Performance paradox in stochastic dynamic matching models

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joint work with J.M. Fourneau (U. Versailles), A. Busic and A. Cadas (Inria Paris)

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December 19, 2023

Outline

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Static Matching Models

Question:

Given a set of people with different likings, how to create couples so that the number of uncoupled people is minimized?

Problem formulation

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- $G = (V, E)$, V is the set of people types and E represent the compatibilities (or likings)
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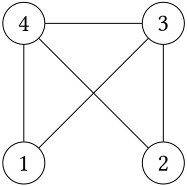
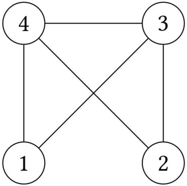


Static Matching Models



Known as the marriage problem (or assignment problem or the dancing problem...)

Widely studied in graph theory

Peterson 1890 and Konig 1937 $\Rightarrow$ Perfect matching

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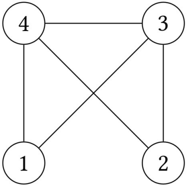

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Questions:

Existence of the perfect matching? How to achieve it?

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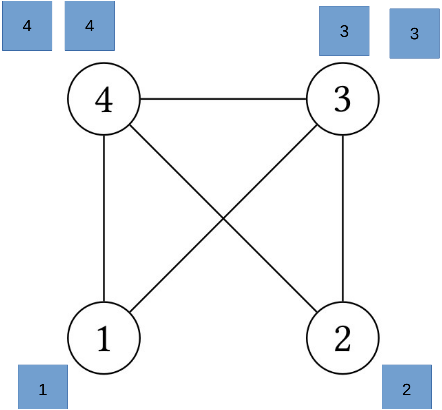

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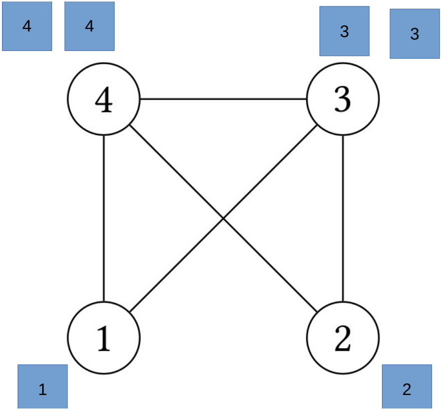

How to achieve the perfect matching?

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- We prioritize matchings of 1 and 2

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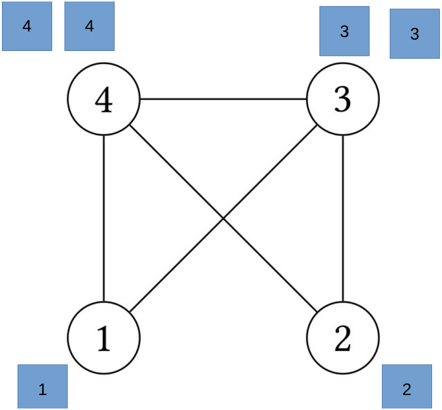

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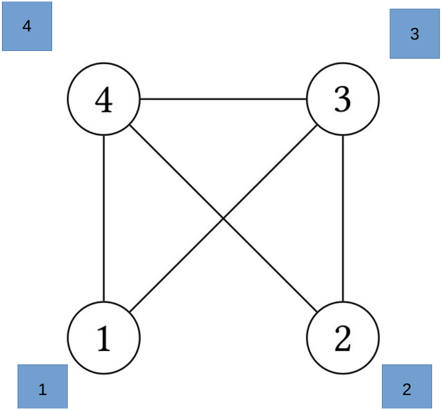

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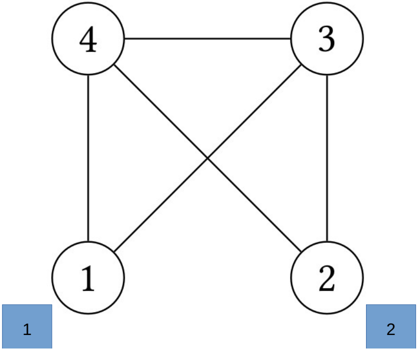

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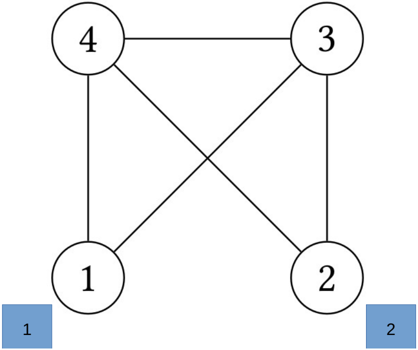

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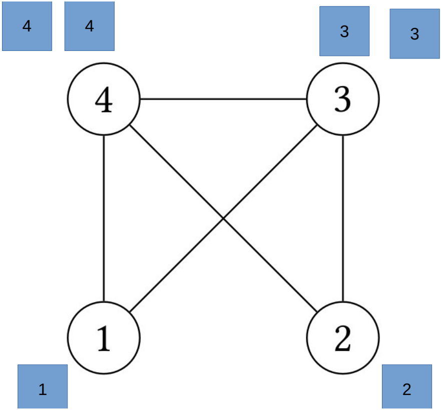

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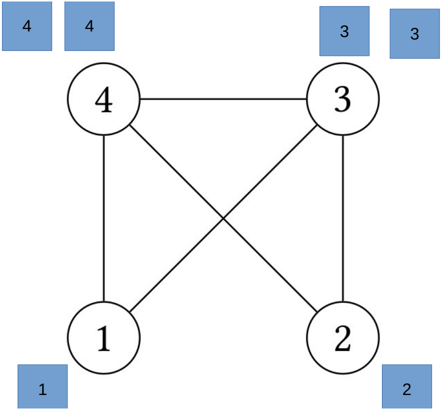

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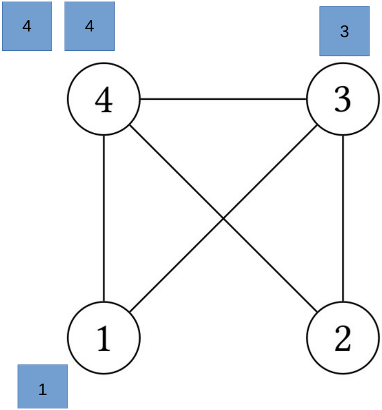

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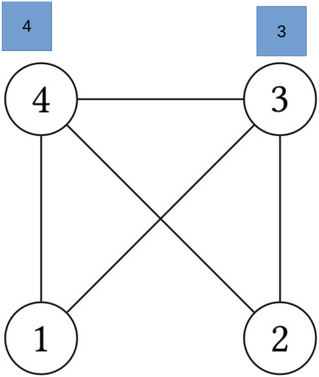

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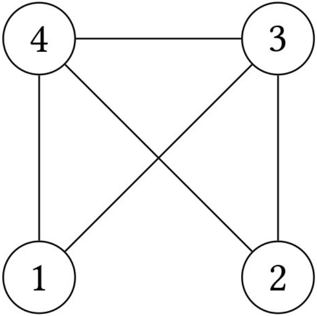

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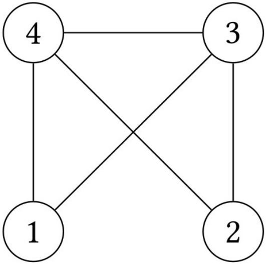
Matching Models



Applications: Not only to marry people

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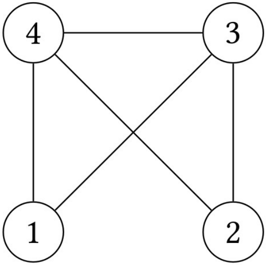

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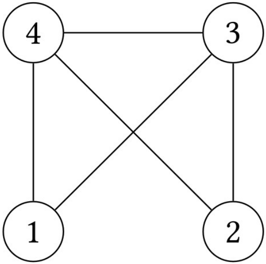

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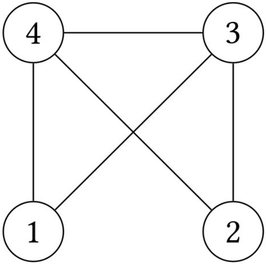

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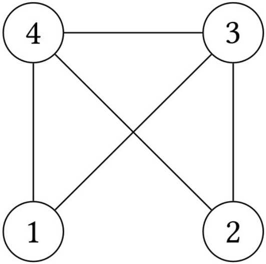

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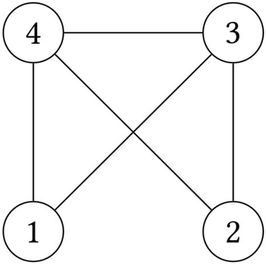

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Items arrive according to a random process

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Introduced in Mairesse et al 2016

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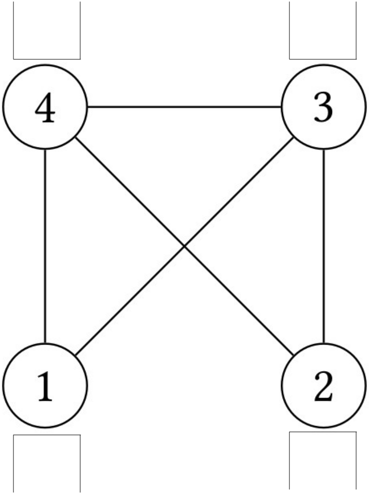
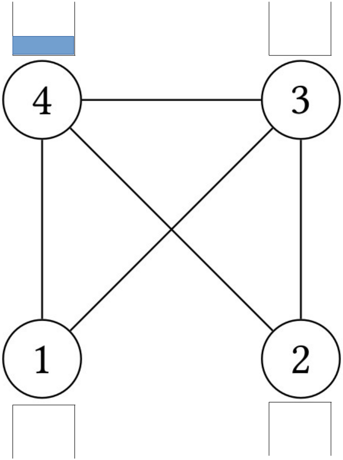


Dynamic Matching Model

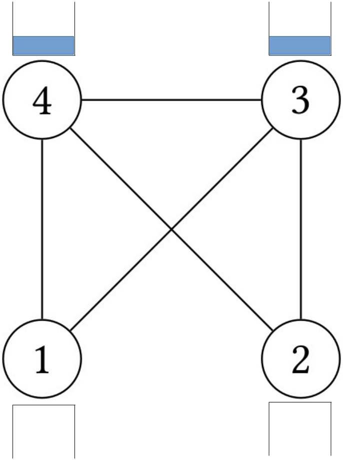
With probability α_4



Dynamic Matching Model

With probability α_3

Compatibles $\Rightarrow$ Match and leave



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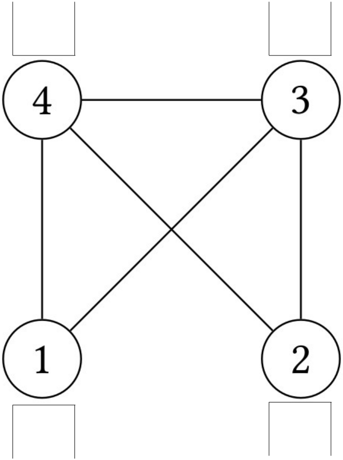
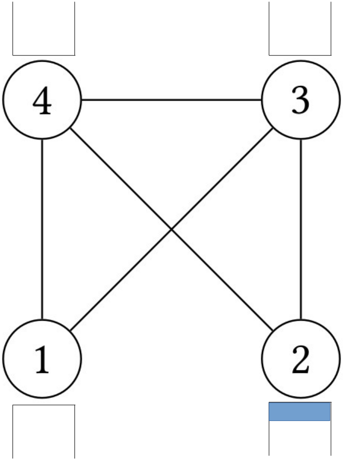


Dynamic Matching Model

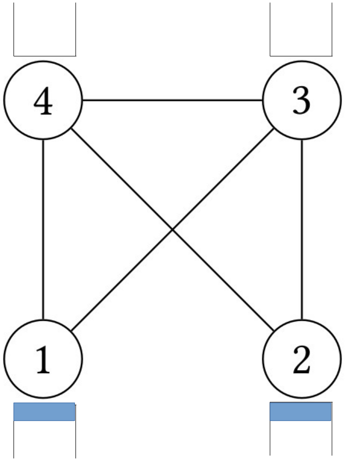
With probability α_2



Dynamic Matching Model

With probability α_1

They are not compatible



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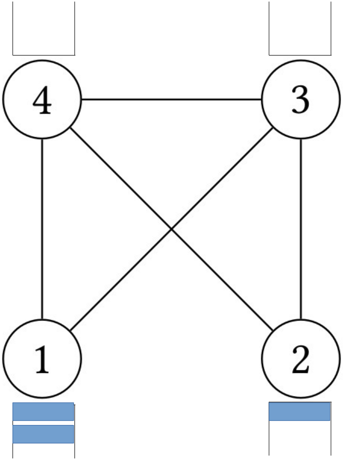

Dynamic Matching Model vs Queueing Models

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QUEUEING THEORY CANNOT BE APPLIED DIRECTLY!

New research challenges!

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Matching Policy

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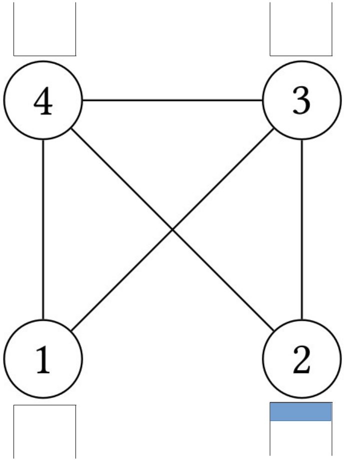
Examples:

- First-Come-First-Matched (FCFM)
- Match the Longest (ML)

Dynamic Matching Model



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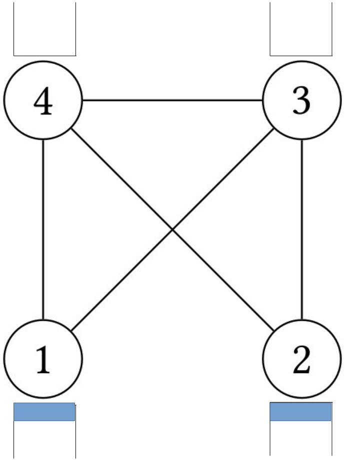

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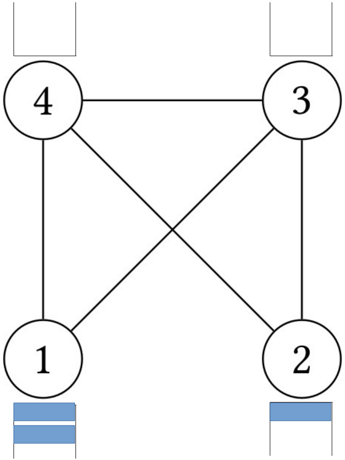

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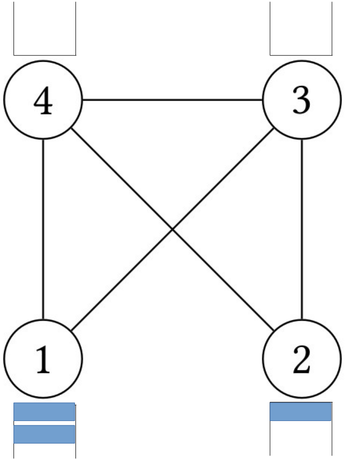

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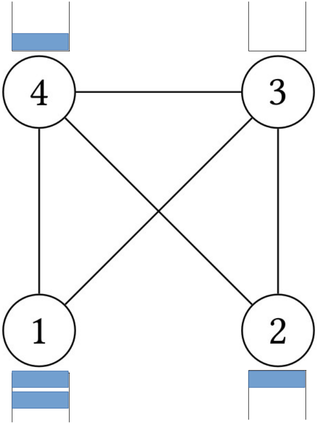
First-Come-First-Matched

When an item of type 4 arrives, it is matched with an item of type 2 (the oldest item)



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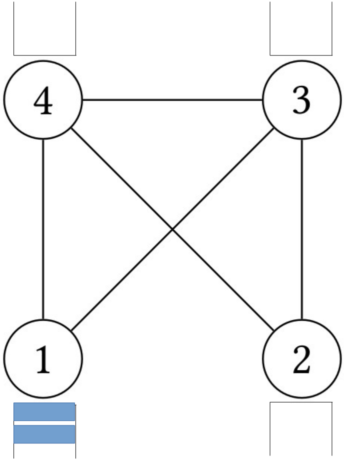
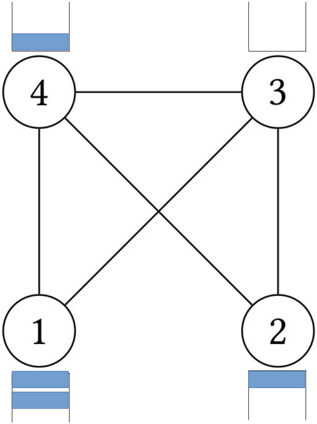


Match the Longest

When an item of type 4 arrives, it is matched with an item of type 1 (the longest queue)



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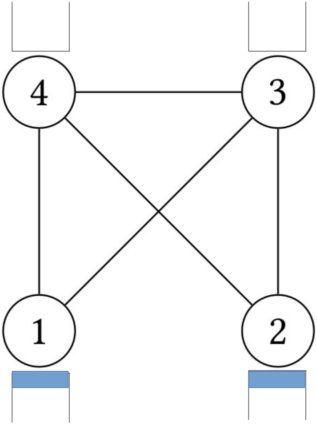

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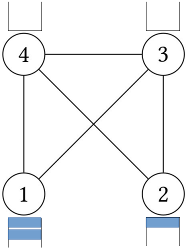
Example: $w = (2, 1, 1)$



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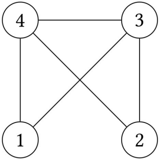

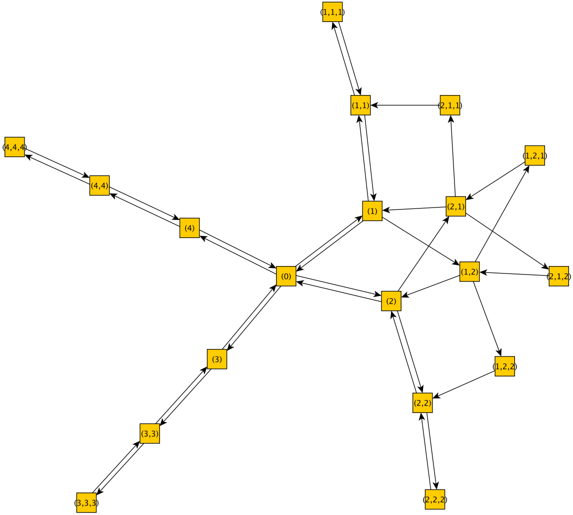

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$(G, \vec{\alpha}, \phi)$

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Definition: Independent set

The nodes that are not connected

$\mathcal{I}$: the set of all independent sets

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How to characterize stability of $(G, \vec{\alpha}, \phi)$ (the Markov chain is recurrent)?

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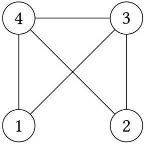

$$\alpha_1 + \alpha_2 < \alpha_3 + \alpha_4$$
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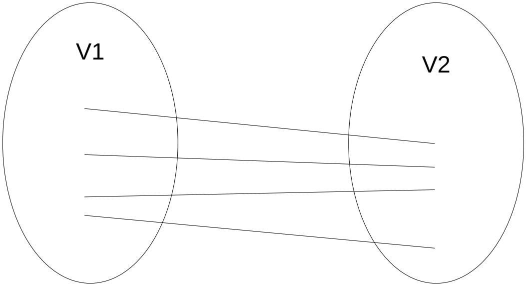

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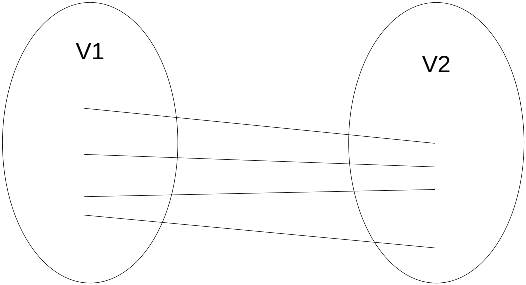

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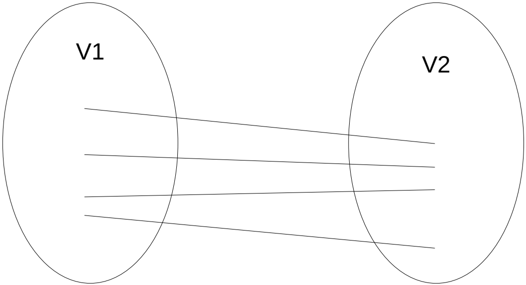

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$\Rightarrow \alpha_{\Gamma(V1)} = \alpha_{V2} > \alpha_{V1}$ and $\alpha_{\Gamma(V2)} = \alpha_{V1} > \alpha_{V2}$ cannot be satisfied together!

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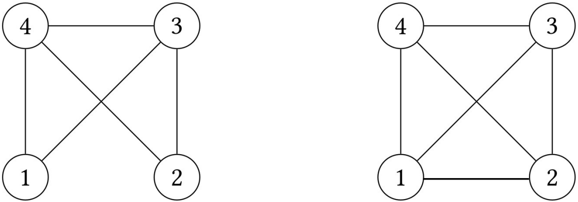

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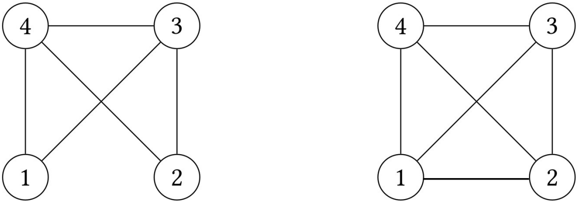
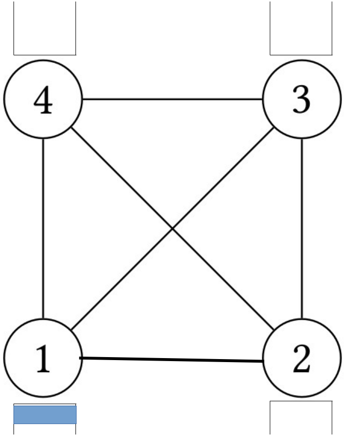


Intuitive Idea

The expected number of unmatched items cannot increase when we add an edge in the compatibility graph (more matchings can be done)

FCFM policy



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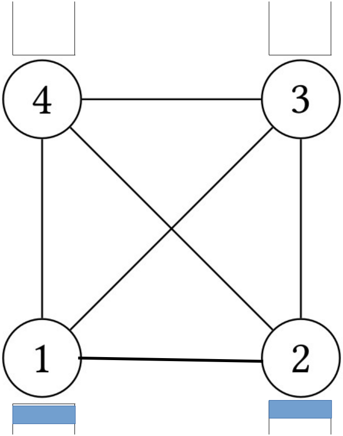

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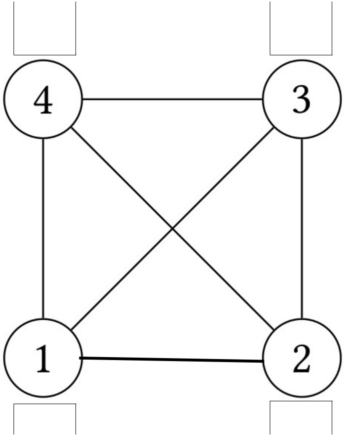

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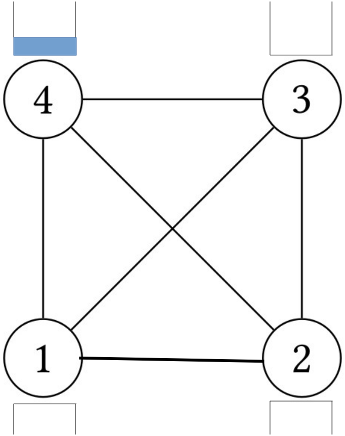

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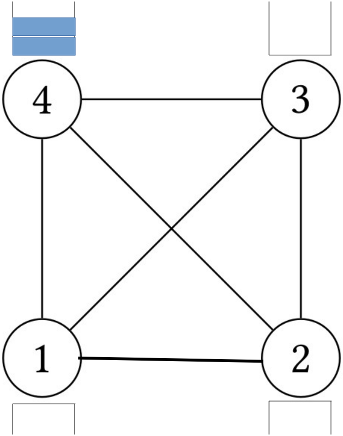

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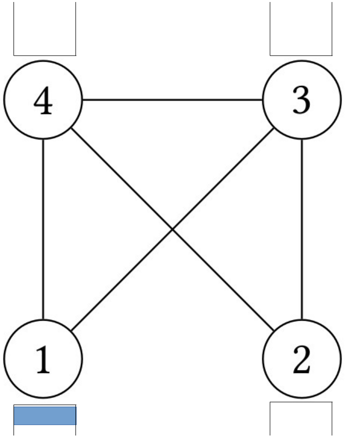

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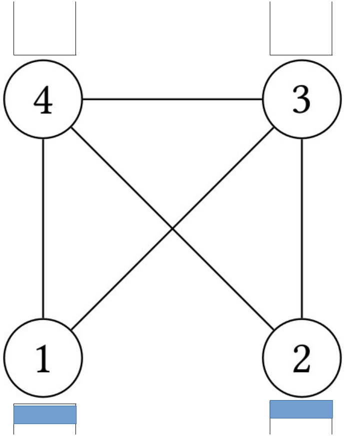

Performance Paradox in Dynamic Matching Models

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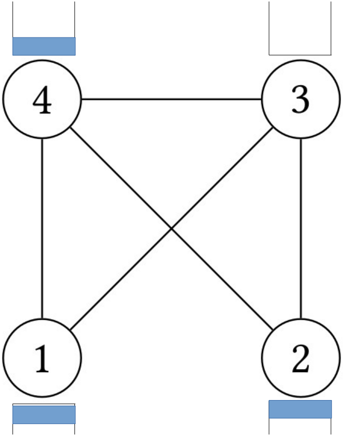

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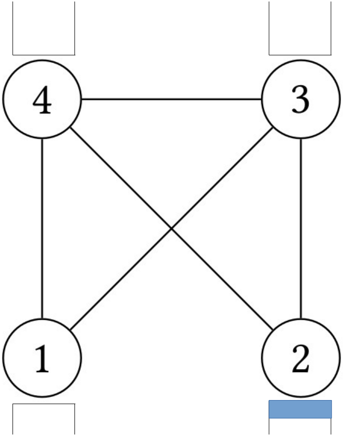

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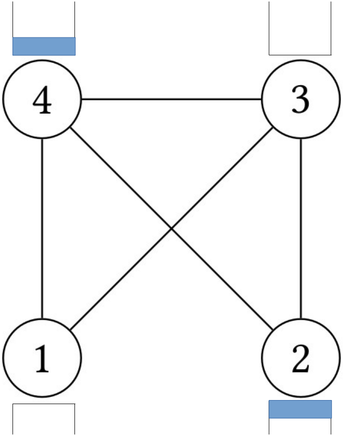

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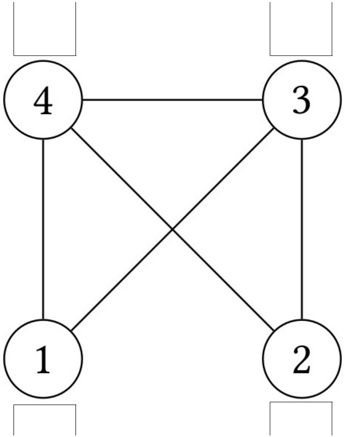

Performance Paradox in Dynamic Matching Models

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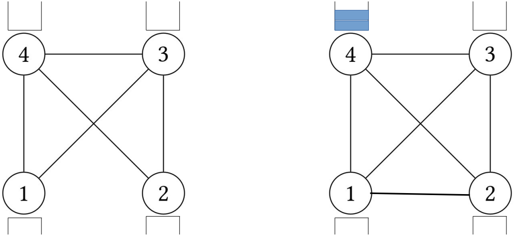

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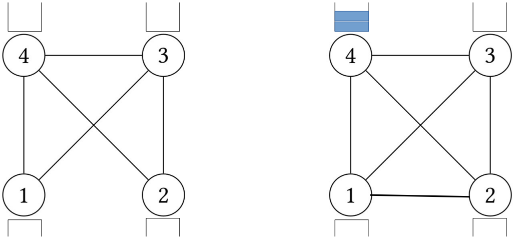

Definition: Performance Paradox

The performance paradox exists in dynamic matching model if, when we add an edge to the compatibility graph, then the expected number of unmatched items increases.

Analogous phenomenon to the Braess paradox

Our Results

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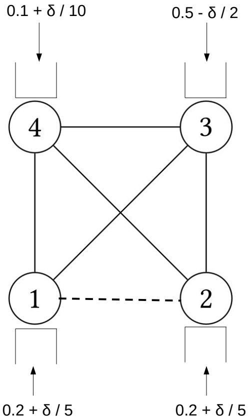

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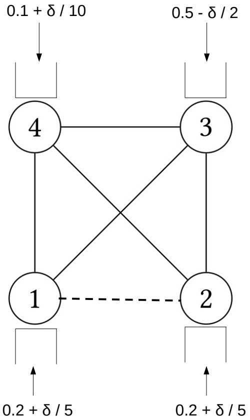

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For the above compatibility graph and FCFM, the performance paradox exists if and only if $\delta < 0.0818$.

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The Markov chains of
- complete graph: single letter words
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Intuitive idea

When $\delta \to 0$, the stability condition in node 3 is nearly satisfied

Our Results: Arbitrary compatibility graph and FCFM

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Definition: Saturated nodes (or in heavy-traffic)

The node i is in saturation if, when $\delta \to 0$, then $\alpha_i - \alpha_{\Gamma(i)} \to 0$

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We assume that there is a unique saturation node.

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When the node in saturation is connected to the nodes where the edge is added, there exists a performance paradox for δ small enough

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Sketch of the proof

- We compute the expected number of unmatched items (Comte 2021)
- We study the obtained expressions when $\delta \to 0$ and check that the condition for the performance paradox is satisfied.

Outline

From FCFM to other matching policies

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Definition: Greedy matching policy

A matching policy is greedy if when an arriving item is compatible with more than one item, it is matched with one of them

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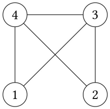
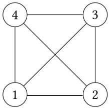
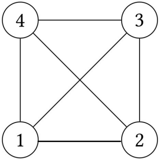
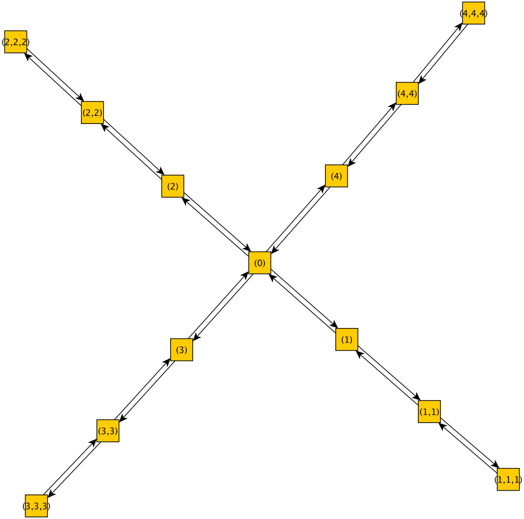
The Markov chains of the below models coincide for any greedy policy (same transitions and states).





(Truncated) Markov chain of the Complete graph





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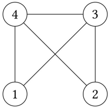

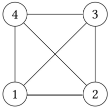

Theorem

The performance paradox for the above graph exists for any greedy policy (because it exists for FCFM).

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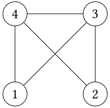

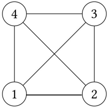

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Large Networks using Aggregation

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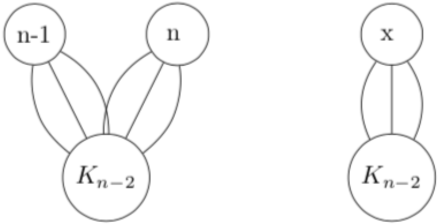

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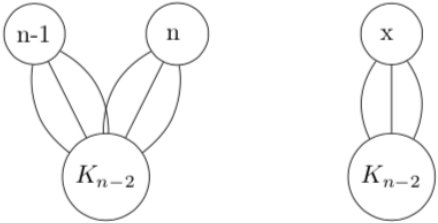

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Assuming that $\alpha_x = \alpha_n + \alpha_{n-1}$, the mean number of unmatched items in both models for any greedy policy is the same.

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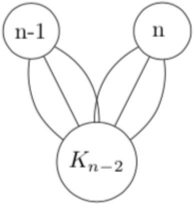

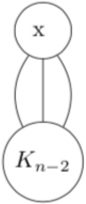

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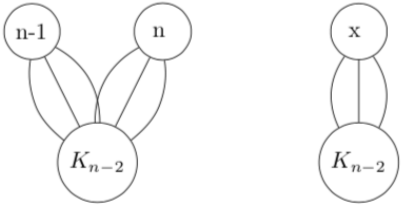
Sketch of the proof

The Markov chain of the left is lumpable in a way that all the items of n and $n-1$ are aggregated
The lumped Markov chain corresponds to that of the right model.

Large Networks using Aggregation



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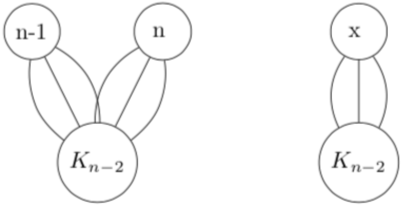

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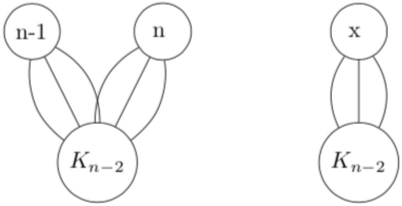
- The proof is valid if we replace K_{n-2} by any other compatibility graph
- The proof is valid if in the left model there are more than one nodes: $n, n-1, n-2, \ldots$

Large Networks using Aggregation



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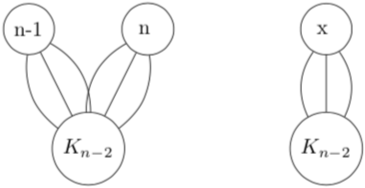

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$\Rightarrow$ Graphs with performance paradox can be as large as we want!

On the Assumption about the Unique Saturated Node

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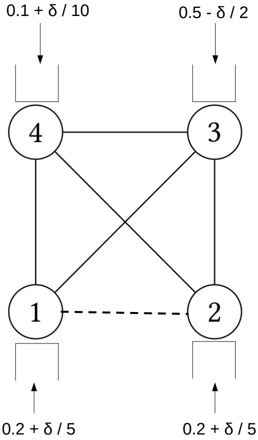
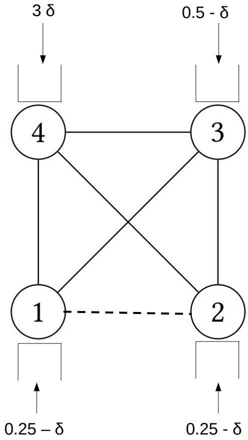


We assume there exists a unique node in saturation

Example: Node 3

On the Assumption about the Unique Saturated Node



The independent sets $\{1, 2\}$ and $\{3\}$ are in saturation

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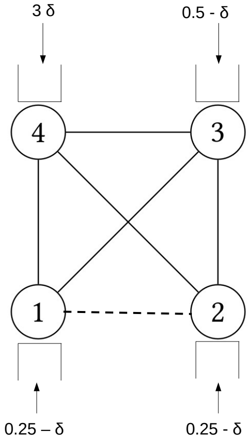

The independent sets $\{1, 2\}$ and $\{3\}$ are in saturation

There exists a performance paradox if and only if 0.0563.

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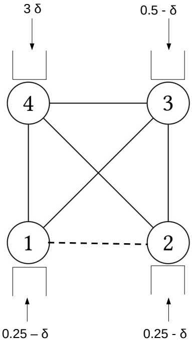


The independent sets $\{1, 2\}$ and $\{3\}$ are in saturation

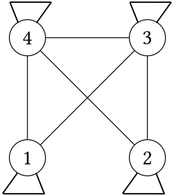
There exists a performance paradox if and only if $\delta < 0.0563$.

The uniqueness of the saturated node is a technical assumption for our main result about general graphs

Outline

Dynamic Matching Model with Loops

We allow self-matchings in the compatibility graph



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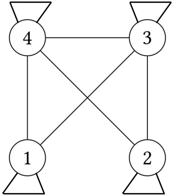

Our Result (Busic et al 2022)

For FCFM, the steady-state distribution of unmatched items has a product form expression

Dynamic Matching Model with Loops

Observation

In a matching model with loops, we have a finite-state Markov chain for any greedy policy and an arbitrary graph
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Current reseach in the following conjecture

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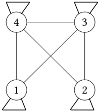
Work in progress: The performance paradox does not exist for FCFM and priorities and for the below graph.



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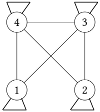

Bipartite Dynamic Matching Models

Assumption: random arrivals and in pairs

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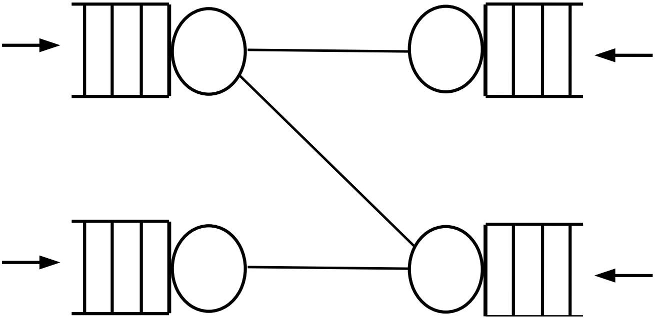

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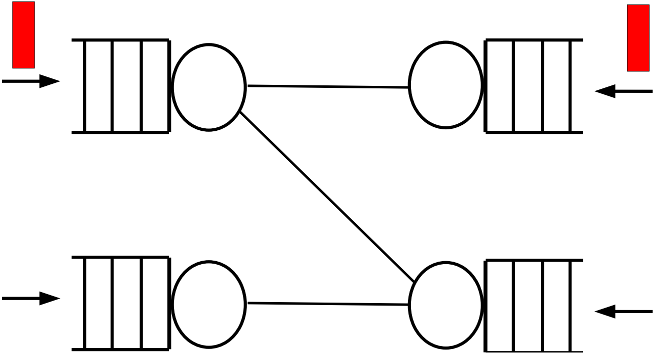

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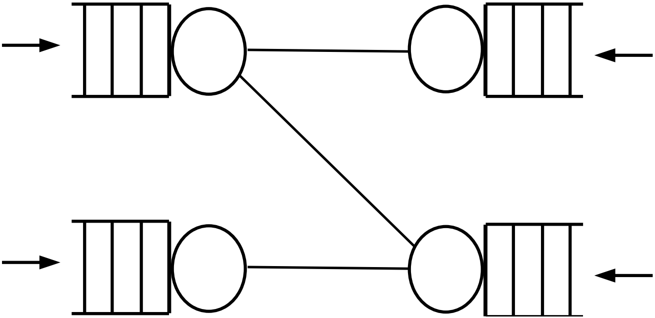

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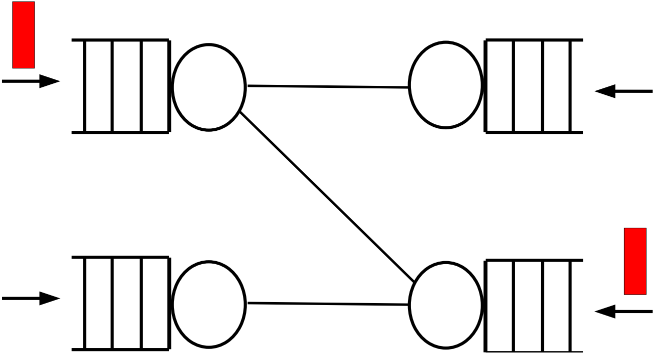

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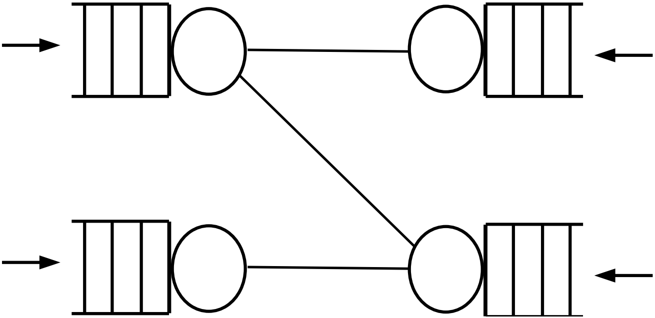

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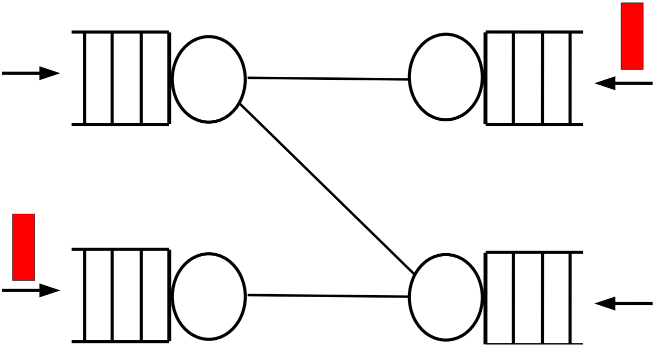

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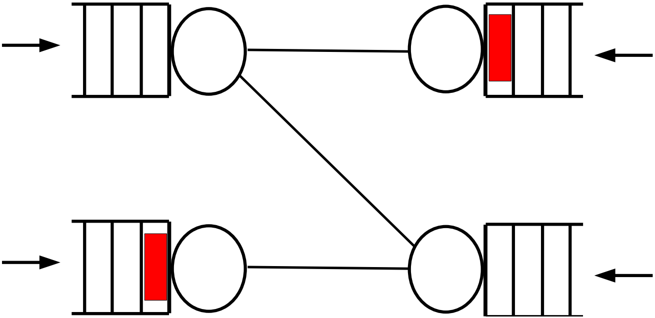

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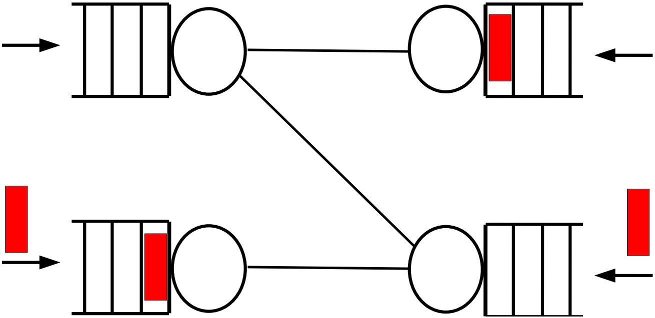

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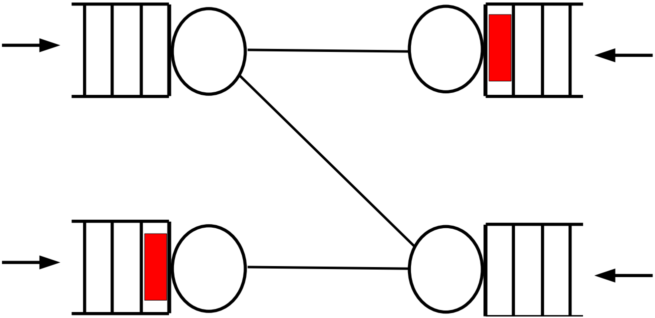

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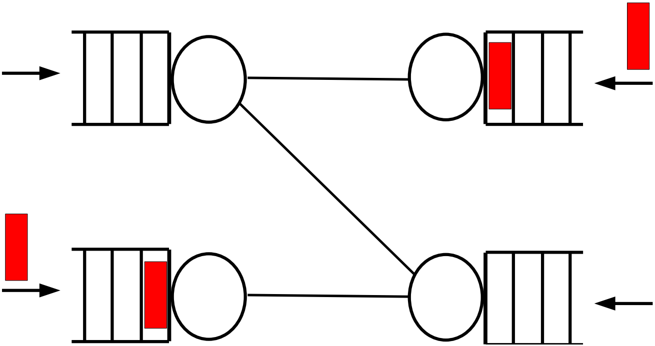

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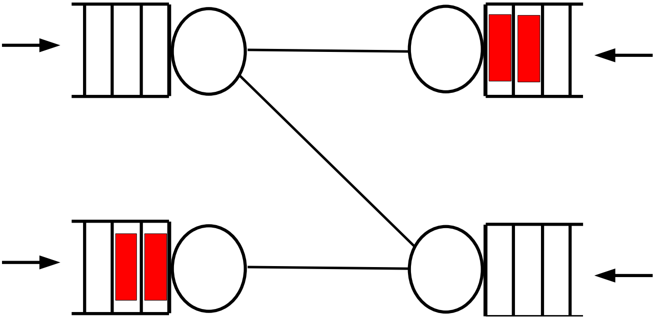

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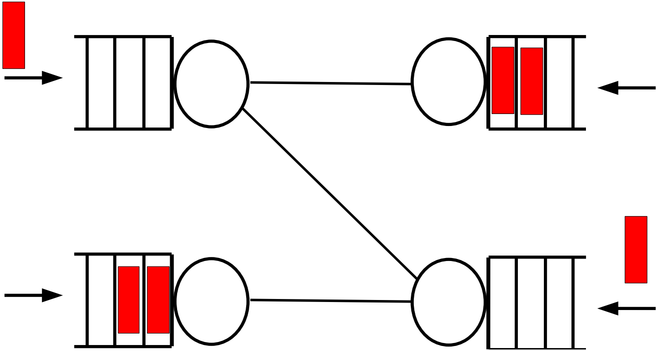

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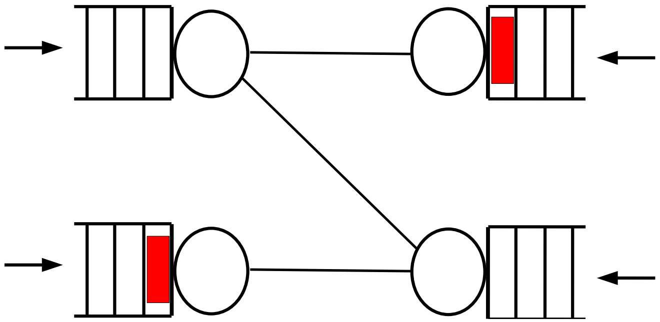

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Bipartite Dynamic Matching Models

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Open question

Does it exhibit the performance paradox in bipartite matching models?

Bipartite Dynamic Matching Models

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The steady-state distribution of unmatched items for FCFM has a product form.

Open question

Does it exhibit the performance paradox in bipartite matching models? For FCFM? For a given network topology?

Outline

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Markov Decision Processes to analyze the optimal policy in matching models

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Abandonments in dynamic matching models

Outline

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Muchas gracias

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