

1) Determinar el menor y mayor valor de la función

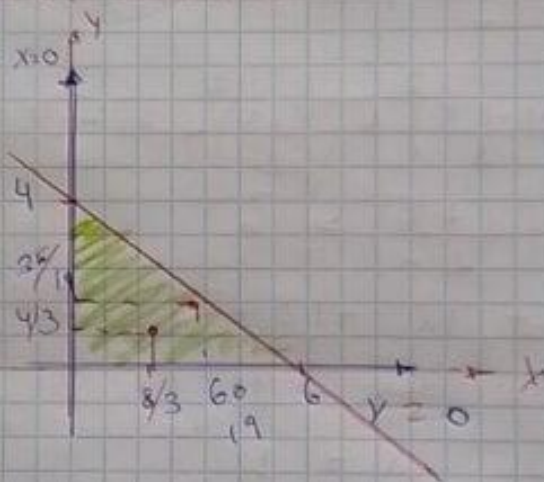
$$z = x^2 - xy + y^2 - 4x$$

en el dominio cerrado cuyo borde son

$$\text{las rectas } x=0, y=0, 2x+3y-12=0$$

$$\text{Dom: } \{(x, y) \in \mathbb{R}^2 / 2x+3y-12=0\}$$

Gráfica del dominio



P.C

$$\frac{\partial z}{\partial x} = 2x - y - 4 = 0$$

$$2x - y = 4 \quad (i)$$

$$\frac{\partial z}{\partial y} = -x + 2y = 0$$

$$2y - x = 0 \quad (ii) \times 2$$

$$\cancel{2x} - y = 4$$

$$4y - \cancel{2x} = 0$$

$$3y = 4$$

$$y = 4/3$$

$$x = 8/3$$

$$(8/3, 4/3) \text{ P.C}$$

Ecuación de Lagrange

$$L(x, y, \lambda) = x^2 - xy + y^2 - 4x + \lambda(2x + 3y - 12)$$

P.C:

$$\frac{\partial L}{\partial x} = 2x - y - 4 + \lambda = 0 \quad (i)$$

$$\frac{\partial L}{\partial y} = -x + 2y + 3\lambda = 0 \quad (ii)$$

$$\frac{\partial L}{\partial \lambda} = 2x + 3y - 12 = 0 \quad (iii)$$

$$(i) \quad x - \frac{3}{2}y + 6 - \lambda = 0$$

$$(ii) \quad -x + 2y + 3\lambda = 0$$

$$-4x + 7y + 6 = 0$$

$$(iii) \quad 2x + 3y - 12 = 0 \quad \times 2$$

$$-4x + 7y + 6 = 0$$

$$4x + 6y - 24 = 0$$

$$P.C \left(\frac{60}{19}, \frac{36}{19} \right)$$

$$\frac{19y}{2} = 18$$

$$y = \frac{36}{19}$$

$$\Rightarrow x = \frac{60}{19}$$

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$$\frac{60}{19}$$

$$f\left(\frac{8}{3}, \frac{4}{3}\right) = -\frac{43}{9} \text{ (Minimum)}$$

$$f\left(\frac{60}{19}, \frac{36}{19}\right) = \frac{2280}{19} \text{ (Maximum)}$$