed	cgm
theta[3]	0.334	0.347
theta[7]	0.242	0.406
theta[5]	0.188	0.011
theta[9]	0.094	0.000
total	0.215	0.191

Boxplots

Histogram of proposed estimates for theta[3]=−0.707
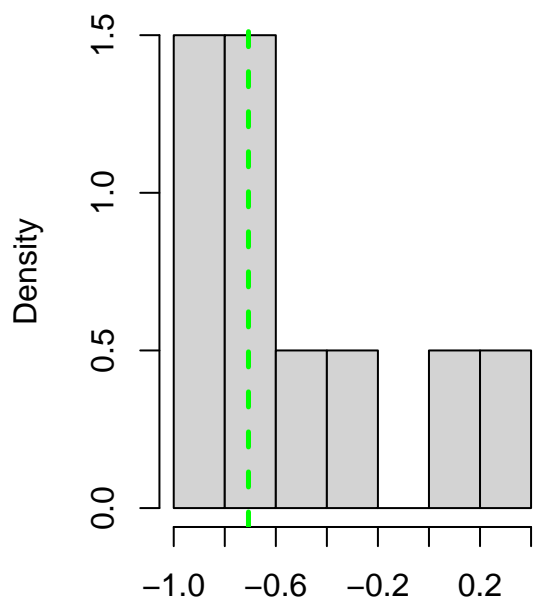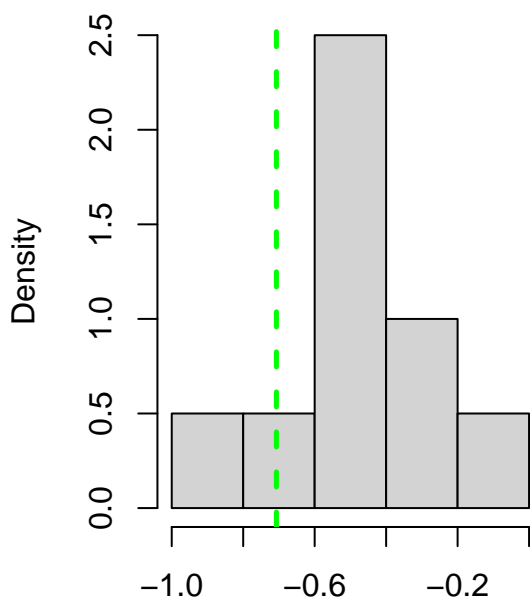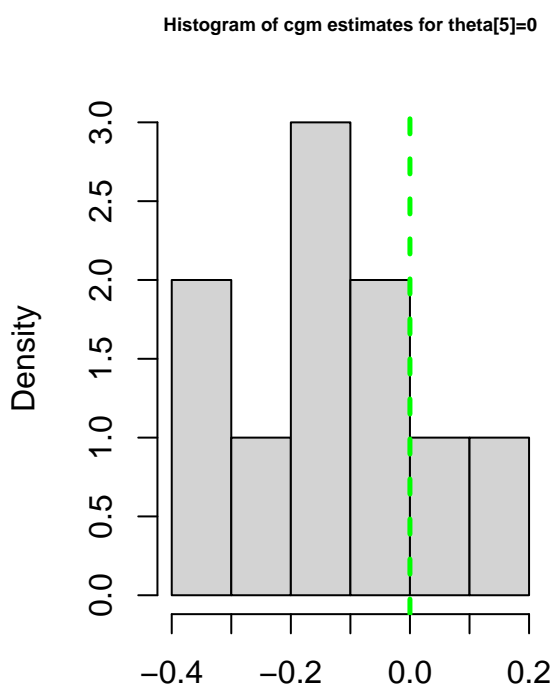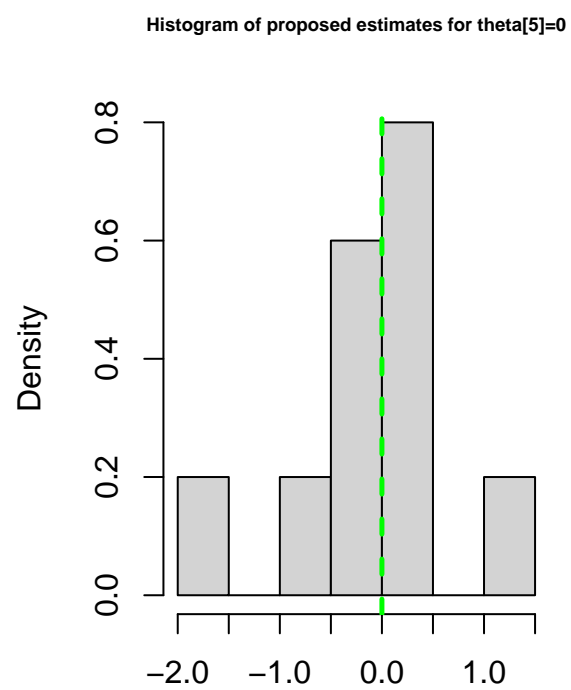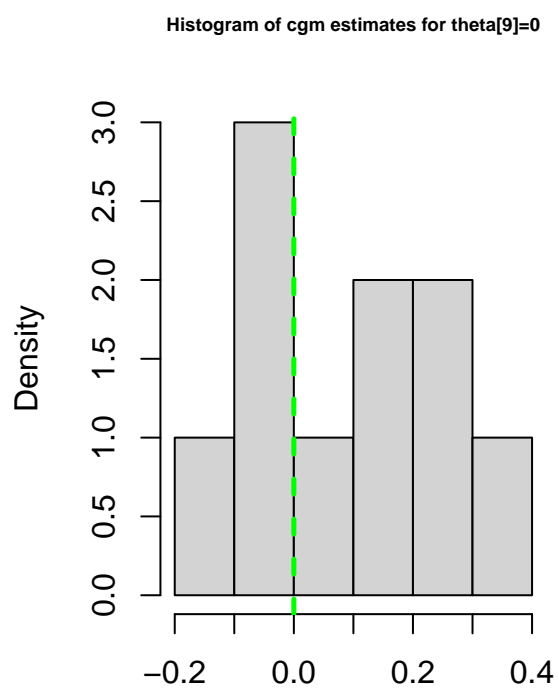

Histogram of cgm estimates for theta[3]=−0.707

Histogram of proposed estimates for theta[7]=−0.707

Histogram of cgm estimates for theta[7]=−0.707

Histogram of proposed estimates for theta[5]=0

Histogram of cgm estimates for theta[5]=0

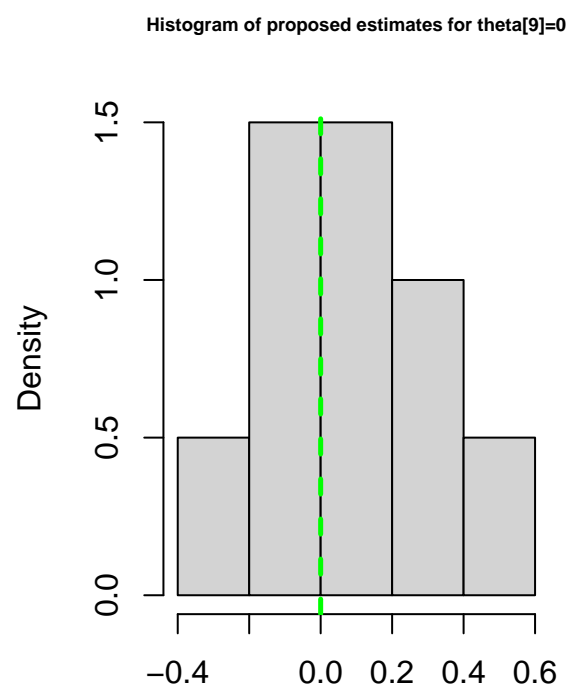**Histogram of proposed estimates for theta[9]=0**
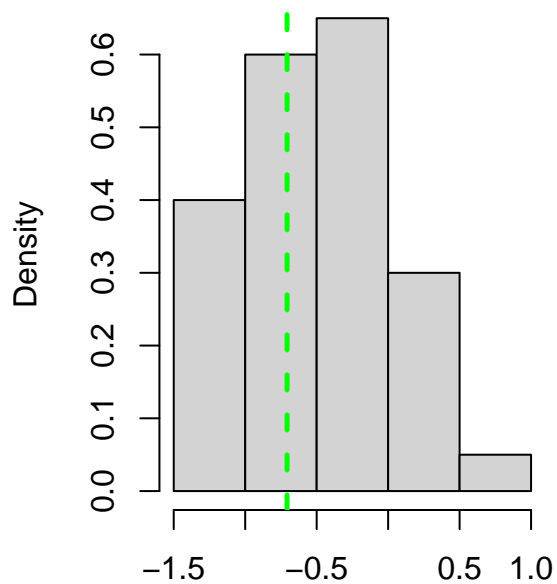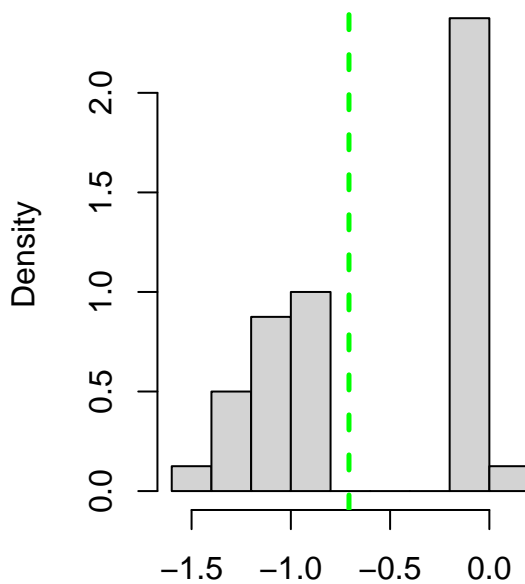

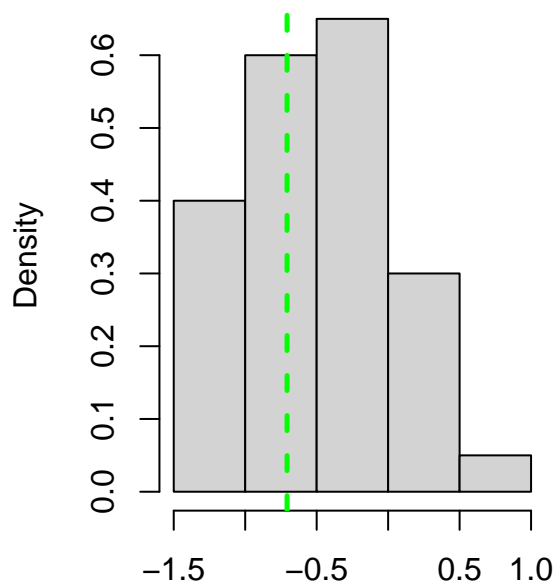**Histogram of cgm estimates for theta[9]=0**

First Step Histograms

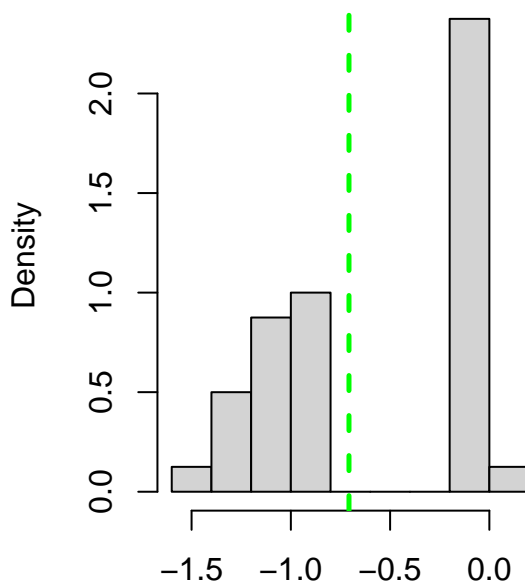**Histogram of proposed first−step estimates for theta[3]=−0.707**

Histogram of cgm first−step estimates for theta[3]=−0.707

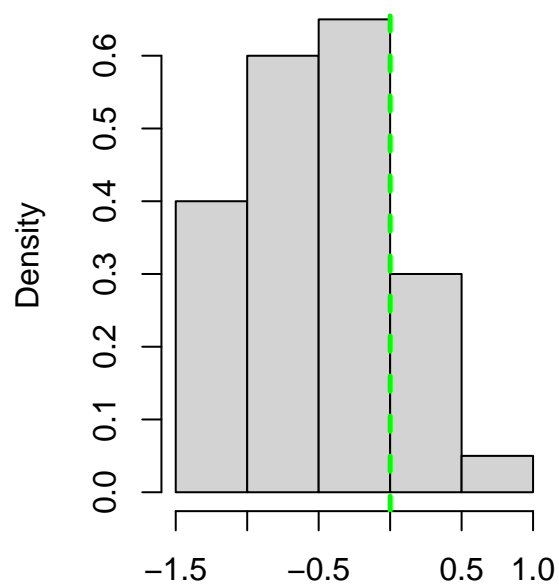**Histogram of proposed first−step estimates for theta[7]=−0.707**
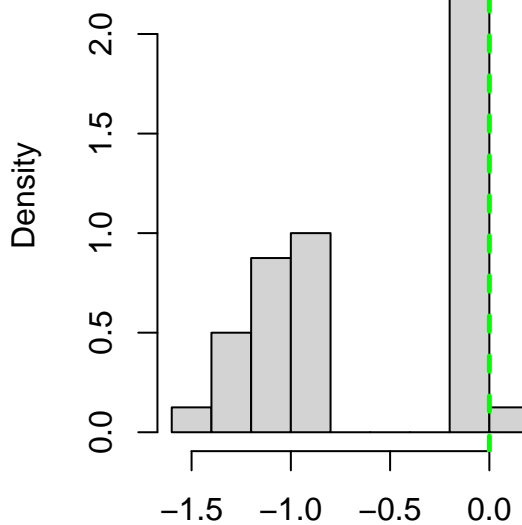

Histogram of cgm first−step estimates for theta[7]=−0.707
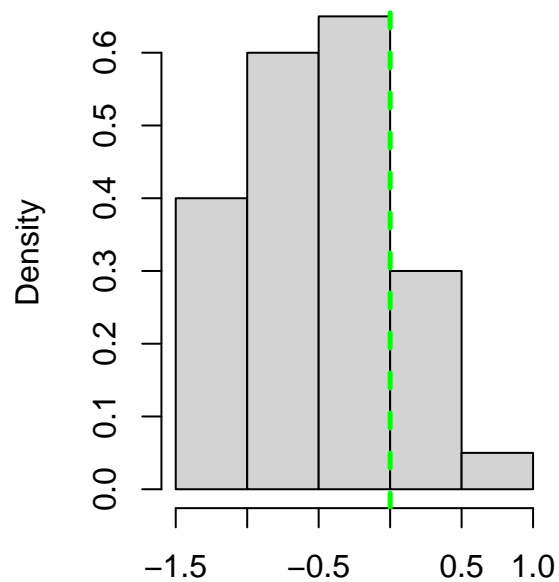
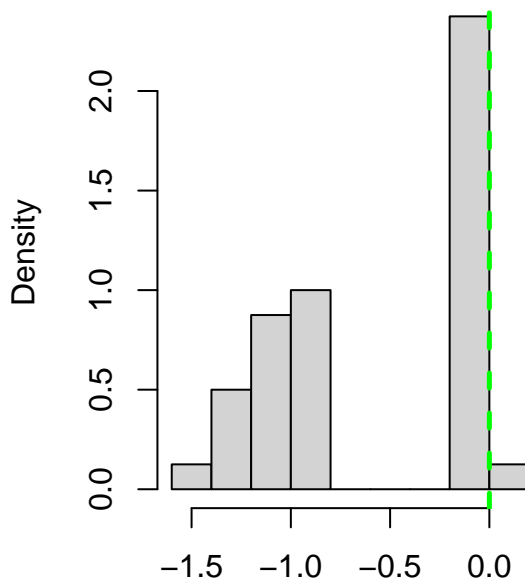**Histogram of proposed first−step estimates for theta[5]=0**   **Histogram of cgm first−step estimates for theta[5]=0**

Histogram of proposed first−step estimates for theta[9]=0   **Histogram of cgm first−step estimates for theta[9]=0**

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 5: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[3]	-0.975	-0.704	0.253	-0.973	0.238
theta[7]	-1.055	-0.769	-0.096	-1.041	-0.159
theta[5]	-1.571	-0.032	1.140	-1.375	0.949
theta[9]	-0.391	0.086	0.423	-0.318	0.383

Table 6: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[3]	-0.898	-0.517	-0.144	-0.840	-0.185
theta[7]	-1.009	-0.411	0.469	-0.997	0.355
theta[5]	-0.320	-0.132	0.144	-0.319	0.127
theta[9]	-0.109	0.065	0.378	-0.104	0.352

Statistics for Theoretical 95% Confidence Intervals

Table 7: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[3]	-0.537	0.240	-1.009	-0.066	0.7
theta[7]	-0.682	0.238	-1.148	-0.216	0.9
theta[5]	-0.107	0.203	-0.505	0.290	0.6
theta[9]	0.068	0.187	-0.300	0.435	0.8

Table 8: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[3]	-0.495	0.192	-0.871	-0.119	0.9
theta[7]	-0.411	0.204	-0.811	-0.011	0.5
theta[5]	-0.122	0.127	-0.370	0.126	0.8
theta[9]	0.075	0.130	-0.180	0.330	0.9