Yoseph A Kebede

Dr. Reza Monfaredi

ENPM662 – Intro To Robotic Modeling

Fall 2022

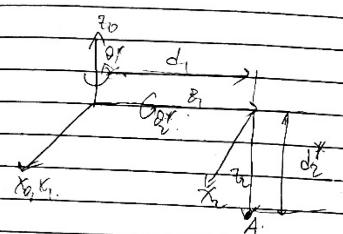
Final Exam

Final Exam Answer Sheet

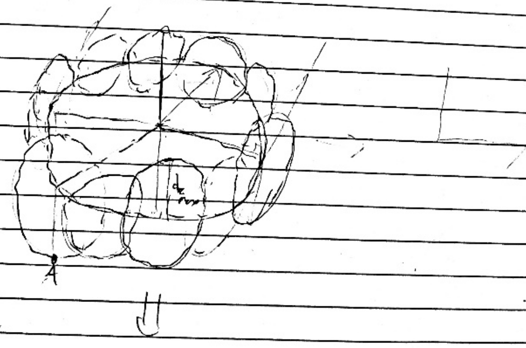
1.

Final Exam

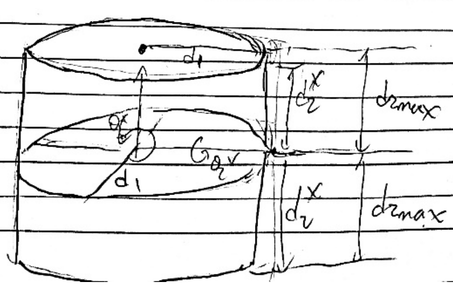
1) Workspace & Velocity Kinematics



1) Specify set of points reachable by end point A when θ_1, θ_2 rotate freely & $0 \le d_2 \le d_{2max}$

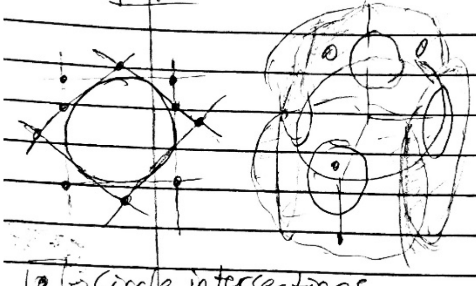


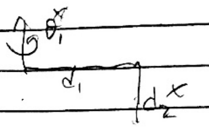
All points lie in this cylindrical space



2) Specify regions in workspace w/ regions based on no. of ways a pt in region can be reached

Top view Front





- By rotating link 3 to various angles and adjusting d_2 to be greater than d_1
$d_2 > d_1$.

- θ_1 rotating in circular motion

[0] → Circle intersections (lines coming out of paper)

3) Derive Analytical Jacobian for pt A

D-H parameters

	Link	a	α	d	θ
0→1	1	0	$-\pi/2$	0	$\theta_1(t)$
1→2	2	d_1	$-\pi/2$	d_1	$\theta_2(t)$
2→3	3	0	0	$d_2(t)$	0

Assume-Let $d_1 = 5$. $d_2 = 4$ $\theta_1(t) = 0$, $\theta_2(t) = \pi/2$

$$A_1 = \begin{bmatrix} \cos\theta_1(t) & -\sin(\theta_1(t))\cos(-\frac{\pi}{2}) & \sin(\theta_1(t))\sin(-\frac{\pi}{2}) & 0 \times \cos\theta(t) \\ \sin\theta_1(t) & \cos\theta(t)\cos(-\pi/2) & -\cos(\theta_1(t))\sin(-\frac{\pi}{2}) & d_1 \times \sin\theta(t) \\ 0 & \sin(-\pi/2) & \cos(-\pi/2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1^0 = A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = A_1 A_2 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 5 \\ -1 & 0 & 0 & 5 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \cos\theta_2(t) & -\sin\theta_2(t)\cos(-\frac{\pi}{2}) & \sin\theta_2(t)\sin(-\frac{\pi}{2}) & 0 \times \cos\theta(t) \\ \sin\theta_2(t) & \cos\theta_2(t)\cos(-\pi/2) & -\cos(\theta_2 t)\sin(-\frac{\pi}{2}) & d_1 \times \sin\theta(t) \\ 0 & \sin(-\frac{\pi}{2}) & \cos(-\pi/2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = A_1 A_2 A_3$$

$$T_3^0 = \begin{bmatrix} 0 & 0 & -1 & +4 \\ 0 & -1 & 0 & 5 \\ -1 & 0 & 0 & -5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 5 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix}$$

$$J_i = \begin{bmatrix} z_{i-1} \times (O_n - O_{i-1}) \\ z_{i-1} \end{bmatrix}$$

$$J_1 = \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left(\begin{bmatrix} -4 \\ 5 \\ -5 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right) \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ -4 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_2 = \begin{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \times \left(\begin{bmatrix} -4 \\ 5 \\ -5 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right) \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \\ 4 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$J_3 = \begin{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 4 - 0 \\ 5 - 5 \\ -5 - 5 \end{bmatrix} \\ -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{J = \begin{bmatrix} -5 & -5 & 0 \\ -4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}}$$

For
$d_1 = 5$, $d_2 = 4$
$\theta_1(t) = 0$ $\theta_2(t) = \pi/2$

2. 3D-3D Registration

Write a python code to estimate the 3D rigid transform that will optimally align the corresponding points in a set. Briefly describe your steps in the report as you Tabulate each of these transforms H, along with the mean error after aligning the point clouds i.e., 1 N PN i=1 ||x ' i – Hxi ||.

First, the three datasets are saved as a list and read into arrays as data sets.

Therefore, since each dataset contains 2 arrays holding points for the two-point cloud in each dataset, a total of 6 arrays are saved storing the 6-point clouds used as a comparison.

Then, by looping through each of the datasets, the below steps are followed to find the mean error for the aligning the points cloud.

$$\text{Lets assume } x = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \text{ and } x' = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \text{ in homogeneous coordinates}$$

$$\text{and } H = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix}$$

$$c \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

By eliminating c we can formulate the above equation in the form

$$Ah = 0$$

$$\text{where } A = \begin{pmatrix} -x & -y & -1 & 0 & 0 & 0 & ux & uy & u \\ 0 & 0 & 0 & -x & -y & -1 & vx & vy & v \end{pmatrix}$$
$$\text{and } h = \begin{pmatrix} h_1 & h_2 & h_3 & h_4 & h_5 & h_6 & h_7 & h_8 & h_9 \end{pmatrix}^T$$

Fig 1 How to find Homography[1]

1. The z coordinate for each data set is normalized such that homogenous coordinates are obtained as shown in figure 1 above, i.e. X = [x/z, y/z, 1] and X' = [x1/z1, y1/z1, 1]
2. Then, the matrix A was created by following the formula in figure 1 above.
3. After obtaining the A matrix, SVD(A) is calculated via

$$SVD(A) = U\ E\ V^T$$

$$U = A\ A^T$$

$$V = A^TA$$

Minimum Eigenvalue = min eigenvalue (V)

H = SVD(A) = eigenvector with smallest eigen value

4. H is reshaped into (3,3) matrix
5. The mean error used to align the point cloud is computed by:

- Multiplying the original point set with the homogenous matrix
  - H * x
- Then the squareroot of mean error is found for each point in the sets, then averaged
  - $\frac{1}{N}\sum_{i=1}^{N} ||x_i' - Hx_i||$

Thus, by performing the above 5 steps on each of the remaining datasets, the mean error is computed.

"Rotational motion of rigid bodies" – November 28, 2008

Reference: Analytical Dynamics – Graduate center CUNY – Fall 2008
Professor Dmitry Garanin

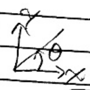
3. Dynamics

3.1 Part 1.

Given
-> Rigid body undergoing pure rotation
-> No external forces

Req
Prove
> $I_{xx}\dot{\omega}_x + (I_{zz} - I_{yy})\omega_y\omega_z = 0$
> $I_{yy}\dot{\omega}_y + (I_{xx} - I_{zz})\omega_z\omega_x = 0$
> $I_{zz}\dot{\omega}_z + (I_{yy} - I_{xx})\omega_x\omega_y = 0$

Assumptn
-> I_{xx}, I_{yy}, I_{zz} Principal moment of Inertia
-> $\omega_x, \omega_y, \omega_z$ angular velocity



Solⁿ

-> Total kinetic Energy of system

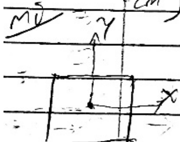
$$T = \underbrace{\left(\frac{\sum_{i=1}^{n} m_i}{2}\right)(\dot{x}^2 + \dot{y}^2)}_{\text{Linear}} + \underbrace{\left(\frac{\sum_{i=1}^{n} m_i a_i^2}{2}\right)\dot{\theta}^2}_{\text{Rotational}}$$

ρ – Mass density

-> To find C.M. x & y positions for 2D

$$x_{cm} = \int x\rho(x,y)\,dx \text{ or } x_{cm} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i} \text{ and } y_{cm} = \frac{\sum_{i=1}^{n} m_i y_i}{\sum_{i=1}^{n} m_i} \text{ or } y_{cm} = \int y\rho(x,y)\,dy$$

MI



-> Moment of Inertia about C.M. will be

$$M = \sum_{i=1}^{n} m_i a_i^2 = \sum_{i=1}^{n} m_i\left((x_i - x_{cm})^2 + (y_i - y_{cm})^2\right)$$

-> In the z-axis, Moment is

$$M_z = \iint (x - x_{cm})^2 + (y - y_{cm})^2 \rho(x,y)\,dx\,dy$$

limits are bounds of rigid mass

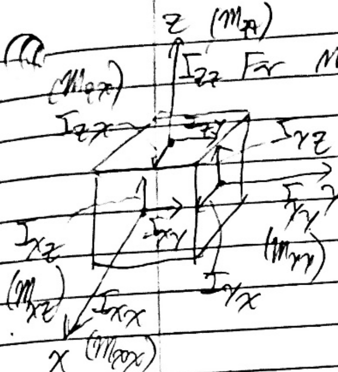
-> Other Moments of Inertia of m_i at (x_i, y_i)

$$M_{xy} = \sum_{i=1}^{n} m_i (x_i - x_{cm})(y_i - y_{cm})$$

-> Generalizing to continuous mass

$$I_{xy} = M_{xy} = \iint (x - x_{cm})(y - y_{cm})\rho(x,y)\,dx\,dy$$

integrate on the area facing X-dire. parallel to Y-axis



For Mass m_i - in 3D

$$x = \frac{\sum_{i=1}^{n+k} m_i x_i}{\sum_{i=1}^{n+k} m_i}, \quad y = \frac{\sum_{i=1}^{n+k} m_i y_i}{\sum_{i=1}^{n+k} m_i}, \quad z = \frac{\sum_{i=1}^{n+k} m_i z_i}{\sum_{i=1}^{n+k} m_i}$$

$$I_{zz} = M_z = \sum_{i=1}^{n+k} m_i\left((x_i - x)^2 + (y_i - y)^2\right)$$

3 Moments of Inertia about primary axes x, y, z:

$$I_{zz}\,(M_{zz}) = \iint (x - x_{cm})^2 + (y - y_{cm})^2 \rho(x,y)\,dx\,dy$$

$$I_{xx}\,(M_{xx}) = \iint (y - y_{cm})^2 + (z - z_{cm})^2 \rho(y,z)\,dy\,dz$$

$$I_{yy}\,(M_{yy}) = \iint (x - x_{cm})^2 + (z - z_{cm})^2 \rho(x,z)\,dx\,dz$$

Product of inertia one for each plane:

$$I_{xy}\,(M_{xy}) = \iint (x - x_{cm})(y - y_{cm})\,\rho(x,y)\,dx\,dy$$

$$I_{xz}\,(M_{xz}) = \iint (x - x_{cm})(z - z_{cm})\,\rho(x,z)\,dx\,dz$$

$$I_{yz}\,(M_{yz}) = \iint (y - y_{cm})(z - z_{cm})\,\rho(y,z)\,dy\,dz$$

→ Inertial tensor w/ C.M as origin

$$M_{CM} = \begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix}$$

→ Total KE of rigid body w/o Constraint: $\vec{V} = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}$

$$T = \frac{1}{2} m V^T V + \frac{1}{2} \omega^T M_{cm} \omega \qquad \vec{\omega} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

where V - linear speed of C.M ; ω - angular velocity of Cm

M_{cm} - Inertial tensor w.r.t C.M

→ Constraint: Body is undergoing pure rotation i.e. $V = 0$

→ Only $KE_{rotational}$: $T = \frac{1}{2} \omega^T M_{cm} \omega$

(ignore)

$$T = \frac{1}{2}\begin{bmatrix}\omega_x & \omega_y & \omega_z\end{bmatrix}\begin{bmatrix}I_{xx} & I_{xy} & I_{xz}\\ I_{yx} & I_{yy} & I_{yz}\\ I_{zx} & I_{zy} & I_{zz}\end{bmatrix}\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix}$$

$$\rightarrow T = \frac{1}{2}\begin{bmatrix}\omega_x & \omega_y & \omega_z\end{bmatrix}\begin{bmatrix}I_{xx}\omega_x + I_{xy}\omega_y + I_{xz}\omega_z\\ I_{yx}\omega_x + I_{yy}\omega_y + I_{yz}\omega_z\\ I_{zx}\omega_x + I_{zy}\omega_y + I_{zz}\omega_z\end{bmatrix}$$

Lagrange Eqs

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_1}\right) - \frac{\partial T}{\partial q_1} = Q_1$$

☆ no external force

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_1}\right) - \frac{\partial T}{\partial q_1} = 0$$

$$T = \frac{1}{2}\left[I_{xx}\omega_x^2 + I_{xy}\omega_x\omega_y + I_{xz}\omega_x\omega_z\right] + \left[I_{yx}\omega_x\omega_y + I_{yy}\omega_y^2 + I_{yz}\omega_y\omega_z\right] + \left[I_{zx}\omega_x\omega_z + I_{zy}\omega_y\omega_z + I_{zz}\omega_z^2\right]$$

$$T_x = \frac{1}{2}\left(I_{xx}\omega_x^2 + I_{xy}\omega_x\omega_y + I_{xz}\omega_x\omega_z\right)$$

$$T_y = \frac{1}{2}\left(I_{yx}\omega_x\omega_y + I_{yy}\omega_y^2 + I_{yz}\omega_y\omega_z\right)$$

$$T_z = \frac{1}{2}\left(I_{zx}\omega_x\omega_z + I_{zy}\omega_y\omega_z + I_{zz}\omega_z^2\right)$$

$$\omega = \dot{\theta}(t)$$

$$x: \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\omega}_x}\right) = 0 \qquad \frac{\partial T_x}{\partial \omega_x} = I_{xx}\omega_x + \frac{I_{xy}\omega_y}{2} + \frac{I_{xz}\omega_z}{2}$$

$$\rightarrow I_{xx}\omega_x + \frac{I_{xy}\omega_y}{2} + \frac{I_{xz}\omega_z}{2} = 0$$

$$y: \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\omega}_y}\right) = 0 \qquad \frac{\partial T_y}{\partial \omega_y} = \frac{I_{yx}\omega_x}{2} + I_{yy}\omega_y + I_{yz}\omega_z$$

$$\rightarrow \frac{I_{yx}\omega_x}{2} + I_{yy}\omega_y + I_{yz}\omega_z = 0$$

$$z: \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\omega}_z}\right) = 0 \qquad \frac{\partial T_z}{\partial \omega_z} = \frac{I_{zx}\omega_x}{2} + \frac{I_{zy}\omega_y}{2} + I_{zz}\omega_z^2 = 0$$

$$\rightarrow \frac{I_{zx}\omega_x}{2} + \frac{I_{zy}\omega_y}{2} + I_{zz}\omega_z^2 = 0$$

☆ No external force simplifies to principal axes

$$\frac{\partial T_{tot}}{\partial \omega} = I_{xx}\omega_x + I_{yy}\omega_y + I_{zz}\omega_z$$

Checking via

Angular momentum

$$L = M_{cm}\,\omega = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

★ There is No Ext Force

∴ Torque = 0

L (momentum) = Const.

→ The relation simplifies to principal coordinate sys

$$L = I_{xx}\omega_x + I_{yy}\omega_y + I_{zz}\omega_z$$

$$T_{rot} = \frac{1}{2}\left(\frac{L_x^2}{I_x} + \frac{L_y^2}{I_y} + \frac{L_z^2}{I_z}\right) \to \frac{1}{2}\left(I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2\right)$$

$$L = \frac{\partial T_{rot}}{\partial \omega} \to L_x = \frac{\partial T_{rot}}{\partial \omega_x} = I_x\omega_x$$

$$L_y = \frac{\partial T_{rot}}{\partial \omega_y} = I_y\omega_y$$

$$L_z = \frac{\partial T_{rot}}{\partial \omega_z} = I_z\omega_z$$

→ Now, taking (t)-time as a variable, i.e. angular momentum and the angular speed depend on time

differentiating → $L = \underbrace{I_{xx}\omega_x}_{L_1} + \underbrace{I_{yy}\omega_y}_{L_2} + \underbrace{I_{zz}\omega_z}_{L_3}$

$$\frac{d}{dt}(L) = \dot{L}_x x + \dot{L}_y y + \dot{L}_z z + L_x\dot{x} + L_y\dot{y} + L_z\dot{z}$$

$$L_x(\omega \times x) \quad L_y(\omega \times y) \quad L_z(\omega \times z)$$

$\hat{x} \times \hat{y} = \hat{z}$

$$\omega = \omega_x\hat{x} + \omega_y\hat{y} + \omega_z\hat{z}$$

$$\omega \times \hat{x} = \hat{y}\omega_z - \hat{z}\omega_y$$

$$\omega \times \hat{y} = \hat{z}\omega_x - \hat{x}\omega_z$$

$$\omega \times \hat{z} = \hat{x}\omega_y - \hat{y}\omega_x$$

Using $L_a = I_a\omega_a$, substituting to $\dot{L}$ one gets

$$\dot{L} = \left[\dot{L}_x - L_y\omega_z + L_z\omega_y\right]\hat{x} + \left[\dot{L}_y - L_z\omega_x + L_x\omega_z\right]\hat{y} + \left[\dot{L}_z - L_x\omega_y + L_y\omega_x\right]\hat{z}$$

$$\dot{L} = \left[I_{xx}\dot{\omega}_x - (I_{yy} - I_{zz})\omega_y\omega_z\right]\hat{x} + \left[I_{yy}\dot{\omega}_y - (I_{zz} - I_{xx})\omega_x\omega_z\right]\hat{y} + \left[I_{zz}\dot{\omega}_z - (I_{xx} - I_{yy})\omega_x\omega_y\right]\hat{z}$$

Since, L = const b/c of NO Ext. Force: $\dot{L} = 0$

Thus, pure angular rotation w/ no External Force
Generalized coordinates comes out to be

$\hat{x}: \; I_{xx}\dot{\omega}_x + (I_{zz} - I_{yy})\omega_y \omega_z = 0$

$\hat{y}: \; I_{yy}\dot{\omega}_y + (I_{xx} - I_{zz})\omega_z \omega_x = 0$

$\hat{z}: \; I_{zz}\dot{\omega}_z + (I_{yy} - I_{xx})\omega_x \omega_y = 0$

Part 2

· Derive Euler-Lagrange Eqs for 3 link planar RPR robot

Given
a_1: 1st link length
a_{1c}: center of 1st link
a_2: 2nd link length
a_{2c}: center of 2nd link
a_3: 3rd link length
a_{3c}: center of 3rd link

Req
-Derive Euler Lagrange eqs

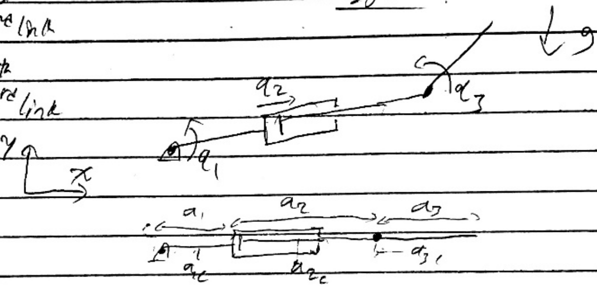
Assumptions
-gravity included

$L = T - V$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = Q_k$

No External Force or Moment

Sol'n



DOF
3
q_1
q_2
q_3

link 1 cm: $\begin{bmatrix} a_{1c}\cos q_1 \\ a_{1c}\sin q_1 \\ 0 \end{bmatrix} \Rightarrow Vel_{cm,1} = \begin{bmatrix} -a_{1c}\sin q_1 \dot{q}_1 \\ a_{1c}\cos q_1 \dot{q}_1 \\ 0 \end{bmatrix}$

link 2 cm: $\begin{bmatrix} (a_{1c} + a_{2c} + q_2)\cos q_1 \\ (a_{1c} + a_{2c} + q_2)\sin q_1 \\ 0 \end{bmatrix} \Rightarrow V_2 = \begin{bmatrix} \dot{q}_2 \cos q_1 - (a_{1c} + a_{2c} + q_2)\sin q_1 \dot{q}_1 \\ \dot{q}_2 \sin q_1 + (a_{1c} + a_{2c} + q_2)\cos q_1 \dot{q}_1 \\ 0 \end{bmatrix}$

link 3 cm: $\begin{bmatrix} (a_{1c} + a_{2c} + q_2)\cos q_1 + a_{3c}\cos q_3 \\ (a_{1c} + a_{2c} + q_2)\sin q_1 + a_{3c}\sin q_3 \\ 0 \end{bmatrix}$

$Velocity_3 = \begin{bmatrix} \dot{q}_2\cos q_1 - (a_{1c} + a_{2c} + q_2)\sin q_1 \dot{q}_1 - a_{3c}\sin q_3 \dot{q}_3 \\ \dot{q}_2\sin q_1 + (a_{1c} + a_{2c} + q_2)\cos q_1 \dot{q}_1 + a_{3c}\cos q_3 \dot{q}_3 \\ 0 \end{bmatrix}$

$a_{12} = a_{1c} + a_{2c}$

$$T = T_{cm} + T_{rot}$$
$$= \frac{1}{2}m(\dot{x}_1^2 + \dot{y}_1^2) = \frac{1}{2}m(a_{1c}^2 s_1^2 \dot{q}_1^2 + a_{1c}^2 c_1^2 \dot{q}_1^2) = \frac{1}{2}m_1 a_{1c}^2 \dot{q}_1^2$$

$V_1 = m_1 g a_{1c} s_1$

$$\frac{\partial T_1}{\partial \dot{q}_1} = m_1(a_{1c}^2 s_1^2 + a_{1c}^2 c_1^2)\dot{q}_1 = m_1 a_{1c}^2 (s_1^2 + c_1^2)\dot{q}_1$$
$$\frac{\partial T_1}{\partial \dot{q}_1} = m_1 a_{1c}^2 \dot{q}_1 \rightarrow \frac{d}{dt}\left(\frac{\partial T_1}{\partial \dot{q}_1}\right) = m_1 a_{1c}^2 \ddot{q}_1$$

$$T_2 = \frac{1}{2}m(\dot{x}_2^2 + \dot{y}_2^2) = \frac{1}{2}m\left((\dot{q}_2 c_1 - (a_{12}+q_2)s_1\dot{q}_1)^2 + (\dot{q}_2 s_1 + (a_{12}+q_2)c_1\dot{q}_1)^2\right)$$

$$T_2 = \frac{1}{2}m\left(\dot{q}_2^2 c_1^2 - 2\dot{q}_2 c_1 (a_{12}+q_2)s_1\dot{q}_1 + (a_{12}+q_2)^2 s_1^2 \dot{q}_1^2 + \dot{q}_2^2 s_1^2 + 2\dot{q}_2 s_1 (a_{12}+q_2)c_1\dot{q}_1 + (a_{12}+q_2)^2 c_1^2 \dot{q}_1^2\right)$$
$$= \frac{1}{2}m\left(\dot{q}_2^2(c_1^2 + s_1^2) + \dot{q}_1^2(s_1^2(a_{12}+q_2)^2 + (a_{12}+q_2)^2 c_1^2)\right) + \dot{q}_2\dot{q}_1(\cancel{2s_1c_1(a_{12}+q_2) - 2s_1c_1(a_{12}+q_2)})$$

$$T_2 = \frac{1}{2}m\left(\dot{q}_2^2 + \dot{q}_1^2(a_{12}+q_2)\right)$$

$$\frac{\partial T_2}{\partial \dot{q}_1} = m(a_{12}+q_2)\dot{q}_1 \rightarrow \frac{d}{dt}\left(\frac{\partial T_2}{\partial \dot{q}_1}\right) = m(a_{12}+q_2)\ddot{q}_1$$

$$\frac{\partial T_2}{\partial q_1} = 0, \quad \frac{\partial T_2}{\partial \dot{q}_2} = m\dot{q}_2 \rightarrow \frac{d}{dt}\left(\frac{\partial T_2}{\partial \dot{q}_2}\right) = m\ddot{q}_2$$

$V_2 = m_2 g (a_{12} + a_{2c} + q_2) s_1$

$$\frac{\partial T_2}{\partial q_2} = q_2 \dot{q}_1^2$$

$$T_3 = \frac{1}{2}m\left((\dot{q}_2 c_1 - (a_{12}+q_2)s_1\dot{q}_1 - a_{3c}s_3\dot{q}_3)^2 + (\dot{q}_2 s_1 + (a_{12}+q_2)c_1\dot{q}_1 + a_{3c}c_3\dot{q}_3)^2\right)$$

$$\dot{x}^2 = \dot{q}_2^2 c_1^2 - \dot{q}_2 c_1(a_{12}+q_2)s_1\dot{q}_1 - \dot{q}_2 c_1 a_{3c} s_3 \dot{q}_3 - \dot{q}_2 c_1 (a_{12}+q_2)s_1\dot{q}_1 + (a_{12}+q_2)^2 s_1^2 \dot{q}_1^2$$
$$- (a_{12}+q_2)s_1\dot{q}_1 a_{3c} s_3 q_3 - \dot{q}_2 c_1 a_{3c} s_3 \dot{q}_3$$
$$+ (a_{12}+q_2)s_1\dot{q}_1 a_{3c} s_3 \dot{q}_3 + a_{3c}^2 s_3^2 \dot{q}_3^2$$

$$\boxed{\dot{x}^2 = \dot{q}_2^2 c_1^2 - 2\dot{q}_2 c_1 a_{3c} s_3 \dot{q}_3 + (a_{12}+q_2)^2 s_1^2 \dot{q}_1^2 + a_{3c}^2 s_3^2 \dot{q}_3^2}$$

$$\dot{y}^2 = \dot{q}_2^2 s_1^2 + \dot{q}_2 s_1 (a_{12}+q_2) c_1 \dot{q}_1 + \dot{q}_2 s_1 a_{3c} c_3 \dot{q}_3 + \dot{q}_2 s_1 (a_{12}+q_2) c_1 \dot{q}_1 + (a_{12}+q_2)^2 c_1^2 \dot{q}_1^2 + (a_{12}+q_2)c_1\dot{q}_1 a_{3c} c_3$$
$$+ a_{3c} c_3 \dot{q}_3 \dot{q}_2 s_1 + (a_{12}+q_2)c_1\dot{q}_1 a_{3c} c_3 \dot{q}_3 + a_{3c}^2 c_3^2 \dot{q}_3^2$$

$$\boxed{\dot{y} = \dot{q}_2^2 s_1^2 + 2\dot{q}_2 s_1 (a_{12}+q_2) c_1 \dot{q}_1 + 2\dot{q}_2 s_1 a_{3c} s_3 \dot{q}_3 + (a_{12}+q_2)^2 c_1^2 \dot{q}_1^2 + 2(a_{12}+q_2)c_1\dot{q}_1 a_{3c} c_3 \dot{q}_3 + a_{3c}^2 c_3^2 \dot{q}_3^2}$$

$$T_3 = \frac{1}{2} m_3 \Big(\dot{q}_2^2 c_1^2 - 2\dot{q}_2 c_1 a_{3c} s_3 \dot{q}_3 + (a_{12}+q_2)^2 s_1^2 \dot{q}_1^2 + a_{3c}^2 s_3^2 \dot{q}_3^2 + \dot{q}_2^2 s_1^2 + 2\dot{q}_2 s_1 (a_{12}+q_2) c_1 \dot{q}_1$$
$$+ 2\dot{q}_2 s_1 a_{3c} s_3 \dot{q}_3 + (a_{12}+q_2)^2 c_1^2 \dot{q}_1^2 + 2(a_{12}+q_2) c_1 \dot{q}_1 a_{3c} c_3 \dot{q}_3$$
$$+ a_{3c}^2 c_3^2 \dot{q}_3^2 \Big)$$

$$T_3 = \frac{1}{2} m_3 \Big(\dot{q}_1^2 \big((a_{12}+q_2)^2 s_1^2 + (a_{12}+q_2)^2 c_1^2 \big) + \dot{q}_2^2 (c_1^2 + s_1^2) + \dot{q}_3^2 (a_{3c}^2 s_3^2 + a_{3c}^2 c_3^2) + 2\dot{q}_1 \dot{q}_2 \big(s_1 c_1 (a_{12}+q_2) \big)$$
$$+ 2\dot{q}_1 \dot{q}_3 \big((a_{12}+q_2) c_1 a_{3c} c_3 \big) + 2\dot{q}_2 \dot{q}_3 \big(s_1 a_{3c} s_3 - c_1 a_{3c} s_3 \big) \Big)$$

$$T_3 = \frac{1}{2} m_3 \Big((a_{12}+q_2)^2 \dot{q}_1^2 + \dot{q}_2 + a_{3c}^2 \dot{q}_3^2 + 2 s_1 c_1 (a_{12}+q_2) \dot{q}_1 \dot{q}_2 + 2(a_{12}+q_2) c_1 a_{3c} c_3 \dot{q}_1 \dot{q}_3$$
$$+ 2 a_{3c} s_3 (s_1 - c_1) \dot{q}_2 \dot{q}_3 \Big)$$

$s_1 c_1 a_{12} + s_1 c_1 q_2$

$$\frac{\partial T_3}{\partial \dot{q}_1} = m_3 (a_{12}+q_2) \dot{q}_1 + 2 s_1 c_1 (a_{12}+q_2) \dot{q}_2 + 2(a_{12}+q_2) c_1 a_{3c} c_3 \dot{q}_3$$

$$\frac{d}{dt}(\ ) = m_3 (a_{12}+\dot{q}_2) \ddot{q}_1 + \dot{q}_2 + (c_1^2 a_{12} - s_1^2 a_{12} + c_1^2 q_2 - s_1^2 q_2 + s_1 c_1 q_2) \dot{q}_2$$
$$+ \big(2 s_1 c_1 (a_{12}+q_2) \big) \ddot{q}_2 + \dot{q}_2 a_{12} a_{3c} (-s_1 c_3 \dot{q}_3 - c_1 s_3 \dot{q}_3 + c_1 c_3 \ddot{q}_3)$$

$$\frac{d}{dt}\left(\frac{\partial T_3}{\partial \dot{q}_1} \right) = m_3 \Big((a_{12}+\dot{q}_2) \ddot{q}_1 + \big(2 s_1 c_1 (a_{12}+q_2) \big) \ddot{q}_2 - 2 a_{12} a_{3c} c_1 c_3 \dot{q}_2 \ddot{q}_3$$
$$+ \dot{q}_2 \big((a_{12}+q_2) \cos 2q_1 + s_1 c_1 q_2 \big) \Big)$$

$$\frac{\partial T_3}{\partial q_1} = 2(a_{12}+q_2) \dot{q}_1 \dot{q}_2 (\cos 2q_1) - 2(a_{12}+q_2) a_{3c} c_3 s_1 \dot{q}_1 \dot{q}_3$$
$$+ 2 a_{3c} s_3 (c_1 + s_1) \dot{q}_2 \dot{q}_3$$

$$\frac{\partial T_3}{\partial \dot{q}_2} = \frac{1}{2} m_3 \Big(1 + 2 s_1 c_1 (a_{12}+q_2) \dot{q}_1 + \underset{2 a_{3c} s_3 s_1 \dot{q}_3}{2 a_{3c} s_3 (s_1 - c_1) \dot{q}_3} \Big)$$

$$\frac{d}{dt}\left(\frac{\partial T_3}{\partial \dot{q}_2} \right) = \frac{m_3}{2} \Big((c_1^2 a_{12} - s_1^2 a_{12} + c_1^2 q_2 - s_1^2 q_2 + s_1 c_1 q_2) \dot{q}_1 + \big(2 s_1 c_1 (a_{12}+q_2) \big) \ddot{q}_1 + 2 a_{3c} \big(c_3 (s_1 - c_1) \dot{q}_3$$
$$+ s_3 (c_1 + s_1) \dot{q}_3 + s_3 (s_1 - c_1) \ddot{q}_3 \big) \Big)$$

$$\frac{\partial T_3}{\partial q_2} = 2(a_{12}+q_2) \dot{q}_1^2 + 2 s_1 c_1 \dot{q}_1 \dot{q}_2 + 2 c_1 a_{3c} c_3 \dot{q}_1 \dot{q}_3$$

$$\frac{\partial T_3}{\partial \dot{q}_3} = m_3 \Big(a_{3c}^2 \dot{q}_3 + 2(a_{12}+q_2) c_1 a_{3c} c_3 \dot{q}_1 + 2 a_{3c} s_3 (s_1 - c_1) \dot{q}_2 \Big)$$

$$\frac{d}{dt}\left(\frac{\partial T_3}{\partial \dot{q}_3} \right) = m_3 \Big(a_{3c}^2 \ddot{q}_3 + 2 a_{3c} \big(\dot{q}_2 c_1 c_3 \dot{q}_1 - (a_{12}+q_2) s_1 c_3 \dot{q}_1 - (a_{12}+q_2) c_1 s_3 \dot{q}_1 + (a_{12}+q_2) c_1 c_3 \ddot{q}_1 \big)$$
$$+ 2 a_{3c} \big(c_3 (s_1 - c_1) \dot{q}_2 + s_3 (c_1 + s_1) \dot{q}_2 + s_3 (s_1 - c_1) \ddot{q}_2 \big) \Big)$$

∂q_1

$$\frac{\partial T_3}{\partial q_3} = m_3\left(-(a_{12}+q_2)c_1 a_{3c} s_3 \dot{q}_1 \dot{q}_3 + a_{3c} c_3 (s_1 - c_1)\dot{q}_2 \dot{q}_3\right)$$

$$V = V_1 + V_2 + V_3 = g\left(m_1 a_{1c} s_1 + m_2(a_{12}+q_2)s_1 + m_3((a_{12}+q_2)s_1 + a_{3c} s_3)\right)$$

$$\frac{\partial V}{\partial q_1} = g(m_1 a_{1c} + m_2(a_{12}+q_2)c_1 + m_3(a_{12}+q_2)c_1$$

$L = T - V$

$Q_k = 0$ No ext. Force or torque

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = Q_k$$

$$\frac{\partial V}{\partial q_2} = g(m_2 s_1 + m_3 s_1)$$

Link 1

$$0 = m_1 a_{1c}^2 \ddot{q}_1 - g(m_1 a_{1c} + m_2(a_{12}+q_2)c_1 + m_2(a_{12}+q_2)c_1$$

$$\frac{\partial V}{\partial q_3} = g(m_3 a_{3c} c_3)$$

Link 2

$$0 = m_2(a_{12}+q_2)\ddot{q}_1 + m\ddot{q}_2 - \left(q_2 \dot{q}_1^2 - g(m_2 s_1 + m_3 s_1)\right)$$

Link 3

$$0 = \left(\frac{d}{dt}\left(\frac{\partial T_3}{\partial \dot{q}_1}\right) + \frac{d}{dt}\left(\frac{\partial T_3}{\partial \dot{q}_2}\right) + \frac{d}{dt}\left(\frac{\partial T_3}{\partial \dot{q}_3}\right)\right) - \left(\left(\frac{\partial T_3}{\partial q_1} + \frac{\partial T_3}{\partial q_2} + \frac{\partial T_3}{\partial q_3}\right) - g(m_3 a_{3c} c_3)\right)$$

Look at previous page for full eqn

To Find Relation of entire system add eqns 1 2 & 3 then set to 0

4. Summary of Research

Dynamic Identification of the KUKA LBR iiwa Robot With Retrieval of Physical Parameters Using Global Optimization

Tian et al's paper on dynamic identification of a robotic manipulator got its basis from robotic applications that work in solving robot motion, human robot interactions and collision detection mechanisms. Because all of these activities deal with controlling the robot with accurate precision there became a need to develop a much more robust technique that can predict joint torques, for example to validate accuracy. So far researchers have linearized dynamic models to identify inertia parameters, to deal with singularities, avoid noise and the like. In this paper, however, Tian et al attempt to add universally global optimization framework that account for parametric bounds and constraints in addition to calculating objective error. Approach used was to measure joint positions and torques with filters applied to reduce noise. Then relative standard deviations were used to weigh contribution of parameters to dynamics model. Finally, saved parameters were used to calculate objective function. Although improvements were seen to limit error in computing joint position and torque, the bounds used were not strictly accurate which limited the resolution of obtaining true physical parameters, thus more work need to be done to support claim. The work initiated in this paper has been used as a starting point for other research that try to identify, for example, dynamics of 7 DoF robotic manipulator via population-based meta-heuristics[2], and setting up coordinate measurement system for links in a robot via pull wire sensor[3].

Improving the Absolute Positioning Accuracy of Robot Manipulators

Industrial robots need to be accurate when tasked to move items from one place to another, and at sites performing repetitive tasks. Hence, position accuracy is vital when robots are instructed to reach a given position. So far, the approaches used to characterize joint positioning such as DH method that use homogenous transformations to control serially linked manipulators have been found to have deficiencies for parallel consecutive joint axis. Whereas Mooring uses parametric identification techniques along with rotation unit vectors, although the parameters might not be consistently using with the control algorithm. Thus, Hayati and Mirmirani proposed to set tolerances for link parameter thereby increasing end effector positioning algorithm. First, error sources of transformation computations from each link were mapped to a common frame through use of error vector. Then, by assuming consecutive links as non parallel, transformation matrixes were computed upon which error equation between frames were computed relating nominal and actual transformations, which were then mapped to the robots base frame. For near parallel links, a plane was introduced and rotated to identify errors between adjacent frames. When methodology was tested on Standard Arm and PUMA 560, there was a careful delineation between running the algorithm for near parallel and non-parallel links which as a result led to in accurate estimation when performing error calculations for the robots that had both alignment in a given configuration. Since publication of this paper, topics on accurately determining robot end effector position discussed here has been a springboard to works that use single camera and computer vision to reduce pose errors [4], and research that attempts to simultaneously calibrate kinematic parameters of robot as well as robot mounted measurement devices[5], all aiming to reduce the error of moving robot end effector to specific location.

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