Supplementary Material

D Proof of Sufficiency of MLRP

Lemma 11. *Suppose that the rules and the equilibrium choice of the escalation game satisfy MLRP. Then, upward incentive constraints together with downward adjacent incentive constraints are sufficient for (global) downward incentive constraints.*

Proof. Take a $\mathcal{CM}$ that satisfies downward adjacent incentive constraints and (global) upward incentive constraints. We show that the global downward incentive constraints are necessarily satisfied.

Take i and any θ and θ' such that $\theta > \theta' - 1$. We establish that $\Pi_i(\theta, \theta) \geq \Pi_i(\theta', \theta)$. We proceed by induction.

Step 1: Basis. Take any θ and θ' such that $\theta - \theta' = 2$. Then $\Pi(\theta, \theta) \geq \Pi(\theta', \theta)$.

We show that

$$z_i(\theta) - z_i(\theta - 2) \geq \gamma_i(\theta - 2) U_i(\theta - 2, \theta, \mathcal{B}(\sigma)) - \gamma_i(\theta) U_i(\theta, \theta, \mathcal{B}(\sigma)). \tag{2}$$

By hypothesis all downward adjacent incentive constraints are satisfied, that is,

$$z_i(k) - z_i(k - 1) \geq \gamma_i(k - 1) U_i(k - 1, k, \mathcal{B}(\sigma)) - \gamma_i(k) U_i(k, k, \mathcal{B}(\sigma)).$$

Therefore,

$$\begin{aligned} z_i(\theta) - z_i(\theta - 2) \geq & \gamma_i(\theta - 1) U_i(\theta - 1, \theta, \mathcal{B}(\sigma)) - \gamma_i(\theta) U_i(\theta, \theta, \mathcal{B}(\sigma)) \\ & + \gamma_i(\theta - 2) U_i(\theta - 2, \theta - 1, \mathcal{B}(\sigma)) - \gamma_i(\theta - 1) U_i(\theta - 1, \theta - 1, \mathcal{B}(\sigma)). \end{aligned} \tag{3}$$

The RHS of equation (3) is larger than the RHS of equation (2) if

$$\frac{U_i(\theta - 1, \theta - 1, \mathcal{B}(\sigma)) - U_i(\theta - 1, \theta, \mathcal{B}(\sigma))}{U_i(\theta - 2, \theta - 1, \mathcal{B}(\sigma)) - U_i(\theta - 2, \theta, \mathcal{B}(\sigma))} \leq \frac{\gamma_i(\theta - 2)}{\gamma_i(\theta - 1)}. \tag{4}$$

Upward and downward adjacent incentive constraints of $\theta - 1, \theta - 2$ jointly imply that

$$\frac{\gamma_i(\theta - 2)}{\gamma_i(\theta - 1)} \geq \frac{U_i(\theta - 1, \theta - 2, \mathcal{B}(\sigma)) - U_i(\theta - 1, \theta - 1, \mathcal{B}(\sigma))}{U_i(\theta - 2, \theta - 2, \mathcal{B}(\sigma)) - U_i(\theta - 2, \theta - 1, \mathcal{B}(\sigma))}.$$

Thus, equation (4) is true if

$$\begin{aligned} & \frac{U_i(\theta - 1, \theta - 2, \mathcal{B}(\sigma)) - U_i(\theta - 1, \theta - 1, \mathcal{B}(\sigma))}{U_i(\theta - 2, \theta - 2, \mathcal{B}(\sigma)) - U_i(\theta - 2, \theta - 1, \mathcal{B}(\sigma))} \\ \geq & \frac{U_i(\theta - 1, \theta - 1, \mathcal{B}(\sigma)) - U_i(\theta - 1, \theta, \mathcal{B}(\sigma))}{U_i(\theta - 2, \theta - 1, \mathcal{B}(\sigma)) - U_i(\theta - 2, \theta, \mathcal{B}(\sigma))} \end{aligned} \tag{5}$$

Equation (5) is satisfied by the definition of the MLRP condition.

Step 2: Induction Hypothesis. Take θ, θ' such that $\theta - \theta' = n$. Then $\Pi(\theta, \theta) \geq \Pi(\theta', \theta)$.

Step 3: Inductive Step. Next, we show that for any θ and θ' such that $\theta - \theta' = n + 1$ it holds that $\Pi(\theta,\theta) \geq \Pi(\theta{-}n-1,\theta)$. That is, we establish

$$z_i(\theta) - z_i(\theta{-}n-1) \geq \gamma_i(\theta{-}n-1)U_i(\theta{-}n-1,\theta,\mathcal{B}(\sigma)) - \gamma_i(\theta)U_i(\theta,\theta,\mathcal{B}(\sigma)). \tag{6}$$

By the induction hypothesis and the fact that local incentive constraints hold, we have that

$$\begin{aligned}
&z_i(\theta) - z_i(\theta{-}n-1) = z_i(\theta) - z_i(\theta{-}1) + z_i(\theta{-}1) - z_i(\theta{-}n-1) \\
&\geq \gamma_i(\theta{-}1)U_i(\theta{-}1,\theta,\mathcal{B}(\sigma)) - \gamma_i(\theta)U_i(\theta,\theta,\mathcal{B}(\sigma)) \\
&+ \gamma_i(\theta{-}n-1)U_i(\theta{-}n-1,\theta{-}1,\mathcal{B}(\sigma)) - \gamma_i(\theta{-}1)U_i(\theta{-}1,\theta{-}1,\mathcal{B}(\sigma)).
\end{aligned} \tag{7}$$

We will show that the RHS of equation (7) is larger than the RHS of equation (6). This is true if

$$\frac{\gamma_i(\theta{-}n-1)}{\gamma_i(\theta{-}1)} \geq \frac{U_i(\theta{-}1,\theta{-}1,\mathcal{B}(\sigma)) - U_i(\theta{-}1,\theta,\mathcal{B}(\sigma))}{U_i(\theta{-}n-1,\theta{-}1,\mathcal{B}(\sigma)) - U_i(\theta{-}n-1,\theta,\mathcal{B}(\sigma))}. \tag{8}$$

To verify equation (8) we use that upward adjacent incentive constraints and downward adjacent incentive constraints jointly imply that

$$\frac{\gamma_i(k)}{\gamma_i(k{+}1)} \geq \frac{U_i(k{+}1,k,\mathcal{B}(\sigma)) - U_i(k{+}1,k{+}1,\mathcal{B}(\sigma))}{U_i(k,k,\mathcal{B}(\sigma)) - U_i(k,k{+}1,\mathcal{B}(\sigma))}. \tag{9}$$

By iteratively applying equation (9) the LHS of equation (8) can be bounded from below by

$$\frac{\gamma_i(\theta{-}n-1)}{\gamma_i(\theta{-}1)} \geq \prod_{k=\theta-n}^{\theta-1} \frac{U_i(k,k{-}1,\mathcal{B}(\sigma)) - U_i(k,k,\mathcal{B}(\sigma))}{U_i(k{-}1,k{-}1,\mathcal{B}(\sigma)) - U_i(k{-}1,k,\mathcal{B}(\sigma))} \tag{10}$$

We thus need to establish that

$$\prod_{k=\theta-n}^{\theta-1} \frac{U_i(k,k{-}1,\mathcal{B}(\sigma)) - U_i(k,k,\mathcal{B}(\sigma))}{U_i(k{-}1,k{-}1,\mathcal{B}(\sigma)) - U_i(k{-}1,k,\mathcal{B}(\sigma))} \geq \frac{U_i(\theta{-}1,\theta{-}1,\mathcal{B}(\sigma)) - U_i(\theta{-}1,\theta,\mathcal{B}(\sigma))}{U_i(\theta{-}n{-}1,\theta{-}1,\mathcal{B}(\sigma)) - U_i(\theta{-}n{-}1,\theta,\mathcal{B}(\sigma))},$$

or equivalently that

$$\begin{aligned}
&\frac{U_i(\theta{-}n-1,\theta{-}1,\mathcal{B}(\sigma)) - U_i(\theta{-}n-1,\theta,\mathcal{B}(\sigma))}{U_i(\theta{-}1,\theta{-}1,\mathcal{B}(\sigma)) - U_i(\theta{-}1,\theta,\mathcal{B}(\sigma))} \\
&\geq \prod_{k=\theta-n}^{\theta-1} \frac{U_i(k{-}1,k{-}1,\mathcal{B}(\sigma)) - U_i(k{-}1,k,\mathcal{B}(\sigma))}{U_i(k,k{-}1,\mathcal{B}(\sigma)) - U_i(k,k,\mathcal{B}(\sigma))}.
\end{aligned} \tag{11}$$

The MLRP allows us to bound the RHS of equation (11) from above by

$$\prod_{k=\theta-n}^{\theta-1} \frac{U_i(k{-}1,\theta{-}1,\mathcal{B}(\sigma)) - U_i(k{-}1,\theta,\mathcal{B}(\sigma))}{U_i(k,\theta{-}1,\mathcal{B}(\sigma)) - U_i(k,\theta,\mathcal{B}(\sigma))}.$$

Straightforward algebra verifies that this bound is equal to the LHS of equation (11). $\square$

E Statement of the General Problem and of the Lagrangian Objective

For any i, θ, the constraints to the minimization problem are

$$\forall\theta' \neq \theta \qquad -\left(z_i(\theta) - z_{-i}(\theta')\right) - y_i(\theta,\theta) + y_i(\theta',\theta) \leq 0, \tag{IC}$$

$$-z_i(\theta) - y_i(\theta,\theta) + \upsilon_i(\theta) \leq 0, \tag{PC_i}$$

$$-1 + \sum_i \sum_\theta \rho^0(\theta) z_i(\theta) + Pr(\mathcal{G}) \leq 0, \tag{RC}$$

$$-z_i(\theta) \leq 0, \tag{EPI}$$

$$\gamma(\theta_1,\theta_2) - 1 \leq 0, \tag{F}$$

$$\text{and } \forall Q \subset \Theta$$

$$\sum_i \sum_{\theta\in Q_i} z_i(\theta)\rho^0(\theta) + \sum_{(\theta_1,\theta_2)\in\bar{Q}} (1-\gamma(\theta_1,\theta_2))\rho^0(\theta_1)\rho^0(\theta_2) - 1 + Pr(\mathcal{G}) \leq 0. \tag{EP}$$

We now derive the Lagrangian representation of the optimization problem. First, we state the complementary slackness conditions and the respective Lagrangian multipliers

$$[z_i(\theta) - z_{-i}(\theta') + y_i(\theta,\theta) - y_i(\theta',\theta)]\nu^i_{\theta,\theta'} = 0, \qquad \nu^i_{\theta,\theta'} \geq 0;$$

$$[z_i(\theta) + y_i(\theta,\theta) - \upsilon_i(\theta)]\lambda^i_\theta = 0, \qquad \lambda^i_\theta \geq 0;$$

$$\Big[1 - \sum_i \sum_\theta \rho^0(\theta) z_i(\theta) - Pr(\mathcal{G})\Big]\delta = 0, \qquad \delta \geq 0;$$

$$z_i(\theta)\zeta^i_\theta = 0, \qquad \zeta^i_\theta \geq 0;$$

$$[1-\gamma(\theta_1,\theta_2)]\mu_{\theta_1,\theta_2} = 0, \quad \mu_{\theta_1,\theta_2} \geq 0;$$

$$\Big[-\sum_i \sum_{\theta\in Q_i} z_i(\theta)\rho^0(\theta) - \sum_{\substack{(\theta_1,\theta_2)\\ \in\bar{Q}}} (1-\gamma(\theta_1,\theta_2))\rho^0(\theta_1)\rho^0(\theta_2) + 1 - Pr(\mathcal{G})\Big]\eta_Q = 0, \qquad \eta_Q \geq 0.$$

For any Lagrangian multiplier, say t, we introduce the following notation $\tilde{t} \equiv \frac{t}{\delta}$. Let Q^2 be the set of all combinations of Q. Let $\tilde{e}^i_\theta := \rho^0(\theta) \sum_{Q\in Q^2|\theta\in Q_i} \tilde{\eta}_Q$ and define

$$\tilde{\Lambda}^i(\theta) := \sum_{k=1}^{\theta} \tilde{\lambda}^i_k, \qquad \tilde{E}^i(\theta) := \sum_{k=1}^{\theta} \tilde{e}^i_k, \qquad \tilde{Z}^i(\theta) := \sum_{k=1}^{\theta} \tilde{\zeta}^i_\theta \tag{12}$$

Next, we characterise the solution in terms of the Lagrangian objective. Using Lemma 4 we optimize over the match-probabilities, which we denote (with abuse of notation) by $\rho(\theta_1,\theta_2)$. $\boldsymbol{\rho}(\sigma)$ is the collection these probabilities, conditional on escalation and signal realization σ. Further, consider the symmetrized problem, that is a problem in which for each signal σ and associated probabilities $\rho(\theta_1,\theta_2|\sigma)$ there exists signal realization σ' such that $Pr(\sigma) = Pr(\sigma')$ and $\rho(\theta_1,\theta_2|\sigma) = \rho(\theta_2,\theta_1|\sigma')$ for all (θ_1,θ_2).[31]

[31] We can symmetrize the problem in this way without loss of generality as players are symmetric ex-ante.

Lemma 12. *The lottery $\{Pr(\sigma), \boldsymbol{\rho}(\sigma)\}_\sigma$ is an optimal solution to the designers problem if and only if there are Lagrangian multipliers that satisfy complementary slackness given the lottery and the lottery includes every $\boldsymbol{\rho}(\sigma)$ that maximizes*

$$
\begin{aligned}
\widehat{\mathcal{L}}(\mathcal{B}(\sigma)) := \sum_i \Bigg[& \sum_{\theta=1}^{K} \rho_i(\theta)(\frac{\mathsf{m}^{\mathsf{i}}_\theta}{\rho^0(\theta)})U_i(\theta, \mathcal{B}(\sigma)) \\
& + \sum_{\theta=1}^{K-1} \sum_{k=\theta+1}^{K} \frac{\mathsf{M}^{\mathsf{i}}(\theta) + \nu^i_{k,\theta} - \nu^i(\theta)}{\rho^0(\theta)} \rho_i(\theta)(U_i(\theta,\theta,\mathcal{B}(\sigma)) - U_i(\theta,k,\mathcal{B}(\sigma))) \\
& - \sum_{\theta=1}^{K} \sum_{k=1}^{\theta-1} \frac{\mathsf{M}^{\mathsf{i}}(\theta) + \nu^i_{k,\theta} - \nu^i(\theta)}{\rho^0(\theta)} \rho_i(\theta)[U_i(\theta,k,\mathcal{B}(\sigma)) - U_i(\theta,\theta,\mathcal{B}(\sigma))] \Bigg] \\
& + \mathcal{T}(\mathcal{B}(\sigma)),
\end{aligned} \tag{13}
$$

where $\mathsf{m}^{\mathsf{i}}_\theta := \rho^0(\theta) + \tilde{e}^i_\theta - \tilde{\zeta}^i_\theta$, $\mathsf{M}^{\mathsf{i}}(\theta) := \tilde{\Lambda}^i(\theta) - \sum_{k=1}^{k=\theta} \rho^0(k) - \tilde{E}^i(\theta) + Z^i(\theta)$, $\nu^i(\theta) := \sum_{k=1}^{\theta} \sum_{v=\theta+1}^{K} [\tilde{\nu}^i_{k,v} - \tilde{\nu}^i_{v,k}]$ *and*

$$
\begin{aligned}
\mathcal{T}(\mathcal{B}(\sigma)) := & \sum_{Q \in Q^2} \sum_{(\theta_1,\theta_2) \in \tilde{Q}} [\rho(\theta_1)\beta_1(\theta_2|\theta_1,\sigma)]\tilde{\eta}_Q \\
& - \sum_{\theta_1 \times \theta_2} \frac{\rho_1(\theta_1,\sigma)\beta_1(\theta_2|\theta_1,\sigma)}{\rho^0(\theta_1)\rho^0(\theta_2)} \tilde{\mu}_{\theta_1,\theta_2}.
\end{aligned} \tag{14}
$$

Hence, $\boldsymbol{\rho} = \sum_\sigma Pr(\sigma)\boldsymbol{\rho}(\sigma)$ is a maximizer of the concave hull of the above function. Moreover, the following is true at the optimum:

- *The* (RC) *constraint is always binding, i.e., $\delta > 0$.*
- *If the Border constraints are redundant, then $\tilde{e}^i_\theta = \tilde{E}_i(\theta) = 0 = \tilde{Z}^i(\theta) = \tilde{\zeta}^i_\theta$.*
- *If $\tilde{\Lambda}^i(\theta) + \tilde{Z}^i(\theta) - \sum_{v=1}^{v=\theta} \rho^0(v) - \tilde{E}^i(\theta) > 0$, then the* downward incentive constraints *are* binding. *If in addition the upward incentive constraints are redundant, then $\tilde{v}^i_{\theta,k} = 0$ for all $k \geq \theta$.*
- *If $\tilde{\Lambda}^i(\theta) + \tilde{Z}^i(\theta) - \sum_{v=1}^{v=\theta} \rho^0(v) - \tilde{E}^i(\theta) < 0$, the* upward incentive *constraints are* binding. *If in addition the downward incentive constraints are redundant, then $\tilde{v}^i_{k,\theta} = 0$ for all $k < \theta$.*
- *If local incentive constraints are sufficient, then $\nu^i(\theta) = \tilde{\nu}^i_{\theta,\theta+1} - \tilde{\nu}^i_{\theta+1,\theta}$. In this case, $\tilde{\nu}^i_{k,\theta} - \nu^i(\theta) = -M^i(\theta)$ for any k such that $k > \theta + 1$ or $k < \theta + 1$.*

Proof. The first part of the proof is along the heuristics below Proposition 1. We manipulate the Lagrangian, $\mathcal{L}$, and derive a tractable dual problem. The second part verifies that the optimum is on the concave hull of the objective. The Lagrangian takes the form

$$\begin{aligned}
\mathcal{L} &= Pr(\mathcal{G}) + \delta[-1 + \sum_i \sum_{\theta=1}^{K} \rho^0(\theta) z_i(\theta) + Pr(\mathcal{G})] \\
&+ \sum_i \sum_{\theta=1}^{K} [-z_i(\theta) - y_i(\theta,\theta) + \upsilon_i(\theta)] \lambda_\theta^i \\
&+ \sum_i \sum_{\theta=1}^{K} \sum_{k\in\Theta\setminus\theta} [z_i(\theta) - z_i(k) - y_i(k,k) + y_i(\theta,k)] \nu_{k,\theta}^i \\
&+ \sum_{Q\in Q^2} \Big[\sum_i \sum_{\theta\in Q_i} z_i(\theta)\rho^0(\theta) + \sum_{(\theta_1,\theta_2)\in\bar{Q}} (1-\gamma(\theta_1,\theta_2))\rho^0(\theta_1)\rho^0(\theta_2) - 1 + Pr(\mathcal{G}) \Big] \eta_Q \\
&+ \sum_{\theta_1\times\theta_2} [\gamma(\theta_1,\theta_2) - 1]\mu_{\theta_1,\theta_2} - \sum_i \sum_\theta z_i(\theta)\zeta_\theta^i
\end{aligned} \tag{15}$$

Using Lemma 3 we optimize over $\{z_i(\cdot), \gamma^\sigma(1,1), \boldsymbol{\rho}(\sigma)\}$, with $\gamma^\sigma(1,1){:=}Pr(\mathcal{G},\sigma|\theta_1{=}1,\theta_2{=}1)$.

Step 1: Eliminating $z_i(\cdot)$ using first order conditions. Define $\nu_{K+1,K}^i := 0 =: \nu_{1,0}^i = \nu_{0,1}^i$ for ease of notation. The FOC w.r.t. $z_i(k)$ are

$$\rho^0(k)\delta - \lambda_k^i - \sum_{k\in\Theta\setminus k} [\nu_{k,v}^i - \nu_{v,k}^i] + \rho^0(k) \sum_{Q\in Q^2|k\in Q_i} \eta_Q - \zeta_k^i = 0. \tag{16}$$

Summing over all K conditions in (16) and recalling definition (12) yields

$$1 = \tilde{\Lambda}^i(K) - \tilde{E}^i(K) + \tilde{Z}^i(k), \tag{17}$$

(16) holds for all θ if and only if

$$\nu^i(\theta) := \sum_{k=1}^{\theta} \sum_{v=\theta+1}^{K} [\tilde{\nu}_{k,v}^i - \tilde{\nu}_{v,k}^i] = \sum_{v=1}^{v=\theta} \rho^0(v) - \tilde{\Lambda}^i(\theta) + \tilde{E}^i(\theta) - Z^i(\theta). \tag{18}$$

Moreover, all terms involving $z_i(\cdot)$ cancel out from (15) via (16).

Step 2: Reformulation Lagrangian. Given the above necessary conditions, we manipulate the Lagrangian objective to derive a more tractable maximization problem. Define $\eta := \sum_{Q\in Q^2} \eta_Q$ and $\tilde{\eta} := \sum_{Q\in Q^2} \tilde{\eta}_Q$. Next, using Bayes' rule together with Lemma 3, applying algebra and using the first-order-conditions it is straightforward to show that (15) admits the following representation

$$\mathcal{L} = Pr(\mathcal{G})(1+\delta+\eta) - \delta C - \delta \sum_\sigma \Pr(\Gamma,\sigma)\widehat{\mathcal{L}}(\mathcal{B}(\sigma)), \tag{19}$$

where C is a constant that is independent of the choice variables,

$$C{:=}1 + \tilde{\eta} - \sum_{Q\in Q^2} \sum_{(\theta_1,\theta_2)\in\tilde{Q}} \rho^0(\theta_1)\rho^0(\theta_2)\tilde{\eta}_Q - \sum_i \sum_\theta \tilde{\lambda}_\theta \upsilon_i(\theta) + \sum_{\theta_1\times\theta_2} \tilde{\mu}_{\theta_1,\theta_2} < 0.$$

Define $\gamma^\sigma(\theta_1,\theta_2) := Pr(\mathcal{G},\sigma|\theta_1,\theta_2)$. From Lemma 3, $\gamma^\sigma(\theta_1,\theta_2) = f(\mathcal{B}(\sigma),\theta_1,\theta_2)\gamma^\sigma(1,1)$, where $f(\mathcal{B}(\sigma), \theta_1, \theta_2)$ is a positive real number. Thus, $Pr(\mathcal{G}, \sigma) = \gamma^\sigma(1,1)R(\mathcal{B}(\sigma))$ with $R(\mathcal{B}(\sigma)) := \sum_{\theta_1 \times \theta_2} \rho^0(\theta_1)\rho^0(\theta_2)f(\mathcal{B}(\sigma), \theta_1, \theta_2)$. Plugging into (19) yields

$$\mathcal{L} = \sum_\sigma \gamma^\sigma(1,1)R(\mathcal{B}(\sigma))(1 + \delta + \tilde{\mu}) - \delta C - \delta \sum_\sigma \gamma^\sigma(1,1)R(\mathcal{B}(\sigma))\widehat{\mathcal{L}}(\mathcal{B}(\sigma)). \tag{20}$$

The FOC of (20) w.r.t. $\gamma^\sigma(1,1)$ is

$$R(\mathcal{B}(\sigma))\Big((1 + \delta + \eta) - \delta\widehat{\mathcal{L}}(\mathcal{B}(\sigma))\Big) = 0, \tag{21}$$

for each signal. By Assumption 2 $R(\mathcal{B}(\sigma)) > 0$ and thus, $\widehat{\mathcal{L}}(\mathcal{B}(\sigma)) - 1 > 0$ if $\gamma^\sigma(1,1) > 0$. Therefore, $\delta = (1 + \eta)(\widehat{\mathcal{L}}(\mathcal{B}(\sigma)) - 1)^{-1}$. As δ is independent of σ, $\widehat{\mathcal{L}}(\mathcal{B}(\sigma))$ takes the same value for each signal realization. Substituting into (20) and simplifying yields

$$\mathcal{L} = \frac{-C(1 + \eta)}{\widehat{\mathcal{L}}(\mathcal{B}(\sigma)) - 1} \tag{22}$$

which is minimized if and only if $\widehat{\mathcal{L}}(\mathcal{B}(\sigma))$ is maximized.[32]

Thus, for the optimal multipliers one constructs the concave hull of $\widehat{\mathcal{L}}$ by taking spreads over those $\mathcal{B}(\sigma)$ that are a global maximum of $\widehat{\mathcal{L}}$. If there are multipliers and a unique maximizer, $\mathcal{B}(\sigma)$, that satisfies the complementary slackness conditions, signals do not improve. If there are multiple global optima and there is a spread that satisfies the complementary slackness conditions, signals improve. □

F Binding constraint for Games with Linearity in Types

In this part we state a more general version of Lemma 1 if Games satisfy linearity in types, but Assumption 3 and MLRP may not hold. The proof is analogous to that of Lemma 1. We then characterise the set of binding conttraints equation (Z) under this relaxed condition.

Lemma 13. *The following holds for the optimal mechanism*

i. all incentive compatibility constraints not concerning adjacent types are redundant,
ii. if both adjacent incentive compatibility constraints are redundant for type θ_i, then her participation constraint is satisfied with equality or $z_i(\theta_i) = 0$,
iii. the participation constraints for at least one type of every player is binding.

Although the set of binding constraints may depend on the exact location of the optimum, Lemma 13 provides enough structure to determine settlement values as a function of escalation values. This reduces the dimensionality of the choice set.

Corollary 4. *Consider the escalation values $\{y_i(\theta_i, \theta_i), y_i(\theta_i{+}1, \theta_i), y_i(\theta_i{-}1, \theta_i)\}$ for all $\theta_i \in \Theta$ of the optimal mechanism. Then there is a partition of $P(\Theta) = \{\Theta^{z_i=0}, \Theta^{PC}, \Theta^{IC^+}, \Theta^{IC^-}\}$*

[32]The Lagrangian multipliers are necessarily such that C is negative at the optimum. Otherwise (15) and (22) imply that $Pr(\mathcal{G})$ is negative, a contradiction to Assumption 2 and /or the fact that a feasible solution to the minimization problem always exists: take a degenerate signal distribution and set $\gamma(\theta_1, \theta_2) = 1$ for all type profiles.

such that

$$z_i(\theta_i) = z_i(\tilde{\theta}_i) + y_i(\tilde{\theta}_i, \theta_i) - y_i(\theta_i, \theta_i), \quad \forall \theta_i \in \{\Theta^{IC^+}, \Theta^{IC^-}\} \tag{Z}$$

with

$$\tilde{\theta}_i = \begin{cases} \theta_i + 1 & \textit{if } \theta_i \in \Theta^{IC^+} \\ \theta_i - 1 & \textit{if } \theta_i \in \Theta^{IC^-}. \end{cases}$$

Moreover $z_i(\theta_i) = 0$ *for* $\theta_i \in \Theta^{z_i=0}$, *and* $z_i(\theta_i) = v(\theta_i) - y_i(\theta_i, \theta_i)$ *for* $\theta_i \in \Theta^{PC}$.