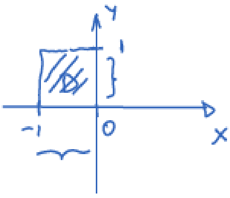
Solución integrales dobles con cambio

1. Calcular $\iint_D \frac{3x^2\sqrt{y}-4y}{x^2-4}\,dA$, $D=[-1,0]\times[0,1]$



$-1\le x\le 0$
$0\le y\le 1$

$$\int_{-1}^{0}\int_{0}^{1}\frac{3x^2\sqrt{y}-4y}{x^2-4}\,dy\,dx=\int_{-1}^{0}\left[\frac{\frac{2}{3}3x^2y^{3/2}-\frac{4y^2}{2}}{x^2-4}\right]_0^1 dx$$

$$=\int_{-1}^{0}\frac{2x^2-2}{x^2-4}\,dx=\int_{-1}^{0}\left(2+\frac{6}{x^2-4}\right)dx$$

$$=\int_{-1}^{0}\left(2+6\left(\frac{1/4}{x-2}+\frac{-1/4}{x+2}\right)\right)dx$$

$$=\left[2x+\frac{3}{2}\ln|x-2|-\frac{3}{2}\ln|x+2|\right]_{-1}^{0}$$

$$=\cancel{\frac{3}{2}\ln(2)-\frac{3}{2}\ln(2)}-\left(-2+\frac{3}{2}\ln(3)-\cancel{\frac{3}{2}\ln(1)}_{0}\right)$$

$$=2-\frac{3}{2}\ln(3)$$

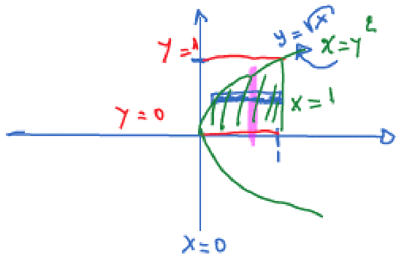
O.A.

$$\begin{array}{r|l} 2x^2-2 & x^2-4 \\ -2x^2+8 & 2 \\ \hline 6 & \end{array}$$

- $\frac{1}{x^2-4}=\frac{A}{x-2}+\frac{B}{x+2}$
- $\frac{1}{x^2-4}=\frac{A(x+2)+B(x-2)}{x^2-4}$

$\Rightarrow 1=A(x+2)+B(x-2)$
- $x=-2 \rightarrow B=-1/4$
- $x=2 \rightarrow A=1/4$

2. Calcular $\int_0^1\int_{y^2}^{1} y^3\,\text{sen}(x^3)\,dx\,dy=\int_0^1\int_0^{\sqrt{x}} y^3\,\text{sen}(x^3)\,dy\,dx=\int_0^1\left[\frac{y^4}{4}\text{sen}(x^3)\right]_0^{\sqrt{x}}dx$



$$\boxed{\begin{array}{l}0\le y\le 1\\ y^2\le x\le 1\end{array}}$$

$\downarrow$

$0\le x\le 1$
$0\le y\le \sqrt{x}$

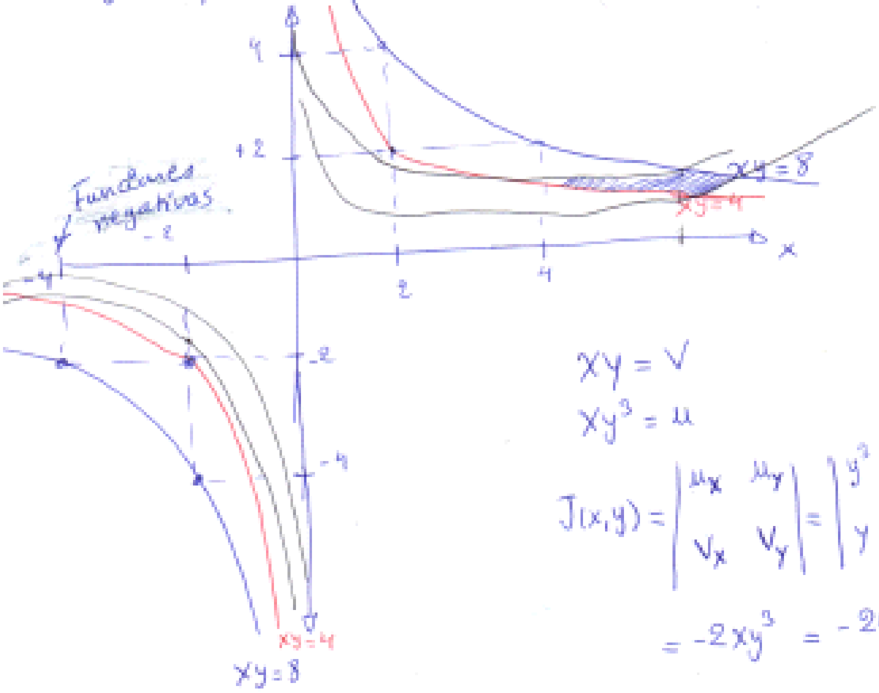
$$=\frac{1}{3}\int_0^1 \frac{3x^2}{4}\,\text{sen}(x^3)\,dx$$

$$=\frac{1}{3}\left[\frac{-\cos(x^3)}{4}\right]_0^1=\frac{1}{12}\left(-\cos(1)+\cancel{\cos(0)}_{1}\right)$$

$$=-\frac{\cos(1)}{12}+\frac{1}{12}$$

4. Hallar el área de la región limitada por las curvas

$xy = 4$, $xy = 8$, $xy^3 = 15$, $xy^3 = 5$



En el primer y tercer cuadrante tenemos dos regiones iguales

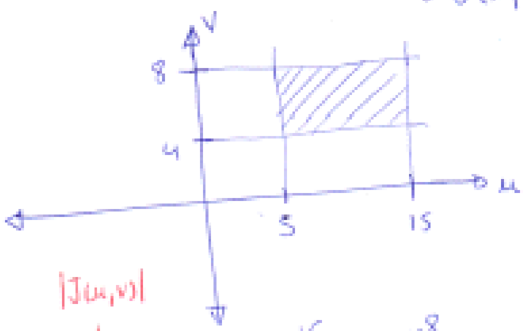
$xy = v$

$xy^3 = u$

$$J(x,y) = \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix} = \begin{vmatrix} y^3 & 3xy^2 \\ y & x \end{vmatrix} = xy^3 - 3xy^3$$

$$= -2xy^3 = -2u$$

$$\Rightarrow J(u,v) = \frac{1}{J(x,y)} = \frac{1}{-2u}$$



$$\Rightarrow \int_5^{15}\int_4^8 \underbrace{\frac{1}{2u}}_{|J(u,v)|} \, dv\,du = \int_5^{15}\left[\frac{v}{2u}\right]_4^8 du = \int_5^{15}\frac{2}{u}\,du = \Big[2\ln|u|\Big]_5^{15}$$

$$= 2\left(\ln(3.5) - \ln(5)\right)$$

$$= 2\ln(3)$$

Área total $= 4\ln(3)\,u^2$