Managing A Conflict*

Benjamin Balzer[†] Johannes Schneider[‡]

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Abstract

Two players conflict over a pie. They have the option to voluntarily participate in conflict management. In case conflict management cannot settle the conflict, it escalates to a costly Bayesian default game. Private information is only relevant in the default game which serves as both an endogenous outside option and a screening device. We show that optimal conflict management is equivalent to optimal post-escalation belief management. We characterize the set of feasible information structures post-escalation and link the mechanism design approach of eliciting information to the information design approach of processing information. We characterize the price of information revelation to the designer and show that additional public signals only play a minor role. Using two distinct examples we show how optimal conflict management links to the underlying games: while lottery games call for optimal sorting, contest games advocate type-independent solutions.

1 Introduction

Conflict management is a tool used to solve a disputes without letting them escalate to a costly fight. Its primary goal is to settle the conflict at little or no cost. However, conflict management is typically both voluntary and not always successful. If conflict management fails and the conflict escalates, players can use the information obtained about their opponent during conflict management when making further decisions. If optimal behaviour after escalation depends on players' information, players may use the information externality of conflict management strategically to extract information useful to them after escalation.

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[†]UT Sydney, benjamin.balzer@uts.edu.au

[‡]Carlos III de Madrid, jschneid@eco.uc3m.es

Thus, optimal conflict management must take behavioural effects of its information externality into account.

In this paper we study conflict management mechanisms when escalation is informative and triggers a non-cooperative Bayesian game. We consider escalation-minimizing conflict management when each player's strength in the post-escalation game is private information and unrelated to her preferences over outcomes. Escalation is informative and players adjust their strategic behaviour post-escalation. These adjustments also influence the players' continuation utilities, since continuation-strategies depend on the beliefs players hold *after* learning that the conflict escalates. We show that the informational externality of escalation must not be ignored if players' optimal continuation strategies depend on their beliefs. In such cases escalation serves as an endogenous, belief-dependent outside option to the mechanism. We show generally that optimal conflict management is equivalent to optimally managing the beliefs players hold after escalation.

Examples of conflict management in which information is useful after escalation are abound. They include alternative dispute resolution escalating to litigation, mediated union-employer bargaining escalating to strikes, peace negotiations escalating to wars, or trade agreements escalating to retaliations. All of the examples have three properties in common: (i) players can enforce the default game instead of conflict management unilaterally by constitution (litigation and strikes) or sovereignty (war and trade retaliation); (ii) conflict management aims at minimizing escalation because of a negative social externality on the legal system, the economy, global stability or global trade respectively, and (iii) after escalation, players play a given default game non-cooperatively. Further, parties typically hold private information about their individual strength within the default game which is payoff irrelevant in case of a settlement solution.

Taking into account the informational externality of optimal conflict management, we show that finding the optimal mechanism has a dual of finding the optimal beliefs players hold after escalation. We call the solution to the dual *optimal belief management*. This duality facilitates the problem significantly. Instead of solving a *mechanism design* problem with a complicated information externality we can instead focus on solving an *information design* problem that implies the solution to the mechanism design problem in a straight forward fashion. Optimal belief management provides an intuitive formulation of the economic problem the designer faces. Within the dual problem we identify the designer's two main motives. The information structure after escalation should ensure (i) discrimination (the *screening motive*), and (ii) little inefficiency (the *welfare motive*) in the continuation game. We establish additively separable measures for both motives as the objective of the dual.

The connection between mechanism design and information design is immediate. Players can use the rules of conflict management to Bayesian update their beliefs. The designer, in turn, can influence the updating procedure choosing these rules. *Belief management* then refers to mechanisms precisely set up to implement a particular information structure after escalation.

We connect optimal conflict management directly to the properties of the Bayesian default game via the belief management approach. This connection characterizes the general relation between the default way of conflict resolution and optimal conflict management. Further, the dual allows us to address the role of additional information the designer can release complementary to escalation. We provide simple sufficient conditions when such public signals are superfluous conditioning only on the set of binding constraints.

When a player forms her strategies for the continuation game, she uses any information she obtains during conflict management. In particular, she takes into account that her opponent updates her beliefs, too. Thus, when calculating the continuation value of escalation the entire belief system becomes relevant for expected choices and hence for the outcome.

Conflict management makes the solution trivially less inefficient since parties can save on investments into the continuation game under settlement. However, given that parties need to mutually agree on conflict management either party can veto and enforce escalation. If strong parties expect to prevail in the default game anyhow they participate in conflict management only if they expect a large enough settlement valuation. A large settlement valuations attracts weaker types as strength is irrelevant under peaceful settlement. If pooling of *all* types is not affordable, escalation is the only screening device and conflict management uses escalation precisely for these purposes.

To separate types, the designer needs to offer strong players enough for participation while keeping weak players from mimicking the strong. To raise the necessary resources she has two options: settle more conflicts, or decrease the inefficiency post-escalation. A player's expected payoff from the mechanism is the sum of the expected settlement value, financed by forgone fights, and the expected escalation value, determined by the expected utility in the default game. Simultaneously the post-escalation game needs to sufficiently discriminate between player-types for separation.

Consider for example a standard war of attrition as the default game. Suppose both players expect to be of similar strength leading to large investment and small aggregate surplus. Compare this to a situation in which one player expects an opponent of similar strength, but the opponent instead expects the player to be (on average) weak. Given the logic of the war of attrition, the opponent invest less compared to the situation with similar expectations. As a response, the player, too, reduces her investment making the overall surplus larger than in the case of symmetric expectations. Not only strategies but also, expected individual utility and overall expected (utilitarian) welfare in the continuation game depend thus on the entire *belief system* and not only on the player's individual belief about her opponent's type.

A change in beliefs impacts the potential for deviation within conflict management. Action choices after escalation can be adjusted after deviations, too. This provides an additional incentive for deviation. The reason is that any strategic adjustment a player makes subsequent to conflict management remains unresponded as the deviation is not detected. Suppose for example that a player expects a weaker opponent after deviation. After deviation and subsequent escalation the player adjusts her continuation-strategy to the relevant expected strength of her opponent, but – different than on the equilibrium-path – *without* expecting her opponent to react. Thus, deviation becomes more attractive.

We use our results to compare two examples of underlying games. Games in which the optimal continuation-strategy is independent of the beliefs (e.g. Hörner, Morelli, and Squintani (2015)) and games in which it the strategy is sensible to beliefs (e.g. an all-pay auction).

We show that conflict management in games with belief-independent continuation strategies induces *sorting* by the mechanism. That is, conflict management identifies easy to solve matches and promises them settlement while sending difficult matches to the default game.

Due to the belief-dependent behavioural adjustments the result differs if strategies react to beliefs, a case not considered much in the literature so far. We show for the example of the all-pay auction, that sorting would undermine the designer's screening motive. Instead, and to induce truth-telling, she promises each player the same information set independent of their behaviour. Further we show that, different to the first example, the solution generically involves an asymmetric type distribution in any continuation game even with ex-ante symmetric players.

1.1 Related Literature

The mechanism design literature on conflict preemption builds on the classical literature on trade mechanisms going back to Myerson and Satterthwaite (1983). Our general setup is related to Compte and Jehiel (2009) within that literature. We assumes a division of a pie as an outcome, a budget constraint mechanism, and an outside option that potentially depends players' private information.

In line with the Spier (1994), Bester and Wärneryd (2006), and Hörner, Morelli, and Squintani (2015) we depart from classic bargaining mechanism in that the status quo (i.e. escalation) in our model cannot be avoided completely although it is always more inefficient than settlement. The design of the mechanism in our model influences players' belief about both their opponent's type *and action* in the continuation game. Thus, the mechanism involves an informational externality.[1]

Informational externalities of a mechanism in different contexts, are present in bargaining models with interdependent valuations (Jehiel and Moldovanu, 2001), and auctions with resale as in Zheng (2002) who considers a sequence of mechanisms each proposed by a different agent.

Closer related to this paper are Philippon and Skreta (2012) and Tirole (2012) who study the informational externality of a bailout mechanism on future market behaviour. Similar to our approach they consider a model in which the design of the mechanism influences the interpretation of observed behaviour and thus subsequent behaviour in the market. A similar approach is taken by the literature on aftermarkets to auctions (Lauermann and Virág, 2012; Atakan and Ekmekci, 2014; Dworczak, 2016) that considers how informational externalities of the mechanism influence behaviour after the auction and thus the final outcome. The main difference to us is that the design of the mechanism influences the belief system in the market only one-sided and beliefs after the mechanism are type-independent by design. In our model, type-dependent beliefs are possible since the same players meet in the aftermarket and private information is two-sided.

We contribute to the literature on conflict preemption (Spier, 1994; Bester and Wärneryd, 2006; Hörner, Morelli, and Squintani, 2015). Contrary to the literature on aftermarkets, this literature involves type-dependent beliefs. Standard models consider, however, type-specific lotteries as continuation-games such that action choices after escalation become irrelevant. We nest these models, but allow for general Bayesian continuation-games in which players action choices are influenced by the mechanism and determine the outcome.[2] Our key contribution is to provide a general approach to conflict management by identifying a channel that links the properties of Bayesian game to the optimal mechanism.

We provide a complement to Zheng (2016) who studies necessary and sufficient conditions for full settlement given a Bayesian contest as the underlying game. We characterise the optimal mechanism when these conditions do not hold. Meirowitz et al. (2015) study the effect of last-minute conflict management on early investment, while we focus on early stage conflict management that saves on these investment. Beliefs in Hörner, Morelli, and Squintani (2015) are only important in case of settlement due to the limited commitment of the players. In our model, commitment is not an issue by design, and we focus on the role of beliefs in the case of escalation.

[1] Interim dependence of the outside option on beliefs is also present in Jehiel and Moldovanu (2001), Fieseler, Kittsteiner, and Moldovanu (2003), and Compte and Jehiel (2009) as they allow for interdependent values. Different to us, however, there is no post-escalation action choice and thus no relevance of higher order beliefs.

[2] As shown by example in Celik and Peters (2011), Bayesian default games can make full-participation non-optimal. Their channel is not present in the main part of our paper, and we provide an extension taking their considerations into account.

Our model is an augmented information design problem (Bergemann and Morris, 2016a,b; Taneva, 2016). The designer chooses an expansion of the initial information structure with binary signals such that one realisation maps into settlement and the other into escalation. The main difference to the existing literature is that the designer can influence the basic game under settlement, but not under escalation. When discussing the role of public singals we use the tools of concavification (Aumann and Maschler, 1995) in a similar fashion as Kamenica and Gentzkow (2011).

Roadmap. The remainder of the paper is structured as follows. We describe the model and some preliminary simplifications in section 2. In section 3 we derive our main result and establish the duality between conflict management and belief management. In section 4 we illustrates the power of our approach by comparing two classes of default games and how their properties map into the optimal mechanism. In section 5 we discuss the robustness of our findings to the relaxation of several of our initial assumptions. We conclude in section 6. All formal proofs are relegated to the appendix.

2 Setup

General Setup and Basic Events. Consider two ex-ante identical, risk-neutral players, with linear preferences over basic outcomes. The players have a conflict over the distribution of a pie worth 1 to each player. Players can either mutually agree on a conflict management mechanism to solve the conflict or engage in a non-cooperative Bayesian default game. Independent of her valuation of the pie, each player i is endowed with a type θ_i independently drawn from $\Theta = \{1, 2, ..., K\}$. We say θ_i is player i's *strength in the default game.* The known ex-ante probability of being θ_i is $\rho^0(\theta_i)$.

A basic outcome of our model is an element of te two-dimensional simplex, representing the distribution of (parts of) the pie.[3] We categorize the basic outcomes in our model into two sets: the set $\mathcal{X}$ consists of all outcomes conditional on the event of settlement and the set $\mathcal{G}$ consists of all outcomes conditional on the event of escalation. An element in $\mathcal{X}$ defines the shares attributed to each player under settlement. In the event of escalation the default game decides over the division of the pie. An element in $\mathcal{G}$ is given by the image of a fundamental 2-player Bayesian game Γ mapping from type profiles and an action pairs to outcomes.

Conflict Management. Conflict management is a mechanism proposed by a non-strategic third-party, the designer. An outcome of conflict management is either identified as settlement (i.e., in the set $\mathcal{X}$), or escalation (i.e., in the set $\mathcal{G}$). The revelation principle implies

[3]Players are risk neutral, which is why we make no restrictions whether the distribution refers to an actual division of the pie, or whether players engage in a (fixed) lottery about the pie as a whole.

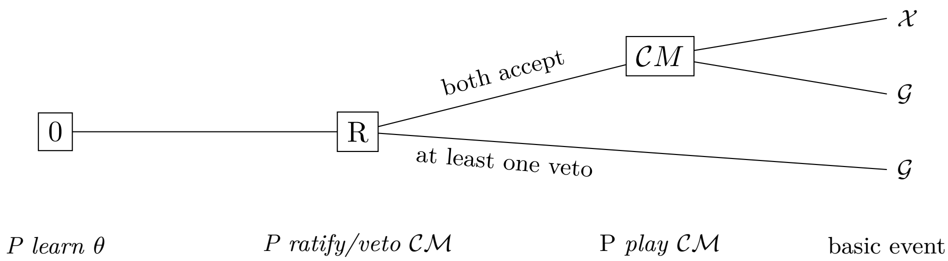

Figure 1: Timing of events.

that it is without loss of generality to focus on direct revelation mechanisms. The set of (stochastic) conflict management mechanisms $\mathcal{CM}$ is defined as a mapping from the type space into the outcome space, that is

$$\mathcal{CM}(\cdot) = (\gamma(\cdot), X(\cdot), \Sigma(\cdot)) : \Theta^2 \mapsto [0,1] \times \mathcal{X} \times \mathcal{S}. \tag{$\mathcal{CM}$}$$

The first component, $\gamma(\cdot)$, defines the probability with which the conflict escalates, the second component, $X = (x_1(\cdot), x_2(\cdot))$ defines the allocation conditional on settlement. In addition, the designer can commit to a public signal Σ. A signal is a random variable mapping the players' type reports into a stochastic, payoff irrelevant, outcome. A typical realization is described by $\sigma \in \mathcal{S}$. We are looking for conflict management mechanisms that minimizes the ex-ante probability of escalation.

Timing. At the initial stage (0) players privately learn their type. At the ratification stage (R) players decide simultaneously whether they want to participate in conflict management. If both agree on $\mathcal{CM}$, they report their types and $\mathcal{CM}$ results either in settlement or escalation. Under settlement, players receive a share and the game ends. Under escalation, players learn the signal realization σ, update their beliefs according to Bayes' rule and non-cooperatively decide on their action in the Bayesian game.[4] If at least one player vetoes conflict management at (R), the conflict escalates immediately to the Bayesian game. The rules of the grand game described in section 2 are common knowledge. We assume that the designer has full commitment power, and players can commit to accepting any outcome in $\mathcal{X}$ at the ratification stage.

Solution Concept and Beliefs. We use perfect Bayesian equilibrium as solution concept. Thus, whenever possible, players use all information available to update their beliefs according to Bayes' rule. Suppose escalation is announced signal σ realises. Now, player i who reported m during conflict management, assigns a conditional probability to each possible type of player $-i$. This conditional probability is denoted by $\beta_i(\theta_{-i}|m, \sigma)$, and the

[4] In principle, signals could also realize if the outcome is $\mathcal{X}$. However, in the baseline model with full commitment signals have no effect on the final outcome in all settlement events.

collection of conditional probabilities $\beta_i(\cdot|m,\sigma)$ is the *individual belief* of that player. By the revelation principle the on-path individual belief $\beta_i(\cdot|\theta_i,\sigma)$ of each player θ_i is common knowledge. We call the collection of all individual on-path beliefs for both players, $\mathcal{B}(\sigma)$, the *belief system* given σ. For notational simplicity we distinguish between two individual beliefs $\beta_i(\cdot|\theta_i,\sigma)$ and $\beta_i'(\cdot|\theta_i,\sigma)$ only if player θ_i appears with positive probability given σ. To distinguish between the belief system induced by the escalation rule γ, and those by the signal distribution Σ we say that $\mathcal{B}(\sigma)$ is the realized belief system given σ, and define the marginal belief system post-escalation as $\mathcal{B} := \sum_{\sigma\in\Sigma} Pr(\sigma)\mathcal{B}(\sigma)$. A special case of $\mathcal{B}$ is the prior belief system $\mathcal{B}^0$ in which $\beta_i^0(\theta_{-i}|\theta_i) = \rho^0(\theta_{-i})$ for any θ_i.

2.1 Preliminary simplifications

The On-Path Continuation Game. The fundamental game Γ consists of an action set A and a function $\bar{u} : \Theta^2 \times A^2 \mapsto \mathbb{R}^2$ that maps from type and action profiles into payoffs. Given escalation and the realization of σ, the (on-path) information structure is entirely determined by the belief system $\mathcal{B}(\sigma)$. Up to the choice of equilibrium, Γ and $\mathcal{B}(\sigma)$ are thus sufficient to determine the equilibrium outcome. In particular, take any fundamental game Γ, and any belief system $\mathcal{B}(\sigma)$ for which a unique equilibrium with some properties $(*)$ exists, then there is a function, $\mathbf{s}^* : \mathcal{B}(\sigma) \mapsto \Delta(A)^{2K}$, that fully describes the equilibrium (mixed-)strategies in that equilibrium. Given the equilibrium strategies, the von-Neumann-Morgenstern equilibrium utility of player θ_i that is matched with type θ_{-i} is denoted by $u_i(\theta_i,\theta_{-i},\mathcal{B}(\sigma)) := u_i(\theta_i,\theta_{-i},\mathbf{s}^*(\mathcal{B}(\sigma)))$ and the expected on-path utility is denoted by $U_i(\theta_i,\mathcal{B}(\sigma)) := \sum_{i=1}^K \beta_i(k|\theta_i,\sigma)u_i(\theta_i,k,\mathcal{B}(\sigma))$. We assume that an equilibrium exists for any realization of $\mathcal{B}(\sigma)$. Its selection is known by the designer.

The Off-Path Continuation Game. Similar to the on-path continuation game we can describe the off-path continuation game given Γ as a mapping from belief systems to outcomes. If a player deviates by misreporting her type, this deviation is undetected. Therefore, a deviation of player i in $\mathcal{CM}$ does not change the behavior of the non-deviating player in case the conflict escalates. Consequently, $-i$'s (continuation) strategy in the continuation-game after deviation is a function of the on-path belief system $\mathcal{B}(\sigma)$. We define the continuation utility of a deviating type θ_i who reports to be type m as the limiting utility of what the deviator could obtain when choosing her strategy optimally, i.e.,

$$U_i(m,\theta_i,\mathcal{B}(\sigma)) := \sup_{s_i} \sum_{i=1}^{K} \beta_i(k|m,\sigma)u_i(\theta_i,k,s_i,\mathbf{s}^*_{-i}(\mathcal{B}(\sigma))).$$

Observe that if $m = \theta_i$ the above equation describes the on path utility and $\mathbf{s}_i^*$ is determined by $\mathcal{B}(\sigma)$. Thus, we can simplify both on-path and off-path continuation utilities to the expression $U_i(m, \theta_i, \mathcal{B}(\sigma))$ which describes the maximum continuation-utility of player θ_i, reporting to be type m after signal realization σ. The designer can neither influence Γ nor the choice of equilibrium. In what follows we treat the function $U_i(m_i, \theta_i, \mathcal{B}(\sigma))$ as a primitive to the optimal mechanism. Similar to the belief system we define the marginal utility $\widehat{U}_i(m, \theta_i, \mathcal{B}, \Sigma) := \sum_{\sigma \in \Sigma} U_i(m, \theta_i, \mathcal{B}(\sigma))$. We refer to $\hat{U}$ as the ex-ante expected utility post-escalation.

Assumptions on the underlying game. To proceed we impose structure on U_i. Since U_i is entirely determined by an equilibrium of Γ, the assumptions we make are essentially assumptions on the default game. We assume upper hemi-continuity of U_i in $\mathcal{B}$ and an *anonymous conflict.* Define

$$\mathcal{B}_-(\sigma) = \Big\{ \tilde{\beta}_i(\cdot|m, \sigma) : \ \tilde{\beta}_i(\cdot|m, \sigma) = \beta_{-i}(\cdot|m, \sigma), \ \beta_{-i}(\cdot|m, \sigma) \in \mathcal{B}(\sigma) \Big\}.$$

Definition 1 (Anonymity)**.** The game Γ and the equilibrium choice rule satisfy *anonymity* if for any $\mathcal{B}(\sigma)$ and any type profile (θ_i, θ_{-i}) it holds that $u_i(\theta_i, \theta_{-i}, \mathcal{B}(\sigma)) = u_{-i}(\theta_i, \theta_{-i}, \mathcal{B}_-(\sigma))$.

Definition 2 (Conflict)**.** A game Γ and an equilibrium choice rule describe a *conflict* if for any $\mathcal{B}(\sigma)$ it holds that

i. *(non-productiveness.)* $\sum_i u_i(\theta_i, \theta_{-i}, \mathcal{B}(\sigma)) \leq 1$ for any $(\theta_i, \theta_{-i}) \in \Theta^2$.
ii. *(monotonicity in own type.)* $U_i(\theta_i, \mathcal{B}(\sigma))$ is non-increasing in θ_i.
iii. *(monotonicity in own belief.)* $U_i(\theta_i, \mathcal{B}(\sigma))$ increases if $\beta_i(\theta_i, \sigma)$ increases in the first order stochastic dominance sense.

Following Definition 2, we order types such that type 1 is *the strongest* and type K is the *weakest.* Non-productiveness guarantees that escalation always reduces the amount of resources available in the economy. Thus ex-ante conflict is never desirable from a utilitarian point of view compared to any settlement solution that distributes the entire pie. The two monotonicity properties ensure that the player's type is a sufficient statistic for the player's ability in the conflict game. While the first property states that higher ability results in higher expected utility, the second property ensures that it is always preferable to play against a weaker opponent.

Discussion of the Assumptions. We provide a detailed discussion of our assumptions in section 5. In this part we discuss the two assumptions most important to our model. First, we assume the designer has full commitment power. In reality, institutions designing conflict management tend to act repetitively and thus rely on reputations which gives them an (unmodeled) incentive to commit.[5]

[5]Our assumption of full commitment power of all participants is in line with most of the mechanism design literature. A notable exemption is Bester and Strausz (2001) who consider a single agent limited

Second, a key point to our model is that the player's strength under escalation is orthogonal to her preferences over outcomes. We use this assumption to motivate escalation after conflict management that is not replicable directly via the mechanism. Our results can be seen as a benchmark on the possibilities of third party conflict management in two ways. (i) It is one of the limiting cases of a more general model in which the private information is about preferences *and* strength, the other limit being a classical trade mechanism with interdependent values á la Fieseler, Kittsteiner, and Moldovanu (2003) without transfers in which preferences and ability coincides. (ii) A second benchmark is on the technology of conflict management. We assume that the designer has no technology to test the players private information other than escalation. Thus, settlement relies only on soft information. In reality, such mechanisms prvovide the cheapest and fastest solution to the dispute. If known to the players when deciding on participation in conflict management, however, we can subsume any more complicated mechanism in the escalation stage making our model a benchmark on what can be achieved via the exchange of soft information only.

3 Analysis

In this section we develop our two main results. First, the optimal conflict management mechanism is entirely determined by the choice of the optimal marginal belief system and a signal distribution Σ. Second, a dual to the escalation minimization problem exists. The dual consists of maximizing the combination of a measure of discrimination between types (i.e., a screening measure), and a measure of welfare in the post-escalation game over the marginal belief system post-escalation. The two measures are each determined by the fundamental game Γ, the prior $\boldsymbol{\rho}^0$ and the choice of the marginal belief $\mathcal{B}$ system only. Thus, they provide a direct link of the post-escalation game and the optimal mechanism.

We proceed in four steps. We first reduce the choice set with help of binding constraints. We then transform the problem into reduced form, and define the set of consistent marginal belief systems. Finally, we use the first order approach to derive the dual and state the equivalence result.

3.1 Binding Constraints

Assumptions on the Value of Vetoing. Each party can trigger escalation unilaterally by vetoing $\mathcal{CM}$. Then the conflict escalates immediately. Let $\mathcal{B}^V$ be the belief system after a veto by player i and $v_i(\theta_i)$ the value of vetoing, that is, the expected utility from Γ under the belief system $\mathcal{B}^V$. In particular, $\mathcal{B}^V$ contains the prior type distribution ρ^0 as belief

commitment model. Evidence supporting our assumption of commitment on the designers' side can also be found in the online appendix of Hörner, Morelli, and Squintani (2015).

for player i, the *individual veto belief* β^V which is the same for any type of player $-i$. We treat β^V and thus v_i as given. We make the following assumption which guarantees full participation.[6]

Assumption 1. $v_i(\theta)$ is convex with respect to ρ^0 given β^V for every θ.

The next assumption is sufficient and necessary for a non-trivial conflict management mechanism.

Assumption 2. $v_i(1) > 1/2$.

A violation of assumption 2 allows a 50/50 sharing rule that is accepted by both players and leads to full settlement. Zheng (2016) provides a detailed study of assumption 2 for general contest games.

Relevant Constraints. Define the value from participation and the announcement m_i in a given mechanism as

$$\Pi_i(m_i, \theta_i) = \underbrace{(1-\gamma_i(m_i))x_i(m_i)}_{=:z_i(m_i)\text{(settlement value)}} + \underbrace{\gamma_i(m_i)\widehat{U}_i(m_i, \theta_i, \mathcal{B})}_{=:y_i(m_i,\theta_i)\text{(escalation value)}}, \tag{1}$$

that is, the (interim expected) utility of player θ_i, who participates in the mechanism, reports type m_i, and behaves optimally in the continuation game after escalation. We call the first part the settlement value, $z_i(m_i)$, and the second part the escalation value or $y(m_i, \theta_i)$. Note that the values $\gamma_i(m_i)$ and $x_i(m_i)$ correspond to θ_i's expected probability of escalation and her expected settlement share, and depend only on her report m_i.[7] Since preferences over outcomes are identical, the settlement value depends on the report only, but the escalation value depends on both the reported, and the actual strength in the default game. By the revelation principle and Assumption 1 the set of participation constraints

$$\Pi_i(\theta_i, \theta_i) \geq v_i(\theta_i), \tag{PC}$$

and the set of incentive compatibility constraints

$$\Pi_i(\theta_i, \theta_i) \geq \Pi_i(m_i, \theta_i) \quad \forall m_i, \theta_i \in \Theta,\ i \in \{1, 2\}, \tag{IC}$$

are satisfied at the optimum. Using (1) we can interpret $y_i(m_i, \theta_i)$ as the screening parameter, while $z_i(m_i)$ is a numeraire good. We proceed by identifying redundant constraints.

[6]Concerns on the full participation assumption are addressed in Celik and Peters (2011). We show robustness of our model with respect to Assumption 1 (at cost of notation) in section 5. For a discussion on the choice of β^V, see Cramton and Palfrey (1995).

[7]Note that the expected settlement share $x_i(m_i)$ depends also on the probability of escalation, γ, since shares are not independent of the escalation rule.

Lemma 1. *The following holds for the optimal mechanism*

i. all incentive compatibility constraints not concerning adjacent types are redundant,

ii. if both adjacent incentive compatibility constraints are redundant for type θ_i, then her participation constraint is satisfied with equality or $z_i(\theta_i) = 0$,

iii. the participation constraints for at least one type of every player is binding.

Although the set of binding constraints may depend on the exact location of the optimum, Lemma 1 provides enough structure to determine settlement values as a function of escalation values. This reduces the dimensionality of the choice set. We can specify $2K$ linear equations which uniquely identify the settlement values as a function of the settlement values with help of Lemma 1.

Corollary 1. *Take any set of escalation values $\{y_i(\theta_i, \theta_i), y_i(\theta_i{+}1, \theta_i), y_i(\theta_i{-}1, \theta_i)\}$ for all $\theta_i \in \Theta$. Then there is a partition of $P(\Theta) = \{\Theta^{z_i=0}, \Theta^{PC}, \Theta^{IC^+}, \Theta^{IC^-}\}$ such that*

$$z_i(\theta_i) = z_i(\tilde{\theta}_i) + y_i(\tilde{\theta}_i, \theta_i) - y_i(\theta_i, \theta_i), \tag{Z}$$

with

$$\tilde{\theta}_i = \begin{cases} \theta_i + 1 & \text{if } \theta \in \Theta^{IC^+} \\ \theta_i - 1 & \text{if } \theta \in \Theta^{IC^-}. \end{cases}$$

Moreover $z_i(\theta_i) = 0$ for $\theta_i \in \Theta^{z_i=0}$, and $z_i(\theta_i) = v(\theta_i) - y_i(\theta_i, \theta_i)$ for $\theta_i \in \Theta^{PC}$.

Reduced Form Representation. Using Corollary 1 we can construct settlement values from escalation values using the set of binding constraints. Using z_i instead of the sharing rule X_i implies that we use a reduced-form approach. The reduced-form approach ignores implementation of the reduced-form z_i via a settlement rule X_i and the escalation rule γ_i. We use an adapted version of the general implementation condition (GI) in Border (2007) to state a necessary and sufficient condition when our reduced-form approach is indeed valid. Abusing notation slightly, we denote the ex-ante probability of escalation with $Pr(\mathcal{G})$. Le $Q_i \subseteq \Theta$ be any subset of the type space and define $\bar{Q} = \{(\theta_1, \theta_2) \in \Theta^2 | \theta_i \notin Q_i \text{ for } i = 1, 2\}$ for each (Q_1, Q_2).

Lemma 2 (Sufficiency of Reduced-Form Mechanism)**.** *Fix a feasible $\gamma(\cdot, \cdot)$ and $z_i(\cdot) \geq 0$. An ex-post feasible X that implements the reduced form allocation z_i exists if and only if*

$$\sum_i \sum_{\theta_i \in Q_i} z_i(\theta)\rho^0(\theta_i) \leq 1 - Pr(\mathcal{G}) - \sum_{(\theta_1,\theta_2) \in \bar{Q}} (1 - \gamma(\theta_1, \theta_2))\rho^0(\theta_1)\rho^0(\theta_2), \quad \forall Q_1, Q_2 \subseteq \Theta. \tag{GI}$$

Lemma 2 implies that the (expected) resource constraint holds at the optimum, that is,

$$\sum_i \sum_m \rho_m^0 \; z_i(m) \le 1 - Pr(\mathcal{G}). \tag{AF}$$

We treat the condition (GI) as an additional constraint of the designer.

3.2 Conflict Management and Belief Management

The option value of escalation depends on the belief system at the point in which players choose their actions in the continuation game. In this part we characterize the set of possible belief systems and show that given the optimal signal structure, finding the optimal belief system is isomorphic to finding the optimal mechanism. We thus establish an equivalence between conflict management and belief management on the level of the marginal belief system. We start with a straight-forward observation.

Observation 1. Any escalation rule γ uniquely determines a belief system $\mathcal{B}$.

Recall that the escalation rule, $\gamma(\theta_1, \theta_2)$, is defined as the probability of conflict escalation conditional on the realization of type profile (θ_1, θ_2). Thus, whenever escalation occurs, each player θ_i uses γ to update the conditional probability of facing player θ_{-i}, $\beta_i(\theta_{-i}|\theta_i)$. The rule γ, the updating procedure of every player, and hence the belief system $\mathcal{B}$ at the point of escalation are common knowledge. The reverse statement to Observation 1 is not true for two reasons: First, the belief system is determined by *relative* escalation probabilities only. If γ implements $\mathcal{B}$, so does any $\alpha\gamma$ for any $\alpha \in (0, 1]$. Second, not every possible belief system is consistent with some γ. Any $\mathcal{B}$ implementable by some γ must be (i) internally consistent since $\gamma(\theta_1, \theta_2)$ influences both $\beta_1(\cdot|\theta_1)$ and $\beta_2(\cdot|\theta_2)$, and (ii) consistent with the prior since $\beta_i(\cdot|\theta_i)$ is a function of γ and the prior, ρ^0.

Using a network approach we can, however, find necessary and sufficient conditions when a given belief system is consistent with some escalation rule. Intuitively, fix any $\gamma(1, 1) > 0$ and some belief system $\mathcal{B}$. Ignore the natural constrain $\gamma(\cdot, \cdot) \in [0, 1]$ for the moment. Our aim is now to construct all function values $\gamma(\theta_1, \theta_2)$ from $\mathcal{B}$ and the anchor $\gamma(1, 1)$. To construct $\gamma(\theta_1, \theta_2)$ we can use two paths. We can construct $\gamma(1, \theta_2)$ and from that $\gamma(\theta_1, \theta_2)$, or we can construct $\gamma(\theta_1, 1)$ and from that $\gamma(\theta_1, \theta_2)$. If and only if both path yield the same value for any $\gamma(\theta_1, \theta_2)$, then $\mathcal{B}$ is consistent with some γ. Constructing $\gamma(\theta_1, \theta_2)$ as sketched above may lead to some $\gamma(\theta_1, \theta_2) > 1$. However, since $\mathcal{B}$ determines γ only up to a constant, and we start with some fixed $\gamma(1, 1) > 0$, we can always find an $\alpha \in (0, 1]$ such that all $\alpha\gamma(\theta_1, \theta_2) \le 1$. More formally we can identify all belief systems consistent with some escalation rule by introducing a set of equations, (C). The construction outlined above is only valid if $\mathcal{B}$ is "sufficiently interior". To cover boundary cases we can use the continuity in the mapping from γ to $\mathcal{B}$.[8] We say that a belief system is *interior* if all individual beliefs have full support over the type space.

Lemma 3. *A belief system $\mathcal{B}$ is consistent with some γ, if and only if there exists a sequence of interior belief systems, $\mathcal{B}_n$, such that*

$$\beta_{2,n}(\theta_1|\theta_2)\beta_{1,n}(\theta_1|1)\beta_{1,n}(1|\theta_1)\beta_{2,n}(1|1) - \beta_{1,n}(\theta_2|\theta_1)\beta_{2,n}(\theta_2|1)\beta_{2,n}(1|\theta_2)\beta_{1,n}(1|1) = 0, \quad \text{(C)}$$

for every type-profile (θ_1, θ_2) and n, where $\lim_{n\to\infty} \mathcal{B}_n = \mathcal{B}$ element-wise.

Lemma 3 provides a set of $(K-1)^2$ independent equations via (C) which allow us to define any consistent $\mathcal{B}$ by choosing the two objects $\boldsymbol{\rho}^e$ and $\boldsymbol{\beta}_1^e$. The first, $\boldsymbol{\rho}^e = (\rho_1^e, \rho_2^e)$, describes the ex-ante expected distribution of types in the continuation game, where $\rho_i^e(\theta_i)$ is the likelihood of a player being type θ_i in the continuation game. The second object, $\tilde{\boldsymbol{\beta}}_1^e = \{\beta_1(\cdot|\theta)\}_{\theta<K}$, collects the individual beliefs of the $K-1$ strongest types of player 1, that is the conditional probability distributions over the opponent's type for each such player. We say $B = \boldsymbol{\rho}^e \cup \boldsymbol{\beta}_1^e$ is consistent whenever $\mathcal{B}$ implemented through B is consistent. An interpretation of $\boldsymbol{\rho}^e$ is that it determines the distribution of types after escalation while β^e can be interpreted as a matching technology that determines how the different types are matched to realized continuation matches.

Lemma 4. *There exists a continuous bijection h such that $h(B) = \mathcal{B}$ for any consistent B and $\mathcal{B}$.*

Lemma 5. *Take any consistent $\mathcal{B}$ with $\gamma(1,1) > 0$. Any escalation rule γ implementing $\mathcal{B}$ is represented by some $\alpha' g$ with $\alpha' \in (0,1]$ and $g : \Theta^2 \mapsto \mathbb{R}$ such that $g(1,1) = 1$. If γ satisfies* (AF)*, then there exists $0 < \alpha^* \leq \alpha'$ such that $\alpha^* g$ satisfies* (AF) *with equality and any escalation rule $\gamma^\alpha = \alpha g$ with $\alpha \geq \alpha^*$ satisfies* (AF) *and implements $\mathcal{B}$. Moreover, α^* is continuous in $\mathcal{B}$.*

Lemma 6. *Any γ that does not satisfy* (AF) *with equality is not an optimal escalation rule.*

Lemmas 4 to 6 are the final results to establish the equivalence in choice sets. Any B determines γ only up to a scalar α. The probability of escalation $Pr(\mathcal{G})$ is monotone increasing in α while (AF) becomes tighter as α decreases. Thus, at the optimum (AF) holds with equality, and α^* provides the anchor needed to construct γ from B.

Remark. Observe that we restrict the argument to $\gamma(1,1) > 0$ in Lemma 5. This restriction is purely for notational purposes, but formally restricts the set of possible conflict management mechanisms to the open choice set $\mathcal{CM}^+ \subset \mathcal{CM}$. By continuity, the infimum of the

[8] The continuity follows directly from Bayes' rule.

choice problem over interior escalation rules corresponds to the minimum of the original problem allowing for any escalation rule. Without loss of generality, we thus restrict our choice to $\mathcal{CM}^+$ and solve for the infimum.[9]

Theorem 1. *There is a one-to-one mapping between the optimal mechanism $\mathcal{CM}$ and set of realized belief systems post-escalation, $\mathcal{B}(\sigma)$.*

Theorem 1 states the isomorphism between conflict management and belief management, that is any mechanism that optimally manages the belief system post-escalation also optimally manages the conflict. The result has several immediate implications. First, it highlights the close link between the optimal belief system and the optimal mechanism. Understanding this link already points towards the importance of understanding the continuation game for the optimal mechanism. In particular it highlights that the optimal mechanism is snesitive to the default game. Second, the result shows that the main role of the mechanism is that of an informational gatekeeper. It is crucial for the success of conflict management that privacy of the players can be protected and at the same time information can be transmitted to the players. Third, on a more abstract level this finding links our approach to the literature on information design (Bergemann, Brooks, and Morris, 2016; Bergemann and Morris, 2016a,b; Taneva, 2016). We show that optimal conflict management is essentially characterized by choosing an extension of the initial information structure that determines two sets of outcomes $\mathcal{X}$ and $\mathcal{G}$. Given Σ, the marginal belief system $\mathcal{B}$ depends only on γ, and the problem reduces to finding a (binary) spread over the initial information structure that induces the optimal $\mathcal{B}$ in one of its instances. Finally, the result provides the foundation for characterizing the optimal mechanism as a simple function of the underlying game. By transforming the choice set to $\mathcal{B}$ the designer's choice is now an object that directly effects the results in the underlying game. Therefore, the effect on the continuation game can be understood without calculating the optimal mechanism. In the next part we build on this isomorphism to characterize the optimal mechanism.

3.3 The Optimal Mechanism

In this part we study the designer's problem. If $\mathcal{C}$ is the set of constraints, the primal problem of the designer is given by

Definition 3 (Minimization of Conflict Escalation)**.**

$$\inf_{\mathcal{CM}^+} Pr(\mathit{\Gamma}) \quad \text{s.t. } \mathcal{C}. \tag{P$_{min}$}$$

[9]Consider a convergent sequence of interior belief-systems $\mathcal{B}_n$ as in Lemma 3. $\gamma_n(\theta_1, \theta_2) = \frac{\beta^0(\theta_1)}{\beta^0(1)} \frac{\beta^0(\theta_2)}{\beta^0(1)} \frac{\beta_{2,n}(\theta_1|\theta_2)}{\beta_{2,n}(1|\theta_2)} \frac{\beta_{1,n}(\theta_1|1)}{\beta_{1,n}(1|1)} \gamma_n(1,1)\ \forall(\theta_1, \theta_2)$. By consistency, this is simply $\gamma_n(\theta_1, \theta_2) = \gamma_n(\theta_1, \theta_2)\ \forall n \Rightarrow \lim_{n\to\infty} \gamma_n(\theta_1, \theta_2) \in [0, 1]$ iff $\gamma_n(\theta_1, \theta_2) \in (0, 1)$ for n large.

In what follows, we show that the characterization of the optimal mechanism corresponds to maximizing the sum of two measures on the continuation game: a measure on the degree of discriminating types in the continuation game (the (expected) ability premium) and a measure on the inefficiency in the continuation game (the (expected) welfare). To convey intuition we restrict attention to an environment in which the strongest type is sufficiently privileged from an ex-ante point of view. The intuition remains for other games but involves cumbersome case distinctions. We discuss it in section 5.

Assumption 3. $2 \sum\limits_{\theta \in Q} \rho^0(\theta) v(\theta) < \sum\limits_{\theta \in Q} \rho^0(\theta)$, for any $Q \subseteq \Theta$ and $Q \neq 1$.

Using Assumption 3 we can state the following corollary to Lemmas 1 and 5

Lemma 7. *Suppose* **??** *1–3 hold. Then, the following is true at the optimum*

i. downward adjacent incentive compatibility constraints are binding,

ii. $z_i(k) > 0$ *for any* k*,*

iii. only the participation constraint for the strongest type is binding.

Under Assumption 3 the designer faces the following trade off: She must provide the strongest type a sufficient amount of expected value from $\mathcal{CM}$, while keeping weaker types from imitating the strongest. To solve this trade off, two motives are relevant: a *screening motive* and a *welfare motive*. Making the continuation game discriminatory and thus unattractive for weaker types deters imitation (the screening motive). Making the continuation game attractive for all types saves on ressources by increasing its value in case of failure (the welfare motive).

The classical mechanism design literature uses information rents to describe the designer's promises to weak types to deter imitation. In our setup the discriminatory power of the Bayesian continuation game can be seen as an inverse to the information rent. Intuitively, the better the continuation game discriminates the less information rents have to be paid. A measure of the (relevant) discriminatory power is the *ability premium*.

Definition 4 (Ability Premium)**.** The ability premium, $\psi_i(\theta)$ is the difference in expected utility after escalation between a type θ and the a next strongest deviating type θ+1. That is, $\psi_i(\theta_i, \sigma) = U_i(m{=}\theta_i, \theta_i, \mathcal{B}(\sigma)) - U_i(m{=}\theta_i, \theta_i{+}1, \mathcal{B}(\sigma)), \quad \forall \theta_i < K.$

Definition 5 (Weighted Ability Premium)**.** The *weighted ability premium* is $\Psi_i(\theta_i, \sigma) := w(\theta_i)\psi_i(\theta_i, \sigma)$, with $w(\theta_i) = (1 - \sum\limits_{k=1}^{\theta} \rho^0(k))(\rho^0(\theta))^{-1}$ the inverse hazard rate of type θ given the prior.

The ability premium measures the distance between two adjacent types in the continuation game if the weaker type had pretended to be the stronger type during conflict management. Recall that deviation has two effects. First, the deviator faces a different distribution of opponents after escalation than under truth-telling. Second, the deviator induces a situation of non-common knowledge as any adjustments she makes in the off-path continuation game remain undetected by the complying opponent. The ability premium of type θ_i captures the premium the continuation game pays to a complying player compared to the "closest deviator", and thus measures the discriminatory power of the underlying game. Observe that monotonicity guarantees that the ability premium is weakly positive.

The larger the ability premium the less attractive mimicking behaviour, and the lower the pressure on (IC). Thus, all else equals, the designer desires a high ability premium. How important a particular player's ability premium is depends on the prior distribution which weighs the ability premium with the inverse hazard rate.

Optimizing over the (weighted) ability premium is, however, only one of two pillars of the optimal mechanism. Due to the welfare motive the ombudsman also has an incentive to decrease inefficiencies in the continuation game. Intuitively, the lower such inefficiencies, the lower the compensation under settlement to reach any value from participation Π. The relevant measure for this is expected welfare in the continuation game after escalation.

The set of binding constraints according to Lemma 7 consists of the downward adjacent incentive compatibility constraints and strongest types' participation constraints. We call this set $C_R \subset \mathcal{C}$. Further, we denote $C_F := \mathcal{C} \setminus C_R$. Define $\mathbb{E}[f_i|\mathcal{G}] = \sum_{\theta\in\Theta} \rho_i(\theta) f_i(\theta)$ as the expected value of f_i conditional on escalation

Proposition 1 (Duality)**.** *Suppose* **??** *1–3 hold and fix the set of signal realizations to a singleton. Then $\mathcal{CM}$ solves* (P_{min}) *if and only if it solves*

$$\sup_{\mathcal{B}} \sum_{i\in\{1,2\}} \mathbb{E}[\Psi_i|\mathcal{G}] + \mathbb{E}[U_i|\mathcal{G}] \quad \text{s.t. } C_F \tag{P_{max}}$$

over the set of consistent $\mathcal{B}$.

Ignoring public signals, Proposition 1 characterizes the solution linking it directly to properties of Γ. It provides a tractable solution approach to finding the optimal mechanism. Given the equilibrium is characterized for any consistent $\mathcal{B}$, the only additional object needed is the utility of a type θ_i+1 that is endowed with the individual belief θ_i and faces an opponent who expects equilibrium play according to $\mathcal{B}$. These utilities are easily calculated as they can be formulated as simple univariate decision problems. Observe further that it also provides the optimal solution given any public signal structure if we reformulate the underlying game and incorporate public signals directly on the level of the Bayesian game.[10]

[10]The only difference to that approach is that it also changes the value of vetoing and may violate Assumption 1. However, these concerns are of second order, as discussed in section 5.

The characterization also highlights the designer's fundamental motives: the welfare motive and the screening motive. The formulation provides an additively separable notion of both these motives. Finally, the characterization determines the relative weights on screening motive and the welfare motive as functions of the primitives of the model, i.e., the prior distribution: While the (weighted) ability premium depends on the prior distribution, the expected utility is independent of it.

In Proposition 1 we ignore the role of public signals which may not lead to the optimal mechanism. However, under some restrictions signals are superfluous. We say a solution to (P_{max}) is least constraint if the solution calculated ignoring C_F, satisfies every constraint in this set.

Corollary 2. *Consider the solution to* (P_{max}). *If it is least constraint, then the optimal signal structure is a singleton.*

The result is an outcome of our integrative approach. Using Lemma 1 we can incorporate the most relevant constraints directly in (P_{max}). The marginal belief system is the first posterior after the announcement of escalation. This posterior acts as a "prior" to the continuation game. As well known from the information design literature (Aumann and Maschler, 1995; Kamenica and Gentzkow, 2011; Bergemann and Morris, 2016a), an extension over this prior increases the objective if the objective is convex around the prior. The interior outcome of the maximization is by definition, however, concave. Ignoring additional constraints, the choice set of the designer is equivalent to the set of realizations any public signal can produce. Thus, the designer can always pick the optimal realization as a prior of the continuation game directly, making signals superfluous.

This notion changes once the solution found when solving (P_{max}) does not satisfy the constraints in C_F. In such a case the choice set of the designer is constraint by C_F. The ignored constraints in C_F, however, only need to hold at the expected level which potentially leads to more than one signal realization at the optimum. The designer can use these to get beyond the constraint (for some realisations) while satisfying it on average.

Finding the optimal marginal belief system can, however, still be separated from from the problem of optimal signals. That is, we can redefine Γ to $\hat{\Gamma}$ such that $\hat{\Gamma}$ contains the same rules as Γ, but includes an initial move by nature that implements the optimal signal Σ, in that nature announces publicly announces σ drawn from Σ. With help of concavification à la Aumann and Maschler (1995) we can, without explicitly calculating the optimal signal structure state our second main result in terms of the marginalized belief system, $\mathcal{B}$ only.

We say that a consistent belief system $\mathcal{B}$ is in the set of admissible belief system $\mathcal{B}^a$ if (i) there is a signal structure Σ for which $\mathcal{B}$ is the marginal belief-system, and (ii) the constraints $\mathcal{C}_F$ hold for $\mathcal{B}^\alpha$

Theorem 2 (Duality of problems)**.** *Suppose* **??** *1–3 hold. Any mechanism* $\mathcal{CM}$ *solves equation* (P$_{min}$) *if and only if it also solves*

$$\sup_{\mathcal{B}\in\mathcal{B}^a} cav\left(\sum_{i\in\{1,2\}} \mathbb{E}[\Psi_i|\mathcal{G}] + \mathbb{E}[U_i|\mathcal{G}]\right), \tag{P$_{max}$}$$

where $cav(f)$ *is the smallest function that is concave and weakly larger than* $f(\mathcal{B})$ *on the domain of consistent belief systems* $\mathcal{B}$.

The economic interpretation of the optimization problem is straight forward. Mimicking behaviour has two effects on the continuation game: The deviator (i) inherits the posterior distribution over the opponents types from the mimicked, and (ii) gains an informational advantage as she is the only one aware of entering an off-path game. In the continuation game, the deviator is not forced to adopt the strategy of the mimicked, but can freely choose her behaviour. Any such choice remains unresponded by the opponent who plays as-if she is in the on-path game. The information advantage of the deviator reduces the discrimination of the continuation game and makes it particularly attractive to mimic seldom types. To account for that, seldom types receive a large ability premium and thus the choice of ability premium depends on the prior distribution. It is also intuitive why the welfare motive is independent of the prior. It considers only on-path behaviour and is thus an aggregate measure of (in)efficiency post-escalation.

Theorem 2 provides several economic insights. First, although the designers is agnostic about the outcome once the conflict escalates, optimal conflict management is entirely determined by the information choice post-escalation. Interestingly, there is a direct incentive to induce as little inefficiencies as possibly after escalation to save on resources for additional settlement.

Second, the combination of Theorems 1 and 2 caters to the understanding of the economic problem. Theorem 1 shows that the informational externality of the mechanism matters and the characteristics of the optimal mechanism should depend on the role of information in the continuation game. Theorem 2 uses this insight and specifies the role of information. It shows that the the choice set in conflict management can be reduced to finding the right information structure post-escalation based only on the expected performance in the continuation game. In particular, it identifies, separates, and quantifies the screening motive and the welfare motive of the designer. Theorem 2 provides an intuitive and tractable notion of both motives.

Finally, Theorem 2 offers a direct analogy to the (single player) persuasion models á la Kamenica and Gentzkow (2011). Allowing for an additional signal concavifies the objective over the set of consistent beliefs. However, this becomes only relevant if none of the additional constraints are binding. The main reason for this is that the posterior of the mechanism is in fact a choice of the designer. The posterior of the mechanism acts as a prior to the continuation game. Bayesian persuasion arguments typically take the prior as given and ask whether signals can improve the objective around this prior. Since we optimize by choosing the prior, it is immediate that the designers objective is necessarily concave around the chosen prior in every interior equilibrium such that public signals are of little use. The designer can choose any signal realization as a marginal directly, if she ignores the additional constraints. Including these constraints may, however, restrict her choice set. Using that these constraints only need to hold in expectations allows her to improve upon the "no-signal" mechanism by increasing the value of the objective to that of its concave hull via public signals.

4 Examples

In this section we apply our results to two different underlying games. An exogenous type-dependent lottery á la Hörner, Morelli, and Squintani (2015) and an all-pay-auction. The two examples highlight the link of the optimal mechanism to the underlying game. In each, we can directly identify the channel driving the results.

The first example focuses on the distribution channel. We show that games similar to that used in Hörner, Morelli, and Squintani (2015) result in a *sorting mechanism*. That is, conflict managmeent identifies "easy to settle" matches and guarantees settlement for these matches while other types are referred back to the conflict game.

Contrasting this, the second example focuses on the informational advantage of a deviator. We show that if the conflict game is an all-pay-auction no sorting as in the lottery case takes places. Instead, the optimal mechanism is always *type independent*. That is, conflict management ensures that each type holds the same conditional distribution over the opponents types. Without type independence the mechanism would always provide an incentive to deviate to "steal" another types individual belief and use the information advantage in the off-path continuation-game. While beliefs are the same accross types for a given player and, they differ across players for a given type.

4.1 Type Dependent Lotteries

Consider a generalization of the default game in Hörner, Morelli, and Squintani (2015) which we call lotteries.

Definition 6 (Lottery)**.** A game Γ is called a *lottery* if the von-Neuman Morgenstern utility of a match is independent of the belief system, that is $u_i(\theta, \theta', \mathcal{B}) = u_i(\theta, \theta')$.

Lotteries are in particular relevant for "last-minute" conflict management, that is settlement negotiations at the day of the trial, or peace negotiations at the verge of war. In such situations, the action choice post-escalation is limited and typically involves a dominant strategy such as showing all evidence collected or unleashing the troops. Thus, the expected outcome depends solely on the individual belief about the opponents type rathher than the entire belief system.

The expected utility for any belief system $\mathcal{B}(\sigma)$ is

$$U_i(m, \theta, \mathcal{B}(\sigma)) = \sum_{\theta_{-i} \in \Theta} \beta_i(\theta_{-i}|m, \sigma) u_i(\theta, \theta_{-i}). \tag{U^L}$$

The utility depends only on the individual belief and is entirely linear. Thus, we can abstract from public signals, and denote expected welfare as

$$E[U_i|\mathcal{G}] = \sum_{\theta_i \in \Theta} \rho(\theta_i) \sum_{\theta_{-i} \in \Theta} \beta_i(\theta_{-i}|\theta_i) u(\theta_i, \theta_{-i}), \tag{2}$$

and similar for the expected ability premium. We define $\rho(\theta, k) = \rho_1(\theta)\beta_1(k|\theta)$ the ex-ante probability of a particular match using Bayes' rule with $\sum_{(\theta,k)} \rho(\theta, k) = 1$. Rewriting the maximization problem from Theorem 2 yields

$$\max_{\rho(\cdot,\cdot)} \sum_{(\theta,k) \in \Theta \setminus (K,K)} \rho(\theta, k) \underbrace{(\omega(\theta)A(\theta, k) + \omega(k)A(k, \theta) + W(k, \theta) - W(K, K))}_{\widetilde{VV}(\theta,k)} \tag{3}$$

with match ability premium and match aggregate utility[11]

$$A(\theta, k) = u(\theta, k) - u(\theta{+}1, k) \text{ and } W(\theta, k) = u(\theta, k) + u(k, \theta), \tag{4}$$

respectively. It includes the model of Hörner, Morelli, and Squintani (2015) as a special case with constants $W(\theta, k) = \bar{W}$ and $A(\theta, k) = \bar{A}$ for any type and player. Thus, the welfare motive is shut down and escalation always leads to the same aggregate payoff. In general, linearity directly implies that the optimal solution is entirely symmetric. Further, given Assumption 3 we maximize equation (3) subject to

$$\alpha R \left(\underbrace{\sum_i E[\Psi_i|\Gamma] + E[U_i|\Gamma]}_{=(3)} - 1 \right) \geq 2v(1) - 1, \text{ with } R = \frac{\rho^0(1)^2}{\rho(1,1)}, \tag{AF}$$

[11]We assume that $A(K, k)$ is some finite, positive real number to avoid case distinction.

and

$$\sum_{k}^{K} \left(\rho^0(2)\rho(1,k) - \rho^0(1)\rho(2,k)\right) A(1,k) \geq 0. \tag{IC$_F$}$$

We assume that $\widetilde{VV}(\theta, k)$ is weakly decreasing.

Proposition 2. *Suppose* **??** *1–3 hold and the default game is a lottery. The optimal reduced-form conflict management has the following characteristics*

- *two weak types settle for sure,*
- *two strong types escalate with positive probability,*
- *signals never improve on the "no-signals" solution.*

Moreover, there exists a $\bar{\rho} \in (0,1)$, *such that whenever the prior* $\rho^0(1) \leq \bar{\rho}$ *only the two strongest types meet post-escalation.*

The optimal solution in lotteries requires the mechanism to "sort" matches. In particular the mechanism can guarantee mutual weak matches settlement while mutual strong matches escalate with high probability. The consequence is that beliefs in case of escalation are generically not the same for each type. While the weakest type only expects to compete with stronger types if the conflict escalates, stronger types may meet the weakest even after escalation.

The main reason for this result is that higher order beliefs do not affect the expected outcome as players actions in the continuation game are independent of the prospects from it. Thus a potential deviator has no informational advantage, but at most distributional one. Using our framework, the linearity of the problem immediately identifies a "virtual valuation of matches" for the designer and a linear program for maximization. It also shows that the optimal mechanism in Hörner, Morelli, and Squintani (2015) is robust and depends mainly on the assumption that the default game is indeed a lottery.[12]

4.2 All-Pay Auction

Consider an all-pay auction with binary private information. The prize is normalized to 1 and players have constant marginal bidding cost $c_i \in \{1, \kappa\}$ with $\kappa > 2$. The probability that a player has type $c_i = 1$ is given by ρ^0. To simplify assume $\rho^0 := \rho^0(1) = \delta\bar{\rho}$ with $\bar{\rho} = (\kappa - 2)/(2\kappa - 2)$ and $\delta \in [0.7, 1]$.[13]

[12]Proposition 2 nests Lemma 1 of Hörner, Morelli, and Squintani (2015) as a special case. If affordable, we put all mass on the highest $\widehat{VV}$ (part (4) in their Lemma 1). If we cannot gneerate enough ressources to support this program, we need to shift mass to asymmetric matches to ease (AF) (part (3) of that Lemma). In their setup however (IC$_F$) never binds, while this may be the case in the more general setup.

[13]The upper bound on ρ^0 guarantees that Assumption 2 is satisfied, while the lower bound is sufficient that the optimal belief system ignoring (AF) is implementable satisfying (AF). Cases in which the latter does not hold are discussed in Balzer and Schneider (2015b).

All-pay auctions have been frequently used to model situations of conflict, including legal disputes, international conflicts, strikes, and patent races.[14] In light of the equilibrium characterization in Siegel (2014) and Rentschler and Turocy (2016), it is immediate that –unlike the lottery case above – strategies in all-pay auctions are sensitive to beliefs. The nature of the top-down algorithm in Siegel (2014) already suggests that not only the players individual belief, but the belief system is relevant for the equilibrium strategies. Thus, the information advantage of a deviating player becomes relevant. We now state the result for optimal conflict management and discuss the intuition thereafter.

Proposition 3. *Suppose the default game is the all-pay auction above. Then optimal conflict management has the following characteristics*

- *all matches escalate with positive probability,*
- *the individual belief when entering the continuation-game is independent of the player's behaviour during conflict management,*
- *in each realization of the continuation-game, one player appears to be stronger than her opponent,*
- *signals improve on the optimal no-signal solution if and only if $\rho^0 > 1/3$. The optimal signal randomizes which player takes the role of "player 1" in the underlying game.*

The difference between conflict management for the all-pay auction and that for a lottery is significant in several dimensions.

First, the result that all matches escalate with positive probability, is tightly connected with that of type-independent beliefs. The reason for this lies in the ability premium. Fix some signals and the ex-ante expected type distributions $\rho_1^e(\sigma)$ and $\rho_2^e(\sigma)$ after signal realization σ. The only variable left to determine $\mathcal{B}(\sigma)$ according to Lemma 4 is the individual belief of player $c_1 = 1$, that is $\tilde{\beta}(\sigma) := \beta_1(1|1, \sigma)$.

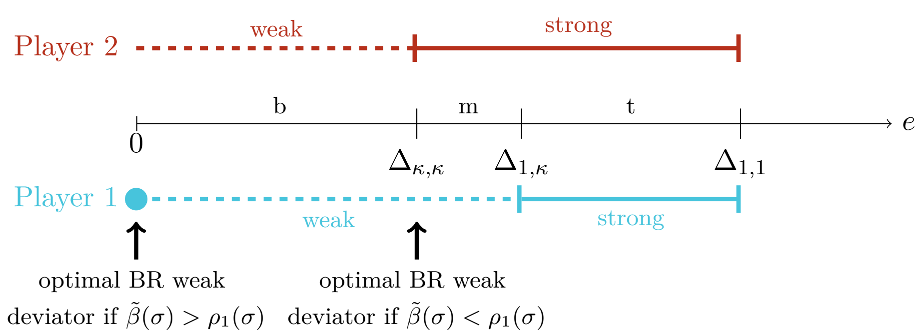

Figure 2: Equilibrium strategies in the all-pay auction assuming $\rho_1 > \rho_2$. Bold lines depict the suport for the storng player, dashed lines that of the weak. The dot indicates a mass point.

[14] See Konrad (2009) for a general discussion also on related games.

Changing $\tilde{\beta}(\sigma)$ has two effects on the ability premium. It reduces the stronger players expected utility in the continuation-game by monotonicity, but it also decreases the deviator's expected utility in the continuation-game as she, too, faces a stronger opponent. As depicted in 2 the deviator's best response changes at $\tilde{\beta}(\sigma) = \rho_1^e(\sigma)$. The reason is straight-forward. Whenever the deviator expects a stronger opponent than on the equilibrium path, that is $\tilde{\beta}(\sigma) > \rho_1^e(\sigma)$, her marginal gain from increasing her bid is reduced and vice versa. If $\tilde{\beta}(\sigma) = \rho_1^e(\sigma)$ she is indifferent as on the equilibrium path. This change in deviation strategies, provides a kink in the expected utility from the continuation game after deviation.

In the all-pay auction this kink is so strong that independent of the choice of $\rho_1^e(\sigma), \rho_2^e(\sigma)$, the ability premium increases for $\tilde{\beta}(\sigma) < \rho_1^e(\sigma)$ as the negative effect on $U_i(1, \kappa, \mathcal{B})$ is stronger than that on $U_i(1, 1, \mathcal{B})$. The ability premium decreases for $\tilde{\beta}(\sigma) > \rho_1^e(\sigma)$ as the negative effect on $U_i(1, \kappa, \mathcal{B})$ is weaker than that on $U_i(1, 1, \mathcal{B})$. Further, straightforward calculations show, that this property also survives if we include the welfare motive. Thus type-independency, $\tilde{\beta}(\sigma) = \rho_1^e(\sigma)$ is always desired.[15]

The third property comes directly from the welfare motive. Shared with many other competition formats, the all pay auction has the feature that an asymmetric distribution creates less aggressive bidding behaviour. This increases the overall expected welfare of the all-pay auction, as we assume bids are lost. Thus, increasing asymmetry maybe helpful. Increased asymmetry comes at a cost, as it reduces the ability premium. However, the optimal mechanism always involves some degree of asymmetry. The level of asymmetry increases in the prior ρ^0, as this reduces (relative) weight of the ability premium compared to aggregate welfare.[16]

Finally, since the condition (GI) is always satisfied, signals can only improve if the strong type's (IC) holds with equality. None of the two binds with equality for any $\rho^0 < 1/3$, and they never bind simultaneously. Providing a symmetrizing signal, that is a coin-flip that then publicly announces who takes the role of player 1 is sufficient to achieve the same solution as in the case ignoring the strong type's (IC).

5 Discussion

In this section we discuss the robustness of our result with respect to several of our assumptions.

Arbitrary sets of constraints binding. Using Assumption 3 we have focused on

[15] A technical advantage of our approach is that the expected utilities in every realization of the escalation game are piecewise linear in $\beta_1(1|1, \sigma)$ for any $\rho_i^e(\sigma)$ providing us immediately with the optimality of a corner solution. This emphasizes the tractability gained by theorems 1 and 2. Details are found in the appendix.

[16] In our binary example $E[\Psi_i|\mathcal{G}] = \rho_i^e(1)\Psi_i(1) = \rho_i^e(1)(1 - \rho^0)/\rho^0\psi_i(1)$.

monotone, well-behaved problems. We can eliminate Assumption 3 by redifining the ability premium. Lemma 1, remains valid and the upward adjacent incentive constraint for some of the types may bind. In this case the ability premium is not only an upward measure, but instead involve a downward component, too. Moreover, more than one type could have a binding participation constraint. We can use an auxiliary linear minimization problem to determine the set of binding constraints for each consistent $\mathcal{B}$. This translates into step functions for the ability premium and the ressource constraint. The ability premium for any type θ_i is given as

$$\widehat{\psi}_i(\theta_i) = \begin{cases} \widehat{U}(\theta_i, \theta_i, \mathcal{B}) - \widehat{U}(\theta_i, \theta_i{+}1, \mathcal{B}) & \text{if } (IC^{+}_{\theta_i+1}) \text{ binds,} \\ \widehat{U}(\theta_i, \theta_i, \mathcal{B}) - \widehat{U}(\theta_i{-}1, \theta_i, \mathcal{B}) & \text{if } (IC^{-}_{\theta_i}) \text{ binds,} \\ 0 & \text{else.} \end{cases}$$

The constant component in the resource constraint consists of $\sum_i \sum_{\theta \in \mathcal{A}(\mathcal{B})} f_\theta(\rho^0, \mathcal{B}) v_i(\theta)$ where $\mathcal{A}(\mathcal{B}) = \{\theta |\ (\text{PC})_\theta \text{ holds with equality given } \mathcal{B}\}$, and a set of functions f_θ s.t. $\sum_{\theta \in \mathcal{A}(\mathcal{B})} f_\theta(\rho^0, \mathcal{B}) = 1$. We can describe the set of binding constraints as a function of the belief system and augment the problem accordingly. Everything else remains as analysed in the main part. A solution approach to this more general problem is discussed in Appendix B

Non Convex Veto-Values. If the value of vetoing is not convex with respect to the prior given the veto off-path belief, we cannot guarantee that full-participation is optimal.[17] However, if we augment the nature of the conflict management game slightly, we can restore full participation. Assume the following changes to the model: Instead of sequentially ratifying conflict management and then communicating their type, assume players do this simultaneously. Assume further that the mechanism has the possibility to send a public signal about the participating player and even a vetoing player cannot ex-ante commit to ignore this information. Then, using the tools from Kamenica and Gentzkow (2011) the designer can always find a set of signals such that the value of vetoing is convex with respect to the prior given the deviator expects to receive these signals in case of vetoing. See Balzer and Schneider (2015a) for details on this.

Private Signals sent by the Designer. In our main analysis we abstract from private signals sent by the designer. However, our observations made in the analysis of public signals shows how private signals could be added to the problem. We could merge the optimal private signals into the description of the default game (by a move of nature e.g.) such that $\widehat{U}$ constitutes the expected utility from participation before the private

[17] An example when optimality of full participation fails in an otherwise different problem is discussed in Celik and Peters (2011).

signals are realised. If the perturbed game is still an anonymous conflict, our analysis remains valid subject to optimal signalling thereafter. Thus, we provide a notion of the benefits a designer that can offer to circumvent the costly game has *on top* of choosing the optimal signal structure. Further we provide a way to separate the pure information design problem downstream in the default game and the hybrid mechanism. Unfortunately, we cannot provide a sensible, yet general characterization when such signals are redundant due to the complexity of communication equilibria.

Limited Commitment by the Players. If players cannot commit ex-ante to obey the mediators settlement decision, the design of the mechanism may change. If players learn enough from the settlement decision, they may expect an easy victory if they decide to enter the default game despite the decision of the mechanism. However, if such rejection cannot credibly be announced publicly, the mechanism has a set of tools to deter players from deviation. Observe that different to false reports a rejection of an offer is observed by the mechanism. We can thus, similar to Hörner, Morelli, and Squintani (2015), assume a different version of our model in which the mechanism triggers escalation by making an unacceptable settlement offer (e.g. 0) with a recommendation to reject to one of the players. If the mechanism communicates this information privately to the players, a deviating player cannot be sure that her deviation indeed triggered escalation. The mechanism can use this to make both players believe they play an on-path escalation game which makes them choose strategies accordingly and punishes the deviating player. Thus the idea of Hörner, Morelli, and Squintani (2015) carries over to some extend. A detailed example of such a situation is discussed in Balzer and Schneider (2015b).[18]

Different Objective. In our paper we assume that conflict management aims at minimizing escalation. While we belief this is the most sensible assumption in our context, we could extend our analysis to that of minimizing (aggregate) inefficiencies from the viewpoint of the parties. While the main motives for the designer remain, the objective of Theorem 2 is no longer valid. In particular the welfare motive receives larger weight. Notice that given settlement resources are scarce (condition (AF) is binding), any settlement solution is by definition the least inefficient outcome. However, the designer in the baseline model gives equal weight to any failed settlement attempt and cares for efficiency post-escalation only to increase resources. If we were to change our objective to a utilitarian maximization over parties ex-ante expected utility, the welfare component would receive higher weight, increasing the pressure on the designer to decrease inefficiencies there. Generically this would not give the same solution as the one carried out here, but the designer would sac-

[18]This construction provides a particular example of private signals. In both examples in section 4 such a twist to the model solves the potential commitment problem (almost) entirely (cf. Balzer and Schneider (2015b) and Hörner, Morelli, and Squintani (2015)). This notiion may, however, not be true in a more general class of games.

rifice some efficient settlements to provide a less inefficient escalation game in many cases. Reformulating the (maximization) objective accordingly, give us a gradient that weighs the two components asymmetrically.[19]

$$\frac{\partial \sum_i E[\Psi_i|\mathcal{G}]}{\partial \mathcal{B}} h_\Psi(\mathcal{B}) + \frac{\partial \sum_i E[U_i|\mathcal{G}]}{\partial \mathcal{B}} h_U(\mathcal{B}), \text{ with } h_U \geq h_\Psi.$$

Transfers and Correlation. So far, we assumed that utility is not directly transferable. While, the settlement value serves as a numeraire good in our analysis there are two additional constraints necessary to guarantee implementation. The first is that z_i can never be negative. The second is captured by (GI) since we need to find a sharing rule X that implements a particular z_i. If we instead allow utilities to be directy transferable we can ignore these additional constraints. Thus the reduced form mechanism is always implementable and z_i can take negative values.

Related to transferable utility is the case of correlated types. If types are correlated, the designer could use techniques similar to those in Crémer and McLean (1988) to exploit correlation and thus to achieve a higher settlement rate. In the baseline model this exploitation is however limited through the the (GI) constraints. Allowing for unlimited transfers (and giving up the ex-post budget constraint), however, would allow a first-best solution á la Crémer and McLean (1988).

6 Conclusion

The main contribution of this paper is to provide a tractable approach to optimal conflict management for a large class of Bayesian games. We offer an economically intuitive dual to the problem that directly links properties of the underlying game to the optimal mechanism. We show that optimal conflict management is completely characterized by the optimal belief system in the event of escalation. We postulate two measures on the continuation-game post-escalation. A measure of discrimination and one of aggregate welfare. We show that the optimal mechanism can be derived by simply maximizing the sum of the two.

We use our general results to characterize the optimal solution in two examples. The first are lottery default games often used in the literature. We show that for the class of lottery games optimal conflict management reduces to a sorting mechanism. The mechanism escalates conflicts with the highest virtual valuations only as players' continuation-strategies are invariant to changes in the information structure.

As a second example we study the all-pay auction. Contrary to lottery games, strategies

[19] Note that in the case of Hörner, Morelli, and Squintani (2015) the two objectives coincide since expected aggregate welfare after escaltion is invariant to changes in the belief system. This is not the case in the all-pay auciton.

depend heavily on the belief system in the all-pay auction. We show that in this case, there is an incentive to misreport during conflict management to extract information from the mechanism. The main channel is the informational advantage a deviator receives by deviating during conflict management and using the undetected deviation to adjust behaviour in the continuation-game without fearing a response. We show that the optimal solution always prevents such deviations by offering type independent beliefs such that the player receives the same information about her opponent independent of her behaviour in the mechanism.

Our results suggest a number of directions for future research. The most obvious lies in the application to specific conflict management problems. As argued in the introduction most of this literature focuses on lottery games mainly for reasons of tractability. Our approach facilitates inclusion of more complicated Bayesian underlying games and thus a better understanding of optimal conflict management. As the optimal solution is characterized by the post-escalation continuation game only, our results provide an approach to link observations on post-escalation behaviour to the quality of the mechanism. A second road is to allow for a more dynamic game. In particular in early stage conflict management players may expect exogenous news arrival in case conflict management fails. Then, the question arises whether there is potential for sequential conflict management that takes this news evolution into account. A different trajectory when moving to a more dynamic game would be to consider repeated interaction between players with (partially) persistent types. That is, players can use some conflicts to learn something about their opponent which may be helpful for future conflicts. We are confident that our simple approach provides a valid starting point and benchmark for such complex models.

Appendix

Structure: In Appendix A we prove Theorem 1 and discuss the general relationship between conflict management and belief management in greater detail. Appendix B provides a formal discussion of the general problem discussed in section 5 and proves Theorem 2. All proofs not provided in these two sections are given in Appendix C.3.

A Conflict Management and Belief Management

A.1 Conflict Management and Belief Management

Fix am interior belief-system. We first derive a representation that links the belief system to the underlying feasible escalation rule.

We interpret every type profile as a node in a network. Node (θ_1, θ_2) has the value $\gamma(\theta_1, \theta_2)$. The (values of the) nodes are linked through a *transition* function in the following way:

Observation 2. Consider the type profiles (θ_i, θ'_{-i}), (θ_i, θ_{-i}) and the *transition* function $q_i(\theta'_{-i}, \theta_{-i}|\theta_i) = \frac{\rho^0(\theta_{-i})}{\rho^0(\theta'_{-i})}\frac{\beta_i(\theta'_{-i}|\theta_i)}{\beta_i(\theta_{-i}|\theta_i)}$. Then,

$$\gamma(\theta'_1, \theta_2) = q_2(\theta'_1, \theta_1|\theta_2)\gamma(\theta_1, \theta_2),$$

$$\gamma(\theta_1, \theta'_2) = q_1(\theta'_2, \theta_2|\theta_1)\gamma(\theta_1, \theta_2).$$

Proof. Applying Bayes' rule it follows that

$$q_i(\theta'_{-i}, \theta_{-i}|\theta_i) = \frac{\rho^0(\theta_{-i})}{\rho^0(\theta'_{-i})}\frac{\gamma(\theta_i, \theta'_{-i})}{\gamma(\theta_i, \theta_{-i})}\frac{\rho^0(\theta'_{-i})}{\rho^0(\theta_{-i})}.$$

□

Fix two nodes in the network, say (θ_1, θ_2) and (k_1, k_2). There are many paths that connect the two nodes. For example, starting from (θ_1, θ_2) we can go to (k_1, θ_2) and then to (θ_1, θ_2). Or, starting from (k_1, k_2) we can approach (θ_1, θ_2) through (θ_1, k_2). Bayes' rule implies that both paths give rise to the same length, or, the resultant values of the nodes are the same. That is, using Observation 2 we have that

$$\gamma(k_1, k_2) = q_1(k_2, \theta_2|k_1)q_2(k_1, \theta_1|\theta_2)\gamma(\theta_1, \theta_2)$$

$$\gamma(k_1, k_2) = q_2(k_1, \theta_1|k_2)q_1(k_2, \theta_2|\theta_1)\gamma(\theta_1, \theta_2)$$

Definition 7 (Bayes' consistent)**.** We consider a belief-system as Bayes' consistent if and only if for every (θ_1, θ_2) and (k_1, k_2) the following is true:

$$q_1(k_2, \theta_2|k_1)q_2(k_1, \theta_1|\theta_2) = q_2(k_1, \theta_1|k_2)q_1(k_2, \theta_2|\theta_1) \tag{5}$$

To derive a more tractable condition we impose a certain structure on the way we move in the network. Let $(1, 1)$ be the origin of the network. Any node (θ_i, θ_{-i}) can be reached from a path that starts at the origin.

Definition 7 boils down to the following requirement

Definition 8 (Consistent)**.** A belief-system is consistent if and only if for every (θ_1, θ_2) the following is true:

$$q_1(\theta_2, 1|\theta_1)q_2(\theta_1, 1|1) = q_2(\theta_1, 1|\theta_2)q_1(\theta_2, 1|1) \tag{6}$$

Lemma 8. *A belief-system is Bayes' consistent if and only if it is consistent.*

Proof. Take any $\gamma(k_1, k_2)$ and $\gamma(\theta_1, \theta_2)$. We want to show that consistency implies

$$q_1(k_2, \theta_2|k_1)q_2(k_1, \theta_1|\theta_2) = q_2(k_1, \theta_1|k_2)q_1(k_2, \theta_2|\theta_1),$$

that is,

$$\frac{\beta_1(k_2|k_1)}{\beta_1(\theta_2|k_1)}\frac{\beta_2(k_1|\theta_2)}{\beta_2(\theta_1|\theta_2)} = \frac{\beta_2(k_1|k_2)}{\beta_2(\theta_1|k_2)}\frac{\beta_1(k_2|\theta_1)}{\beta_1(\theta_2|\theta_1)},$$

or

$$\frac{\beta_1(k_2|k_1)}{\beta_2(k_1|k_2)}\frac{\beta_1(\theta_2|\theta_1)}{\beta_2(\theta_1|\theta_2)} = \frac{\beta_1(\theta_2|k_1)}{\beta_2(k_1|\theta_2)}\frac{\beta_1(k_2|\theta_1)}{\beta_2(\theta_1|k_2)}. \tag{7}$$

Observe that consistency for $\gamma(k_1, \theta_2)$ implies

$$q_1(\theta_2, 1|k_1)q_2(k_1, 1|1) = q_2(k_1, 1|\theta_2)q_1(\theta_2, 1|1),$$

i.e.,

$$\frac{\beta_1(\theta_2|k_1)}{\beta_2(k_1|\theta_2)} = \frac{\beta_1(1|k_1)}{\beta_2(1|\theta_2)}\frac{\beta_1(\theta_2|1)}{\beta_1(1|1)}\frac{\beta_2(1|1)}{\beta_2(k_1|1)}. \tag{8}$$

Consistency for $\gamma(\theta_1, k_2)$ implies

$$q_1(k_2, 1|\theta_1)q_2(\theta_1, 1|1) = q_2(\theta_1, 1|k_2)q_1(k_2, 1|1),$$

i.e.,

$$\frac{\beta_1(k_2|\theta_1)}{\beta_2(\theta_1|k_2)} = \frac{\beta_1(1|\theta_1)}{\beta_2(1|k_2)}\frac{\beta_1(k_2|1)}{\beta_1(1|1)}\frac{\beta_2(1|1)}{\beta_2(\theta_1|1)}. \tag{9}$$

Consistency for $\gamma(\theta_1, \theta_2)$ implies

$$q_1(\theta_2, 1|\theta_1)q_2(\theta_1, 1|1) = q_2(\theta_1, 1|\theta_2)q_1(\theta_2, 1|1),$$

i.e.,

$$\frac{\beta_1(\theta_2|\theta_1)}{\beta_2(\theta_1|\theta_2)} = \frac{\beta_1(1|\theta_1)}{\beta_2(1|\theta_2)}\frac{\beta_1(\theta_2|1)}{\beta_1(1|1)}\frac{\beta_2(1|1)}{\beta_2(\theta_1|1)}. \tag{10}$$

Consistency for $\gamma(k_1, k_2)$ implies

$$q_1(k_2, 1|k_1)q_2(k_1, 1|1) = q_2(k_1, 1|k_2)q_1(k_2, 1|1),$$

i.e.,

$$\frac{\beta_1(k_2|k_1)}{\beta_2(k_1|k_2)} = \frac{\beta_1(1|k_1)}{\beta_2(1|k_2)} \frac{\beta_1(k_2|1)}{\beta_1(1|1)} \frac{\beta_2(1|1)}{\beta_2(k_1|1)}. \tag{11}$$

Equation (7) is satisfied because the right-hand side of Equation (11) times the right-hand side of Equation (10) is equal to the right-hand side of equation Equation (8) times the right-hand side of Equation (9).

□

By Lemma 8 a consistent belief system can be implemented by some escalation rule if and only if the escalation rule feasible, i.e., every node (θ_1, θ_2) has a value weakly below 1. Using our network interpretation, the value of a node is given by the length of the path connecting the node with the origin, i.e.,

$$\gamma(\theta_1, \theta_2) = q_1(\theta_2, 1|\theta_1)q_2(\theta_1, 1|1)\gamma(1, 1) \tag{12}$$

Definition 9. Take any consistent belief-system. We say that the system is feasible, if and only there exists an escalation rule that implements it.

Lemma 9. *A consistent belief-system is feasibly if and only if there exists* $\gamma(1, 1) \in (0, 1)$ *such that for all* (θ_1, θ_2) *the following is true:*

$$\frac{\rho^0(1)}{\rho^0(\theta_2)} \frac{\rho^0(1)}{\rho^0(\theta_1)} \beta_1(\theta_2|\theta_1)\beta_2(\theta_1|1)\gamma(1, 1) \leq \beta_1(1|\theta_1)\beta_2(1|1). \tag{13}$$

Proof. Equation (12) implies that feasibility is satisfied if and only if

$$\frac{\rho^0(1)}{\rho^0(\theta_2)} \frac{\beta_1(\theta_2|\theta_1)}{\beta_1(1|\theta_1)} \frac{\rho^0(1)}{\rho^0(\theta_1)} \frac{\beta_2(\theta_1|1)}{\beta_2(1|1)} \gamma(1, 1) \leq 1,$$

which in turn implies Equation (13). □

Observation 3. A interior belief-system can be implemented by some feasible escalation rule γ if and only if it is *implementable*, that is, (i) feasible and (ii) consistent.

Proof. The only if part follows trivially from the above exposition.

Suppose that an interior belief-system satisfies (i) and (ii). Consider the following set of equations:

$$\gamma_1(\theta_2)\beta_1(\theta_2|\theta_1) = \rho^0(\theta_2)\gamma(\theta_1, \theta_2)$$

$$\gamma(\theta_1, \theta_2) = \frac{\beta_1(\theta_2|\theta_1)}{\rho^0(\theta_2)(1 - \beta_2(\theta_2|\theta_1))} \sum_{k \neq \theta_2} \gamma(\theta_1, k)\rho^0(k) \tag{14}$$

This defines a system of equations.

We show that a consistent belief system satisfies these equations. By definition we know that a consistent belief-system satisfies Equation (12). Substituting into Equation (14) we get

$$q_1(\theta_2, 1|\theta_1)q_2(\theta_1, 1|1) = \frac{\beta_1(\theta_2|\theta_1)}{\rho^0(\theta_2)(1 - \beta_1(\theta_2|\theta_1))} \sum_{k \neq \theta_2} q_1(k, 1|\theta_1)q_2(\theta_1, 1|1)\rho^0(k),$$

which can be simplified to

$$q_1(\theta_2, 1|\theta_1) = \frac{\beta_1(\theta_2|\theta_1)}{\rho^0(\theta_2)(1 - \beta_1(\theta_2|\theta_1))} \sum_{k \neq \theta_2} q_1(k, 1|\theta_1)\rho^0(k) \tag{15}$$

Using the definition of $q_1(\cdot, \cdot|\theta_1)$ Equation (15) becomes

$$\frac{\rho^0(1)}{\rho^0(\theta_2)} \frac{\beta_1(\theta_2|\theta_1)}{\beta_1(1|\theta_1)} = \frac{\beta_1(\theta_2|\theta_1)}{\rho^0(\theta_2)(1 - \beta_1(\theta_2|\theta_1))} \sum_{k \neq \theta_2} \frac{\beta_1(k|\theta_1)}{\beta_1(1|\theta_1)}\rho^0(1).$$

Using algebra we see that

$$1 = \frac{\sum_{k \neq \theta_2} \beta_1(k|\theta_1)}{1 - \beta_1(\theta_2|\theta_1)}, \tag{16}$$

which is correct because $\beta_1(\cdot|\theta_1)$ is a belief. Therefore, a consistent belief system uniquely determines the escalation rule up to $\gamma(1, 1)$. Because the belief-system is feasible, we can find a $\gamma(1, 1)$ so that every $\gamma(\cdot, \cdot)$ is a real number between 0 and 1. □

Given Observation 3 we prove Lemma 3.

A.2 Proof of Lemma 3 and Lemma 4

Proof. Suppose there exist a implementable sequence $\mathcal{B}_n \to \mathcal{B}$. Because $\mathcal{B}_n$ is consistent, Observation 3 implies the existence of some continuous function, say $f : \mathcal{B} \to [0, 1]^{K \times K}$, such that $f(\mathcal{B}_n) = \gamma_n$ with γ_n being feasible and implementing $\mathcal{B}_n$. Because $f(\cdot)$ is continuous it follows that $\lim_{n\to\infty} f(\mathcal{B}_n) = f(\lim_{n\to\infty} \mathcal{B}_n) = \gamma$. Moreover, $\mathcal{B}$ satisfies Equation (C): Equation (C) can be rewritten as

$$g_L(\mathcal{B}) = g_R(\mathcal{B}), \tag{17}$$

where both $g_L(\cdot)$ and $g_R(\cdot)$ are continuous functions that map the belief system into a real number. Because $g_L(\mathcal{B}_n) - g_H(\mathcal{B}_n) = 0$, we can conclude that $g_L(\mathcal{B}) - g_R(\mathcal{B}) = \lim_{n\to\infty}[g_L(\mathcal{B}_n) - g_R(\mathcal{B}_n)] = 0$.

Conversely, take any $\mathcal{B}$ being implemented by some γ. We show that we can find a sequence of interior belief-systems that are implementable and converge to $\mathcal{B}$: Call γ^* the escalation rule that implements $\mathcal{B}$. Choose a sequence of escalation rules that lies in the interior and converges to γ^*. By Bayes' rule every element of the above sequence, say γ_n, implements some belief system $\mathcal{B}_n$. Moreover, there is a continuous function, say $f^{-1} : [0,1]^{K\times K} \to [0,1]^{K\times K}$, such that $f^{-1}(\gamma_n) = \mathcal{B}_n$. Because $\mathcal{B}_n$ is implemented by some γ_n Observation 3 implies that the system satisfies Equation (C). □

Definition 10. The type-profile probability post-escalation, $\rho(\cdot,\cdot)$ is defined up to $\gamma(1,1)$ as follows:

$$\frac{\rho(\theta_1,\theta_2)}{\rho(k_1,k_2)} \equiv \frac{\gamma(\theta_1,\theta_2)}{\gamma(k_1,k_2)} \frac{\rho^0(\theta_1)\rho^0(\theta_2)}{\rho^0(k_1)\rho^0(k_2)} \tag{18}$$

Lemma 10. *The type-profile probability post-escalation, $\rho(\cdot,\cdot)$, is a sufficient statistic for $\mathcal{B}$. $\rho(\cdot,\cdot)$ is implementable if and only if it satisfies*

$$\frac{\rho^0(1)}{\rho^0(\theta_2)} \frac{\rho^0(1)}{\rho^0(\theta_1)} \frac{\rho(\theta_1,\theta_2)}{\rho(1,1)} \gamma(1,1) \leq 1, \tag{19}$$

for every type profile (θ_1,θ_2).

Proof. Using Bayes' rule we can calculate the belief system from $\rho(\cdot,\cdot)$ according to

$$\beta_i(\theta_{-i}|\theta_i) = \frac{\rho(\theta_i,\theta_{-i})}{\sum_{k_{-i}\in\Theta_{-i}} \rho(\theta_i,k_{-i})},$$

which trivially satisfies Equation (C). Moreover, Equation (13) is satisfied if and only if:

$$\frac{\rho^0(1)}{\rho^0(\theta_2)} \frac{\rho^0(1)}{\rho^0(\theta_1)} \frac{\rho(\theta_1,\theta_2)}{\rho(\theta_1,1)} \frac{\rho(\theta_1,1)}{\rho(1,1)} \gamma(1,1) \leq 1$$

□

Lemma 11. *B is implementable if and only if (i)*

$$\rho_2(\theta_2) = \sum_{\theta_1\in\Theta_1} \rho_1(\theta_1)\beta_1(\theta_2|\theta_1), \tag{20}$$

for all θ_2, and (ii)

$$\frac{\rho^0(1)}{\rho^0(\theta_2)} \frac{\rho^0(1)}{\rho^0(\theta_1)} \frac{\rho_1(\theta_1)\beta_1(\theta_2|\theta_1)}{\rho_1(1)\beta_1(1|1)} \gamma(1,1) \leq 1,$$

for all (θ_1, θ_2).

Note that B and ρ determine (i) $\rho_1(K)$, $\rho_2(K)$ (through the requirement that $\sum_K \rho_i(k) = 1$), and (ii) $\beta_1(\cdot|K)$ through Equation (20).

Proof. Observe that $\rho(\theta_1, \theta_2) = \rho_1(\theta_1)\beta_1(\theta_2|\theta_1)$. Therefore, the proof follows from applying Lemma 10. □

A.3 Proof of Lemma 5

Proof. It follows from Observation 3, Lemma 3 and Lemma 4 that there is a one-to-one relation between the escalation rule and the consistent belief-system that is implemented, up to the choice of $\gamma(1,1)$. Moreover, for a given consistent belief-system Equation (12) defines the corresponding escalation rule, which is linear in $\gamma(1,1)$. Since both the right-hand side and the left-hand side of Equation (AF) are linear in the escalation rule, given a consistent belief-system, both sides are linear in $\gamma(1,1)$. □

A.4 Proof of Lemma 6

Proof. This observation follows from Lemma 13. □

A.5 Proof of Theorem 1

Proof. This follows directly from Lemma 6 and Lemma 5. □

B General Problem

In this part of the appendix we construct a solution algorithm to general case. Using this general approach directly provides a proof for Theorem 2.

B.1 The general Problem

We first use the arguments of Luenberger, 1969 chapter 8 theorem 1 to show that the Lagrangian methodology can be applied to solve the general problem.

Our choice variables are a finite set of signals Σ (together with realization probabilities), the $\gamma(\cdot,\cdot)$ and z. Let the choice set be CS, with element cs.

Lemma 12. *The Lagrangian approach yields the unique optimum.*

Proof. Let T be the space of Lagrangian multiplier, with element t.

Define

$$\mathrm{w}(t) := inf\{Pr(\Gamma)|cs = (\gamma, z, \Sigma) \in CS, G(cs) \leq t\},$$

with $G(\cdot)$ being the set of inequality constraints and $Pr(\Gamma)(cs)$ being a function from the choice variable in the probability of escalation.

Observe that $\mathrm{w}(t)$ is convex in t: Assume that $\mathrm{w}(t_0)$ is not convex at t_0.

Then, there is t_1 and t_2 with $\alpha t_1 + (1-\alpha)t_2 = t_0$ so that $\alpha \mathrm{w}(t_1) + (1-\alpha)\mathrm{w}(t_2) < \mathrm{w}(t_0)$. Denote by cs_j the optimal solution, so that $Pr(\Gamma)(cs_j) = \mathrm{w}(t_j)$.

Then, consider the choice cs_0 so that $z^0(\cdot) = \alpha z^1(\cdot) + (1-\alpha)z^2(\cdot)$, $\gamma^0(\cdot,\cdot) = \alpha\gamma^1(\cdot,\cdot) + (1-\alpha)\gamma^2(\cdot,\cdot)$ and $\Sigma = \{1,2\}$, with $Pr(\sigma_1) = \alpha$ and $\gamma^{\sigma_j}(k_1,k_2) = \gamma_j(k_1,k_2)$.

By construction the constraints are satisfied and and the solution value is equals to the convex combination, that is,

$$\mathrm{w}(t_0) = Pr(\Gamma)(cs_0) = \alpha \sum_{\sigma_j} Pr(\Gamma, \sigma_j) = \alpha \mathrm{w}(t_1) + (1-\alpha)\mathrm{w}(t_1)$$

Hence, $\mathrm{w}(t)$ is convex. By Luenberger, 1969 chapter 8 the Lagrangian approach finds a global minimum.

$\square$

Lemma 13. *The optimal conflict management mechanism is the maximizer of the concave hull of*

$$\sum_i \Bigg[\\
\sum_{k=1}^{K} \rho_i(k)(\frac{\rho^0(k) + \tilde{\xi}_k^i + \tilde{e}_k^i - \tilde{\zeta}_k^i}{\rho^0(k)})U_i(k) \\
+ \sum_{k=1}^{K-1} \frac{\tilde{\Lambda}^i(k) - B(k) - \tilde{\Xi}^i(k) - \tilde{E}^i(k) + Z^i(k) - \tilde{v}^i_{k,k+1}}{\rho^0(k)} \rho_i(k)(U_i(k,k) - U_i(k,k+1)) \\
- \sum_{k=1}^{K} \frac{\tilde{\Lambda}^i(k) - B(k) - \tilde{\Xi}^i(k) - \tilde{E}^i(k) + Z^i(k) - \tilde{v}^i_{k+1,k}}{\rho^0(k+1)} \rho_i(k+1)[U_i(k+1,k) - U_i(k+1,k+1)] \\
- \sum_{k=1}^{K} \frac{\rho_i(k)}{\rho^0(k)}\tilde{\xi}_k^i - \sum_{k_1 \times k_2} [-\rho(k_1)\beta_1(k_2|k_1) + \rho_1(k_1) + \rho_2(k_2)]\tilde{\eta}_{k_1,k_2} \\
- \sum_{k_1 \times k_2} \frac{\rho_1(k_1)\beta_1(k_2|k_1)}{\rho^0(k_1)\rho^0(k_2)} \tilde{\mu}_{k_1,k_2} \Bigg],$$

- *The (AF) constraint is always binding.*
- *If the border constraints are redundant, then* $\tilde{\xi}_k^i = \tilde{e}_k^i = \tilde{E}_i(k) = \tilde{\Xi}^i(k) = 0$

- *If* $\tilde{\Lambda}^i(k) + \tilde{Z}^i(k) - B(k) - \tilde{\Xi}^i(k) - \tilde{E}^i(k) > 0$, *then the downward adjacent incentive constraints are binding. If in this case the upward adjacent incentive constraints are redundant* $\tilde{v}^i_{k,k+1} = 0$.
- $\tilde{\Lambda}^i(k) + \tilde{Z}^i(k) - B(k) - \tilde{\Xi}^i(k) - \tilde{E}^i(k) < 0$, *the upward adjacent incentive constraints are binding. If in this case the downward adjacent incentive constraints are redundant* $\tilde{v}^i_{k+1,k} = 0$.

Proof. We minimize $Pr(\Gamma)$ subject to the following constraints, which are required to hold for all i and all k:

$$-(z_i(k) - z_{-i}(k-1)) - y_i(k,k) + y_i(k-1,k) \leq 0 \tag{IC_i^-}$$

$$-(z_i(k) - z_{-i}(k-1)) - y_i(k,k) + y_i(k-1,k) \leq 0 \tag{IC_i^+}$$

$$-z_i(k) - y_i(k,k) + \upsilon_i(k) \leq 0 \tag{PC_i}$$

$$-1 + \sum_i \sum_k \rho^0(k) z_i(k) + Pr(\Gamma) \leq 0 \tag{AF}$$

$$z_i(k) + \gamma_i(k) - 1 \leq 0 \tag{IF_i}$$

$$-z_i(k) \leq 0 \tag{EPI}$$

$$\gamma(k_1, k_2) - 1 \leq 0 \tag{F}$$

In the following we introduce the complementary slackness conditions and introduce the Lagrangian multiplier of the respective constraint:

$$[z_i(k) - z_{-i}(k-1) + y_i(k,k) - y_i(k-1,k)]\upsilon^i_{k,k-1} = 0, \qquad \upsilon^i_{k,k-1} \geq 0$$

$$[z_i(k) - z_{-i}(k-1) + y_i(k,k) - y_i(k-1,k)]\upsilon^i_{k-1,k} = 0, \qquad \upsilon^i_{(k-1,k)} \geq 0$$

$$[z_i(k) + y_i(k,k) - \upsilon_i(k)]\lambda^i_k, \qquad \lambda^i_k \geq 0$$

$$[1 - \sum_i \sum_k \rho^0(k) z_i(k) - Pr(\Gamma)]\delta = 0, \qquad \delta \geq 0$$

$$[z_i(k) + \gamma_i(k) - 1]\xi^i_k = 0, \qquad \xi^i_k \geq 0$$

$$\left[(1 - \gamma(k_1,k_2))\rho^0(k_1)\rho^0(k_2) - \sum_i \rho^0(k_i)(1 - \gamma_i(k_i) - z_i(k_i))\right]\eta_{k_1,k_2} = 0, \quad \eta_{k_1,k_2} \geq 0$$

$$z_i(k)\zeta^i_k = 0, \qquad \zeta^i_k \geq 0$$

$$[\gamma(k_1,k_2) - 1]\mu_{k_1,k_2} = 0, \qquad \mu_{k_1,k_2} \geq 0,$$

The Lagrangian objective takes the following form:

$$
\begin{aligned}
\mathcal{L} &= Pr(\Gamma) + \delta[-1 + \sum_i \sum_{k=0}^{K} \rho^0(k) z_i(k) + Pr(\Gamma)] \\
&+ \sum_i \sum_{k=1}^{K} [-z_i(k) - y_i(k,k) + \upsilon_i(k)]\lambda_k^i \\
&+ \sum_i \sum_{k=1}^{K-1} [z_i(k-1) - z_i(k) - y_i(k,k) + y_i(k-1,k)]v_{k,k-1}^i \\
&+ \sum_i \sum_{k=2}^{K} [z_i(k) - z_i(k-1) - y_i(k-1,k-1) + y_i(k,k-1)]v_{k-1,k}^i \\
&+ \sum_i \sum_{k=1}^{K} [z_i(k) + \gamma_i(k) - 1]\xi_k^i \\
&+ \sum_{k_1 \times k_2} \left[(1-\gamma(k_1,k_2))\rho^0(k_1)\rho^0(k_2) - \sum_i \rho^0(k_i)(1-\gamma_i(k_i) - z_i(k_i)) \right] \eta_{k_1,k_2} \\
&+ \sum_{k_1 \times k_2} [\gamma(k_1,k_2) - 1]\mu_{k_1,k_2} - \sum_i \sum_k z_i(k)\zeta_k^i
\end{aligned}
\tag{21}
$$

Using Lemma 3 we optimize with respect to $\{z_i(\cdot), \gamma^\sigma(1,1), \mathcal{B}(\sigma)\}$.

<u>Step 1: FOCs w.r.t. $z_i(\cdot)$</u>

We take the first order conditions w.r.t. $z_i(k)$:

Let $v_{K+1,K}^i := 0 =: v_{1,0}^i$, then

$$
\rho^0(k)\delta - \lambda_k^i - v_{k,k-1}^i + v_{k-1,k}^i + v_{k+1,k}^i - v_{k,k+1}^i + \xi_k + \rho^0(k_i) \sum_{k_{-i}} \eta_{k_i,k_{-i}} - \zeta_k^i = 0 \tag{22}
$$

For any Lagrangian multiplier, say t, we introduce the following notation $\tilde{x} \equiv \frac{t}{\delta}$.

Let $e^i(k) := \rho^0(k) \sum_{k_{-i}} \tilde{\eta}_{k,k_{-i}}$ and define

$$
\tilde{\Lambda}^i(k) := \sum_{v=1}^{k} \tilde{\lambda}_v^i, \quad \tilde{\Xi}(k) := \sum_{v=1}^{k} \tilde{\xi}_k, \quad \tilde{E}^i(k) := \sum_{v=1}^{k} \tilde{e}^i(v), \quad \tilde{Z}^i(k) := \sum_{v=1}^{k} \tilde{\zeta}_k^i
$$

Summing over the K Equation (22) yields

$$
1 = \tilde{\Lambda}^i(K) - \tilde{\Xi}(K) - \tilde{E}^i(K) - \tilde{Z}^i(k) \tag{23}
$$

Moreover, Equation (22) is satisfied if and only if

$$
\tilde{v}_{k-1,k}^i - \tilde{v}_{k,k-1}^i = B(k-1) - \tilde{\Lambda}^i(k-1) + \tilde{\Xi}(k-1) + \tilde{E}^i(k-1) - Z^i(k-1) \tag{24}
$$

Step 2: Reformulation of the Lagrangian terms

Given the above necessary conditions, we aim to manipulate the Lagrangian in order to derive a tractable maximization problem. As first step consider all terms that involve $z_i(\cdot)$. The first order conditions with respect to $z_i(\cdot)$, i.e., Equation (22), imply that these terms cancel out. Now consider the following term:

$$\begin{aligned}
&\sum_i \sum_{k=1}^{K} [-y_i(k,k)]\tilde{\lambda}_k^i \\
&+ \sum_i \sum_{k=2}^{K} [-y_i(k,k) + y_i(k-1,k)]\tilde{v}_{k,k-1}^i \\
&+ \sum_i \sum_{k=2}^{K} [-y_i(k-1,k-1) + y_i(k,k-1)]\tilde{v}_{k-1,k}^i \\
&+ \sum_i \sum_{k=1}^{K} [\gamma_i(k) - 1]\tilde{\xi}_k \\
&+ \sum_{k_1 \times k_2} [(1-\gamma(k_1,k_2))\rho^0(k_1)\rho^0(k_2) - \rho^0(k_1)(1-\gamma_1(k_1)) - \rho^0(k_2)(1-\gamma_2(k_2))]\tilde{\eta}_{k_1,k_2} \\
&+ \sum_{k_1 \times k_2} [\gamma(k_1,k_2) - 1]\tilde{\mu}_{k_1,k_2}
\end{aligned} \tag{25}$$

Using Equation (22) and adding and subtracting $\tilde{v}_{k-1,k}^i$ and $\tilde{v}_{k-1,k}^i$, Equation (25) reads (for any i)

$$\sum_{k=1}^{K}[-y_i(k,k)]\tilde{\lambda}_k^i + \sum_{k=2}^{K}[-D_i(k-1,k)][\tilde{v}_{k,k-1}^i - \tilde{v}_{k-1,k}^i + \tilde{v}_{k-1,k}^i] + \sum_{k=2}^{K}[-D_i(k,k-1)][\tilde{v}_{k-1,k}^i - \tilde{v}_{k,k-1}^i + \tilde{v}_{k,k-1}^i], \tag{26}$$

with $D_i(\theta,k) = y_i(k,k) - y_i(\theta,k)$.

Using Equation (24) , Equation (27) becomes

$$\begin{aligned}
&\sum_{k=1}^{K}[-y_i(k,k)]\tilde{\lambda}_k^i + \sum_{k=2}^{K}[-D_i(k-1,k)][\tilde{\Lambda}^i(k-1) - B(k-1) - \tilde{\Xi}^i(k-1) - \tilde{E}^i(k-1) + \tilde{Z}^i(k-1) + \tilde{v}_{k-1,k}^i] \\
&+ \sum_{k=2}^{K}[-D_i(k,k-1)][B(k-1) + \tilde{\Xi}^i(k-1) + \tilde{E}^i(k-1) - \tilde{\Lambda}^i(k-1) - \tilde{Z}^i(k-1) + \tilde{v}_{k,k-1}^i]
\end{aligned} \tag{27}$$

We next apply some algebra manipulation in order to arrive at the desired result. Consider first

$$\sum_{k=2}^{K}[-D_i(k-1,k)][\tilde{\Lambda}^i(k-1)-\tilde{\Xi}^i(k-1)-\tilde{E}^i(k-1)-B(k-1)+\tilde{Z}(k-1)+\tilde{v}^i_{k-1,k}] \quad (28)$$

Add

$$\sum_{k=1}^{K} y_i(k,k)(\tilde{\lambda}^i_k-\rho^0(k)+\tilde{v}^i_{k,k+1}-\tilde{v}^i_{k-1,k}-\tilde{\xi}^i_k-\tilde{\eta}^i_k+\tilde{\zeta}^i_k-[\tilde{\lambda}^i_k-\rho^0(k)+\tilde{v}^i_{k,k+1}-\tilde{v}^i_{k-1,k}-\tilde{\xi}^i_k-\tilde{\eta}^i_k+\zeta^i_k]) \quad (29)$$

Then, Equation (28) becomes

$$\begin{aligned}
&-\sum_{k=1}^{K}[y_i(k,k)-y_i(k,k+1)][\tilde{\Lambda}^i(k)-B(k)-\tilde{\Xi}^i(k)-\tilde{E}^i(k)+\tilde{Z}^i(k)\tilde{v}^i_{k-1,k}] \\
&-\sum_{k=1}^{K}[y_i(k,k)][\tilde{\lambda}^i_k-\rho^0(k)+\tilde{v}^i_{k,k+1}-\tilde{v}^i_{k-1,k}-\tilde{\xi}^i_k-\tilde{\eta}^i_k+\tilde{\zeta}^i_k]
\end{aligned} \quad (30)$$

Moreover, consider

$$\sum_{k=2}^{K}[-D_i(k,k-1)][B(k-1)-\tilde{\Lambda}^i(k-1)+\tilde{\Xi}^i(k-1)-\tilde{E}^i(k-1)-\tilde{Z}^i(k-1)+\tilde{v}^i_{k,k-1}] \quad (31)$$

Add

$$\sum_{k=1}^{K} y_i(k,k)(\rho^0(k)-\tilde{\lambda}^i_k+\tilde{v}^i_{k,k+1}-\tilde{v}^i_{k-1,k}+\tilde{\xi}^i_k+\tilde{\eta}^i_k-\tilde{\zeta}^i_k-[(\rho^0(k)-\tilde{\lambda}^i_k+\tilde{v}^i_{k,k+1}-\tilde{v}^i_{k-1,k}+\tilde{\xi}^i_k+\tilde{\eta}^i_k-\tilde{\zeta}^i_k]) \quad (32)$$

Then, Equation (31) becomes

$$\begin{aligned}
&-\sum_{k=1}^{K}(B(k-1)-\tilde{\Lambda}^i(k-1)-\tilde{v}^i_{k-1,k}+\tilde{\Xi}^i(k-1)+\tilde{E}^i(k-1)-\tilde{Z}^i(k-1))[y_i(k,k)-y_i(k+1,k)] \\
&+\sum_{k=1}^{K}(-\tilde{\lambda}^i_k+\rho^0(k)-\tilde{v}^i_{k+1,k}+\tilde{v}^i_{k,k-1}+\tilde{\xi}^i_k+\tilde{\eta}^i_k-\zeta^i_k)y_i(k,k).
\end{aligned} \quad (33)$$

Equation (22) implies that

$$-\tilde{\lambda}^i_k+\rho^0(k)-\tilde{v}^i_{k+1,k}+\tilde{v}^i_{k,k-1}+\tilde{\xi}^i_k+\tilde{\eta}^i_k-\tilde{\zeta}^i_k+\tilde{\lambda}^i_k-\rho^0(k)+\tilde{v}^i_{k,k+1}-\tilde{v}^i_{k-1,k}-\tilde{\xi}^i_k-\tilde{\eta}^i_k+\tilde{\zeta}^i_k \quad (34)$$

$$=-\tilde{v}^i_{k+1,k}+\tilde{v}^i_{k,k-1}+\tilde{v}^i_{k,k+1}-\tilde{v}^i_{k-1,k}=\rho^0(k)-\tilde{\lambda}^i_k+\tilde{\xi}^i_k+\tilde{\eta}^i_k-\tilde{\zeta}^i_k \quad (35)$$

substituting Equation (30), Equation (33), and Equation (35) into Equation (21) and using Bayes' rule, the Lagrangian objective reads

$$\mathcal{L} = Pr(\Gamma)(1+\delta) - \delta(C)$$
$$-\delta \sum_{\sigma} \Pr(\Gamma,\sigma)\Bigg\{ \sum_{i} \Bigg[$$
$$\sum_{k=1}^{K} (\rho_i(k,\sigma)(\frac{\rho^0(k) + \tilde{\xi}^i_k + \tilde{e}^i_k - \tilde{\zeta}^i_k}{\rho_0(k)})U_i(k,\mathcal{B}(\sigma))$$
$$+\sum_{k=1}^{K-1} \frac{\tilde{\Lambda}^i(k) - B(k) - \tilde{\Xi}^i(k) - \tilde{E}^i(k) + Z^i(k) - \tilde{v}^i_{k,k+1}}{\rho^0(k)} \rho_i(k,\sigma)(U_i(k,k,\mathcal{B}(\sigma)) - U_i(k,k+1,\mathcal{B}(\sigma)))$$
$$-\sum_{k=1}^{K} \frac{\tilde{\Lambda}^i(k) - B(k) - \tilde{\Xi}^i(k) - \tilde{E}^i(k) + Z^i(k) - \tilde{v}^i_{k+1,k}}{\rho^0(k+1)} \rho_i(k+1,\sigma)[U_i(k+1,k,\mathcal{B}(\sigma)) - U_i(k+1,k+1,\mathcal{B}(\sigma))\Bigg]$$
$$-\sum_{k=1}^{K} \frac{\rho_i(k,\sigma)}{\rho^0(k)}\tilde{\xi}^i_k] - \sum_{k_1\times k_2} [(-\rho(k_1,\sigma)\beta_1(k_2|k_1,\sigma) + \rho_1(k_1,\sigma) + \rho_2(k_2,\sigma)]\tilde{\eta}_{k_1,k_2}$$
$$-\sum_{k_1\times k_2} \frac{\rho_1(k_1,\sigma)\beta_1(k_2|k_1,\sigma)}{\rho^0(k_1)\rho^0(k_2)}\tilde{\mu}_{k_1,k_2}\Bigg\},$$

where C is a constant that is independent of the choices, i.e.,

$$C := 1 - \sum_i \sum_k \tilde{\lambda}_k \upsilon_i(k) - \sum_{k_1\times k_2} \tilde{\mu}_{k_1,k_2} - \sum_i \tilde{\Xi}^i(K) + \sum_{k_1\times k_2} [\rho^0(k_1)\rho^0(k_2) - \rho^0(k_1) - \rho^0(k_2)]\tilde{\eta}_{k_1,k_2} < 0$$

Define $\hat{\mathcal{L}}(\mathcal{B}(\sigma))$ as solution to

$$\mathcal{L} = \sum_{\sigma} \gamma^{\sigma}(1,1)R(\mathcal{B}(\sigma))(1+\delta) - \delta C - \delta \sum_{\sigma} \gamma^{\sigma}(1,1)R(\mathcal{B}(\sigma))\hat{\mathcal{L}}(\mathcal{B}(\sigma)) \tag{36}$$

and take the derivative of the Lagrangian objective w.r.t. $\gamma^{\sigma}(1,1)$. Then,

$$(1+\delta)R(\mathcal{B}(\sigma)) - \delta R(\mathcal{B}(\sigma))\hat{\mathcal{L}}(\mathcal{B}(\sigma)) = 0, \tag{37}$$

for each signal. Therefore, (i) every signal must give rise to the same value, (ii) $\delta = \frac{1}{\hat{\mathcal{L}}(\mathcal{B}(\sigma))-1}$ and (iii) the Lagrangian objective becomes:

$$\frac{-C}{\hat{\mathcal{L}}(\mathcal{B}(\sigma)) - 1} \tag{38}$$

Finally, observe that every signal must give rise to the same value, δ. Therefore, every $\mathcal{B}(\sigma)$ must maximize $\hat{\mathcal{L}}(\mathcal{B}(\sigma))$.

Therefore, for the optimal multipliers the Lagrangian objective is concave in $\mathcal{B}$. The global optimum of the concave hull then necessarily lies on $\hat{\mathcal{L}}(\cdot)$, but might not be unique. Whenever it is not unique, signals can improve. □

B.2 Proof of Theorem 2

The next lemma implies theorem 2

Lemma 14. *At the optimum we maximize the concave hull of $\hat{\mathcal{L}}(\mathcal{B}(\sigma))$*

Proof. Because of lemma 12 we know that the Lagrangian approach is sufficient to find a global optimum. Applying lemma 13 and using complementary slackness, given lemma 7, the Lagrangian objective ($\hat{\mathcal{L}}(\mathcal{B}(\sigma))$) in lemma 13 is the Lagrangian objective that is associated with the maximization problem stated in theorem 2.

By lemma 12 we know that the optimum lies on the concave hull of the Lagrangian objective. Therefore, the optimal solution to the original problem (stated in theorem 2) lies on the concave hull on the domain of admissible belief systems. □

C Remaining Proofs

C.1 Proof of Lemma 1

Proof. Monotonicity implies that local incentive compatibility constraints are sufficient. The remainder of the claim follows from Lemma 13. □

C.2 Proof of Lemma 2

Proof. The proof follows diretcly from Border (2007) theorem 3. □

C.3 Proof of Lemma 7

Let Q be the set of types so that conflict either (i) the participation constraint is binding, or $z_i(k) = 0$.

Take two adjacent types k,k' in $\hat{Q}$. Let $\hat{Q}(k) := \{k, k+1, ..., k'\}$. Using Corollary 1 the expected sum of shares reads

$$\sum_{k\in\hat{Q}} \sum_{\theta_i\in\hat{Q}(k)} \rho^0(\theta_i)\{[\mathbb{1}_{[\Pi_i(k)=\upsilon_i(k)]}[\Pi_i(k) - y_i(k,k)] + \tilde{h}(\theta_i)\},$$

with

$$\tilde{h}(\theta_i) := \sum_{k=\underline{k}+1}^{\theta_i} g_k\Big(y_i(k,k-1), y_i(k,k+1), y_i(k,k)\Big)$$

or

$$\tilde{h}(\theta_i) := \sum_{k=\theta_i}^{\bar{k}-1} g_k\Big(y_i(k,k-1), y_i(k,k+1), y_i(k,k)\Big)$$

Define

$$h(\gamma) := \sum_i \sum_{k\in\hat{Q}} \sum_{\theta_i\in\hat{Q}(k)} \rho^0(\theta_i)\{\mathbb{1}_{[\Pi_i(k)=\upsilon_i(k)]} y_i(k,k) - \tilde{h}(\theta_i)\} - Pr(\Gamma) \tag{39}$$

Lemma 5 implies that $h(0\times\gamma) = 0$. Therefore, for any γ we can set $h(\cdot)$ arbitrarily close to zero by choosing a small α. Observe that (AF) is satisfied if and only if

$$\sum_{k\in\hat{Q}} \sum_{\theta_i\in\hat{Q}(k)} \rho^0(\theta_i)\{1[\Pi_i(k)=\upsilon_i(k)]\Pi_i(k)\} - 1 \le h(\gamma) \tag{40}$$

Observe that

$$2\sum_{k\in\hat{Q}} \sum_{\theta_i\in\hat{Q}(k)} \rho^0(\theta_i)\{1[\Pi_i(k)=\upsilon_i(k)]\Pi_i(k)\} \le 2\sum_{k\in\hat{Q}} \sum_{\theta_i\in\hat{Q}(k)} \rho^0(\theta_i)\upsilon(k) \tag{41}$$

Finally observe that Assumption 3 implies that the right-hand side is negative, but if $\hat{Q} = \theta_1$. Hence, we have a contradiction to assumption 2.

C.4 Proof of Proposition 1

Proof. The form of the maximization problem follows from applying lemma 13 and using complementary slackness, given lemma 7. □

C.5 Proof of Corollary 2

Proof. Because the solution to the problem is in the set of least constraint solutions, the constraints in the set $\mathcal{C}_F$ are redundant. Consider any $\mathcal{B}^*(\sigma^*)$ that solves problem equation (38). The degenerated signal structure, i.e., $\Sigma = \{\sigma^*\}$, satisfies all constraints by hypothesis. Hence, non-degenerated signal structure cannot improve the solution value of the problem. □

C.6 Proof of Proposition 2

Proof. The first claim is a direct consequence of (3) and monotonicity. Monotonicity ensures that $\widehat{VV} > 0$ such that the designer distributes all mass on matches (θ, k) such that $\theta, k < K$. Monotonicity in $\widehat{VV}$ guarantees positive mass on the type profile $(1,1)$ and linearity of (3),(AF),(IC$_F$) ensures non-convexity at any solution making signals superfluous.

Finally if ρ^0 is small enough, both (AF) and (IC_F) are redundant. Thus, all mass is put on the strongest types and $\rho(1,1) = 1$. □

C.7 Proof of Proposition 3

Proof. **Structure of the proof.** We use a guess and verify approach to proof Proposition 3. A constructive proof is possible, too, but notationally intense. In a companion paper Balzer and Schneider (2015b) we provide a more constructive version. We also omit the proof that the unique equilibrium in the all-pay auction given for the optimal belief system is monotonic. It is however, straight-forward to proof using the equilibrium descriptions of Siegel (2014) and Rentschler and Turocy (2016). We assume without loss of generality that $\rho_1 \geq \rho_2$ and proceed in several steps and make frequent use of the upper bound of interval b in Figure 2 above, $\Delta_{\kappa,\kappa}$. For large parts of the proof we consider the least constraint problem, invoke Proposition 1 such that signals are superfluous.

Part A (Linearity of the Objective).

Step 1: Linearity of individual beliefs. Given ρ_1, ρ_2 use lemma 11 to describe each $\beta_i(\theta_{-i}|\beta_i) \in \mathcal{B}$ as a linear function of $\tilde{\beta}$.

Step 2: Linearity of $\Delta_{\kappa,\kappa}$. The upper bound of $\Delta_{\kappa,\kappa}$ is determined by the mass put on player 1_κ and her belief. Let $F_{i_\theta}(a_i)$ denote the equilibrium probability of i_θ choosing an action smaller than a_i according to Siegel (2014)'s algorithm, then player 2_κ equilibrium support includes a_2 if and only if it maximizes

$$\beta_2(\kappa|\kappa)F_{1_\kappa}(a_2) + \beta_2(1|\kappa)F_{1_1}(a_2) - a_2\kappa.$$

Player 2_κ is indifferent on the interval b in Figure 2. Thus player 1_κ uses a mixed strategy with constant density $f_{1_\kappa}(x) = k/\beta_2(\kappa|\kappa)$ for $x \in b$. Second, again by construction, player 1_κ uses her entire mass on that interval. The length of the interval is

$$\Delta_{\kappa,\kappa} = \frac{\beta_2(\kappa|\kappa)}{\kappa} = \frac{1}{\kappa(1-\rho_2)}\left(1 - \rho_1 - \rho_2 + \tilde{\beta}\rho_1\right),$$

which is linear in $\tilde{\beta}$.

Step 3: Linearity of winning probability at $\Delta_{\kappa,\kappa}$. By construction player 2's winning probability is the same as her belief that player 1 is type 1_κ, i.e. $\beta_2(\kappa|m)$ which is linear in

$\tilde{\beta}$ by step 1. The probability that player 2_κ chooses an action below $\Delta_{\kappa,\kappa}$ is determinde by

$$\begin{aligned} F_{2_k}(\Delta_{\kappa,\kappa}) :=& \underbrace{F_{2_\kappa}(0)}_{\text{independent of } \tilde{\beta}} + \Delta_{\kappa,\kappa} \underbrace{\frac{\kappa}{\beta_1(\kappa|\kappa)}}_{\text{density of } 2_\kappa}. \\ =& F_{2_\kappa}(0) + \frac{1-\rho_2}{1-\rho_1}, \end{aligned} \tag{42}$$

which is independent of $\tilde{\beta}$. The winning probability of player 1_θ is linear in $\tilde{\beta}$ by step 1.

Step 4: Linearity of equilibrium utilities. Given step 2 and 3 it is sufficient to show that all equilibrium utilities can be expressed in the form $F_i(\Delta_{\kappa,\kappa}) - \Delta_{\kappa,\kappa}c_i$. This follows directly from construction for $U_1(1,1,\mathcal{B})$ and $U_2(\kappa,\kappa,\mathcal{B})$. Further, $\Delta_{\kappa,\kappa}$ is the supremum of the equilibrium support of 1_κ and since there are no mass points other than at 0, also $U_1(\kappa,\kappa,\mathcal{B})$ has the desired structure. Finally, $U_2(1,1,\mathcal{B}) = U_1(1,1,\mathcal{B})$ by the common upper bound.

Step 5: Linearity of the Objective. It remains to show (piecewise) linearity in the deviation utilities of κ-types. A deviating κ-type always has either $\Delta_{\kappa,\kappa}$ or 0 in her best response set. Second, such a deviator is only indifferent between several actions if $\tilde{\beta} = \rho_2$ in which case beliefs are type-independent, and the deviator expects the same distribution (and thus utility) as a non-deviating player. If $\tilde{\beta} < (>)\rho_2$ her best response is a singleton at $\Delta_{\kappa,\kappa}$ (0). $\Delta_{\kappa,\kappa}$ and 0 are both linear in $\tilde{\beta}$ and the winning probability is, too. Thus deviating utilities are linear in $\tilde{\beta}$ and have a kink at $\tilde{\beta} = \rho_2$.

Part B (Optimality). We guess the solution at $\tilde{\beta} = \rho_1 = (1+\rho^0)/2$ and $\rho_2 = (1-\rho^0)/2$.

Step 1: Type-independency. Assume, to the contrary that that $\tilde{\beta} < \rho_2$ at the optimum, and rewrite

$$\begin{aligned} \rho^0 \left[E[\Psi(\gamma)|\mathcal{G}] + E[U|\mathcal{G}]\right]\Big|_{\tilde{\beta}<\rho_2} =& F_{2_\kappa}(\Delta_{\kappa,\kappa}) \Big(\beta_1(\kappa|1)\rho_2 + \rho^0(1-\rho_2)\Big) \\ &- \beta_2(\kappa|1) \Big((1-\rho^0)\rho_2 F_{1_\kappa}(\Delta_{\kappa,\kappa})\Big) \\ &+ \underbrace{\Delta_{\kappa,\kappa}}_{=\beta_2(\kappa|\kappa)/\kappa} \Big(\left(\rho_1+\rho_2\right)(\kappa-1) - (1+\rho_2)\kappa\rho^0\Big). \end{aligned} \tag{43}$$

Taking the derivative with respect to $\tilde{\beta}$ and substituting ρ_i with the guessed solution we get

$$\lim_{\tilde{\beta}\to^-\rho_1} \frac{\partial\rho^0 \left[E[\Psi(\gamma)|\mathcal{G}] + E[U|\mathcal{G}]\right]}{\partial\tilde{\beta}}\Bigg|_{\text{cand}} = \frac{\kappa(1-(\rho^0)^2) - (1-(\rho^0)^2)}{\kappa(1+\rho^0)}$$

which is positive. Now, assume $\tilde{\beta} > \rho_2$ at the optimum, and proceed as before, then the derivative of the objective at the candidate solution for ρ_i is

$$\lim_{\tilde{\beta} \to^+ \rho_1} \left. \frac{\partial \rho^0 \left[E[\Psi(\gamma)|\mathcal{G}] + E[U|\mathcal{G}]\right]}{\partial \tilde{\beta}} \right|_{\text{cand}} = -\frac{\kappa(1-(\rho^0)^2) - 1 - (\rho^0)^2}{\kappa(1+\rho^0)},$$

which is negative.

Step 2: Type distribution. Taking the derivative of (43) with respect to ρ_i and evaluating at $\tilde{\beta} = \rho_2$ directly establishes the critical point irrespective of the choice of ρ_{-i}.[20] Second order conditions are satisfied at the desired point and we can conclude that a local optimum exist in case we face a least constraint problem. Due to our assumptions on ρ^0, there always exists an $\alpha \leq 1$ such that the optimal solution satisfies (AF) with equality.

Step 3: Potential for signals. A sufficient condition to satisfy incentive compatibility for i_1 and our candidate can be obtained by plugging into the incentive constraint and noticing that $U(1,1,\mathcal{B}) = U(\kappa,1,\mathcal{B}) \geq U(m,\kappa,\mathcal{B})$. The condition is given as

$$(\gamma_i(1) \geq \gamma_i(\kappa)) \Leftrightarrow \rho_i \geq \rho^0. \tag{44}$$

This always holds for player 1_1, but not for player 2_1 if $\rho^0 > 1/3$. Now consider the following mechanism with public signals. There are two realizations σ_1 and σ_2 both equally likely. Under realization σ_1 the result is as discussed above, under realization σ_2 the roles are flipped. All types of player 1 take the role of player 2 and vice versa. By ex-ante symmetry, the value of (P_{max}) does not change, neither do the binding constraints. The sufficient condition (44) now becomes

$$\frac{1}{2}\left(\gamma_i(1,\sigma_1) + \gamma_i(1,\sigma_2)\right) \geq \frac{1}{2}\left(\gamma_i(\kappa,\sigma_1) + \gamma_i(\kappa,\sigma_2)\right) \Leftrightarrow \frac{1}{2} \geq \rho^0,$$

which holds by Assumption 2. □

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[20] By continuity of the objective the same holds true if we took the objective given $\tilde{\beta} \geq \rho_i$ instead.

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