Simulation Results

2026-01-15

Simulation Setup

This simulation is performed with $n = 200$ and $d = 20$, using the 2-d lattice as the underlying graph. $s = 5$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.2$. The attached results are for a 10-replication simulation. The true values of the parameter vector θ are

```
0 0 0 0 0 0 0 0 -0.4472136 0 -0.4472136 0 0 0 -0.4472136 0 -0.4472136 0 0 -0.4472136 ,
```

but for brevity, our simulation only estimates the indices of θ in $\mathcal{C} = \{ 9, 11, 10, 4\}$ elements of θ. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

Results

```
### Mean-squared error comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat\theta
```

Table 1: Mean-Squared Error of Parameter Estimates

	proposed	cgm
theta[9]	0.063	0.016
theta[11]	0.024	0.035
theta[10]	0.020	0.017
theta[4]	0.014	0.019
total	0.030	0.022

Table 2: Mean-Squared Error of First-Step Parameter Estimates

	proposed	cgm
theta[9]	0.136	0.032
theta[11]	0.107	0.018
theta[10]	0.001	0.017
theta[4]	0.009	0.004
total	0.063	0.018

Mean absolute deviation comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat

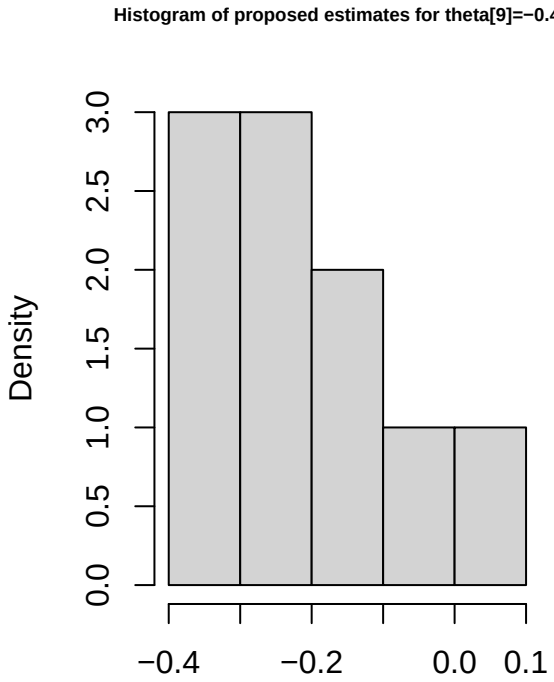
Table 3: Mean Absolute Deviation of Parameter Estimates

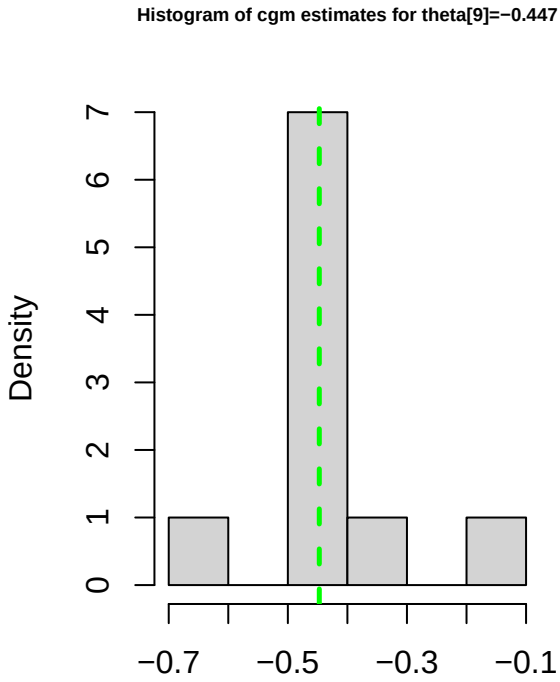
	proposed	cgm
theta[9]	0.217	0.087
theta[11]	0.137	0.150
theta[10]	0.094	0.105
theta[4]	0.095	0.115
total	0.136	0.114

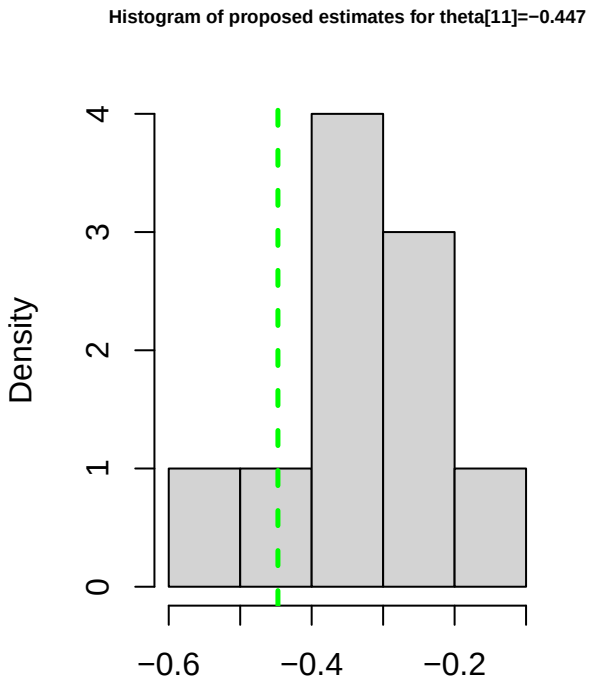
Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

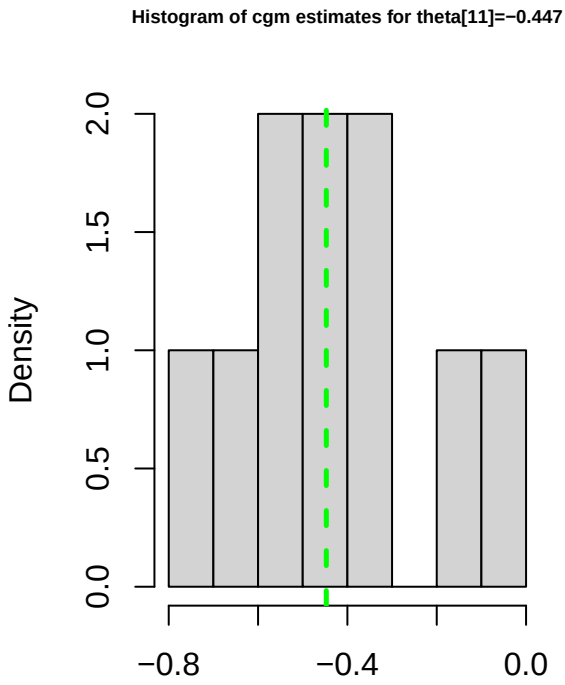
	proposed	cgm
theta[9]	0.359	0.139
theta[11]	0.296	0.103
theta[10]	0.012	0.086
theta[4]	0.036	0.038
total	0.176	0.091

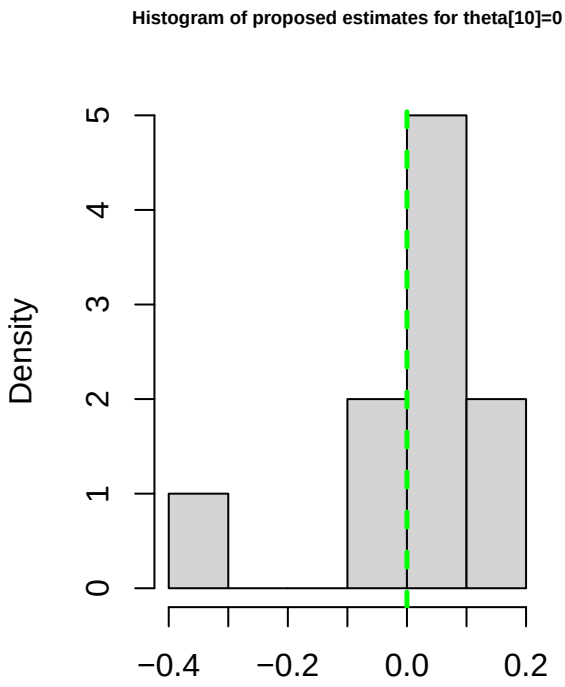
Boxplots

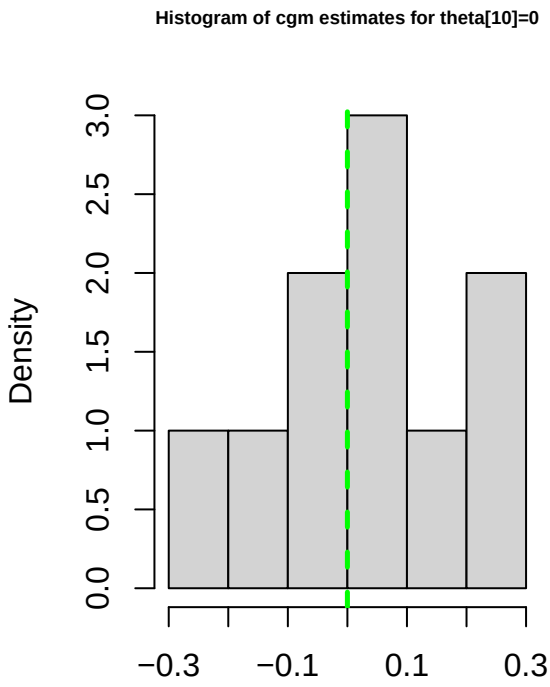

Histogram of proposed estimates for theta[9]=−0.447
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.4
−0.2
0.0
0.1

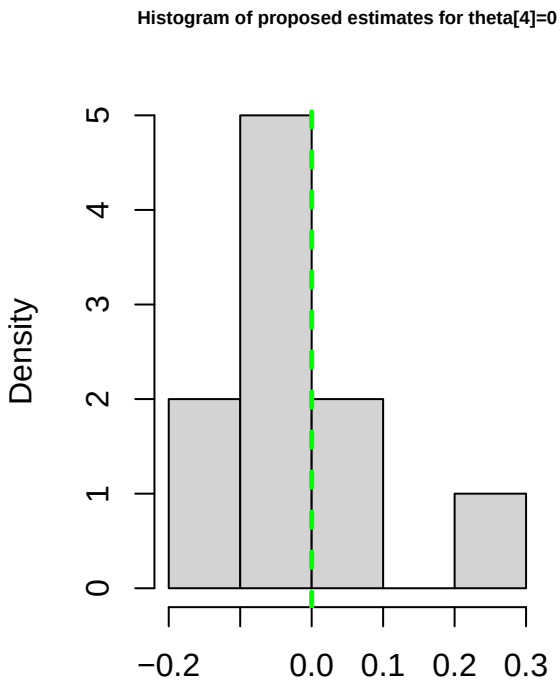

Histogram of cgm estimates for theta[9]=−0.447
Density
0
1
2
3
4
5
6
7
−0.7
−0.5
−0.3
−0.1

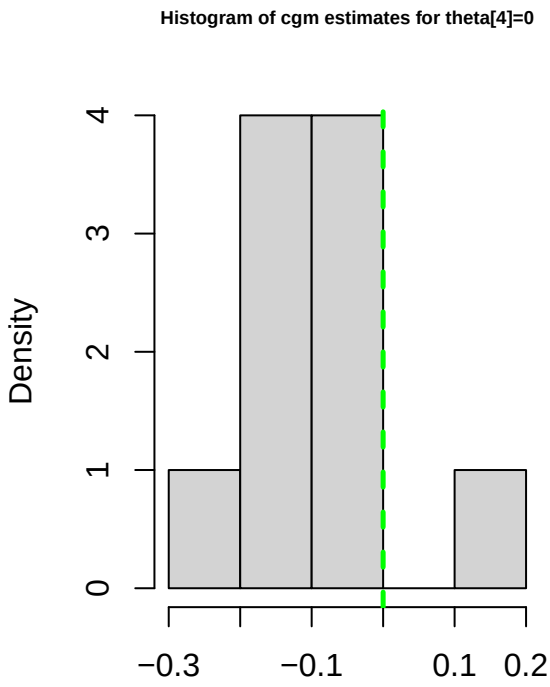

Histogram of proposed estimates for theta[11]=−0.447
Density
0
1
2
3
4
−0.6
−0.4
−0.2


Histogram of cgm estimates for theta[11]=−0.447
Density
0.0
0.5
1.0
1.5
2.0
−0.8
−0.4
0.0

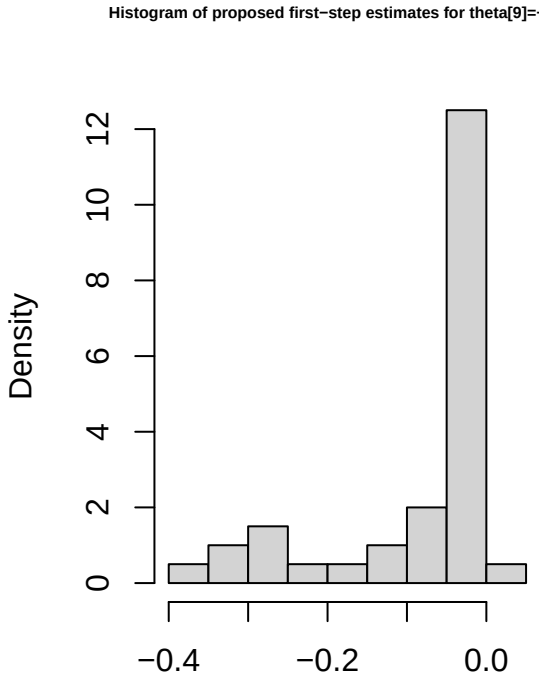

Histogram of proposed estimates for theta[10]=0
Density
0
1
2
3
4
5
−0.4
−0.2
0.0
0.2

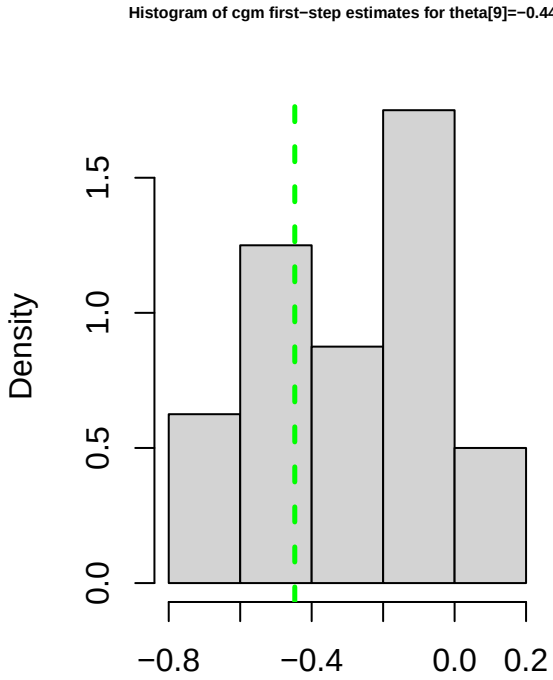

Histogram of cgm estimates for theta[10]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.3
−0.1
0.1
0.3

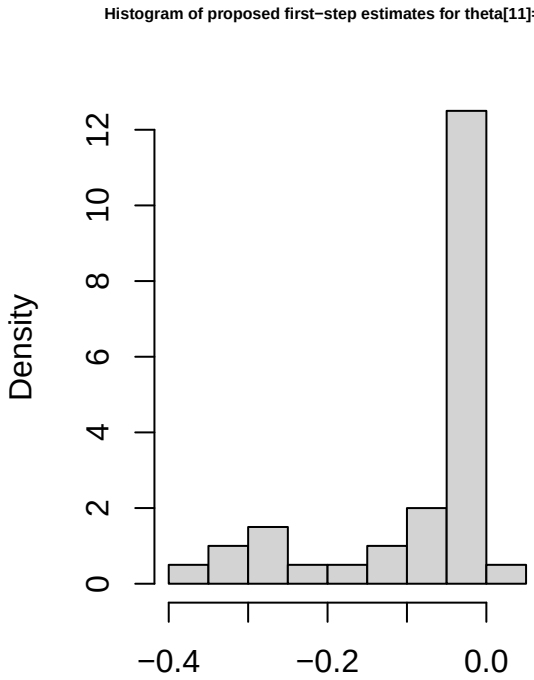

Histogram of proposed estimates for theta[4]=0
Density
0
1
2
3
4
5
−0.2
0.0
0.1
0.2
0.3

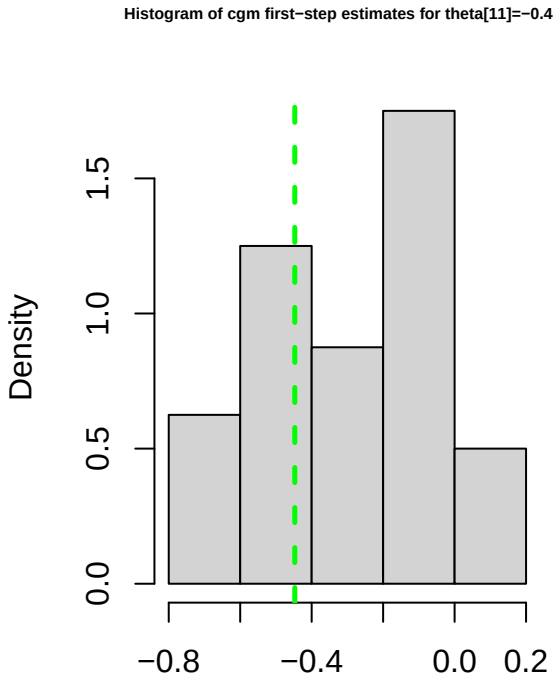

Histogram of cgm estimates for theta[4]=0
Density
0
1
2
3
4
−0.3
−0.1
0.1
0.2

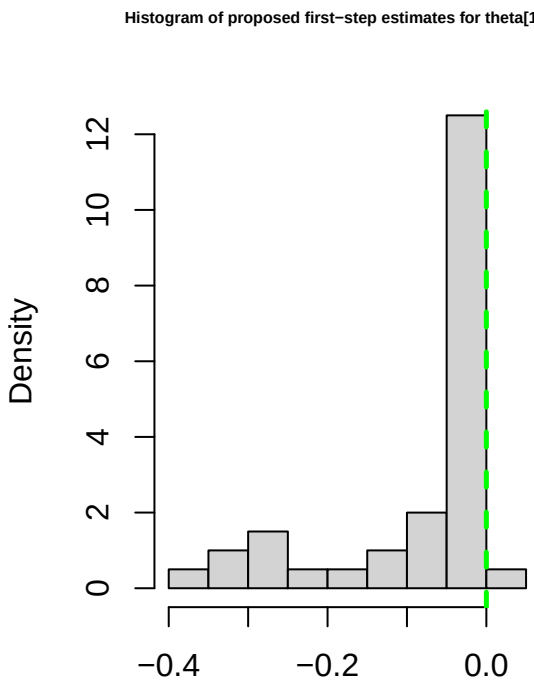
First Step Histograms

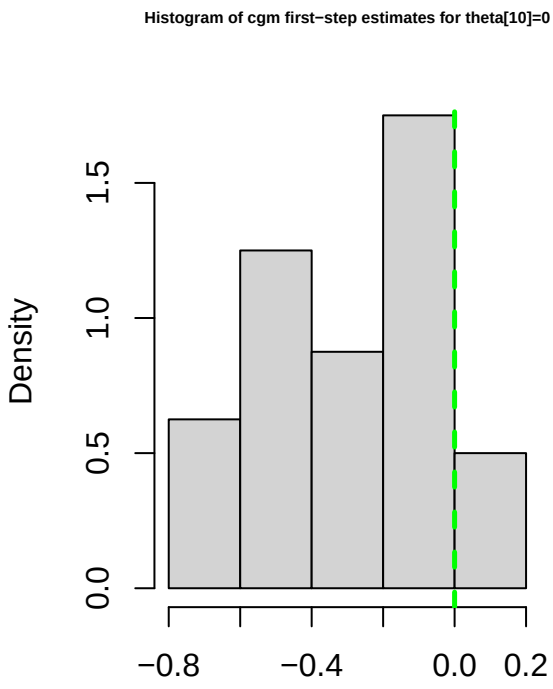


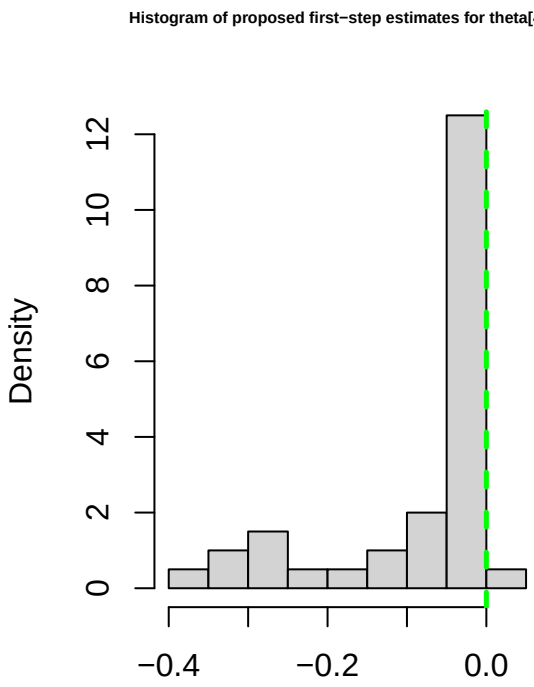


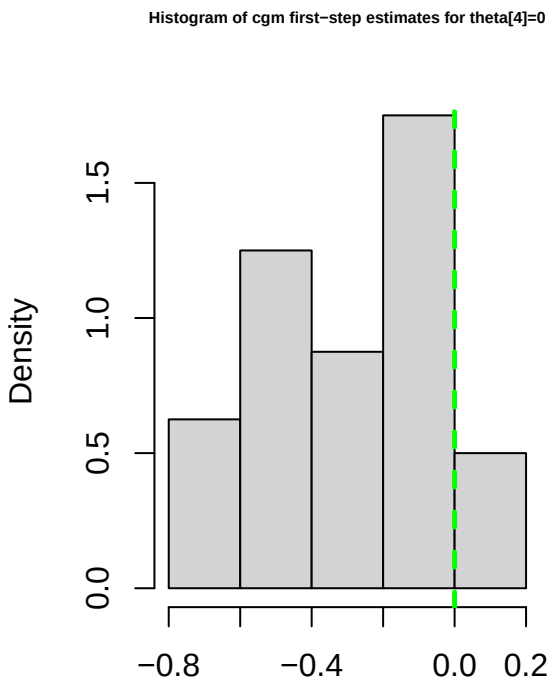





Histogram of proposed first−step estimates for theta[10]=0
Density
0
2
4
6
8
10
12
−0.4
−0.2
0.0


Histogram of cgm first−step estimates for theta[10]=0
Density
0.0
0.5
1.0
1.5
−0.8
−0.4
0.0
0.2


Histogram of proposed first−step estimates for theta[4]=0
Density
0
2
4
6
8
10
12
−0.4
−0.2
0.0


Histogram of cgm first−step estimates for theta[4]=0
Density
0.0
0.5
1.0
1.5
−0.8
−0.4
0.0
0.2

Statistics and 95% Confidence Intervals from per-Replicate Estimates

Table 5: Statistics for proposed Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[9]	-0.392	-0.280	0.020	-0.388	-0.003
theta[11]	-0.578	-0.346	-0.175	-0.549	-0.186
theta[10]	-0.388	0.020	0.139	-0.315	0.134
theta[4]	-0.156	-0.034	0.264	-0.144	0.223

Table 6: Statistics for cgm Estimates

	Min	Median	Max	lower.CI.btsp	upper.CI.btsp
theta[9]	-0.699	-0.419	-0.174	-0.650	-0.205
theta[11]	-0.708	-0.460	-0.100	-0.689	-0.107
theta[10]	-0.201	0.032	0.207	-0.181	0.207
theta[4]	-0.268	-0.086	0.187	-0.251	0.142

Statistics for Theoretical 95% Confidence Intervals

Table 7: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[9]	-0.230	0.123	-0.471	0.011	0.6
theta[11]	-0.337	0.120	-0.574	-0.101	0.8
theta[10]	0.002	0.117	-0.227	0.230	0.9
theta[4]	-0.009	0.122	-0.249	0.231	1.0

Table 8: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

	Estimate	SE	lower.CI	upper.CI	cvg
theta[9]	-0.426	0.123	-0.668	-0.184	0.9
theta[11]	-0.433	0.122	-0.673	-0.194	0.7
theta[10]	0.031	0.112	-0.189	0.251	1.0
theta[4]	-0.077	0.115	-0.303	0.149	0.9