# Simulation Results

2026-01-15

## Simulation Setup

This simulation is performed with $n = 200$ and $d = 20$, using the 2-d lattice as the underlying graph. $s = 5$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0$. The attached results are for a 10-replication simulation. The true values of the parameter vector $\theta$ are

```
0 0 0.4472136 0 0.4472136 0 0 0 0 0 0 0.4472136 0 0 0.4472136 0 0 0 -0.4472136 0 ,
```

but for brevity, our simulation only estimates the indices of $\theta$ in $\mathcal{C} = \{\ 3, 5, 11, 6\}$ elements of $\theta$. Accordingly, **all statistics and visuals are indicative of performance only on the set $\mathcal{C}$.**

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

## Results

```
### Mean-squared error comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat\theta
```

Table 1: Mean-Squared Error of Parameter Estimates

| | proposed | cgm |
|---|---|---|
| theta[3] | 0.045 | 0.012 |
| theta[5] | 0.022 | 0.011 |
| theta[11] | 0.008 | 0.020 |
| theta[6] | 0.012 | 0.035 |
| total | 0.022 | 0.019 |

Table 2: Mean-Squared Error of First-Step Parameter Estimates

| | proposed | cgm |
|---|---|---|
| theta[3] | 0.120 | 0.037 |
| theta[5] | 0.126 | 0.046 |
| theta[11] | 0.001 | 0.000 |
| theta[6] | 0.000 | 0.000 |
| total | 0.062 | 0.021 |

### Mean absolute deviation comparison $(\frac{1}{n.sim}\sum_{i=1}^{n.sim} \frac{1}{|\mathcal C|}\|\hat

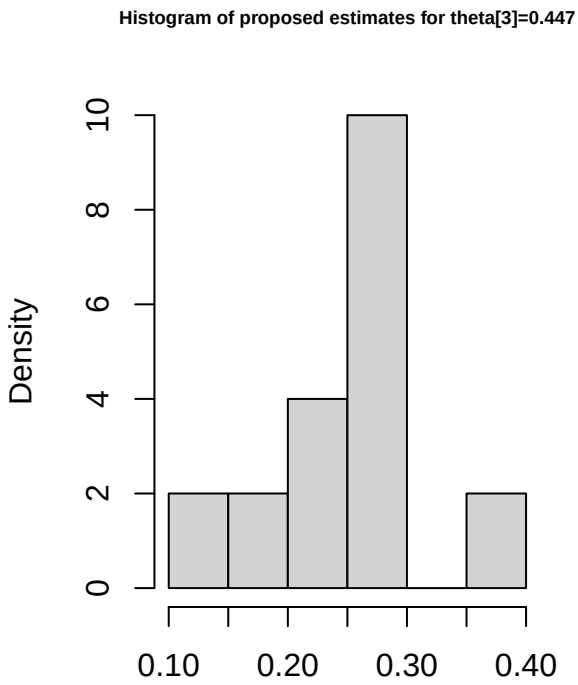
Table 3: Mean Absolute Deviation of Parameter Estimates

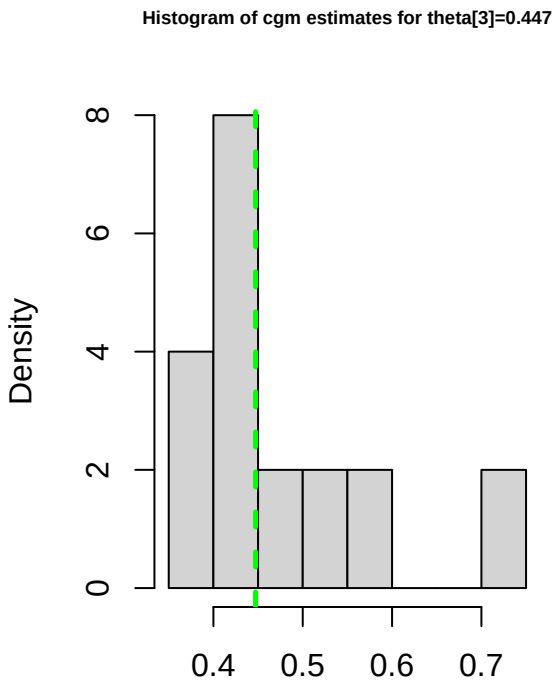
| | proposed | cgm |
|---|---|---|
| theta[3] | 0.199 | 0.073 |
| theta[5] | 0.139 | 0.090 |
| theta[11] | 0.077 | 0.115 |
| theta[6] | 0.091 | 0.149 |
| total | 0.127 | 0.107 |

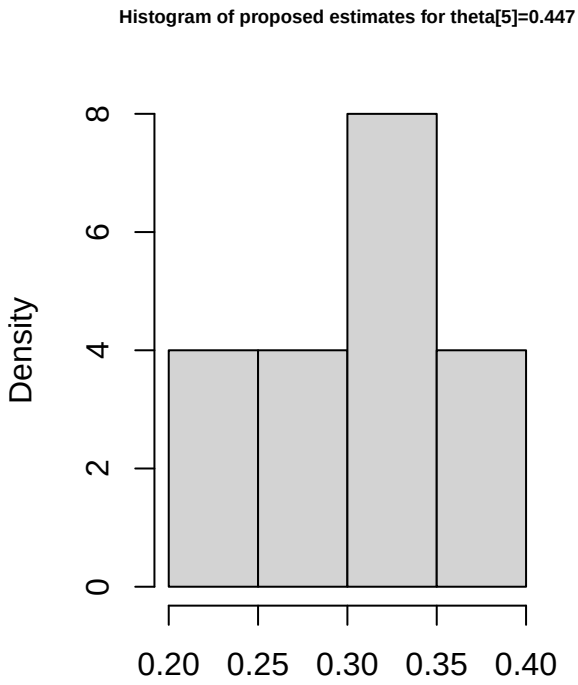
Table 4: Mean Absolute Deviation of First-Step Parameter Estimates

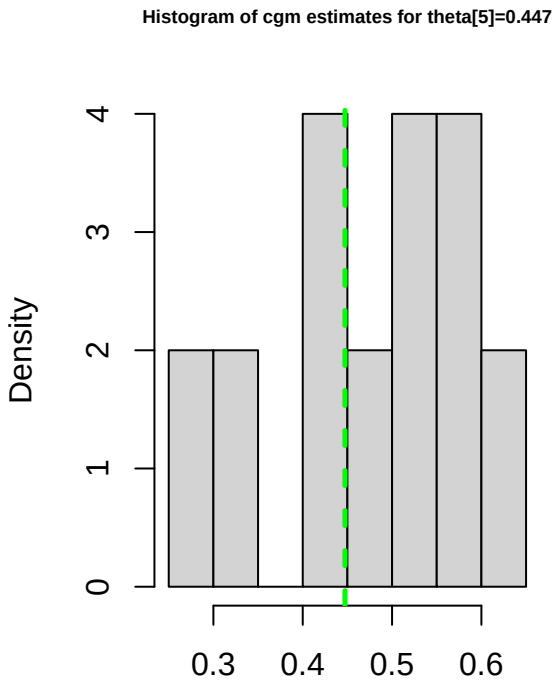
| | proposed | cgm |
|---|---|---|
| theta[3] | 0.332 | 0.154 |
| theta[5] | 0.338 | 0.154 |
| theta[11] | 0.008 | 0.000 |
| theta[6] | 0.005 | 0.001 |
| total | 0.171 | 0.077 |

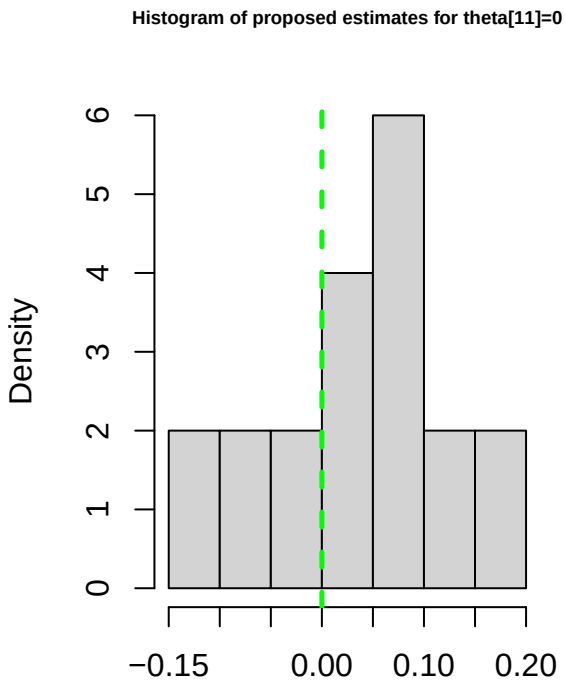
## Boxplots

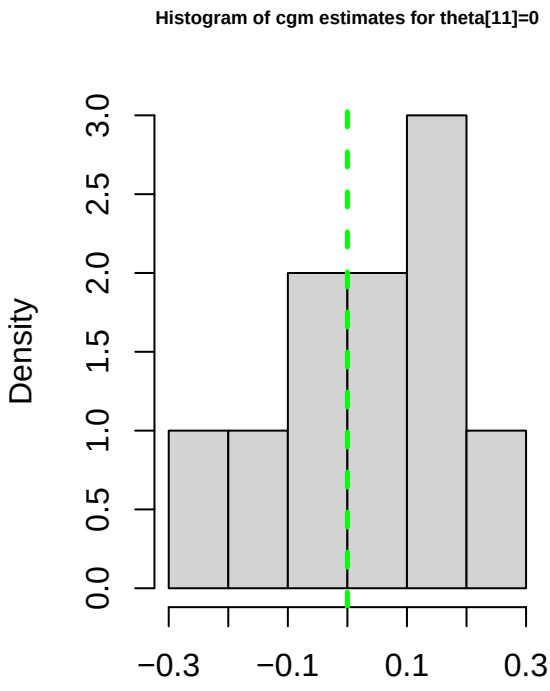

Histogram of proposed estimates for theta[3]=0.447
Density
0
2
4
6
8
10
0.10
0.20
0.30
0.40

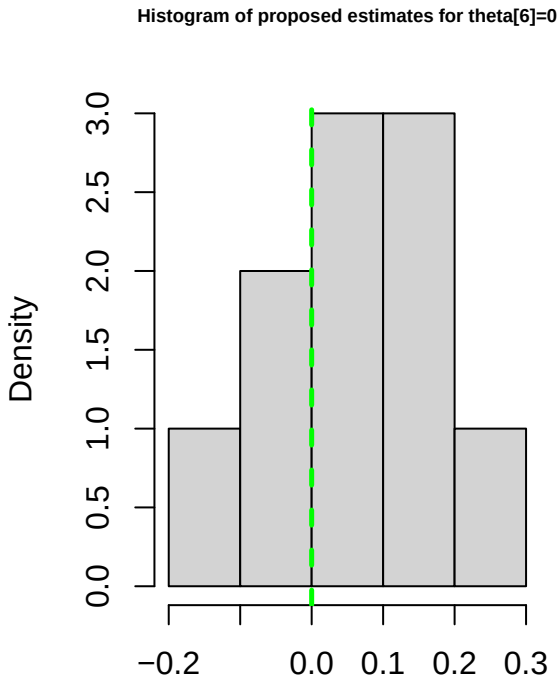

Histogram of cgm estimates for theta[3]=0.447
Density
0
2
4
6
8
0.4
0.5
0.6
0.7

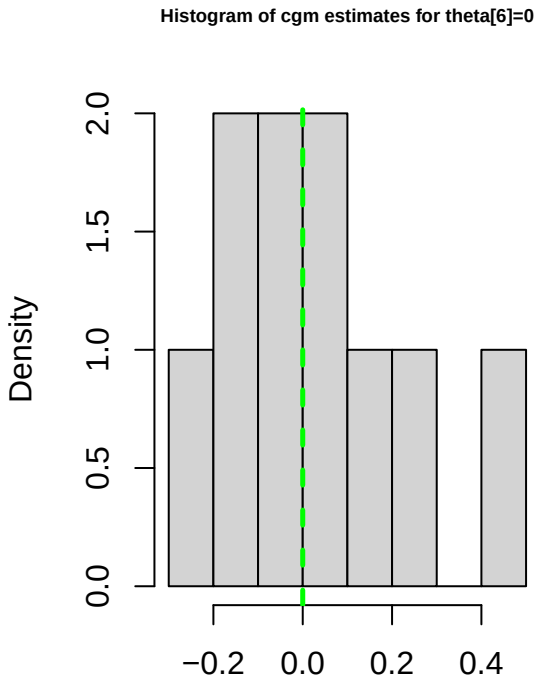

Histogram of proposed estimates for theta[5]=0.447
Density
0
2
4
6
8
0.20
0.25
0.30
0.35
0.40


Histogram of cgm estimates for theta[5]=0.447
Density
0
1
2
3
4
0.3
0.4
0.5
0.6

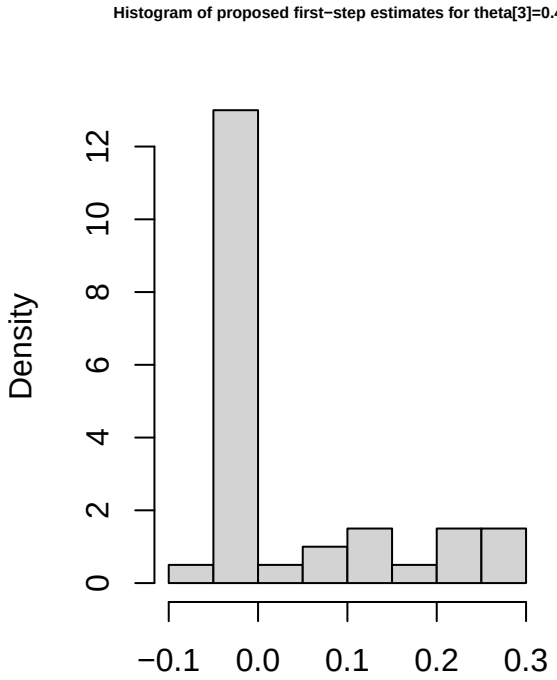

Histogram of proposed estimates for theta[11]=0
Density
0
1
2
3
4
5
6
−0.15
0.00
0.10
0.20

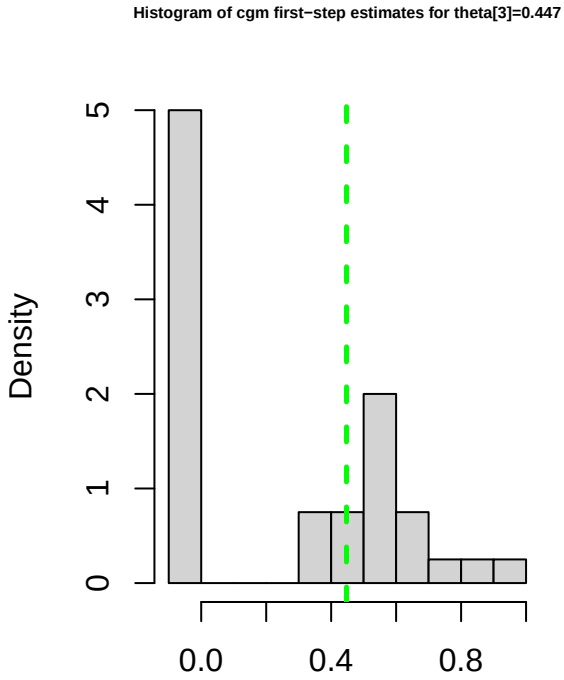

Histogram of cgm estimates for theta[11]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.3
−0.1
0.1
0.3

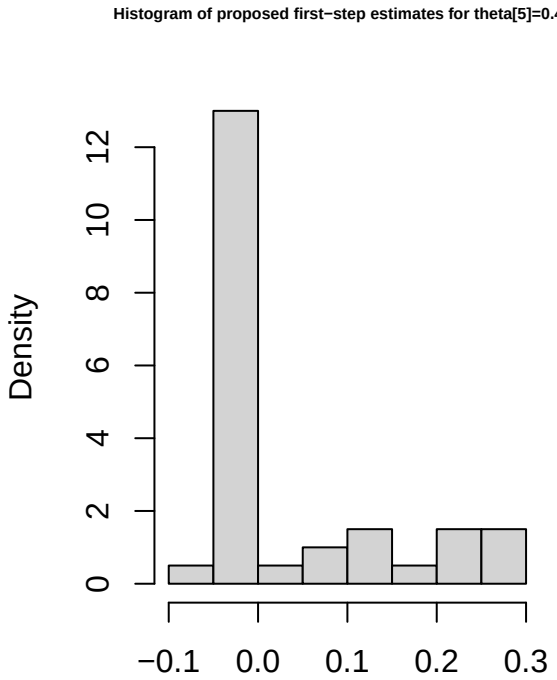

Histogram of proposed estimates for theta[6]=0
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−0.2
0.0
0.1
0.2
0.3

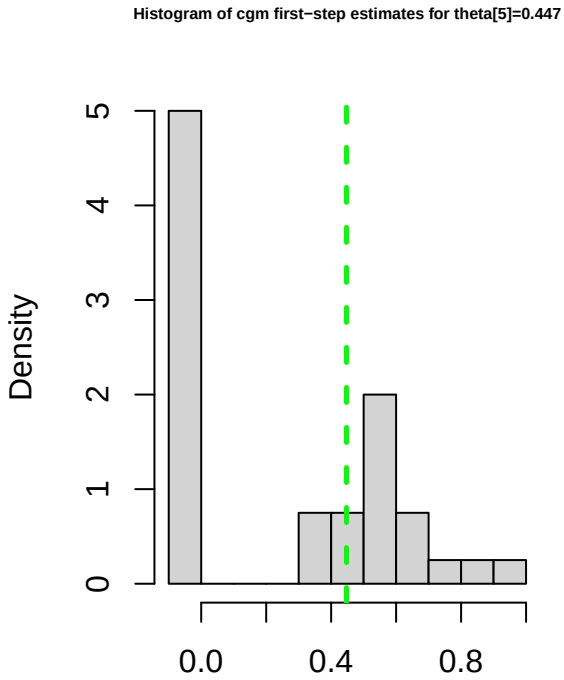

Histogram of cgm estimates for theta[6]=0
Density
0.0
0.5
1.0
1.5
2.0
−0.2
0.0
0.2
0.4

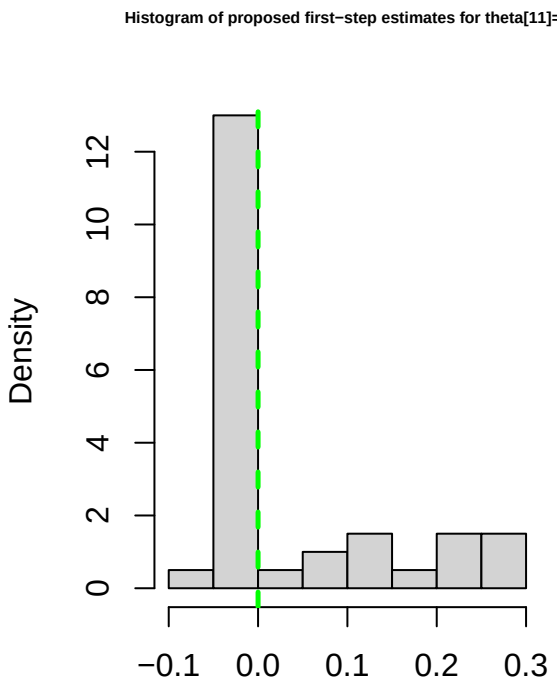
### First Step Histograms

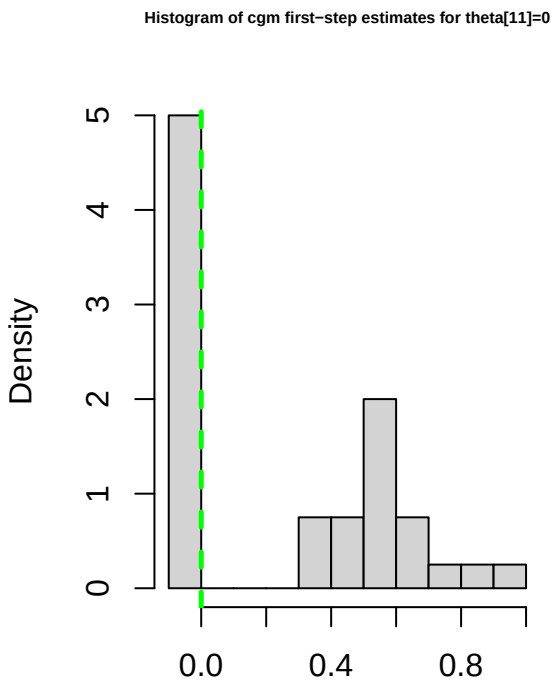


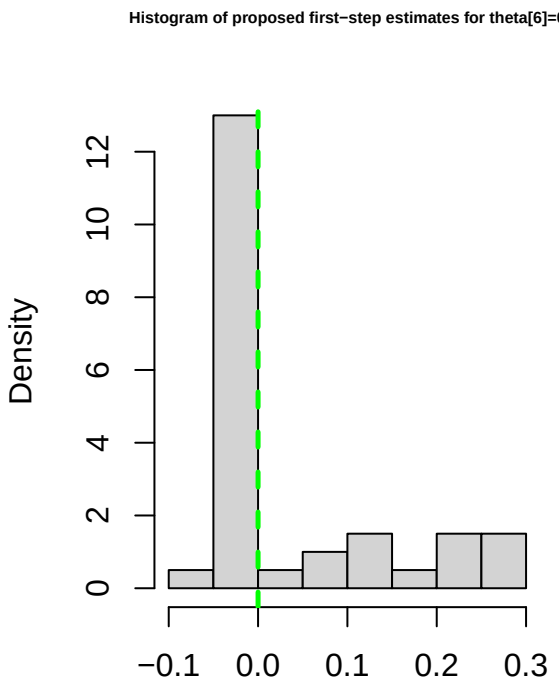



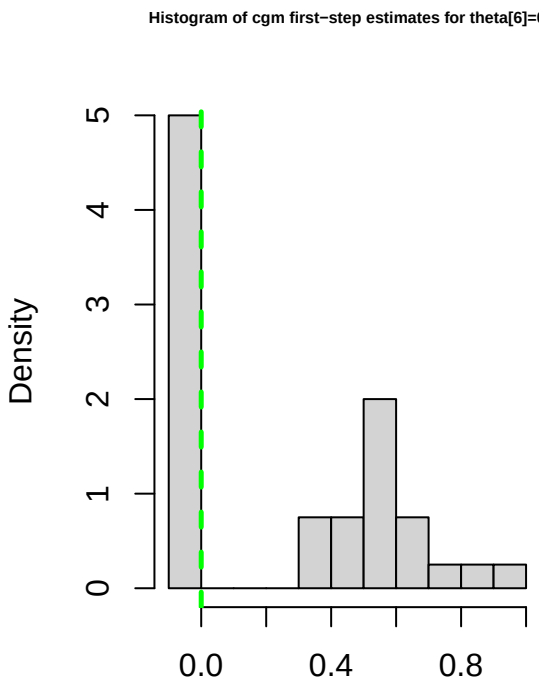




Histogram of proposed first−step estimates for theta[11]=0
Density
0
2
4
6
8
10
12
−0.1
0.0
0.1
0.2
0.3


Histogram of cgm first−step estimates for theta[11]=0
Density
0
1
2
3
4
5
0.0
0.4
0.8


Histogram of proposed first−step estimates for theta[6]=0
Density
0
2
4
6
8
10
12
−0.1
0.0
0.1
0.2
0.3


Histogram of cgm first−step estimates for theta[6]=0
Density
0
1
2
3
4
5
0.0
0.4
0.8

**Statistics and 95% Confidence Intervals from per-Replicate Estimates**

Table 5: Statistics for proposed Estimates

| | Min | Median | Max | lower.CI.btsp | upper.CI.btsp |
|---|---|---|---|---|---|
| theta[3] | 0.108 | 0.257 | 0.394 | 0.123 | 0.372 |
| theta[5] | 0.219 | 0.319 | 0.388 | 0.221 | 0.387 |
| theta[11] | -0.141 | 0.050 | 0.151 | -0.123 | 0.147 |
| theta[6] | -0.112 | 0.061 | 0.252 | -0.101 | 0.220 |

Table 6: Statistics for cgm Estimates

| | Min | Median | Max | lower.CI.btsp | upper.CI.btsp |
|---|---|---|---|---|---|
| theta[3] | 0.383 | 0.438 | 0.729 | 0.385 | 0.699 |
| theta[5] | 0.293 | 0.493 | 0.613 | 0.303 | 0.609 |
| theta[11] | -0.225 | 0.012 | 0.228 | -0.197 | 0.220 |
| theta[6] | -0.207 | 0.005 | 0.424 | -0.205 | 0.375 |

**Statistics for Theoretical 95% Confidence Intervals**

Table 7: Theoretical 95% Confidence Interval Statistics (averaged across replications) for proposed Estimates

| | Estimate | SE | lower.CI | upper.CI | cvg |
|---|---|---|---|---|---|
| theta[3] | 0.249 | 0.116 | 0.021 | 0.476 | 0.8 |
| theta[5] | 0.309 | 0.113 | 0.088 | 0.530 | 0.8 |
| theta[11] | 0.034 | 0.117 | -0.195 | 0.263 | 1.0 |
| theta[6] | 0.051 | 0.119 | -0.183 | 0.285 | 0.9 |

Table 8: Theoretical 95% Confidence Interval Statistics (averaged across replications) for cgm Estimates

| | Estimate | SE | lower.CI | upper.CI | cvg |
|---|---|---|---|---|---|
| theta[3] | 0.478 | 0.126 | 0.232 | 0.725 | 0.9 |
| theta[5] | 0.476 | 0.130 | 0.222 | 0.730 | 1.0 |
| theta[11] | 0.030 | 0.116 | -0.197 | 0.256 | 0.8 |
| theta[6] | 0.018 | 0.118 | -0.213 | 0.249 | 0.9 |