PROGRAMA PARA RESOLVER UN SISTEMA MASA-RESORTE USANDO LA TRANFORMADA DE LAPLACE

mx''+βx'+kx=f(t)

NOTA: la variable "t" se va acambiar por la variable "x" debido a que el programa solo acepta como variable independiente la "x"

ingrese los valores de m, β y k y f(t)

> $m\_ := \frac{1}{2}; \beta := 3; k\_ := 6; f\_ := 40\cdot\cos(3\cdot x)$

$$m\_ := \frac{1}{2}$$
$$\beta := 3$$
$$k\_ := 6$$
$$f\_ := 40\cos(3x) \quad (1)$$

Condiciones iniciales

> $x(0) := -2; xp(0) := 0$

$$x(0) := -2$$
$$xp(0) := 0 \quad (2)$$

>

> $a := 1; b := convert\left(\frac{\beta}{m\_}, rational\right); c := convert\left(\frac{k\_}{m\_}, rational\right); F := \frac{f\_}{m\_};$

$$a := 1$$
$$b := 6$$
$$c := 12$$
$$F := 80\cos(3x) \quad (3)$$

ecuación diferencial

> $a\cdot X'' + b\cdot X' + c\cdot X = F$

$$\frac{d^2}{dx^2}X(x) + 6\left(\frac{d}{dx}X(x)\right) + 12X(x) = 80\cos(3x) \quad (4)$$

>

Transformada de laplace y obtención de L{x}

> $with(inttrans):$

> $lf := laplace(F, x, s)$

$$lf := \frac{80\, s}{s^2+9} \tag{5}$$

> $(s^2 \cdot L(x) - s \cdot x(0) - xp(0)) + b \cdot (s \cdot L(x) - x(0)) + c \cdot L(x) = lf$

$$s^2 L(x) + 2\, s + 12 + 6\, s\, L(x) + 12\, L(x) = \frac{80\, s}{s^2+9} \tag{6}$$

> $Lx := solve((s^2 \cdot L(x) - s \cdot x(0) - xp(0)) + b \cdot (s \cdot L(x) - x(0)) + c \cdot L(x) = lf, L(x))$

$$Lx := -\frac{2\, (s^3 + 6\, s^2 - 31\, s + 54)}{(s^2+9)\, (s^2 + 6\, s + 12)} \tag{7}$$

Fracciones parciales y reescritura de L{x}

> $with(student):$

> $simplify(-2\, (s^3 + 6\, s^2 - 31\, s + 54) = (A \cdot s + B) \cdot (s^2 + 6\, s + 12) + (C \cdot s + \mathrm{D}) \cdot (s^2 + 9))$

$$-2\, s^3 - 12\, s^2 + 62\, s - 108 = A\, s^3 + C\, s^3 + 6\, A\, s^2 + B\, s^2 + \mathrm{D}\, s^2 + 12\, A\, s + 6\, B\, s + 9\, C\, s + 12\, B + 9\, \mathrm{D} \tag{8}$$

> $solve(\{A + C = -2, 6 \cdot A + B + \mathrm{D} = -12, 12 \cdot A + 6 \cdot B + 9 \cdot C = 62, 12 \cdot B + 9 \cdot \mathrm{D} = -108\}, \{A, B, C, \mathrm{D}\})$

$$\left\{A = \frac{80}{111}, B = \frac{480}{37}, C = -\frac{302}{111}, \mathrm{D} = -\frac{1084}{37}\right\} \tag{9}$$

> $Ls := convert(Lx, parfrac, s);$

$$Ls := \frac{1}{111}\, \frac{-302\, s - 3252}{s^2 + 6\, s + 12} + \frac{1}{111}\, \frac{80\, s + 1440}{s^2+9} \tag{10}$$

>

> $Lse := expand(Ls)$

$$Lse := -\frac{302}{111}\, \frac{s}{s^2 + 6\, s + 12} - \frac{1084}{37\, (s^2 + 6\, s + 12)} + \frac{80}{111}\, \frac{s}{s^2+9} + \frac{480}{37\, (s^2+9)} \tag{11}$$

> $Lsec := completesquare(Lse, s)$

$$Lsec := -\frac{302}{111}\, \frac{s}{(s+3)^2 + 3} - \frac{1084}{37\, ((s+3)^2 + 3)} + \frac{80}{111}\, \frac{s}{s^2+9} + \frac{480}{37\, (s^2+9)} \tag{12}$$

Tranformada inversa de Laplace

> $Y := invlaplace(Lsec, s, t)$

$$Y := \frac{80}{111} \cos(3\, t) + \frac{160}{37} \sin(3\, t) - \frac{2}{111}\, \mathrm{e}^{-3\, t} \left(391 \sqrt{3} \sin(\sqrt{3}\, t) + 151 \cos(\sqrt{3}\, t)\right) \tag{13}$$

Solución usando transformada de Laplace

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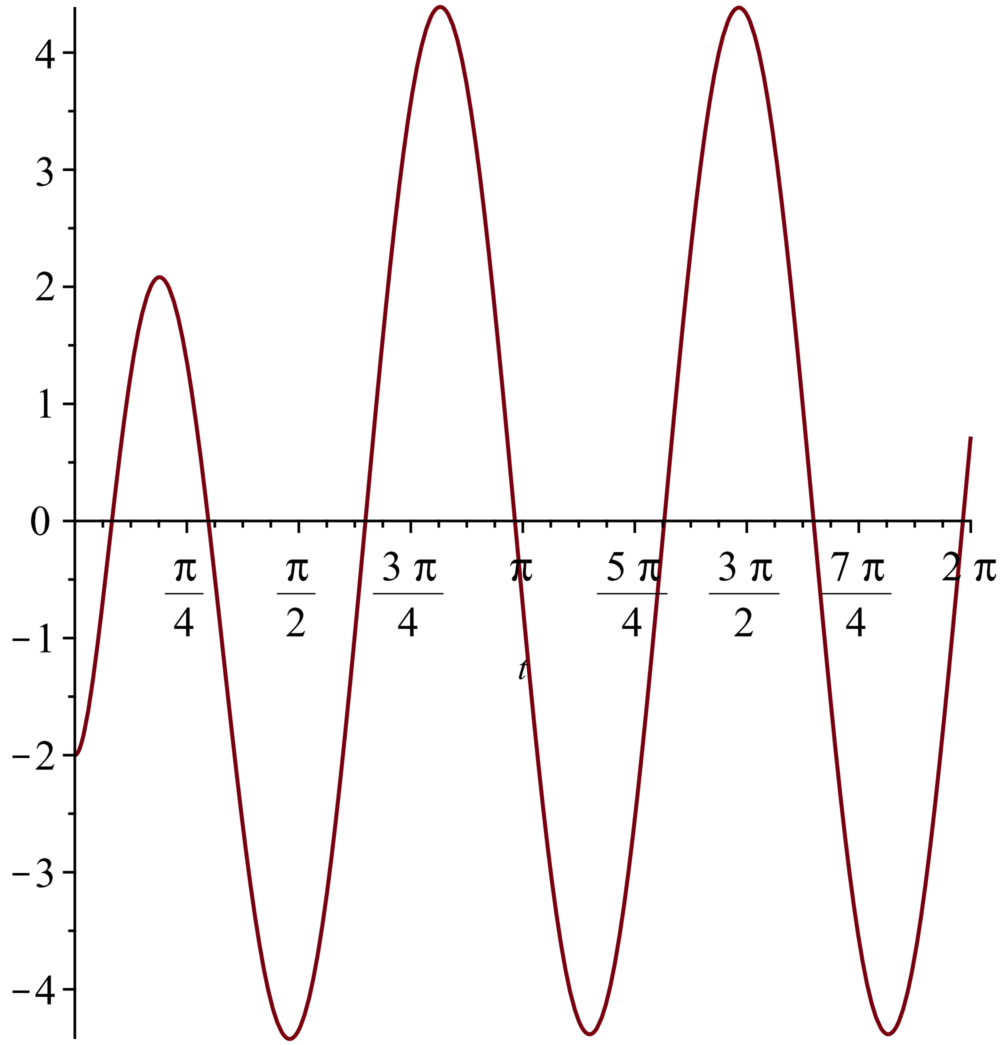
> $sol1 := dsolve([a \cdot X'' + b \cdot X' + c \cdot X = F, X(0) = x(0), \mathrm{D}(X)(0) = xp(0)], \{X(x)\}, method = laplace);$

$$sol1 := X(x) = \frac{80}{111} \cos(3\, x) + \frac{160}{37} \sin(3\, x) - \frac{2}{111}\, \mathrm{e}^{-3\, x} \left(391 \sqrt{3} \sin(\sqrt{3}\, x) + 151 \cos(\sqrt{3}\, x)\right) \tag{14}$$

> $plot(Y, t = 0 ..2 * \mathrm{Pi});$



4
3
2
1
0
−1
−2
−3
−4
π/4
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3 π/4
π
5 π/4
3 π/2
7 π/4
2 π
t