Simulation Results

2026-01-19

Simulation Setup

This simulation is performed with $n = 200$ and $d = 100$, using the 2-d lattice as the underlying graph. $s = 5$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.2$. The attached results are for a 10-replication simulation. The true values of the parameter vector θ are

```
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -0.4472136 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{$ 20, 51, 71, 40$\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

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### Mean-squared error comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat\theta
```

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[20]	0.040	0.050
theta[51]	0.071	0.035
theta[71]	0.015	0.028
theta[40]	0.020	0.041
total	0.036	0.038

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[20]	0.131	0.049
theta[51]	0.117	0.035
theta[71]	0.000	0.005
theta[40]	0.000	0.003
total	0.062	0.023

Mean absolute deviation comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat

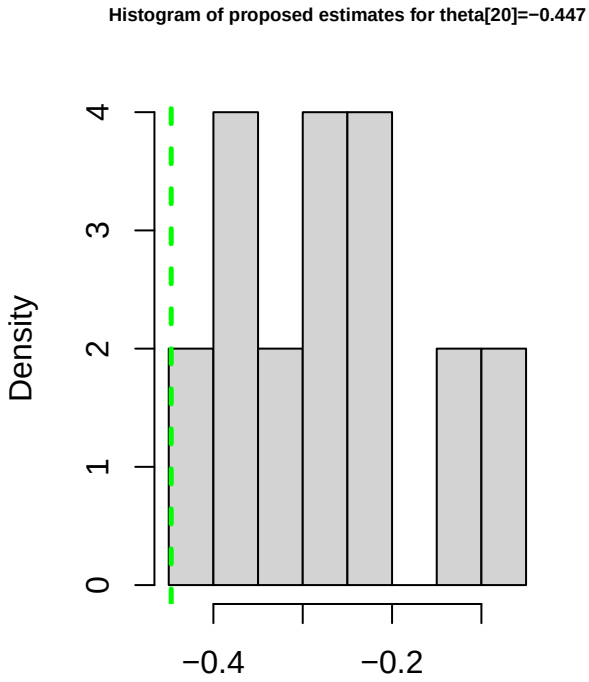
Table 3: Mean Absolute Deviation of Parameter Estimates

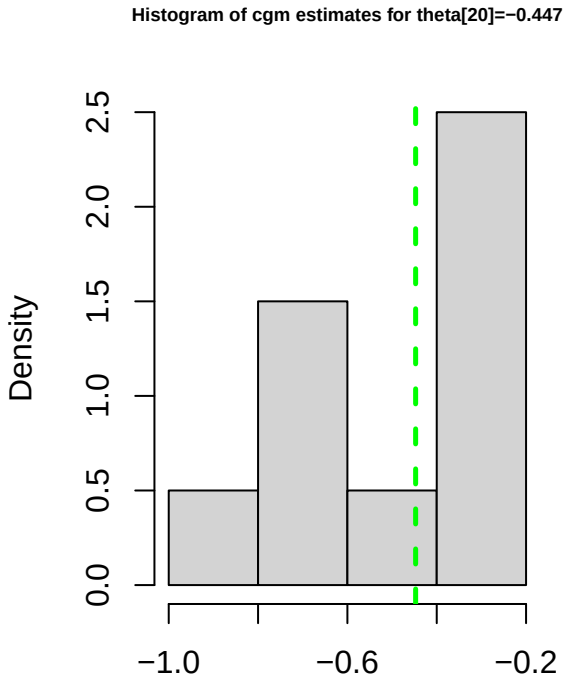
	proposed	cgm
theta[20]	0.168	0.180
theta[51]	0.233	0.169
theta[71]	0.108	0.126
theta[40]	0.087	0.170
total	0.149	0.161

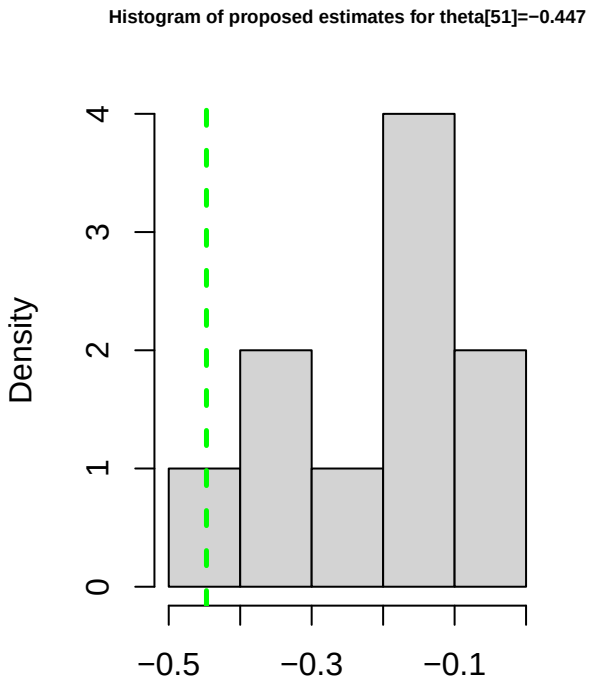
Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

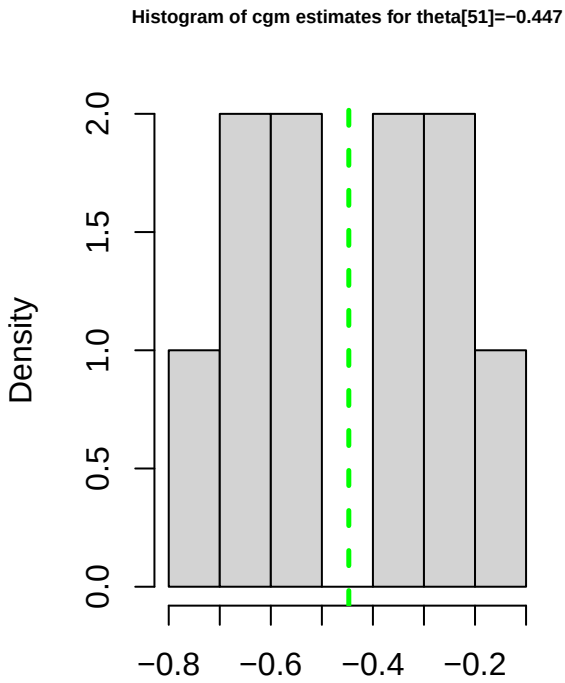
	proposed	cgm
theta[20]	0.352	0.189
theta[51]	0.317	0.151
theta[71]	0.006	0.033
theta[40]	0.005	0.023
total	0.170	0.099

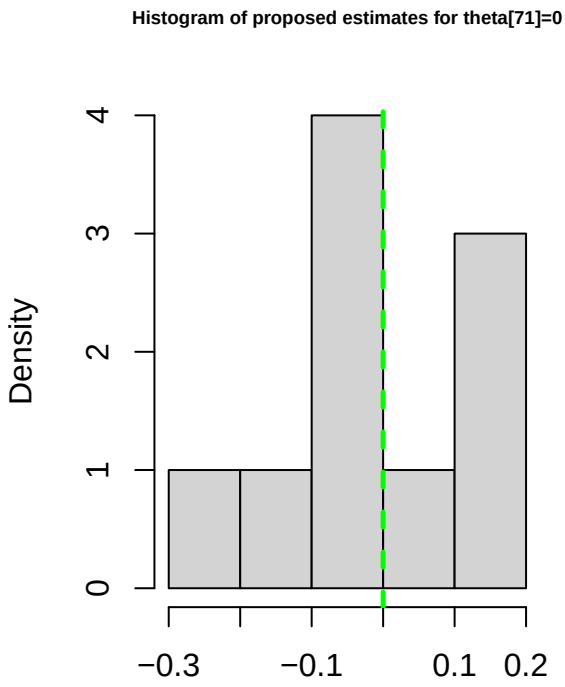
Boxplots

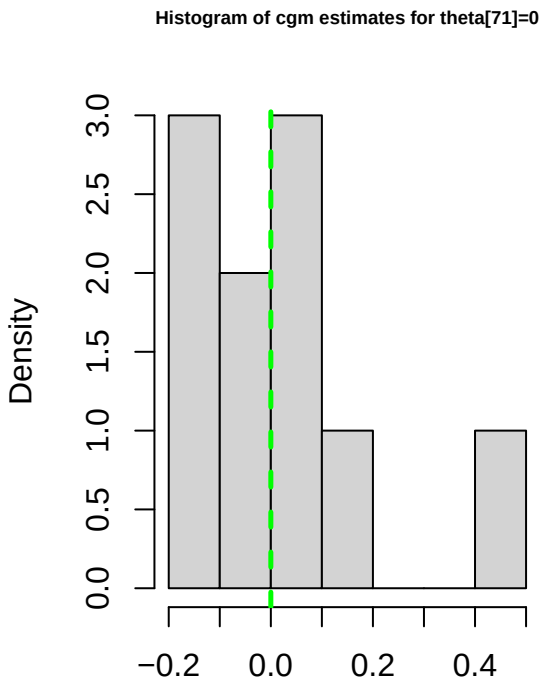

Histogram of proposed estimates for theta[20]=−0.447
Density
0
1
2
3
4
−0.4
−0.2

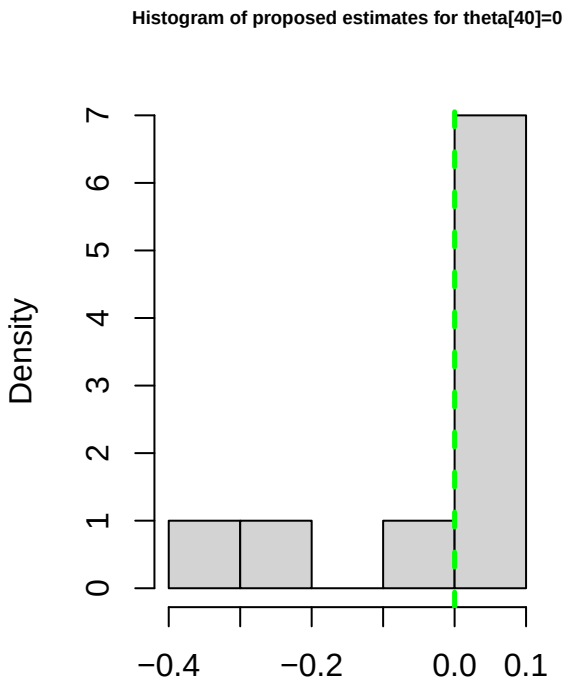

Histogram of cgm estimates for theta[20]=−0.447
Density
0.0
0.5
1.0
1.5
2.0
2.5
−1.0
−0.6
−0.2

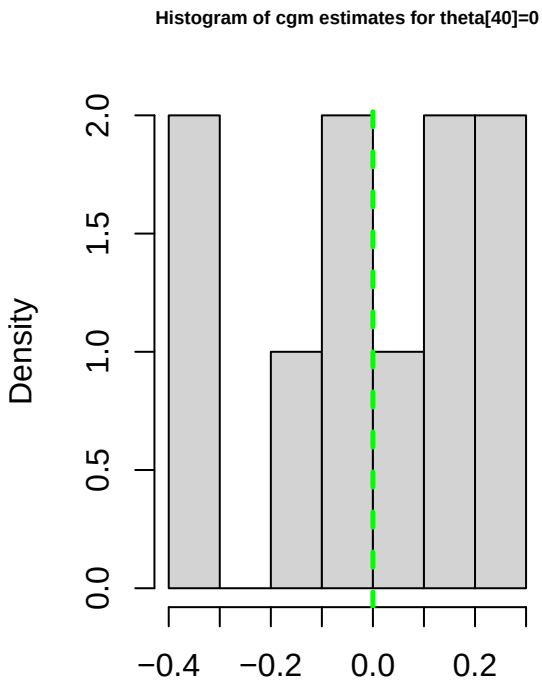

Histogram of proposed estimates for theta[51]=−0.447
Density
0
1
2
3
4
−0.5
−0.3
−0.1


Histogram of cgm estimates for theta[51]=−0.447
Density
0.0
0.5
1.0
1.5
2.0
−0.8
−0.6
−0.4
−0.2

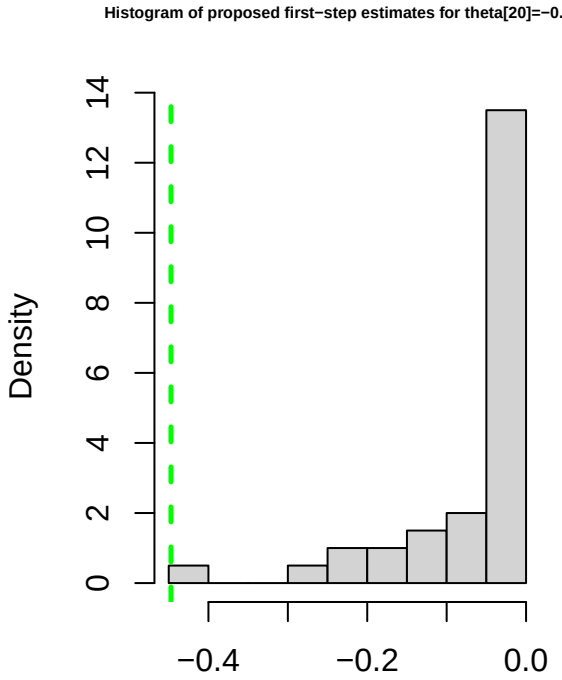

Histogram of proposed estimates for theta[71]=0
Density
0
1
2
3
4
−0.3
−0.1
0.1
0.2

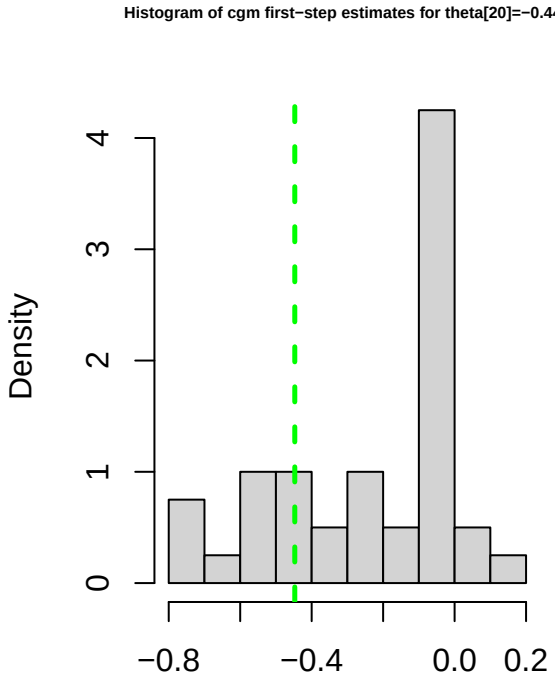

Histogram of cgm estimates for theta[71]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.2
0.0
0.2
0.4

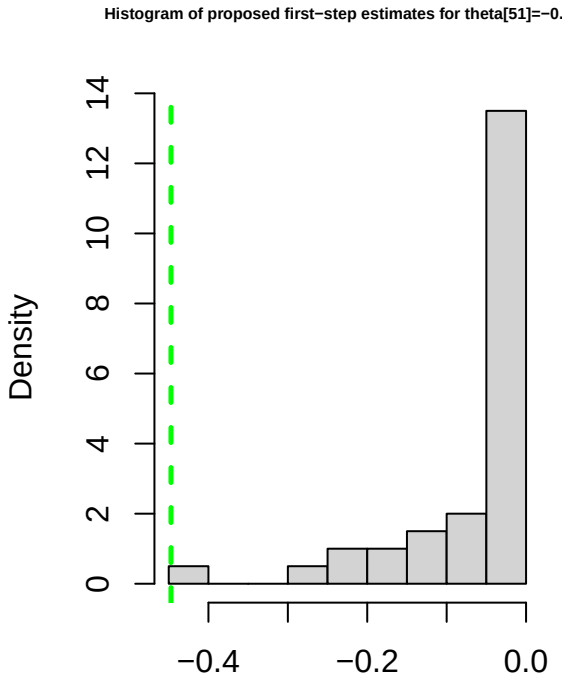

Histogram of proposed estimates for theta[40]=0
Density
0
1
2
3
4
5
6
7
−0.4
−0.2
0.0
0.1

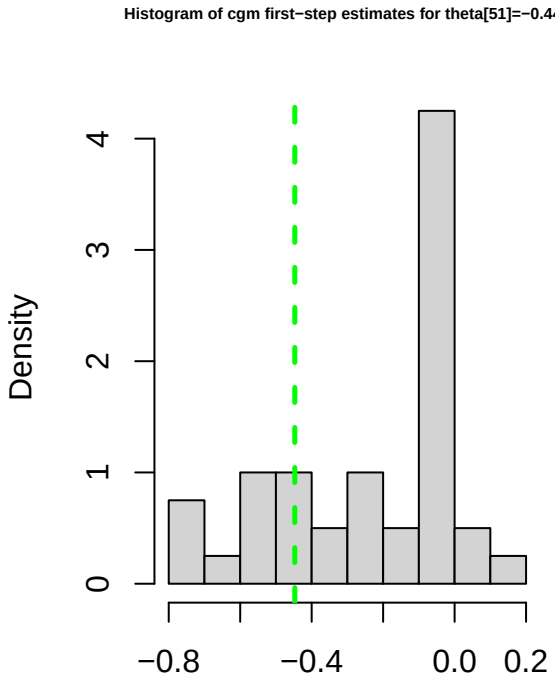

Histogram of cgm estimates for theta[40]=0
Density
0.0
0.5
1.0
1.5
2.0
−0.4
−0.2
0.0
0.2

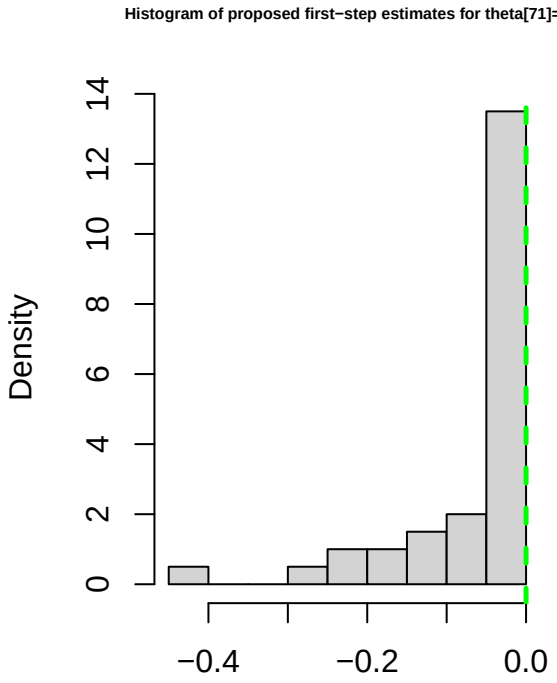
First Step Histograms

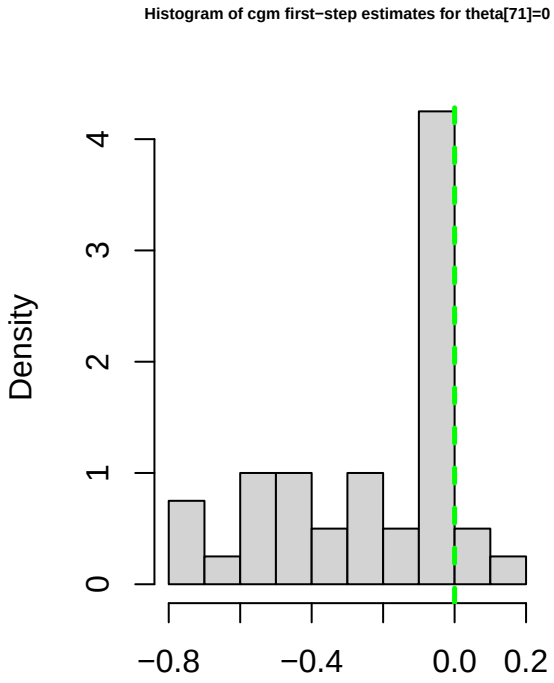


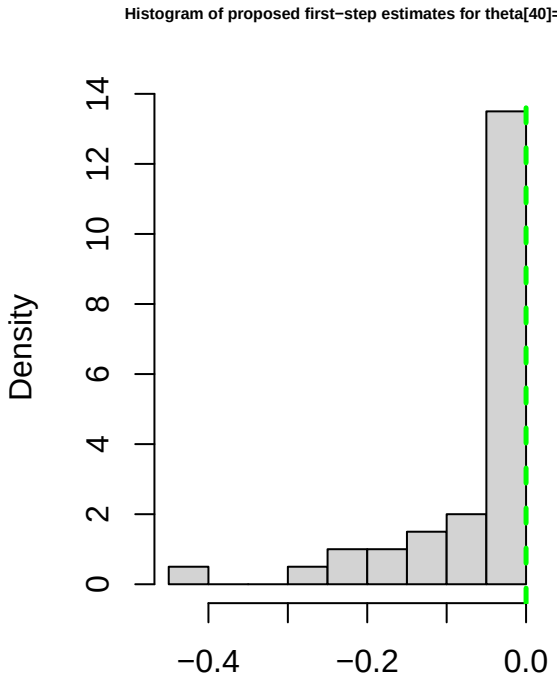


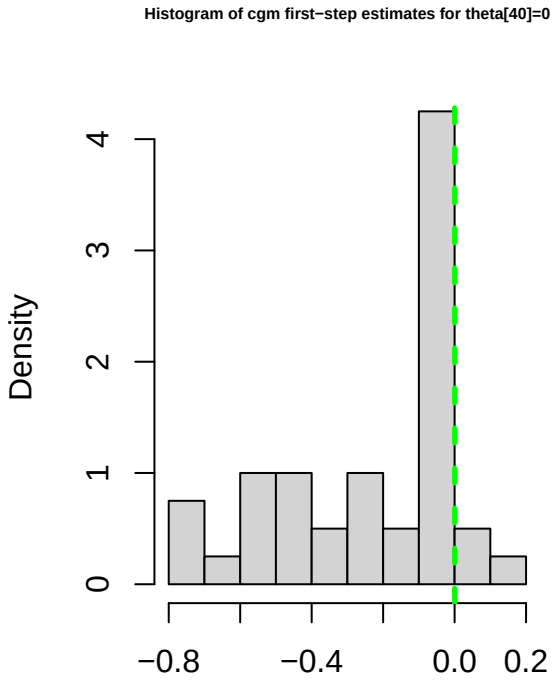





Histogram of proposed first−step estimates for theta[71]=0
Density
0
2
4
6
8
10
12
14
−0.4
−0.2
0.0


Histogram of cgm first−step estimates for theta[71]=0
Density
0
1
2
3
4
−0.8
−0.4
0.0
0.2


Histogram of proposed first−step estimates for theta[40]=0
Density
0
2
4
6
8
10
12
14
−0.4
−0.2
0.0


Histogram of cgm first−step estimates for theta[40]=0
Density
0
1
2
3
4
−0.8
−0.4
0.0
0.2

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 5: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[20]	-0.427	-0.277	-0.084	-0.421	-0.097
theta[51]	-0.465	-0.177	-0.042	-0.446	-0.046
theta[71]	-0.248	-0.057	0.137	-0.221	0.136
theta[40]	-0.357	0.024	0.053	-0.333	0.052

Table 6: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[20]	-0.989	-0.406	-0.275	-0.919	-0.276
theta[51]	-0.771	-0.429	-0.192	-0.748	-0.205
theta[71]	-0.189	-0.023	0.417	-0.181	0.346
theta[40]	-0.332	0.024	0.252	-0.332	0.252

Statistics for Theoretical 95% Confidence Intervals

Table 7: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[20]	-0.279	0.142	-0.557	-0.001	0.9
theta[51]	-0.218	0.131	-0.474	0.038	0.5
theta[71]	-0.019	0.131	-0.276	0.238	1.0
theta[40]	-0.040	0.129	-0.292	0.212	0.9

Table 8: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[20]	-0.489	0.129	-0.743	-0.235	0.9
theta[51]	-0.452	0.132	-0.711	-0.192	0.9
theta[71]	0.013	0.133	-0.249	0.274	0.9
theta[40]	0.002	0.141	-0.273	0.278	0.8