

Simulation Results

2026-01-26

Simulation Setup

This simulation is performed with $n = 400$ and $d = 200$, using the 2-d lattice as the underlying graph. $s = 5$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.2$. The attached results are for a 10-replication simulation. The parameter vector θ has sparse components other than the following:

Parameter.Index	Value
54	0.447
70	0.447
107	0.447
121	0.447
132	0.447

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{ 54, 70, 69, 172 \}$ elements of θ . Accordingly, **all statistics and visuals are indicative of performance only on the set \mathcal{C} .**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

Mean-squared error comparison

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[54]	0.053	0.037
theta[70]	0.029	0.019
theta[69]	0.011	0.004
theta[172]	0.004	0.002
total	0.024	0.015

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[54]	0.153	0.093

	proposed	cgm
theta[70]	0.131	0.091
theta[69]	0.000	0.000
theta[172]	0.000	0.000
total	0.071	0.046

Mean absolute deviation comparison $\frac{1}{n.\text{sim}} \sum_{i=1}^{n.\text{sim}} \frac{1}{|\mathcal{C}|} \|\hat{\theta}_i - \theta\|$

Table 3: Mean Absolute Deviation of Parameter Estimates

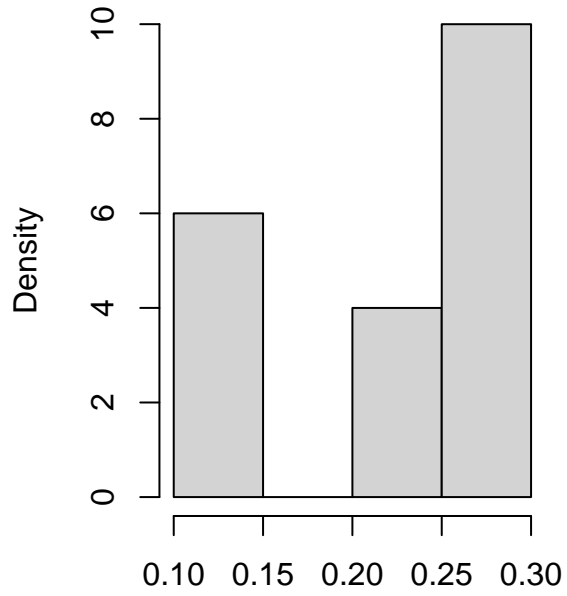
	proposed	cgm
theta[54]	0.221	0.182
theta[70]	0.149	0.127
theta[69]	0.084	0.050
theta[172]	0.046	0.034
total	0.125	0.098

Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

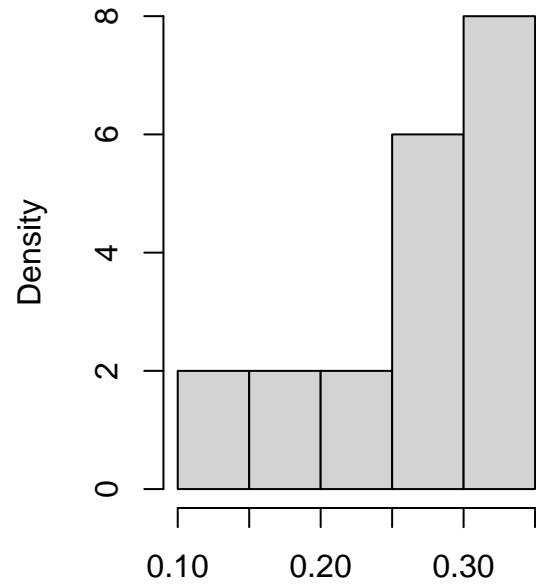
	proposed	cgm
theta[54]	0.382	0.279
theta[70]	0.351	0.279
theta[69]	0.000	0.000
theta[172]	0.001	0.000
total	0.183	0.139

Boxplots

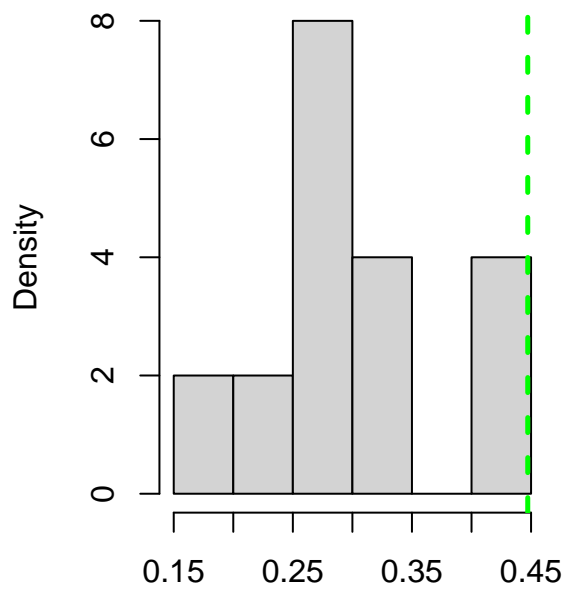
Histogram of proposed estimates for $\theta_{[54]}=0.447$



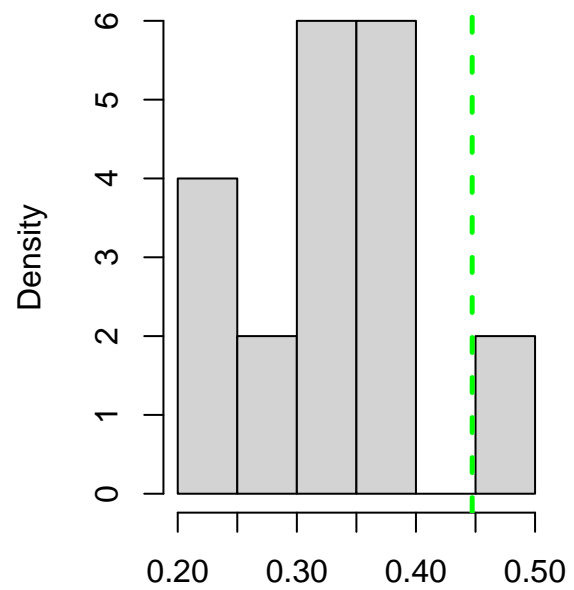
Histogram of cgm estimates for $\theta_{[54]}=0.447$



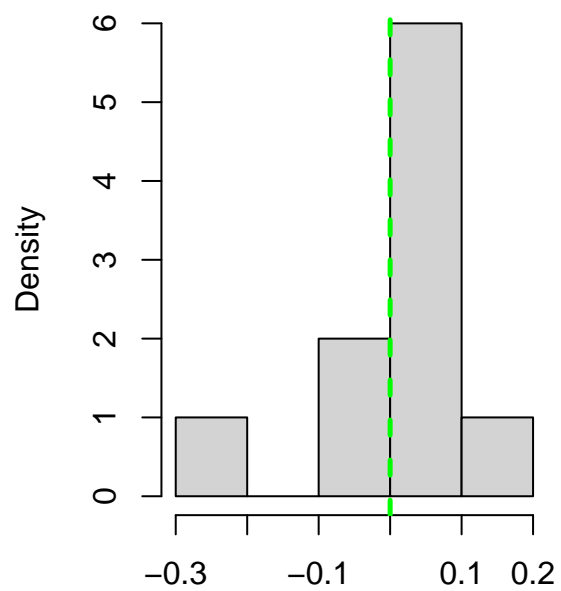
Histogram of proposed estimates for $\theta_{[70]}=0.447$



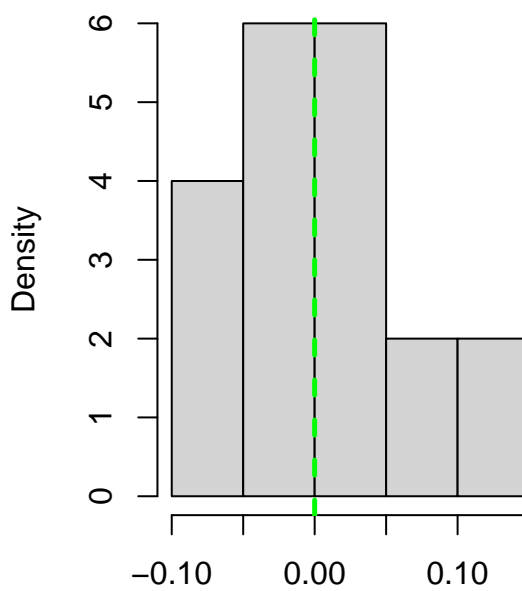
Histogram of cgm estimates for $\theta_{[70]}=0.447$



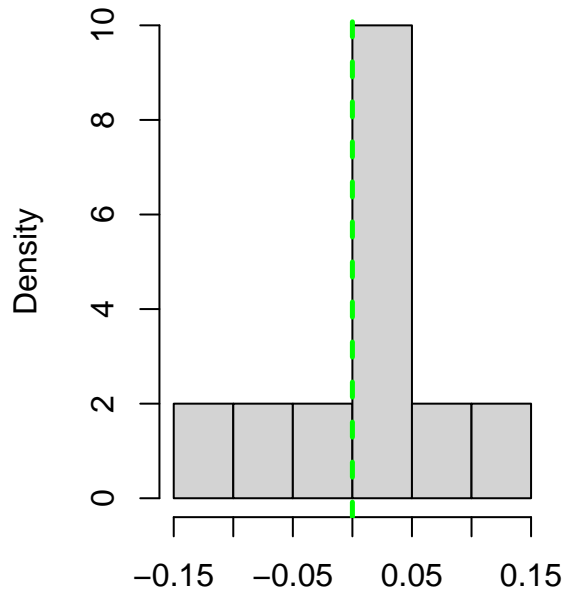
Histogram of proposed estimates for $\theta_{69}=0$



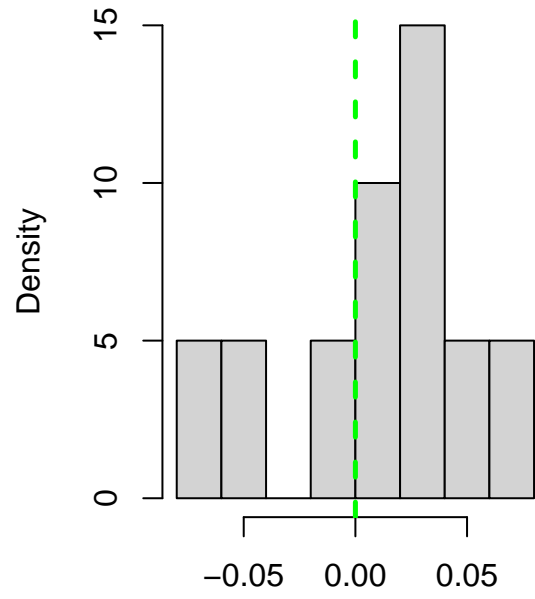
Histogram of cgm estimates for $\theta_{69}=0$



Histogram of proposed estimates for $\theta_{172}=0$



Histogram of cgm estimates for $\theta_{172}=0$



Statistics and 95% Confidence Intervals from per-Replicate Estimates

Statistics for Theoretical 95% Confidence Intervals

Table 5: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[54]	0.227	0.102	0.027	0.426	0.5
theta[70]	0.299	0.097	0.109	0.488	0.6
theta[69]	0.016	0.103	-0.186	0.218	0.9
theta[172]	0.009	0.102	-0.190	0.208	1.0

Table 6: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[54]	0.266	0.068	0.132	0.400	0.3
theta[70]	0.326	0.069	0.190	0.462	0.6
theta[69]	0.008	0.068	-0.125	0.141	0.9
theta[172]	0.009	0.069	-0.127	0.145	1.0