Convolution

Course: Computer Vision

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Outline

Definition

Etymology

Lat. *Convolvere*: *volvere* (roll), *com* (together).

Meaning

- Roll together. Entwine. Merge shapes.
- Combine one function (image) with another (filter).
- Impact of one function onto another.
- Result of blending two functions.

Common Examples

- A shadow is the convolution of a source of light and the shape of an object that obstructs the beams.
- An out-of-focus photography is the convolution of a sharp image and a blur circle.
- The echo is the convolution of a sound and an object that reflects it.

Mathematically

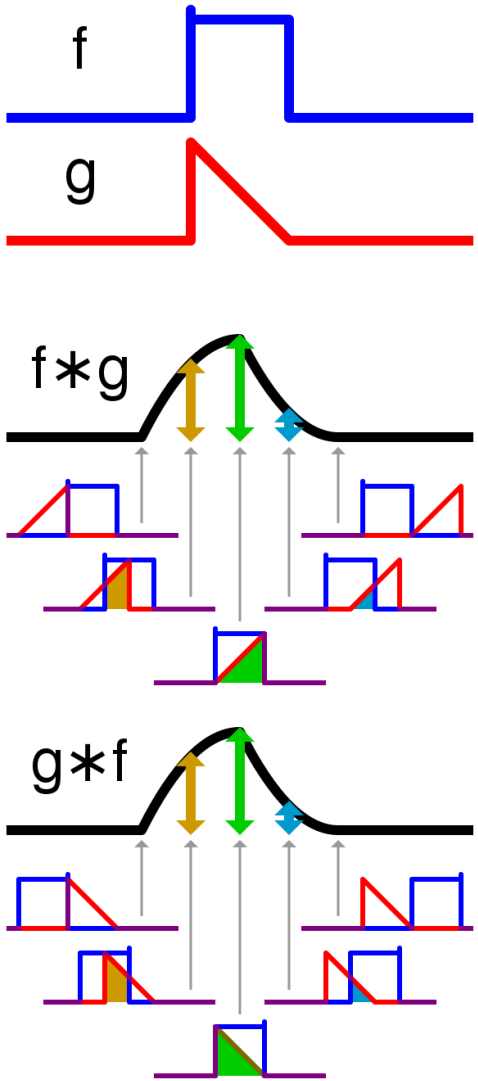
Continuous 1-D:

$$(f * g)(t) = \int_0^t f(t-\tau)g(\tau)d\tau.$$

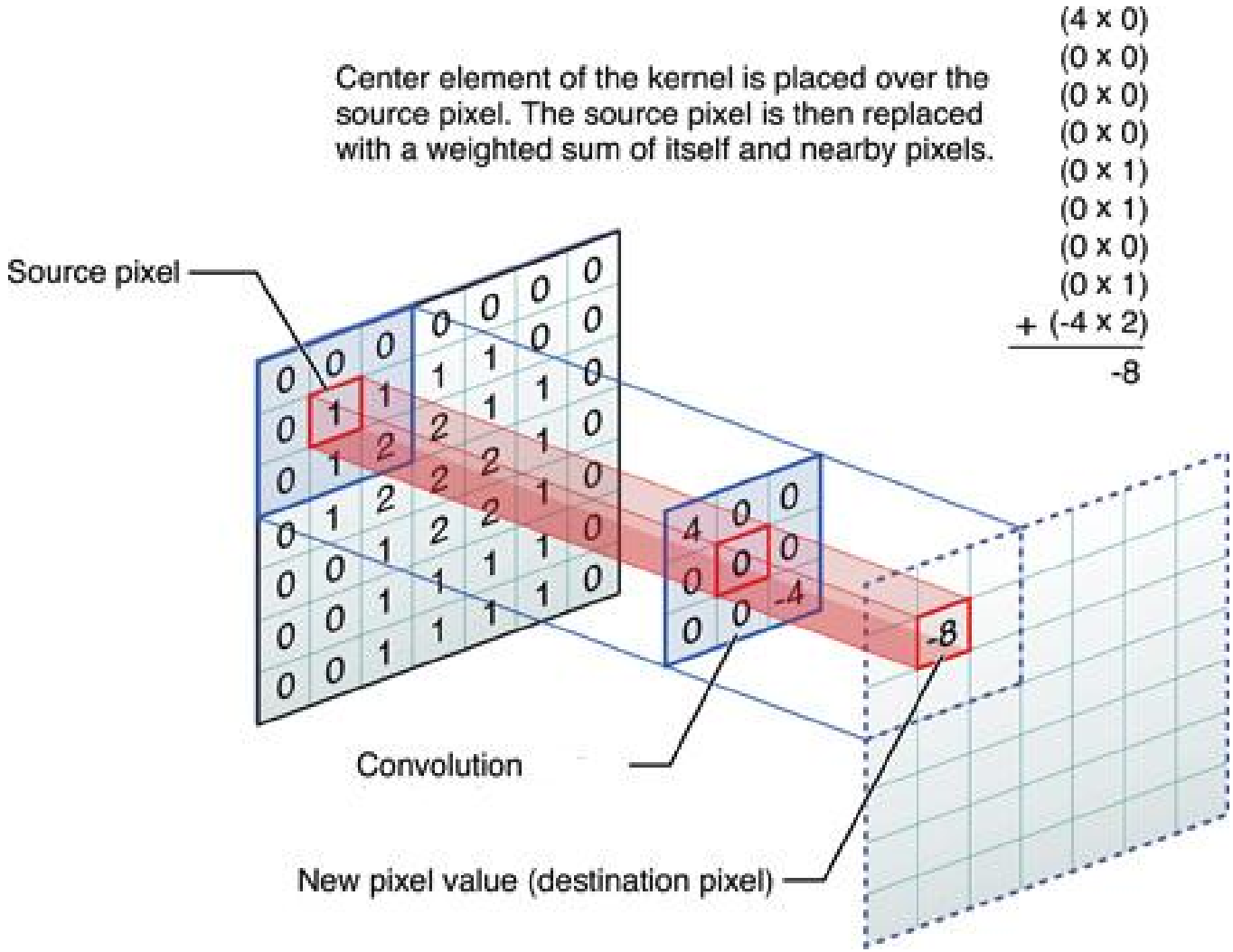
Discrete 2-D:

$$(I * k)[x, y] = \sum_{i,j} I[x-i, y-j]k[i, j].$$

1-D Continuous-Time



2-D Discrete


Center element of the kernel is placed over the source pixel. The source pixel is then replaced with a weighted sum of itself and nearby pixels.
(4 x 0)
(0 x 0)
(0 x 0)
(0 x 0)
(0 x 1)
(0 x 1)
(0 x 0)
(0 x 1)
+ (-4 x 2)
-8
Source pixel
Convolution
New pixel value (destination pixel)

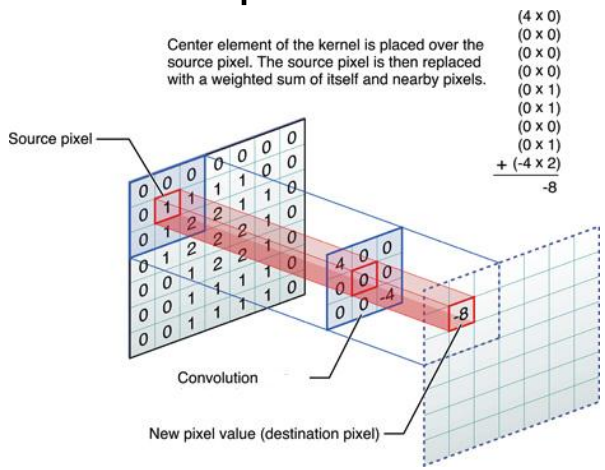
Examples of Image convolution

- Blur.
- Edge detector.
- Sharpening.
- etc.

https://aishack.in/tutorials/image-convolution-examples/

Resulting size

Convolution is not defined for pixels outside of the kernel support.



Resulting size:

$$I[M \times N] * k[m \times n] \rightarrow [M - m + 1 \times N - n + 1].$$

So, we lose some information.

Padding

Add a frame of dummy pixels, so k now fully overlaps I.

- Zero:
  $[5|4|2|3|7] \to [0|5|4|2|3|7|0]$.
- Ones:
  $[5|4|2|3|7] \to [1|5|4|2|3|7|1]$.
- Extended:
  $[5|4|2|3|7] \to [5|5|4|2|3|7|7]$.
- Cyclic:
  $[5|4|2|3|7] \to [7|5|4|2|3|7|5]$.
- Undefined:
  $[5|4|2|3|7] \to [5|4|2|3|7]$ (smaller result).

Properties

- Commutative: $f * g = g * f$.
- Associative: $f * (g * h) = (f * g) * h$.
- Distributive: $f * (g + h) = (f * g) + (f * h)$.
- Scaling: $a \cdot (f * g) = (a \cdot f) * g = f * (a \cdot g)$, a is a scalar.
- Identity: $f * \delta = f$, δ is a unitary impulse.
- Convolutional theorem: $\mathcal{F}\{f * g\} = \mathcal{F}\{f\}\mathcal{F}\{g\}$.

Properties: Laplacian-of-Gaussian I

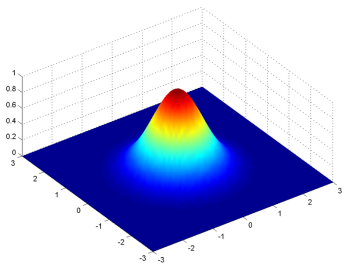
A Laplacian filter is useful to detect borders.

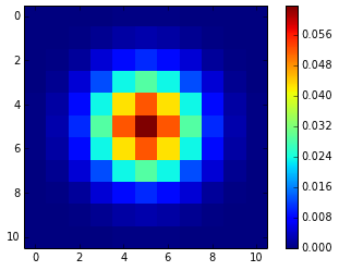
$$L[x, y] = \frac{\delta^2 I}{\delta x^2} + \frac{\delta^2 I}{\delta y^2}.$$

However, it is sensible to even small changes in the image, thus potentially resulting in spurious borders.

To ameliorate this effect, images are often first convolved with a Gaussian smoothing filter.

$$G[x, y] = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/(2\sigma^2)}.$$

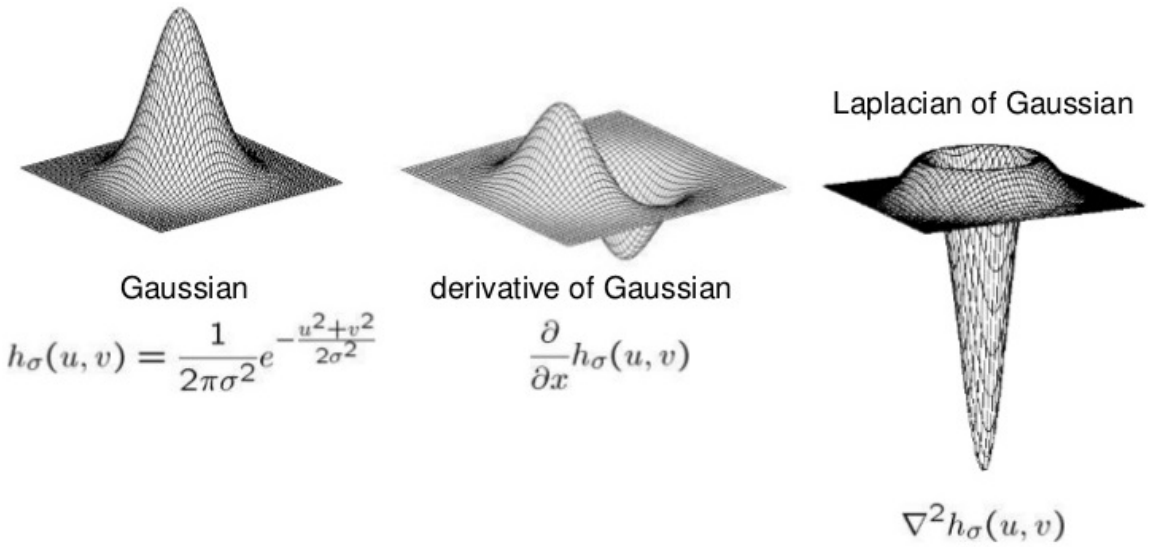




Properties: Laplacian-of-Gaussian II

Pre-convolve both kernels:

$$LoG = L * G.$$



More efficient scanning of the image.

Recap

- Image formation.
- Color spaces.
- Transforms.
- Operations.

Q&A

Thank you!

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