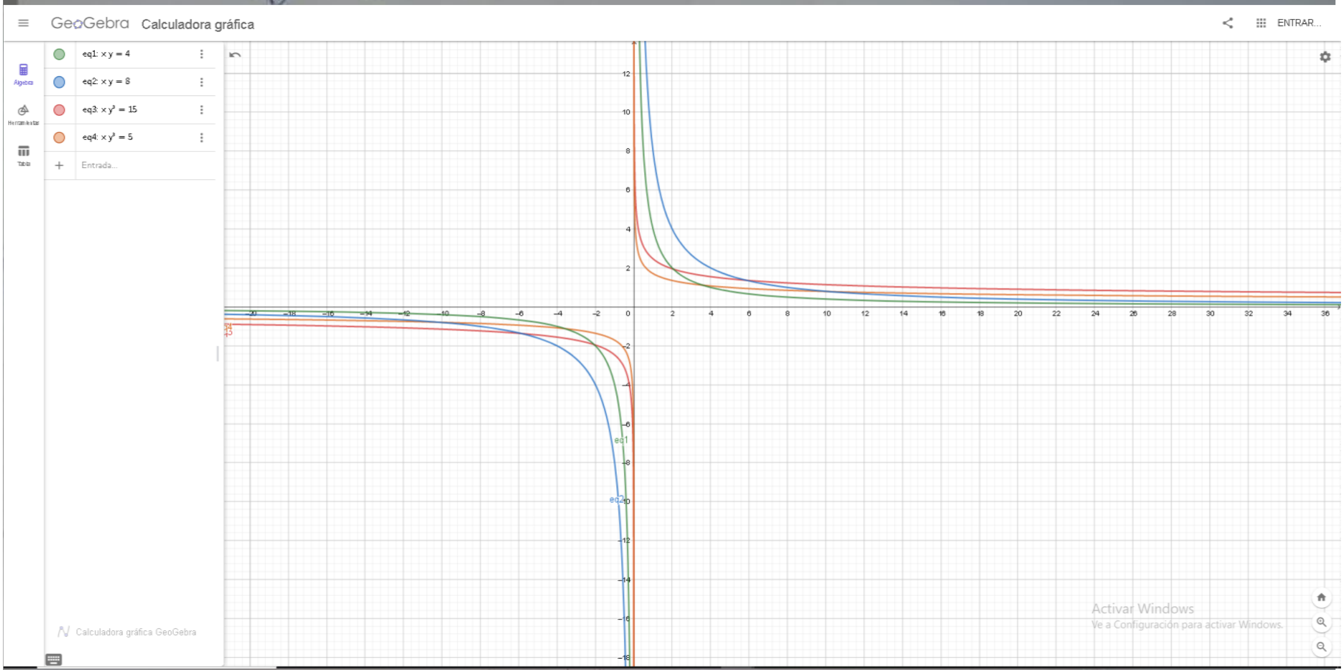
1) Hallar el area de la región limitada por las curvas

- $x y = 4$
- $x y = 8$
- $x y^3 = 15$
- $x y^3 = 5$



b) Cambio de variable

$$u = x y \quad \dots \quad i$$

$$v = x y^3 \quad \dots \quad ii$$

- $4 \leq u \leq 8$
- $5 \leq v \leq 15$

$i \quad x = \dfrac{u}{y}$

$ii \quad v = u \cdot y^2$

- $y = \sqrt{\dfrac{v}{u}}$

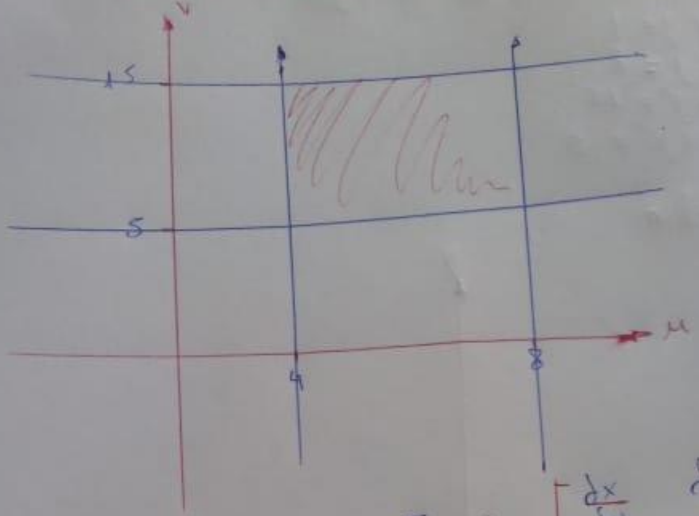
$$y = v^{1/2} \cdot u^{-1/2}$$

- $\Rightarrow \dfrac{\partial x}{\partial u} = \dfrac{3}{2\sqrt{v}} u^{1/2}$
- $\Rightarrow \dfrac{\partial x}{\partial v} = \dfrac{-u^{3/2}}{2} v^{-3/2}$
- $\Rightarrow \dfrac{\partial y}{\partial u} = \dfrac{-\sqrt{v}}{2} u^{-3/2}$
- $\Rightarrow \dfrac{\partial y}{\partial v} = \dfrac{1}{2\sqrt{u}\sqrt{v}}$

- $\dfrac{u^{3/2}}{\sqrt{v}} = x$

$$x = u^{3/2} \cdot v^{-1/2}$$

•> Grafico de la Transformacion



$$J(u,v) = \begin{bmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v} \\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v} \end{bmatrix}$$

Area: $\iint_R 1 \, dA$

•> Transformación

Area: $\int_R \int 1 \, |J(u,v)| \, du \, dv$

Area: $\int_5^{15} \left[\int_4^8 |J(u,v)| \, du \right] dv$

$$J(u,v) = \begin{bmatrix} \dfrac{3}{2}\sqrt{\dfrac{u}{v}} & -\dfrac{u^{3/2}}{2\,v^{3/2}} \\ \dfrac{-\sqrt{v}}{2\,u^{3/2}} & \dfrac{1}{2\sqrt{u}\sqrt{v}} \end{bmatrix}$$

$$J(u,v) = \left| \frac{3}{4v} - \left(\frac{v^{1/2}}{4\,v^{3/2}} \right) \right|$$

$$J(u,v) = \left| \frac{3}{4v} - \frac{1}{4v} \right| = \frac{2}{4v} = \frac{1}{2v}$$

Area: $\int_5^{15} \left[\int_4^8 \frac{1}{2v}\, du\right] dv$

$$\int_5^{15} \left.\frac{u}{2v}\right|_4^8 dv$$

$$\int_5^{15} \frac{8}{2v} - \frac{4}{2v} \, dv$$

$$\int_5^{15} \frac{4}{2v} \, dv$$

$$\int_5^{15} \frac{2}{v} \, dv$$

$$2\int_5^{15} v^{-1} \, dv$$

$$2\int_5^{15} M^{-1} \, dv$$

$$2\left[(\ln(15) - \ln(5))\right]$$

$$2\left[\ln(3)\right]$$

Area : $2\ln(3)\ u^2$ //