Ejercicio 3

3. Si el vector $\vec{x} = (3, -1, 2)$ y $\vec{y} = (1, 1, -4)$; hallar dos vectores $\vec{u}$ y $\vec{v} \in \mathbb{R}^3$ de modo que $\vec{x} = \vec{u} + \vec{v}$, $\vec{y} \cdot \vec{v} = 0$ y $\vec{u} // \vec{y}$

Rpta: $\vec{u} = \frac{1}{3}(-1, -1, 4)$, $\vec{v} = \frac{2}{3}(5, -1, 1)$.

3) $\vec{x} = (3, -1, 2)$

$\vec{y} = (1, 1, -4)$

$\vec{u}$ y $\vec{v} \in \mathbb{R}^3$

1. $\vec{x} = \vec{u} + \vec{v}$
2. $\vec{y} \cdot \vec{v} = 0$
3. $\vec{u} // \vec{y}$

$\vec{u} = (a, b, c)$

$\vec{v} = (d, e, f)$

(1) $(3, -1, 2) = (a, b, c) + (d, e, f)$

- $a + d = 3 \quad \ldots\ldots (i)$
- $b + e = -1 \quad \ldots\ldots (ii)$
- $c + f = 2 \quad \ldots\ldots (iii)$

(2) $(1, 1, -4) \cdot \vec{v} = 0$

$(1, 1, -4)(d, e, f) = 0$

- $d + e - 4f = 0 \quad \ldots\ldots (iiii)$

(3) $\vec{u} // \vec{y}$ si existe $t \neq 0 \in \mathbb{R}$

$\vec{u} = t \cdot \vec{y}$

$(a, b, c) = t \cdot (1, 1, -4)$

$a = t$

$b = t$

$-\frac{c}{4} = t$

$a = b = -\frac{c}{4} \quad \ldots\ldots (iiiii)$

Resolución:

∘> i + ii

$$a+b + \underbrace{d+e}_{} = 2$$

$$(iiii) + (iii)$$

$$-\frac{c}{4} + \left(-\frac{c}{4}\right) + 4f = 2$$

$$\circ \quad -\frac{c}{2} + 4f = 2 \quad \ldots (5)$$

∘> i − ii

$$\circ \quad d = 2f + 2 \quad \ldots (6)$$

$$\langle\circ\rangle \left(-\frac{c}{2} + 4f = 2 \ldots (5)\right) \times (2)$$

$$\langle\circ\rangle \quad c + f = 2 \ldots (iii)$$

$$\begin{aligned} \langle\circ\rangle \quad -c + 8f &= 4 \ldots (5) \quad + \\ \langle\circ\rangle \quad c + f &= 2 \ldots (iii) \\ \hline 9f &= 6 \end{aligned}$$

$$\circ \quad f = \frac{2}{3}$$

$$\circ \quad c = \frac{4}{3}$$

$$\langle\circ\rangle \quad d = 2\left(\frac{2}{3}\right) + 2 \quad \ldots (6)$$

$d = \frac{4}{3} + 2$

$\cdot\ d = \frac{10}{3}$

$\langle\circ\rangle \quad a + \frac{10}{3} = 3 \ \ldots \ (i)$

$\cdot\ a = -\frac{1}{3}$

$\cdot\ a = b$

$b = -\frac{1}{3}$

$\cdot\ -\frac{1}{3} + e = -1 \ \ldots \ (ii)$

$e = -\frac{2}{3}$

Rpta:

$\vec{u} = \left(-\frac{1}{3}, -\frac{1}{3}, \frac{4}{3}\right)$

$\vec{v} = \left(\frac{10}{3}, -\frac{2}{3}, \frac{2}{3}\right)$

Ejercicio 7

7. Los vectores $\vec{u}$ y $\vec{v}$ forman un ángulo de 60^o, sabiendo que $||\vec{u}|| = 5$, $||\vec{v}|| = 8$, determinar $||\vec{u} + \vec{v}||$, $||\vec{u} - \vec{v}||$.   Rpta: $\sqrt{129}$ y 7.

$$\text{Cos } \theta = \frac{\vec{u} \cdot \vec{v}}{||\vec{u}|| . ||\vec{v}||}$$

$$\text{Cos } 60 = \frac{\vec{u} \cdot \vec{v}}{5 . 8}$$

$$\frac{1}{2} = \frac{\vec{u} \cdot \vec{v}}{40}$$

$$20 = \vec{u} \cdot \vec{v}$$

$$||\vec{u} + \vec{v}||^2 = ||\vec{u}||^2 + ||v||^2 + 2\, \vec{u} \cdot \vec{v}$$

$$||\vec{u} + \vec{v}||^2 = 25 + 64 + 2(20)$$

$$||\vec{u} + \vec{v}||^2 = 129$$

$$\boxed{||\vec{u} + \vec{v}|| = \sqrt{129}}$$

$$||\vec{u} - \vec{v}||^2 = ||\vec{u}||^2 + ||v||^2 - 2\, \vec{u} \cdot \vec{v}$$

$$||\vec{u} - \vec{v}||^2 = 25 + 64 - 2(20)$$

$$||\vec{u} - \vec{v}||^2 = 49$$

$$||\vec{u} - \vec{v}|| = \sqrt{49}$$

$$\boxed{||\vec{u} - \vec{v}|| = 7}$$

Ejercicio 10

10. Sean los vectores $\vec{u} = (-3, 4, 1)$ y $\vec{v} = (3, \sqrt{2}, 5)$ determinar un vector $\vec{w}$ ortogonal al vector $(0, 1, 0)$ que satisface las condiciones $\vec{u} \cdot \vec{w} = 6$ y $||Proy_v w|| = 1$
Rpta: $(-4/3, 0, 2)$ o $(-2, 0, 0)$.

10.

$\vec{u} = (-3, 4, 1)$ y $\vec{v} = (3, \sqrt{2}, 5)$

$\vec{\omega} = (a, b, c)$ $\qquad \vec{\omega} \cdot (0, 1, 0) = 0$

$\vec{u} \cdot \vec{\omega} = 6$ $\qquad ||Proy_v \omega|| = 1$

Hallar $\vec{\omega} = ?$

$$(a, b, c) \cdot (0, 1, 0) = 0$$

$$a \cdot 0 + b \cdot 1 + c \cdot 0 = 0$$

$$b = 0$$

$$\vec{u} \cdot \vec{\omega} = 6$$

$$(-3, 4, 1) \cdot (a, 0, c) = 6$$

$$-3a + 0 + c = 6$$

$$-3a + c = 6$$

$$||Proy_{\vec{\omega} \to \vec{v}}|| = 1$$

$$\left|\left| \frac{\vec{\omega} \cdot \vec{v}}{\vec{v} \cdot \vec{v}} \cdot \vec{v} \right|\right| = 1$$

$$\left|\left| \frac{(a, 0, c) \cdot (3, \sqrt{2}, 5)}{(3, \sqrt{2}, 5) \cdot (3, \sqrt{2}, 5)} \cdot (3, \sqrt{2}, 5) \right|\right| = 1$$

$$\left|\left| \frac{3a + 5c}{9 + 2 + 25} \cdot (3, \sqrt{2}, 5) \right|\right| = 1$$

$$\left|\left| \frac{3a + 5c}{36} \cdot (3, \sqrt{2}, 5) \right|\right| = 1$$

$$\left|\left| \left(\frac{(3a + 5c)}{12}, \frac{\sqrt{2}(3a + 5c)}{36}, \frac{(3a + 5c)5}{36} \right) \right|\right| = 1$$

$$\sqrt{\frac{(3a+5c)^2}{12^2}+\frac{2(3a+5c)^2}{36^2}+\frac{25(3a+5c)^2}{36^2}}=1$$

$$\sqrt{\frac{(3a+5c)^2}{12^2}\left(1+\frac{2}{3^2}+\frac{25}{3^2}\right)}=1$$

$$\frac{3a+5c}{12}\sqrt{1+\frac{2}{9}+\frac{25}{9}}=1$$

$$(3a+5c).\sqrt{\frac{9+2+25}{9}}=12$$

$$(3a+5c)\sqrt{\frac{36}{9}}$$

$$(3a+5c).2=12$$

$$3a+5c=6 \qquad 3a+10=6$$

$$-3a+c=6 \qquad 3a=-4$$

$$a=\frac{-4}{3}$$

$$6c=12$$

$$c=2$$

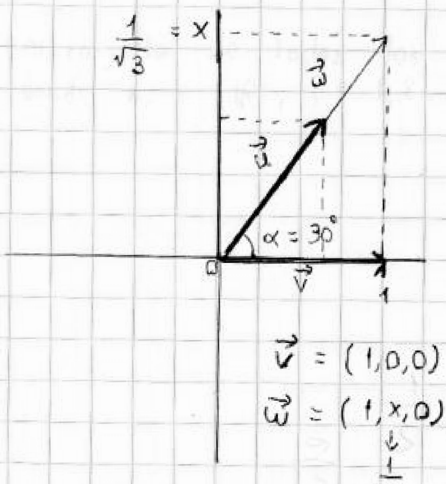
$$\vec{\omega}=\left(-\frac{4}{3},0,2\right)$$

Ejercicio 14

14. Suponga que los vectores $\vec{u}, \vec{v} \in \mathbb{R}^3$ son vectores unitarios que forman entre si un ángulo de $\pi/6$rad. Calcular $||\vec{u} \times \vec{v}||$
Rpta: 0,5.

14. $\vec{u}\ ,\ \vec{v} \in \mathbb{R}^3$

$\|\vec{u}\| = 1 \quad , \quad \|\vec{v}\| = 1 \quad , \quad \alpha = \frac{\pi}{6}$ rad



$\alpha = \frac{\pi}{6} \times \frac{180}{\pi} = 30^\circ$

$\vec{v} = (1, 0, 0)$

$\vec{w} = (1, x, 0)$

$\downarrow$
$\frac{1}{\sqrt{3}}$

$\vec{u}$ es el vector unitario de $\vec{w}$, entonces primero calculamos x.

$1 = a\sqrt{3} \quad , \quad x = a$ por triangulos notables

$a = \frac{1}{\sqrt{3}}$

$x = \frac{1}{\sqrt{3}}$

Calculamos $\vec{u}$, vector unitario de $\vec{w}$

$\vec{w} = (1, \frac{1}{\sqrt{3}}, 0)$

$|\vec{w}| = \sqrt{1^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + 0^2}$

$|\vec{w}| = \sqrt{1 + \frac{1}{3}}$

$|\vec{w}| = \sqrt{\frac{4}{3}}$

$|\vec{w}| = \frac{2}{\sqrt{3}}$

$\vec{u} = \frac{\vec{w}}{|\vec{w}|} = \left(\frac{1}{\frac{2}{\sqrt{3}}}, \frac{\frac{1}{\sqrt{3}}}{\frac{2}{\sqrt{3}}}, 0\right)$

$\vec{u} = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 0\right)$

Calculamos $\vec{u} \times \vec{v}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \sqrt{3}/2 & 1/2 & 0 \\ 1 & 0 & 0 \end{vmatrix}$$

$\vec{u} \times \vec{v} = (0, 0, -1/2)$

Calculamos $|\vec{u} \times \vec{v}|$

$|\vec{u} \times \vec{v}| = \frac{1}{2}$

$|\vec{u} \times \vec{v}| = 0,5$

Ejercicio 22

22. Los vértices de un triángulo son los puntos $A(-1,-2,4)$; $B(-4,-1,2)$ y $C(-5,6,-4)$. Hallar un vector $\overrightarrow{w}$ que es paralelo a la altura trazada del vértice B al lado opuesto, si se sabe que $\|\overrightarrow{w}\| = 2\sqrt{5}$

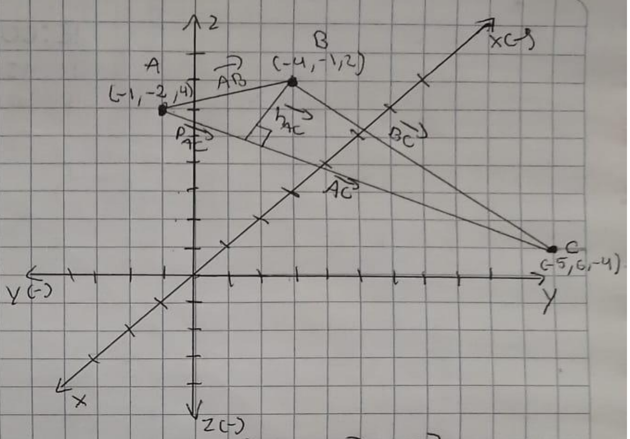
Rpta: $\overrightarrow{w} = (4,2,0)$

22.

$A(-1,-2,4)$

$B(-4,-1,2)$

$C(-5,6,-4)$



$\overrightarrow{AB} = (-3,1,-2)$

$\overrightarrow{AC} = (-4,8,-8)$

$|\overrightarrow{AC}| = \sqrt{(-4)^2+8^2+(-8)^2} = 12$

$\overrightarrow{U_{AC}} = \left(\frac{-4}{12}, \frac{8}{12}, \frac{-8}{12}\right)$

$\overrightarrow{P_{AC}} = (\overrightarrow{AB} \cdot U_{AC}) \cdot U_{AC}$

$\overrightarrow{P_{AC}} = \left[(-3,1,-2) \cdot \left(\frac{-4}{12}, \frac{8}{12}, \frac{-8}{12}\right)\right] \cdot \left(\frac{-4}{12}, \frac{8}{12}, \frac{-8}{12}\right)$

$\overrightarrow{P_{AC}} = \left(1 + \frac{2}{3} + \frac{4}{3}\right) \cdot \left(\frac{-4}{12}, \frac{8}{12}, \frac{-8}{12}\right)$

$\overrightarrow{P_{AC}} = 3 \cdot \left(\frac{-4}{12}, \frac{8}{12}, \frac{-8}{12}\right)$

$\overrightarrow{P_{AC}} = (-1,2,-2)$

$\overrightarrow{h_{AC}} = \overrightarrow{AB} - \overrightarrow{P_{AC}}$

$\overrightarrow{h_{AC}} = (-3,1,-2) - (-1,2,-2)$

$\overrightarrow{h_{AC}} = (-2,-1,0)$ //

Altura

Ahora hallamos el vector paralelo a la altura opuesto con el modulo $|w| = 2\sqrt{5}$ y la altura $\overrightarrow{h_{AC}} = (-2,-1,0)$

$\overrightarrow{w} = K \cdot \overrightarrow{h_{AC}} = K \cdot (-2,-1,0) = (-2K, -1K, 0)$

$|w| = \sqrt{(-2K)^2 + (-1K)^2 + 0^2} = \sqrt{4K^2 + 1K^2 + 0}$

$|w| = 2\sqrt{5} \rightarrow 2\sqrt{5} = \sqrt{5K^2} \Rightarrow -\sqrt{4} = K$

$\sqrt{4} = K$

$\overrightarrow{w} = (-2 \cdot -\sqrt{4}, -1 \cdot -\sqrt{4}, 0) \Rightarrow \boxed{\overrightarrow{w} = (4,2,0)}$ //

$\overrightarrow{w} = (-2 \cdot \sqrt{4}, -1 \cdot \sqrt{4}, 0) \rightarrow \overrightarrow{w} = (-4,-2,0)$