Simulation Results

2025-10-06

Simulation Setup

This simulation is performed with $n = 100$ and $d = 5$, using the 2-d lattice as the underlying graph. $s = 1$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.1$. The attached results are for a 5-replication simulation. The true values of the parameter vector θ are

```
[1] 0 0 1 0 0
```

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

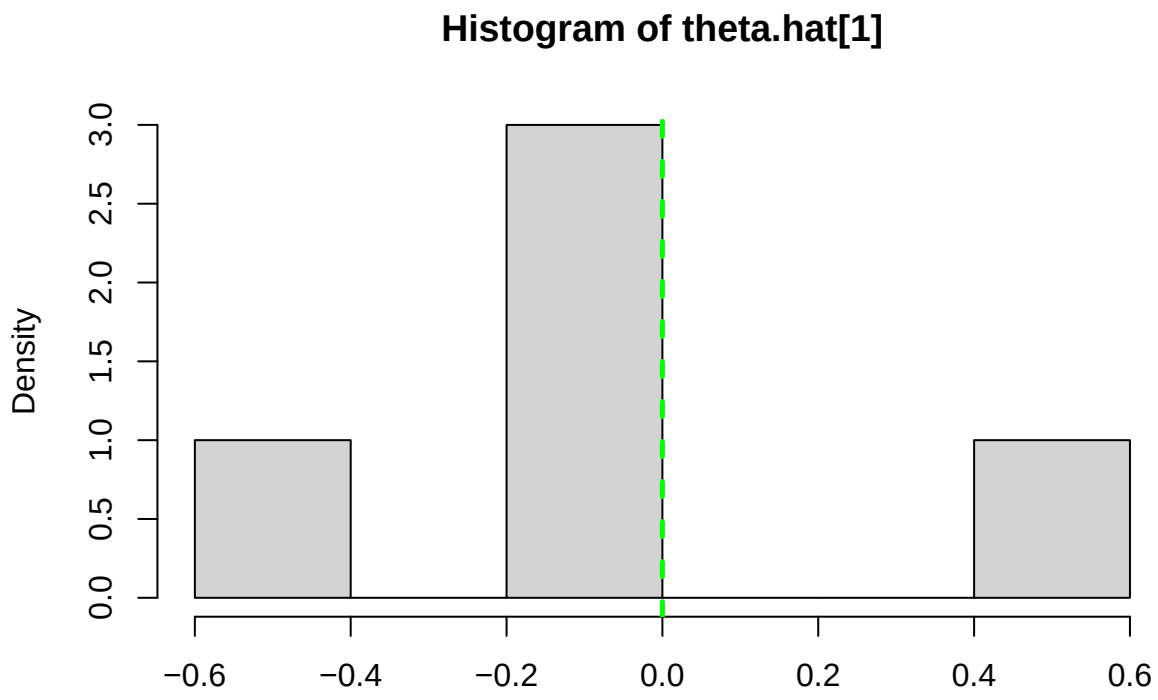
Results

Mean-squared error comparison ($\frac{1}{n.sim}\sum_{i=1}^{n.sim}\frac{1}{d}\|\hat{\theta}_i - \theta\|^2$)

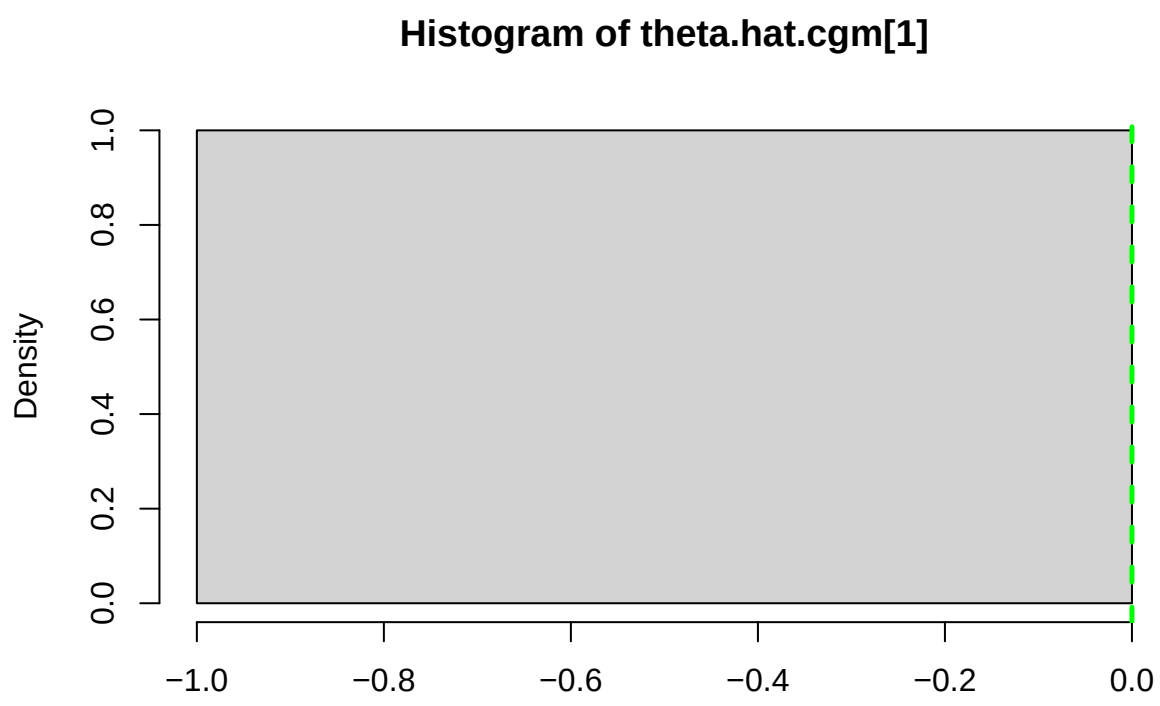
```
# A tibble: 1 x 2
  `MISLE (First-step) MSE` `MISLE MSE`
                     <dbl>       <dbl>
1                    0.424        13.3
```

```
# A tibble: 1 x 2
  `MISLE MSE` `CGM MSE`
        <dbl>     <dbl>
1        13.3     0.121
```

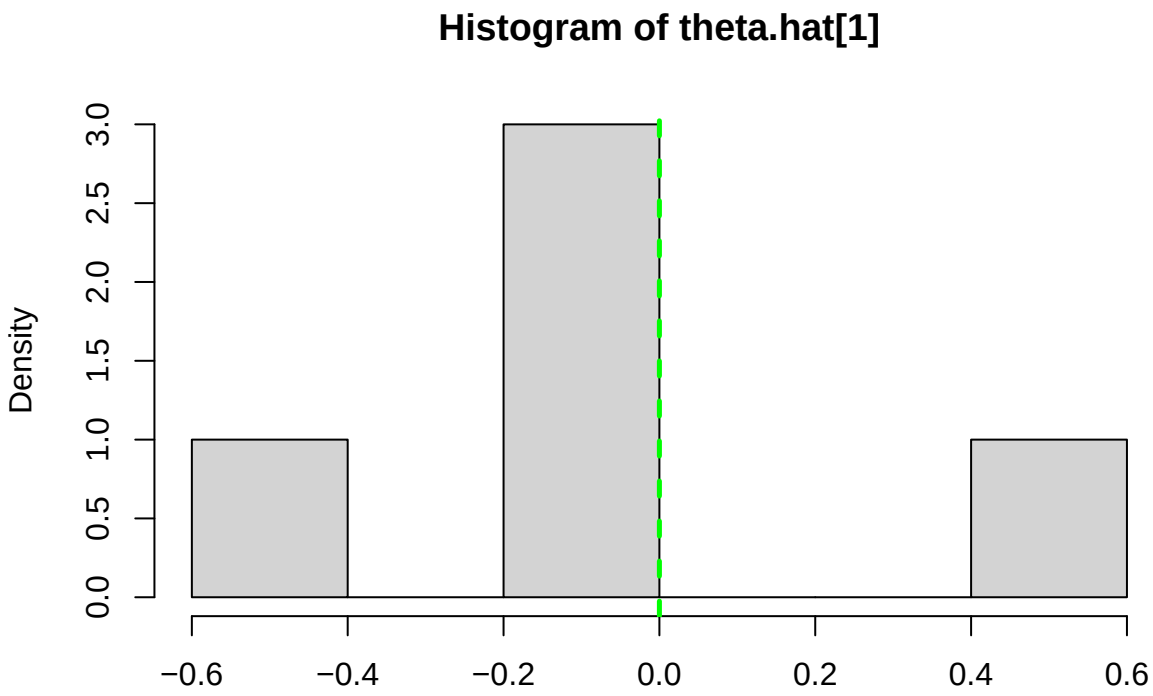
First Step Histograms



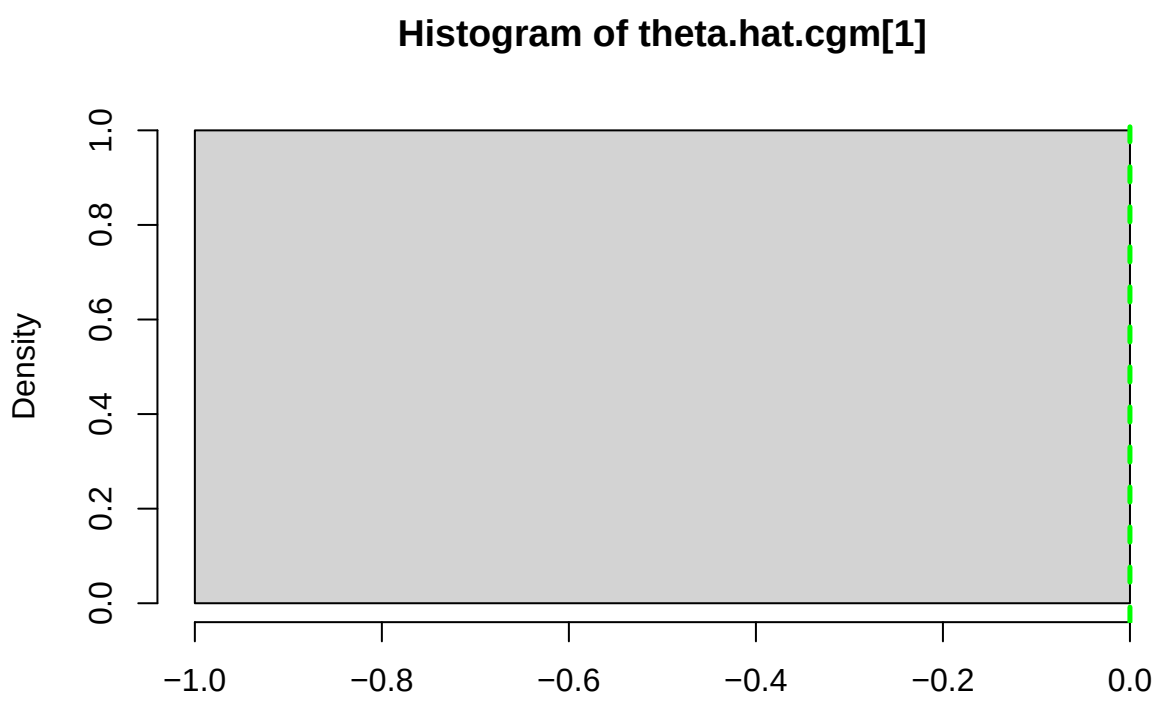

```
[1] "Summary statistics of bootstrap replicates:"
     Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
-0.471404 -0.036868  0.000000  0.001059  0.000000  0.513570
[1] "95% CI based on bootstrap:"
       lower     upper
1 -0.4279508 0.4622126
```



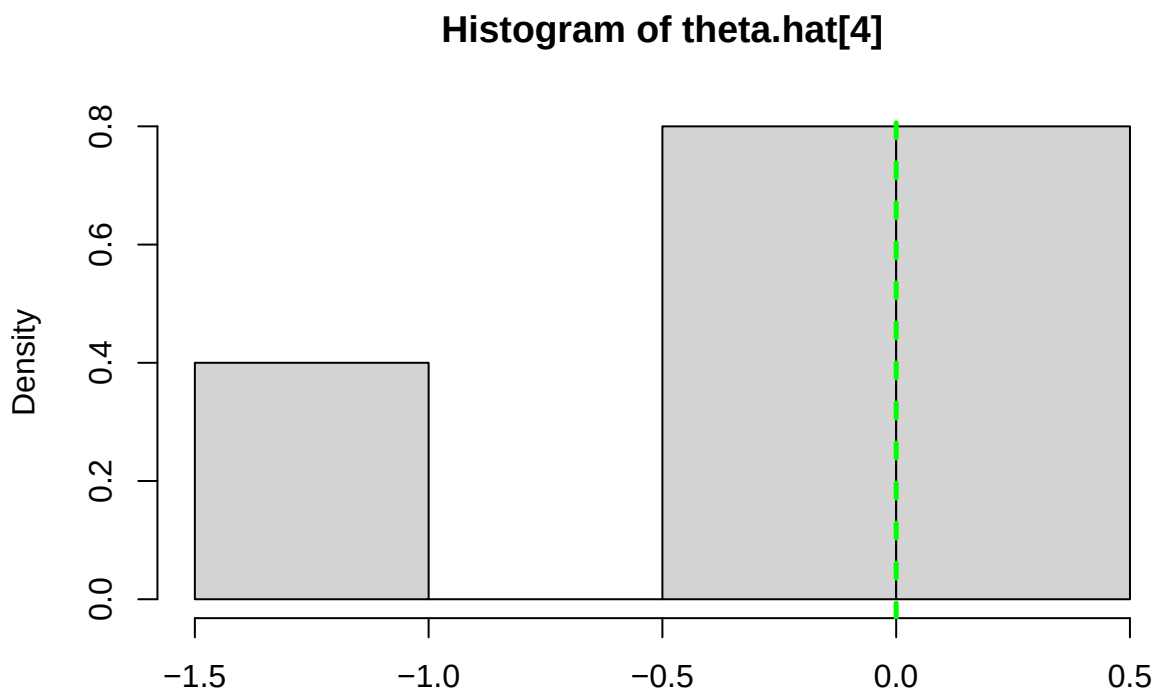

```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
      0       0       0       0       0       0
[1] "95% CI based on bootstrap:"
  lower.cgm upper.cgm
1         0         0
```



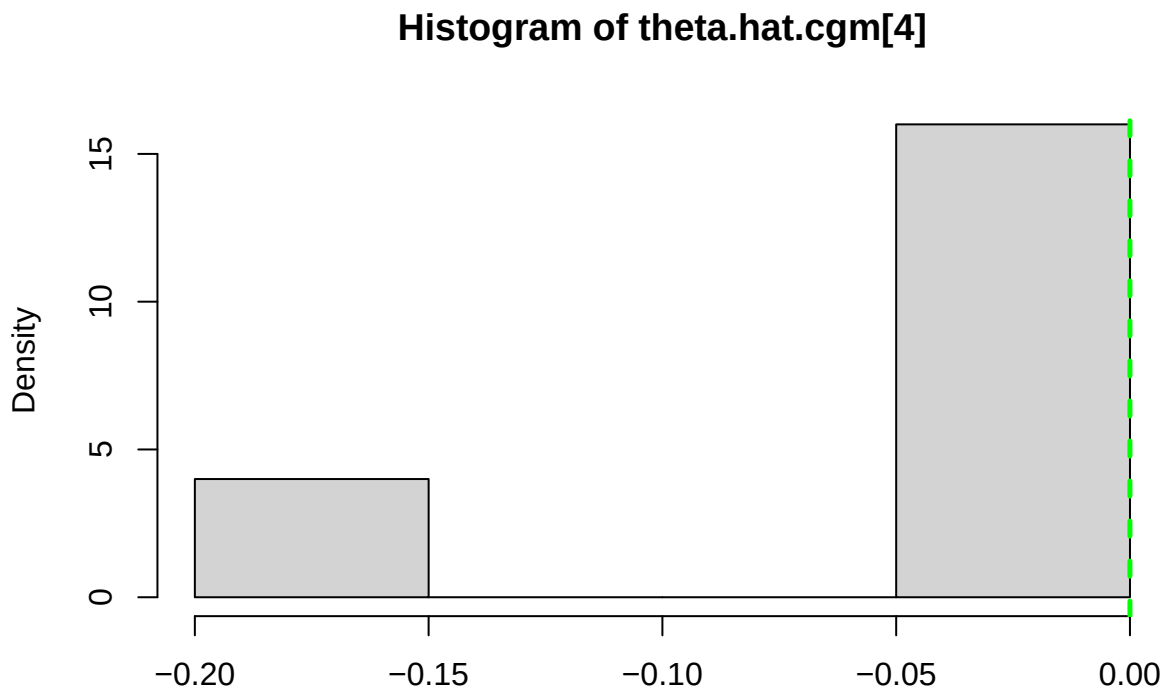

```
[1] "Summary statistics of bootstrap replicates:"
     Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
-0.471404 -0.036868  0.000000  0.001059  0.000000  0.513570
[1] "95% CI based on bootstrap:"
       lower     upper
1 -0.4279508 0.4622126
```




```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
      0       0       0       0       0       0
[1] "95% CI based on bootstrap:"
  lower.cgm upper.cgm
1         0         0
```

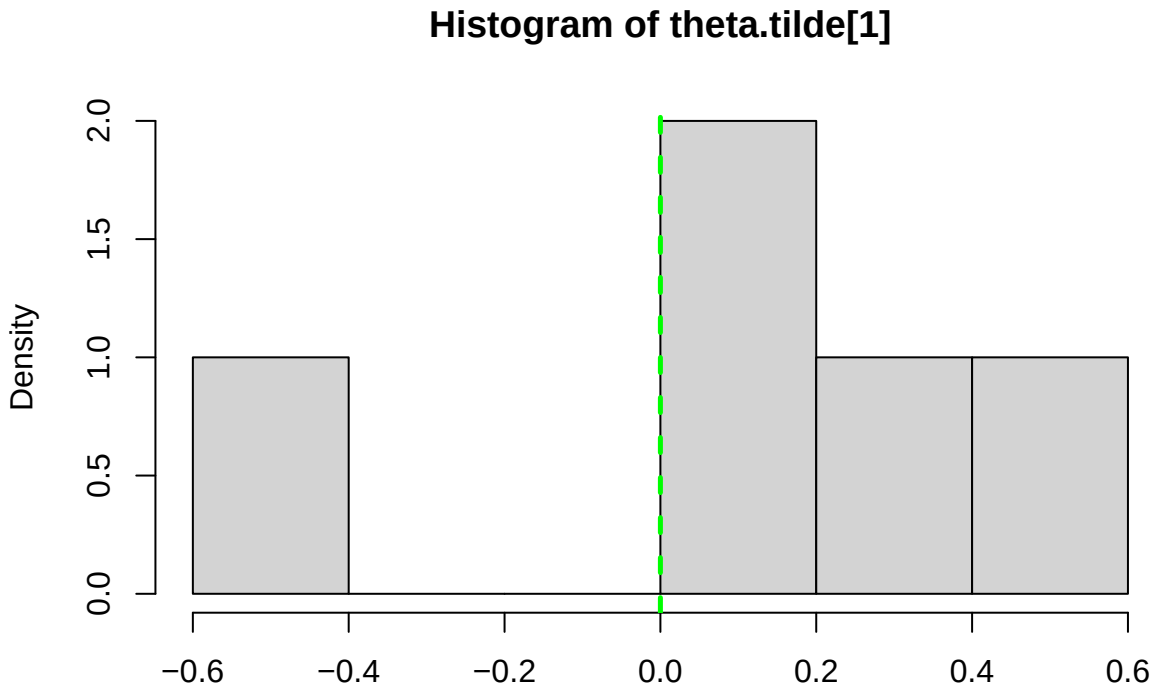



```
[1] "Summary statistics of bootstrap replicates:"
     Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
-1.000306 -0.311037 -0.004819 -0.183599  0.081261  0.316907
[1] "95% CI based on bootstrap:"
       lower     upper
1 -0.9313786 0.2933422
```

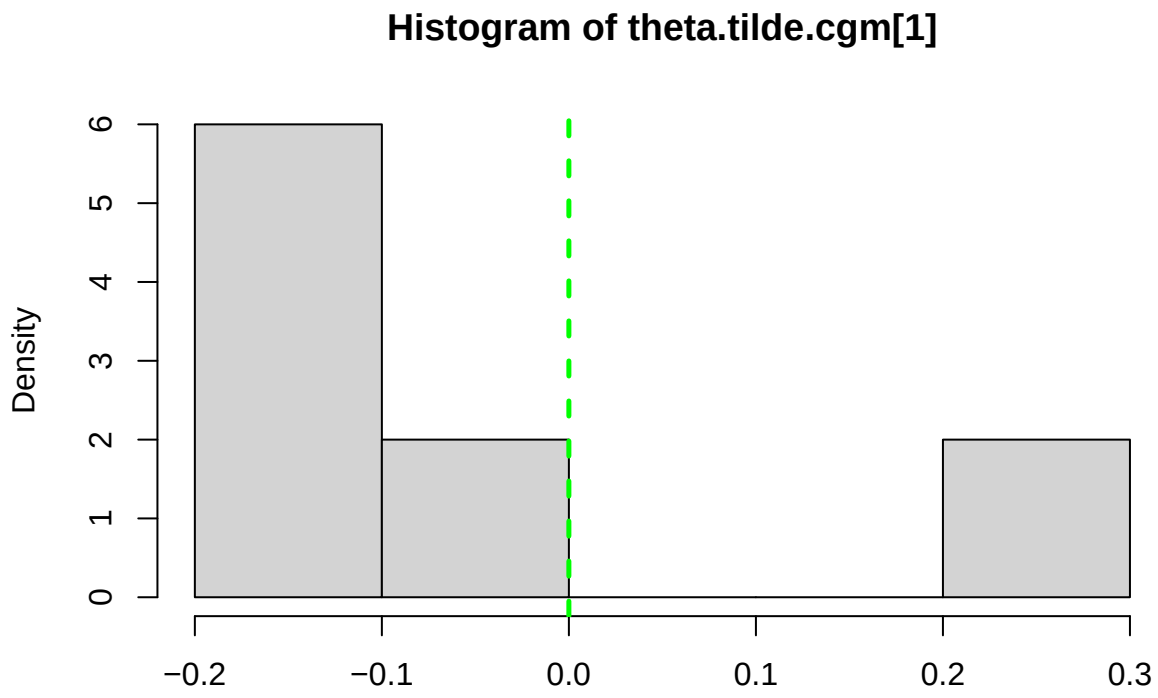



```
[1] "Summary statistics of bootstrap replicates:"
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-0.15629  0.00000  0.00000 -0.03126  0.00000  0.00000
[1] "95% CI based on bootstrap:"
   lower.cgm upper.cgm
1 -0.1406627         0
```

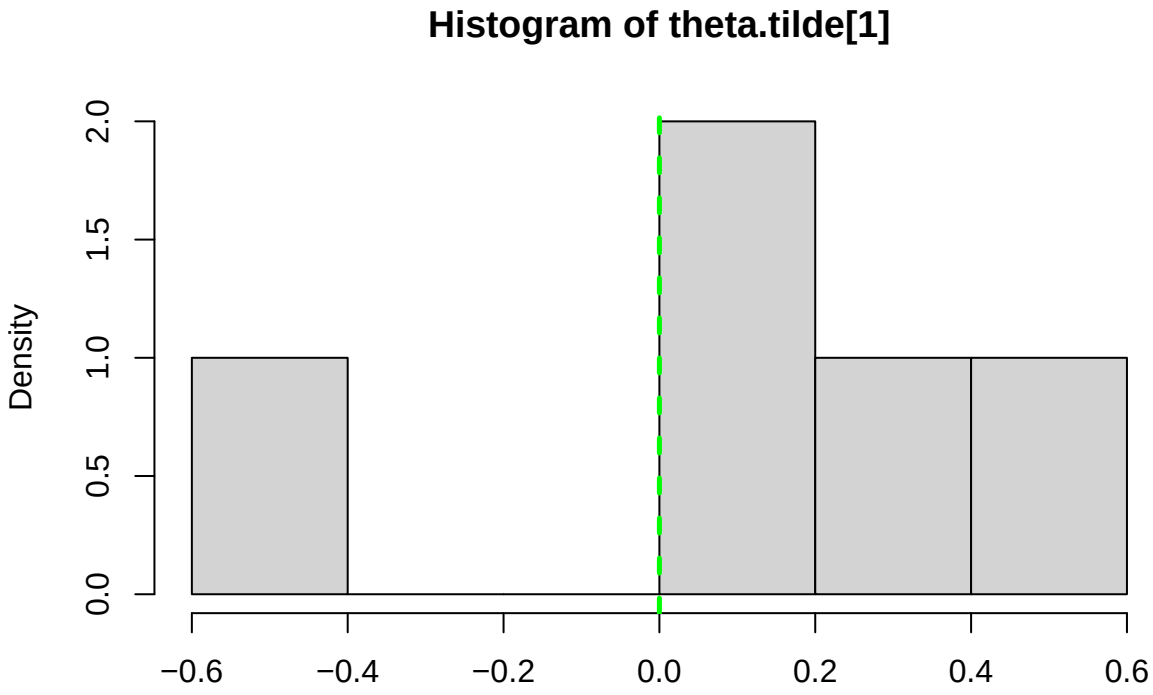
Statistics and 95% Confidence Intervals from per-Replicate Estimates



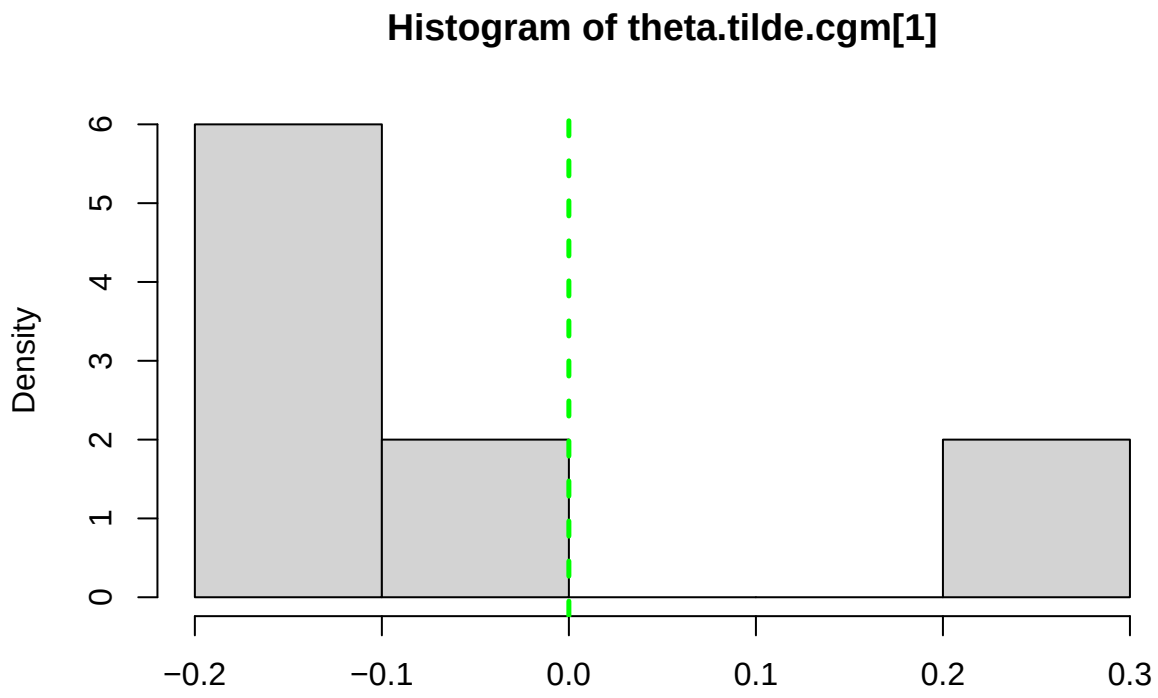

```
[1] "Summary statistics of bootstrap replicates:"
     Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
-0.530159  0.009647  0.074520  0.066001  0.248474  0.527525
[1] "95% CI based on bootstrap:"
       lower   upper
1 -0.4761783 0.49962
```



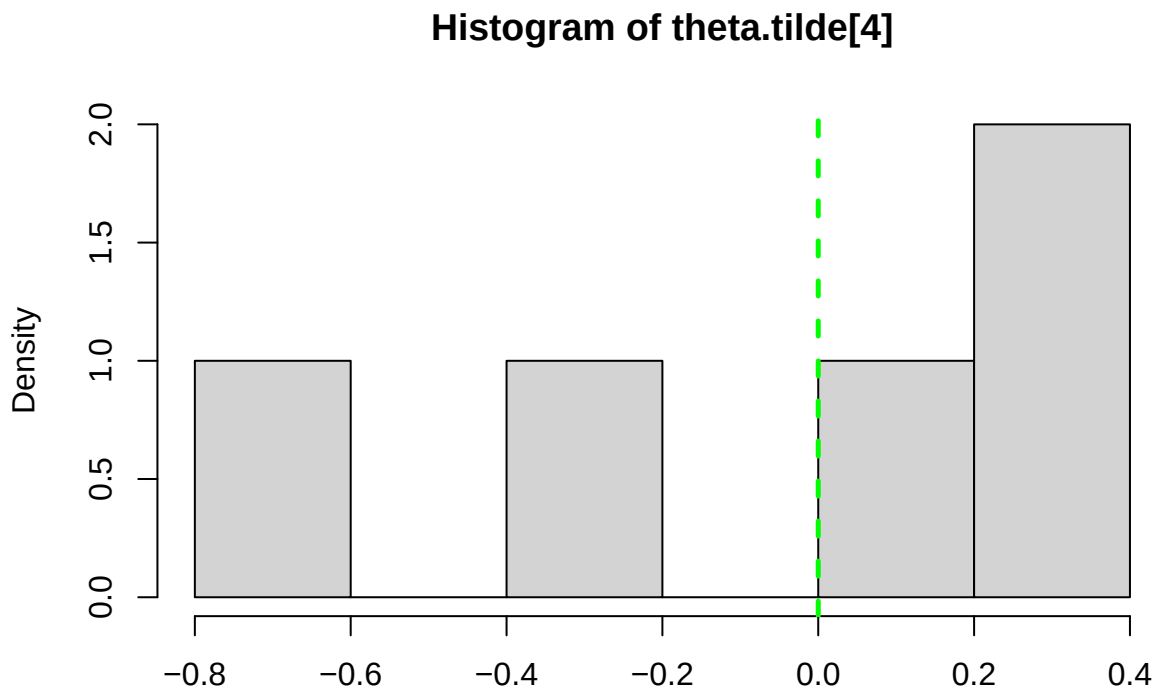

```
[1] "Summary statistics of bootstrap replicates:"
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-0.17888 -0.16491 -0.13916 -0.05467 -0.07586  0.28546
[1] "95% CI based on bootstrap:"
   lower.cgm upper.cgm
1 -0.1774808 0.2493242
```



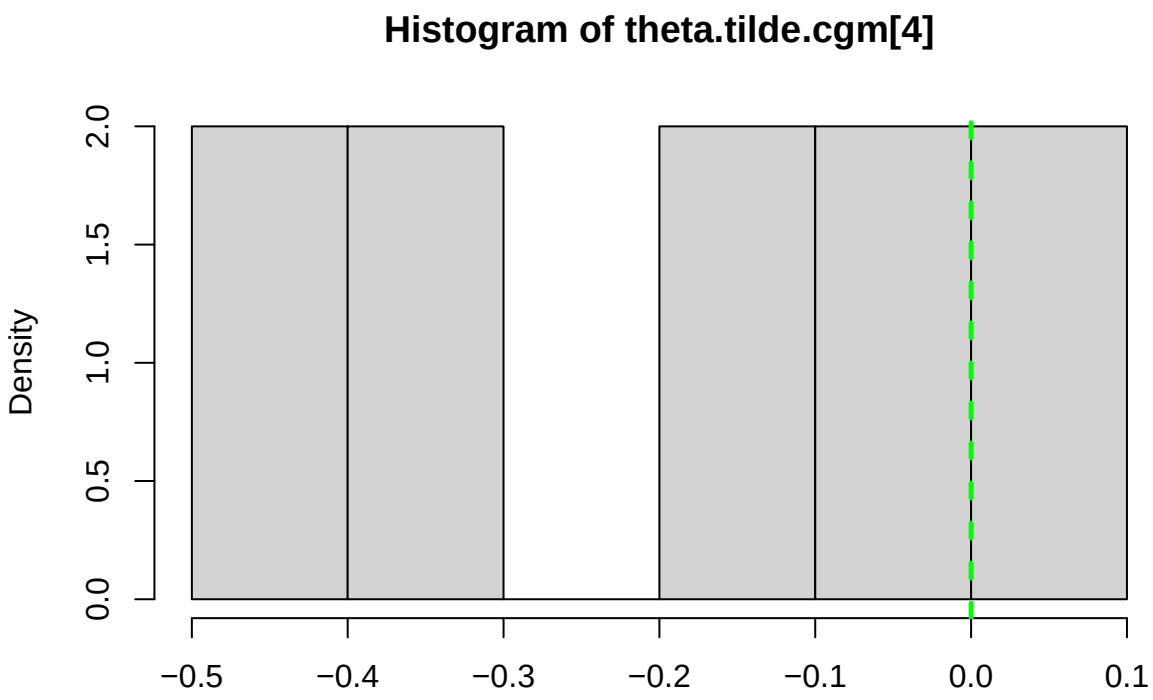

```
[1] "Summary statistics of bootstrap replicates:"
     Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
-0.530159  0.009647  0.074520  0.066001  0.248474  0.527525
[1] "95% CI based on bootstrap:"
       lower   upper
1 -0.4761783 0.49962
```




```
[1] "Summary statistics of bootstrap replicates:"
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-0.17888 -0.16491 -0.13916 -0.05467 -0.07586  0.28546
[1] "95% CI based on bootstrap:"
   lower.cgm upper.cgm
1 -0.1774808 0.2493242
```




```
[1] "Summary statistics of bootstrap replicates:"
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-0.69819 -0.26020  0.16550 -0.07147  0.20600  0.22956
[1] "95% CI based on bootstrap:"
       lower     upper
1 -0.6543879 0.2272047
```




```
[1] "Summary statistics of bootstrap replicates:"
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-0.41557 -0.34096 -0.11139 -0.18421 -0.08568  0.03256
[1] "95% CI based on bootstrap:"
   lower.cgm  upper.cgm
1 -0.4081053 0.02073759
```

Statistics for Theoretical 95% Confidence Intervals

```
[1] Length of Confidence Intervals for theta[1]
[1] Coverage proportion: 0.8
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.7309  0.8271  0.8422  0.9256  0.9056  1.3223
[1] Length of Confidence Intervals for theta[1] (CGM Method)
[1] Coverage proportion: 1
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  2.568   2.864   2.979   3.177   3.362   4.114
[1] Length of Confidence Intervals for theta[1]
[1] Coverage proportion: 0.8
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.7309  0.8271  0.8422  0.9256  0.9056  1.3223
[1] Length of Confidence Intervals for theta[1] (CGM Method)
[1] Coverage proportion: 1
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  2.568   2.864   2.979   3.177   3.362   4.114
[1] Length of Confidence Intervals for theta[4]
[1] Coverage proportion: 0.8
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.7125  0.7809  0.8293  0.8646  0.9002  1.1000
[1] Length of Confidence Intervals for theta[4] (CGM Method)
[1] Coverage proportion: 1
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  2.335   2.953   2.962   3.211   3.497   4.311
```