A retirement planner and withdrawal optimization model

Introduction:

What is the problem we are concerned with here?

For most of us the amount we are able to build up for our retirement is less than what we would like. This creates challenges for us in our retired years. In order to make the most of what we are able to do with these funds we look to our financial advisors. The financial community has understood for a long time that the specific withdrawal patterns can make a significant impact on the amount available to us and how long our funds will last.

One of the first articles I read[[1]](#footnote-1) along these lines discussed 15 separate withdrawal strategies, chief among them is the Common Rule (CR) and its comparison with the winner, starting with withdrawals from a Tax Deferred account up to your level of deductions first (TDD) strategy variant. The common rule, simply stated, is to withdraw from after tax accounts first, then the tax deferred accounts and finally the tax free accounts. The idea is to allow as much growth as possible in accounts sheltering that growth. TDD on the other hand shuffles the order to start with some portion of the tax deferred accounts, followed by after tax accounts, next the tax free accounts and finally the rest from the tax deferred accounts. Here the idea is to withdraw the amount equal to the level of deductions from the tax deferred account effectively withdrawing taxable money tax free. Additionally this lowers the amount that will have to be withdrawn starting at age 70 ½ for the Required Minimum Distribution (RMD) required by the IRS, which may prevent some creeping into higher tax brackets caused by the RMD. The paper finds TDD outperforms CR by $400K more remaining and paying $225K less taxes when starting with $2M (IRA 70%, After Tax Account 20%, Roth 10%) and a $50K yearly withdrawal.

A primary tool for their work as discussed in their paper was a Linear Programming (LP) model that fully accounted for tax implications. However, they do not go into the details of the model. Thankfully, there is a substantial example[[2]](#footnote-2) from 1994 that can serve as a guide for those of us not completely comfortable with LP models.

The main focus in the above work is on where to withdraw funds for each period of retirement in order to get the most from what you have. Another line of thinking that is even more important is focused on how much to withdraw while ensuring your funds do not run out. The 4% rule[[3]](#footnote-3) seems to have been the first systematic approach to this line of thinking and is well worth the read. The basic idea is to withdraw 4% of your funds the first year of retirement. Each subsequent year, increase the 4% by the previous year’s inflation rate. In many cases (dare I say most) following this advice will leave a substantial amount for the estate. In a few cases (have some examples for this) however it may still fail to fund the retirement in its entirety.

Looking to improve the guidance for where to make withdrawals while also ensuring funds will last throughout retirement has pushed LP models forward to be use for retirement planning and year to year execution as with 3-PEAT.[[4]](#footnote-4) ---Now what to say ---

(Give refs and examples…) and a later ref

Now, what about the model and its mathematical representation?

OK, so we are looking at a new object function as well as the original we got from Ragsdale. Let’s start with the new (experimental) object function (S1) and then we will rewrite the Ragsdale object function as a variant (R1):

(S1)

(R1)

The idea here is that we want to maximize the spendable (si) dollars across the retirement years. For Ragsdale we also want to maximize the remaining account balances. However, we don’t want to require that all the available account balances is use up in cases where we set a maximum spending amount. So, in both the cases we define the object function to include both the sum of the yearly spendable amount and the sum of the account balances at the end of the period times a discount rate for each account (DjA). For (S1) though we include a “balancer” to lower the significance of the final balances in the optimization such that spendable funds will be favored. The balancer is using a heuristic of dividing by the sum of the initial account balances. The account discount rate is applied to the final balances to suggest the value of the balance given how the account is taxed; TDRA at 0.85, ROTH at 1.0, and the after tax investment account at 0.9.

The spendable amount (si) is the sum of all withdrawals (wij) plus Social Security (SSi) and other income (oi) minus income tax (xik subject to tk) and capital gains tax (yil subject to tl) as well as money deposited back into the investment account (Di). The following constraint is used to assign the spendable amount to si.

(2)

(2 as imp)

In general we want allowable spending to increase with inflation and to remain steady year to year so we have the following constrain:

(3)

The objective function (R1) consists of two present value components. The first component calculates the sum total present value of the taxpayer’s after tax income for n years. The second component calculates the sum balance of funds remaining in all accounts in nl, one account of each type, in the n+1th year. The account types include:

1. n1, Tax Deferred Retirement Accounts (TDRA) including 401(k), traditional IRA and similar plans,
2. n2, Roth Retirement Accounts (RothRA) including after tax contributions to 401(k) and similar as well as Roth IRAs and
3. n3, After Tax Retirement Savings/Investing Accounts (ATRSI).

Thus, the objective specified by expression (S1) attempts to maximize the NPV of the withdrawals and other taxable income (after tax) made over the taxpayer’s retirement period. Similarly, (R1) does the same while maximizing the NPV of the remaining account balances.

Note: This model uses one account for each type of account such that the balance and withdrawals for the model account represent the sum of the balances / withdrawals for any number of accounts of that type. One sticky point here is when, for example, a 401(k) contains both tax deferred and after tax contributions. In this case the balance must be split and added to the correct account types.

The tax rates (tk) represent the rate for ordinary income for each inflation adjusted tax bracket while tlcg is the capital gains tax rate for each capital gains inflation adjusted tax bracket. Currently, though, tlcg is being modeled as a single bracket with one rate (15%).

Questions to address: Maximizing the after tax yearly income and the TDRA balances at the end may have issues, is this really what I want? Maybe maximize include while keeping estate a certain level? Or meeting an include amount while maximizing the estate? How to handle the estate taxes. I’ve dropped them in the above.

(4)

(5)

A number of constraints are required to constrain the objective function in expression (R1) and (S1) to optimal values while ensuring that IRS rules are followed and taxes and penalties are properly accounted for. Additional constrains will be given to allow for a minimum level of income per year. To this end we define expression (4) to ensure that each year’s spendable amount (si) is at least as much as the desired income (di) for every year in the modeled retirement period.

(6)

Of the three account types, only the TDRA has an IRS requirement for a Minimum Require Distribution (MRD). This requirement applies to all such accounts but the sum of the MRD can be withdrawn from each account, any one of the accounts or some combination of TDRA accounts as long as the full amount of the MRD is withdrawn from TDRA accounts. To ensure withdrawals are at least as much as the IRS minimum required, equation (6), requires withdrawals, starting at age 70 (nr), exceed the balance in the TDRA (bi1) divided by an IRS defined life expectancy value (aij this should include a way to choose which expectancy table is needed).

(7)

(8)

Equation (7) constrains the variable representing the amount of income in tax bracket k (xik) to take on an amount related to the total taxable income (the TDRA withdrawals and other income, oi) minus the deductions (standard deduction and exemptions, sdi). Expression (2) forces income into the lowest possible brackets through the applied tax and need to be maximized. That is, tk is monotonically increasing as k increases in expression (2), which forces the xik in the lowest brackets to fill first. Expression (8) ensures that the xik portion of the income does not exceed the bracket amount (mik). Mik and sdi are inflation adjusted.

(9)

(10)

(11)

(11 Alt)

nsil is to take up any negative slack when the total ordinary taxable amount exceeds the floor of the next capital gains bracket in (11).

In the same manor (9, 10) fill the capital gains tax brackets (yil) with the non-basis portion (pi) of the investment account (ATRSI) withdrawal (wi3). However, the capital gains tax bracket fill must start where the ordinary income bracket fill stopped and continue up from there. In order to do this we add another constraint on yil that may lower the bracket size by making it less than or equal to the floor of the next bracket level (Fl+1cg) minus the total ordinary taxable income (wi1 + oi +SStSSi-sdi). Or as in the alternative constraint (11 Alt) we simply subtract the sum of the ordinary taxable income. One problem with this is it may force yil into negative territory which would cause the model to have no solution so we add nsil to take up any negative amount that the constraint causes.

(12)

(13)

Add: Di which is the Deposits to investment account in year i

Equation (12) ensures that the balance for each account at the beginning of the year (bi+1,j) is equal to the balance of the account at the start of the previous year (bij), minus the previous year’s withdrawals (wij) (modeled as being withdrawn at the beginning of the year) times the return on the investment for the year as rate of return (rij). This is somewhat pessimistic because withdrawals are usually not taken out in one transaction at the beginning of the year but this is a small effect for our purposes here.

(14)

Expression (14) sets the beginning account balances to qj.

(15)

Finally, expression (15) constrains the model variables to be greater than or equal zero.

Transforming our model into a python implementation:

OK for our current work we will use the python scipy library, specifically the function scipy.optimize.linprog(). This requires the model to conform to the following template:

Object function: Minimize ct x

With constraints: A x <= b, and x >= 0

(need to zero base the indices)

Given this we transform our model expressions to match scipy template form as follows:

OK, as above we will first look at the new objective function followed by the Ragsdale objective function.

(S1’)

(R1’)

(2’)

(2’ as imp)

(3a’)

(3b’)

(4’)

(5’)

(6’)

(7a’)

(7b’)

(8’)

(9a’)

(9b’)

(10’)

(11’)

(11’ Alt)

nsil is to take up any negative slack.

(12a’)

(12b’)

(13a’)

(13b’)

Bij supports an extra year

(14a’)

(14b’)

(15’)

To transform our model into the scipy equivalent we have only to use a few operations. To transform the object function, expression (R1’) or (S1’), we minimize the opposite of (R1) or (S1). The rest of the expressions are transformed by multiplying by minus one (-1) to change greater than or equal (≥) to less than or equal (≤), moving constants to the Right Hand Side (RHS) of the expression and doubling up equations to convert from equal (=) into two relations, one with greater than or equal (≤) and the other with less than or equal (≥) (properly transformed) to bring all constraints into standard form (i.e., A x ≤ b). In (10a’ and 10b’) we also multiplied out the c(bij – wij) to (cbij – cwij) to more closely match the matrix coding. Constraint (14a’ and 14b’) set the initial account balances to qj.

Key:

aij IRA life expectancy at age in year i for account j

Bcg Number of capital gains tax brackets

Bt Number of tax brackets

bij balance of account j in year i

di desired minimal before tax income

dmi desire maximal before tax income

Di deposits to investment account in year i (this may be expanded to other accounts pre retirement?)

Flcg floor of the capital gains bracket l

i index for number of retirement years

inf inflation rate

j index for the number of accounts

k index for the tax brackets

l index for capital gains tax brackets

mik size of the kth tax bracket in year i

mcgil size of the lth capital gains bracket in year i

n Number of retirement years

nr year number that retiree is age 70

oi Other income in the ith year (need to break out taxable and non-taxable)

pi the capital gains fraction of investments (i.e., not basis)

pvi Present Value in year i

qj balance for account j at the start of retirement

rij rate of return for account j in year i

rcg capital gains tax rate (temp until cg tax brackets are working)

sdi Standard deduction in year i

si Spendable amount in year i

SSi Social Security income in year i

SSt Social Security faction that is taxable

SSnt Social Security fraction that is NOT taxable

tlcg marginal capital gains tax rate in bracket l

tk marginal tax rate in tax bracket k

wij withdrawal from account j in year i (j=1 TDRA, j=2 Roth, j=3 Investment)

xik ordinary taxable income in year i and bracket k

yil capital gains income in year i and bracket l

Consolidated Todos:

1. Fix Roth (for some reason Roth account is being emptied when optimizing for final estate (or not) which makes no sense. It should not be move from an account that is not taxed to one that is.
2. Max income should cause excess money to be stored in the investment account but this is not working.
3. When optimizing for spending only all funds are yanked from accounts in year 1 without attributing them to spendables or deposits to the investment account. This forces the slack variable to be no zero and the constraint not to be in effect. So this is broken
4. Create a case to use Aeq x == beq as well as Aub x <= bub

To Add:

* other sources of income and whether they are taxed taxed (Wayne Scott’s data code allows for this but I need to use it correctly.
* Capability to handle pre-age 60 effects like a possible 10% penalty for early withdrawal…
* Add upper bound for how much to grow liquid assets (sum of bij)
* Add upper bound for spending (have di need max di or mdi)
* MUST FIX. Like to enable the user to plug in a value for the standard deduction + exemptions.
* (Should I assume any interest yealds??? Currently its just capital gains ) and other taxes (medicare tax, ???) Another constraint for other portion of wi3??? pi is the basisi multiplier for year i. Need to extend the size of the rows to include the new xibt+1 var. Another problem with this is it DOES NOT ALLOW FOR A CHANGE IN THE BASIS while optimization is happening. Why is this important? Because, I want to be able to have excess withdrawals placed in ATRSI (bi3) which would need a corresponding change to the basis bmi this would require it to be a variable (not a constant) but I don’t think this method allows for variable to be multiplied. NEED TO VERIFY

To Do:

* Set up github repository
* Convert the indices to zero based to match the python code
* For oi break out the taxable and non-taxable portions for proper modeling (i.e., use TAX[year])
* Double check all inflation adjustments and where they should be
* ~~Add sd~~~~i~~ ~~to equations (4) in the code~~
* Add separate rate for increasing other income and percent taxed
* ~~Split out social security as a separate income source with its own growth rate and percent taxed~~
* ~~Add separate growth rate for desired income~~
* Add selectable MRD tables (single, joint married, ???)
* Add state taxes
* Add for retirement pre age 60 (TDRA 10% penalties, …)
* Can the tax brackets be changed somehow to significantly lower the number of variables?
  + Want smaller / faster model
* ~~Wrap up the Ragsdale model (store away as is and for the new Auld model)~~
* New Auld model:
  + One account for Traditional IRA, one for Roth, one for Saving (interest baring like) and one for investments
  + Add pre-retirement, how much to add to tIRA, Roth, Savings, Investments
  + Input to optimize:
    - total after tax withdrawals w/ minimum withdrawal rate
    - total after tax withdrawals + final balances w/ minimum withdrawal rate
    - add a maximum spend rate (to for excess withdrawals into Savings/Investment/Roth accounts
    - Capital gains taxes vs. interest vs. income tax
    - Other sources of income include (my SS, spouse SS, yield (is this interest? From?)
    - Ability to output the breakdown of taxes (how much in each bracket, total, how much cap gain…) Like the ORP
    - Like to optimize start year of SS
    - Moving withdrawal income into other investments (conversions?) after tax investment or savings accounts (dependent on how soon the money might be needed).
    - Short term savings goals (car, ….)
    - Emergency fund (6months – 1year, family emergencies,…)
* Ability to run in simulation mode against a defined return rate for each year (ie., a portion of the historical S&P 500 record).
  + Compare against other strategies:
    - 4% initial amount forever, or with inflation, or adjust to 4% each year[[5]](#footnote-5) [[6]](#footnote-6) [[7]](#footnote-7)
    - CR
    - TDD
    - Autopilot (<http://www.marketwatch.com/story/theres-a-better-way-to-plan-retirement-withdrawals-2015-01-13?page=2> )
    - RMD all the way
* Like to be able to consider two peoples accounts more fully. That is to be able to determine things like:
  + When older retires and is withdrawing from TDRA does it make sense for the younger to continue to add money to the TDRA, maybe even enough money for the limit of both.

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2. Ragsdale, Cliff T., Andrew F. Seila, and Philip L. Little. 1994. “An Optimization Model for Scheduling Withdrawals from Tax-Deferred Retirement Accounts.” Financial Services Review 3, 2: 93–108. Retrieved from [www.twenty-first.com/pdf/RagsdaleAn\_Opt\_Model.pdf](http://www.twenty-first.com/pdf/RagsdaleAn_Opt_Model.pdf) [↑](#footnote-ref-2)
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6. Bengen, William P (May 1, 2012) How much is enough? Financial Advisor, Retrieved from: <http://www.fa-mag.com/news/how-much-is-enough-10496.html> [↑](#footnote-ref-6)
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