SOLUCION EX PARCIAL 2

domingo, 4 de julio de 2021 12:44

Diga si la siguiente afirmación es verdadera o falsa:

Si $z = \frac{1}{y}[f(ax+y) + g(ax-y)]$, se satisface $\frac{\partial^2 z}{\partial x^2} = \frac{a^2}{y^2}\frac{\partial}{\partial y}\left(y^2 \frac{\partial z}{\partial y}\right)$

Seleccione una:

- (•) Verdadero
- ( ) Falso

Solución

- $\frac{\partial z}{\partial x} = \frac{1}{y}\left[f'(ax+y)\cdot a + g'(ax-y)\cdot a\right] = \frac{a}{y}\left[f'(ax+y) + g'(ax-y)\right]$
- $\frac{\partial^2 z}{\partial x^2} = \frac{a^2}{y}\left[\underbrace{f''(ax+y) + g''(ax-y)}\right] \quad \dots (2)$
- $\frac{\partial}{\partial y}\left(y^2 \frac{\partial z}{\partial y}\right) = \frac{\partial}{\partial y}\left(y^2\left[-\frac{1}{y^2}\left(f(ax+y) + g(ax-y)\right) + \frac{1}{y}\left(f'(ax+y) - g'(ax-y)\right)\right]\right)$

$$= \frac{\partial}{\partial y}\left(-f(ax+y) - g(ax-y) + y\left[f'(ax+y) - g'(ax-y)\right]\right)$$

$$= \cancel{-f'(ax+y)} + \cancel{g'(ax-y)} + \cancel{f'(ax+y)} - \cancel{g'(ax-y)} + y\overbrace{\left[f''(ax+y) + g''(ax-y)\right]}$$

$\Rightarrow$ Reemp. en (2): $\frac{\partial^2 z}{\partial x^2} = \frac{a^2}{y}\left(\frac{1}{y}\cdot\frac{\partial}{\partial y}\left(y^2\frac{\partial z}{\partial y}\right)\right) = \frac{a^2}{y^2}\frac{\partial}{\partial y}\left(y^2\frac{\partial z}{\partial y}\right) \neq \frac{a^2}{x^2}\frac{\partial}{\partial y}\left(y^2\frac{\partial z}{\partial y}\right)$

Diga si la siguiente afirmación es verdadera o falsa:

Si $z = \frac{1}{y}[f(ax+y) + g(ax-y)]$, se satisface $\frac{\partial^2 z}{\partial x^2} = \frac{a^2}{x^2}\frac{\partial}{\partial y}\left(y^2 \frac{\partial z}{\partial y}\right)$

Seleccione una:

- ( ) Verdadero
- (•) Falso

Observe: $\frac{\partial^2 z}{\partial x^2} = \frac{a^2}{y^2}\frac{\partial}{\partial y}\left(y^2\frac{\partial z}{\partial y}\right) \neq \frac{a^2}{x^2}\frac{\partial}{\partial y}\left(y^2\frac{\partial z}{\partial y}\right)$

Diga si la siguiente afirmación es verdadera o falsa:

Si $z = \frac{1}{x}[f(x-y) + g(x+y)]$, se satisface $\frac{\partial}{\partial x}\left(x^2 \frac{\partial z}{\partial x}\right) = x^2 \frac{\partial^2 z}{\partial y^2}$

Seleccione una:

- (•) Verdadero
- ( ) Falso

Solución

- $\frac{\partial z}{\partial y} = \frac{1}{x}\left[-f'(x-y) + \underbrace{g'(x+y)}\right]$
- $\frac{\partial^2 z}{\partial y^2} = \frac{1}{x}\left[f''(x-y) + g''(x+y)\right]$
- $\frac{\partial}{\partial x}\left(x^2\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial x}\left(x^2\left[-\frac{1}{x^2}\left(f(x-y) + g(x+y)\right) + \frac{1}{x}\left(f'(x-y) + g'(x+y)\right)\right]\right)$

$$\cdot \frac{\partial}{\partial x}\left(x^2 \frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial x}\left(x^2\left[-\frac{1}{x^2}\left(f(x-y)+g(x+y)\right) + \frac{1}{x}\left(f'(x-y)+g'(x+y)\right)\right]\right)$$

$$= \frac{\partial}{\partial x}\left(-f(x-y)-g(x+y)+x\left(f'(x-y)+g'(x+y)\right)\right) = \cancel{-f'(x-y)} \cancel{-g'(x+y)} + \left(\cancel{f'(x-y)} + \cancel{g'(x+y)}\right) + x\left(f''(x-y)+g''(x+y)\right)$$

$$= x\left(\overbrace{f''(x-y)+g''(x+y)}^{2}\right) = x\left(x\frac{\partial^2 z}{\partial y^2}\right) = x^2\frac{\partial^2 z}{\partial y^2}$$

$$\Rightarrow \frac{\partial}{\partial x}\left(x^2\frac{\partial z}{\partial x}\right) = x^2\frac{\partial^2 z}{\partial y^2} \;\#$$

Diga si la siguiente afirmación es verdadera o falsa:

Si $z = \frac{1}{x}[f(x-y)+g(x+y)]$, se satisface $\frac{\partial}{\partial x}\left(x^2\frac{\partial z}{\partial x}\right) = x^2\frac{\partial^2 z}{\partial x^2}$

Seleccione una:

- ○ Verdadero
- ◉ Falso

Observe que

$$\frac{\partial z}{\partial x} = -\frac{1}{x^2}\left(f(x-y)+g(x+y)\right) + \frac{1}{x}\left(f'(x-y)+g'(x+y)\right)$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{1}{x^3}\left(f(x-y)+g(x+y)\right) - \frac{1}{x^2}\left(f'(x-y)+g'(x+y)\right) - \frac{1}{x^2}\left(f'(x-y)+g'(x+y)\right) + \frac{1}{x}\left(f''(x-y)+g''(x+y)\right)$$

$$= \frac{1}{x^3}\left(f(x-y)+g(x+y)\right) - \frac{2}{x^2}\left(f'(x-y)+g'(x+y)\right) + \frac{1}{x}\left(f''(x-y)+g''(x-y)\right) \neq \frac{\partial^2 z}{\partial y^2}$$

Compruebe que la función $F(x,y) = yln(x^2+y^2) - 2xy$ satisface las hipótesis del teorema de la función implícita en el punto $P = (0,1)$, perteneciente al nivel cero de F y obtenga la derivada de la función $y = f(x)$ en el punto dado.

Solución

i) $F(0,1) = \ln(1) = 0$

ii) $\frac{\partial F}{\partial x} = y\frac{1}{x^2+y^2}\cdot 2x - 2y = \frac{2xy}{x^2+y^2} - 2y$

$\frac{\partial F}{\partial y} = \ln(x^2+y^2) + y\cdot\frac{1}{x^2+y^2}\cdot 2y - 2x = \ln(x^2+y^2) + \frac{2y^2}{x^2+y^2} - 2x$

Continuas en $(0,1)$

iii) $\frac{\partial F}{\partial y}(0,1) = \ln(1) + 2 = 2 \neq 0$

$\Rightarrow$ Por el TFI existe $y = f(x)$ tal que $F(x, f(x)) = 0$, $\forall x \in B_\delta(0)$ y

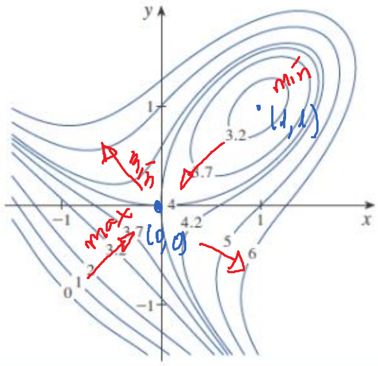
$$y'\Big|_{(0,1)} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}}\Bigg|_{(0,1)} = -\frac{\frac{2xy}{x^2+y^2} - 2y}{\ln(x^2+y^2) + \frac{2y^2}{x^2+y^2} - 2x}\Bigg|_{(0,1)}$$

$$y'\Big|_{(0,1)} = -\frac{-2}{2} = 1 \;\#$$

$$y'\Big|_{(0,1)} = -\frac{-2}{2} = 1$$

Use las curvas de nivel en la figura para predecir la ubicación de los puntos críticos de f y si f tiene un punto silla o un máximo o mínimo local en cada punto crítico. Explique su razonamiento

$$f(x, y) = 4 + x^3 + y^3 - 3xy$$



Solución

① PC. : $\begin{cases} \dfrac{\partial f}{\partial x} = 3x^2 - 3y = 0 \\ \dfrac{\partial f}{\partial y} = 3y^2 - 3x = 0 \end{cases} \Rightarrow \begin{matrix} x^2 - y = 0 \\ y^2 - x = 0 \end{matrix} \Rightarrow \begin{matrix} y = x^2 \\ x^4 - x = 0 \end{matrix} \Rightarrow x(x^3-1) = 0$

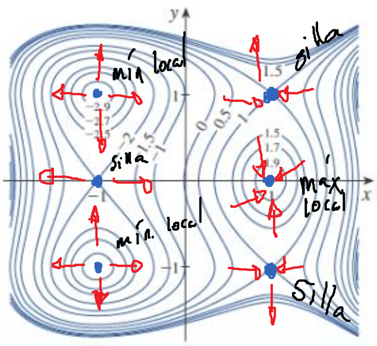
$x = 0$ $\boxed{\begin{matrix} x = 0 \\ y = 0 \end{matrix}}$; $x^3 - 1 = 0$ $\boxed{\begin{matrix} x = 1 \\ y = 1 \end{matrix}}$

② En el punto (1,1) la función tiene un mínimo local.

En el punto (0,0) la función tiene un punto de silla.

Use las curvas de nivel en la fi gura para predecir la ubicación de los puntos críticos de f y si f tiene un punto silla o un máximo o mínimo local en cada punto crítico. Explique su razonamiento.

$$f(x, y) = 3x - x^3 - 2y^2 + y^4$$



Solución

① PC. : $\begin{cases} \dfrac{\partial f}{\partial x} = 3 - 3x^2 = 0 \\ \dfrac{\partial f}{\partial y} = -4y + 4y^3 = 0 \end{cases} \Rightarrow \begin{cases} 1 - x^2 = 0 \\ y^3 - y = 0 \end{cases} \Rightarrow \begin{matrix} (x-1)(x+1) = 0 \\ y(y^2 - 1) = y(y-1)(y+1) = 0 \end{matrix} \Rightarrow \begin{matrix} x = -1 \lor x = 1 \\ y = 0 \lor y = -1 \lor y = 1 \end{matrix}$

$\Rightarrow$ (−1,0) silla ; (−1,−1) mín. local ; (−1,1) mín. local ; (1,0) máx. local ; (1,−1) silla ; (1,1) silla

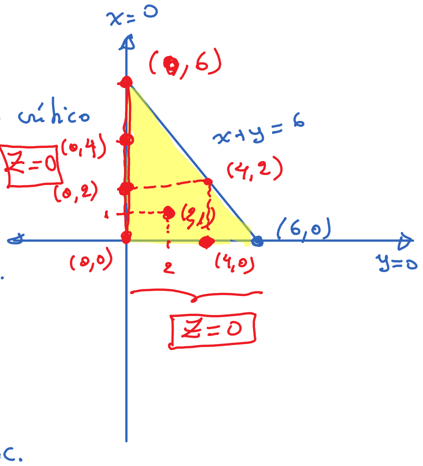
Hallar los valores máximos de la función $z = x^2y(4 - x - y)$ en el triángulo limitado por las rectas x=0, y=0, x+y=6.

Hallar los valores máximos de la función $z = x^2y(4-x-y)$ en el triángulo limitado por las rectas x=0, y=0, x+y=6.

Solución

1. $z = 4x^2y - x^3y - x^2y^2$

P.C. $\begin{cases} \dfrac{\partial z}{\partial x} = 8xy - 3x^2y - 2xy^2 = 0 \\ \dfrac{\partial z}{\partial y} = 4x^2 - x^3 - 2x^2y = 0 \end{cases} \Rightarrow \begin{cases} xy(8-3x-2y) = 0 \\ x^2(4-x-2y) = 0 \end{cases} \Rightarrow \begin{cases} x=0 \vee y=0 \vee 8-3x-2y=0 \\ x=0 \vee 4-x-2y=0 \end{cases}$



- Si $x=0 \wedge x=0 \Rightarrow (0,y)$; $\forall y \in \mathbb{R}$ puntos críticos.
- Si $x=0 \wedge 4-x-2y=0 \Rightarrow 4-2y=0 \Rightarrow \boxed{y=2,\ x=0} \Rightarrow (0,2)$ punto crítico
- Si $y=0 \wedge x=0 \Rightarrow (0,0)$ punto crítico
- Si $y=0 \wedge 4-x-2y=0 \Rightarrow 4-x=0 \Rightarrow \boxed{x=4,\ y=0} \Rightarrow (4,0)$ p.c.
- Si $8-3x-2y=0 \wedge x=0 \Rightarrow 8-2y=0 \Rightarrow \boxed{y=4,\ x=0} \Rightarrow (0,4)$ p.c.
- Si $8-3x-2y=0 \wedge 4-x-2y=0 \Rightarrow$

$$\begin{array}{r} 8-3x-2y=0 \\ -\ (4-x-2y=0) \\ \hline 4-2x \qquad =0 \end{array}$$

$\boxed{\begin{matrix} x=2 \\ y=1 \end{matrix}} \Rightarrow (2,1)$ p.c.

2. $\max\ z = x^2y(4-x-y)$

s.a. $x+y-6=0$

$L(x,y,\lambda) = x^2y(4-x-y) + \lambda(x+y-6)$

P.C. $\begin{cases} \dfrac{\partial L}{\partial x} = 2xy(4-x-y) + x^2y(-1) + \lambda = 0 & (1) \\ \dfrac{\partial L}{\partial y} = x^2(4-x-y) + x^2y(-1) + \lambda = 0 & (2) \\ \dfrac{\partial L}{\partial \lambda} = x+y-6 = 0 & (3) \end{cases}$

De (1) y (2):

$2xy(4-x-y) - x^2(4-x-y) = 0$

$x(4-x-y)(2y-x) = 0 \quad \dots (4)$

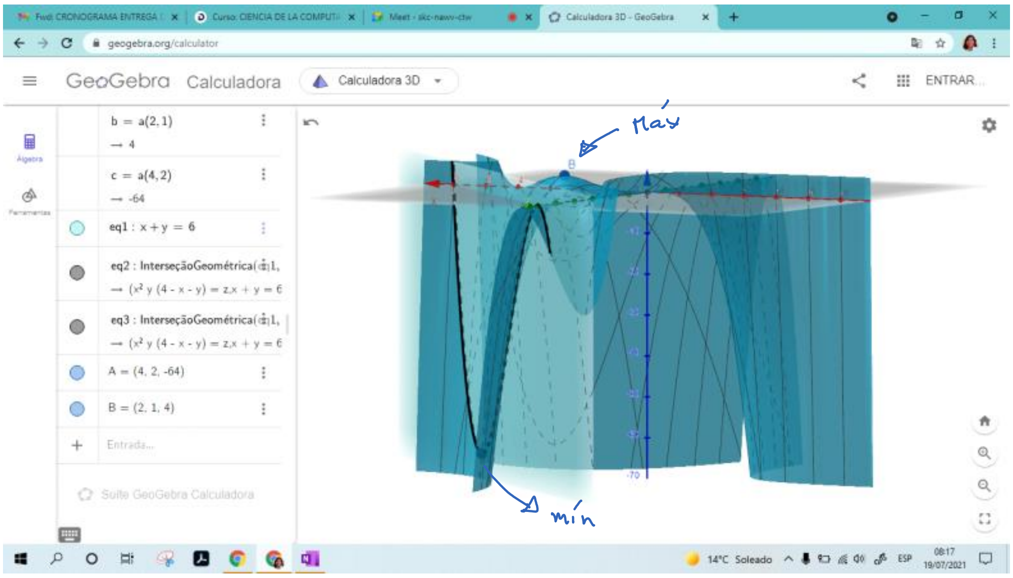
De (3) y (4)

$\begin{cases} x(4-x-y)(2y-x) = 0 \\ x+y-6 = 0 \end{cases} \Rightarrow \begin{cases} x=0 \vee 4-x-y=0 \vee 2y-x=0 \\ x+y-6=0 \end{cases}$

- Si $x=0 \wedge x+y-6=0 \Rightarrow y=6 \Rightarrow (0,6)$ p.c.
- Si $\begin{cases} 4-x-y=0 \\ x+y-6=0 \end{cases} \Rightarrow \emptyset$
- Si $\begin{cases} 2y-x=0 \\ x+y-6=0 \end{cases}$

$$\begin{array}{r} 2y-x=0 \\ x+y-6=0 \\ \hline 3y-6=0 \end{array}$$

$\boxed{y=2,\ x=4} \Rightarrow (4,2)$ p.c.

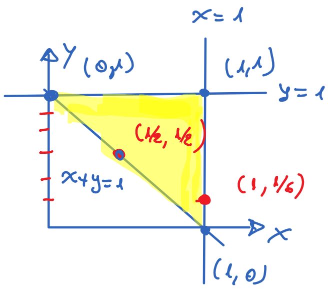


Hallar los valores máximos y mínimos de la función $z = x^2 + 3y^2 + x - y$ en el triángulo cuyo borde son las rectas x=1, y=1, x+y=1.

Solución

1. $z = x^2 + 3y^2 + x - y$

P.C. $\left\{ \begin{array}{l} \dfrac{\partial z}{\partial x} = 2x + 1 = 0 \\ \dfrac{\partial z}{\partial y} = 6y - 1 = 0 \end{array} \right. \Rightarrow \begin{array}{l} x = -1/2 \\ y = 1/6 \end{array} \Rightarrow \cancel{(-1/2, 1/6)}$



2. $x = 1 \Rightarrow z = 3y^2 - y + 2$

$\dfrac{\partial z}{\partial y} = 6y - 1 = 0 \Rightarrow \boxed{\begin{array}{l} y = 1/6 \\ x = 1 \end{array}} \Rightarrow (1, 1/6)$

3. $y = 1 \Rightarrow z = x^2 + x + 2$

$\dfrac{\partial z}{\partial x} = 2x + 1 = 0 \Rightarrow \boxed{\begin{array}{l} x = -1/2 \\ y = 1 \end{array}} \Rightarrow \cancel{(-1/2, 1)}$

4. max/min $z = x^2 + 3y^2 + x - y$

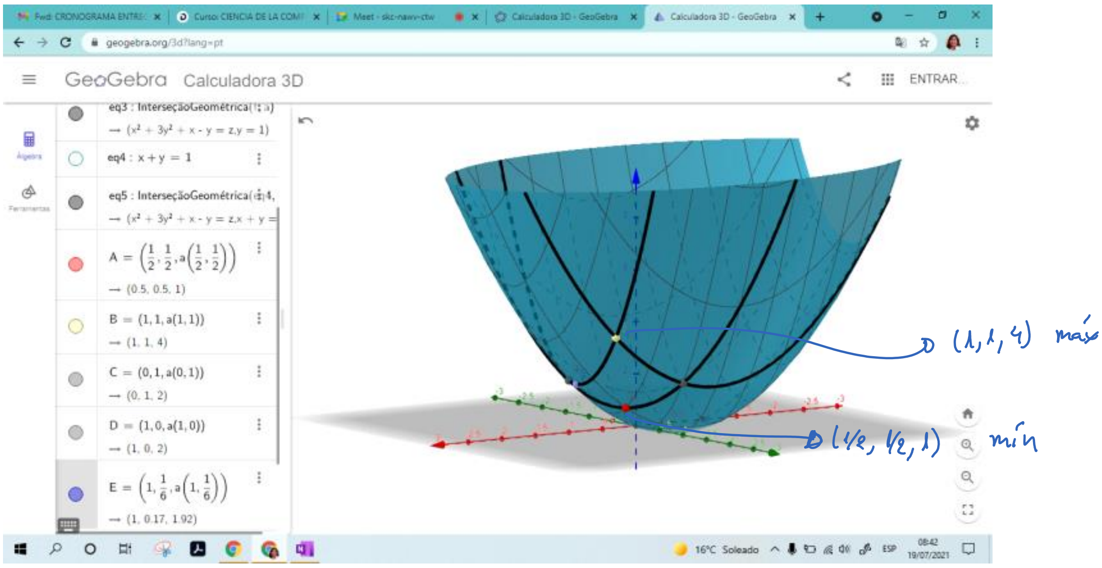
s.a. $x + y - 1 = 0$

$L(x, y, \lambda) = x^2 + 3y^2 + x - y + \lambda(x + y - 1)$

P.C.

$$- \left\{ \begin{array}{l} \dfrac{\partial L}{\partial x} = 2x + 1 + \lambda = 0 \\ \dfrac{\partial L}{\partial y} = 6y - 1 + \lambda = 0 \\ \dfrac{\partial L}{\partial \lambda} = x + y - 1 = 0 \end{array} \right. \Rightarrow \begin{array}{r} \left. \begin{array}{l} 2x - 6y + 2 = 0 \\ x + y - 1 = 0 \end{array} \right\} -2 \\ \hline -8y + 4 = 0 \end{array}$$

$$\boxed{\begin{array}{l} y = 1/2 \\ x = 1/2 \end{array}} \Rightarrow (1/2, 1/2)$$


GeoGebra Calculadora 3D
ENTRAR
eq3 : InterseçãoGeométrica
eq4 : x + y = 1
eq5 : InterseçãoGeométrica
A = (1/2, 1/2, a(1/2, 1/2))
→ (0.5, 0.5, 1)
B = (1, 1, a(1, 1))
→ (1, 1, 4)
C = (0, 1, a(0, 1))
→ (0, 1, 2)
D = (1, 0, a(1, 0))
→ (1, 0, 2)
E = (1, 1/6, a(1, 1/6))
→ (1, 0.17, 1.92)
(1,1,4) máx
(1/2, 1/2, 1) mín