

Simulation Results

2025-10-14

Simulation Setup

This simulation is performed with $n = 400$ and $d = 10$, using the 2-d lattice as the underlying graph. $s = 2$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.1$. The attached results are for a 5-replication simulation. The true values of the parameter vector θ are

```
[1] 0 0 0 0 0 0 0 0 1 -1 0
```

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

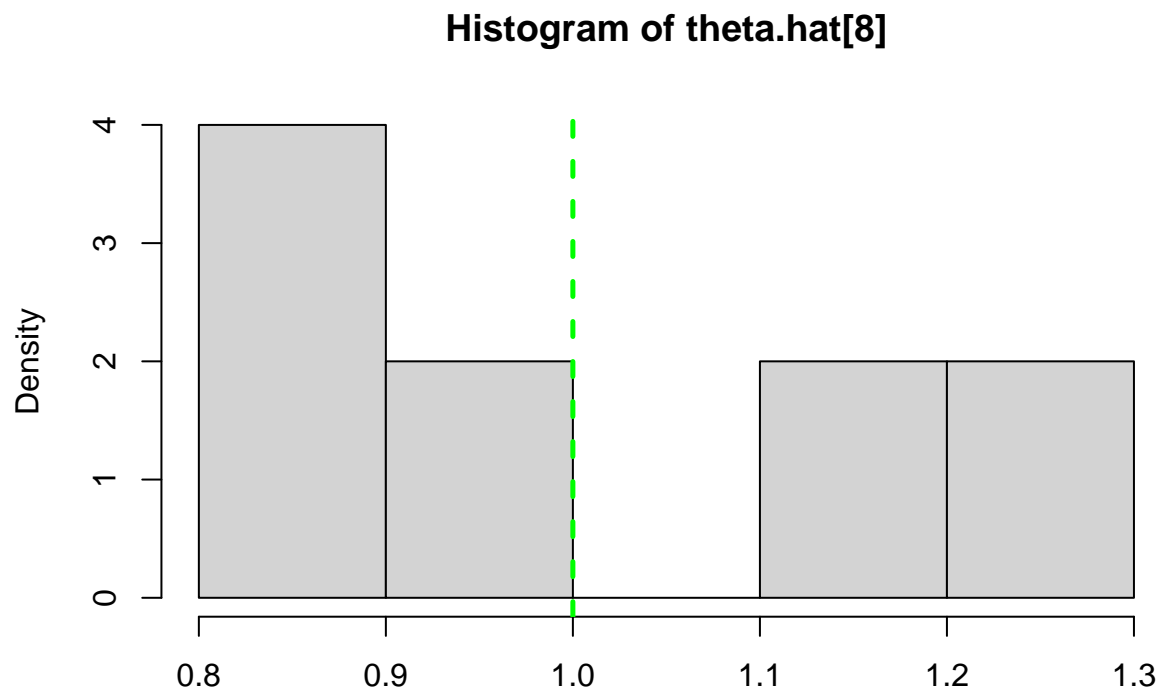
Results

Mean-squared error comparison ($\frac{1}{n.sim} \sum_{i=1}^{n.sim} \frac{1}{d} \|\hat{\theta}_i - \theta\|^2$)

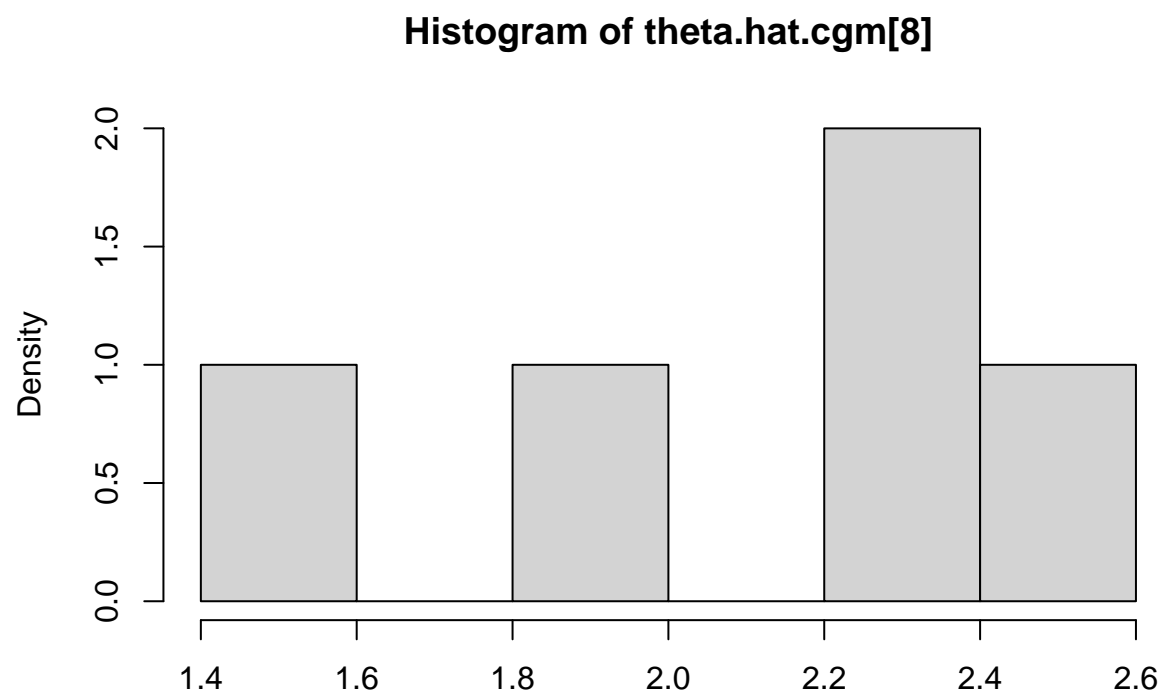
```
# A tibble: 1 x 2
  `MISLE (First-step) MSE` `MISLE MSE`
      <dbl>      <dbl>
1      0.0187      0.0272
```

```
# A tibble: 1 x 2
  `MISLE MSE` `CGM MSE`
      <dbl>      <dbl>
1      0.0272      2.61
```

First Step Histograms

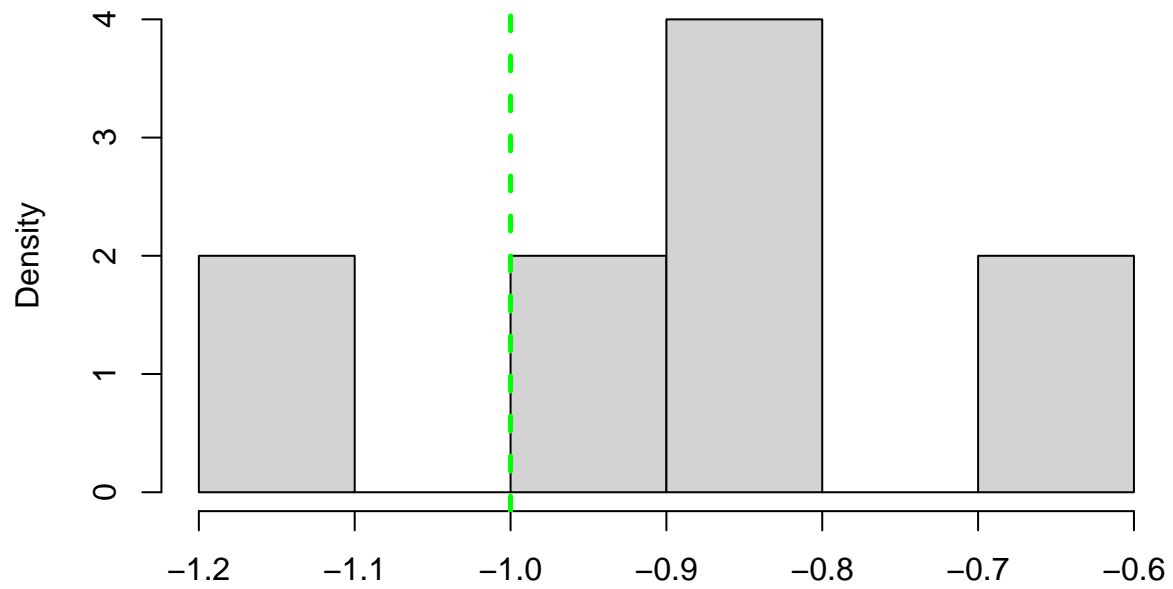


```
[1] "Summary statistics of bootstrap replicates:"  
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.  
0.8603  0.8830  0.9275  1.0136  1.1062  1.2909  
[1] "95% CI based on bootstrap:"  
      lower  upper  
1 0.8625514 1.272444
```

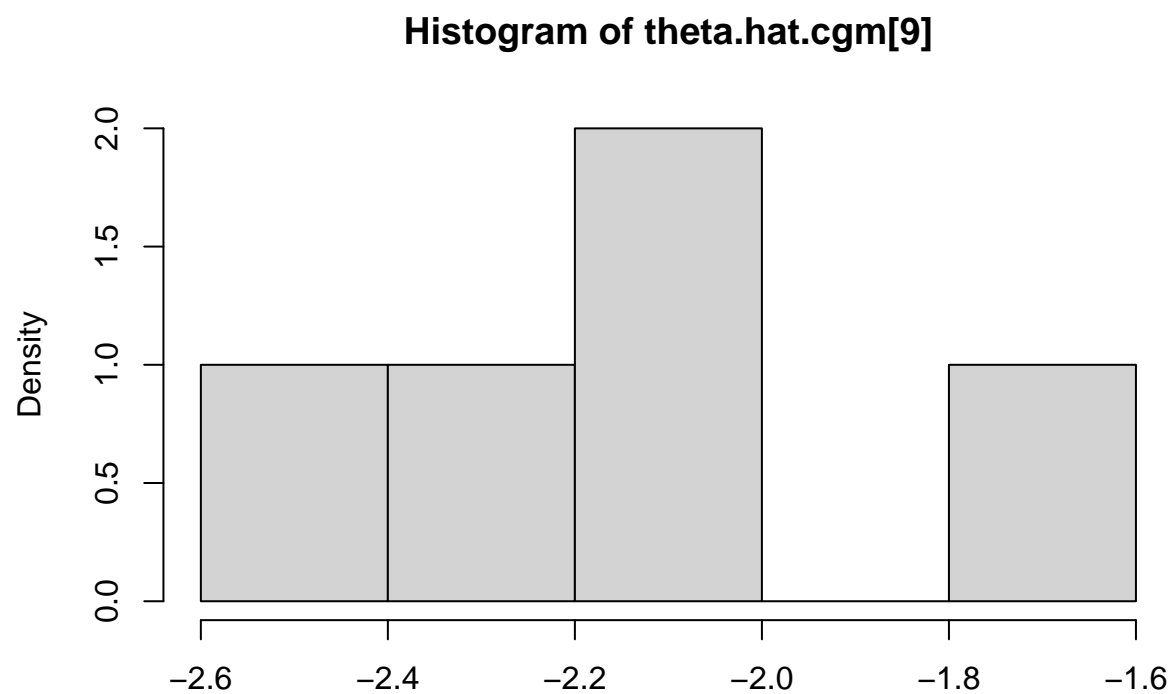


```
[1] "Summary statistics of bootstrap replicates:"  
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.  
1.476  1.881   2.228   2.099  2.379   2.531  
[1] "95% CI based on bootstrap:"  
      lower.cgm upper.cgm  
1  1.516124  2.515654
```

Histogram of theta.hat[9]

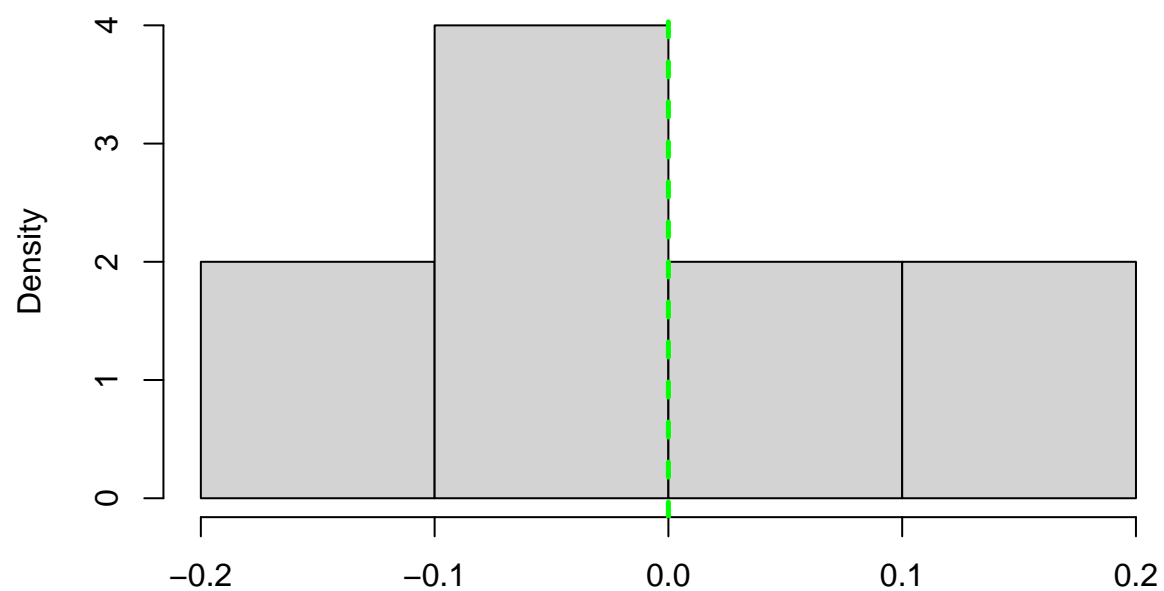


```
[1] "Summary statistics of bootstrap replicates:"  
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.  
-1.1681 -0.9963 -0.8534 -0.9063 -0.8151 -0.6985  
[1] "95% CI based on bootstrap:"  
      lower      upper  
1 -1.150889 -0.7101637
```



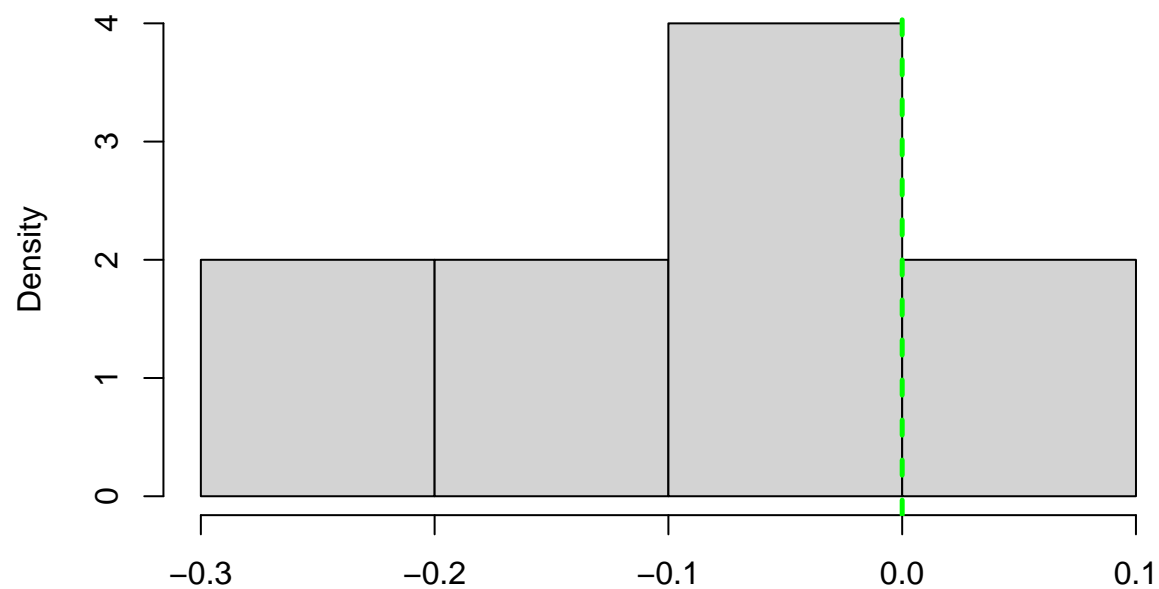
```
[1] "Summary statistics of bootstrap replicates:"  
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.  
-2.564 -2.381  -2.191  -2.206  -2.153  -1.740  
[1] "95% CI based on bootstrap:"  
      lower.cgm upper.cgm  
1 -2.545526 -1.780897
```

Histogram of theta.hat[1]



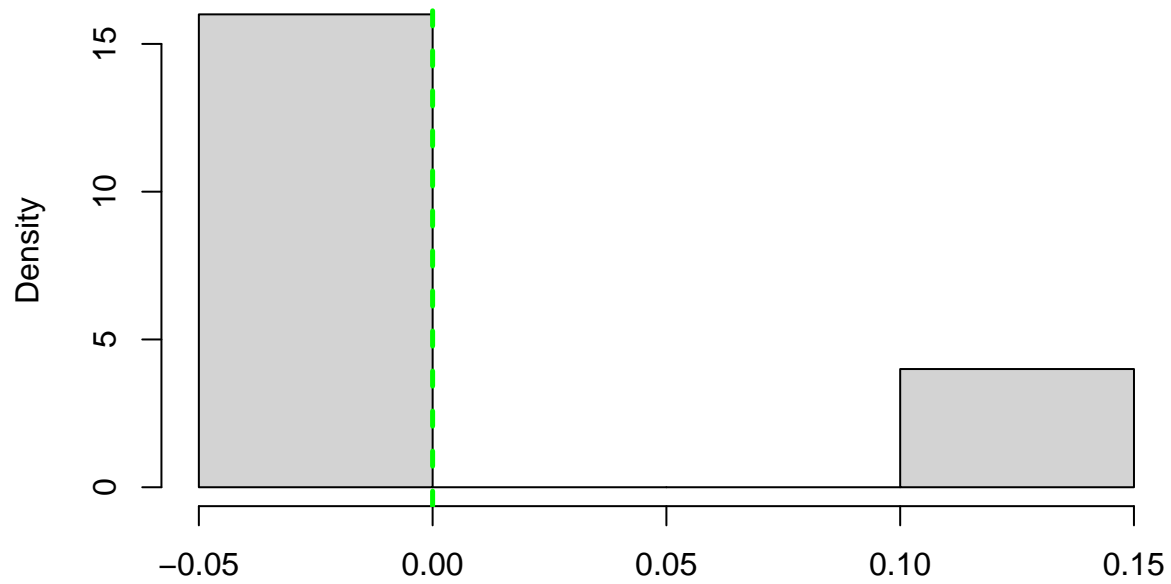
```
[1] "Summary statistics of bootstrap replicates:"  
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.  
-0.14009 -0.08891 -0.02909 -0.02276 0.00420 0.14010  
[1] "95% CI based on bootstrap:"  
      lower    upper  
1 -0.1349692 0.1265063
```

Histogram of theta.hat.cgm[1]

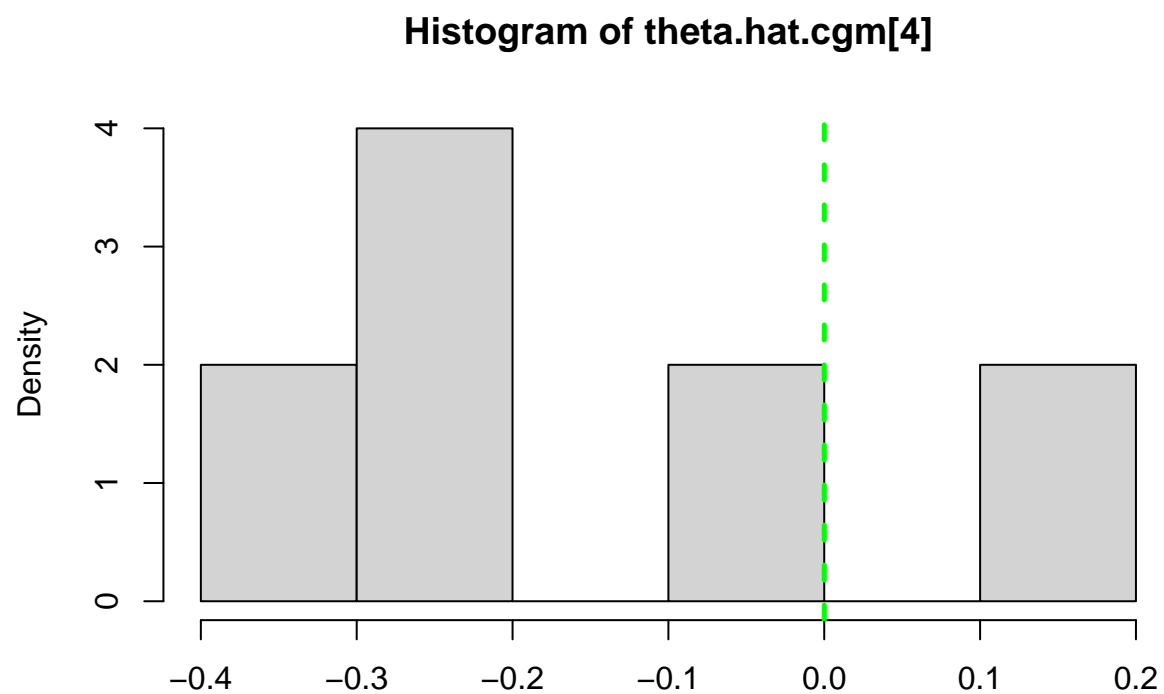


```
[1] "Summary statistics of bootstrap replicates:"
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-0.29922 -0.18235 -0.09957 -0.10136  0.00000  0.07432
[1] "95% CI based on bootstrap:"
      lower.cgm upper.cgm
1 -0.2875298  0.06688716
```

Histogram of theta.hat[4]

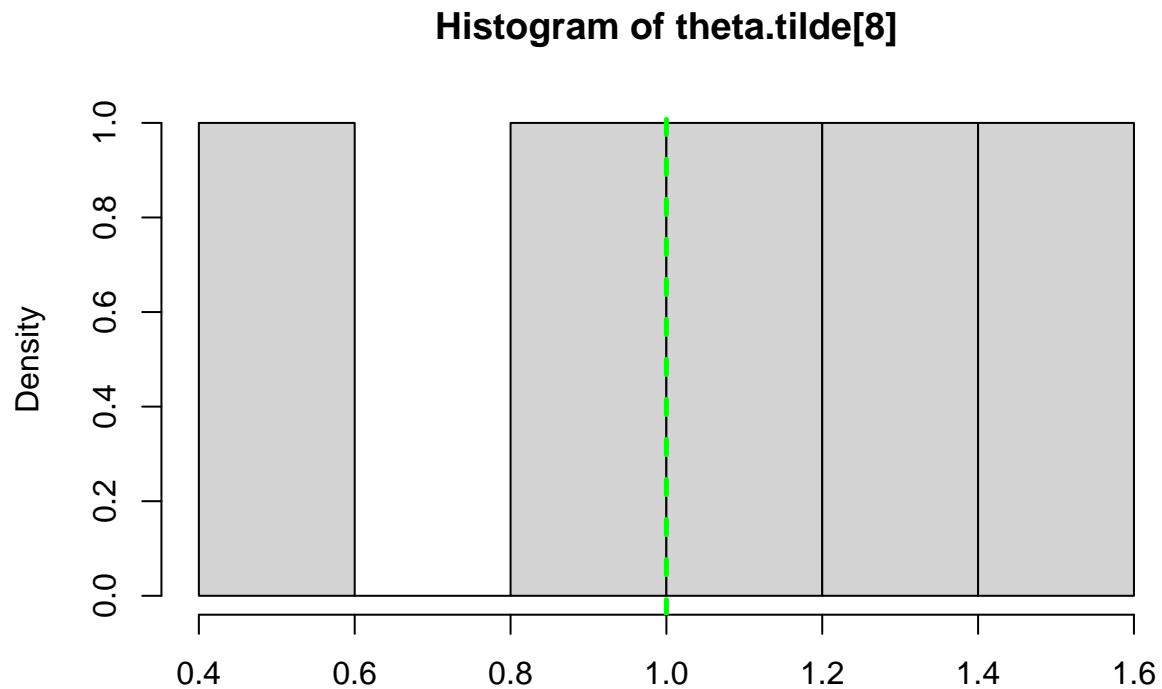


```
[1] "Summary statistics of bootstrap replicates:"
      Min.   1st Qu.   Median     Mean   3rd Qu.     Max.
-0.0472829 -0.0465674 -0.0147296 -0.0001757  0.0000000  0.1077016
[1] "95% CI based on bootstrap:"
      lower      upper
1 -0.04721133  0.09693141
```

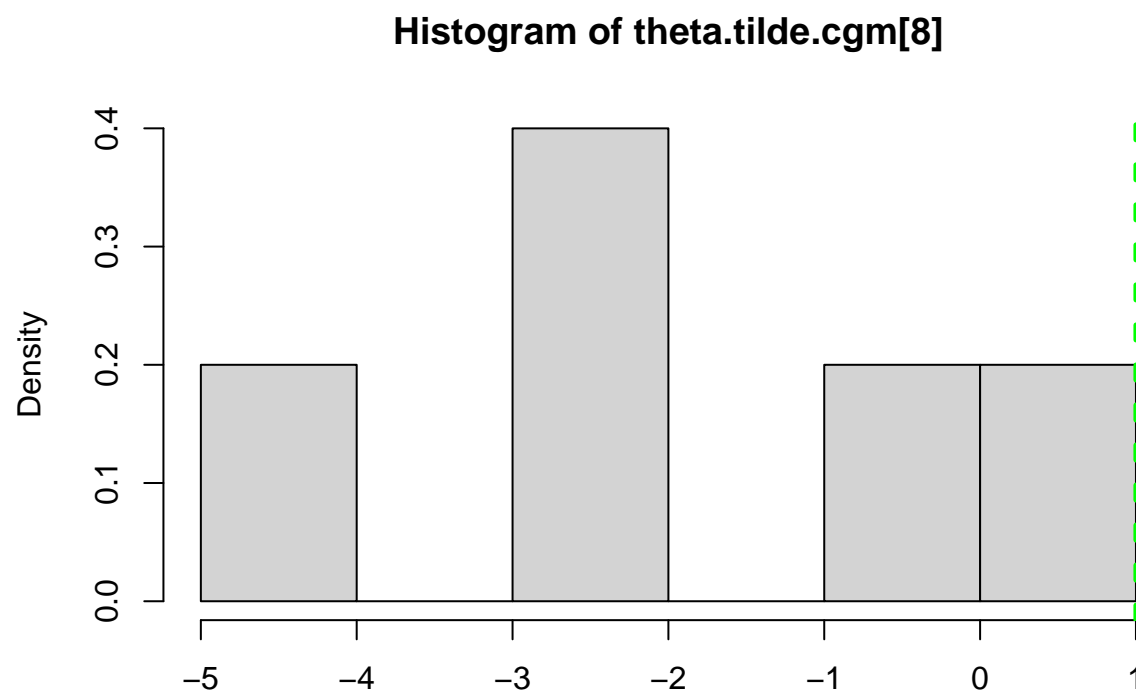


```
[1] "Summary statistics of bootstrap replicates:"
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-0.3941 -0.2787 -0.2552 -0.1637  0.0000  0.1094
[1] "95% CI based on bootstrap:"
      lower.cgm  upper.cgm
1 -0.3825272  0.09841997
```

Statistics and 95% Confidence Intervals from per-Replicate Estimates

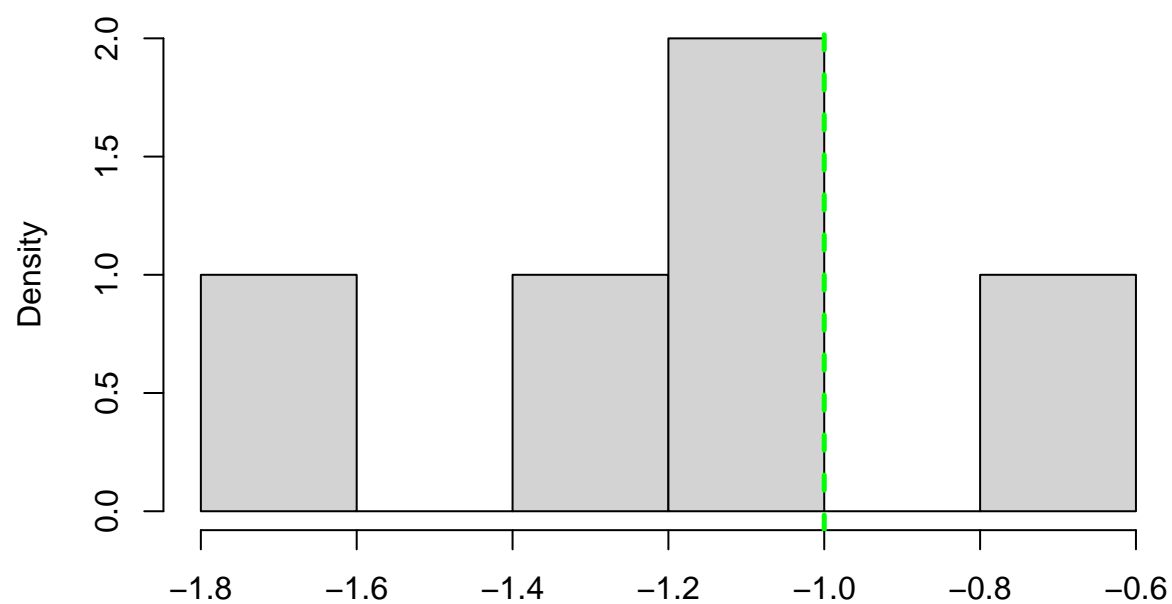


```
[1] "Summary statistics of bootstrap replicates:"  
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.  
0.5688  0.8843  1.0421  1.0747  1.3808  1.4974  
[1] "95% CI based on bootstrap:"  
      lower  upper  
1 0.6003282 1.485786
```



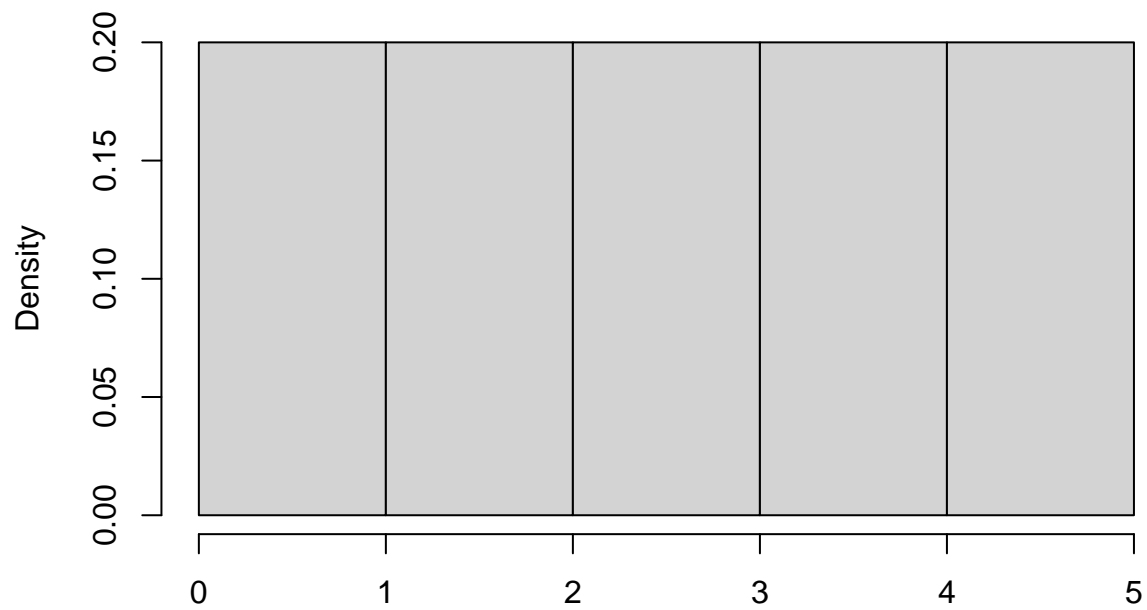
```
[1] "Summary statistics of bootstrap replicates:"  
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.  
-4.9666 -2.6021 -2.4412 -2.1121 -0.6569  0.1064  
[1] "95% CI based on bootstrap:"  
      lower.cgm upper.cgm  
1 -4.730149  0.03004351
```

Histogram of theta.tilde[9]



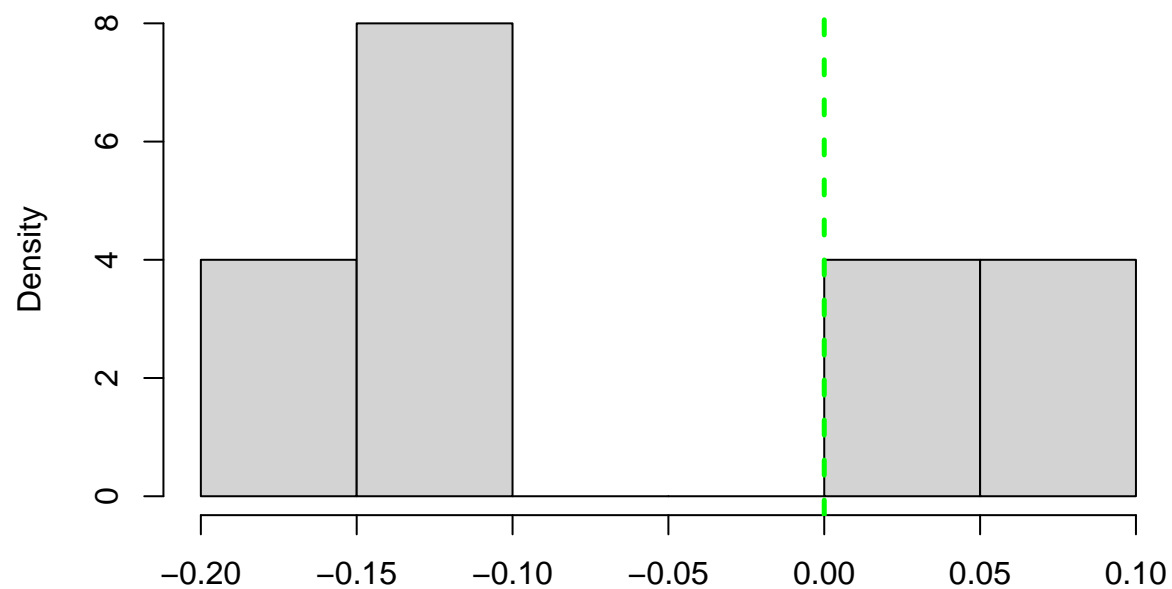
```
[1] "Summary statistics of bootstrap replicates:"
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-1.6409 -1.2229 -1.1138 -1.1359 -1.0933 -0.6088
[1] "95% CI based on bootstrap:"
      lower      upper
1 -1.599121 -0.6572445
```

Histogram of theta.tilde.cgm[9]



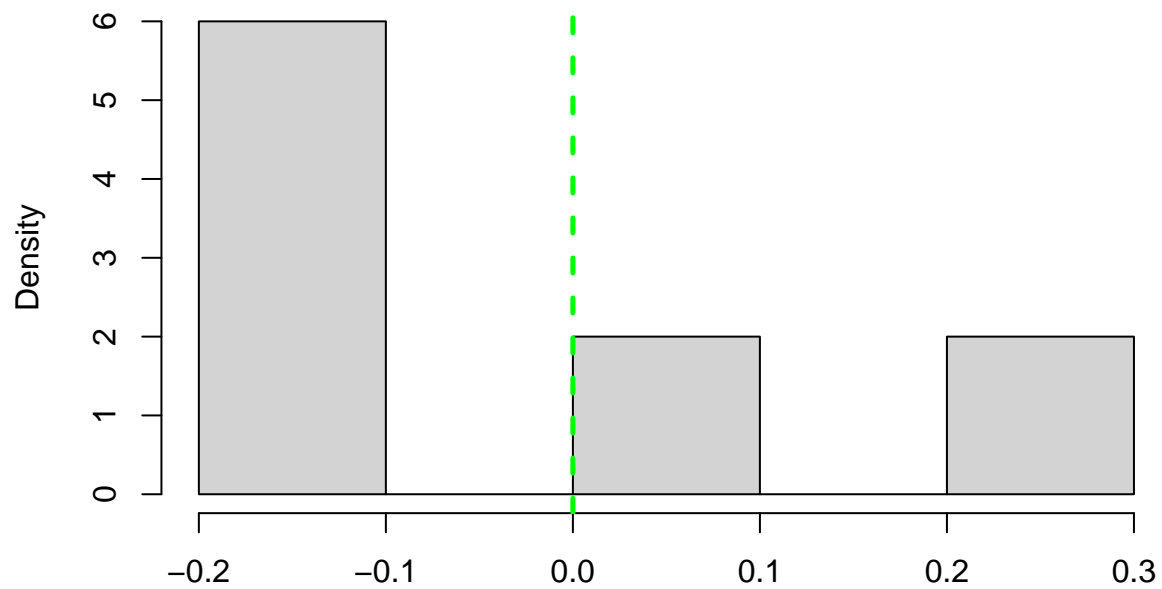
```
[1] "Summary statistics of bootstrap replicates:"  
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.  
0.4224  1.0728  2.1920  2.2949  3.4078  4.3794  
[1] "95% CI based on bootstrap:"  
      lower.cgm upper.cgm  
1  0.487402  4.282282
```

Histogram of theta.tilde[1]



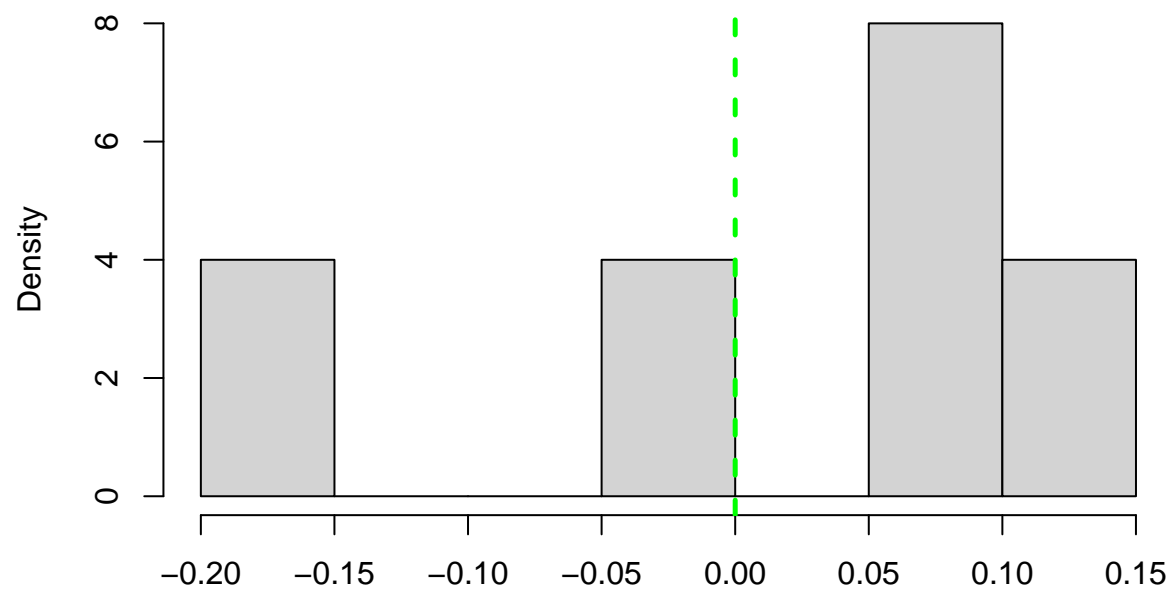
```
[1] "Summary statistics of bootstrap replicates:"
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-0.17824 -0.14891 -0.14088 -0.07520 0.03717 0.05488
[1] "95% CI based on bootstrap:"
      lower      upper
1 -0.1753078 0.05310652
```

Histogram of theta.tilde.cgm[1]



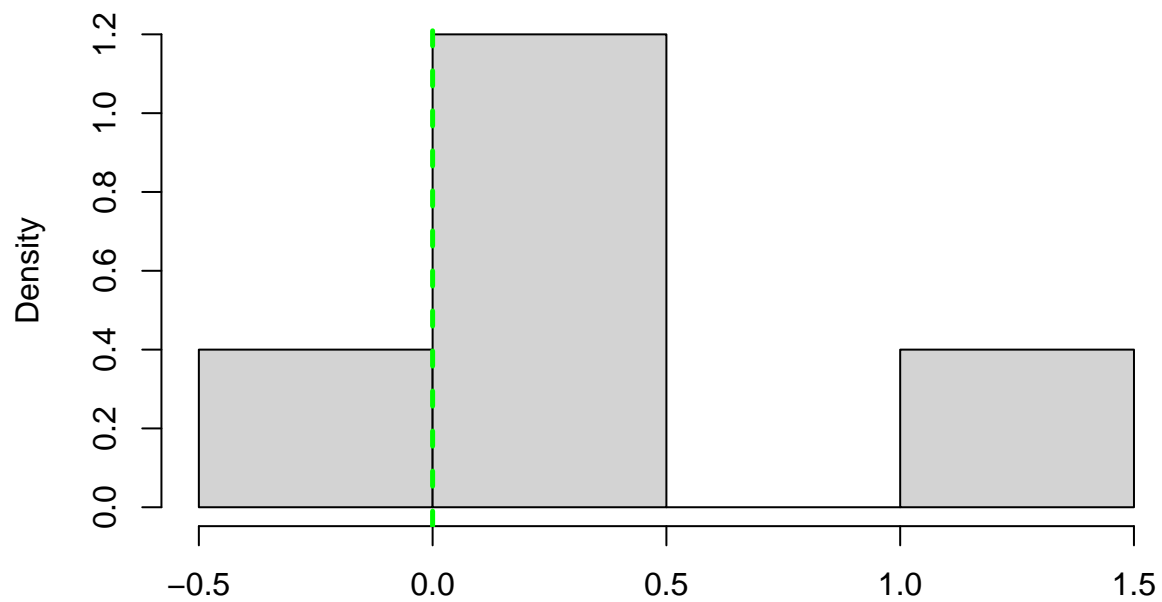
```
[1] "Summary statistics of bootstrap replicates:"  
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.  
-0.18009 -0.16501 -0.14664 -0.03553 0.09441  0.21970  
[1] "95% CI based on bootstrap:"  
      lower.cgm upper.cgm  
1 -0.1785809 0.2071715
```

Histogram of $\theta.tilde[4]$



```
[1] "Summary statistics of bootstrap replicates:"
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-0.16987 -0.02448  0.08017  0.01515  0.08157  0.10836
[1] "95% CI based on bootstrap:"
      lower  upper
1 -0.1553294 0.1056838
```

Histogram of theta.tilde.cgm[4]



```
[1] "Summary statistics of bootstrap replicates:"  
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.  
-0.10835  0.02857  0.10464  0.32851  0.38194  1.23575  
[1] "95% CI based on bootstrap:"  
      lower.cgm upper.cgm  
1 -0.09465751  1.15037
```

Statistics for Theoretical 95% Confidence Intervals

```

[1] Length of Confidence Intervals for theta[8]
[1] Coverage proportion: 0.6
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
0.7022  0.7176  0.7392  0.8223  0.9382  1.0143
[1] Length of Confidence Intervals for theta[8] (CGM Method)
[1] Coverage proportion: 1
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
6.305   6.534   7.603   8.285  10.415  10.566
[1] Length of Confidence Intervals for theta[9]
[1] Coverage proportion: 0.6
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
0.6242  0.7352  0.7519  0.7931  0.8545  0.9998
[1] Length of Confidence Intervals for theta[9] (CGM Method)
[1] Coverage proportion: 1
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
10.63   15.22   16.87   15.45  16.90  17.60
[1] Length of Confidence Intervals for theta[1]
[1] Coverage proportion: 1
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
0.4939  0.5475  0.5571  0.5595  0.5770  0.6220
[1] Length of Confidence Intervals for theta[1] (CGM Method)
[1] Coverage proportion: 1
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
3.620   4.349   4.467   4.481   4.633   5.337
[1] Length of Confidence Intervals for theta[4]
[1] Coverage proportion: 1
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
0.5161  0.5165  0.5295  0.5727  0.6041  0.6972
[1] Length of Confidence Intervals for theta[4] (CGM Method)
[1] Coverage proportion: 1
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
3.847   4.406   4.550   5.073   4.763   7.800

```