>

ECUACIONES DIFERENCIALES CON MAPLE

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PROGRAMA PARA RESOLVER ECUACIONES DIFERENCIALES LINEALES NO HOMOGENEAS CON COEFICIENTES CONSTANTES MÉTODO DE VARIACIÓN DE PARÁMETROS DE CAUCHY EULER

ay''+by'+cy=f(x)

ingrese la ecuación diferencial

> $x^2 \cdot y'' + 2 \cdot x \cdot y' + y = x^2$

$$x^2 \left(\frac{d^2}{dx^2} y(x)\right) + 2x\left(\frac{d}{dx} y(x)\right) + y(x) = x^2 \quad (1)$$

> *dsolve*(**(1)**, { $y(x)$ })

$$y(x) = \frac{\sin\left(\frac{1}{2}\sqrt{3}\ln(x)\right) \_C2}{\sqrt{x}} + \frac{\cos\left(\frac{1}{2}\sqrt{3}\ln(x)\right) \_C1}{\sqrt{x}} + \frac{1}{7}x^2 \quad (2)$$

> *dsolve*[':-interactive']$(x^2 \cdot y'' + 2 \cdot x \cdot y' + y = x^2)$

$$y(x) = \frac{16}{21}\frac{\sin\left(\frac{1}{2}\sqrt{3}\ln(x)\right)\sqrt{3}}{\sqrt{x}} + \frac{6}{7}\frac{\cos\left(\frac{1}{2}\sqrt{3}\ln(x)\right)}{\sqrt{x}} + \frac{1}{7}x^2 \quad (3)$$

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ingrese los valores de a,b,c

> $a := 1; b := 2; c := 1$

$$a := 1$$
$$b := 2$$
$$c := 1 \quad (4)$$

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Ingrese f(x) para encontar la solución particular :

> $f := x^2$

$$f := x^2 \quad (5)$$

> $F := \frac{f}{a \cdot x^2}$

$$F := 1 \quad (6)$$

> **if** $(b-a)^2 - 4 \cdot a \cdot c \geq 0$ **then**

$$m1 := \frac{-(b-a)+\sqrt{(b-a)^2-4\cdot a\cdot c}}{2\cdot a};\ m2 := \frac{-(b-a)-\sqrt{(b-a)^2-4\cdot a\cdot c}}{2\cdot a}$$

elif $(b-a)^2-4\cdot a\cdot c<0$ **then** $m1 := \frac{-(b-a)}{2\cdot a};\ m2 := \frac{\sqrt{\left|(b-a)^2-4\cdot a\cdot c\right|}}{2\cdot a}$

end if

$$m1 := -\frac{1}{2}$$

$$m2 := \frac{1}{2}\sqrt{3} \tag{7}$$

> **if** $(b-a)^2-4\cdot a\cdot c>0$ **then** $y1 := x^{m1}; y2 := x^{m2}$ **elif** $(b-a)^2-4\cdot a\cdot c=0$ **then** $y1 := x^{m1}$; $y2 := x^{m2}\cdot\ln(x)$ **elif** $(b-a)^2-4\cdot a\cdot c<0$ **then** $y1 := x^{m1}\cdot\cos(m2\cdot\ln(x)); y2 := x^{m1}\cdot\sin(m2\cdot\ln(x))$ **end if**

$$y1 := \frac{\cos\left(\frac{1}{2}\sqrt{3}\ln(x)\right)}{\sqrt{x}}$$

$$y2 := \frac{\sin\left(\frac{1}{2}\sqrt{3}\ln(x)\right)}{\sqrt{x}} \tag{8}$$

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la solución de la ecuación complemetaria es:

> $Yc := c1\cdot y1 + c2\cdot y2$

$$Yc := \frac{c1\cos\left(\frac{1}{2}\sqrt{3}\ln(x)\right)}{\sqrt{x}} + \frac{c2\sin\left(\frac{1}{2}\sqrt{3}\ln(x)\right)}{\sqrt{x}} \tag{9}$$

> $with(VectorCalculus):$

> $W := Wronskian([y1, y2], x)$

$$W := \left[\left[\frac{\cos\left(\frac{1}{2}\sqrt{3}\ln(x)\right)}{\sqrt{x}}, \frac{\sin\left(\frac{1}{2}\sqrt{3}\ln(x)\right)}{\sqrt{x}}\right], \left[-\frac{1}{2}\frac{\cos\left(\frac{1}{2}\sqrt{3}\ln(x)\right)}{x^{3/2}} - \frac{1}{2}\frac{\sin\left(\frac{1}{2}\sqrt{3}\ln(x)\right)\sqrt{3}}{x^{3/2}}, -\frac{1}{2}\frac{\sin\left(\frac{1}{2}\sqrt{3}\ln(x)\right)}{x^{3/2}} + \frac{1}{2}\frac{\cos\left(\frac{1}{2}\sqrt{3}\ln(x)\right)\sqrt{3}}{x^{3/2}}\right]\right] \tag{10}$$

> $with(LinearAlgebra):$

> $w := combine(Determinant(W))$

$$w := \frac{1}{2}\frac{\sqrt{3}}{x^2} \tag{11}$$

> $W1 := \begin{bmatrix} 0 & y2 \\ F & \frac{d}{dx}\left(y2\right) \end{bmatrix}$

$$W1 := \begin{bmatrix} 0 & \dfrac{\sin\left(\frac{1}{2}\sqrt{3}\ln(x)\right)}{\sqrt{x}} \\ 1 & -\dfrac{1}{2}\dfrac{\sin\left(\frac{1}{2}\sqrt{3}\ln(x)\right)}{x^{3/2}} + \dfrac{1}{2}\dfrac{\cos\left(\frac{1}{2}\sqrt{3}\ln(x)\right)\sqrt{3}}{x^{3/2}} \end{bmatrix} \quad \textbf{(12)}$$

> $w1 := combine(Determinant(W1))$

$$w1 := -\frac{\sin\left(\sqrt{3}\ln\left(\sqrt{x}\right)\right)}{\sqrt{x}} \quad \textbf{(13)}$$

> $W2 := \begin{bmatrix} y1 & 0 \\ \frac{d}{dx}y1 & F \end{bmatrix}$

$$W2 := \begin{bmatrix} \dfrac{\cos\left(\frac{1}{2}\sqrt{3}\ln(x)\right)}{\sqrt{x}} & 0 \\ -\dfrac{1}{2}\dfrac{\cos\left(\frac{1}{2}\sqrt{3}\ln(x)\right)}{x^{3/2}} - \dfrac{1}{2}\dfrac{\sin\left(\frac{1}{2}\sqrt{3}\ln(x)\right)\sqrt{3}}{x^{3/2}} & 1 \end{bmatrix} \quad \textbf{(14)}$$

> $w2 := combine(Determinant(W2))$

$$w2 := \frac{\cos\left(\sqrt{3}\ln\left(\sqrt{x}\right)\right)}{\sqrt{x}} \quad \textbf{(15)}$$

> $u1 := simplify\left(\int \frac{w1}{w}\,dx, trig\right)$

$$u1 := \int\left(-\frac{2}{3}x^{3/2}\sin\left(\frac{1}{2}\sqrt{3}\ln(x)\right)\sqrt{3}\right)dx \quad \textbf{(16)}$$

> $u2 := simplify\left(\int \frac{w2}{w}\,dx, trig\right)$

$$u2 := \int\frac{2}{3}x^{3/2}\cos\left(\frac{1}{2}\sqrt{3}\ln(x)\right)\sqrt{3}\,dx \quad \textbf{(17)}$$

> $yp := u1\cdot y1 + u2\cdot y2$

$$yp := \frac{\left(\int\left(-\frac{2}{3}x^{3/2}\sin\left(\frac{1}{2}\sqrt{3}\ln(x)\right)\sqrt{3}\right)dx\right)\cos\left(\frac{1}{2}\sqrt{3}\ln(x)\right)}{\sqrt{x}} \quad \textbf{(18)}$$

$$+\frac{\left(\int \frac{2}{3}\,x^{3/2}\cos\left(\frac{1}{2}\sqrt{3}\,\ln(x)\right)\sqrt{3}\,\mathrm{d}x\right)\sin\left(\frac{1}{2}\sqrt{3}\,\ln(x)\right)}{\sqrt{x}}$$

la solución de la ecuación particular es:

> $yps := simplify(yp, trig)$

$$yps := \frac{1}{\sqrt{x}}\left(\left(\int \frac{2}{3}\,x^{3/2}\cos\left(\frac{1}{2}\sqrt{3}\,\ln(x)\right)\sqrt{3}\,\mathrm{d}x\right)\sin\left(\frac{1}{2}\sqrt{3}\,\ln(x)\right)+\left(\int\left(-\frac{2}{3}\,x^{3/2}\sin\left(\frac{1}{2}\sqrt{3}\,\ln(x)\right)\sqrt{3}\right)\mathrm{d}x\right)\cos\left(\frac{1}{2}\sqrt{3}\,\ln(x)\right)\right) \quad \textbf{(19)}$$

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solución general de la ecuacion diferencial

> $S := Yc + yps$

$$S := \frac{c1\cos\left(\frac{1}{2}\sqrt{3}\,\ln(x)\right)}{\sqrt{x}}+\frac{c2\sin\left(\frac{1}{2}\sqrt{3}\,\ln(x)\right)}{\sqrt{x}}+\frac{1}{\sqrt{x}}\left(\left(\int \frac{2}{3}\,x^{3/2}\cos\left(\frac{1}{2}\sqrt{3}\,\ln(x)\right)\sqrt{3}\,\mathrm{d}x\right)\sin\left(\frac{1}{2}\sqrt{3}\,\ln(x)\right)+\left(\int\left(-\frac{2}{3}\,x^{3/2}\sin\left(\frac{1}{2}\sqrt{3}\,\ln(x)\right)\sqrt{3}\right)\mathrm{d}x\right)\cos\left(\frac{1}{2}\sqrt{3}\,\ln(x)\right)\right) \quad \textbf{(20)}$$

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ingrese los valores iniciales para plantear las ecuaciones

> $simplify(eval(S, x=1)=1)$

$$c1+\frac{1}{10}=1 \quad \textbf{(21)}$$

> $simplify\left(eval\left(\frac{\mathrm{d}}{\mathrm{d}x}S, x=1\right)=1\right)$

$$\frac{3}{10}+c2=1 \quad \textbf{(22)}$$

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solución del sistema de ecuaciones

> $solve\left(\left\{eval(S, x=1)=1, eval\left(\frac{\mathrm{d}}{\mathrm{d}x}S, x=1\right)=1\right\}, \{c1, c2\}\right)$

$$\left\{c1=\frac{9}{10}, c2=\frac{7}{10}\right\} \quad \textbf{(23)}$$

solucion final

$$y(x)=\frac{72}{203}\sqrt[4]{x}\,sin\left(\frac{1}{4}\sqrt{7}\,ln(x)\right)\sqrt{7}+\frac{28}{29}\sqrt[4]{x}\,cos\left(\frac{1}{4}\sqrt{7}\,ln(x)\right)+\frac{1}{29}\,x^{4}$$

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