

$$1) F(u, v) = f\left(uv, \frac{1}{2}(u^2 - v^2)\right)$$

$$F(u, v) = f(x, y)$$

$$F = f(x, y)$$

$$x = uv$$

$$y = \frac{1}{2}(u^2 - v^2)$$

• Demonstrar

$$(u^2 + v^2) \left( \left( \frac{\partial F}{\partial x} \right)^2 + \left( \frac{\partial F}{\partial y} \right)^2 \right) = \left( \frac{\partial F}{\partial u} \right)^2 + \left( \frac{\partial F}{\partial v} \right)^2$$

$$\frac{\partial F}{\partial u} = \frac{\partial F}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial F}{\partial y} \cdot \frac{\partial y}{\partial u}$$

$$\rightarrow \frac{\partial F}{\partial u} = \frac{\partial F}{\partial x} \cdot v + \frac{\partial F}{\partial y} \cdot u$$

$$\frac{\partial F}{\partial v} = \frac{\partial F}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial F}{\partial y} \cdot \frac{\partial y}{\partial v}$$

$$\rightarrow \frac{\partial F}{\partial v} = \frac{\partial F}{\partial x} \cdot u + \frac{\partial F}{\partial y} \cdot (-v)$$

$$\frac{\partial F}{\partial x} = \frac{\partial f}{\partial x}$$

$$\wedge \frac{\partial F}{\partial y} = \frac{\partial f}{\partial y}$$

Comprobando

$$(u^2 + v^2) \left[ \left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2 \right] = (av + bu)^2 + (av - bu)^2$$

$$\rightarrow a = \frac{\partial F}{\partial x} = \frac{\partial f}{\partial x}$$

$$\rightarrow b = \frac{\partial F}{\partial y} = \frac{\partial f}{\partial y}$$

$$(u^2 + v^2) \left[ \left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2 \right] = (u^2 + v^2) \left[ \left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2 \right]$$

Rpta : Se Comprobo