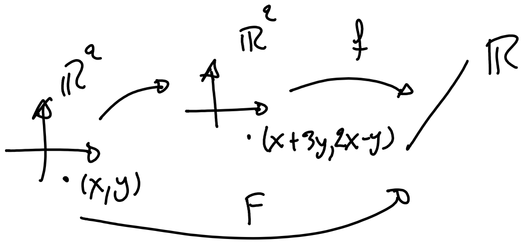
Solución práctica calificada

miércoles, 2 de junio de 2021 08:03

Sea $F(x, y) = f(x+3y, 2x-y)$ donde $f : \mathbb{R}^2 \to \mathbb{R}$ es diferenciable. Suponga que $\nabla f(0,0) = (4, -3)$. Determine la derivada direccional de la función F en el origen en la dirección del vector $v = (1, 1)$.



Solución

- $\dfrac{\partial F}{\partial v}(0,0) = \nabla F(0,0)\cdot v$; $v = \dfrac{1}{\sqrt{2}}(1,1)$ ← vector unitario

- $\nabla F(x,y) = \nabla f(x+3y, 2x-y)\cdot\begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix} \Rightarrow \nabla F(0,0) = \nabla f(0,0)\cdot\begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix} = \underbrace{(4 \quad -3)}_{1\times 2}\underbrace{\begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}}_{2\times 2} = (-2 \quad 15)_{1\times 2}$

- $\dfrac{\partial F}{\partial v}(0,0) = \nabla F(0,0)\cdot v = \dfrac{1}{\sqrt{2}}(-2, 15)\cdot(1,1) = \dfrac{1}{\sqrt{2}}(-2+15) = \dfrac{13\sqrt{2}}{2}$

Dado el nivel cero de la función $F(x, y)$. Compruebe que esta función satisface las hipótesis del Teorema de la Función Implícita en el punto indicado (perteneciente al nivel cero). Obtenga la derivada de la función $y = f(x)$ en el punto dado

$$F(x, y) = \underbrace{xe^x + ye^y - 2x - 2y} = 0,\ P(0, 0)$$

Solución

Debemos ver que se cumplen las hipótesis del Teorema de la Función Implícita:

i) $F(0,0) = 0$

ii) $\left.\begin{array}{l} \dfrac{\partial F}{\partial x} = e^x + xe^x - 2 \\ \dfrac{\partial F}{\partial y} = e^y + ye^y - 2 \end{array}\right\}$ son funciones continuas ~~sobre~~ $\mathbb{R}^2$

iii) $\dfrac{\partial F}{\partial y}(0,0) = -1 \neq 0$

$\Rightarrow$ Por el TFI tenemos $y' = f'(0) = -\dfrac{\frac{\partial F}{\partial x}(0,0)}{\frac{\partial F}{\partial y}(0,0)} = -\dfrac{-1}{-1} = -1$

D-E Sea $z = x\varphi(x+y) + y\psi(x-y)$, donde φ y ψ son dos funciones reales de variable real, dos veces derivables. Demuestre que

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 2\frac{\partial^2 z}{\partial x \partial y}$$

Solución

1° $\dfrac{\partial z}{\partial x} = \varphi(x+y) + x\varphi'(x+y) + y\psi'(x-y)$

$\dfrac{\partial z}{\partial y} = x\varphi'(x+y) + \psi(x-y) - y\psi'(x-y)$

2°
$$\frac{\partial^2 z}{\partial x^2} =$$
$$\frac{\partial^2 z}{\partial y^2} =$$
$$\frac{\partial^2 z}{\partial x \partial y} =$$

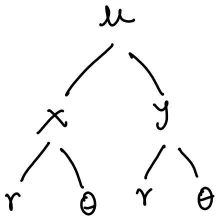
A-H/ B-G Dadas las funciones $u(x, y)$ y $v(x, y)$, verificar que las ecuaciones de Cauchy-Riemann

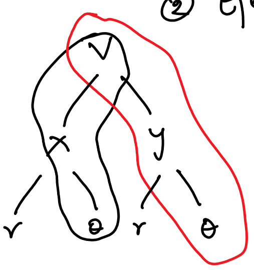
$$\boxed{\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}} \text{ y } \underbrace{\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}}$$

pueden escribirse en coordenadas polares como

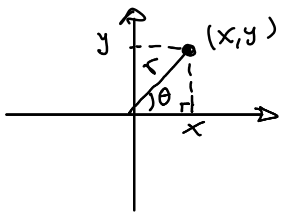
$$\underbrace{\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}}_{1} \text{ y } \underbrace{\frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}}_{2} \text{ Ejercicio}$$

Solución





$$\boxed{x = r\cos\theta}$$
$$\boxed{y = r\,\text{sen}\,\theta}$$



$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial r} = \frac{\partial u}{\partial x}\cos\theta + \frac{\partial u}{\partial y}\,\text{sen}\,\theta$$

$$\frac{\partial v}{\partial \theta} = \frac{\partial v}{\partial x}\cdot\frac{\partial x}{\partial \theta} + \frac{\partial v}{\partial y}\cdot\frac{\partial y}{\partial \theta} = \frac{\partial v}{\partial x}(-r\,\text{sen}\,\theta) + \frac{\partial v}{\partial y}(r\cos\theta) = r\left(\overbrace{-\frac{\partial v}{\partial x}}^{\frac{\partial u}{\partial y}}\text{sen}\,\theta + \overbrace{\frac{\partial v}{\partial y}}^{\frac{\partial u}{\partial x}}\cos\theta\right) = r\frac{\partial u}{\partial r}$$

$$\Rightarrow \boxed{\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}} \checkmark$$