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ENPM673

Homework 1

Problem 1:

Given:

d (width sensor) = 14mm

f (focal length) = 25mm

camera resolution = 5 MP

Required:

1) Field of View >> Horizontal and vertical direction

2) Min. no. of pixels for object given: object detected is square, width = 5cm, D (Distance) = 20m.

Solution

1) $\varphi = 2 * \tan^{-1}(d/2*f)$

$\varphi = 2* \tan^{-1}(14/(2*25))$

$\varphi = 31.28^0$

since the camera sensor is square shaped, the horizontal and vertical direction FOV should be the same.

2) camera Area to Pixel ratio = 14^2 mm^2 / 5 MP = 1 mm^2 / 25,511 pixels

For every 1 mm^2 area, there are about 25,511 pixels on the camera sensor.

Using similar shapes between camera and object, we find:

f (focal length) / D (Distance between camera and object) = x (length in camera) / w (object width)

x (length in camera) = 25 * 50 mm^2 / 20, 000 mm = 1/16 mm

Area = $(1/16)^2$ mm^2

# of pixels in camera = $(1/16)^2$ mm^2 * 25,511 pixels/ mm^2

# of pixels in camera = 100

Therefore, a minimum number of 100 pixels are needed to occupy object image.

Problem 2:

Given

Dataset from topmost and bottommost pixel of ball in video frame

Path of ball is parabolic

Required:

Use Standard Least Squares Solutions to fit curves

Solution:

Look at code for explanation of implementation of steps mentioned here:

$ax^2+bx+c = y$

A V = y

$$\begin{bmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ x_m^2 & x_m & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_m \end{bmatrix}$$

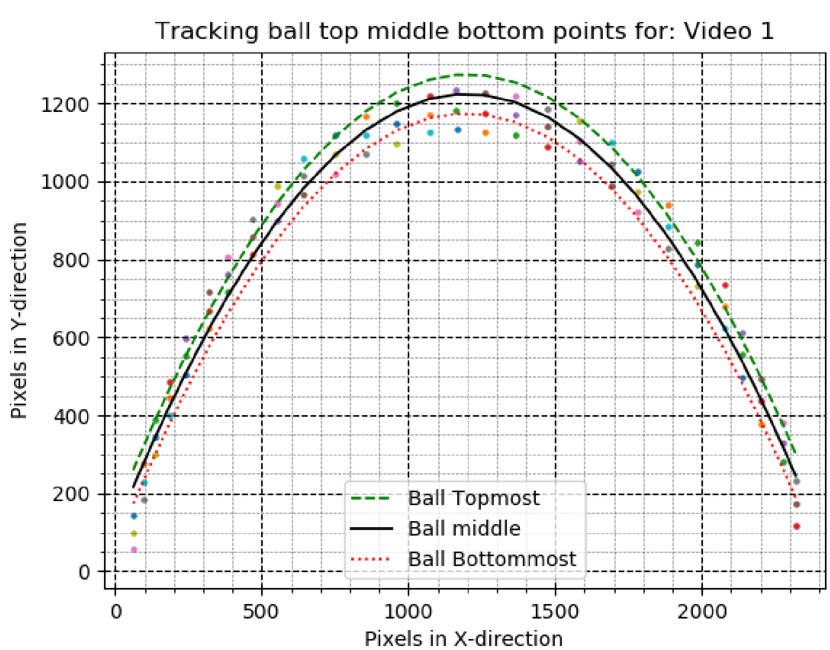
Pseudoinverse of the data was computed to evaluate the slope and intercept of least square regression line

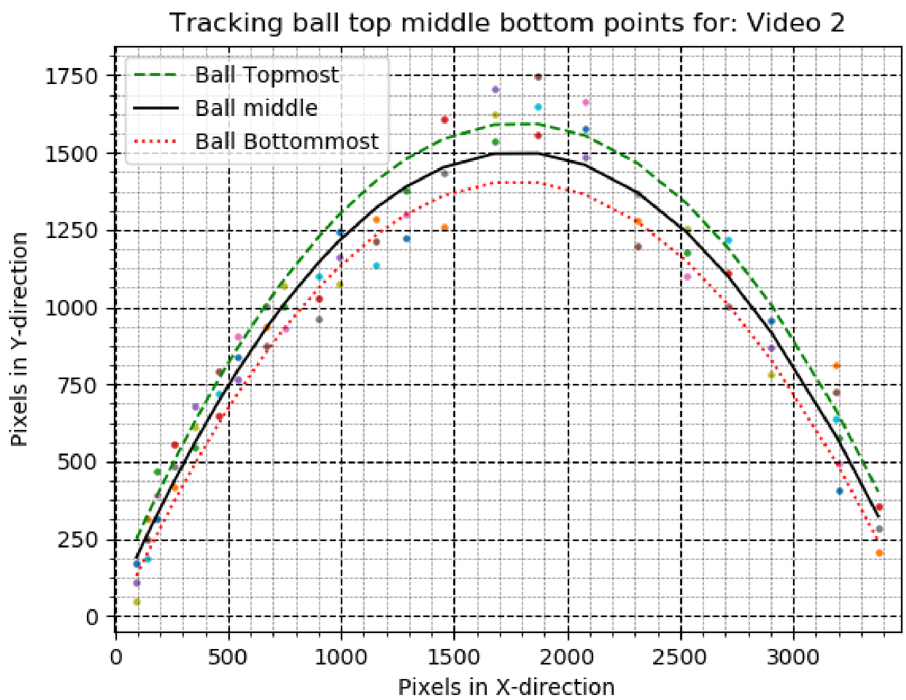
$V = (A^T A)^{-1} * A^T * y$

a = V[0] ; b = V[1] ; c = V[2]

Equation of Least Squares method Line will then be:

$a*x^2 + b*x + c = y$





Problem 3

Discussion of Least Square, Total Least Square, and RANSAC methods

Solution used to obtain regression methods

1) Capture data as an array and compute covariance matrix

2) obtained eigenvectors and eigenvalues from covariance matrix (part a. where eigenvectors were drawn with data)

3) pseudoinverse of the data was computed to evaluate the slope and intercept of least square regression line (part b. Least Sq line drawn with data)

4) eigenvectors and eigen values of the pseudo inverses were computed and data was adjusted by subtracting its mean from it. (part b. Total Least Sq. plotted with data)

5) for RANSAC method, random points were selected to create lines passing through the data for a selected number of iterations where the distance of data points from line were computed. Inliers were counted by giving error threshold, then the line with the most number of inliers were selected as the best fit for the given amount of iterations (part b. that selected best fit curve was drawn on the data)

Method used	Pros	Cons
Least Square Method	Easy to compute, gives an average distance between extremum points,	Does not catch outliers, not helpful for dispersed datasets
Total Least Square	Relatively easy to	Can miss in and outlier

Method	compute, tracks direction of data better than Least Square Method, uses normal lines between regression line and data set points which bettor estimates errors	samples as it gets drawn to data trend trying to align the line with data growth rate, not helpful for randomly dispersed data, ie if data spread not as obvious
RANSAC	Applies many attempts to obtain the best fit curve that captures the area where most data points reside in. Secludes outlier data far more precisely than the least square methods, gives higher confidence when analyzing data as more regression lines are computed to obtain error between data and trend	Time consuming, precision depends with number of iteration, ie not helpful for quick data analysis.

Conclusion: I recommend using RANSAC for this situation. Since the number of data points aggregating together far outweigh the outliers, and due to the outlying points are much distant from the grouped samples which the other methods could perturb as the line inclinations deviate towards those outliers or just lie right in between the group of samples and the outliers; I believe RANSAC confidently and far more accurately describes the trend than the other methods.

Problem 4

Find Homography matrix (Symbolic explanation. See the values in the code for exact numbers of the vector V which comprises of the homography matrix as indicated below).

A =

```
[[   -5    -5    -1     0     0     0   500   500   100]
 [    0     0     0    -5    -5    -1   500   500   100]
 [ -150    -5    -1     0     0     0 30000  1000   200]
 [    0     0     0  -150    -5    -1 12000   400    80]
 [ -150  -150    -1     0     0     0 33000 33000   220]
 [    0     0     0  -150  -150    -1 12000 12000    80]
 [   -5  -150    -1     0     0     0   500 15000   100]
 [    0     0     0    -5  -150    -1  1000 30000   200]]
```

A = U * E * V.T

U = AA.T

V = A.TA

E = diagonal of (eigValues(A.T A))

Finally SVD is found by combining the lowest column from E and lowest column from V.T

SVD = [E(last column), V.T(last column)]

X will be equal to V

So the homography matrix will be:

H = [V11 V12 V13}

[V21 V22 V23]

[V31 V32 V33] where V= [V1…. V9] numbered V11 V12 V13 V21…. V33