Forehead idiot game

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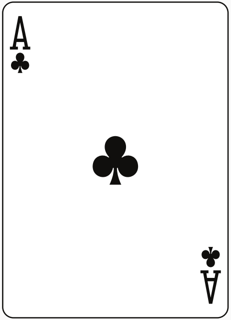
Multi agent systems

Introduction

Introduction



- Drinking game
- Players have to avoid getting smashed

Game explanation

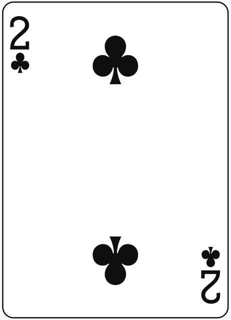
- ♣ Playing a round
- ♣ Strategies

Types of logic

- Epistemic logic
- Public announcement

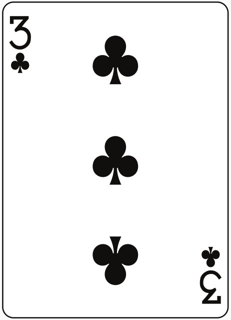
Formalizing the game

Kripke structure



- $M = \langle S, \pi, R \rangle$
- States are given by $\mathbf{s} = (s_1, s_2, \ldots, s_m) \in S$ where $s_i \in D = \{2, 3, 4, 5, 6, 7, 8, 9, 10, j, q, k, a\}$.
- Relations are defined as $\mathbf{s} R_i \mathbf{t}$ such that $s_j = t_j$ for all $j \neq i$.

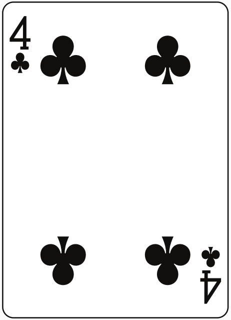
Kripke structure II



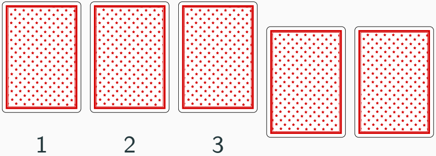
- Propositions
  $\mathbf{P} = \{p_i c_2, p_i c_3, \ldots, p_i c_a, w_i, l_i\}_{i=1}^{m}$
- We have $\pi(\mathbf{s})(p_i c_j) = \mathbb{t}$ iff $s_i = j$, so $\pi(\mathbf{s})(p_i c_j) = \mathbb{f}$ iff $s_i \neq j$.
- Furthermore, we have $\pi(\mathbf{s})(w_i) = \mathbb{t}$ and $\pi(\mathbf{s})(l_i) = \mathbb{f}$ iff $s_i > s_j$ for all $j \neq i$.

Analysis

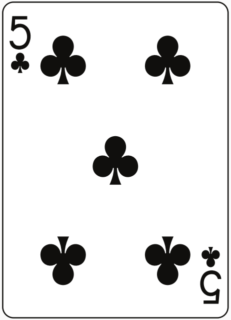
Combinatorial analysis



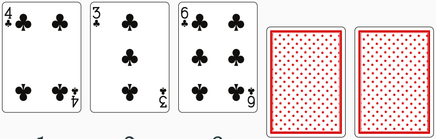
- Combinatorial explosion?
- Suppose we take a subset of the total deck $D' \subseteq D$ with $|D'| = n$.
- Let's say we have m players. Before anything is known, there are $g = \dfrac{n!}{(n-m)!}$ possible games.



Combinatorial analysis II

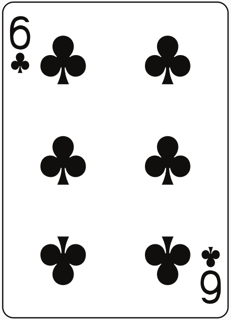


- Once they are drawn it isn't that bad anymore
- Each player has $n - m + 1$ alternatives for what he is seeing
- Hence, there are $m(n - m + 1) - m + 1$ possible worlds left



1 2 3

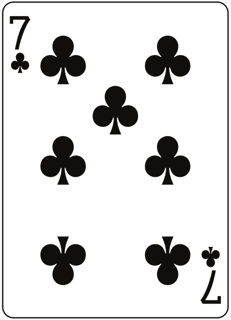
Game analysis



- First scenario: n cards game with $m = n$ players.
- After cards are drawn only one possibility is left:

$$m(n - m + 1) - m + 1 = 1$$

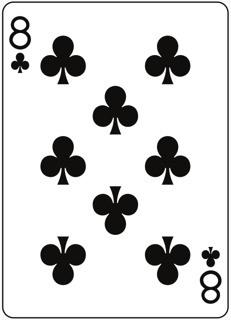
Game analysis II



- Second scenario: n cards with $m < n$ players
- Let $T_i \subseteq S$ be the subset of states such that $\mathbf{s}R_i\mathbf{t}$
- Estimating probability to win:

$$\mathcal{P}_i^{(w_i)} = \frac{|T_i^{(w_i)}|}{|T_i|} = \frac{|T_i^{(w_i)}|}{n - m + 1}$$

Strategies



- Let $\mathcal{G}^{(k)}$ be the gain of winning the in betting round k. We define the utility for winning as:

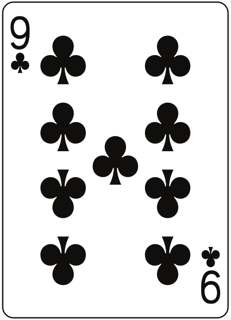
$$U_i^{(w_i)} = \mathcal{G}_i^{(k)} \mathcal{P}_i^{(w_i)}$$

- And for losing:

$$U_i^{(l_i)} = -\mathcal{L}_i^{k} (1 - \mathcal{P}_i^{(w_i)})$$

in which $\mathcal{L}_i^{(k)}$ denotes the loss

Deciding to call

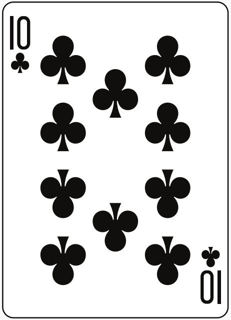


- Let's simplify things by saying that there is only one round for players to bet
- A player decides to call with 1 sip iff

$$\gamma_i U_i^{(w_i)} + U_i^{(l_i)} \geq 0$$

- The γ_i parameter determines the strategy of a player
- Risky: $\gamma_i > 1$, safe: $\gamma_i < 1$, optimal $\gamma_i = 1$

Inferring another player's strategy



- ♣ We add three propositions for each player:

$$\mathbf{P}' = \mathbf{P} \cup \{r_i, o_i, h_i\}_{i=1}^{m}$$

- ♣ Each player knows that a player has either one of three strategies: risky $\gamma_i = 1.1$, optimal $\gamma_i = 1$ or harmless (safe) $\gamma_i = 0.9$:

$$M \models K_i r_i \lor K_i o_i \lor K_i h_i$$

$$M \models K_i \bigwedge_{j=1}^{m} \left(\begin{aligned} &(r_j \land \neg o_j \land \neg h_j) \lor \\ &(\neg r_j \land o_j \land \neg h_j) \lor \\ &(\neg r_j \land \neg o_j \land h_j) \end{aligned} \right)$$

Ending a round

- If a player called, the others can know that

$$\gamma_i U_i^{(w_i)} + U_i^{(l_i)} \geq 0$$

$$\gamma_i \geq -\frac{U_i^{(l_i)}}{U_i^{(w_i)}}$$

- Similarly, when a player folded:

$$\gamma_i U_i^{(w_i)} + U_i^{(l_i)} < 0$$

$$\gamma_i < -\frac{U_i^{(l_i)}}{U_i^{(w_i)}}$$

Playing multiple rounds

- We can consider these events as announcements about r_i, o_i and h_i
- Suppose we have played three rounds in which $\gamma_i < 1.5$, $\gamma_i \geq 0.93$ and $\gamma_i < 1.06$.

$$M \models [r_i \vee o_i \vee h_i][\neg h_i][\neg r_i] \bigwedge_{j=1}^{m} K_j o_i$$

Simulation of the game

http://jostosh.github.io/website/index.html

Playing the game

- ♣
- ♣

Conclusion

Questions?