ECUACIONES DIFERENCIALES CON MAPLE

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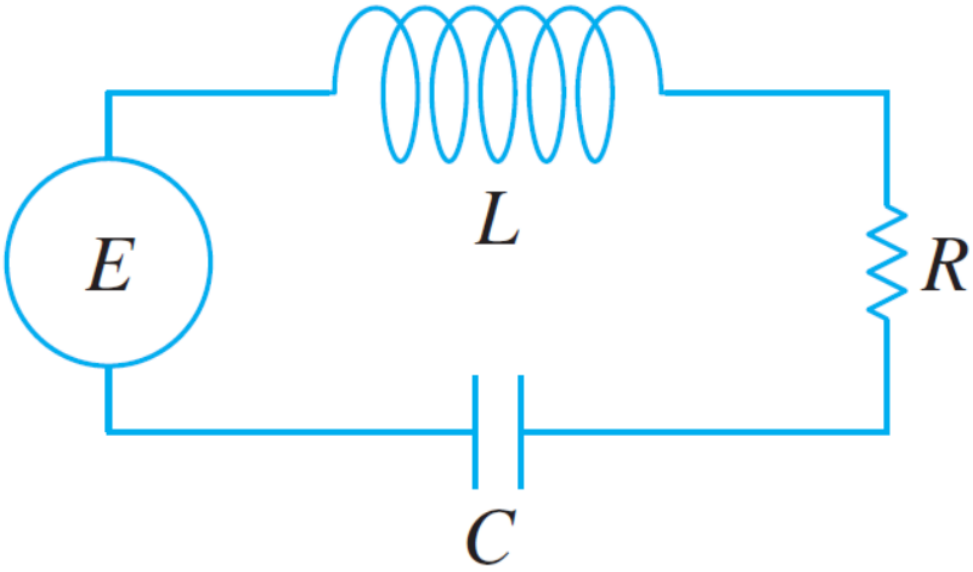
CIRCUITO EN SERIE RLC



$$Lq''+Rq'+\frac{1}{C}q=E(t)$$

Ingrese los valores de R, L, C y E

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> L := 0.5; R := 0; C := 0.005; E := cos(20·x)
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$$L := 0.5$$
$$R := 0$$
$$C := 0.005$$
$$E := \cos(20\,x) \quad \textbf{(1)}$$

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> a := 1; b := convert(R/L, rational); c := convert(1/(C·L), rational); F := convert(E/L,
      rational);
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$$a := 1$$
$$b := 0$$
$$c := 400$$
$$F := 2\cos(20\,x) \quad \textbf{(2)}$$

>

Ecuación diferencial

> $a \cdot q'' + b \cdot q' + c \cdot q = F$

$$\frac{d^2}{dx^2} q(x) + 400\, q(x) = 2\cos(20\, x) \quad (3)$$

>

Solución ecuación complementaria

> $a \cdot m^2 + b \cdot m + c = 0$

$$m^2 + 400 = 0 \quad (4)$$

> $solve(a \cdot m^2 + b \cdot m + c = 0, m)$

$$20\,I,\ -20\,I \quad (5)$$

Conjunto de soluciones ecuación homogenea

> **if** $b^2 - 4 \cdot a \cdot c \geq 0$ **then**

$$m1 := \frac{-b + \sqrt{(b)^2 - 4 \cdot a \cdot c}}{2 \cdot a} ; m2 := \frac{-b - \sqrt{(b)^2 - 4 \cdot a \cdot c}}{2 \cdot a} \textbf{ elif } b^2 - 4 \cdot a \cdot c < 0$$

$$\textbf{then } \alpha := \frac{-b}{2 \cdot a} ; \beta := \frac{\sqrt{|b^2 - 4 \cdot a \cdot c|}}{2 \cdot a}$$

end if

$$\alpha := 0$$

$$\beta := 20 \quad (6)$$

> **if** $b^2 - 4 \cdot a \cdot c > 0$ **then** $q1 := \exp(m1 \cdot x); q2 := \exp(m2 \cdot x)$ **elif** $b^2 - 4 \cdot a \cdot c = 0$ **then** $q1 := \exp(m1 \cdot x); q2 := x \cdot \exp(m2 \cdot x)$ **elif** $b^2 - 4 \cdot a \cdot c < 0$ **then** $q1 := \exp(\alpha \cdot x) \cdot \cos(\beta \cdot x); q2 := \exp(\alpha \cdot x) \cdot \sin(\beta \cdot x)$ **end if**

$$q1 := \cos(20\, x)$$

$$q2 := \sin(20\, x) \quad (7)$$

Solución complementaria (transitoria)

> $qc := c1 \cdot q1 + c2 \cdot q2$

$$qc := c1\cos(20\, x) + c2\sin(20\, x) \quad (8)$$

Calculo de wronskianos: w, w1 y w2

> $with(VectorCalculus)$:

> $W := Wronskian([q1, q2], x)$

$$W := \begin{bmatrix} \cos(20\, x) & \sin(20\, x) \\ -20\sin(20\, x) & 20\cos(20\, x) \end{bmatrix} \quad (9)$$

> $with(LinearAlgebra)$:

> $w := combine(Determinant(W))$

$$w := 20 \quad (10)$$

> $W1 := \begin{bmatrix} 0 & q2 \\ F & \frac{d}{dx}(q2) \end{bmatrix}$

$$W1 := \begin{bmatrix} 0 & \sin(20\,x) \\ 2\cos(20\,x) & 20\cos(20\,x) \end{bmatrix} \quad (11)$$

> $w1 := simplify(Determinant(W1), trig)$

$$w1 := -2\sin(20\,x)\cos(20\,x) \quad (12)$$

> $W2 := \begin{bmatrix} q1 & 0 \\ \frac{d}{dx} q1 & F \end{bmatrix}$

$$W2 := \begin{bmatrix} \cos(20\,x) & 0 \\ -20\sin(20\,x) & 2\cos(20\,x) \end{bmatrix} \quad (13)$$

> $w2 := combine(Determinant(W2), trig)$

$$w2 := \cos(40\,x) + 1 \quad (14)$$

Calculo de funciones: u1 y u2

> $u1 := simplify\left(\int \frac{w1}{w}\, dx, trig\right)$

$$u1 := \frac{1}{400}\cos(20\,x)^2 \quad (15)$$

> $u2 := simplify\left(\int \frac{w2}{w}\, dx, trig\right)$

$$u2 := \frac{1}{800}\sin(40\,x) + \frac{1}{20}\,x \quad (16)$$

>

Solución particular (estado estable)

> $qp := combine(u1 \cdot q1 + u2 \cdot q2, trig)$

$$qp := \frac{1}{400}\cos(20\,x) + \frac{1}{20}\sin(20\,x)\,x \quad (17)$$

Solución general de la ecuación de carga

> $dsolve(\{a \cdot q'' + b \cdot q' + c \cdot q = F, \{q(x)\}\})$

$$q(x) = \sin(20\,x)\,\_C2 + \cos(20\,x)\,\_C1 + \frac{1}{400}\cos(20\,x) + \frac{1}{20}\sin(20\,x)\,x \quad (18)$$

> $q\_ := qc + qp$

$$q\_ := c1\cos(20\,x) + c2\sin(20\,x) + \frac{1}{400}\cos(20\,x) + \frac{1}{20}\sin(20\,x)\,x \quad (19)$$

Solución general de la ecuación de corriente

> $i := \frac{d}{dx}(q\_)$

$$i := -20\,c1\,\sin(20\,x) + 20\,c2\,\cos(20\,x) + \cos(20\,x)\,x \tag{20}$$

Condiciones iniciales

> $q0 := 0;\ i0 := 0$

$$q0 := 0$$
$$i0 := 0 \tag{21}$$

Sistema de ecuaciones

> $simplify(eval(q\_, x = 0) = q0)$

$$c1 + \frac{1}{400} = 0 \tag{22}$$

> $simplify(eval(i, x = 0) = i0)$

$$20\,c2 = 0 \tag{23}$$

Solución sistema de ecuaciones

> $solve(\{eval(q\_, x = 0) = 0, eval(i, x = 0) = 0\}, \{c1, c2\})$

$$\left\{c1 = -\frac{1}{400}, c2 = 0\right\} \tag{24}$$

> $C1 := -\frac{1}{400};\ C2 := 0$

$$C1 := -\frac{1}{400}$$
$$C2 := 0 \tag{25}$$

Función de carga

> $dsolve[':\text{-}interactive'](a \cdot q'' + b \cdot q' + c \cdot q = F)$

$$q(x) = \frac{1}{20}\,\sin(20\,x)\,x \tag{26}$$

> $Q := eval(q\_, \{c1 = C1, c2 = C2\})$

$$Q := \frac{1}{20}\,\sin(20\,x)\,x \tag{27}$$

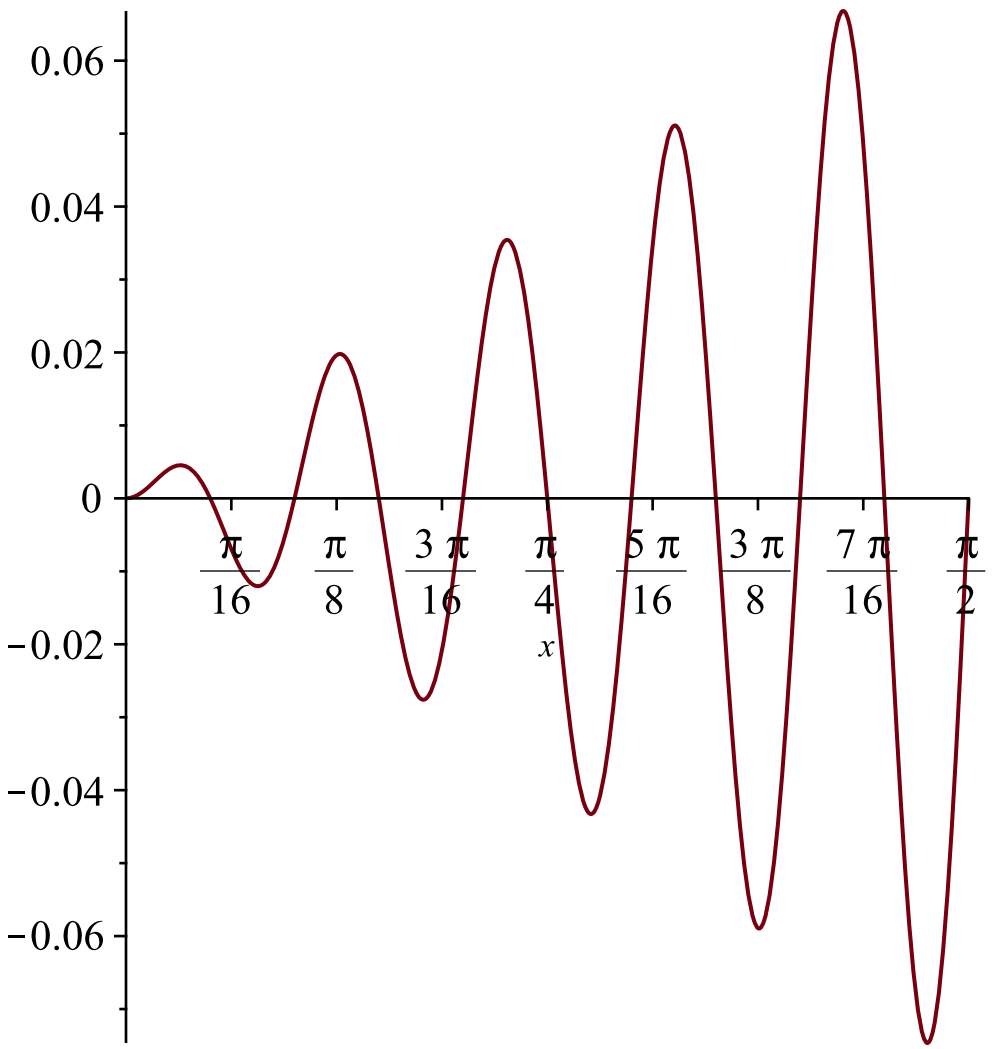
Función de corriente

> $i\_ := \frac{d}{dx} Q$

$$i\_ := \cos(20\,x)\,x + \frac{1}{20}\,\sin(20\,x) \tag{28}$$

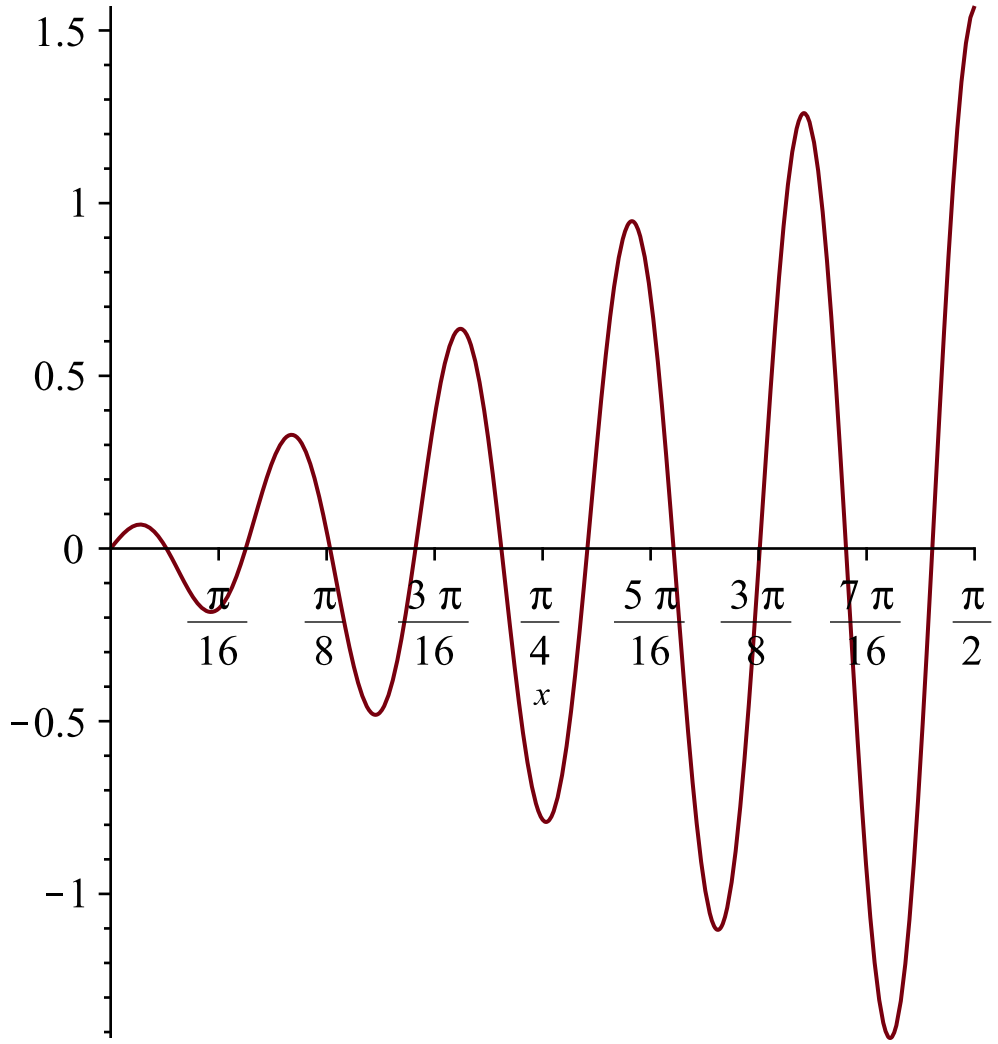
Grafica de la función de carga

> $plot\left(\{Q\}, x = 0 .. \frac{Pi}{2}\right)$



Grafica de la función de corriente

> $plot\left(\{i\_\}, x = 0 .. \frac{\mathrm{Pi}}{2} \right)$

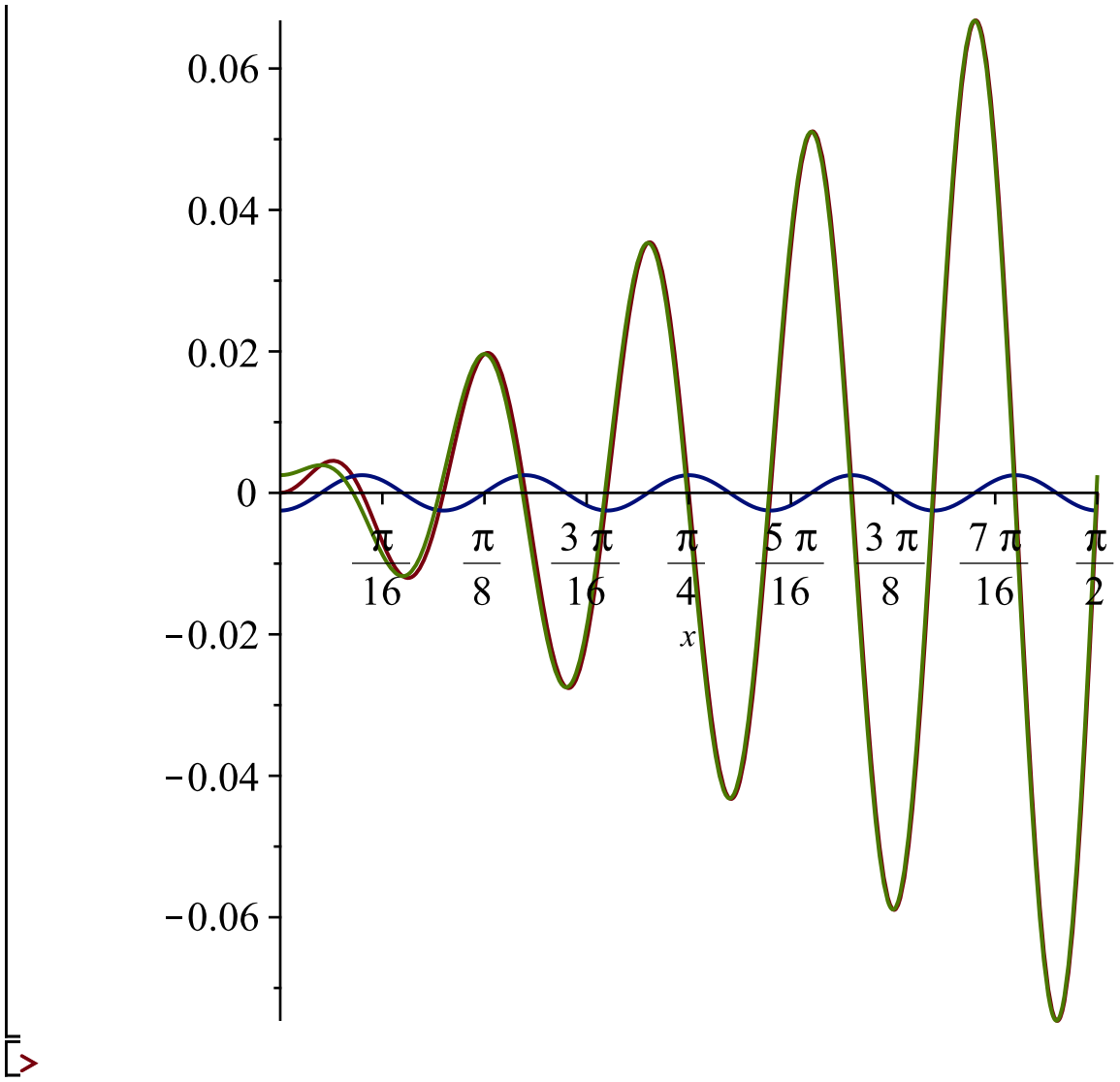


Grafica de la función de carga, solución transitoria y solucion particular

> $qc\_ := eval(qc, \{c1 = C1, c2 = C2\})$

$$qc\_ := -\frac{1}{400}\cos(20\,x) \tag{29}$$

> $plot\left(\{Q, qc\_, qp\}, x = 0 .. \frac{\mathrm{Pi}}{2}\right)$


0.06
0.04
0.02
0
−0.02
−0.04
−0.06
π/16
π/8
3π/16
π/4
5π/16
3π/8
7π/16
π/2
x