Examen parcial 3

1)

b)

1b

b) $k+1 \leq n$

$$\binom{n}{k+1} = \frac{n-k}{k+1}\binom{n}{k}$$

$$\frac{n!}{(n-k-1)!\,(k+1)!} = \frac{n-k}{k+1}\left(\frac{n!}{(n-k)!\,(k)!}\right)$$

$$\frac{n!\,\cancel{(n-k)}}{\cancel{(n-k)}\,(n-k-1)!\,\underline{(k))\,(k+1}}$$

$$\frac{n!}{(n-k-1)!\,(k+1)!} = \frac{n!}{(n-k-1)!\,(k+1)!}$$

//

2)

a)

2) a) $n \geq 3$ Usando Binomio de Newton

$$\sum_{k=3}^{n}\left[\underbrace{\binom{n-3}{k-3}+\binom{n-3}{k-2}}+\binom{n-2}{k-1}\right]$$

$$C^{n-3}_{k-3}+C^{n-3}_{k-2}+C^{n-2}_{k-1}$$

$$\frac{(n-3)!}{(k-3)!(n-k)!}+\frac{(n-3)!}{(k-2)!(n-k-1)!}+\frac{(n-2)!}{(k-1)!(n-k-1)!}$$

$$\frac{(n-3)!}{(k-3)!(n-k-1)!}\left[\frac{1}{n-k}+\frac{1}{k-2}\right]$$

Por propiedad

$$\binom{n}{k}+\binom{n}{k-1}=\binom{n+1}{k}$$

$$\frac{(n-3)!}{(k-3)!(n-k-1)!}\left(\frac{n-2}{(n-k)(k-2)}\right)$$

Respuesta: $\sum_{k=3}^{n}\left[\binom{n-1}{k-1}\right]$ //

$$\frac{(n-2)!}{(n-k)!(k-2)!}+\frac{(n-2)!}{(k-1)!(n-k-1)!}$$

$$\frac{(n-2)!}{(k-2)!(n-k-1)!}\left(\frac{1}{n-k}+\frac{1}{k-1}\right)$$

$$\sum_{k=3}^{n}\left[\frac{(n-1)!}{(n-k)!(k-1)!}\right]=\sum_{k=3}^{n}\left[\binom{n-1}{k-1}\right]$$

b)

b) $(\underbrace{x^2}_{a} - \underbrace{3x^{-1}}_{b})^6 = (a+b)^6$

Mx^6 y Nx^3 ¿$M+N$?

Mx^6

1° hallamos exponentes

• $Mx^6 = (x^2)^y(-3x^{-1})^{6-y}$

$Mx^6 = Mx^{2y+y-6}$

$Mx^6 = Mx^{3y-6}$

$y = 4$

• $Nx^3 = (x^2)^z(-3x^{-1})^{(6-z)}$

$Nx^3 = x^{2z}(Px^{z-6}$

$Nx^3 = P.Tx^{3z-6}$

$Nx^3 = Nx^{3z-6}$

$z = 3$

2° hallamos Coeficientes

$C^6_2(x^2)^4(-3x^{-1})^2$

$\frac{6!}{2(4)!}x^8(9 \cdot x^{-2})$

$15 \cdot 9x^6 = 135x^6$

$M = 135$

$C^6_3(x^2)^3(-3x^{-1})^3$

$\frac{6!}{3!\cdot 3!}(-27)x^3$

$-540x^3$

$N = -540$

• Rpta $M+N = 135 - 540 = -405$ //

3)

3)

$$\sum_{k=0}^{n} \binom{n+1}{k+1} = 2^{n+1} - 1$$

→ $n = 1$

$$\sum_{k=0}^{1} \binom{n+1}{k+1} = 2^{n+1} - 1$$

$$\binom{2}{1} + \binom{2}{2} = 2^{2} - 1$$

$$2 + 1 = 3$$

$$3 = 3$$

→ $n = m$

$$\sum_{k=0}^{m} \binom{m+1}{k+1} = 2^{m+1} - 1$$

→ $n = m + 1$

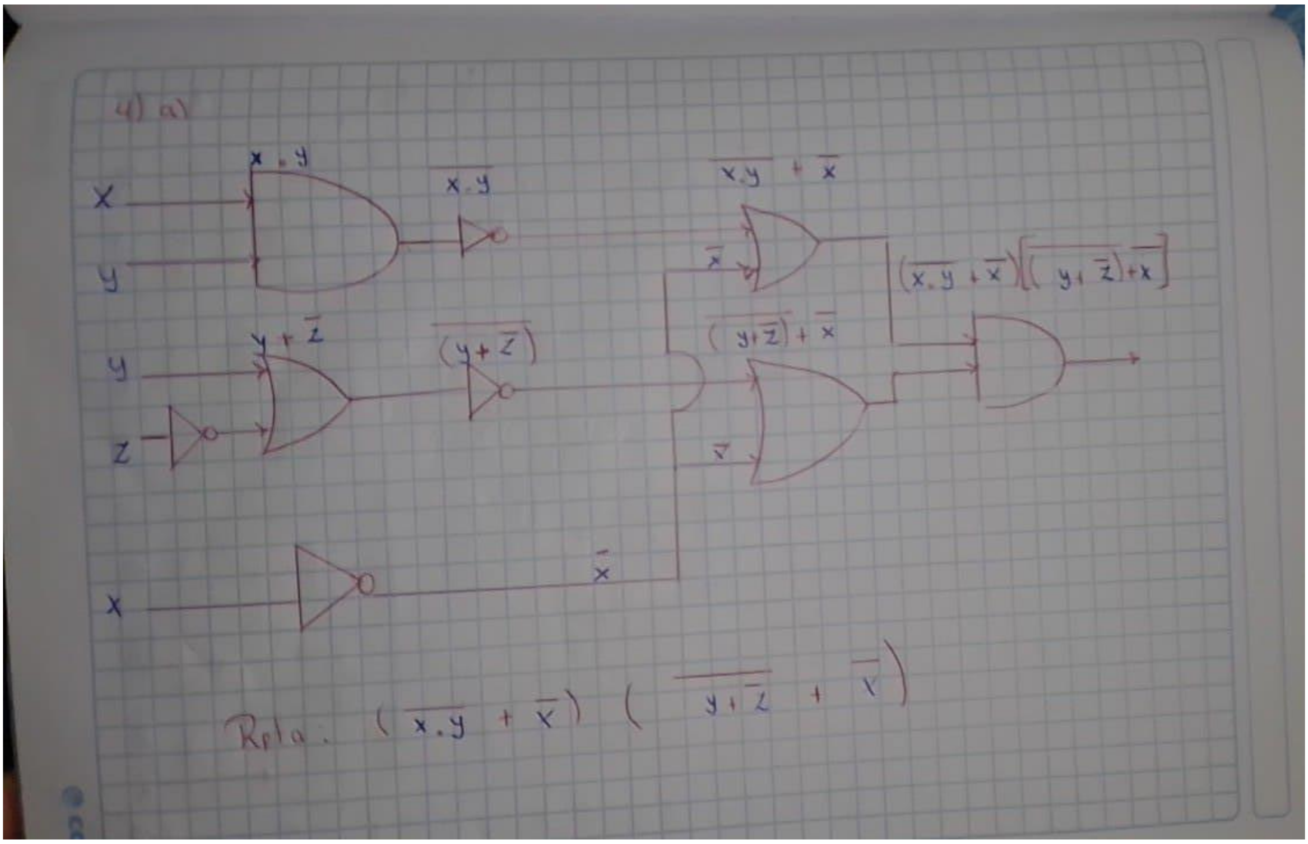
$$2^{m+1} - 1 + \binom{m+2}{m+2} = 2^{m+2} - 1$$

$$2^{m+1} + 1 = 2^{m+2}$$

//

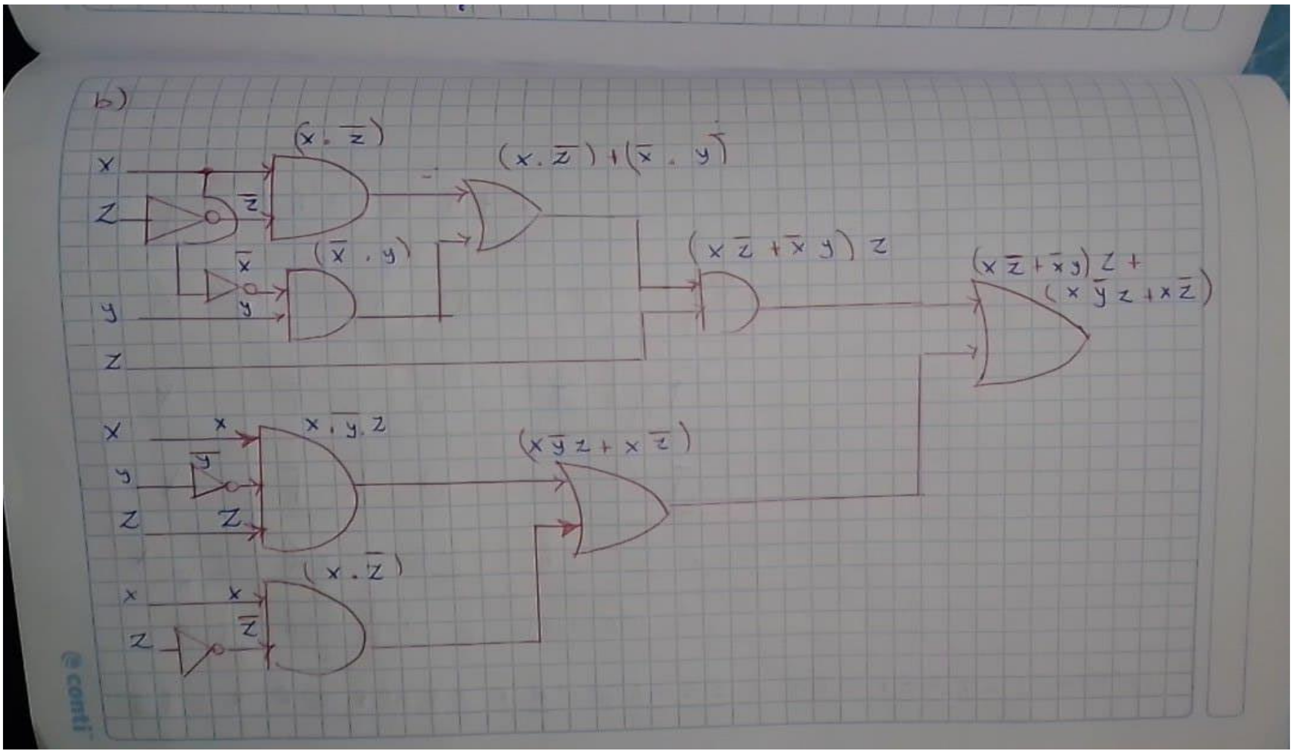
4)

a)



4) a)
Rpta: $(\overline{x.y}+\overline{x})(\overline{y+\overline{z}}+\overline{x})$

b)



b)
$(x.\overline{z})+(\overline{x}.y)$
$(x\overline{z}+\overline{x}y)z$
$(x\overline{y}z+x\overline{z})$

5)

a)

5) $xyz + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}\bar{y}\bar{z} + x\bar{y}z + \bar{x}\bar{y}z = F(x,y,z)$

a) Dibujar Mapa de Karnaugh

$x \backslash yz$	yz	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$
x	1	0	1	1
$\bar{x}$	1	0	1	1

b)

b) Aplicando un "redondeo" o agrupación de "1".

$F(x,y,z) = \bar{y} + z$ //