ECUACIONES DIFERENCIALES CON MAPLE

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PROGRAMA PARA RESOLVER ECUACIONES DIFERENCIALES LINEALES NO HOMOGENEAS CON COEFICIENTES CONSTANTES MÉTODO DE VARIACIÓN DE PARÁMETROS

ay''+by'+cy=f(x)

ingrese la ecuación diferencial

> $y''+4\cdot y=\frac{1}{2}\cdot\cos(2\cdot x)$

$$\frac{d^2}{dx^2}y(x)+4\,y(x)=\frac{1}{2}\cos(2\,x) \quad (1)$$

> $dsolve(\,(1), \{\,y(x)\,\}\,)$

$$y(x)=\sin(2\,x)\,\_C2+\cos(2\,x)\,\_C1+\frac{1}{16}\cos(2\,x)+\frac{1}{8}\sin(2\,x)\,x \quad (2)$$

> $dsolve['{:}\text{-}interactive']\left(y''+4\cdot y=\frac{1}{2}\cdot\cos(2\cdot x)\right)$

$$y(x)=\frac{1}{2}\sin(2\,x)+\frac{1}{8}\sin(2\,x)\,x \quad (3)$$

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ingrese los valores de a,b,c

> $a:=1;\,b:=0;\,c:=4$

$$a:=1$$
$$b:=0$$
$$c:=4 \quad (4)$$

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Ingrese f(x) para encontar la solución particular :

> $f:=\frac{1}{2}\cdot\cos(2\cdot x)$

$$f:=\frac{1}{2}\cos(2\,x) \quad (5)$$

> $F:=\frac{f}{a}$

$$F:=\frac{1}{2}\cos(2\,x) \quad (6)$$

> **if** $b^2 - 4\cdot a\cdot c \geq 0$ **then**

$$m1 := \frac{-b+\sqrt{(b)^2-4\cdot a\cdot c}}{2\cdot a}\ ;\ m2 := \frac{-b-\sqrt{(b)^2-4\cdot a\cdot c}}{2\cdot a}\ \textbf{elif}\ b^2-4\cdot a\cdot c<0$$

$$\textbf{then}\ m1 := \frac{-b}{2\cdot a}\ ;\ m2 := \frac{\sqrt{|b^2-4\cdot a\cdot c|}}{2\cdot a}$$

end if

$$m1 := 0$$
$$m2 := 2 \quad \textbf{(7)}$$

> **if** $b^2 - 4\cdot a\cdot c > 0$ **then** $y1 := \exp(m1\cdot x); y2 := \exp(m2\cdot x)$ **elif** $b^2 - 4\cdot a\cdot c = 0$ **then** $y1 := \exp(m1\cdot x); y2 := x\cdot\exp(m2\cdot x)$ **elif** $b^2 - 4\cdot a\cdot c < 0$ **then** $y1 := \exp(m1\cdot x)\cdot\cos(m2\cdot x);$ $y2 := \exp(m1\cdot x)\cdot\sin(m2\cdot x)$ **end if**

$$y1 := \cos(2\,x)$$
$$y2 := \sin(2\,x) \quad \textbf{(8)}$$

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la solución de la ecuación complemetaria es:

> $Yc := c1\cdot y1 + c2\cdot y2$

$$Yc := c1\cos(2\,x) + c2\sin(2\,x) \quad \textbf{(9)}$$

> $with(VectorCalculus)$:

> $W := Wronskian([y1, y2], x)$

$$W := \begin{bmatrix} \cos(2\,x) & \sin(2\,x) \\ -2\sin(2\,x) & 2\cos(2\,x) \end{bmatrix} \quad \textbf{(10)}$$

> $with(LinearAlgebra)$:

> $w := combine(Determinant(W))$

$$w := 2 \quad \textbf{(11)}$$

> $W1 := \begin{bmatrix} 0 & y2 \\ F & \frac{d}{dx}(y2) \end{bmatrix}$

$$W1 := \begin{bmatrix} 0 & \sin(2\,x) \\ \frac{1}{2}\cos(2\,x) & 2\cos(2\,x) \end{bmatrix} \quad \textbf{(12)}$$

> $w1 := simplify(Determinant(W1), trig)$

$$w1 := -\frac{1}{2}\sin(2\,x)\cos(2\,x) \quad \textbf{(13)}$$

> $W2 := \begin{bmatrix} y1 & 0 \\ \frac{d}{dx}y1 & F \end{bmatrix}$

(14)

$$W2 := \begin{bmatrix} \cos(2x) & 0 \\ -2\sin(2x) & \frac{1}{2}\cos(2x) \end{bmatrix} \quad (14)$$

> $w2 := simplify(Determinant(W2), trig)$

$$w2 := \frac{1}{2}\cos(2x)^2 \quad (15)$$

> $u1 := simplify\left(\int \frac{w1}{w}\,dx, trig\right)$

$$u1 := \frac{1}{16}\cos(2x)^2 \quad (16)$$

> $u2 := simplify\left(\int \frac{w2}{w}\,dx, trig\right)$

$$u2 := \frac{1}{16}\sin(2x)\cos(2x) + \frac{1}{8}x \quad (17)$$

> $yp := u1 \cdot y1 + u2 \cdot y2$

$$yp := \left(\frac{1}{16}\sin(2x)\cos(2x) + \frac{1}{8}x\right)\sin(2x) + \frac{1}{16}\cos(2x)^3 \quad (18)$$

la solución de la ecuación complemetaria es:

> $yps := combine(yp, trig)$

$$yps := \frac{1}{16}\cos(2x) + \frac{1}{8}\sin(2x)\,x \quad (19)$$

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solución general de la ecuacion diferencial

> $S := Yc + yps$

$$S := c1\cos(2x) + c2\sin(2x) + \frac{1}{16}\cos(2x) + \frac{1}{8}\sin(2x)\,x \quad (20)$$

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ingrese los valores iniciales para plantear las ecuaciones

> $simplify(eval(S, x = 0) = 0)$

$$c1 + \frac{1}{16} = 0 \quad (21)$$

> $simplify\left(eval\left(\frac{d}{dx} S, x = 0\right) = 1\right)$

$$2\,c2 = 1 \quad (22)$$

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solución del sistema de ecuaciones

> $solve\left(\left\{eval(S, x = 0) = 0, eval\left(\frac{d}{dx} S, x = 0\right) = 1\right\}, \{c1, c2\}\right)$

$$\left\{c1 = -\frac{1}{16}, c2 = \frac{1}{2}\right\} \quad (23)$$

solucion final

> $y(x) = \frac{1}{2} \sin(2x) + \frac{1}{8} \sin(2x) x$

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