Volumen de

Intersección

$z = 3x^2 + y^2 \wedge z = 4 - x^2$

$3x^2 + y^2 = 4 - x^2$

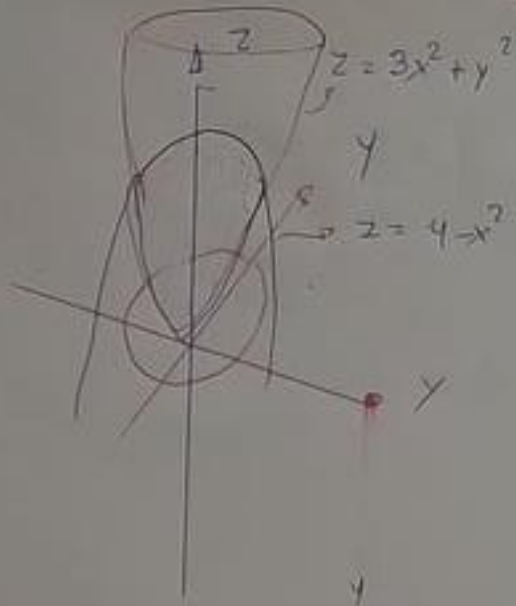
$4x^2 + y^2 = 4$

Elipse

Cambio a Coordenadas Cilíndricas

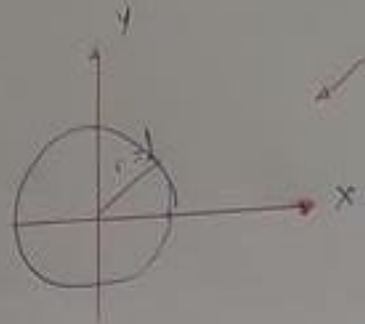
$x^2 + \frac{y^2}{4} = 1$

$x^2 + \left(\frac{y}{2}\right)^2 = 1$



$x = r\cos\theta$

$\frac{y}{2} = r\,\text{sen}\,\theta \Rightarrow y = 2r\,\text{sen}\,\theta$



$0 \leq r \leq 1$

$0 \leq \theta \leq 2\pi$

$3x^2 + y^2 \leq z \leq 4 - x^2$

$r^2\,\text{sen}^2\theta + 3r^2 \leq z \leq 4 - r^2\cos^2\theta$

$$J(r,\theta,z) = \begin{vmatrix} \cos\theta & -r\,\mathrm{sen}\,\theta & 0 \\ 2\,\mathrm{sen}\,\theta & 2r\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = 2r\cos^2\theta + 2r\,\mathrm{sen}^2\theta$$

$$\begin{matrix} \cos\theta & -r\,\mathrm{sen}\,\theta & 0 \\ 2\,\mathrm{sen}\,\theta & 2r\cos\theta & 0 \end{matrix}$$

$$J(r,\theta,z) = 2r$$

$$V = \int_0^1 \int_0^{2\pi} \int_{r^2 sen^2\theta + 3r^2}^{4 - r^2\cos^2\theta} |J(r\theta,z)| \, dz \, d\theta \, dr$$

$$V = \int_0^1 \int_0^{2\pi} \int_{r^2 sen^2\theta + 3r^2}^{4 - r^2\cos^2\theta} 2r \, dz \, d\theta \, dr$$

$$V = 2 \int_0^1 \int_0^{2\pi} \int_{r^2 sen^2\theta + 3r^2}^{4 - r^2\cos^2\theta} r \, dz \, d\theta \, dr$$

$$V = 2 \int_0^1 \int_0^{2\pi} r \left(4 - r^2\cos^2\theta - r^2 sen^2\theta - 3r^2 \right) d\theta \, dr$$

$$V = 2 \int_0^1 \int_0^{2\pi} r \left(4 - 3r^2 - r^2 \right) d\theta \, dr$$

$$V = 2 \int_0^1 \int_0^{2\pi} 4r - 4r^3 \, d\theta \, dr$$

$$V = 8 \int_0^1 \int_0^{2\pi} r - r^3 \, d\theta \, dr$$

$$V = 8 \int_0^1 r\theta - r^3\theta \Big|_0^{2\pi} dr$$

$$V = 8 \int_0^1 \left(2\pi r - 2\pi r^3\right) dr$$

$$V = 16\pi \int_0^1 r - r^3 \, dr$$

$$V = 16\pi \left[\frac{r^2}{2} - \frac{r^4}{4}\right]\Big|_0^1$$

$$V = 16\pi \left(\frac{1}{2} - \frac{1}{4}\right)$$

$$V = 16\pi \left(\frac{1}{4}\right)$$

$$V = 4\pi \, u^3$$