

# Simulation Results

2026-01-14

## Simulation Setup

This simulation is performed with  $n = 200$  and  $d = 20$ , using the 2-d lattice as the underlying graph.  $s = 2$  parameters are set to be nonzero, and the beta parameter is chosen to be  $\beta = 0$ . The attached results are for a 10-replication simulation. The true values of the parameter vector  $\theta$  are

0 0 0 0 0 0 0.7071068 0 0 0 0.7071068 0 0 0 0 0 0 0 0 ,

but for brevity, our simulation only estimates the indices of  $\theta$  in  $\mathcal{C} = \{7, 11, 4, 8\}$  elements of  $\theta$ . Accordingly, **all statistics and visuals are indicative of performance only on the set  $\mathcal{C}$ .**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

## Results

### Mean-squared error comparison  $\frac{1}{n.\text{sim}} \sum_{i=1}^{n.\text{sim}} \frac{1}{|\mathcal{C}|} \|\hat{\theta}_i - \theta\|^2$

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[7]	0.210	0.062
theta[11]	0.173	0.132
theta[4]	0.168	0.007
theta[8]	0.108	0.047
total	0.165	0.062

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[7]	0.076	0.042
theta[11]	0.168	0.119
theta[4]	0.022	0.001
theta[8]	0.063	0.004
total	0.082	0.042

### Mean absolute deviation comparison  $\frac{1}{n.\text{sim}} \sum_{i=1}^{n.\text{sim}} \frac{1}{|\mathcal{C}|} \|\hat{\theta}_i - \theta\|$

Table 3: Mean Absolute Deviation of Parameter Estimates

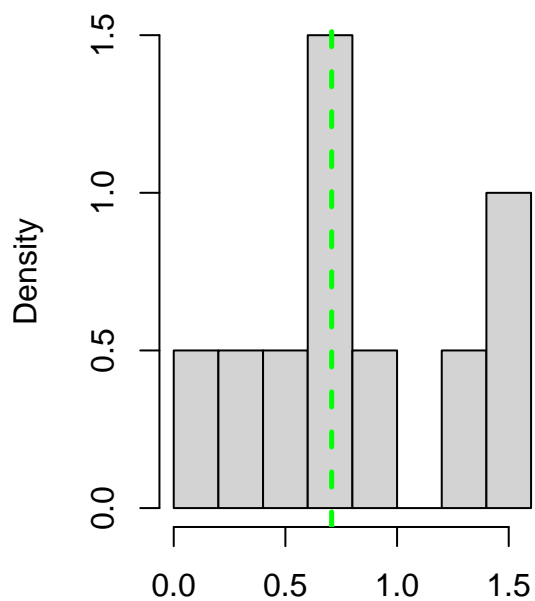
	proposed	cgm
theta[7]	0.366	0.168
theta[11]	0.262	0.264
theta[4]	0.305	0.061
theta[8]	0.258	0.170
total	0.298	0.166

Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

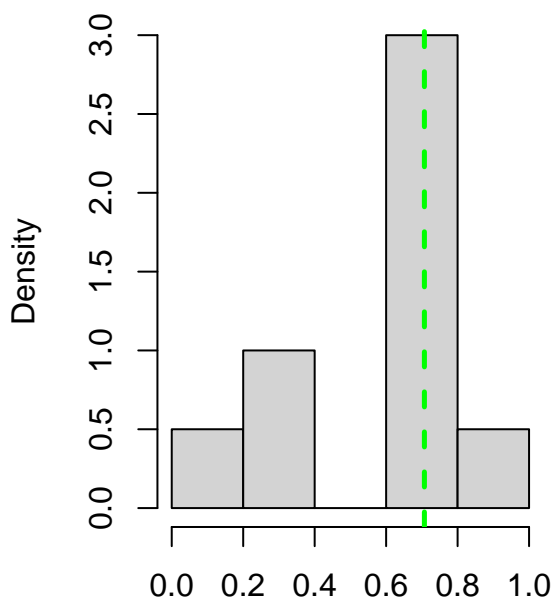
	proposed	cgm
theta[7]	0.219	0.183
theta[11]	0.331	0.264
theta[4]	0.096	0.013
theta[8]	0.179	0.045
total	0.206	0.126

## Boxplots

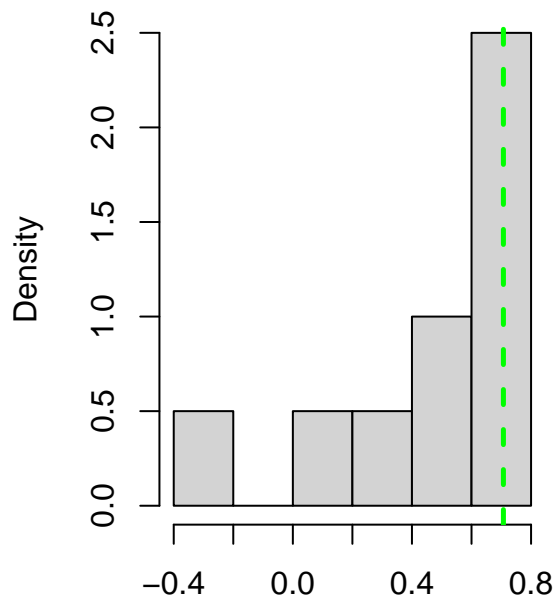
Histogram of proposed estimates for  $\theta[7]=0.707$



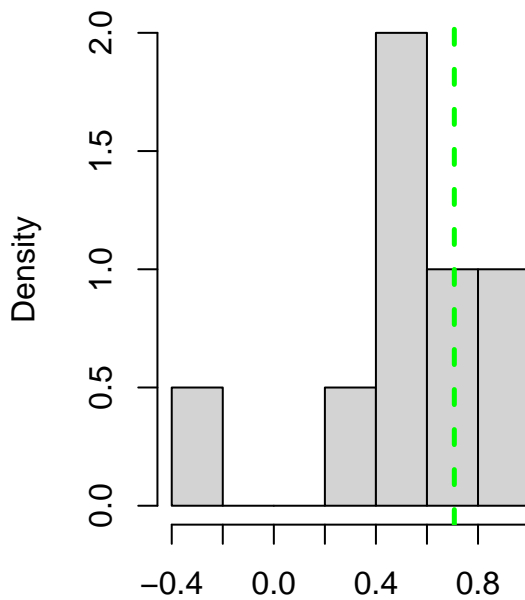
Histogram of cgm estimates for  $\theta[7]=0.707$

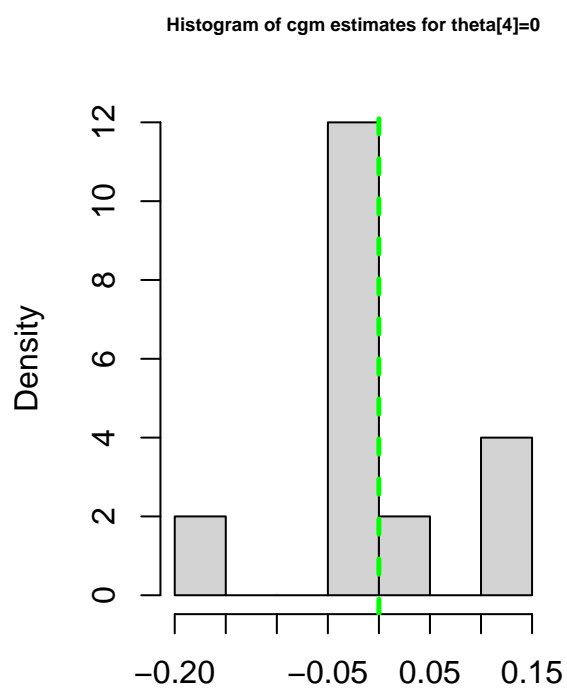
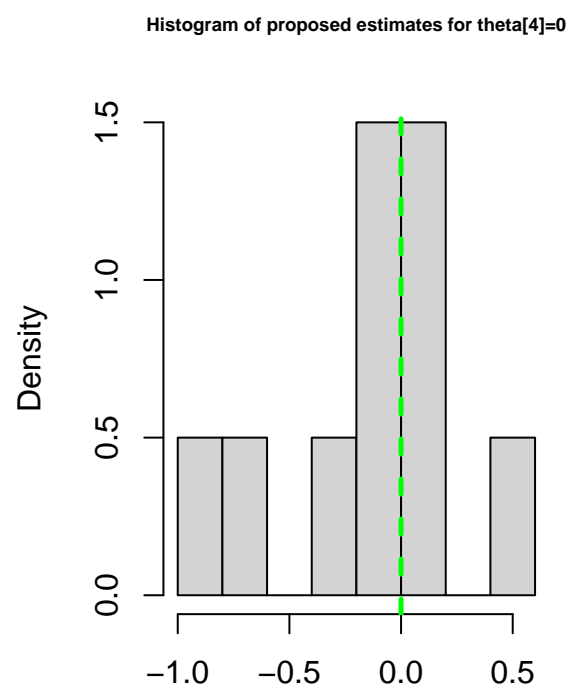


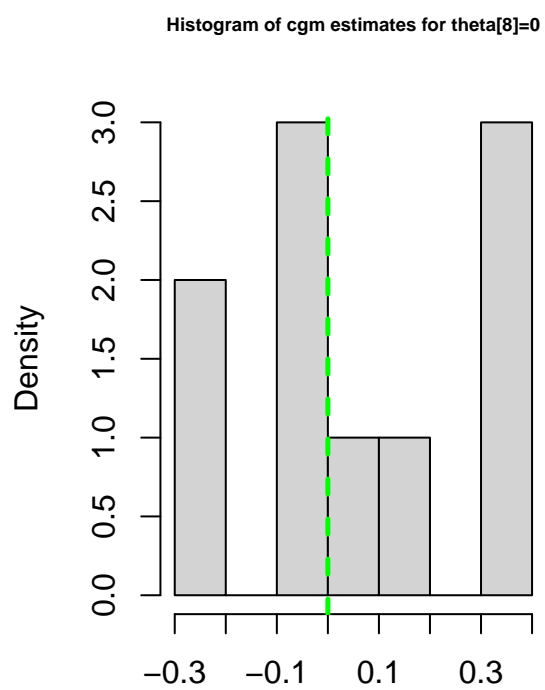
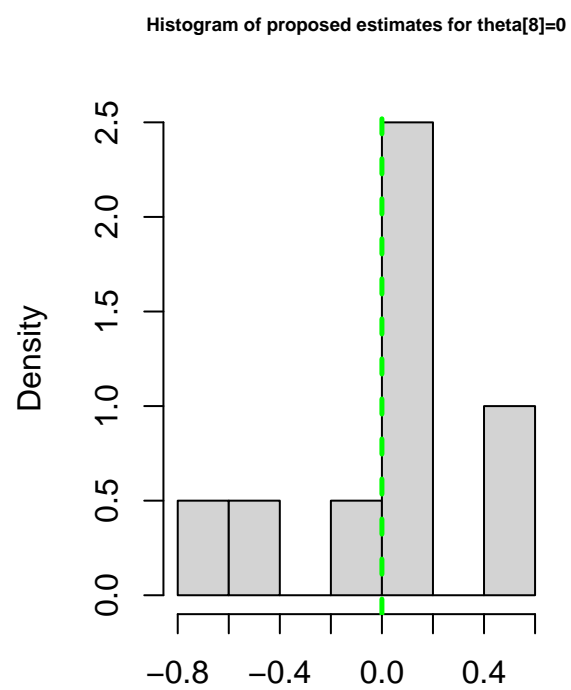
Histogram of proposed estimates for  $\theta[11]=0.707$



Histogram of cgm estimates for  $\theta[11]=0.707$

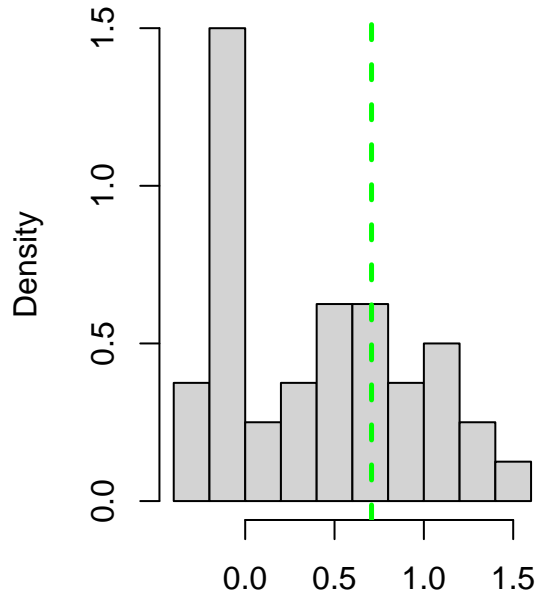




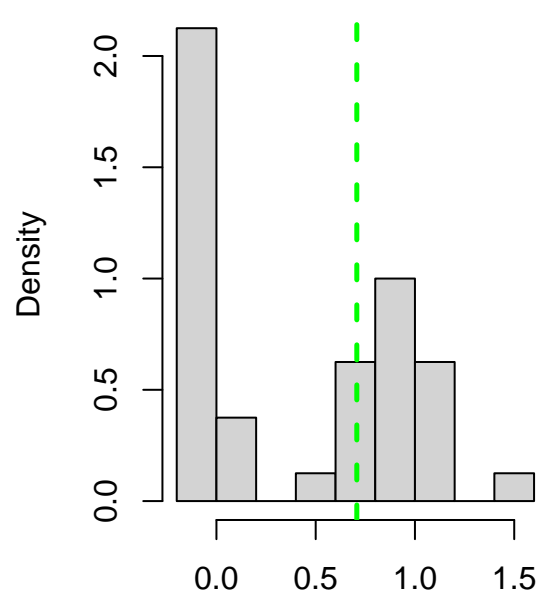


## First Step Histograms

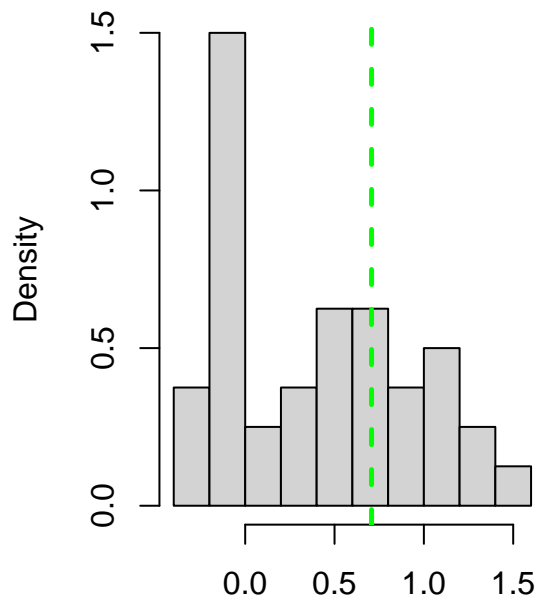
Histogram of proposed first-step estimates for  $\theta[7]=0.707$



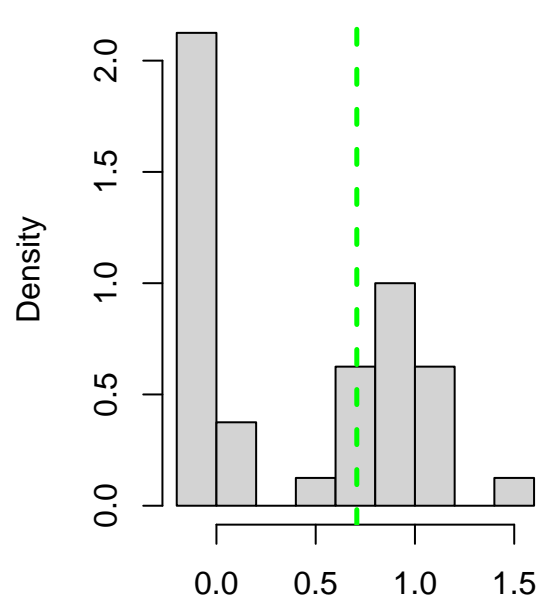
Histogram of cgm first-step estimates for  $\theta[7]=0.707$



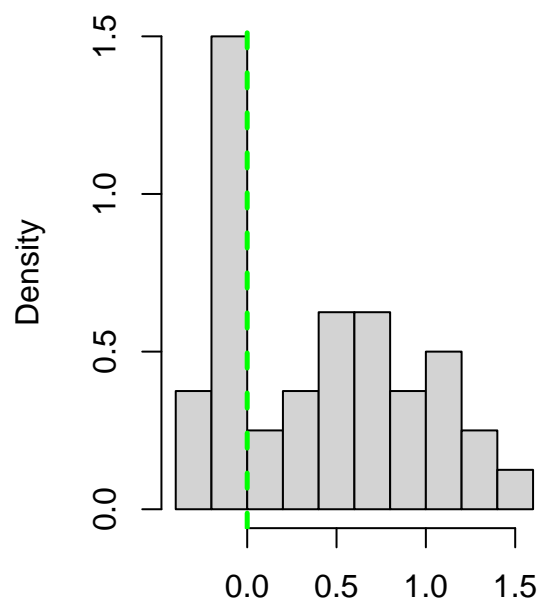
Histogram of proposed first-step estimates for  $\theta[11]=0.707$



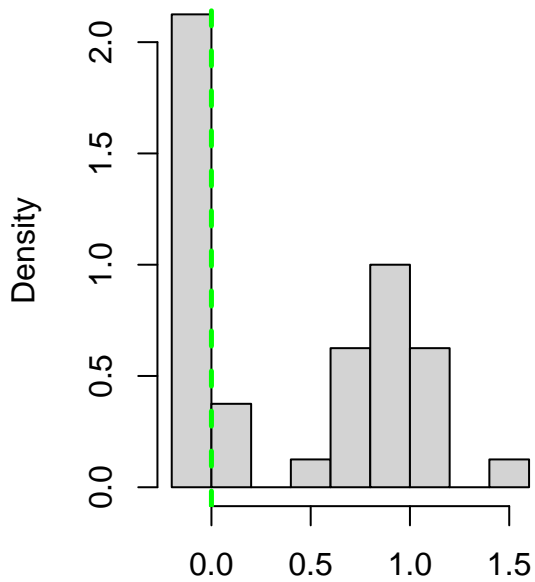
Histogram of cgm first-step estimates for  $\theta[11]=0.707$



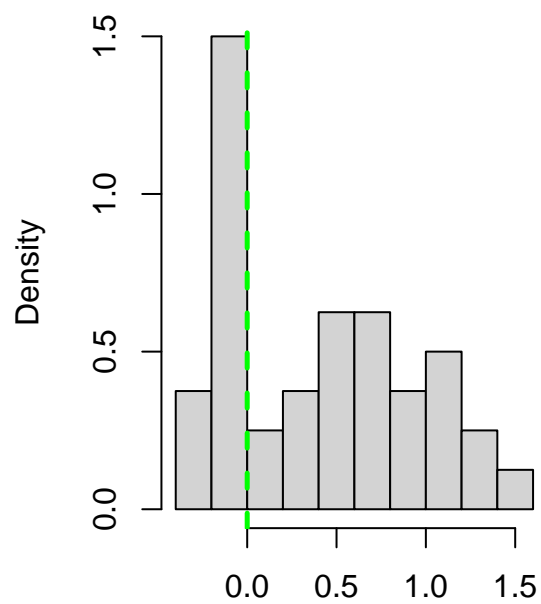
Histogram of proposed first-step estimates for  $\theta_4=0$



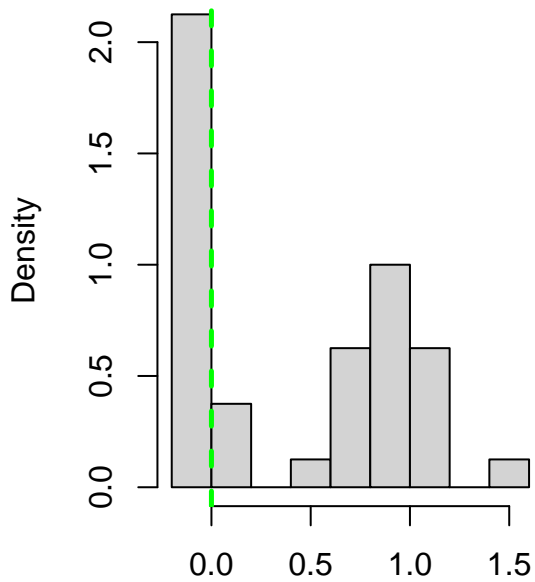
Histogram of cgm first-step estimates for  $\theta_4=0$



Histogram of proposed first-step estimates for  $\theta[8]=0$



Histogram of cgm first-step estimates for  $\theta[8]=0$





## Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 5: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[7]	0.155	0.779	1.519	0.205	1.511
theta[11]	-0.337	0.612	0.772	-0.247	0.766
theta[4]	-0.899	-0.096	0.402	-0.856	0.347
theta[8]	-0.662	0.098	0.464	-0.606	0.453

Table 6: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[7]	0.172	0.693	0.880	0.192	0.858
theta[11]	-0.246	0.546	0.948	-0.122	0.917
theta[4]	-0.190	-0.026	0.120	-0.157	0.116
theta[8]	-0.237	0.022	0.393	-0.229	0.376

## Statistics for Theoretical 95% Confidence Intervals

Table 7: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[7]	0.833	0.206	0.429	1.238	0.5
theta[11]	0.478	0.217	0.053	0.903	0.8
theta[4]	-0.164	0.195	-0.547	0.218	0.8
theta[8]	0.033	0.177	-0.313	0.379	0.7

Table 8: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[7]	0.600	0.162	0.283	0.916	0.8
theta[11]	0.524	0.175	0.181	0.866	0.8
theta[4]	-0.013	0.128	-0.264	0.237	1.0
theta[8]	0.069	0.130	-0.185	0.323	0.7