Managing A Conflict*

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Abstract

We characterize the design of mechanisms aiming to settle conflicts that otherwise escalate to a costly game. Participation is voluntary. Players have private information about their strength in the escalation game. The designer fully controls settlement negotiations but has no control over the escalation game. We transform the mechanism-design problem of conflict management to the information-design problem of belief management conditional on escalation. The transformed problem identifies how the properties of the escalation game influence the optimal mechanism. We use our general results to study optimal alternative dispute resolution in the shadow of a legal contest.

1 Introduction

Conflict management that aims to settle a conflict at little or no cost often operates in the shadow of some default resolution mechanism. Should an intermediary fail to settle the conflict, it escalates to a fight, that is, a non-cooperative game beyond the intermediary's control. If the conflicting parties hold private information regarding their ability in that escalation game, continuation strategies depend on the information obtained during conflict management. Thus, when designing conflict management, the intermediary has to take into account that information revelation during conflict management influences players' behavior in the escalation game.

Consider, for example, alternative dispute resolution (ADR) in legal disputes aiming to find out-of-court settlement solutions. Should ADR fail to settle the conflict, the conflict escalates, and disputants revert to formal litigation in court. The procedural rules of litigation are beyond control of the ADR mechanism's designer. However, the

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designer's choices influence how informative the process of ADR is to disputants. Naturally, disputants' strategic choices in litigation following a failed ADR attempt depend on the information they obtain during ADR. Thus, the designer must account for the effect of information revelation *during* ADR on disputants' behavior *after* escalation. A disputant who decides whether to participate in an ADR mechanism takes into account how information revelation affects both her and her opponent's continuation strategy.

Examples of conflict management in the shadow of an escalation game abound. Besides legal disputes they include a mediator in collective bargaining that operates in the shadow of a strike, peace negotiators concerned about escalation to war, or negotiations over trade agreements limited by a sovereign country's right to impose tariffs.

In this paper, we characterize optimal conflict management when two players dispute over the allocation of a given pie. Conflict management's aim is to prevent escalation to a game beyond the designer's control. Continuation strategies – and hence expected outcomes – depend on the players' beliefs about the distribution of ability in the escalation game. We address the following questions for a broad class of escalation games: *Which aspects of the escalation game are most important for optimal conflict management? What is the role of information in the relation between conflict management and the escalation game? What influences players' strategic choices before, during and after conflict management?*

We characterize optimal conflict management for a broad class of escalation games. The designer has to consider how her mechanism influences players' strategic choices should the conflict escalate. Both the structure of the escalation game and information revelation during conflict management determine the players' choices.

To gain intuition for the designer's problem, consider the ADR example from above. In formal litigation disputants provide evidence to convince a judge or a jury. Evidence provision is costly and destroys part of the joint surplus. A disputant's cost function is her private information. Absent ADR, a low-cost disputant expects to succeed in litigation easily. She requires a *favorable settlement* to participate in ADR. Absent escalation, a high-cost disputant may bluff by claiming low cost as no evidence is provided and actual costs are irrelevant. Occasional *escalation* to litigation *deters* such *bluffs*.

To understand how continuation strategies react to the information structure, consider an environment in which ADR cannot guarantee full settlement. The structure of ADR and the event of escalation jointly reveal information about the distribution of cost functions. Disputants use this information to adjust their continuation strategies. Suppose a plaintiff *expects* that on average the defendant has a much lower cost than herself. Then, her chances of winning are low and she reduces evidence provision in litigation to save costs.

Part of the mechanism-design problem is to design the information structure at the beginning of the escalation game. That information-design problem interacts with the mechanism in a non-trivial way. A player has an incentive to use conflict management only to *plant false information* into her opponent's mind. If, for example, a high-cost plaintiff convinces the defendant that she has low cost, the defendant may respond by reducing the amount of evidence. The plaintiff then exploits this change in behavior. Naturally, optimal conflict management takes these incentives into account.

Results and Implications. Our main result, Theorem 2, establishes a duality between the primal mechanism-design problem of conflict management and a dual information-design problem of belief management. It characterizes optimal conflict management as the solution to optimal belief management in the escalation game.

In the dual problem, the designer chooses the information structure in the event of escalation. Thus, she picks the "prior" of the escalation game. Her objective is to maximize the sum of two measures, both directly defined on the properties of the escalation game. The first measure relates to effectiveness of screening. It quantifies the discrimination between types, that is, the importance of a player's exogenous type on continuation payoffs as a function of the information structure. The second measure determines how attractive it is to participate in conflict management. It is aggregate welfare conditional on escalation.

The characterization facilitates the analysis in several dimensions. First, it significantly simplifies the problem. The primal mechanism-design problem involves non-trivial effects of the design choices on the players' behavior in the escalation game. Design choices influence the settlement stage, the escalation stage, and the probability of either occurring. The dual information-design problem instead concentrates entirely on the event of escalation. The information structure solving the dual problem is a sufficient statistic for the optimal (reduced-form) mechanism. That is, there is a one-to-one mapping between possible solutions to the information-design problem and candidate mechanisms. As a byproduct, the formulation disentangles the information-revelation part from the "game-design" part of the problem. The separation identifies the channel that connects the escalation game with the mechanism.

Second, the characterization highlights the economic channel through which optimal conflict management operates. The role of the escalation game is twofold. It serves as a last resort to the designer to verify claims during conflict management by making private information payoff relevant. In addition, it ensures that players expect to obtain some payoff even if conflict management fails to settle the conflict. The dual objective identifies, and quantifies the importance of these two effects as a function of the escalation game's properties. The effect of the information structure on the discrimination measure captures the first effect, that on the welfare measure captures the second effect.

Finally, the characterization illustrates that understanding the role of information in the escalation game is crucial for the design of optimal conflict management. We study optimal ADR in the shadow of litigation in courts to demonstrate that last point. In line with the literature we model formal litigation as a legal contest and characterize the mechanism that settles as many cases as possible outside court.

The ADR-model highlights two channels through which we contribute to the literature. First, we consider behavioral externalities of ADR on subsequent litigation. They are of first-order importance. Consequently, existing models that abstract from such externalities are lacking an important point. Second, classic approaches in mechanism design do not apply directly to our problem because it is generically non-convex. Using belief management, however, we characterize the solution via an information-design problem. That information-design problem is both tractable and economically intuitive.

The first channel is relevant because existing models on conflict-management mechanisms assume that the escalation game has an ex-post equilibrium, on which the players coordinate.[1] This assumption implies that a change in the information structure *only* changes the likelihood that a particular type-profile occurs. Thus, the information structure only has *distributional consequences* on expected payoffs. Absent an ex-post equilibrium the information structure, however, also has *behavioral consequences*. Players adjust their continuation strategies in response to information, and expect their opponents to do alike. In (legal) contests, strategies are particularly sensitive to information, and behavioral consequences are the designer's main concerns.

The second channel is relevant because including behavioral externalities is non-trivial from a technical point of view. It implies that a change in the information structure not only affects a player's expectations over possible outcomes, but through changes in behavior also directly these outcomes. Then, the mechanism's effect on the continuation payoff is non-separable from the details of the underlying game. Constraints are generically non-convex and standard techniques do not apply directly. Moreover, interpreting results becomes increasingly difficult since a change in the parameters of the mechanism affects the outcome along many dimensions.

Using our belief-management approach, we characterize the solution directly via the resulting escalation game. Our approach not only overcomes the tractability problem, but also offers a straight-forward economic explanation for the results.

Separately inspecting the two terms of the dual problem provides an economic intuition. First, the discrimination term relates to incentive compatibility and depends on how well the information structure in the escalation game discriminates between different types sharing the *same belief*. Optimal ADR ensures that a potential deviator has *no information advantage* once settlement negotiations fail. Second, the welfare term relates to players' incentive to participate in ADR. The legal conflict should be asymmetric such that *both* players *reduce wasteful effort*. Thus, litigation after a failed ADR attempt is less costly to the players than without having tried ADR at all.

Structure of the Argument. We first derive the general equivalence result, and then apply it to the specific case of alternative dispute resolution. We proceed in steps. We describe the setup of the general model in Section 2. In Section 3 we first derive

[1]See Spier (2007) for legal disputes, Jackson and Morelli (2011) for international conflicts, and Hörner, Morelli, and Squintani (2015) and references therein for more recent contributions.

a condition, Proposition 1, that determines in which cases conflict management can guarantee a settlement solution. We show that the question whether full settlement is implementable reduces to: *Can the mechanism implement an equal split of surplus?*

Next, we consider cases in which full settlement is not implementable. Using binding constraints and properties of Bayes' rule, Theorem 1 establishes a one-to-one mapping between the information structure in case of escalation and a set of reduced-form mechanisms. For a given information structure, the mapping identifies the optimal mechanism implementing this information structure. Thus, effectively, Theorem 1 reduces the problem to: *What is the optimal information structure conditional on escalation?*

We proceed by transforming the designer's objective function. In Proposition 2, we state necessary and sufficient conditions when a reduced-form mechanism is implementable. We then use first-order conditions to develop an equivalent formulation of the problem, the dual. The dual is the sum of the two economic measures discussed above, discrimination and welfare. Both depend on the primitives of the escalation game and take the information structure at the event of escalation as their argument. Using the dual objective we characterize the optimum as the answer to the question: *Which information structure maximizes discrimination and welfare in the escalation game?*

In Section 4 we apply the general result to a model of alternative dispute resolution. We discuss various extensions in Section 5 and conclude in Section 6.

Related Literature. We contribute to the growing literature on conflict-resolution mechanisms. In line with Zheng (2017), we derive a necessary and sufficient condition for full settlement. Beyond that, our analysis addresses the optimal second-best mechanism in case this condition is not met and escalation occurs on the equilibrium path. We nest the existing models of second-best mechanisms without behavioral externalities (e.g. Bester and Wärneryd, 2006; Hörner, Morelli, and Squintani, 2015; Spier, 1994), but allow for interactions between the design of the mechanism and players' action choices *after* the mechanism. Players obtain information through the mechanism and use it to adjust their continuation strategies. The existing literature acknowledges this informational content but abstracts from an effect on players' behavior.

We show that economic forces differ substantially if continuation strategies are instead information sensitive. More specifically, we show that including a behavioral effect of information provides an additional benefit to a deviating player that previous models do not capture. We show that this effect is of first-order importance if the escalation game takes the form of a contest. We identify, and quantify the mechanism's effect on subsequent behavior complementing Meirowitz et al. (2017) who study how conflict management affects behavior prior to it.

Methodologically, we build on the mechanism-design problem of Compte and Jehiel (2009). We assume the division of a pie as an outcome, a budget-constraint mechanism, and continuation payoffs that depend on the information structure. We include interdependent valuations in the sense of Jehiel and Moldovanu (2001) that lead to an information externality. Two aspects are different from these papers. First, we allow for a continuation payoff that is non-linear in beliefs.[2] Departure from linearity has important consequences on optimal design. Second, we shift focus by characterizing the second-best mechanism. We depart from standard solution approaches and consider players' information sets. Thereby, we characterize and interpret the optimal solution using the properties of the escalation game directly.

Close in spirit is the literature on bailout mechanisms (Philippon and Skreta, 2012; Tirole, 2012), as well as that on aftermarkets in mechanism-design problems (Atakan and Ekmekci, 2014; Dworczak, 2017; Lauermann and Virág, 2012; Zhang, 2014). A common theme in these models is that the mechanism typically affects the information structure only one-sidedly and beliefs are type-independent by construction. In contrast, an important feature in our model is that the same players meet within and after the mechanism. Thus, design choices influences the entire information structure and type-dependent beliefs are possible.

Our *belief-management approach* to the mechanism-design problem is related to the approach of Grossman and Hart (1983) to contracting. Similar to them, we offer an implementation approach. In a first step, we determine the cost of implementing a particular information structure to the designer. In the second step, we determine a program to select the least costly information structure. In fact, our formulation of the second step shares strong similarities to the formulation of the optimal auction problem in Myerson (1981). Our results allow us to separate the cost and benefits of manipulating the information structure.

Thus, although conflict management is inherently a mechanism-design problem a la Myerson (1982), we present a dual (pure) information-design problem (Bergemann and Morris, 2016), and a mapping from its solution to the optimal mechanism. Moreover, the belief-based techniques of Mathevet, Perego, and Taneva (2017) directly apply to our reformulated problem.[3] In fact, their techniques help us to determine both the optimal information structure after the mechanism *and* the role of additional signals the designer may want to communicate. We complement these papers by emphasizing the first-order importance of information-design techniques for the literature on mechanism design with endogenous outside options.

[2]Fieseler, Kittsteiner, and Moldovanu (2003) and the literature on second-best conflict preemption (Bester and Wärneryd, 2006; Hörner, Morelli, and Squintani, 2015; Spier, 1994) considers similar models. In all of those, continuation payoffs are linear in beliefs.

[3]Important for us, they generalize the findings of Aumann and Maschler (1995) and Kamenica and Gentzkow (2011) to multiple players with interdependent values. Similar to the information-design literature we repeatedly use the fact documented in Kamenica and Gentzkow (2011) that optimization over posterior beliefs improves tractability substantially.

2 Model

Grand Game. Two ex-ante identical, risk-neutral players compete for a pie worth 1 to each player. There are two ways to solve the dispute. The players can engage in an exogenously given game of conflict, $\mathcal{V}$, such as litigation, or they can seek conciliation via some form of conflict management, $\mathcal{CM}$, such as alternative dispute resolution (ADR). Conflict management takes place only if *both* parties agree to participate. Otherwise they non-cooperatively play $\mathcal{V}$. The veto structure captures, for example, the idea of the rule of law providing a constitutional right to enforce a lawsuit.

Conflict Management. Conflict management is a mechanism proposed by a non-strategic third party, the designer, at the beginning of the game. It leads either to a settlement solution, $\mathcal{Z}$, or to escalation, $\mathcal{G}$. A settlement solution awards player i a share of the pie, x_i, such that $x_1 + x_2 \leq 1$.[4] Escalation triggers a non-cooperative game $\mathcal{G}$. The two games $\mathcal{G}$ and $\mathcal{V}$ may be identical, but do not have to be.[5] In any case, $\mathcal{G}$, too, is beyond the designer's control. That is, once the conflict escalates the designer controls neither players' action choices nor the payoff rule that $\mathcal{G}$ implements. Conflict management thus describes a mechanism-design problem within a greater strategic environment beyond the designer's control, e.g., designing ADR within the legal system.

Initial Information Structure. Each player i is endowed with type θ_i independently drawn from the same distribution over $\Theta = \{1, 2, ..., K\}$. The probability of being a specific type θ_i is $p(\theta_i) > 0$. The type is payoff relevant in $\mathcal{V}$ and $\mathcal{G}$, but not under settlement. That is, we assume that players have identical preferences for the good, but may differ in their *ability* to play the non-cooperative games. In a legal dispute θ_i represents the cost of providing formal evidence in litigation which is unrelated to a disputant's value of winning the case. All, but the realization of θ_i is common knowledge.

Payoffs. Under settlement, each player receives a payoff equal to her share of the pie, x_i. Conditional on escalation, payoffs depend on the play of the escalation game $\mathcal{G}$. Let $A^{\mathcal{G}} \subset \mathbb{R}^2$ describe the space of joint actions in $\mathcal{G}$. We assume that the escalation game is more inefficient than settlement. That is, its payoff function is a mapping $(u_1, u_2) : \Theta^2 \times A^{\mathcal{G}} \to (-\infty, 1]^2$ with $u_1 + u_2 \leq 1$. Similarly, the payoff function for the veto game $\mathcal{V}$ is $(v_1, v_2) : \Theta^2 \times A^{\mathcal{V}} \to (-\infty, 1]^2$ with $v_1 + v_2 \leq 1$.

We assume that u_i and v_i are weakly decreasing in θ_i capturing the interpretation of types as (marginal) *cost-of-effort parameters* in both $\mathcal{V}$ and $\mathcal{G}$. Throughout the paper we use the terms stronger (weaker) to indicate lower (higher) values of θ_i. For simplicity we assume symmetry, that is, $u_i(\theta_i, \theta_{-i}, a_i, a_{-i}) = u(\theta_i, \theta_{-i}, a_i, a_{-i})$ and $v_i = v$.

Timing. After learning their type, players simultaneously decide whether to ratify or to veto conflict management. If at least one player vetoes, her identity is revealed, players play $\mathcal{V}$ and payoffs realize. If both players ratify conflict management, the mechanism

[4] If the pie is indivisible, we interpret x_i as probability of winning the whole pie.

[5] In the legal context the rules of formal litigation after a failed attempt of ADR may differ from those if ADR is rejected altogether. See Prescott, Spier, and Yoon (2014) for a specific example.

is played and either implements a settlement solution, or the conflict escalates to $\mathcal{G}$. For simplicity we focus on arbitration and assume full commitment upon ratification. This assumption implies in particular that the mechanism can directly or indirectly trigger the escalation game by creating an environment sufficiently hostile so that players refuse to settle. We discuss ways to relax this assumption in Section 5.

Solution Concept. We use perfect Bayesian equilibrium (Fudenberg and Tirole, 1988). We aim to find the mechanism that maximizes the probability of settlement. The choice of the objective is driven by applications in which either escalation implies externalities on the society, e.g., by limiting access to the legal system, or quality of conflict management is judged by "cases solved" rather than by details of settlement contracts. In addition, our objective emphasizes that the designer's main concern is about behavior in the escalation game, despite her being genuinely agnostic about the resulting outcomes.[6]

Key Modeling Choices. A key feature of our model is that a player's strength under escalation is orthogonal to her preferences regarding outcomes. The motivation behind this modeling choice lies in the institutional specifics of the environment. Often, the institutional environment requires certain skills that are not directly connected to the value of winning. For example, in litigation, there are typically aspects determining access to evidence that are uncorrelated with the value of winning the case. All results extend to settings that include additional private information about preferences.

A second important assumption is that all settlement solutions are based on soft information only. In reality, such mechanisms are frequent as they are the least costly solutions. In case of ADR, the outcome function of any more complicated ADR mechanism that includes provision of evidence *within* ADR can be subsumed in the escalation stage with our mechanism focusing on initial exchange of soft information only.

Finally, the assumption that the mechanism can ex-ante commit to her protocol is in line with the vast majority of the mechanism-design literature, but nevertheless restrictive. We ignore important cases in which renegotiation *through the designer* is anticipated by parties in advance. In reality, however, out-of-court settlement solutions result mainly from interaction with retired judges or mediators who are trained in conflict management and who provide their services repeatedly and having an incentive to commit.

3 Analysis

3.1 First-Best Benchmark: Full-Settlement Solutions

We first provide conditions for a full-settlement mechanism. Invoking the revelation principle we restrict attention to direct revelation mechanisms. Any such mechanism implements a decision rule based on players' type reports. The decision rule is in-

[6]Maximizing settlement is *not equivalent* to maximizing utilitarian welfare. Our main insights remain under such an alternative objective. See Section 5 for a discussion.

centive compatible and satisfies participation constraints. Types are payoff irrelevant under settlement. Thus, any mechanism completely avoiding both $\mathcal{V}$ and $\mathcal{G}$ is a pooling mechanism.

Any pooling mechanism implements a particular sharing rule (x_1, x_2) independent of the players' reports. Whether such a rule satisfies the participation constraints depends on the expected payoff from vetoing the mechanism. A player's optimal strategy in $\mathcal{V}$ maximizes her expected payoff over action choices conditional on her belief about the opponent's type *and* corresponding strategy. The entire information structure becomes relevant, since the opponent's strategy is a function of the opponent's beliefs.

Suppose player i vetoes the mechanism. Then, she learns nothing about her opponent and continues to use the common prior p to evaluate $-i$'s type distribution. The non-vetoing player $-i$, to the contrary, may learn from i's veto. She is going to use the probability function ρ^V over i's type. Via the "no-signaling-what-you-don't-know" condition of perfect Bayesian equilibrium ρ^V is independent of θ_{-i}.[7] The information structure after i vetoes is (p, ρ^V). Behavior in the continuation game after a veto forms a Bayes Nash equilibrium given (p, ρ^V). Player i's expected continuation payoff is

$$V_i(\theta_i, (p, \rho^V)) := \max_{a_i \in A_i^{\mathcal{V}}} \sum_{\theta_{-i}} p(\theta_{-i}) \int_{A_{-i}^{\mathcal{V}}} v(\theta_i, \theta_{-i}, a_i, a_{-i}) \mathrm{d}F(a_{-i}|\theta_{-i}, (p, \rho^V)), \tag{V}$$

where $F(a_{-i}|\theta_{-i}, (p, \rho^V))$ is the conditional distribution over equilibrium action choices of θ_{-i} given information structure (p, ρ^V). Our first result is in the spirit of Zheng (2017). It determines a necessary and sufficient condition for a full-settlement mechanism.

Proposition 1 (Full Settlement Mechanisms)**.** *The optimal conflict-management mechanism guarantees full settlement if and only if there is a probability mass function $\tilde{\rho}$ over the type space such that $V_i(1, (p, \tilde{\rho})) \leq 1/2$.*

The result combines three constraints. Settlement has no screening power, thus the designer offers a pooling mechanism; conflict management is budget constraint, thus settlement divides the pie; and participation is voluntary, thus the designer incentivizes players via a sufficiently large share. Players are ex-ante symmetric, and a first-best mechanism exists only for $\mathcal{V}$ sufficiently costly. Proposition 1 reduces existence of the first-best to the simple question: *Can we implement an equal split of the pie?*

3.2 Second-Best Mechanisms: The Designer's Problem

We now analyze the second-best mechanism. We impose a set of assumptions on the veto game $\mathcal{V}$ to facilitate the analysis. The set of functions V_i describes a reduced-form of $\mathcal{V}$, and enters the designer's problem as a primitive. The designer has no direct control over the players' decisions, given (ρ_i, ρ_{-i}). As we are going to see, a good knowledge of

[7] In the first-best benchmark this belief is off the equilibrium path. In principle nothing changes if a veto occurs on the equilibrium path, but that ρ^V is derived via Bayes' rule.

the game's equilibrium properties is crucial when designing the mechanism. Hence, we make our (technical) assumptions directly on the functional form of V_i. Let $conv_x f(t,x)$ be the largest function weakly smaller than $f(t,x)$, and convex in x.

Assumption 1. V_i exists for all $\mathcal{B}$ and satisfies the following.

(HC) Upper hemicontinuity. V_i is upper hemicontinuous in (ρ, ρ').

(S) Symmetry. $V_1(\theta, (\rho, \rho')) = V_2(\theta, (\rho', \rho))$ for any θ, (ρ, ρ').

(OST) Optimistic strongest type. $V_i(1, (p, \rho^V)) > 1/2$ for any ρ^V.

(CONV) Convex envelope. $V_i(\theta, (p, \rho^V)) = conv_p V_i(\theta, (p, \rho^V))$ for any θ, ρ^V.

Property (HC) implies that the designer's objective is continuous in her choices. (S) assumes a symmetric, anonymous equilibrium. (HC) guarantees existence of an optimum and (S) significantly reduces the notational burden. (OST) ensures that Proposition 1 does not apply. The property (CONV) (together with Assumption 2 made below) avoids tedious case distinctions by ensuring that no player vetoes the optimal mechanism. We extend our analysis relaxing (CONV) in Section 5.

If full settlement is not implementable and no player vetoes on the equilibrium path, second-best conflict management uses escalation as a screening device. Once the conflict escalates, the designer's influence ceases and $\mathcal{G}$ is played non-cooperatively. We apply the revelation principle of Myerson (1982) to account for the strategic interaction after escalation. Thus, it is without loss of generality to restrict the set of mechanisms to functions mapping type reports into (i) a probability that the conflict escalates, γ, (ii) a sharing rule under settlement, X, and (iii) an additional public signal, i.e., a random variable, Σ, with realization σ. That is, conflict management is a mapping

$$\mathcal{CM}(\cdot) = (\gamma(\cdot), X(\cdot), \Sigma(\cdot)) : \Theta^2 \to [0,1] \times [0,1]^2 \times \Delta(\mathscr{S}), \qquad (\mathcal{CM})$$

where $\Delta(\mathscr{S})$ is the set of probability distributions over an arbitrary countable set $\mathscr{S}$. Although public signals are necessary to invoke the revelation principle, the effect of the implicit signal sent via γ is sufficient to describe the economic intuition. Hence, to maintain simplicity we suppress signals notationally in the exposition whenever convenient. All formal results include the choice of the signaling function. We focus on public signals in the main part and extend our results to private signals in Section 5.[8]

Let θ_i's value from participating in the mechanism and reporting m_i be $\Pi_i(m_i, \theta_i)$. Further, let the probability that conflict management breaks down and the conflict escalates be $Pr(\mathcal{G})$. Then, the optimal mechanism with full participation solves

$$\begin{aligned} &\min_{(\gamma, X, \Sigma)} Pr(\mathcal{G}) \text{ s.t.} \\ &\Pi_i(\theta_i; \theta_i) \geq \Pi_i(m_i; \theta_i), \qquad \forall (m_i; \theta_i) \in \Theta^2,\ i \in \{1,2\} \\ &\Pi_i(\theta_i; \theta_i) \geq V_i(\theta_i, (p, \rho^V)), \qquad \forall \theta_i \in \Theta,\ i \in \{1,2\}. \end{aligned} \qquad (\mathrm{P}_{min})$$

[8]Conceptually, although we impose the restriction to public signals, our setting nests all game forms with observable actions including all forms of decentralized and bilateral negotiation.

The first set of constraints ensures incentive compatibility, the second full participation.[9] The value from participation, Π_i, depends on behavior in conflict management assuming optimal behavior in the continuation game. Next, we add more structure to Π_i using the equilibrium properties of the escalation game.

3.3 Incentives and Information

There is a conceptual difference between the information sets in games $\mathcal{V}$ and $\mathcal{G}$. Different to the veto game, a player knows her own report at the beginning of $\mathcal{G}$, and thus enters with a private history of past action. Hence, (equilibrium) beliefs are not the same across types. We describe the information structure using a $K \times 2$ matrix of probability distributions. An element of this matrix, $\beta_i(\cdot|\theta_i)$, is θ_i's *individual belief.* The individual belief describes θ_i's information about player $-i$'s type at the start of the (on-path) escalation game. The collection of all types' beliefs is a K-dimensional vector, $\boldsymbol{\beta}_i$, of probability distributions. The on-path information structure, $\mathcal{B} := (\boldsymbol{\beta}_1, \boldsymbol{\beta}_2)$, contains all possible on-path beliefs. We refer to $\mathcal{B}$ as a *belief system.* By the revelation principle, the escalation rule γ determines $\mathcal{B}$, and $\mathcal{B}$ is common knowledge.

At the beginning of the grand game the designer is uninformed about the players' type. Thus, γ conditions only on *reports.* Consequently, individual beliefs differ with their report. A player's private information at the beginning of the escalation game consists both of her *true payoff type* and her *reported type.* She expects no more than

$$U_i(m_i; \theta_i, \mathcal{B}) := \sup_{a_i \in A_i^{\mathcal{G}}} \sum_{\theta_{-i}} \beta_i(\theta_{-i}|m_i) \int_{A_{-i}^{\mathcal{G}}} u(\theta_i, \theta_{-i}, a_i, a_{-i}) \mathrm{d}G(a_{-i}|\theta_{-i}, \mathcal{B}), \tag{U}$$

where $G(a_{-i}|\theta_{-i}, \mathcal{B})$ is θ_{-i}'s distribution over equilibrium actions.[10] We impose simplifying assumptions on $\mathcal{G}$ for notational convenience.

Assumption 2. U_i exists and satisfies (HC) and (S) for all $\mathcal{B}$. In addition, for any information sets such that both U_i and V_i are defined, $U_i \geq V_i$.[11]

The last part of Assumption 2 implies that players are not *exogenously* punished for attending conflict management. Together with Assumption 1 it is sufficient for the revelation principle to hold. That is, an optimum with full participation always exists.

The continuation payoff (U) illustrates the main features of our model. First, the individual belief *depends on the type report only.* Second, the belief about each type's action depends on the entire on-path belief system $\mathcal{B}$. On the equilibrium path each player expects her opponent to report truthfully and to follow an equilibrium strategy in the escalation game. Third, the designer influences the continuation game only through

[9] Assumption 2 below ensures that the optimal mechanism indeed involves full participation.

[10] We assume that an equilibrium exists in any game so that whenever $m_i = \theta_i$ a maximum exists. However, a maximum may not exist if $m_i \neq \theta_i$ which is why we choose the more flexible supremum.

[11] Formally, define the projection $I^{\mathcal{G}}(I^{\mathcal{V}}) := \{(m_i; \theta_i, \mathcal{B}) | m_i{=}\theta_i,\ \mathcal{B}{=}(\rho_i, \rho_{-i})_K, \text{ and } (\theta_i, (\rho_i, \rho_{-i})) = I^{\mathcal{V}}\}$, with $(\rho_i, \rho_{-i})_K$ a $K \times 2$-matrix such that each row equals to (ρ_i, ρ_{-i}). Then, $U_i(I^{\mathcal{G}}(I^{\mathcal{V}})) \geq V_i(I^{\mathcal{V}})$.

$\mathcal{B}$ which itself is determined by the escalation rule. All other elements in (U) are beyond the designer's control, in particular the functional form of U_i. Therefore we treat U_i as a primitive to the problem.

Finally, (U) highlights the main difference between veto and escalation. Players cannot always replicate escalation in the veto game even if $V_i = U_i$. Any information set post-veto only depends on the vetoing player's *identity*. The escalation rule γ can implement a richer set. In particular, it induces individual beliefs that depend on *identity and report*. Thus, different types may have different individual beliefs. For the special case that beliefs are constant in m_i, the continuation payoff U_i is constant in m_i, too.

To address the incentive problem, let us consider the expected payoff from participating in conflict management. For now, assume that the mechanism releases no information beyond the escalation decision, that is, no additional public signal is sent. Then, the expected payoff from participation is

$$\Pi_i(m_i;\theta_i) = \underbrace{\sum_{\theta_{-i}} p(\theta_{-i})(1-\gamma_i(m_i,\theta_{-i}))x_i(m_i,\theta_{-i})}_{=:z_i(m_i)(\text{settlement value})} + \underbrace{\gamma_i(m_i)U_i(m_i;\theta_i,\mathcal{B})}_{=:y_i(m_i;\theta_i)(\text{escalation value})}, \tag{1}$$

where $\gamma_i(m_i) := \sum_{\theta_{-i}} p(\theta_{-i})\gamma_i(m_i,\theta_{-i})$ is the probability of escalation when reporting m_i. Conceptually, the expected payoff can be split in two parts. The settlement value, z_i, depending on a player's report only, and the escalation value, $y_i(m_i;\theta_i)$, which is a function of the report and the payoff type of the player.

To illustrate the novelty in our model, consider two adjacent types θ_i, $\theta_i{+}1$ both reporting θ_i during conflict management. The difference in their expected payoffs is

$$\Pi_i(\theta_i;\theta_i) - \Pi_i(\theta_i;\theta_i{+}1) = \gamma_i(\theta_i)\big(\underbrace{U_i(\theta_i;\theta_i,\mathcal{B}) - U_i(\theta_i;\theta_i{+}1,\mathcal{B})}_{=:D_i(\theta_i;\theta_i,\mathcal{B})}\big).$$

We refer to the difference $D_i(\theta_i;\theta_i,\mathcal{B})$ as player θ_i's *ability premium*. It is the difference between player i's on-path continuation payoff and that of the next strongest player mimicking θ_i. It measures how much of that payoff is due to θ_i's payoff type. Different to the existing literature, a change in $\mathcal{B}$ affects the ability premium non-linearly. Misreporting alters both the beliefs and as a consequence the deviator's action. Moreover, misreporting always provides an informational advantage to the deviator. Deviators remain undetected by definition and operate under superior knowledge. Their action is a best response to the opponent's on-path strategy. The reverse, however, does not need to hold as the opponent expects equilibrium play.

The expected payoff from conflict management, (1), also illustrates the main difficulty of the problem. Any choice of γ has a non-linear effect on U_i via $\mathcal{B}$. Standard methods are thus not directly applicable and we proceed with a change of variable.

3.4 Belief Management

In this part we show that the choice of the optimal post-escalation belief system is sufficient to determine the optimal mechanism. Thus, we reduce the problems' dimensionality and transform the conflict-management problem into a problem of managing beliefs in the escalation game. The transformation not only reduces complexity, but also separates the mechanism-design part of eliciting information from the information-design part of distributing that information. We later use the belief-management approach to characterize the optimal mechanism directly via the properties of the escalation game $\mathcal{G}$. Belief management identifies the channel through which information in the escalation game enters the considerations of the designer.

Two steps yield that result. First, we determine the reduced form of a mechanism (Border, 2007). This step allows us to work with expected shares, rather than ex-post shares. Second, we construct a one-to-one mapping between an information structure and a candidate for the optimal reduced-form mechanism.

Definition 1 (Reduced-Form Mechanism)**.** A tuple (z, γ) is the *reduced-form mechanism* of a mechanism (γ, X) if each element of $z_i \in z$ takes the functional form $z_i(m_i) = \sum_{\theta_{-i}} p(\theta_{-i})(1 - \gamma_i(m_i, \theta_{-i}))x_i(m_i, \theta_{-i})$.

We introduce two more concepts before stating our result. First, we define the set of *consistent belief systems* which describes all belief systems the designer can implement given the prior. Consistency allows for $K + 1$ independent belief distributions.[12]

Definition 2 (Consistency)**.** The set of *consistent belief systems*, $\{\mathcal{B}\}_p$, contains all $\mathcal{B}$ for which an escalation rule γ exist such that $\mathcal{B}$ follows from the prior and Bayes' rule.

Second, we extend consistency to settings with an additional public signal, Σ.

Definition 3 (Random Consistent Belief System)**.** A random variable, $\mathcal{B}(\Sigma)$, is a random consistent belief system, if it maps any realization of the public signal Σ into a consistent belief system.

The random variable $\mathcal{B}(\Sigma)$ links the realization σ directly to a consistent belief system. Thus, if σ is induced with $Pr(\sigma)$ via Σ, so is the corresponding consistent belief system. Any (stochastic) mechanism $\mathcal{CM}$ trivially induces a random consistent belief system. Our first theorem shows a similar statement holds in reverse. Any random consistent belief system, $\mathcal{B}(\Sigma)$, determines at most one candidate for the optimal (reduced-form) mechanism.

Theorem 1. *Under Assumption 1 and 2 the set of feasible random belief systems $\mathcal{B}(\Sigma)$ is well-defined. Moreover, for any feasible $\mathcal{B}(\Sigma)$ the optimal reduced-form mechanism, (z, γ), is unique.*

[12] Consistency corresponds to Bayes' plausibility in a Bayesian persuasion setting. We provide an in-depth discussion and a constructive characterization in the supplementary material, appendix G.

The proof of Theorem 1 is constructive. It characterizes the function $CM : \mathcal{B}(\Sigma) \mapsto (z, \gamma)$ that identifies a *unique candidate* for any consistent belief system. The intuition behind the construction does not rely on the choice of the public signal, Σ. We suppress it notationally in the main text. We organize our discussion using a set of observations that correspond to the steps in the formal proof.

Observation 1. Belief system, $\mathcal{B}$, and continuation payoff, U_i, are homogeneous of degree 0 with respect to the escalation rule. Escalation value, y_i, and escalation probability, γ_i, are homogeneous of degree 1 with respect to the escalation rule.

Suppose player i submits a report m_i and learns that the conflict escalates. Then, the probability that she faces type $\tilde{\theta}_{-i}$ is $p(\tilde{\theta}_{-i})\gamma(m_i, \tilde{\theta}_{-i}) / \sum_{\theta_{-i}} p(\theta_{-i})\gamma(m_i, \theta_{-i})$ which is determined by the *relative* likelihood of escalation among the different possible *report profiles*. Thus, if γ implements $\mathcal{B}$ so does $\alpha\gamma$. The externality that γ imposes on the continuation payoff in $\mathcal{G}$ is entirely expressed by the belief system that γ induces. Thus, U_i is invariant to any scaling of the escalation rule. Finally, the probability of reaching escalation and hence the escalation value depend linearly on the escalation rule.

Observation 2. The *worst escalation rule* implementing a given belief system is unique.

Take any escalation rule that implements $\mathcal{B}$ and pick the largest scalar $\overline{\alpha}$ such that $\overline{\alpha}\gamma(\theta_1, \theta_2) \leq 1$ for all (θ_1, θ_2). Then, the rule $\overline{g}_{\mathcal{B}} := \overline{\alpha}\gamma$ maximizes escalation and is thus the worst that implements $\mathcal{B}$. Identifying the worst escalation rule is sufficient to characterize *all escalation rules* implementing $\mathcal{B}$. The set of all γ implementing $\mathcal{B}$ is $\{\alpha\overline{g}_{\mathcal{B}} : \alpha \in (0, 1]\}$. Given $\mathcal{B}$, the problem reduces to finding the *lowest* α compatible with an incentive compatible mechanism $\mathcal{CM}$ satisfying participation constraints.

Observation 3. For any type θ_i with positive settlement value it is without loss to assume that either an incentive constraint, or the participation constraint binds in the second-best mechanism. Moreover, given $\mathcal{B}$, constraints are linear in α and z.

The first part of the observation follows since no first-best mechanism exists. Full settlement fails because the designer is budget constraint under settlement. The second part is immediate when combining a player's expected payoff from a mechanism, expression (1), with Observation 2. Observation 3 implies that $\mathcal{B}$ captures the entire non-linear part of the constraint. Furthermore, given $\mathcal{B}$ the set of constraints consists of $2K$ linear equations with $2K + 1$ unknowns, the $2K$ settlement values and the scalar α. To close the problem we thus need one more equation. We use the resource constraint of the designer at the level of expected settlement payments,

$$\sum_i \sum_{\theta_i} p(\theta_i) z_i(\theta_i) \leq 1 - Pr(\mathcal{G}). \tag{2}$$

Observation 4. A reduced-form mechanism is feasible only if it satisfies (2).

The resource constraint, (2), is an immediate consequence of the designers budget constraint. The designer can only allocate the pie if there is settlement. Thus, the total share allocated cannot be greater than the probability of settlement.

By Observation 3 z_i is linear in α, and thus $\sum_i \sum_{\theta_i} p(\theta_i)z_i(\theta_i)$ is linear, too. By Observation 1 the same holds for $1 - Pr(\mathcal{G})$. Since Proposition 1 does not apply, the resource constraint binds at the optimum and is our final equation. For any consistent $\mathcal{B}$ there is a unique tuple (z^*, α^*) satisfying all binding constraints with equality.

Via Observation 1 and 2, a feasible escalation rule implementing $\mathcal{B}$ exists if and only if the corresponding $\alpha^* \leq 1$. We partition the set of consistent belief systems into two subsets, $\{\mathcal{B}\}_\emptyset := \{\mathcal{B}|$ no feasible (z, γ) exists$\}$, and the remainder $\{\mathcal{B}\}_p \setminus \{\mathcal{B}\}_\emptyset$.

Finally, we construct a function $CM : \mathcal{B} \mapsto (z, \gamma)$ that identifies a *unique candidate* (z, γ) for any $\mathcal{B} \notin \{\mathcal{B}\}_\emptyset$ and points to the origin otherwise. The function CM is continuous in the interior of the support and given by

$$CM(\mathcal{B}) := \begin{cases} (z^*, \alpha^* \overline{g}_{\mathcal{B}}) & \text{if } \mathcal{B} \notin \{\mathcal{B}\}_\emptyset \\ 0 & \text{if } \mathcal{B} \in \{\mathcal{B}\}_\emptyset. \end{cases}$$

Discussion of Theorem 1. The conflict-management problem (P_{min}) contains both a mechanism-design part and an information-design part. The designer decides on the game form and acts as a mechanism designer. Yet if she invokes escalation, her actions are restricted to the distribution of information. Thus, she acts as an information designer in the escalation game. However, prior to distribution she has to elicit the information by designing the settlement game. The (promised) information distribution, in turn, influences the cost of information elicitation.

As main implication, Theorem 1 separates elicitation and distribution of information. For any belief system, the function CM determines whether that belief system is affordable. Moreover, CM determines the escalation rule that minimizes the price (in terms of lost settlement) to elicit the necessary information. Once the price of information is determined, the mechanism-design problem reduces to an information-design problem asking: *What is the optimal post-escalation information structure?*

3.5 A Dual Problem to Optimal Conflict Management

What remains is to determine (i) when a reduced-form mechanism is implementable and (ii) how to find the optimal belief system. To address the first issue, we borrow the general implementation condition from Border (2007). The second issue depends, in general, on the details of the escalation game. Our characterization is constructive and focuses on what we call a *regular* environment. We address the two issues in turns.

First, we translate the results of Border (2007) to our setup and state necessary and sufficient conditions to implement a reduced-form mechanism (z, γ) via some $\mathcal{CM}$. Let $Q \subset \Theta^2$ be any subset of the type space and define $Q_i := \{\theta_i | \exists \theta_{-i} : (\theta_i, \theta_{-i}) \in Q\}$ and $\tilde{Q} := \{(\theta_1, \theta_2) \in \Theta^2 | \theta_i \notin Q_i \text{ for } i = 1, 2\}$.

Proposition 2 (Border (2007))**.** *Take any reduced-form mechanism* (z, γ)*. An ex-post feasible* X_i *that implements* z *exists if and only if*

$$\sum_i \sum_{\theta_i \in Q_i} z_i(\theta_i)p(\theta_i) \leq 1 - Pr(\mathcal{G}) - \sum_{(\theta_1,\theta_2)\in\bar{Q}} (1-\gamma(\theta_1,\theta_2))p(\theta_1)p(\theta_2), \quad \forall Q \subseteq \Theta^2. \quad \text{(GI)}$$

Proposition 2 imposes a set of additional constraints. Following Border (2007) we call this set the general implementation condition, (GI).

Next, we define a regular environment in our setting. We state sufficient conditions that guarantee that the following constraints bind at the optimum: (i) the strongest types' participation constraints, and (ii) local downward incentive constraints.

The first requirement is related to the veto game. Regularity implies an environment in which the strongest type is sufficiently privileged in the veto game, for example, because she occurs sufficiently rare. Then, a strong type expects to face a weak opponent. She is optimistic about her prospects outside conflict management. The mechanism has to compensate the strongest type, making her participation constraint binding. The second requirement is related to the escalation game. Roughly put, it requires that the weaker a player is, the more important is her fundamental type θ_i. Recall that the ability premium, $D_i(m;\theta_i,\mathcal{B}) = U_i(m;\theta_i,\mathcal{B}) - U_i(m;\theta_i{+}1,\mathcal{B})$, is the difference in the continuation payoff of two adjacent types θ_i and $\theta_i{+}1$ reporting the same type m.

Definition 4 (MDR)**.** The game $\mathcal{G}$ satisfies the monotone difference ratio condition **(MDR)** if $D_i(m;\theta_i,\mathcal{B})/D_i(m+1;\theta_i,\mathcal{B})$ is non-decreasing in θ_i.

Definition 5 (Regularity)**.** Let $\underline{\rho}^V := \arg\min_{\rho^V} V_i(1,(p,\rho^V))$. In a regular environment

(i) $2 \sum_{\theta_i\in\hat{Q}} p(\theta_i)V_i(\theta_i,(p,\underline{\rho}^V)) < \sum_{\theta_i\in\hat{Q}} p(\theta_i)$, for any $\hat{Q} \subseteq \Theta$ and $\hat{Q} \neq \{1\}$, and

(ii) $\mathcal{G}$ satisfies (MDR).

Assumption 3 (Regularity)**.** The environment is regular.

Regularity, in particular (MDR), imposes strong restrictions on the nature of the escalation game, but allows us to focus on the economics when setting up the dual problem. Main insights carry over to a more general environment. In Section 5 we discuss the irregular case and its relation to the characterization provided in this section.

Lemma 1. *Suppose Assumption 1 to 3 hold. At the optimum local downward incentive constraints bind for any type* $\theta_i > 1$*. In addition, for each player the only binding participation constraint is that of type 1.*

Combining Theorem 1 with Lemma 1 leaves the designer with a two-fold objective. First, she wants to provide a belief system that maximizes the aggregate surplus in the continuation game. The larger the aggregate surplus in the continuation game the larger the escalation values. The expected payoff from participation is the sum of both the escalation value and the settlement value. Therefore, a larger settlement value relaxes the participation constraint. Second, the designer provides a belief system that allows her to *discriminate* between types. The ability premium, D_i, measures the discriminatory power of a belief system at the beginning of $\mathcal{G}$. Thus, the designer has an incentive to choose a belief system that leads to a relatively high ability premium. A high ability premium reduces the information rent the designer has to pay to weaker types. A smaller information rent, in turn, reduces the need to screen via escalation.

Definition 6 (Virtual Rent)**.** Player θ_i's virtual rent is $\Psi_i(\theta_i, \mathcal{B}) := w(\theta_i) D_i(\theta_i; \theta_i, \mathcal{B})$, with $w(\theta_i) := \Big(1 - \sum_{k=1}^{\theta_i} p(k)\Big) / \Big(p(\theta_i)\Big)$.

We now derive a dual problem to (P_{min}) for the case that additional signals are superfluous. The set C contains all constraints in problem (P_{min}). The set of unambiguously binding constraints, $C_R \subset C$, are those defined in Lemma 1. These constraints are incorporated in the objective of the dual. We define $C_F := (C \setminus C_R) \cup (CM(\mathcal{B}) \neq 0)$, where the latter restricts the set of belief systems to those implementable by a feasible reduced-form mechanism. These constraints are outside the dual's objective. The conditional distribution of a player's type after escalation, ρ, is the solution to the following system of linear equations: $\rho_i(\theta_i) = \sum_{\theta_{-i}} \beta_{-i}(\theta_i|\theta_{-i}) \rho_{-i}(\theta_i)\ \forall \theta_i$.

The dual objective is the simple sum of an ex-ante expected measures of discrimination, $\mathbb{E}[\Psi_i|\mathcal{B}] = \sum_{\theta_i} \rho_i(\theta_i)\Psi_i(\theta_i, \mathcal{B})$, and one of welfare, $\mathbb{E}[U_i|\mathcal{B}] = \sum_{\theta_i} \rho_i(\theta_i) U_i(\theta_i; \theta_i, \mathcal{B})$,

$$\max_{\mathcal{B} \in \{\mathcal{B}\}_P} \sum_i \mathbb{E}[\Psi_i|\mathcal{B}] + \mathbb{E}[U_i|\mathcal{B}] \quad s.t.\ C_F. \tag{$\mathrm{P}^{\mathcal{B}}_{max}$}$$

Proposition 3 (Duality)**.** *Suppose Assumption 1 to 3 hold and fix the set of signal realizations to a singleton. A mechanism solves* (P_{min}) *if and only if its reduced form,* $(z, \gamma) = CM(\mathcal{B}^*)$*, and* $\mathcal{B}^*$ *solves* ($\mathrm{P}^{\mathcal{B}}_{max}$)*.*

Proposition 3 follows from rearranging the first-order conditions and Theorem 1. We summarize the argument assuming $\gamma(1,1) \geq \gamma(k,n)$, that is, the two strongest types are the most likely to escalate. The escalation rule at the optimum is $\gamma(1,1)\overline{g}_{\mathcal{B}}$, where $\overline{g}_{\mathcal{B}}$ is the worst escalation rule for $\mathcal{B}$. Using Lemma 1 we rewrite the settlement values

$$z_i(\theta_i) = V_i(1, (\rho, \underline{\rho}^V)) + \sum_{k=2}^{\theta_i} y_i(k{-}1; k) - \sum_{k=1}^{\theta_i} y_i(k; k).$$

At the optimum the probability of settlement corresponds to the probability weighted sum of settlement values. Thus, substituting for y_i, and forming expectations we get

$$\sum_i \sum_{\theta_i} p(\theta_i) z_i(\theta_i) = 2V_i(1, (\rho, \underline{\rho}^V)) + \gamma(1,1)\mathcal{Q}(\mathcal{B}) \overset{!}{=} 1 - Pr(\mathcal{G}) = 1 - \gamma(1,1) R(\mathcal{B}) \tag{3}$$

which implicitly determines $\mathcal{Q}(\mathcal{B})$ and $R(\mathcal{B})$. Solving equation (3) for $\gamma(1,1)$ and plugging into the objective yields

$$Pr(\mathcal{G}) = \Big(\sum_i V_i(1,(\rho,\underline{\rho}^V)) - 1\Big)\frac{R(\mathcal{B})}{\mathcal{Q}(\mathcal{B}) - R(\mathcal{B})} = \Big(\sum_i V_i(1,(\rho,\underline{\rho}^V)) - 1\Big)\frac{1}{\frac{\mathcal{Q}(\mathcal{B})}{R(\mathcal{B})} - 1}.$$

Any $\mathcal{B}$ that solves (P_{min}), solves $\sup \mathcal{Q}(\mathcal{B})/R(\mathcal{B})$. Multiply the latter by $\gamma(1,1)/\gamma(1,1)$ and substitute back for $\gamma(1,1)\mathcal{Q}(\mathcal{B})$. Bayes' rule implies $\gamma(1,1)R(\mathcal{B}) = Pr(\mathcal{G})$ and $\gamma_i(\theta_i)/Pr(\mathcal{G}) = \rho_i(\theta_i)/p(\theta_i)$. Finally, substituting for z_i and y_i yields ($\mathrm{P}^{\mathcal{B}}_{max}$).

We postpone further discussion of Proposition 3 and first state the general case including public signals. Concavification results from the Bayesian persuasion literature do not apply directly as they build on the assumption of type-independent beliefs. The belief system $\mathcal{B}$, to the contrary, specifies type-dependent beliefs. Instead we use a two-step procedure.[13] We define the mean of $\mathcal{B}(\Sigma)$ as $\overline{\mathcal{B}} := \sum_{\sigma\in\Sigma} Pr(\sigma)\mathcal{B}(\sigma)$.

Definition 7 (Admissible Means)**.** The mean, $\overline{\mathcal{B}}$, is in the set of admissible means, $\{\overline{\mathcal{B}}\}^a$ if $\mathcal{B}(\Sigma)$ satisfies the constraints C_F.

For any mean, the set of signal structures, Σ, such that $\mathcal{B}(\Sigma)$ satisfies C_F is $\mathcal{S}(\overline{\mathcal{B}})$. Given objective $\mathcal{O}$ the value of $\overline{\mathcal{B}} \in \{\overline{\mathcal{B}}\}^a$ is

$$\mathcal{W}(\overline{\mathcal{B}}, \mathcal{O}(\mathcal{B}(\sigma))) := \max_{\Sigma\in\mathcal{S}(\overline{\mathcal{B}})} \sum_{\sigma\in\Sigma} Pr(\sigma)\mathcal{O}(\mathcal{B}(\sigma)). \tag{4}$$

Using (4) we generalize ($\mathrm{P}^{\mathcal{B}}_{max}$) to,

$$\max_{\overline{\mathcal{B}}\in\{\overline{\mathcal{B}}\}^a} \mathcal{W}\left(\overline{\mathcal{B}}, \sum_i \Big(\mathbb{E}[\Psi_i|\mathcal{B}(\sigma)] + \mathbb{E}[U_i|\mathcal{B}(\sigma)]\Big)\right). \tag{$\mathrm{P}^{\mathcal{B}(\Sigma)}_{max}$}$$

Theorem 2 (Duality of problems)**.** *Suppose Assumption 1 to 3 hold. A mechanism solves* (P_{min}) *if and only if its reduced form* $(z,\gamma) = CM(\mathcal{B}^*(\Sigma))$ *and* $\mathcal{B}^*(\Sigma)$ *solves* ($\mathrm{P}^{\mathcal{B}(\Sigma)}_{max}$).

Every consistent belief system specifies a value of the objective. The value of the objective is no sufficient statistic for the optimal belief system because of additional binding constraints. Furthermore, we cannot solve the problem by choosing a signal structure such that $\{\mathcal{B}(\sigma)\}_\sigma$ spans the concave closure of the objective. The reason such a concavification approach fails is that $\{\mathcal{B}(\sigma)\}_\sigma$ spanning the concave closure may violate the binding constraints. Instead we introduce the lottery-means approach, which has no economic meaning per se. Mathematically, however, it exploits the linearity of equation (4) representing the maximization problem in a concise manner.

As a corollary to Theorem 2 we state a sufficient condition to when public signals are superfluous and Proposition 3 applies directly.

Corollary 1. *Consider the solution to* ($\mathrm{P}^{\mathcal{B}}_{max}$) *ignoring* C_F*. If this solution does not violate* C_F*, the optimal signal structure is degenerate.*

[13] In the supplementary material, appendix E, we use a Lagrangian approach and show that the optimal solution can be found by maximizing a concave closure of the supporting Lagrangian function.

The intuition for Corollary 1 is straightforward. Via Theorem 1 the most relevant constraints are included in the objective of ($\mathrm{P}^{\mathcal{B}}_{max}$). For a given information structure, signals might improve by linearizing convexities at the optimum (Aumann and Maschler, 1995). Convexities do not exist if the solution to ($\mathrm{P}^{\mathcal{B}}_{max}$) satisfies all remaining constraints by definition of a maximum. Thus, signals are superfluous. That is, instead of implementing a spread over information structures, the designer can implement the most preferred information structure with probability 1.

Discussion of Theorem 2. Theorem 2 provides a tractable dual to the conflict-management problem. The economic interpretation of its objective is intuitive. Mimicking behavior has two effects on the continuation game: The deviator (i) inherits the posterior distribution over the opponent's types from the mimicked type, and (ii) gains an informational advantage being the only one aware of entering an off-path game. The deviator is not forced to adopt the strategy of the mimicked type in the continuation game, and can freely adjust her strategy. Choices remain unresponded by the opponent who plays *as if* she were on the equilibrium path. The deviator's information advantage reduces discrimination after escalation providing an additional incentive to deviate.

The problem ($\mathrm{P}^{\mathcal{B}}_{max}$) has a direct analogue to that of an optimal auction (Myerson, 1981). The main difference is that – although types are ordered due to Assumption 2 and 3 – the term $\Psi(\theta_i, \mathcal{B}) + U_i(\theta_i; \theta_i, \mathcal{B})$ is non-linear in the designer's choice. Consequently and different to the auction-design problem, it is not possible to derive a simple characterization of the optimal mechanism without considering the details of $\mathcal{G}$.

Economically the dual problem provides important implications. First, to save on resources the designer reduces inefficiencies in the escalation game despite being agnostic about outcomes after escalation.

Second, combining Theorem 1 and 2 via ($\mathrm{P}^{\mathcal{B}(\Sigma)}_{max}$) we identify how details of the escalation game influence the optimal mechanism. More specifically, we define an information-design problem. Its intuitive objective identifies and quantifies the designer's motives.

Tractability is guaranteed even in the presence of public signals. Theorem 2 describes a "backward-induction" approach to that general problem. First, we relax incentive constraints via information design in the continuation game assuming an omniscient designer. Second, we solve the core mechanism-design problem that determines the price of information to the designer.

4 Application: Alternative Dispute Resolution

In this section we apply our general results to a simple model of alternative dispute resolution (ADR) in the shadow of formal litigation, a legal contest. We show that behavioral externalities are the main driving force for the optimal design of ADR. Further, we characterize ADR using the approach derived in Section 3. Optimal ADR has two main features. First, disputants cannot influence the information structure in litigation via their actions within ADR. Second, whenever the litigation game is triggered it is asymmetric in the sense that it is common knowledge that on of the disputants is stronger in expectations. As a a consequence of these features, optimal ADR has to maintain a risk of escalation to *any* possible type profile.

4.1 Institutional Background

Alternative Dispute Resolution (ADR) is a tool introduced into many countries' legal system to reduce inefficiencies by settling as many cases as possible outside the court. ADR itself can take many forms and describes a third-party mechanism other than formal litigation to solve the conflict. However, ADR typically cannot overturn the rule of law, such that parties return to the litigation track once ADR fails.

The court system of most developed countries is heavily overburdened. As an example, the average judge in a US district court in 2017 receives 639 newly filed cases per year. At the same time she has a stock of around 744 pending cases.[14] Most jurisdictions thus encourage disputants to engage in some form of ADR before starting the formal litigation process.

The U.S. Alternative Dispute Resolution Act of 1998 states that courts should provide disputants with ADR-options in all civil cases. ADR is defined as "any process or procedure, other than an adjudication by a presiding judge, in which a neutral third party participates to assist in the resolution of issues in controversy" (Alternative Dispute Resolution Act, 1998). However, ADR supplements the "rule of law" rather than replacing it. Ultimately, each disputant has the right to return to formal litigation.[15] Hence, ADR indeed happens "in the shadow of the court." When ADR does not settle the conflict, disputants return to the traditional litigation path.

ADR is very effective and has success rates substantially above 50% across time, jurisdictions, and case characteristics. Furthermore, disputants report that ADR has an impact on the continuation of the litigation process even when unsuccessful (Anderson and Pi, 2004; Genn, 1998). Informational spillovers to litigation influence the design of optimal ADR. If a disputant can influence the information she receives via the information she provides, there is an incentive to strategically provide information *within ADR* to manipulate behavior in *litigation* once ADR breaks down.

We follow a large literature dating back to Posner (1973), and model litigation as a legal contest. The party providing the most convincing evidence wins the case. In such a contest, the optimal amount of evidence the plaintiff provides is a function not only of her own cost of evidence provision, but also of her beliefs about the defendant's evidence choice and vice versa. Hence, continuation strategies after failed ADR are a function of the entire information structure.

[14]There were 677 judgeships in 2017, but also 1266 vacant judgeship months. To calculate the average judge's yearly incoming cases, we use the non-vacant judgeships months.

[15]For a detailed discussion on this, see Brown, Cervenak, and Fairman (1998).

4.2 A stylized model

Two ex-ante identical economic agents have a legal dispute. With abuse of notation we assume a disputant's type realizes independently as 1 ("strong") or $K > 2$ ("weak"). The ex-ante probability of being type 1 is p. If a disputant vetoes or ADR breaks down, a game of litigation is triggered. Formally, we assume $v \equiv u$ such that $\mathcal{V}$ and $\mathcal{G}$ are identical given an information structure. We choose p low enough for Assumption 1 to hold. We refer to the realization (θ_1, θ_2) as a *match* of types θ_1 and θ_2.

Litigation is a legal contest where disputants compete to provide evidence to a judge or jury. That is, disputant i chooses the quality level of the evidence she provides, $a_i \in [0, \infty)$. Whoever provides the highest quality of evidence wins the lawsuit. The marginal cost of increasing the quality of evidence is θ_i. Type θ_i's ex-post utility from evidence levels (a_i, a_{-i}) is

$$u(\theta_i, a_i, a_{-i}) = \begin{cases} 1 - \theta_i a_i & \text{if } a_i > a_{-i} \\ -\theta_i a_i & \text{if } a_i < a_{-i} \\ 1/2 - \theta_i a_i & \text{if } a_i = a_{-i}. \end{cases}$$

4.3 Optimal ADR

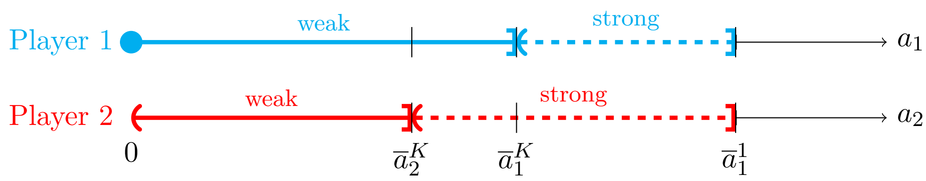
To simplify notation we use $b_i(m_i) := \beta_i(1|m_i)$ for i's belief about $-i$ conditional on escalation after the report m_i during ADR. We assume without loss of generality that $b_1(1) \geq b_2(1)$. We follow Siegel (2014) in characterizing equilibrium behavior conditional on escalation. Figure 1(a) sketches the equilibrium strategy support in the unique monotonic equilibrium. Types mix piecewise uniformly on disjoint intervals and at most one disputant has a mass point at 0. We use $F_i^{\theta_i}$ for θ_i's distribution over actions and $f_i^{\theta_i}$ for the density. Disputants influence the information structure via their report during conflict management because escalation probabilities, and thus updated beliefs, depend on these reports. Disputant i's winning probability at quality level a_i is a function of her previous report and *her opponent's equilibrium strategy.* We denote this winning probability as the conditional distribution

$$F_{-i}(a_i|m_i) = b_i(m_i)F_{-i}^1(a_i) + (1 - b_i(m_i))F_{-i}^K(a_i).$$

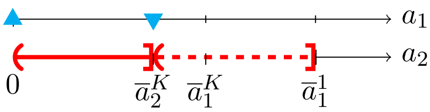
Quality $a_i^*(m_i)$ after report m_i therefore implies a continuation payoff

$$U_i(m_i; \theta_i, \mathcal{B}) = F_{-i}(a_i^*(m_i)|m_i) - \theta_i a_i^*(m_i).$$

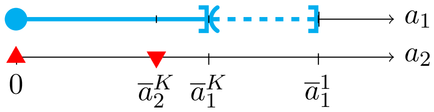
The common upper bound on optimal evidence provision implies identical continuation payoffs for strong types. Applying the properties of best responses from figure 1(a)



(a) Equilibrium effort support in the on-path continuation game. *All types (partially) mix. Solid lines denote the intervals of $\theta_i{=}K$, dashed lines those of $\theta_i{=}1$. The equilibrium effort distribution is non-atomistic, apart from the mass point at* 0 *for disputant* 1*, type* K*.*



(b) Continuation effort after disputant 1, K deviates. *The non-deviator follows her equilibrium strategy. The deviator chooses action* ▲ *if* $b_2(1) > b_2(K)$ *and* ▼ *if* $b_2(1) < b_2(K)$*.*



(c) Continuation effort after disputant 2, K deviates. *The non-deviator follows her equilibrium strategy. The deviator chooses action* ▲ *if* $b_1(1) > b_1(K)$ *and* ▼ *if* $b_1(1) < b_1(K)$*.*

Figure 1: Continuation strategies for different histories if $b_1(1) \geq b_2(1) \neq b_2(K)$*.*

we obtain

$$\begin{aligned}
U_i(1;1,\mathcal{B}) &= F_1(\overline{a}_2^K|1) - \overline{a}_2^K \\
U_1(K;K,\mathcal{B}) &= 0 \\
U_2(K;K,\mathcal{B}) &= F_1(0|K) \\
U_1(1;K,\mathcal{B}) &= 0 + \mathbb{1}_{\leq}\left(F_2(\overline{a}_2^K|1) - K\overline{a}_2^K\right) \\
U_2(1;K,\mathcal{B}) &= F_1(0|1) + \mathbb{1}_{\leq}(F_1(\overline{a}_2^K|1) - K\overline{a}_2^K - F_1(0|1)),
\end{aligned} \tag{5}$$

with $\mathbb{1}_{\leq}$ an indicator function with value 1 if $b_1(1) \leq b_1(K)$ and 0 otherwise.

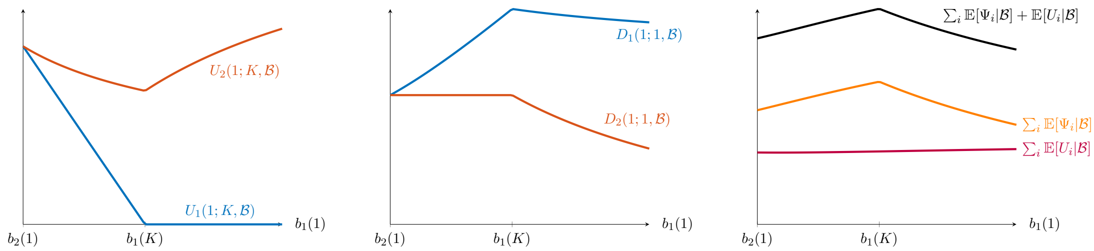
Recall that disputant i's incentive constraint implies that for any $\hat{\theta}$

$$z_i(\theta_i) + \gamma_i(\theta_i)U_i(\theta_i;\theta_i,\mathcal{B}) \geq z_i(\hat{\theta}) + \gamma_i(\hat{\theta})U_i(\hat{\theta};\theta_i,\mathcal{B}),$$

with $z_i(m) = p(1-\gamma_i(m,1))x_i(m,1) + (1-p)(1-\gamma_i(m,K))x_i(m,K)$.

Solving for optimal conflict management by a standard first-order approach is difficult since changes in γ also influence z, and lead to non-linear changes in $\mathcal{B}$, which in turn lead to non-linear changes in U_i. Thus, a careful consideration of U_i is inevitable. The belief-management approach simplifies the problem by directing attention exclusively to the well-defined function U_i. We provide a detailed description of all relevant terms in the appendix. Here we focus on the conceptual intuition.

Consistency implies $\mathrm{sgn}(b_1(1){-}b_1(K)) = \mathrm{sgn}(b_2(1){-}b_2(K))$. Moreover, whenever $b_1(1) \neq b_1(K)$ a deviation during ADR implies an informational advantage. If $b_1(1) \neq b_1(K)$, a deviator correctly estimates her opponent's type distribution and best-responds using $F_{-i}(\cdot|m_i)$. To the contrary, the non-deviating opponent is unaware of the deviation



(a) Deviation payoff. (b) Ability premium. (c) Objective Function.

Figure 2: Deviation in relation to $b_1(1)$. *Beliefs* $b_2(1), b_1(K)$ *are fixed,* $b_2(K)$ *adjusts endogenously ensuring consistency of* $\mathcal{B}$.

and uses her on-path estimate $F_i(\cdot|\theta_{-i})$. Her action is thus a best-response to faulty beliefs. In the following we discuss type K's behavior after a deviation case by case.

If $b_1(1) < b_1(K)$, type-K deviator expects a weaker opponent than on the equilibrium path. She reacts by putting full mass on $\overline{a}_2^K$. If $b_1(1) > b_1(K)$, she expects a tougher opponent. She reacts by putting full mass (close) to 0.[16] In the first case, she shifts mass upwards to obtain a higher probability of winning. In the second case, she shifts mass downwards to cut losses.

If, however, $b_1(1) = b_1(K)$ a deviation during ADR leads to *no informational advantage* since $F_{-i}(\cdot|1) = F_{-i}(\cdot|K)$. A deviator faces the same information structure as an on-path disputant and remains indifferent between any quality choice on the interval $(0, \overline{a}_2^K]$. The terms after the indicator $\mathbb{1}_{\leq}$ collapse to 0, and $U_i(1; K, \mathcal{B}) = U_i(K; K, \mathcal{B})$.

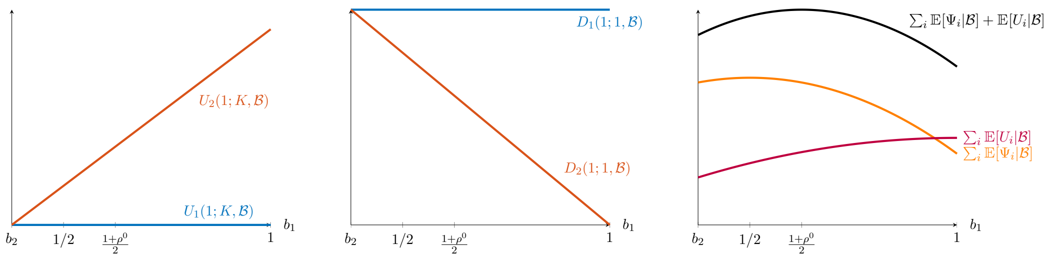
We graph optimal post-deviation actions in figures 1(b) and 1(c) with triangles. Figure 2(a) graphs the corresponding deviation payoffs as a function of $b_1(1)$. Deviation payoffs are minimized if $b_1(1) = b_1(K)$, the case that eliminates a deviator's information advantage.

Obviously, the information advantage influences the ability premium. The ability premium has a maximum if the informational advantage is minimized,

$$\begin{aligned} D_1(1;1,\mathcal{B}) &= \begin{cases} F_1(\overline{a}_2^K|1) - \overline{a}_2^K & \text{if } b_1(1) > b_1(K) \\ F_1(\overline{a}_2^K|1) - F_2(\overline{a}_2^K|1) + (K-1)\overline{a}_2^K & \text{if } b_1(1) \leq b_1(K) \end{cases} \\ D_2(1;1,\mathcal{B}) &= \begin{cases} F_1(\overline{a}_2^K|1) - F_1(0|1) - \overline{a}_2^K & \text{if } b_1(1) > b_1(K) \\ (K-1)\overline{a}_2^K & \text{if } b_1(1) \leq b_1(K). \end{cases} \end{aligned} \tag{6}$$

Figure 2(b) depicts the ability premium as a function of $b_1(1)$. If a deviator expects to face a weaker opponent, i.e., $b_1(1) < b_1(K)$, an increase in $b_1(1)$ has a (weakly) positive effect on the ability premium. It reduces the size of the deviator's information advantage. If, however, a deviator expects a stronger opponent, the opposite is true. If

[16] Recall that continuation payoffs have a tight upper bound in the supremum defined in equation (U), Section 3.3 which is obtained as a_i goes to 0.



(a) Deviation payoff. (b) Ability premium. (c) Objective Function.

Figure 3: Effect of asymmetry. *Increasing b_1 from $b_1 = b_2$ while holding beliefs type independent.*

$b_1(1)$ increases, then the deviator's information advantage increases. Moreover, if $b_1(1)$ increases, strong types expect tougher competition also on the equilibrium path. Thus, the ability premium shrinks.

The effect prevails in the expected virtual rent, $\sum_i \mathbb{E}[\Psi_i|\mathcal{B}]$, graphed in figure 2(c). The expected continuation welfare, $\sum_i \mathbb{E}(U_i|\mathcal{B})$, changes only marginally in $b_1(1)$. Hence, the optimal mechanism eliminates the information advantage for deviators. Each disputants information set is independent of her behavior within ADR.

Lemma 2. *Optimal alternative dispute resolution induces type independent beliefs in the escalation game, that is, $b_i(1) = b_i(K)$.*

Applying Lemma 2 we define $b_i := b_i(1) = b_i(K)$. The design problem reduces to the question: *should the escalation game be symmetric, $b_1 = b_2$, or asymmetric, $b_1 > b_2$?*

The effect of asymmetry on the expected virtual rent is ambiguous. Symmetry guarantees symmetric strategies which imply symmetric payoffs. That is, in the symmetric case we have symmetric intervals $\overline{a}_2^K = \overline{a}_1^K$, and no mass point, $F_1^K(0) = 0$. Both weak disputants have a continuation payoff after escalation of 0 regardless of their report. The effect of asymmetric beliefs is sketched in figure 3. Starting at $b_1 = b_2$ and gradually increasing b_1 has no effect on the continuation payoff of a weak disputant 1, but increases the continuation payoff of weak disputant 2.

The larger the asymmetry, the more likely the event that disputant 1 provides no evidence and thus the higher the expected payoff of a type-K disputant 2. Only b_2 determines the strong types' payoff which remains constant in b_1. Consequently, D_1 remains constant while D_2 decreases in b_1. Finally, type-independence implies that type distributions conditional on escalation coincide with beliefs that condition on escalation *and report*. That is, $b_1 = \rho_2(1)$ and $b_2 = \rho_1(1)$, and increasing b_1 puts higher weight on D_2.

The effect of asymmetry on on-path payoffs in contests is well-known. We graph it in Figure 3(c). The more asymmetric a contest, the lower the evidence level in equilibrium. Combining both effects yields a hump-shaped objective. Maximizing this objective then determines the degree of asymmetry.

Proposition 4. *Optimal alternative dispute resolution induces an on-path belief system in the escalation game with beliefs that are*

- *type independent,* $b_i = b_i(1) = b_i(K)$*, and*
- *have full support,* $b_i \in (0, 1)$*.*

Moreover, beliefs are always asymmetric*,* $b_1 \neq b_2$*. Ignoring public signals is without loss of generality if and only if* $p \leq 1/3$*.*

Ignoring C_F, we obtain an optimum at $\widehat{\mathcal{B}}$ described by $b_1^* = 1/2 + p/2$ and $b_2^* = 1/2 - p/2$. If the prior $p \in [\underline{r}, 1/3]$ with $\underline{r} := (2(K-1) - \sqrt{8 - 4K + K^2})/(2 + 3K)$ then that optimum satisfies C_F. As a consequence, Corollary 1 applies and public signals do not improve. If $p < \underline{r}$, then the reduced form mechanism implementing $\widehat{\mathcal{B}}$ is not feasible and we have to adjust the belief system accordingly. While a closed-form solution may fail to exist, the qualitative statements of Proposition 4 prevail and signals do not improve.

If, however, $p > 1/3$, $\widehat{\mathcal{B}}$ violates the incentive constraint for a strong disputant 1: By regularity and Lemma 2, U_i is independent of a previous report and incentive constraints of weak types are binding. Moreover weak types of disputant 1 expect no continuation payoff in case of escalation. Thus, settlement values are the same across types and a strong type of disputant 1 picks the report that guarantees her the largest likelihood of escalation. If, however, $p > 1/3$ information structure $\widehat{\mathcal{B}}$ implies a higher likelihood of escalation for type K than for type 1, violating type 1's incentive constraint.

However, the same does not hold for disputant 2. Weak disputant-2 types gain positive continuation payoffs under escalation and the strong type's incentive constraint holds with strict inequality. We enhance ADR by a coin-flip that determines who takes the role of disputant 1. This coin-flip is carried out after the reports but before the outcome of ADR realizes. Together with the realization the mechanism announces the result of the coin-flip. Then, incentive constraints for strong types are always satisfied. Formally, optimal ADR is a stochastic mechanism implementing with probability 1/2 $\widehat{\mathcal{B}}$, and otherwise its mirror image $\breve{\mathcal{B}} = \{\hat{\beta}_2, \hat{\beta}_1\}$.[17]

4.4 Interpretation

Proposition 4 emphasizes the importance of the informational spillover of ADR on disputant's behavior in litigation. If information is dependent on previous behavior, litigants have an incentive to misreport the quality of their case as this leads to a correct belief of the deviator while the non-deviator's beliefs are faulty. The reason is precisely the behavioral externality ADR imposes on litigation. Once ADR fails disputants reason about the cause of escalation and adapt continuation strategies accordingly. Only by

[17]Obviously, such a *symmetrizing* signal is always feasible and ensures ex-ante symmetry. However, only in case $p > 1/3$ a signal is *necessary* at the optimum. In any case the realized escalation game is played under asymmetric beliefs.

making that reasoning independent of a disputants previous behavior ensures that they do not try to manipulate the information structure.

That first result resembles the intuition from a second-price auction. There, to ensure incentive compatibility, the expected payment conditional on winning is independent of a bidder's type report. Similarly here, the expected continuation payoff conditional on escalation is independent of a disputant's type report.

Our second main result also relies on the fact that ADR imposes an externality on behavior in litigation. By guaranteeing to treat the asymmetric matches $(1, K)$ and $(K, 1)$ differently, ADR guarantees that disputants learn sufficiently through the process of ADR. That is, escalation is informative in that it offers disputants' a signal who has the better case. That way, it resolves some of the uncertainty and reduces aggregate evidence provision. Thus, going through the ADR process allows parties to reduce expected legal expenditure which in turn provides an incentive to participate in ADR even when preferences for a settlement solution are weak.

That second result follows from the observation that the outcome of litigation depends on the information structure. The second-best litigation outcome cannot be implemented in an ex-post equilibrium. Consequently, expected (aggregate) welfare varies non-linearly with the information structure.

4.5 Relationship to Existing Models

There are two major difference in our litigation model compared to the literature (e.g. Bester and Wärneryd, 2006; Fey and Ramsay, 2011; Hörner, Morelli, and Squintani, 2015). First, these papers abstract from action choices *after* escalation. Second, they assume a lump-sum destruction of surplus once the conflict escalates.

The first abstraction is without loss of generality only in two cases. Either conflicting parties are incapable of adjusting equilibrium strategies after they learn about the outcome of ADR, or the escalation game results in an ex-post equilibrium (Crémer and McLean, 1985), that is, equilibrium action choices are constant in the information set. The second simplification implicitly assumes that the cost of escalation is independent of any previous history of negotiations.

Formally, these modeling choices reduce the escalation game to a type-dependent lottery over the remaining surplus. A litigant's likelihood to win the lottery depends on her relative strength compared to her opponent. Such an escalation game has constant aggregate welfare. Thus, our belief-management approach reveals that the optimal belief system maximizes only the sum of expected virtual rents, $E[\Psi_1|\mathcal{B}] + E[\Psi_2|\mathcal{B}]$ over the belief system. In addition, these expected virtual rents are linear in beliefs such that the entire problem collapses to a linear programming exercise.

In the supplementary material, appendix H, we provide a solution algorithm in case the distribution satisfies some monotonicity condition. The result of this algorithm is a sorting mechanism that promises escalation to strong matches, settlement to weak matches, and an intermediate escalation probability to intermediate matches.

In contrast to the results of Proposition 4 that mechanism has the following features: weak matches obtain sure settlement, players can influence the information structure through their report and introducing asymmetries is never beneficial.[18]

5 Extensions

In this section we discuss robustness of our findings by proposing several extensions.

The Irregular Case. Assumption 3 implies that incentive compatibility holds locally in one direction and global deviations are non-profitable. The problem's properties thus closely match those of standard, monotone mechanism-design problems. Applying the guess-and-verify approach common in mechanism design, we can without loss relax Assumption 3 such that (MDR) is required only to hold *at the optimum.* Arguably the assumption remains strong nonetheless.

Games as outside option necessarily lead to a deviation from the quasi-linearity paradigm used in most mechanism-design problems. Assumption 3 recovers some of the properties from quasi linearity and thus ensures tractability. An alternative, perhaps closer, way to restore these properties is to assume *linearity in types.*

Definition 8 (Linearity in Types)**.** Let G^* be player θ_{-i}'s equilibrium distribution of actions. Then, the escalation game is linear in types if there is a pair of functions n, t such that for any m_i, θ_i, and $\mathcal{B}$

$$\sup_{a_i \in A_i^{\mathcal{G}}} \sum_{\theta_{-i}} \beta_i(\theta_{-i}|m_i) \int_{A_{-i}^{\mathcal{G}}} u(\theta_i, \theta_{-i}, a_i, a_{-i}) \mathrm{d}G^*(a_{-i}|\theta_{-i}, \mathcal{B}) = n(m; \mathcal{B})\theta_i + t(m; \mathcal{B}).$$

For a game that is linear in types, the insights of Section 3 remain and the content of Theorem 2 changes only slightly. Depending on binding participation constraints, the ability premium for a particular type may be either defined upwards *or* downwards but never both. The optimal mechanism aims at increasing discrimination between a type and the *next worst* type. All other arguments prevail.

Even if neither of the conditions hold, the procedure in Section 3 is a necessary first step to solve the problem. If the optimal solution satisfies all global constraints, we have found an optimum. If the optimal solution violates any omitted constraints, we replace the objective with a Lagrangian objective including the global constraint. Results from Theorem 1 and 2 remain under the adjusted objective.[19]

Non-Convex Veto-Values. Assumption 1 imposes several properties on the veto game. Apart from property (CONV), which states that the value of vetoing is on the convex closure at the prior, all of the properties are either common in the literature

[18] See appendix H for further details on the model of Hörner, Morelli, and Squintani (2015) from a belief-management perspective.

[19] A formal treatment of the general Lagrangian is in the supplementary material to this paper.

((HC) and (S)) or serve a well-defined purpose (OST). Property (CONV) is special to mechanism-design problems in which at least one party can unilaterally enforce a given veto game. In an otherwise different problem Celik and Peters (2011) provide an example where (CONV) fails and, as a result, some types reject the optimal mechanism on the equilibrium path. We avoid such failure of the revelation principle assuming (CONV) to hold at the prior. In most of the games considered in the literature, and in our ADR model, too, (CONV) is satisfied *for all* possible priors.

However, we can eliminate assumption (CONV) completely if we assume the designer has access to a signaling device a la Kamenica and Gentzkow (2011) that realizes independently of players' participation decisions. In addition we have to change the timing slightly: Instead of sequentially ratifying conflict management and then communicating, players do both simultaneously. The designer promises to publish a Bayes' plausible signal over the information she receives in case either of the players rejects the mechanism. This promise (or threat) alone ensures full participation and allows us to employ our techniques even absent of (CONV).[20]

Enlarging the Designer's Signal Space. We restrict the designer's signal space to public messages to focus on the duality of the problems. In the dual problem, the designer has to solve an information-design problem for example via the methods from Mathevet, Perego, and Taneva (2017). The dual remains under a larger signal space, and results change neither conceptually nor qualitatively.

Limited Commitment by the Players. The main focus of our analysis is to highlight the differences in conflict management mechanisms that are embedded in a greater strategic environment beyond the designer's control. Apart from this deviation we aim at staying as close as possible to standard mechanism-design problems. In particular, we assume that players have full-commitment power once the mechanism is accepted. In our litigation example we cover what is known as "arbitration". A second important mode known as "mediation" allows players to invoke litigation at any point during conflict management rather than only at the beginning.

Conceptually, mediation implies that the participation constraints hold ex-post, that is, after players learn the outcome of conflict management. Hörner, Morelli, and Squintani (2015) show for a special case of our model that optimal mediation can achieve the same results as optimal arbitration. The techniques in Hörner, Morelli, and Squintani (2015) do not generalize to arbitrary escalation games. We propose an alternative extension. Suppose instead of offering the settlement shares publicly, the designer can *privately offer* each party their share. Then, on-path escalation may be triggered via an unacceptable settlement offer and a private recommendation to reject it. Hence, accepting an offer does not imply settlement. The opponent may still reject.

Furthermore, the designer creates private information on her own. She can exploit this private information by initiating "seemingly unnecessary escalations" with a

[20] A general treatment of such signaling mechanisms is discussed in Balzer and Schneider (2018).

small probability on the equilibrium path. That is, breakdown may occur regardless of players' reporting behavior. As the probability of "seemingly unnecessary escalations" approaches 0, the value of the objective converges to that without such escalations. As long as the probability remains positive, however, the designer can react to *any* deviation by implementing the worst possible belief system of the deviating type.[21]

For the case of Section 4 such an augmented mechanism is sufficient to guarantee a settlement ratio of optimal *mediation* that is arbitrarily close to that of optimal *arbitration*. Further details are in a companion paper (Balzer and Schneider, 2017).

Different Objective. Our choice of objective implies that the designer's preferences are as simple as possible. She focuses exclusively on achieving settlement. In principle, however, the designer may have preferences about the outcome of the escalation game, too. In legal conflicts, the designer may be willing give up some settlement solutions to decrease overall inefficiency in the continuation game.

Adjusting ($\mathrm{P}^{\mathcal{B}}_{max}$) is straightforward. Suppose the designer cares about efficiency in the escalation game. Her objective now assigns more weight on the second term, $\mathbb{E}[U_i|\mathcal{B}]$, of the objective. In the case of the all-pay auction such preferences would lead to *larger asymmetries* in the continuation game, while type-independence prevails. More generally, the derivative of the objective of ($\mathrm{P}^{\mathcal{B}}_{max}$) changes to

$$\frac{\partial \sum_i \mathbb{E}[\Psi_i|\mathcal{B}]}{\partial \mathcal{B}} h_\Psi(\mathcal{B}) + \frac{\partial \sum_i \mathbb{E}[U_i|\mathcal{B}]}{\partial \mathcal{B}} h_U(\mathcal{B}), \qquad \text{with } h_U \geq h_\Psi \geq 0,$$

if the designer cares about reducing inefficiencies conditional on escalation, too.

Implementing Reduced-Form Mechanisms. Our main results are on reduced-form mechanisms. The general implementation conditions, (GI), determine whether the solution is implementable. They are redundant in the model of Section 4 as well as in those from the literature. In principle, however, they serve as an additional constraint to the problem. Two extensions to our model are particularly related to these conditions.

The first considers correlated types. If types are correlated, the designer exploits correlation via the settlement value in the same way as in Crémer and McLean (1988). She offers a "side bet" over the opponent's types to relax incentive compatibility. To achieve full efficiency unlimited utility transfers are necessary. Thus, in our setup first-best is not implementable even with correlated types. All ex-post settlement outcomes split the pie, a restriction governed by (GI). Naturally, the conditions bind when types are correlated. Otherwise, the logic of Crémer and McLean (1988) applies.

The second extension concerns additional transferable utility. If the designer could impose (binding) transfers of any sort in addition to the settlement allocation she is no longer restricted by the general implementability condition, (GI). Then, any reduced-form mechanism is implementable ex-post with the choice of appropriate transfers. However, such a transfer rule may then include that players ex-post utility is negative, as

[21] Correia-da-Silva (2017) and Gerardi and Myerson (2007) use similar techniques in another context.

they might lose the good *and* pay a transfer to their opponent.[22]

6 Conclusion

We provide a general, yet tractable approach to optimal conflict management within a strategic environment partially beyond the designer's control. We propose an economically intuitive dual problem to the conflict-management problem that links properties of the escalation game to the optimal mechanism. The dual highlights that optimal conflict management is driven by how information release during conflict management effects action choices under escalation. We reduce the main objective of optimal conflict management to the choice of an information structure conditional on escalation.

We apply our general result to optimal alternative dispute resolution in the shadow of a court. We show that optimal ADR ensures that a disputant's behavior within ADR has no effect on the information she obtains about her opponent. Applying this rule is the only way to ensure that a deviator gains no information advantage by deviating. In addition, optimal ADR guarantees an asymmetric information structure in case no settlement solution results. That way, litigation after a failed settlement attempt is less inefficient than absent ADR, and disputant's incentives to participate are stronger.

Both results are absent in existing models in the literature which restricts attention to escalation games with ex-post equilibria neglecting the behavioral externality. We include this option, and our belief-management approach provides an economically intuitive objective function exclusively based on the escalation game. That way, we not only ensure tractability, but straight-forwardly suggest the intuition behind optimal conflict management for a given escalation game.

Our results suggest a number of directions for future research. In this paper we focus on conflict management that aims to avoid a costly resolution of a dispute. However, the property that mechanisms interact with a greater strategic environment prevails beyond conflict management. In fact, a designer in most real-world problems only controls part of the strategic environment, but information obtained while interacting within a mechanism is relevant beyond the mechanism itself. Problems along this line include antitrust measures, financial regulation, and international treaties of any sort.

Similar to our discussion here, information obtained during negotiations becomes valuable in future interactions, making the continuation game information sensitive. This effect, in turn, influences the design of institutions. Although many details may differ, our results suggest that a connection exists between the mechanism-design problem in restricted environments and the information-design problem in the mechanism's surroundings. Extending our results in that direction is natural, but beyond the scope of this paper. We leave it to future research.

[22] Naturally, combining the two extensions leads to full settlement. However, in real world scenarios such a combination is at most a rough benchmark.

Appendix

Organization: Appendix A proves Theorem 1. Appendix B proves Theorem 2. Appendix C contains further discussion on Section 4.

A Belief Management and Proof of Theorem 1

A.1 Proof of Proposition 1

Proof. Full settlement implies pooling. Hence, the sum of players' expected payoffs is constant in their types and can be set to 1. For given p, $V_i(\theta_i, (p, \tilde{p}))$ is decreasing in θ_i. Thus, if $\sum_i V_i(1, (p, \tilde{p})) > 1$ for all $\tilde{p}$, then all pooling solutions violate at least one player's participation constraints. Conversely, the pooling solution $x_i = V_i(1, (p, \tilde{p}))$ implements full settlement. By symmetry a pooling solution exists if and only if an equal split pooling solution exists. $\square$

A.2 Conflict Management and Belief Management

We prove Theorem 1 in steps. The steps correspond to the observations in the main text. Different to the text, we include the designer's ability to provide a public signal in the formal argument. Each realization σ occurs with probability $Pr(\sigma)$. The corresponding realization of the random variable $\mathcal{B}(\Sigma)$ is $\mathcal{B}(\sigma)$.

Step 0: Extending Definitions. The signaling function Σ determines a joint probability of escalation and the realization of σ, $\gamma^\sigma(\theta_i, \theta_{-i})$, as a function of players' reports. Individual beliefs, $\beta_i(\theta_{-i}|\theta_i, \sigma)$, are probability distributions conditional on own reports and σ. Other expressions extend in the natural way, $\gamma_i^\sigma(\theta_i) := \sum_{\theta_{-i}} p(\theta_{-i})\gamma^\sigma(\theta_i, \theta_{-i})$, and $y_i^\sigma(m_i; \theta_i) := \gamma^\sigma(m_i)U_i(m_i; \theta_i, \mathcal{B}(\sigma))$. Players' commitment power to accept settlement solutions implies that realizations average out in $z_i(\theta_i)$. Absent additional signals all expressions collapse to those in the main text.

Definition 9. An escalation rule γ^σ implements $\mathcal{B}(\sigma)$ if $\mathcal{B}(\sigma)$ is consistent with Bayes' rule under γ^σ.

Step 1: Homogeneity. We show $\mathcal{B}(\sigma)$ is homogeneous of degree 0 w.r.t. γ^σ via the following claim.

Claim. γ^σ implements $\mathcal{B}(\sigma)$ iff every escalation rule $\hat{g}_{\mathcal{B}(\sigma)} = \alpha\gamma^\sigma$ implements $\mathcal{B}(\sigma)$ where α is a scalar.

Proof. Suppose γ^σ implements $\mathcal{B}(\sigma)$. Homogeneity of Bayes' rule implies that any escalation rule $\hat{g}_{\mathcal{B}(\sigma)} = \alpha\gamma^\sigma$ implements $\mathcal{B}(\sigma)$. For the reverse suppose $\alpha\gamma^\sigma$ implements $\mathcal{B}(\sigma)$ and set $\alpha = 1$. If γ^σ is an escalation rule it implements $\mathcal{B}(\sigma)$. $\square$

If $\mathcal{B}(\sigma)$ is homogeneous w.r.t. γ^σ so is U_i; $\gamma_i^\sigma(\theta_i)$ is homogeneous of degree 1 by definition and so is $y_i^\sigma(m_i; \theta_i)$.

Step 2: Worst escalation rule. We show that $\mathcal{B}(\Sigma)$ determines $Pr(\mathcal{G})$ up to $|\Sigma|$ real numbers. That is, the set of all escalation rules implementing a given lottery, $\mathcal{B}(\Sigma)$, is defined up to the real numbers $\{\alpha^\sigma\}_\sigma$. The escalation probability is linear in any α^σ. If the lottery is degenerate, then the worst-escalation rule is uniquely defined.

Fix a random consistent belief systems $\mathcal{B}(\Sigma)$. For each $\mathcal{B}(\sigma)$ take *some* escalation rule $\hat{\gamma}^\sigma$ that implements the belief system. Step 1 implies that the set of escalation rules implementing $\mathcal{B}(\Sigma)$ satisfies $Pr(\mathcal{G}) = \sum_{(\theta_1,\theta_2)} p(\theta_1)p(\theta_2)\Big(\sum_\sigma \alpha^\sigma \hat{\gamma}^\sigma(\theta_1,\theta_2)\Big)$. Let $\mathcal{A}$ be the set of all $\{\alpha^\sigma\}_\sigma$, with α^σ such that $\forall(\theta_1,\theta_2)$, $\alpha^\sigma\hat{\gamma}^\sigma(\theta_1,\theta_2) \leq 1$ and $\hat{\gamma}(\theta_1,\theta_2) = \sum_\sigma \alpha^\sigma\hat{\gamma}^\sigma(\theta_1,\theta_2) \leq 1$. $\mathcal{A}$ determines all escalation rules implementing $\mathcal{B}(\Sigma)$. If $\mathcal{B}(\Sigma)$ is a singleton, the largest element of $\mathcal{A}$ determines the worst escalation rule uniquely.

Step 3a: Linearity of constraints in $\{\alpha^\sigma\}_\sigma$. Consider the optimal mechanism.

Claim. For any θ_i with $z_i(\theta_i) > 0$ either the participation or an incentive constraint is satisfied with equality.

Proof. To the contrary, suppose neither the participation constraint nor an incentive constraint holds with equality. Then, we can reduce $z_i(\theta_i)$ until either $z(\theta_i) = 0$ or one of the above constraints is satisfied with equality, and all constraints remain satisfied. □

If $\Theta_i^{IC}\subset\Theta$ is the set of types with some binding incentive constraint, and Θ_i^{PC}, Θ_i^0 its analogues for participation and non-negativity constraints, then $\Theta_i^{PC}\cup\Theta_i^{IC}\cup\Theta_i^0 = \Theta$. In addition, let $\Theta_i^I(\theta_i) \subset \Theta$ be the set of types such that θ_i's incentive constraints regarding these types holds with equality. We say $\widehat{\Theta}_i \subset \Theta_i^{IC}$ describes a *cycle* if for any $\theta_i \in \widehat{\Theta}_i$, it holds that $\theta_i \notin \Theta_i^{PC} \cup \Theta_i^0$ and $\Theta_i^I(\theta_i) \subset \widehat{\Theta}_i$.

Claim. It is without loss of generality to assume no cycles exist.

Proof. Suppose $\widehat{\Theta}_i$ describes a cycle. Reducing $z_i(\theta_i)$ for all $\theta_i \in \widehat{\Theta}_i$ under condition $z_i(\theta_i) - z(\theta_i') = y_i(\theta_i';\theta_i) - y_i(\theta_i;\theta_i)$ for any $\theta' \in \Theta^I(\theta_i)$ is possible without violating any other constraint since $\Theta_i^I(\theta_i) \cap \{\Theta_i^{PC} \cup \Theta_i^0 \cup \{\Theta_i^I(k)\}_{k\notin\widehat{\Theta}_i}\} = \emptyset$. □

Claim. z_i is linear in α^σ given $\mathcal{B}(\Sigma)$.

Proof. If $\theta_i \in \Theta_i^0$, z_i is constant and thus linear in α^σ. Now consider $\theta_i \in \Theta_i^{PC}$. Then, $z_i(\theta_i) = V_i(\theta_i,(p,\rho^V)) - y_i(\theta_i;\theta_i)$. The first term of the RHS is a constant, the second is linear in α^σ since $y_i(m_i;\theta_{-i}) = \sum_{\sigma\in\Sigma} y_i^\sigma(m_i;\theta_i)$ which is linear by step 1. Finally, for any $\theta_i \in \Theta_i^{IC}$, the incentive constraint is $z_i(\theta_i) = z_i(\theta_i') + y_i(\theta_i';\theta_i) - y_i(\theta_i;\theta_i)$ for any $\theta_i' \in \Theta_i^I(\theta_i)$. Given $z_i(\theta_i')$, linearity holds because y_i is linear in α^σ by step 1. Now, either $\theta_i' \in \Theta_i^{PC} \cup \Theta_i^0$, or, $z_i(\theta_i')$ is linear given some $z_i(\theta''{}_i)$ with $\theta_i'' \in \Theta_i^I(\theta_i)$. No cycles exist so that recursively applying the last step yields the desired result. □

Step 3b: Homogeneity of the expected shares. Using the results from step 3a, let $\mathbb{P}_i(\Theta)$ describe the finest partition of Θ into subsets $\{\Theta_i^p\}_p$ such that for every $\theta_i \in \Theta_i^p$, $\Theta^I(\theta_i) \in \Theta_i^p$. Again using step 3a, $\exists\ \theta_i \in \Theta_i^p$ s.t. $\theta_i \in \Theta_i^{PC} \cup \Theta_i^0$ and each z_i is entirely determined by additively separable, linear elements $y_i(\cdot;\cdot)$ and $V_i(\cdot,(\cdot,\cdot))$. V_i is independent of σ and each y_i is a weighted sum of all y_i^σ. Substituting into the expected settlement share and collecting terms, we can find a set of functions $H_i(\gamma^\sigma)$ solving

$$\begin{aligned}&\sum_{\theta_i} p(\theta_i)z_i(\theta_i) = \\ &\quad -\sum_\sigma H_i(\gamma^\sigma) + \sum_{\theta_i\in\Theta_i^{PC}} p(\theta_i)V_i(\theta_i,(p,\rho^V)) + \sum_{\theta_i\in\Theta_i^{IC}} p(\theta_i) \sum_{k\in\Theta^I(\theta_i)} V_i(k,(p,\rho^V))\end{aligned} \tag{7}$$

Let $H_i(\{\gamma^\sigma\}_\sigma) := \sum_\sigma H_i(\gamma^\sigma)$. Further let $P_i(\Theta_i^0) := \sum_{\theta_i\in\Theta_i^0} p(\theta_i)$. Straight-forward algebra implies $H_i(\{\alpha^\sigma\gamma^\sigma\}_\sigma) = \sum_\sigma \Big(P_i(\Theta_i^0)(\alpha^\sigma - 1)H_i(\gamma^\sigma) + H_i(\gamma^\sigma)\Big)$. Thus, $H_i(\{\alpha^\sigma\gamma^\sigma\}_\sigma)$ is linearly increasing in α^σ given γ^σ.

Step 4: Determining $\{\alpha^\sigma\}_\sigma$ via resource constraint. A conflict management outcome is only feasible if the ex-ante expected settlement values are weakly lower than the probability of settlement, (2). That is, $\sum_i\sum_{\theta_i} p(\theta_i)z_i(\theta_i) \leq 1 - Pr(\mathcal{G})$, where the RHS is strictly lower than 1 by Assumption 1. By step 1 any escalation rule $\{\alpha^\sigma\gamma^\sigma\}_\sigma$ implements the same $\mathcal{B}(\Sigma)$. If each $\alpha^\sigma\gamma^\sigma$ is feasible then $\{\alpha^\sigma\gamma^\sigma\}_\sigma$ satisfies (2). By step 3b we can rewrite (2) as

$$\sum_i v_i(V_i(\Theta,(p,\rho^V))) - 1 \leq \sum_\sigma \left(\sum_i \Big(P_i(\Theta_i^0)(\alpha^\sigma - 1)H_i(\gamma^\sigma) + H_i(\gamma^\sigma)\Big) - Pr(\mathcal{G},\sigma)\right) \tag{2'}$$

where $v_i(V_i(\Theta,(p,\rho^V))) := \sum_i\sum_{\theta_i\in\Theta_i^{IC}}\sum_{k\in\Theta_i^I(\theta_i)} p(k)\left[\mathbb{1}_{PC}(\theta_i)V_i(\theta_i,(p,\rho^V))\right]$ is a probability weighted sum of veto values for types with binding participation constraint. Given $\Theta_i^{PC}, \Theta_i^{IC}, \Theta_i^0$, and $\{\Theta_i^I(\theta_i)\}_{\theta_i}$ the LHS is independent of the designer's choice.

Let $\{\alpha^\sigma\gamma^\sigma\}_\sigma$ implement $\mathcal{B}(\Sigma)$, then we can write the RHS as

$$\underbrace{\sum_\sigma \left(\sum_i \Big(P_i(\Theta_i^0)(\alpha^\sigma - 1)H_i(\gamma^\sigma) + H_i(\gamma^\sigma)\Big) - \alpha^\sigma \sum_{(\theta_1,\theta_2)\in\Theta} p(\theta_1)p(\theta_2)\gamma^\sigma(\theta_1,\theta_2)\right)}_{=:h(\{\alpha^\sigma\gamma^\sigma\}_\sigma)}.$$

Moreover, using the definition of H_i it follows that h is linear in α, since $y_i(\theta_i;\theta_i)$ and $Pr(\mathcal{G})$ are homogeneous in $\{\gamma^\sigma\}_\sigma$. In particular, $h(\sum_\sigma \alpha^\sigma\gamma^\sigma)$ converges to a weakly positive number if every α^σ is sufficiently small. Observe that $\sum_\sigma \alpha^\sigma\gamma^\sigma \to 0$ is the full settlement solution. Thus, Assumption 1 implies that the LHS of (2') is strictly positive. In turn, $h(\{\alpha^\sigma\gamma^\sigma\}_\sigma) > 0$, because $\{\alpha^\sigma\gamma^\sigma\}_\sigma$ is part of an implementable mechanism. Therefore, the optimal $\{\alpha^\sigma\}_\sigma$ equates LHS and RHS. Thus, for any $\mathcal{B}(\Sigma)$ the minimal $Pr(\mathcal{G})$ uses an $\{a^\sigma\}_\sigma$ at the boundary of $\mathcal{A}$.

B Optimal Conflict Management and Proof of Theorem 2

We construct a solution algorithm to solve for $\mathcal{CM}$. We use it to prove Theorem 2.

Remark. Our argument throughout this section assumes that $\overline{g}_{\mathcal{B}(\sigma)}(1,1) = 1$. This normalization is without loss. For cases in which $0 < \overline{g}_{\mathcal{B}(\sigma)}(1,1) < 1$ relabeling provides the missing step. The remaining cases with $\gamma(1,1) = 0$ are covered by continuity of $\mathcal{B}$ in

γ. Lemma 9 in the supplementary material provides the corresponding formal argument.

B.1 Proof of Proposition 2

Proof. The proof follows directly from Border (2007), Theorem 3. □

B.2 Proof of Lemma 1

Proof. The MDR property implies that local downward incentive compatibility is sufficient for global downward incentive compatibility.[23] Now, assume there is a type θ_i for which both incentive constraints are redundant. Then, it is possible to reduce $z_i(\theta_i)$ at no cost for the designer until either an incentive constraint binds, i.e., $\theta_i \in \Theta_i^{PC}$, the participation constraint starts to bind, i.e., $\theta_i \in \Theta_i^{PC}$, or $z_i = 0$, i.e., $\theta_i \in \Theta_i^0$.

Assumption 1 implies that the set of types with binding participation constraints is non-empty. Otherwise full settlement is feasible. If there is exactly one type of one player with a binding participation constraint, the designer can offer an alternative mechanism: The mechanism determines at random who is assigned the role of player i and who that of $-i$ *after* players have submitted their report. Each of the two realizations satisfies the constraints and players are symmetric, and so does the combination. Under the alternative mechanism, no participation constraint is binding. A contradiction.

To see that exactly 1's participation constraint is binding and that $z_i(k) \geq 0$ does not bind, consider the designer's resource constraint. Focus on the formulation (2') in the proof of Theorem 1, step 4. Assume by contradiction that the set of types with binding participation constraint $\Theta_i^{PC} \neq \{1\}$. An upper bound on LHS of (2') is

$$\sum_i \sum_{\substack{\theta_i \in \\ \Theta_i^{PC} \cup \Theta_i^0}} \sum_{\substack{k \in \\ \Theta_i^I(\theta_i)}} p(k) \left[\mathbb{1}_{PC}(\theta_i) V_i(\theta_i, (p, \rho^V))\right] - 1 \leq \sum_i \sum_{\substack{\theta_i \in \\ \Theta_i^{PC} \cup \Theta_i^0}} \sum_{\substack{k \in \\ \Theta_i^I(\theta_i)}} p(k) V_i(\theta_i, (p, \rho^V)) - 1.$$

Assumption 3, part (i), implies a negative upper bound if $\Theta_i^{PC} \neq \{1\}$ contradicting Assumption 1. Given the set of types with binding participation constraint, $\Theta_i^{PC} = \{1\}$, it is without loss to focus on mechanisms in which the downward local incentive compatibility binds: Suppose $\theta_i > 1$'s downward IC is redundant. The designer can reduce $z_i(\theta_i)$ (and potentially burn the share) without violating any other constraint. □

B.3 The Lagrangian Problem

The designer's choice is $cs = (\Sigma, \gamma, z)$. The choice set is CS.

Lemma 3. *The Lagrangian approach yields the global optimum.*

Proof. We use theorem 1 in Luenberger (1969) to show that the Lagrangian approach is sufficient. Let T be the set of Lagrangian multiplier, with element t. Further, let $G(\cdot)$ be the set of inequality constraints, and $Pr(\mathcal{G})$ a function from choices to escalation probabilities. Define $\mathrm{w}(t) := inf\{Pr(\mathcal{G}) | cs = (\gamma, z, \Sigma) \in CS, G(cs) \leq t\}$. The Lagrangian is sufficient for a global optimum if $\mathrm{w}(t)$ is convex.

[23]The proof is along the standard argument that the monotone likelihood ratio implies sufficiency of local incentive compatibility in standard mechanism design problems. Our version of the proof is in appendix D in the supplementary material.

Assume for a contradiction that $\mathrm{w}(t_0)$ is not convex at t_0. Then, there is t_1, t_2 and $\lambda \in (0,1)$ such that $\lambda t_1 + (1-\lambda)t_2 = t_0$ and $\lambda \mathrm{w}(t_1) + (1-\lambda)\mathrm{w}(t_2) < \mathrm{w}(t_0)$. For $j \in \{1,2\}$ let $cs_j = (\gamma[j], z[j], \Sigma[j])$ describe the optimal solution, such that $Pr(\mathcal{G})(cs_j) = \mathrm{w}(t_j)$. Then, consider the choice cs_0 such that $z[0] = \lambda z[1] + (1-\lambda)z[2]$, $\gamma[0] = \lambda\gamma[1] + (1-\lambda)\gamma[2]$ and $\Sigma = \{1,2\}$, with $Pr(\sigma = 1) = \lambda$ and $\gamma^{\sigma=j} = \gamma[j]$. By construction constraints are satisfied and and the solution value equals that of the convex combination

$$\mathrm{w}(t_0) = Pr(\mathcal{G})(cs_0) = \sum_{\sigma\in\{1,2\}} Pr(\sigma)Pr(\mathcal{G}|\sigma) = \tilde{\alpha}\mathrm{w}(t_1) + (1-\tilde{\alpha})\mathrm{w}(t_1)$$

A contradiction. $\square$

We continue under Assumption 3. The approach absent Assumption 3 is similar, yet notationally more involved. We describe it in the supplementary material, appendix E. First, we define the conditional type probabilities.

Definition 10 (Conditional Type Probabilities)**.** Let $\rho_i(\cdot|\sigma)$ be the probability distributions over i's types conditional on escalation and σ. It is the solution to the system of linear equations $\rho_i(\theta_i|\sigma) = \sum_{\theta_{-i}} \beta_{-i}(\theta_i|\theta_{-i},\sigma)\rho_{-i}(\theta_{-i}|\sigma)$. The conditional probability distribution of a profile is $\rho(\theta_i, \theta_{-1}|\sigma) := \beta_{-i}(\theta_i|\theta_{-i},\sigma)\rho_{-i}(\theta_{-i}|\sigma)$. The set of all $\rho(\theta_i,\theta_{-1}|\sigma)$ is $\boldsymbol{\rho}(\sigma)$.

Given Σ, $\mathcal{B}(\sigma)$ and $\boldsymbol{\rho}(\sigma)$ are isomorphic. We define the set of Lagrangian multipliers $\nu^i_{\theta,\theta'}$ for the incentive compatibility constraints, λ^i_θ, for the participation constraints, δ, for the designer's resource constraint, ζ^i_θ for non-negativity of z_i, and η_Q for the general implementability constraints of the reduced form mechanism from Proposition 2. Finally, μ_{θ_1,θ_2} is the multiplier on the consistency constraint. We divide all multipliers by δ and obtain the set $\{\tilde{\nu^i_{\theta,\theta'}}, \tilde{\lambda}^i_\theta, 1, \tilde{\zeta}_{\theta^i}, \tilde{\eta}_Q, \tilde{\mu}_{\theta_1,\theta_2}\}$. Let $\tilde{e}_i(\theta) = p(\theta) \sum\limits_{Q|\theta\in Q} \eta_Q$. Furthermore, we define the aggregation up to type θ using capital letters, $\widetilde{\Lambda}^i(\theta) := \sum_k^\theta \tilde{\lambda}^i_k$, and $\widetilde{E}^i(\theta)$ and $\widetilde{Z}^i(\theta)$ in a similar way. Finally, $\mathsf{m}^\mathsf{i}_\theta := p(\theta) + \tilde{e}^i_\theta - \tilde{\zeta}^i_\theta$, $\mathsf{M}^\mathsf{i}(\theta) := \widetilde{\Lambda}^i(\theta) - \sum_{k=1}^{k=\theta} p(k) - \widetilde{E}^i(\theta) + Z^i(\theta)$, $\nu^i(\theta) := \sum_{k=1}^K |(\sum_{l=1}^K \tilde{v}^i_{k+1,l})| - \tilde{v}^i_{\theta,\theta+1}$.

We state the transformed Lagrangian objective as a corollary to the more general solution discussed in the supplementary material appendix E.

Corollary 2. *Suppose Assumption 1 to 3 hold. The lottery* $\{Pr(\sigma), \boldsymbol{\rho}(\sigma)\}_{\sigma\in\Sigma}$ *is an optimal solution to the designers problem if and only if* each $\boldsymbol{\rho}(\sigma)$*, maximizes*

$$\begin{aligned}\widehat{\mathcal{L}}(\mathcal{B}(\sigma)) :=& \mathcal{T}(\mathcal{B}(\sigma)) + \sum_i \Bigg[\sum_{\theta=1}^K \rho_i(\theta|\sigma)(\frac{\mathsf{m}^\mathsf{i}_\theta}{p(\theta)})U_i(\theta;\theta,\mathcal{B}(\sigma)) \\ &+ \sum_{\theta=1}^{K-1} \frac{\mathsf{M}^\mathsf{i}(\theta) - \tilde{\nu}^i_{\theta,\theta+1}}{p(\theta)} \rho_i(\theta|\sigma)(U_i(\theta;\theta,\mathcal{B}(\sigma)) - U_i(\theta;\theta{+}1,\mathcal{B}(\sigma))) \\ &- \sum_{k=1}^{\theta-1} \frac{\mathsf{M}^\mathsf{i}(\theta) + \nu^i_{k,\theta} - \nu^i(\theta)}{p(\theta)} \rho_i(\theta|\sigma)[U_i(\theta;k,\mathcal{B}(\sigma)) - U_i(\theta;\theta,\mathcal{B}(\sigma))] \Bigg],\end{aligned} \tag{8}$$

with

$$\mathcal{T}(\mathcal{B}(\sigma)) := \sum_{Q\in Q^2}\sum_{(\theta_1,\theta_2)\in\tilde{Q}}[\rho(\theta_1|\sigma)\beta_1(\theta_2|\theta_1,\sigma)]\tilde{\eta}_Q - \sum_{\theta_1\times\theta_2}\frac{\rho_1(\theta_1|\sigma)\beta_1(\theta_2|\theta_1,\sigma)}{p(\theta_1)p(\theta_2)}\tilde{\mu}_{\theta_1,\theta_2}. \quad (9)$$

Hence, $\boldsymbol{\rho} := \sum_\sigma Pr(\sigma)\boldsymbol{\rho}(\sigma)$ *is a maximizer of the concave closure of the above function. The following holds by complementary slackness*

- *The resource constraint from equation* (2) *binds, hence* $\delta > 0$.
- *If the optimal reduced form mechanism is implementable, then the constraints from Proposition 2 are redundant and* $\widetilde{E}_i(\theta) = \tilde{e}^i_\theta = \widetilde{Z}^i(\theta) = \tilde{\zeta}^i_\theta = 0$.
- *Downward local incentive constraints bind, thus* $M_i(\theta) > 0$. *If, in addition, all upward incentive constraints are redundant, then* $\tilde{v}^i_{\theta,k} = 0$ *for all* $k \geq \theta$.
- *Local downward incentive constraints are sufficient for global incentive constraints.*

Results follows from algebraic manipulation of the initial Lagrangian objective using Lemma 1 to identify binding constraints. Manipulations proceed alongside the discussion of Proposition 3. A full description is in appendix E of the supplementary material.

B.4 Proof of Proposition 3

Proof. With access to signals the designer can implement spreads over consistent post-escalation belief systems. Then, (i) the Lagrangian approach yields the global maximum, and (ii) the optimal solution lies on the concave closure of the Lagrangian function over consistent post-escalation belief systems. Without access to signals, (i) a critical point of the Lagrangian objective is only necessary but not sufficient for global optimality, (ii) every optimal solution must be a local maximum of the Lagrangian objective (but not of its concave hull), and (iii) constraints have to hold for the ex-post realized belief system (rather than for the lottery over realized belief-systems). Despite these differences, we still can use the form of the Lagrangian function stated in Corollary 2. Take the first two terms of the Lagrangian in Corollary 2 as the objective due to the binding constraints from Lemma 1, set the Border multipliers $\tilde{e}^i_\theta, \zeta^i_\theta, \widetilde{E}^i, Z^i_\theta$ to zero and add the respective constraints from Proposition 2. Consequently, $\frac{\mathsf{m}^i_\theta}{p(\theta)} = 1$ and $(\mathsf{M}^i(\theta) - \tilde{\nu}^i_{\theta,\theta+1})/p(\theta) = w(\theta)$. The last term of (8) boils down to an expression that consists of local downward incentive constraints. The signaling term (9) is implied by consistency and the Border constraints, completing the proof. □

B.5 Proof of Corollary 1

Proof. The solution to the optimization problem, $\mathcal{B}^*$, maximizes (8). By hypothesis, $\mathcal{B}^*$ is in the set of least constraint solutions. Thus the last two terms of equation (8) are 0. Thus, $\mathcal{B}^*$, maximizes

$$\sum_i\left[\sum_{\theta=1}^{K}\rho_i(\theta)U_i(\theta;\theta,\mathcal{B}) + \sum_{\theta=1}^{K-1}\frac{1-\sum_{k=1}^{k=\theta}p(k)}{p(\theta)}\rho_i(\theta)(U_i(\theta;\theta,\mathcal{B}) - U_i(\theta;\theta+1,\mathcal{B}))\right],$$

By construction the optimum is on the concave closure, signals do not improve. □

B.6 Proof of Theorem 2

Proof. The problem collapses to ($\mathrm{P}^{\mathcal{B}}_{max}$) if the optimal signal is degenerate and Corollary 1 applies. Suppose we are at an optimum with a non-degenerate signal. Assume that instead of the continuation game $\mathcal{G}$, an alternative continuation game $\widehat{\mathcal{G}}$ is played. $\widehat{\mathcal{G}}$ differs from $\mathcal{G}$ in that an omniscient nature first draws a realization of a state-dependent random variable Σ and communicates this to the players. Players update to $\mathcal{B}(\sigma)$ and play $\mathcal{G}$ under updated beliefs. The continuation payoff of $\hat{\mathcal{G}}$ is $\widehat{U}(m;\theta,\mathcal{B}(\Sigma))$. If $\mathcal{B}(\Sigma)$ satisfies the constraints, it is implementable. Furthermore, $\mathcal{B}(\Sigma)$ leads to a random expected ability premium, $\mathbb{E}[\widehat{\Psi}|\mathcal{B}(\Sigma)] := \sum_i \sum_\sigma Pr(\sigma)\mathbb{E}[\Psi_i|\mathcal{B}(\sigma)]$, and a random expected welfare, $\mathbb{E}[\widehat{U}|\mathcal{B}(\Sigma)] := \sum_i \sum_\sigma Pr(\sigma)\mathbb{E}[U_i(\theta;\theta,\mathcal{B}(\sigma))|\mathcal{B}(\sigma)]$. Any mean $\overline{\mathcal{B}}$ may have many possible lotteries that support it and are feasible. We select the maximum for each. Hence, $\overline{\mathcal{B}}$ that maximizes $\mathbb{E}[\widehat{\Psi}|\mathcal{B}(\Sigma)] + \mathbb{E}[\widehat{U}|\mathcal{B}(\Sigma)]$ also solves (8). $\square$

C Additional Material to Section 4

C.1 Closed-Form Expression of Effort Distributions

We assume without loss of generality that $b_1(1) \geq b_2(1)$. We identify players via their marginal cost θ_i. We start by deriving equilibrium strategies. For simplicity we focus on interior beliefs. The extension to the boundary cases is straight forward.

Players' Strategies: Densities and Distributions. (cf. Siegel, 2014, for a general discussion). The distribution function $F_i^{\theta_i}(a)$ denotes the probability of θ_i choosing an action smaller than a_i. Player θ_i's support includes a if and only if a maximizes

$$F_{-i}(a|\theta_i) - a\theta_i = (1 - b_i(\theta_i))\, F^K_{-i}(a) + b_i(\theta_i)F^1_{-i}(a) - a\theta_i.$$

Referring to Figure 1 on page 22, the support of players' strategies can be partioned into $I_1 = (0, \overline{a}_2^K]$, $I_2 = (\overline{a}_2^K, \overline{a}_1^K]$ and $I_3 = (\overline{a}_1^K, \overline{a}_1^1]$. We define indicator functions $\mathbb{1}_{\in I_l}$ with value 1 if $a \in I_l$ and 0 otherwise. Similar the indicator function $\mathbb{1}_{>I_l}$ takes value 1 if $a > \max I_l$ and 0 otherwise. Player θ_i mixes such that the opponent's first-order condition holds on the joint support. The densities are

$$f_2^1(a) = \mathbb{1}_{\in I_2}\frac{K}{b_1(K)} + \mathbb{1}_{\in I_3}\frac{1}{b_1(1)}, \qquad f_2^K(a) = \mathbb{1}_{\in I_1}\frac{K}{1 - b_1(K)},$$
$$f_1^1(a) = \mathbb{1}_{\in I_3}\frac{1}{b_2(1)}, \qquad f_1^K(a) = \mathbb{1}_{\in I_1}\frac{K}{1 - b_2(K)} + \mathbb{1}_{\in I_2}\frac{1}{1 - b_2(1)}.$$

This leads to the following cumulative distribution functions:

$$F_2^1(a) = \mathbb{1}_{\in I_2} a\frac{K}{b_1(K)} + \mathbb{1}_{\in I_3}\left(\frac{a}{b_1(1)} + F_2^1(\overline{a}_2^K)\right) + \mathbb{1}_{>I_3},$$
$$F_2^K(a) = \mathbb{1}_{\in I_1} a\frac{K}{1 - b_1(K)} + \mathbb{1}_{>I_1},$$
$$F_1^1(a) = \mathbb{1}_{\in I_3}\frac{a}{b_2(1)} + \mathbb{1}_{>I_3},$$

$$F_1^K(a) = \mathbb{1}_{\in I_1}\left(a\frac{K}{1-b_2(K)} + F_1^K(0)\right) + \mathbb{1}_{\in I_2}\left(\frac{a}{1-b_2(1)} + F_2^K(\overline{a}_2^K)\right) + \mathbb{1}_{>I_2}.$$

Players' Strategies: Interval Boundaries. The densities define the strategies up to intervals' boundaries. These boundaries are determined as follows

1. $\overline{a}_2^K$ is determined using $F_2^K(\overline{a}_2^K) = 1$, i.e., $\overline{a}_2^K f_2^K(a) = 1$ with $a \in I_1$. Substituting yields
$$\overline{a}_2^K = \frac{1-b_1(K)}{K}.$$
2. For any $\overline{a}_1^K$, $\overline{a}_1^1$ is determined using $F_1^1(\overline{a}_1^1) = 1$, i.e., $\left(\overline{a}_1^1 - \overline{a}_1^K\right) f_1^1(a) = 1$ with $a \in I_3$. Substituting yields
$$\overline{a}_1^1 = \overline{a}_1^K + b_2(1).$$
3. $\overline{a}_1^K$ is determined by $F_2^1(\overline{a}_1^K) = 1$. That is, $\left(\overline{a}_1^K - \overline{a}_2^K\right) f_2^1(a) + \left(\overline{a}_1^1 - \overline{a}_1^K\right) f_2^1(a') = 1$ with $a \in I_2, a' \in I_3$. Substituting yields
$$\overline{a}_1^K = \overline{a}_2^K + \left(1 - \frac{b_2(1)}{b_1(1)}\right)\frac{b_1(K)}{K}.$$
4. $F_1^K(0)$ is determined by the condition $F_1^K(\overline{a}_1^K) = 1$, i.e., $F_1^K(0) = 1 - \overline{a}_2^K f_1^K(a) - \left(\overline{a}_1^K - \overline{a}_2^K\right) f_1^K(a')$ with $a \in I_1, a' \in I_2$. Substituting yields
$$F_1^K(0) = 1 - \frac{1-b_1(K)}{1-b_2(K)} - \left(1 - \frac{b_2(1)}{b_1(1)}\right)\frac{b_1(K)}{1-b_2(1)}\frac{1}{K}.$$

Change of Variables. To simplify the argument in the proof of Proposition 4 we express $\mathcal{B}$ entirely using ρ and $b_1(1)$. The probabilities ρ are the solution to the system of linear equations $\rho_i(\theta_i) = \sum_{k=1}^K \beta_{-i}(\theta_i|k)\rho_{-i}^{\mathcal{G}}(k)$. We abuse notation and define $\rho_i := \rho_i(1)$. Given ρ_1, ρ_2 we describe any $b_i(m) \in \mathcal{B}$ as a linear function of $b_1(1)$ using Bayes' rule. That is,

$$b_1(K) = \frac{\rho_2 - \rho_1 b_1(1)}{1-\rho_1}, \quad b_2(K) = \frac{\rho_1}{1-\rho_2}(1 - b_1(1)), \quad \text{and } b_2(1) = \frac{\rho_1}{\rho_2} b_1(1).$$

Closed Form Expressions relative to $b_1(1)$. Any fraction $\beta(\theta_1|\theta_2)/\beta(\theta_2|\theta_1)$ depends only on ρ, and we simplify further

$$\begin{aligned} \overline{a}_2^K &= \frac{1-\rho_2-\rho_1}{K(1-\rho_1)} + \frac{\rho_1}{K(1-\rho_1)} b_1(1), & F_1(0) &= (1 - b_2(m)\left(\frac{(\rho_2-\rho_1)}{(1-\rho_1)}\frac{(K-1)}{K}\right), \\ F_2(\overline{a}_2^K|m) &= (1 - b_1(m)), & F_1(\overline{a}_2^K|m) &= (1 - b_2(m))\left(1 - \frac{(\rho_2-\rho_1)}{(1-\rho_1)}\frac{1}{K}\right). \end{aligned} \tag{10}$$

C.2 Proof of Proposition 4

Structure of the Proof. We prove Lemma 2 together with Proposition 4. We use a guess and verify approach to prove the statements jointly. A constructive proof is possible, but notationally intense. We omit showing that the escalation game has a unique and monotonic equilibrium at the optimum and the case for small priors $p < \underline{r}$. Both aspects are straightforward to verify. A more constructive version including these missing steps is in the companion paper Balzer and Schneider (2017). We start by guessing that the conditions in Corollary 1 hold and no additional constraint binds.

Proof. ***Part A (Piece-wise Linearity).*** $b_1(1) \geq b_2(1)$ implies $\rho_2 > \rho_1$. Take any ρ_i that satisfies this condition. Expressions (10) imply that a player's winning probability, $F_i(\overline{a}_2^K|m)$, is linear in $b_1(1)$, since $(1 - b_i(m))$ is linear in $b_1(1)$. Since $\overline{a}_2^K$ is linear in $b_1(1)$, too, so are the payoffs, (5). The virtual rent, (6), is piecewise linear with a kink at $b_1(1) = \rho_2$.

Thus, the objective is (piece-wise) linear. Multiplying with p for readability yields

$$\begin{aligned}\Xi(b_1(1)) &:= p\left(\sum_i \mathbb{E}[\Psi_i|\mathcal{B}] + \mathbb{E}[U_i|\mathcal{B}]\right)\\ &= (\rho_1+\rho_2)U_1(1;1,\mathcal{B}) - (1-p)\sum_i \rho_i U_i(1;K,\mathcal{B}) + p(1-\rho_2)U_2(K;K,\mathcal{B}).\end{aligned} \tag{11}$$

Part B (Optimality).

Step 1: Type-independence. Linearity immediately implies that the optimal $b_1(1)$ includes a point at the boundary. The relevant boundaries are the lowest value and the highest value such that the solution yields a monotonic equilibrium, and the point at which the virtual rent has a kink. These three points are

$$\underline{b} = \frac{\rho_1}{K(1-\rho_2)+\rho_2}, \qquad \overline{b} = \frac{(K-1)(1-\rho_1)+\rho_2}{K(1-\rho_1)+\rho_1}, \qquad b^* = \rho_2.$$

We guess that the optimum is at $b^* = \rho_2$ taking Lemma 2 for granted and proceed.

Step 2: Type distribution. Given Lemma 2 we now determine the optimal ρ_i. Using the RHS of (11) and substituting $b_1(1) = \rho_2$ yields a quadratic objective in both ρ_is. Moreover, the first-order conditions are independent of each other. The unique solution is $(\rho_1, \rho_2) = ((1-p)/2, (1+p)/2)$.[24] Second-order conditions are satisfied at the desired point and we can conclude that a local optimum exist in case we face a least constraint problem. If $p \geq \underline{r}$, there always exists an α and thus an escalation rule such that the optimal solution satisfies the resource constraint, (2), with equality.

Step 3: Upward Incentive Constraints and Potential for signals. Downward incentive constraints are satisfied with equality by construction. However, so far we have ignored type 1's incentive constraint, $\gamma_i(1)U_i(1;1,\widehat{\mathcal{B}}) + z_i(1) \geq \gamma_i(K)U_i(K;1,\widehat{\mathcal{B}}) + z_i(K)$. Using $z_i(K) - z_i(1) = \gamma_i(1)U_i(1;K,\widehat{\mathcal{B}}) - \gamma_i(K)U_i(K;K,\widehat{\mathcal{B}})$ the condition becomes $\gamma_i(1)U_i(1;1,\widehat{\mathcal{B}}) - \gamma_i(K)U_i(K;1,\widehat{\mathcal{B}}) \geq \gamma_i(1)U_i(1;K,\widehat{\mathcal{B}}) - \gamma_i(K)U_i(K;K,\widehat{\mathcal{B}})$. Using type-independence we have that $U_i(1;k,\widehat{\mathcal{B}}) = U_i(K;k,\widehat{\mathcal{B}})$.

[24]By continuity of the objective the same holds true if we take the objective given $b_1(1) \geq \rho_i$ instead.

Incentive compatibility is thus satisfied if

$$\gamma_i(1) \geq \gamma_i(K) \Leftrightarrow \rho_i \geq p. \tag{12}$$

This always holds for player 1_2, but not for player 1_1 if $p > 1/3$. Now consider the following mechanism with public signals. There are two realizations, σ_1 and σ_2, both equally likely. If σ_1 realizes the mechanism proceeds as above, if σ_2 realizes, the mechanism flips players' identities. By ex-ante symmetry, the value of the problem remains constant and condition (12) holds by Assumption 1 as it becomes

$$(\gamma_i^{\sigma_1}(1) + \gamma_i^{\sigma_2}(1)) \geq (\gamma_i^{\sigma_1}(K) + \gamma_i^{\sigma_2}(K)) \Leftrightarrow \frac{1}{2} \geq p.$$

Step 4: Verifying local optimality. We now verify that type independence yields a local optimum. Assume to the contrary that $b_1(1) < \rho_2$ at the optimum. Substituting the claimed optimum into the objective we observe that

$$\begin{aligned} \Xi(b_1(1))|_{b_1(1)<\rho_2} = {} & F_1^K(\overline{a}_2^K)\left((1-b_1(1))\,\rho_2 + p(1-\rho_2)\right) \\ & - (1-b_2(1))\left((1-p)\rho_2 F_1^K(\overline{a}_2^K)\right) \\ & + \underbrace{\overline{a}_2^K}_{=(1-b_1(K))/K} \left((\rho_1+\rho_2)(K-1) - (1+\rho_2)Kp\right). \end{aligned} \tag{13}$$

The derivative changes sign at $b_1(1) = \rho_2$, since the derivative of the solution evaluated at the optimal point $\rho^* = (\rho_1 = (1-p)/2, \rho_2 = (1+p)/2)$ reads

$$\frac{\partial \Xi(b_1(1))}{\partial b_1(1)}|_{\rho^*} = \begin{cases} \frac{K(1-(p)^2)-(1-(p)^2)}{K(1+p)} & \text{if } b_1(1) < \rho_2 \\ -\frac{K(1-(p)^2)-(1-(p)^2)}{K(1+p)} & \text{if } b_1(1) > \rho_2 \\ \text{undefined} & \text{if } b_1(1) = \rho_2. \end{cases}$$

Step 5: Global Optimality. For global optimality we have to verify that the optimal solution given $b_1(1) \in \{\underline{b}, \overline{b}\}$ is worse than the optimum calculated here. Plugging into Ξ and solving yields the desired result.

Step 6: Implementability. Finally, we have to verify that the reduced-form mechanism is implementable. For that, we plug our solution into the constraints from Proposition 2 and obtain that these are satisfied. ☐

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