PROGRAMA PARA RESOLVER Y SIMULAR UN SISTEMA MASA-RESORTE

no estirado

posición de equilibrio
$mg - ks = 0$

$mX''+\beta X'+kX=f(t)$

Ingrese los valores de m, β y k y f(t)

> $m\_ := \dfrac{1}{4}; \beta\_ := 2; k\_ := 4; f\_ := 5 \cdot \cos(4 \cdot x)$

$$m\_ := \frac{1}{4}$$

$$\beta\_ := 2$$

$$k\_ := 4$$

$$f\_ := 5 \cos(4\,x) \tag{1}$$

> $a := 1; b := convert\left(\dfrac{\beta\_}{m\_}, rational\right); c := convert\left(\dfrac{k\_}{m\_}, rational\right); F := convert\left(\dfrac{f\_}{m\_}, rational\right);$

$$a := 1$$

$$b := 8$$

$$c := 16$$

$$F := 20 \cos(4\,x) \tag{2}$$

Ecuación diferencial

> $a \cdot X'' + b \cdot X' + c \cdot X = F$

$$\tag{3}$$

$$\frac{d^2}{dx^2} X(x) + 8 \left(\frac{d}{dx} X(x) \right) + 16 X(x) = 20 \cos(4\,x) \tag{3}$$

Solución ecuación complementaria

> $a \cdot m^2 + b \cdot m + c = 0$

$$m^2 + 8\,m + 16 = 0 \tag{4}$$

> $solve(a \cdot m^2 + b \cdot m + c = 0, m)$

$$-4, -4 \tag{5}$$

Conjunto de soluciones ecuación homogenea

> **if** $b^2 - 4 \cdot a \cdot c \geq 0$ **then**

$$m1 := \frac{-b + \sqrt{(b)^2 - 4 \cdot a \cdot c}}{2 \cdot a} \; ; m2 := \frac{-b - \sqrt{(b)^2 - 4 \cdot a \cdot c}}{2 \cdot a} \; \textbf{elif} \; b^2 - 4 \cdot a \cdot c < 0$$

$$\textbf{then } \alpha := \frac{-b}{2 \cdot a} \; ; \beta := \frac{\sqrt{|b^2 - 4 \cdot a \cdot c|}}{2 \cdot a}$$

$$\textbf{end if}$$

$$m1 := -4$$
$$m2 := -4 \tag{6}$$

> **if** $b^2 - 4 \cdot a \cdot c > 0$ **then** $X1 := \exp(m1 \cdot x); X2 := \exp(m2 \cdot x)$ **elif** $b^2 - 4 \cdot a \cdot c = 0$ **then** $X1 :=$ $\exp(m1 \cdot x); X2 := x \cdot \exp(m2 \cdot x)$ **elif** $b^2 - 4 \cdot a \cdot c < 0$ **then** $X1 := \exp(\alpha \cdot x) \cdot \cos(\beta \cdot x);$ $X2 := \exp(\alpha \cdot x) \cdot \sin(\beta \cdot x)$ **end if**

$$X1 := e^{-4x}$$
$$X2 := x\,e^{-4x} \tag{7}$$

Solución complementaria (transitoria)

> $Xc := c1 \cdot X1 + c2 \cdot X2$

$$Xc := e^{-4x} c1 + x\,e^{-4x} c2 \tag{8}$$

Calculo de wronskianos: w, w1 y w2

> $with(VectorCalculus):$

> $W := Wronskian([X1, X2], x)$

$$W := \begin{bmatrix} e^{-4x} & x\,e^{-4x} \\ -4\,e^{-4x} & e^{-4x} - 4\,x\,e^{-4x} \end{bmatrix} \tag{9}$$

> $with(LinearAlgebra):$

> $w := simplify(Determinant(W))$

$$w := e^{-8x} \tag{10}$$

> $W1 := \begin{bmatrix} 0 & X2 \\ F & \frac{d}{dx}(X2) \end{bmatrix}$

$$W1 := \begin{bmatrix} 0 & x\,e^{-4x} \\ 20 \cos(4\,x) & e^{-4x} - 4\,x\,e^{-4x} \end{bmatrix} \tag{11}$$

> $w1 := simplify(Determinant(W1), trig)$

$$w1 := -20\, x\, e^{-4x} \cos(4x) \tag{12}$$

> $W2 := \begin{bmatrix} X1 & 0 \\ \dfrac{d}{dx}\left(X1\right) & F \end{bmatrix}$

$$W2 := \begin{bmatrix} e^{-4x} & 0 \\ -4\, e^{-4x} & 20 \cos(4x) \end{bmatrix} \tag{13}$$

> $w2 := combine(Determinant(W2), trig)$

$$w2 := 20\, e^{-4x} \cos(4x) \tag{14}$$

Calculo de funciones: u1 y u2

> $u1 := combine\left(\displaystyle\int \frac{w1}{w}\ dx, trig\right)$

$$u1 := -\frac{5}{2}\, x\, e^{4x} \cos(4x) - \frac{5}{2}\, e^{4x} \sin(4x)\, x + \frac{5}{8}\, e^{4x} \sin(4x) \tag{15}$$

> $u2 := combine\left(\displaystyle\int \frac{w2}{w}\ dx, trig\right)$

$$u2 := \frac{5}{2}\, e^{4x} \cos(4x) + \frac{5}{2}\, e^{4x} \sin(4x) - \frac{5}{2}\, e^{4x} + \frac{5}{2}\, \left(e^{x}\right)^{4} \tag{16}$$

>

Solución particular (estado estable)

> $Xp := simplify(combine(u1\cdot X1 + u2\cdot X2, trig))$

$$Xp := \frac{5}{8}\, \sin(4x) \tag{17}$$

>

Solución general de la ecuación de posición

> $dsolve(\ a\cdot X'' + b\cdot X' + c\cdot X = F,\ \{\ X(x)\ \}\ )$

$$X(x) = e^{-4x}\, \_C2 + x\, e^{-4x}\, \_C1 + \frac{5}{8}\, \sin(4x) \tag{18}$$

> $X\_ := Xc + Xp$

$$X\_ := e^{-4x}\, c1 + x\, e^{-4x}\, c2 + \frac{5}{8}\, \sin(4x) \tag{19}$$

Solución general de la ecuación de velocidad

> $v := \dfrac{d}{dx}\ (X\_)$

$$v := -4\, e^{-4x}\, c1 + e^{-4x}\, c2 - 4\, x\, e^{-4x}\, c2 + \frac{5}{2}\, \cos(4x) \tag{20}$$

Condiciones iniciales

> $X\_0 := 0;\ V\_0 := -3$

$$X\_0 := 0$$

$$\tag{21}$$

$$V\_0 := -3 \tag{21}$$

Sistema de ecuaciones

> $simplify(eval(X\_, x=0) = X\_0)$

$$c1 = 0 \tag{22}$$

> $simplify(eval(v, x=0) = V\_0)$

$$-4\,c1 + c2 + \frac{5}{2} = -3 \tag{23}$$

Solución sistema de ecuaciones

> $solve(\{eval(X\_, x=0) = X\_0,\ (eval(v, x=0) = V\_0)\}, \{c1, c2\})$

$$\left\{ c1 = 0,\ c2 = -\frac{11}{2} \right\} \tag{24}$$

> $C1 := 0;\ C2 := -\dfrac{11}{2}$

$$C1 := 0$$

$$C2 := -\frac{11}{2} \tag{25}$$

Función de posición respecto al tiempo

> $dsolve['{:-interactive'}](\,a \cdot X'' + b \cdot X' + c \cdot X = F)$

$$X(x) = -\frac{11}{2}\,x\,\mathrm{e}^{-4x} + \frac{5}{8}\,\sin(4\,x) \tag{26}$$
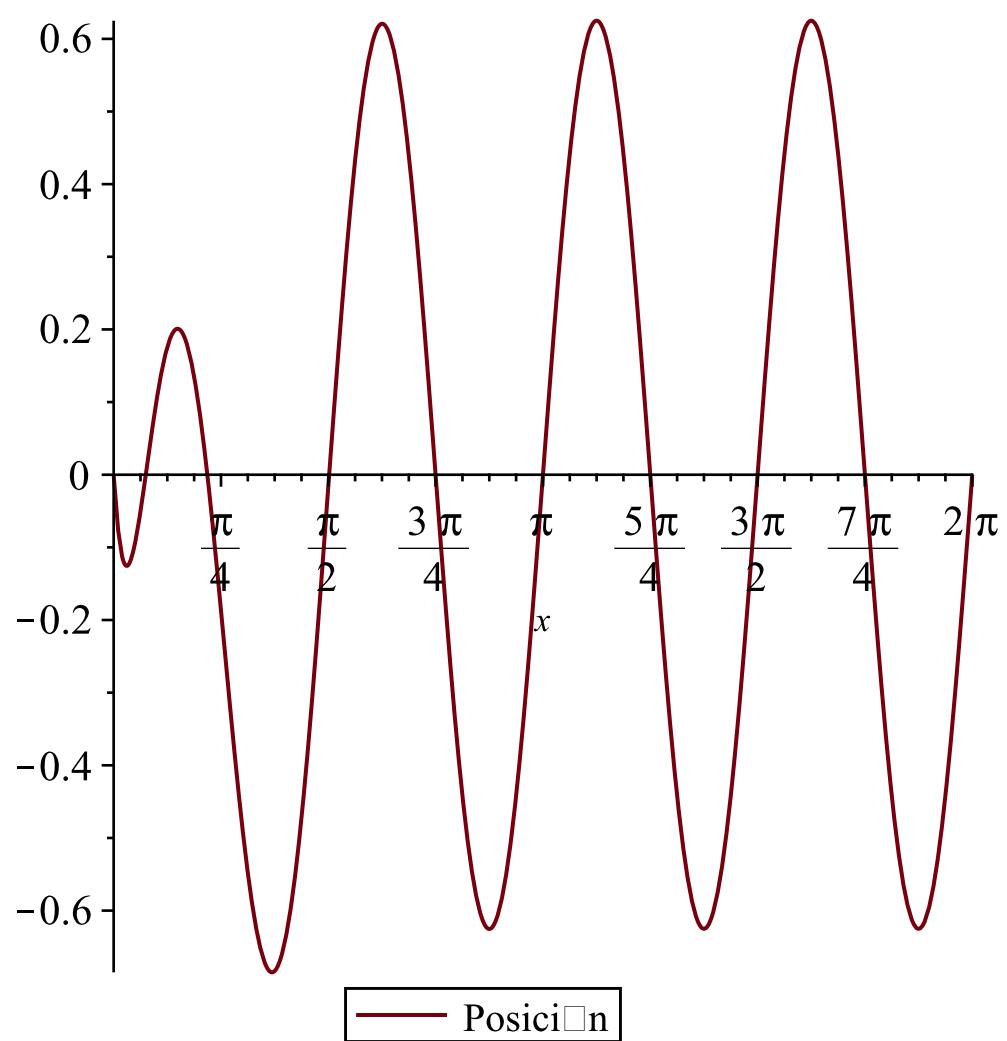
> $x\_ = eval(X\_, \{c1 = C1, c2 = C2\})$

$$x\_ = -\frac{11}{2}\,x\,\mathrm{e}^{-4x} + \frac{5}{8}\,\sin(4\,x) \tag{27}$$

Función de velocidad respecto al tiempo

> $v\_ := eval(v, \{c1 = C1, c2 = C2\})$

$$v\_ := -\frac{11}{2}\,\mathrm{e}^{-4x} + 22\,x\,\mathrm{e}^{-4x} + \frac{5}{2}\,\cos(4\,x) \tag{28}$$
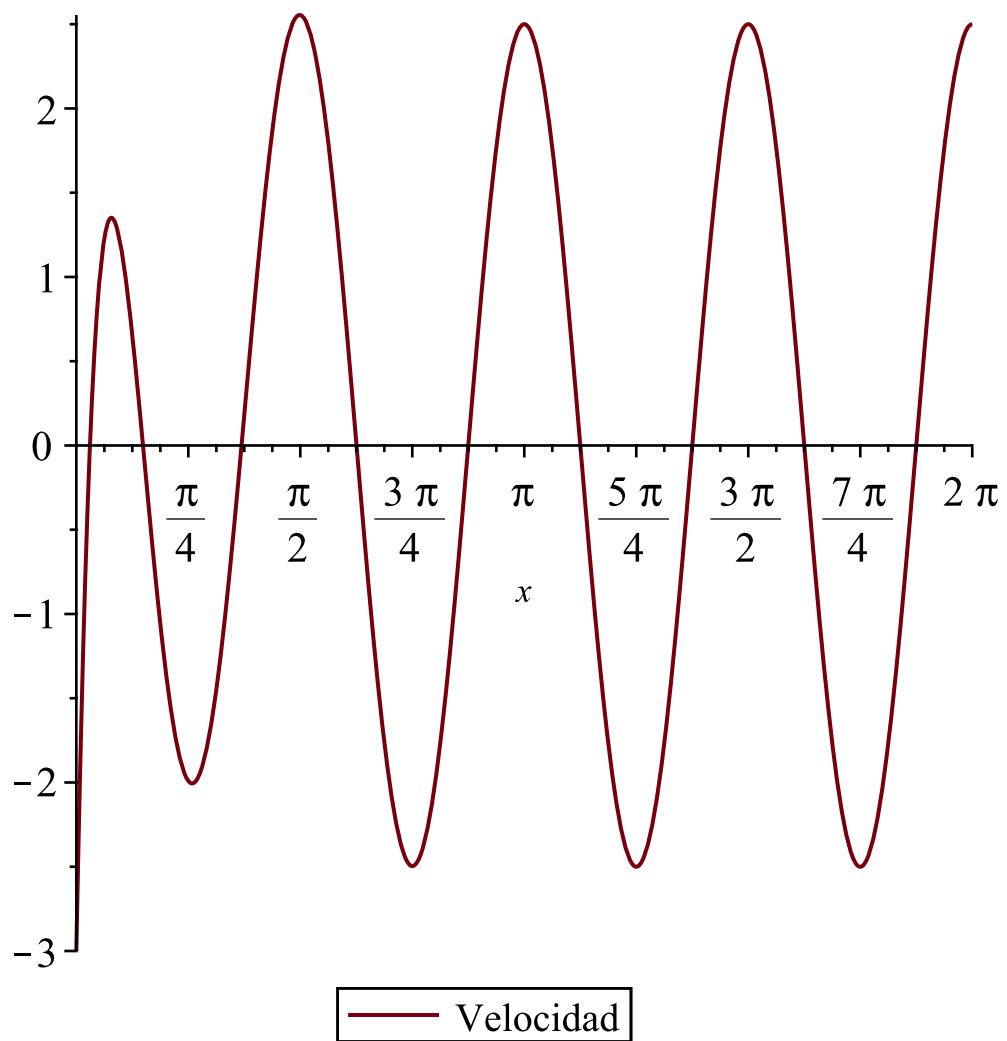
## Grafica de la función de posición respecto al tiempo

> $plot(eval(X\_, \{c1 = C1, c2 = C2\}), x = 0\,..\,2 \cdot Pi)$

Grafica de la función de velocidad respecto al tiempo

> $plot(v\_, x = 0 .. 2 \cdot \text{Pi})$
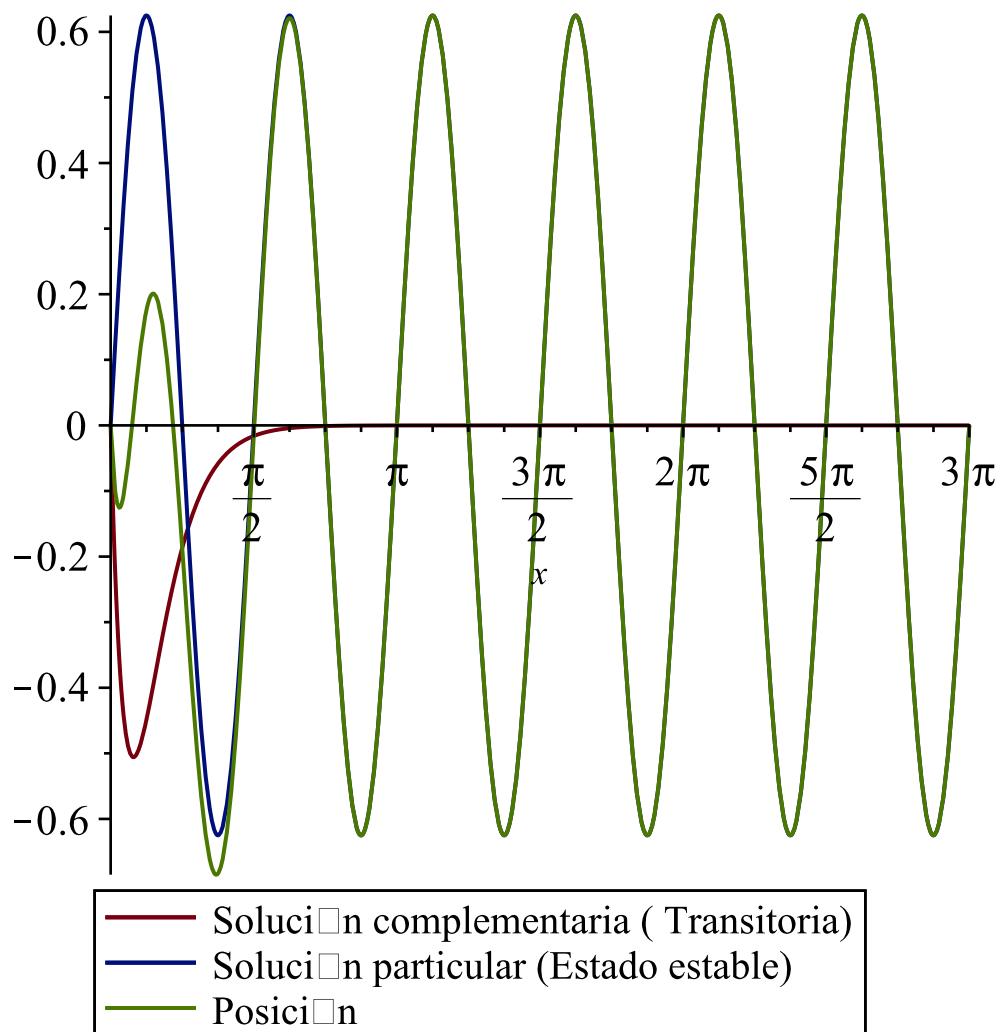
Grafica de la función de posición, solución transitoria y solucion particular

> $Xc\_ := eval(Xc, \{c1 = C1, c2 = C2\})$

$$Xc\_ := -\frac{11}{2} x\, e^{-4x} \tag{29}$$

> $plot(\{eval(X\_, \{c1 = C1, c2 = C2\}), Xc\_, Xp\}, x = 0 .. 3 \cdot \mathrm{Pi})$

——	Soluci□n complementaria (Transitoria)
——	Soluci□n particular (Estado estable)
——	Posici□n

Simulación sistema masa-resorte

```
> mass_spring := proc(m, r, k, x1, x2, x0, xp0, tk, f, n )
    local mass, deq, init, sol, xk, xu, plt1, plt2, plt, pltxk, rect, base, spring,
    spring1, rod, cylinder, piston, fluid, dashpot;
    with(plots) : with(plottools) :
        spring := proc(x1, x2, n) #procedure for the spring
          local p1, p2, p3, p4, pn_1, pn, p;
          p1 := [x1, 0];
          p2 := [x1 + 0.25, 0];
          p3 := [x2 - 0.25, 0];
          p4 := [x2, 0];
          p := i → [x1 + 0.25 + (x2 - x1 - 0.5)/n * i, (-1)^(i + 1) * 0.5];
        plot([p1, p2, seq(p(i), i = 1 ..n - 1), p3, p4], thickness = 2, color = aquamarine);
          end:
    spring1 := x2 → spring(x1, x2, 12);
    rect := t → translate(rectangle([-0.2, -0.2], [0.2, 0.2], color = red), t, xk(t) + x2 + 1);
    #rect: display the position of the mass on the displacement curve x(t)
    mass := x2 → rectangle([x2, -1], [x2 + 2, 1], color = red) :
        if x0 = 0 and xk(0.1) = 0 then
        xu := 1.5
        elif x0 = 0 and xk(0.1) ≠ 0 then
```

```
              xu := 5.0;
          else
              xu := x0;
          fi;
      rod := x2→plot( [ [x2 + 2, 0], [x2 + 2.5 + 2 * xu, 0] ], color = grey, thickness = 2 );
      cylinder := plot( [ [x2 + 2.5 + xu, -1], [x2 + 2.5 + xu, 1], [x2 + 3.5 + 3 * xu, 1], [x2 + 3.5
          + 3 * xu, -1],
      [x2 + 2.5 + xu, -1] ], color = tan, thickness = 2 );
      piston := x2→rectangle( [x2 + 2.5 + 2 * xu, -1], [x2 + 3 + 2 * xu, 1], color = grey );
      fluid := rectangle( [x2 + 2.5 + xu, -1], [x2 + 3.5 + 3 * xu, 1], color = blue );
      dashpot := x2→display( rod(x2), cylinder, piston(x2), fluid );
      deq := m * diff(x(t), t$2) + r * diff(x(t), t) + k * x(t) = f(t);
      print(deq);
      init := x(0) = x0, D(x)(0) = xp0 :
      sol := dsolve( {deq, init}, x(t) );
      print(combine(simplify(sol)));
      xk := unapply(rhs(sol), t);
      #xk=x(t) : the displacement of mass
      pltxk := plot( [xk(t) + x2 + 1, x2 + 1], t = 0 ..max(tk, x2 + 4 + 3 * xu) + 0.5,
      color = [blue, grey], numpoints = 400 );
      base := plot( -1.1, t = 0 ..max(tk, x2 + 4 + 3 * xu) + 0.5, color = brown, thickness = 4 );
      if r = 0 then #undamped case
      plt1 := x→translate(display(spring1(x2 + x), mass(x2 + x), base), 0, -3);
      else #with damping
      plt1 := x→translate(display(spring1(x2 + x), mass(x2 + x), dashpot(x2 + x), base), 0, -3);
      fi;
      plt2 := i→display( pltxk, rect(tk/ n * i) );
      plt := i→display( plt1(xk(tk/ n * i)), plt2(i) );
      display(seq( plt(i), i = 0 ..n), insequence = true, args[11 ..nargs]);
      end:
>   impulse_func :=proc(m, r, k, a, eps, n)
      local h, dlign1, dlign2, sol1, sol2, plt1, plt2, pltd, plt, txt, txtd, kloss, e;
      with(plots) :
      alias(u = Heaviside) :
      h := eps→( u(t-a) - u(t-a-eps) ) / eps;
      dlign1 := eps→m * diff(y(t), t, t) + r * diff(y(t), t) + k * y(t) = 5 * h(eps);
      dlign2 := m * diff(y(t), t, t) + r * diff(y(t), t) + k * y(t) = 5 * Dirac(t-a);
      print(dlign1(eps), epsilon = eps);
      print(dlign2);
      sol1 := eps→simplify(dsolve( {dlign1(eps), y(0) = 0, D(y)(0) = 0}, y(t) ));
      sol2 := dsolve( {dlign2, y(0) = 0, D(y)(0) = 0}, y(t) );
      e := textplot( [6, 1.3, convert( [101], bytes) ], font = [SYMBOL, 12] ) :
      txt := eps→textplot( [6.9, 1.3, cat(` = `, convert(evalf(eps, 2), string)) ]);
      txtd := textplot( [4, 1.4, `Response to Dirac's delta function`], align = {RIGHT, ABOVE});
      pltd := eps→plot(h(eps), t = 0 ..5 * Pi, color = green, thickness = 2 ) :
      plt1 := eps→plot(rhs(sol1(eps)), t = 0 ..5 * Pi, color = red, thickness = 2 ) :
      plt2 := plot(rhs(sol2), t = 0 ..5 * Pi, color = blue, thickness = 2 ) :
      plt := eps→display( pltd(eps), txt(eps), e, txtd, plt1(eps), plt2);
      display(seq( plt(eps- (eps-0.1) / n * i), i = 0 ..n), insequence = true, args[7 ..nargs]);
      end:
```

Ingrese las constantes de la siguiente manera
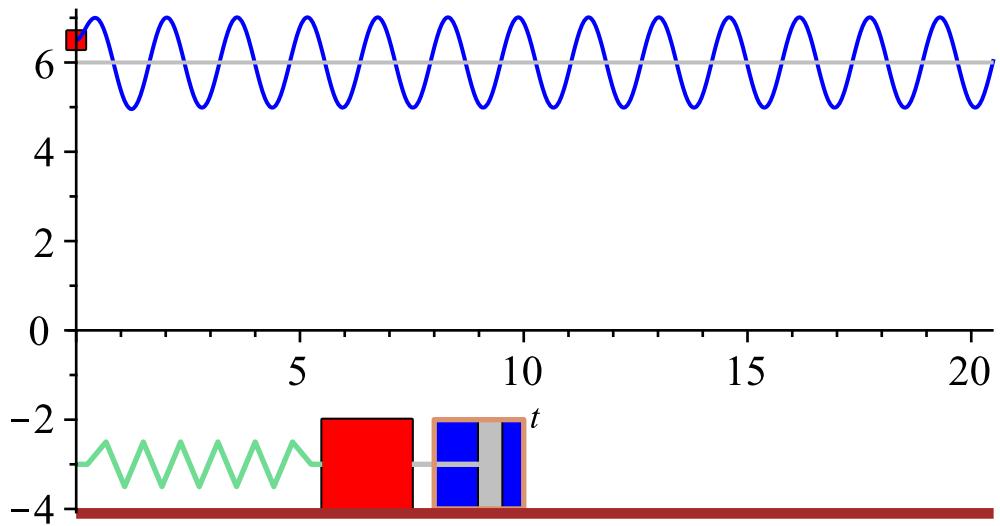
*(m: **masa**, r: **constante de amortiguamiento**, k: **constante elastica del resorte**, x1,x2: **posiciones del resorte** x0 = x(0), xp0=v(0): **condiciones iniciales**,tk:**longitud eje x**, f: **fuerza**, n: **numero de frames**)*

> $f\_ := t \rightarrow 5 \cdot \cos(4 \cdot t)$:

> $mass\_spring\left(\dfrac{1}{5}, 1.2, 2, 0, 5, \dfrac{1}{2}, 0, 20, f\_, 60, scaling = constrained \right);$

$$\frac{1}{5}\,\frac{d^2}{dt^2}\,x(t) + 1.2\left(\frac{d}{dt}\,x(t)\right) + 2\,x(t) = 5\cos(4\,t)$$

$$x(t) = -\frac{86}{51}\,e^{-3t}\sin(t) + \frac{38}{51}\,e^{-3t}\cos(t) - \frac{25}{102}\cos(4\,t) + \frac{50}{51}\sin(4\,t)$$

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