

$$1. \forall x (Fx \rightarrow Gx), \exists x (Hx \wedge Fx) \vdash \exists x (Hx \wedge Gx)$$

1 (1)	$\forall x (Fx \rightarrow Gx)$	S
2 (2)	$\exists x (Hx \wedge Fx)$	S
3 (3)	$Ha \wedge Fa$	S
1 (4)	$Fa \rightarrow Ga$	$\text{E}\forall 1$
3 (5)	Fa	$\text{E}\wedge 3$
3 (6)	Ga	$\text{E}\wedge 3$
1, 3 (7)	Ga	M. Ponens 4, 5
1, 3 (8)	$Ha \wedge Ga$	$\text{I}\wedge 6, 7$
1, 3 (9)	$\exists x (Hx \wedge Gx)$	$\text{I}\exists 8$
1, 2 (10)	$\exists x (Hx \wedge Gx)$	$\text{E}\exists 2, 9$

$$2. \forall x (Fx \rightarrow Gx) \vdash \exists x \sim Gx \rightarrow \exists x \sim Fx$$

La conclusión podría expresarse también como
 $\exists x (\sim Gx \rightarrow \sim Fx)$

1 (1)	$\forall x (Fx \rightarrow Gx)$	S
1 (2)	$Fa \rightarrow Ga$	$\text{E}\forall 1$
1 (3)	$\sim Ga \rightarrow \sim Fa$	Ley de transposición 2
1 (4)	$\exists x (\sim Gx \rightarrow \sim Fx)$	$\text{I}\exists 3$

$$\text{o.o. } \exists x (\sim Gx \rightarrow \sim Fx) \equiv \exists x \sim Gx \rightarrow \exists x \sim Fx$$

$$3. \exists x (Fx \wedge \sim Gx), \forall x (Fx \rightarrow Hx) \vdash \exists x (Hx \wedge \sim Gx)$$

1 (1)	$\exists x (Fx \wedge \sim Gx)$	S
2 (2)	$\forall x (Fx \rightarrow Hx)$	S
3 (3)	$Fa \wedge \sim Ga$	$\text{E}\forall 2$
2 (4)	$Fa \rightarrow Ha$	$\text{E}\wedge 3$
3 (5)	Fa	$\text{E}\wedge 3$
3 (6)	$\sim Ga$	M. Ponens 4, 5
2, 3 (7)	Ha	$\text{I}\wedge 6, 7$
2, 3 (8)	$Ha \wedge \sim Ga$	$\text{I}\exists 8$
2, 3 (9)	$\exists x (Hx \wedge \sim Gx)$	$\text{E}\exists 4, 9$
1, 2 (10)	$\exists x (Hx \wedge \sim Gx)$	