Simulation Results

2025-10-14

Simulation Setup

This simulation is performed with $n = 200$ and $d = 10$, using the 2-d lattice as the underlying graph. $s = 2$ parameters are set to be nonzero, and the beta parameter is chosen to be $\beta = 0.1$. The attached results are for a 5-replication simulation. The true values of the parameter vector θ are

```
 [1]  0  1  0  0  0  0  0  0  0 -1
```

The results from our code are compared to those of Cai, Guo, and Ma (2021).

The attached results include the mean-squared error for each parameter estimate, as well as boxplots for a selection of nonzero and zero-valued parameters. In the boxplots, the green line represents the true value of the estimated parameter.

After these, I show coverage statistics for 95% symmetric confidence intervals for each of the parameters.

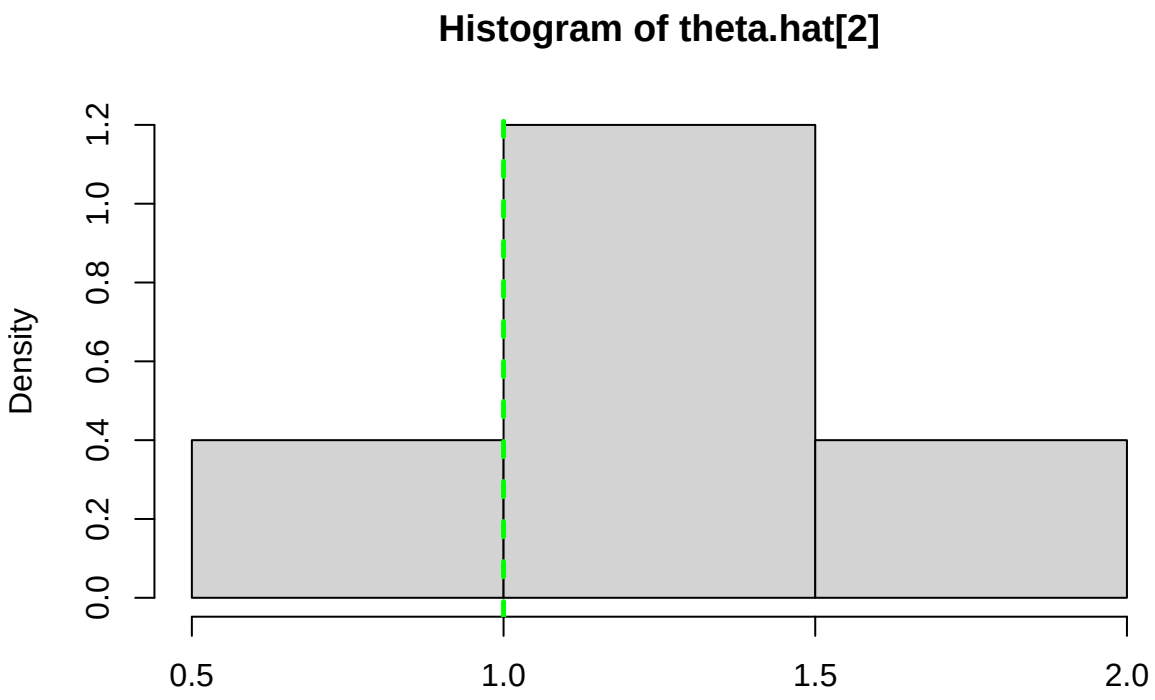
Results

Mean-squared error comparison ($\frac{1}{n.sim}\sum_{i=1}^{n.sim}\frac{1}{d}\|\hat{\theta}_i - \theta\|^2$)

```
# A tibble: 1 x 2
  `MISLE (First-step) MSE` `MISLE MSE`
                     <dbl>       <dbl>
1                    0.115      0.0458
```

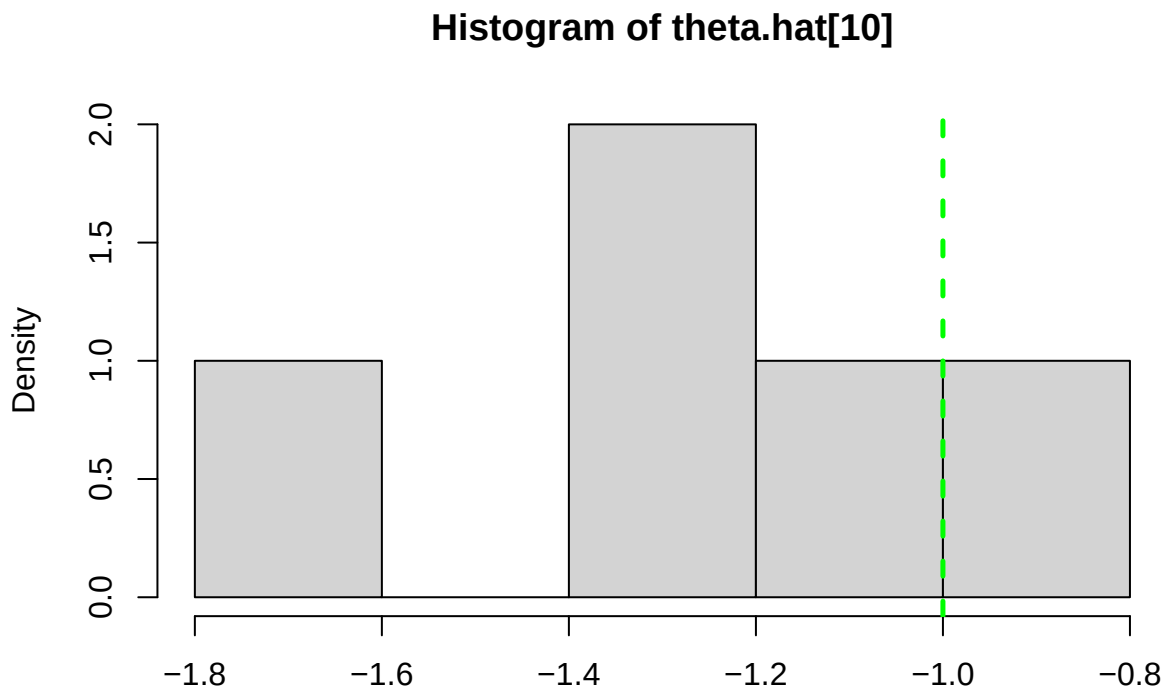
```
# A tibble: 1 x 2
  `MISLE MSE` `CGM MSE`
        <dbl>     <dbl>
1      0.0458      3.45
```

First Step Histograms

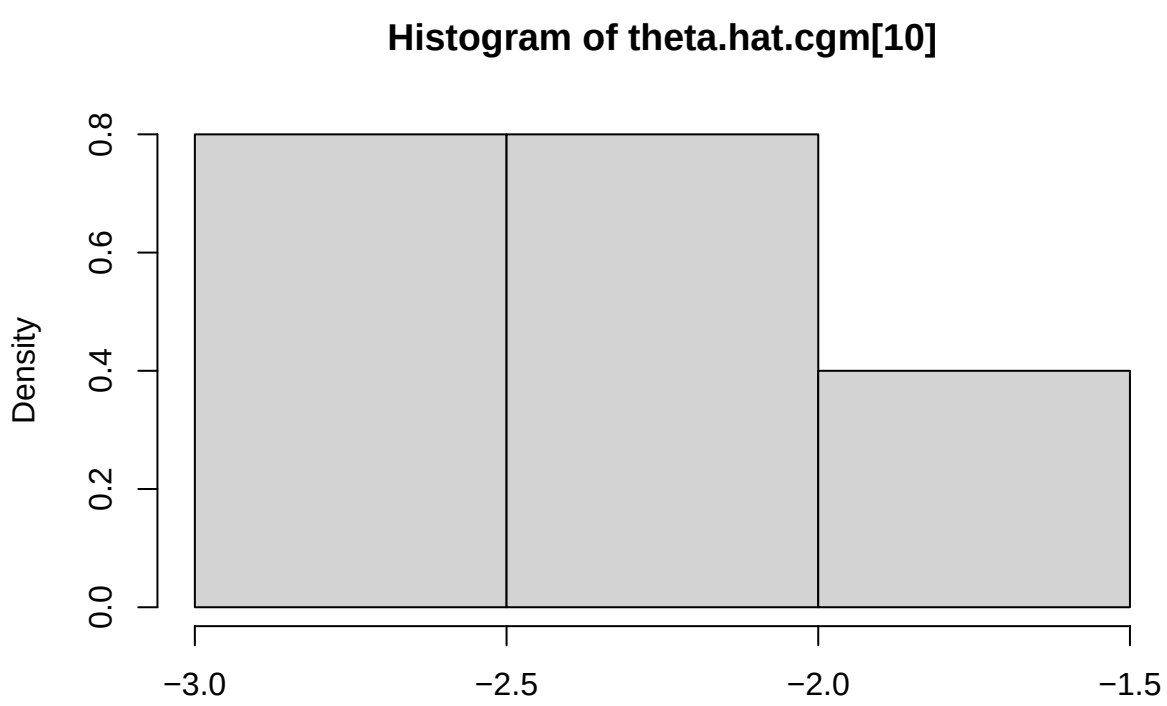



```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  0.810   1.032   1.082   1.270   1.490   1.936
[1] "95% CI based on bootstrap:"
      lower   upper
1 0.8322902 1.89176
```

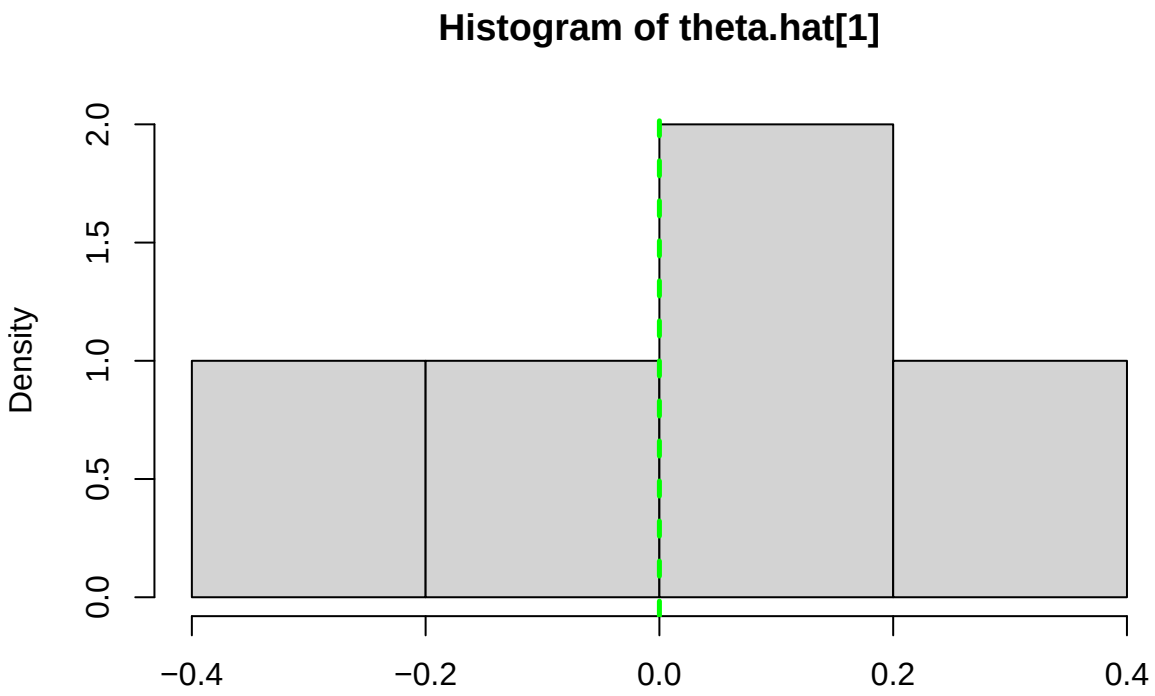

```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  1.306   1.840   1.982   1.940   2.099   2.472
[1] "95% CI based on bootstrap:"
  lower.cgm upper.cgm
1  1.359157  2.434995
```



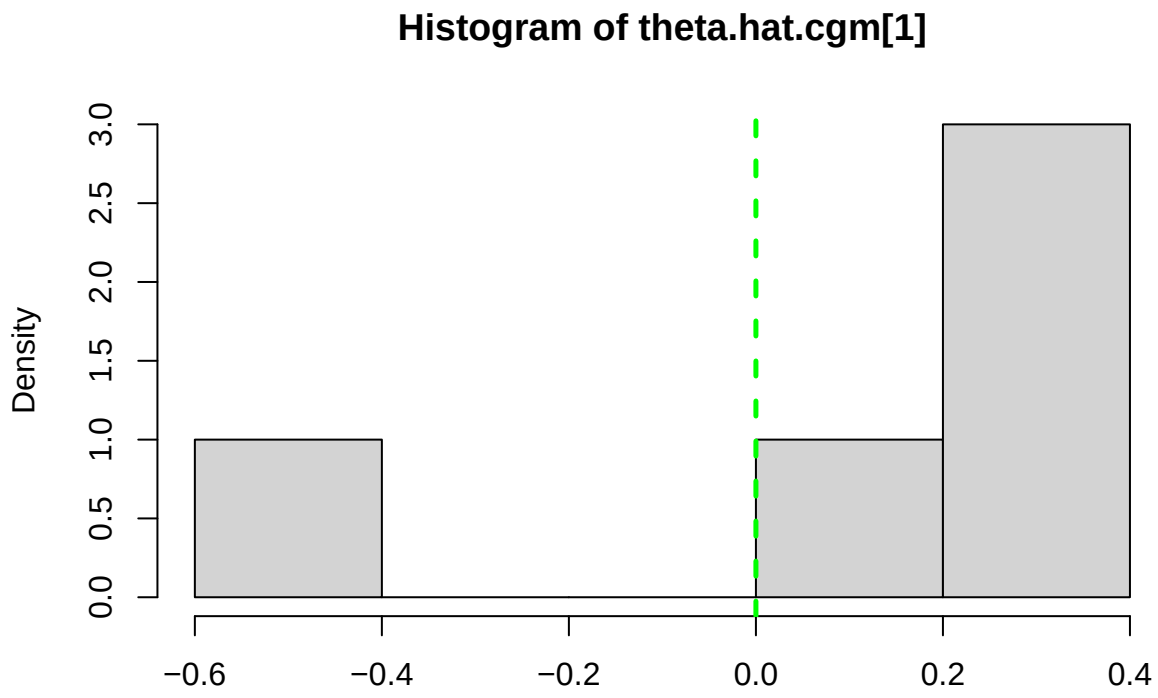

```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-1.6959 -1.3902 -1.3302 -1.2821 -1.0237 -0.9707
[1] "95% CI based on bootstrap:"
      lower      upper
1 -1.665325 -0.9759733
```



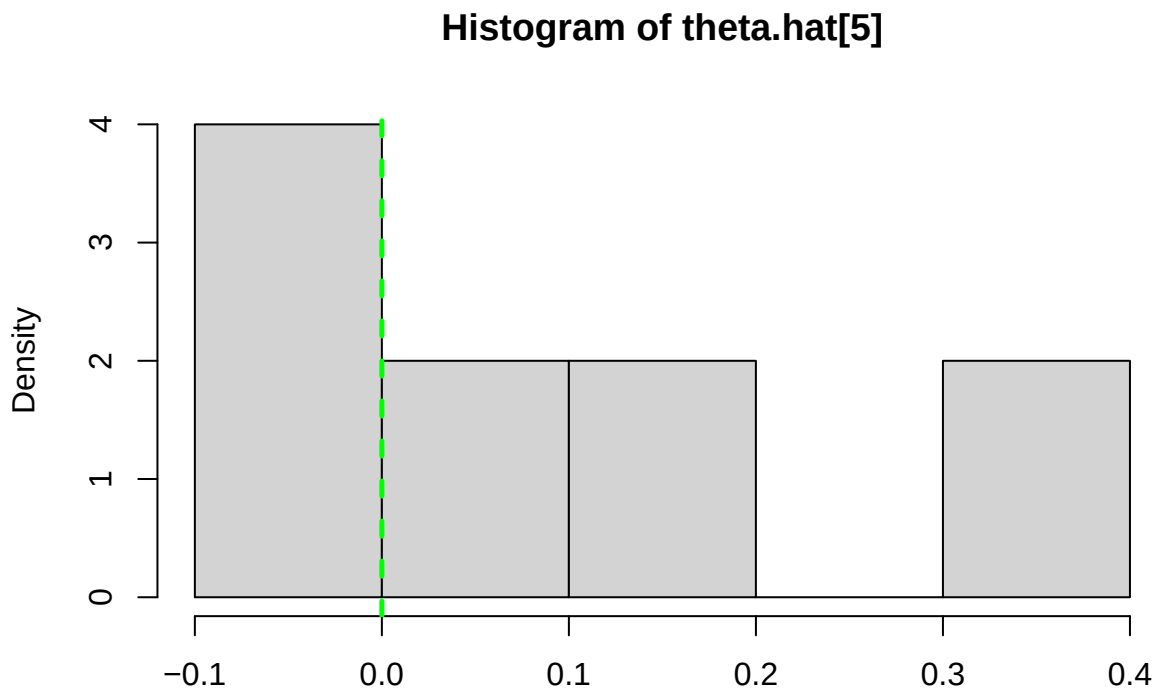

```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 -2.741  -2.557  -2.413  -2.289  -2.119  -1.613
[1] "95% CI based on bootstrap:"
  lower.cgm upper.cgm
1 -2.722696 -1.663977
```



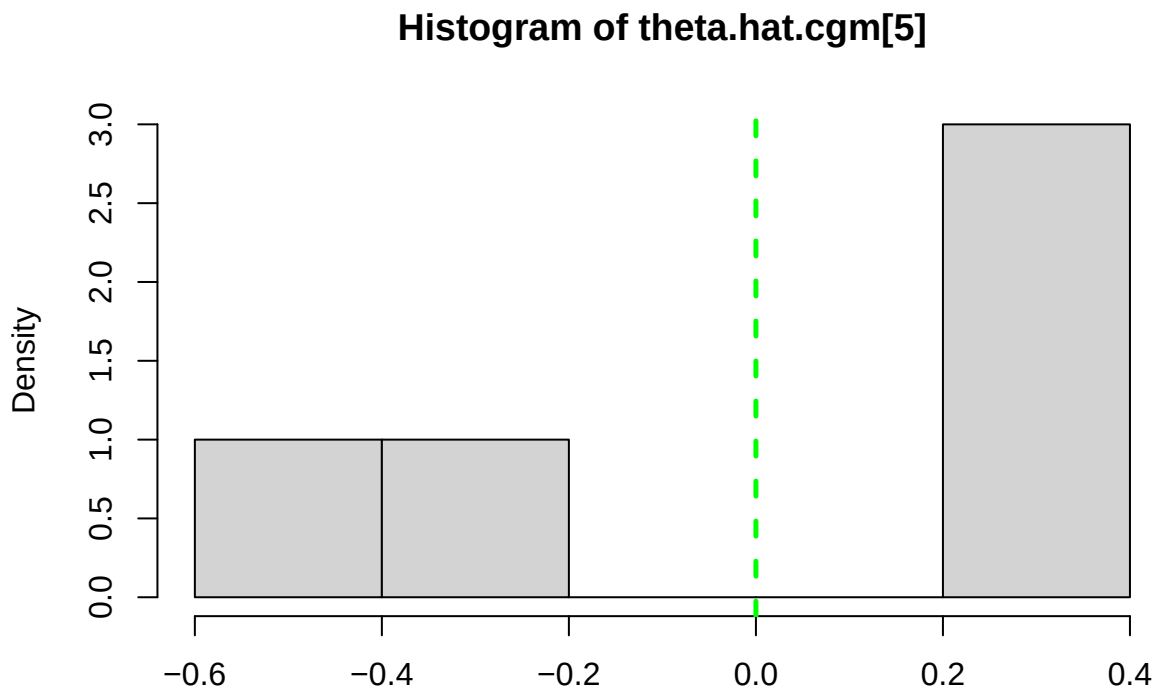

```
[1] "Summary statistics of bootstrap replicates:"
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-0.33707 -0.04881  0.02514  0.02274  0.12796  0.34649
[1] "95% CI based on bootstrap:"
       lower     upper
1 -0.3082476 0.3246414
```




```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-0.4142  0.1284  0.2259  0.1285  0.3143  0.3881
[1] "95% CI based on bootstrap:"
   lower.cgm upper.cgm
1 -0.3599113 0.3807547
```

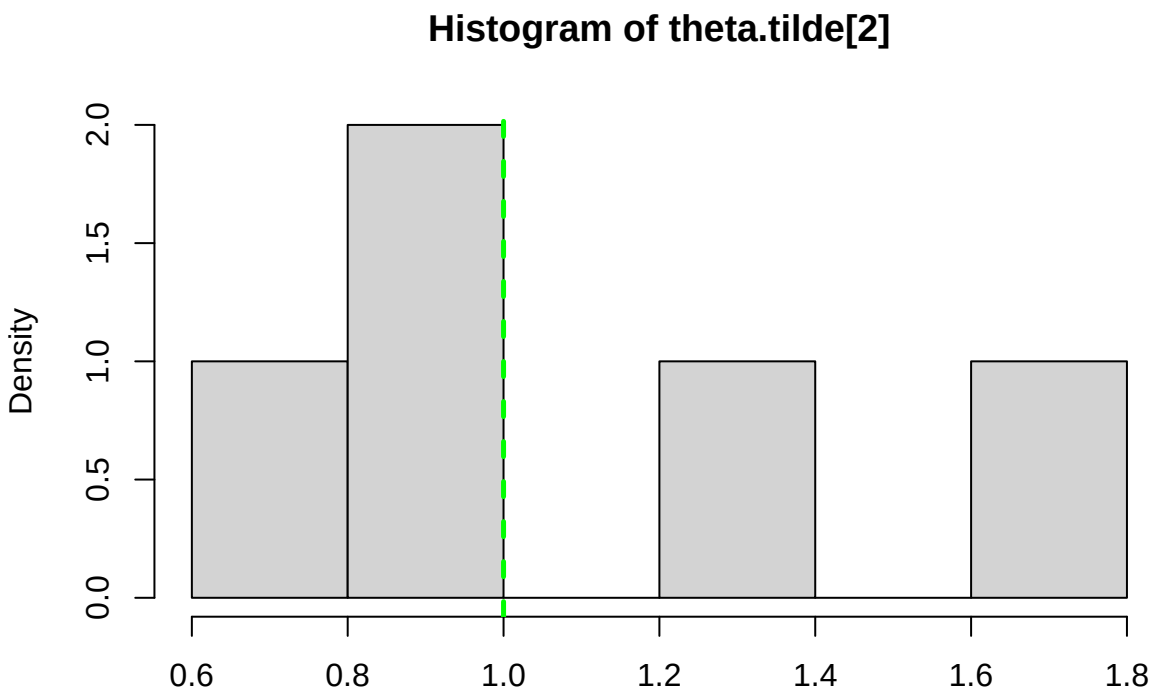



```
[1] "Summary statistics of bootstrap replicates:"
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-0.05951  0.00000  0.03871  0.09312  0.13185  0.35456
[1] "95% CI based on bootstrap:"
        lower    upper
1 -0.05355728 0.332287
```

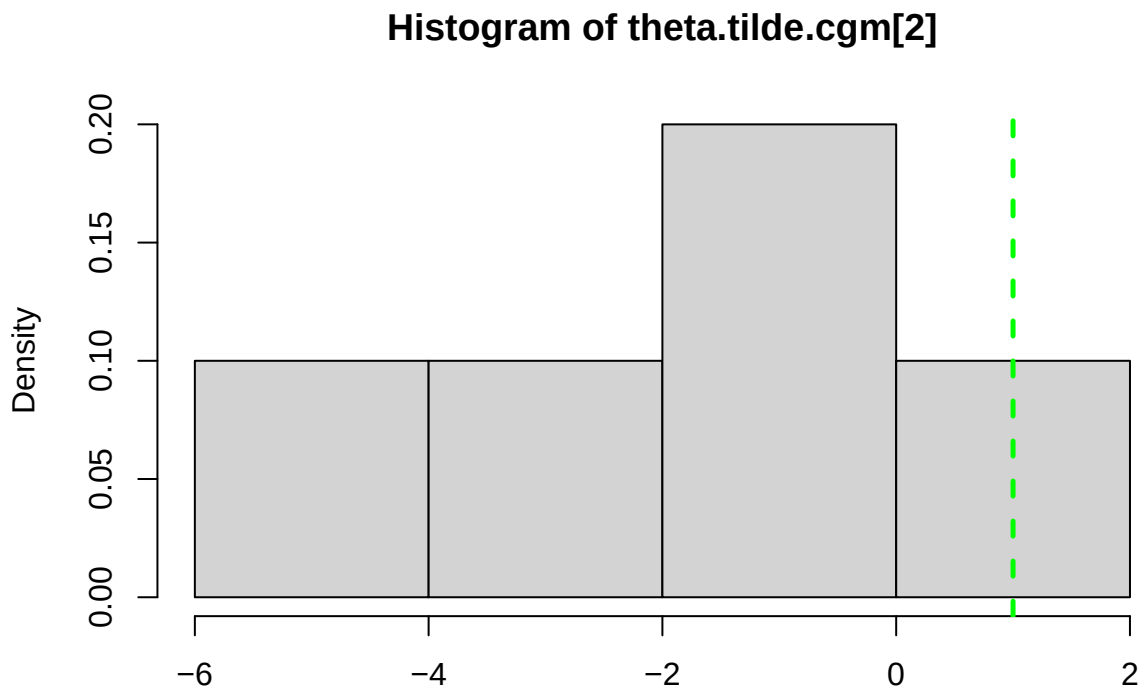



```
[1] "Summary statistics of bootstrap replicates:"
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-0.49412 -0.25649  0.24477  0.02262  0.25308  0.36587
[1] "95% CI based on bootstrap:"
   lower.cgm upper.cgm
1 -0.4703601 0.3545882
```

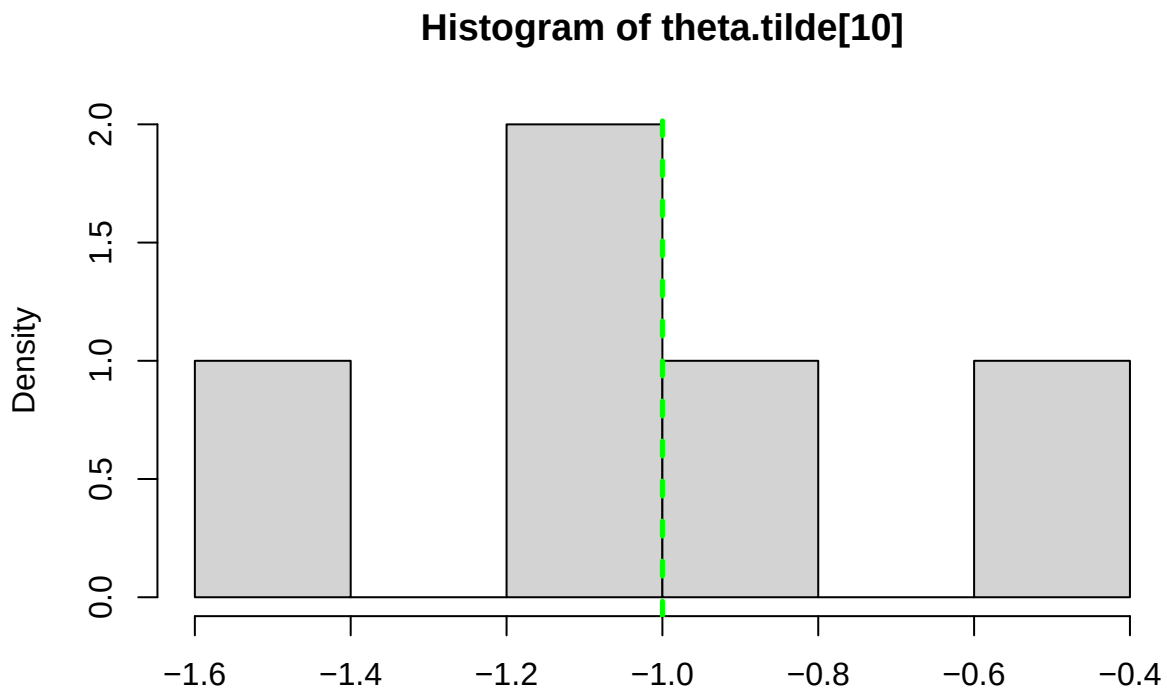
Statistics and 95% Confidence Intervals from per-Replicate Estimates



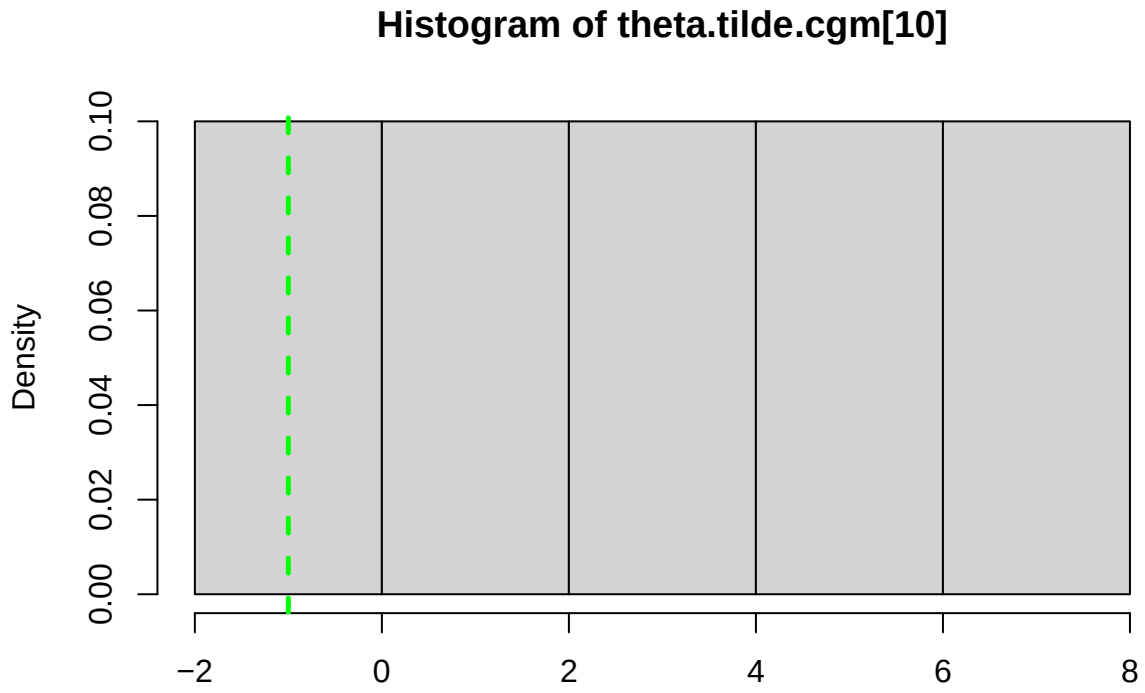

```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.7590  0.8398  0.9232  1.1280  1.3478  1.7700
[1] "95% CI based on bootstrap:"
     lower    upper
1 0.767042 1.727795
```



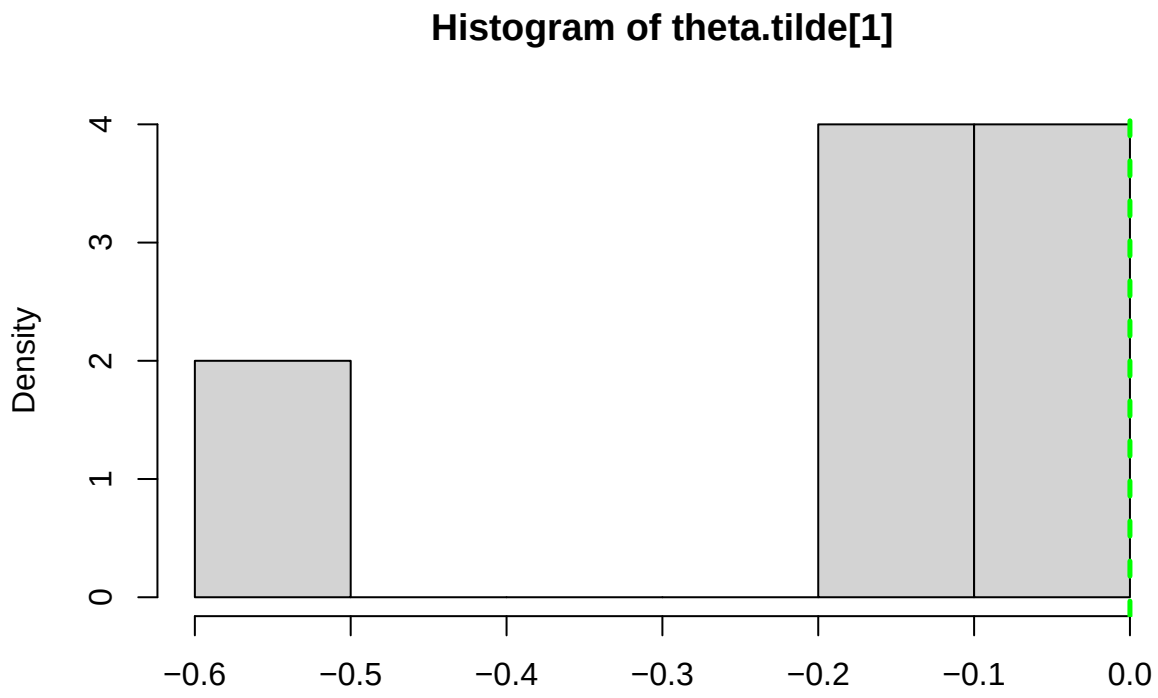

```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-4.9208 -2.9313 -1.0977 -1.6710 -0.1467  0.7413
[1] "95% CI based on bootstrap:"
  lower.cgm upper.cgm
1  -4.72186 0.6525374
```



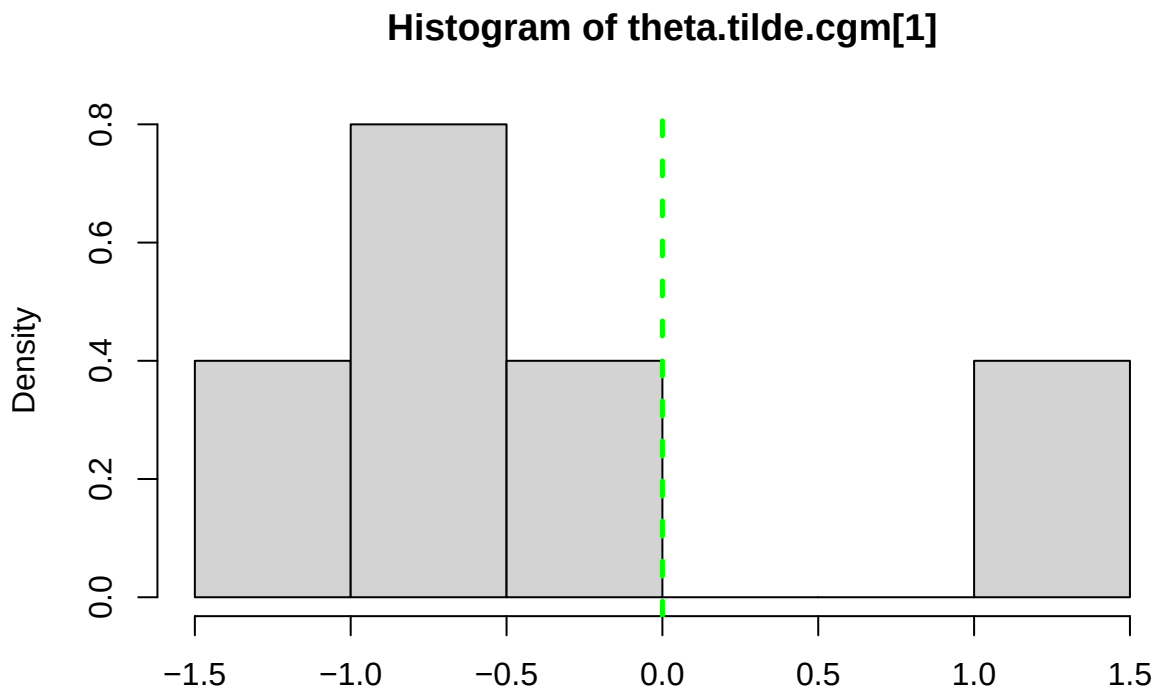

```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-1.5618 -1.1256 -1.1165 -1.0487 -0.9951 -0.4443
[1] "95% CI based on bootstrap:"
      lower      upper
1 -1.518203 -0.4993911
```



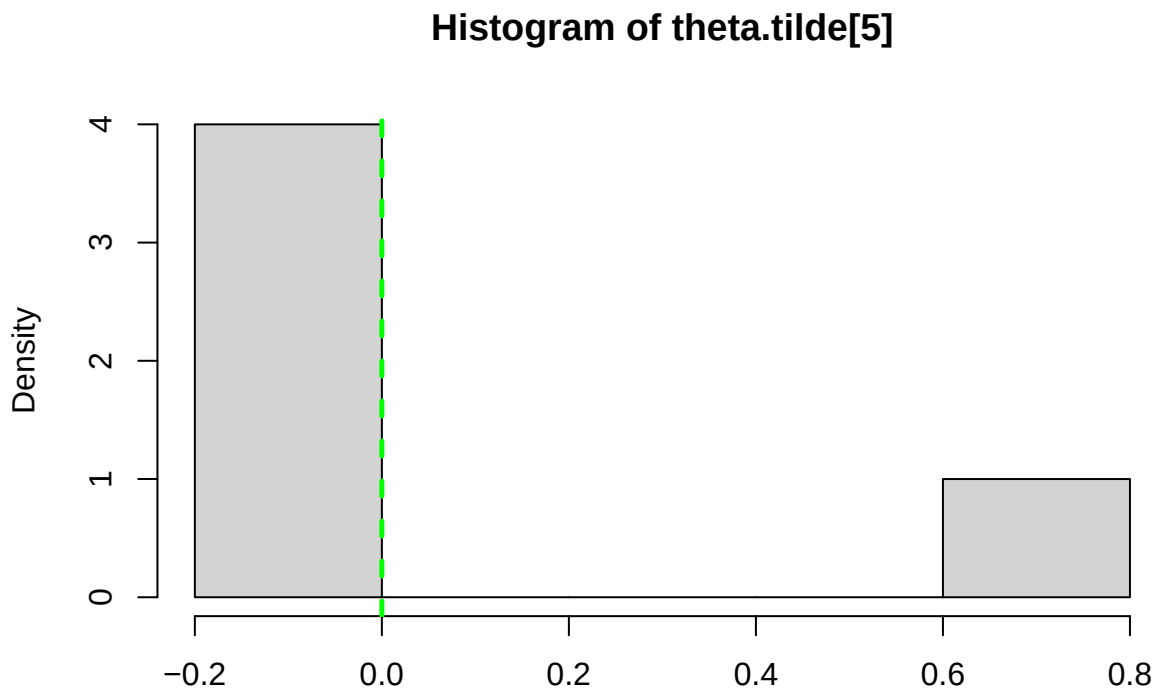

```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-0.8916  0.4021  2.4864  2.7086  5.1626  6.3835
[1] "95% CI based on bootstrap:"
   lower.cgm upper.cgm
1 -0.7622234  6.261431
```



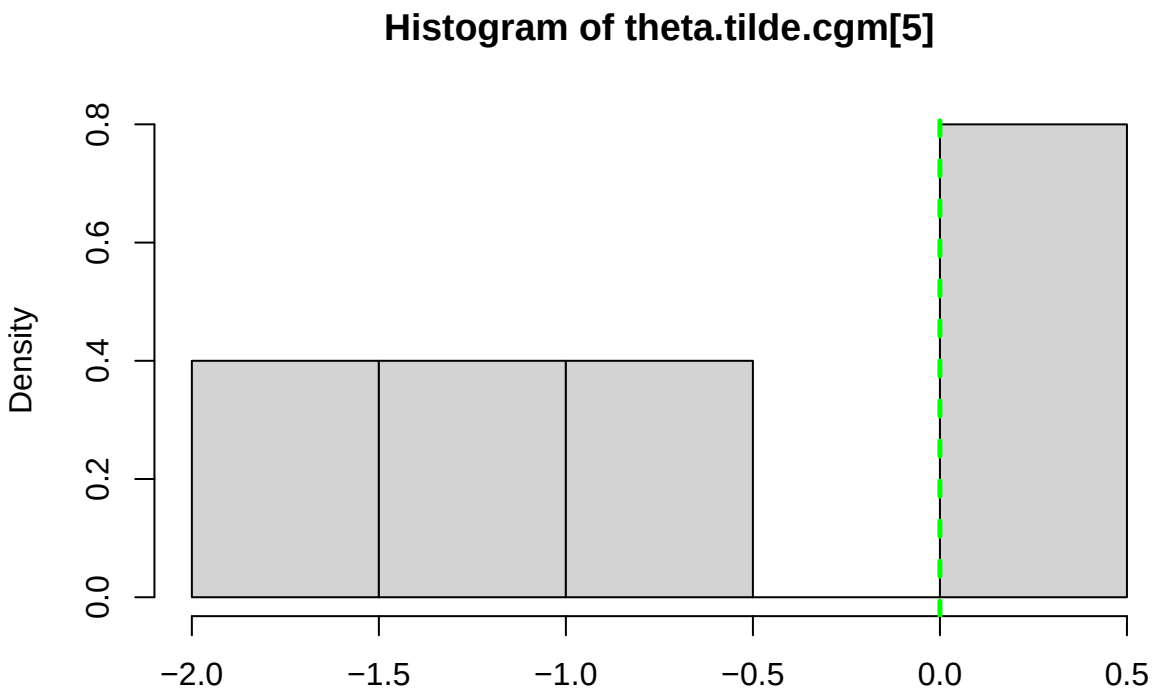

```
[1] "Summary statistics of bootstrap replicates:"
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-0.56600 -0.11406 -0.10488 -0.17792 -0.07955 -0.02510
[1] "95% CI based on bootstrap:"
       lower       upper
1 -0.5208052 -0.03054902
```




```
[1] "Summary statistics of bootstrap replicates:"
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-1.4696 -0.9926 -0.5941 -0.4118 -0.1546  1.1520
[1] "95% CI based on bootstrap:"
  lower.cgm upper.cgm
1  -1.42188   1.02137
```




```
[1] "Summary statistics of bootstrap replicates:"
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-0.18545 -0.15540 -0.05391  0.03963 -0.05317  0.64606
[1] "95% CI based on bootstrap:"
       lower     upper
1 -0.1824444 0.5761363
```




```
[1] "Summary statistics of bootstrap replicates:"
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-1.74891 -1.11967 -0.52210 -0.66560  0.01972  0.04294
[1] "95% CI based on bootstrap:"
  lower.cgm  upper.cgm
1 -1.685986 0.04061747
```

Statistics for Theoretical 95% Confidence Intervals

```
[1] Length of Confidence Intervals for theta[2]
[1] Coverage proportion: 1
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.7818  0.7986  0.9183  1.0325  1.0998  1.5641
[1] Length of Confidence Intervals for theta[2] (CGM Method)
[1] Coverage proportion: 1
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  6.205   7.732   7.948   8.632   9.862  11.412
[1] Length of Confidence Intervals for theta[10]
[1] Coverage proportion: 1
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.7593  0.8460  1.0851  1.0611  1.1779  1.4371
[1] Length of Confidence Intervals for theta[10] (CGM Method)
[1] Coverage proportion: 1
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  8.999  12.642  17.232  16.903  22.410  23.231
[1] Length of Confidence Intervals for theta[1]
[1] Coverage proportion: 0.8
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.6128  0.6523  0.7274  0.7075  0.7592  0.7860
[1] Length of Confidence Intervals for theta[1] (CGM Method)
[1] Coverage proportion: 1
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  3.616   3.892   3.913   4.885   5.120   7.882
[1] Length of Confidence Intervals for theta[5]
[1] Coverage proportion: 0.8
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.5495  0.5570  0.5846  0.6977  0.8716  0.9260
[1] Length of Confidence Intervals for theta[5] (CGM Method)
[1] Coverage proportion: 1
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  3.444   4.045   4.482   4.596   5.446   5.561
```