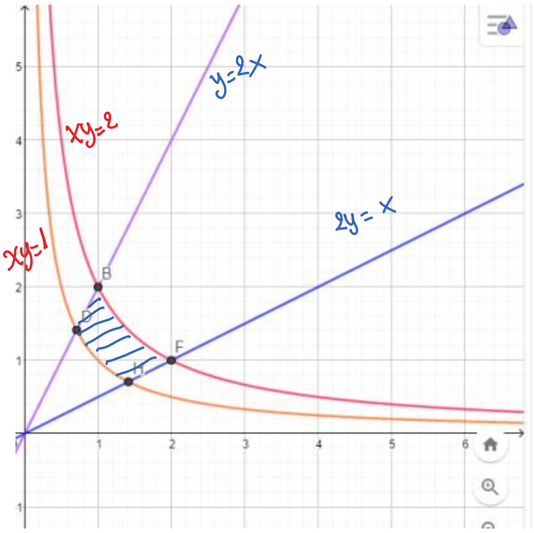
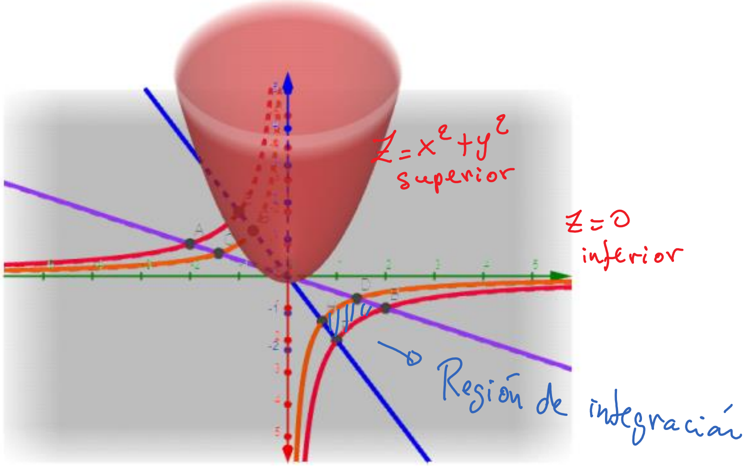
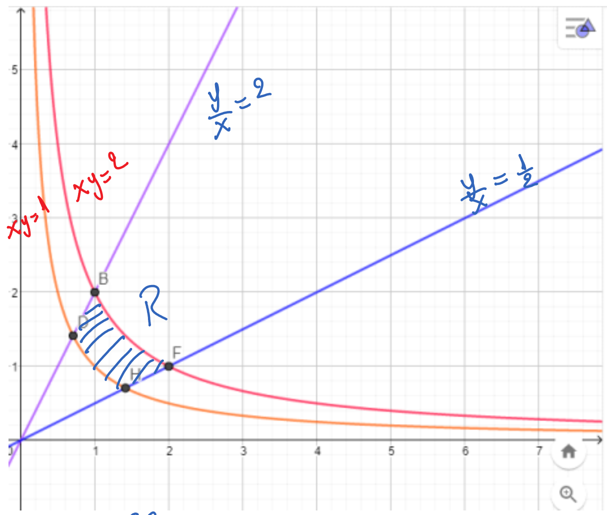
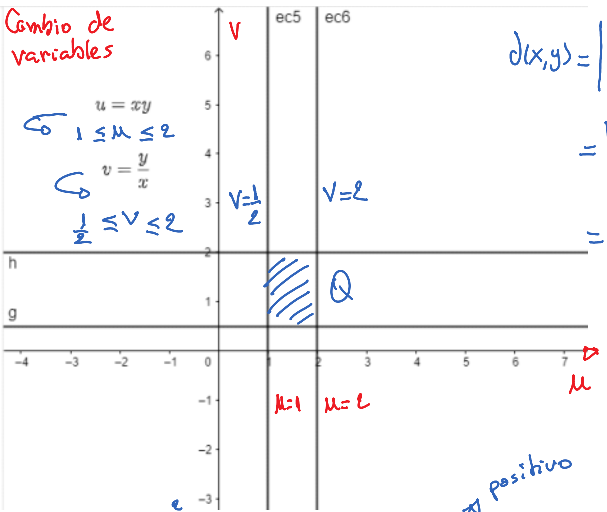
14. Calcular el volumen del cuerpo limitado por las superficies $z = x^2 + y^2$, $xy = 1$, $xy = 2$, $2y = x$, $2x = y$, $z = 0$. $(x \geq 0)$ y $(y \geq 0)$ Rpta. $9/4$.









$$J(x,y) = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} y & x \\ -\frac{y}{x^2} & \frac{1}{x} \end{vmatrix} = \frac{y}{x} + \frac{y}{x} = 2\frac{y}{x}$$

$$\Rightarrow J(x,y) = 2\left(\frac{y}{x}\right) = 2v$$

$$\Rightarrow J(u,v) = \frac{1}{J(x,y)} = \frac{1}{2v}$$

$$\Rightarrow V = \iint_R z_{sup} - z_{inf}\, dA = \iint_R (x^2+y^2-0)\,dx\,dy = \int_{1/2}^{2}\int_{1}^{2}\left(\frac{u}{v} + uv\right)\left|\frac{1}{2v}\right| du\,dv$$

positivo

$$= \int_{1/2}^{2}\int_{1}^{2}\left(\frac{u}{2v^2} + \frac{u}{2}\right) du\,dv = \int_{1/2}^{2}\left[\frac{u^2}{4v^2} + \frac{u^2}{4}\right]_1^2 dv = \int_{1/2}^{2}\left(\frac{1}{v^2} + 1 - \frac{1}{4v^2} - \frac{1}{4}\right) dv$$

$$= \int_{1/2}^{2} \frac{3}{4}v^{-2} + \frac{3}{4}\, dv = \frac{3}{4}\left(-v^{-1} + v\right)\Big|_{1/2}^{2} = \frac{3}{4}\left(-\frac{1}{2} + 2 + 2 - \frac{1}{2}\right) = \frac{9}{4}u^3$$

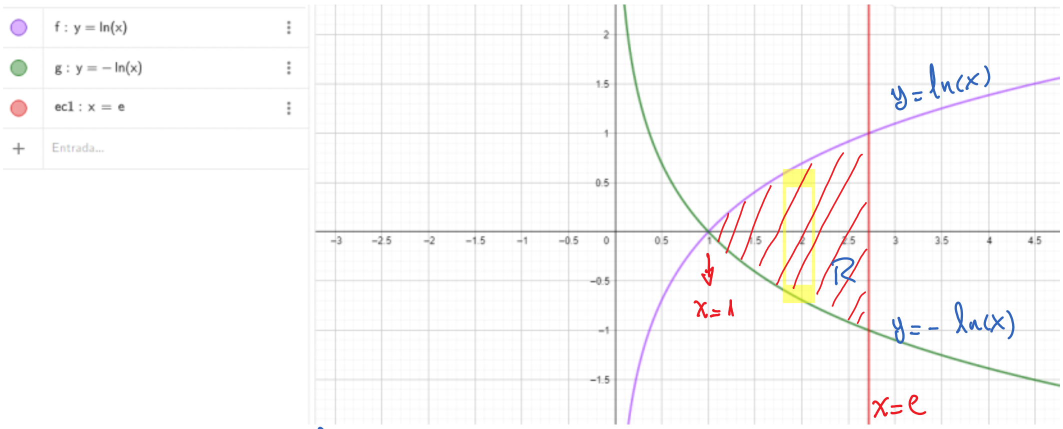
O.A.

$$u \cdot v = \cancel{x} y \cdot \frac{y}{\cancel{x}} = y^2$$

$$\frac{u}{v} = \frac{x\cancel{y}}{\frac{\cancel{y}}{x}} = x^2$$

Para más detalles sobre los gráfico visitar el siguiente link https://www.geogebra.org/classic/wnbwwrbm

15. Calcular el área de la región limitada por las curvas $y = \ln(x)$, $y = -\ln(x)$, $x = e$. Rpta. 2.



$$\text{Área} = \iint_R 1\,dy\,dx = \int_1^e \int_{-\ln(x)}^{\ln(x)} dy\,dx = \int_1^e \Big[y\Big]_{-\ln(x)}^{\ln(x)} dx = \int_1^e 2\ln(x)\,dx = 2\int_1^e \ln(x)\,dx$$

<u>O.A.</u>

$$u = \ln(x) \qquad \int dv = \int dx$$
$$du = \frac{1}{x}dx \qquad v = x$$

$$\Rightarrow \int \ln x\,dx = x\ln(x) - \int x\cdot\frac{1}{x}dx = x\ln x - x + C$$

$$\Rightarrow \text{Área} = 2\Big[x\ln x - x\Big]_1^e = 2\left(e\,\cancel{\ln e}^{\,1} - e - \cancel{\ln 1}^{\,0} + 1\right) = 2\,u^2$$