An improved method for modeling of leaf venation patterns

Wenbiao Jin
The School of Science,
Hangzhou Dianzi University,
Hangzhou, 310018, China

Wenzhe Gu
College of Computer Science and Technology,
Chongqing University of Posts and Telecommunications,
Chongqing,China

Zhifeng Zhang
The School of Science,
Hangzhou Dianzi University,
Hangzhou, China

***Abstract*—In this paper, an improved method is presented for modeling of leaf venation patterns. Combined Runions' modeling method with L-systems, the growth of main and lateral veins is accelerated. Moreover, a new dart-throwing algorithm is exploited to place auxin sources in each simulation loop. The results verify that the improved method result in venation patterns of higher reality more effectively.**

Keywords-L-systems; venation pattern; nutrition weight; Poisson-disk

I. INTRODUCTION

With the rapid development of computer techniques, modeling and visualization of plants scenes is becoming a hot research topic of computer graphics. It bridges disciplines including biology, morphogenesis, mathematics and computer graphics, etc. In the case of rendering plants scenes, leaf venation is almost a necessary component. Existing plant leaf venation patterns on earth show high diversity. Yet, leaf simulation is very difficult with traditional methods. This makes the modeling of venation patterns a particularly challenging problem.

In this paper, an improved method is presented for modeling of leaf venation patterns. To treat a defect of Runions'[1] modeling method, L-systems are introduced, which accelerate the growth of main and lateral veins. A new dart-throwing algorithm is used to place auxin sources in each simulation loop, which effectively speeds the algorithm. The results verify that the improved method can get more realistic venation pattern more quickly.

II. BACKGROUND AND RELATED WORK

A. Leaf shape description

Dengler and Kang[2] verified that venation patterns are strongly correlated with leaf shapes and thus must be considered in modeling. Judd[3] proposed the terminology for describing leaf shape (Figure 1). A typical leaf consists of a leaf blade, a petiole and a stem. A simple leaf only has one single blade, while compound leaves have blades partitioned into separate subunits called leaflets. In this paper, only the individual leaflets venation modeling is considered.

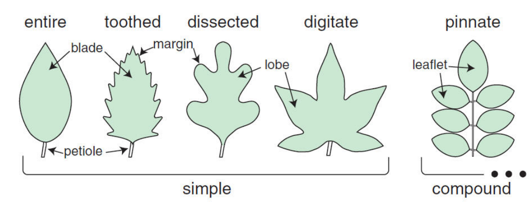


Figure 1. Relevant terminology to describe the shape of leaves

The growth of venation patterns are also closely correlated with the growth of leaf blade. The growth of blade can be characterized by growth tensor field, which is a generalization of the relative elementary rate of growth (RERG)[4]. It specifies the magnitude of the expansion of infinitesimal surface regions in various

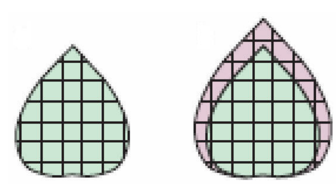


Figure 2. A sample leaf and its marginal growth

directions. In this paper, only simple special cases are focused on, in which growth is marginal(Figure 2).

B. *Venation patterns and modeling methods*

Judd[3] proposed the terminology for describing leaf shape (Figure 3). Leaf venations include primary veins (midvein), secondary veins (lateral) and high order veins (veinlets).

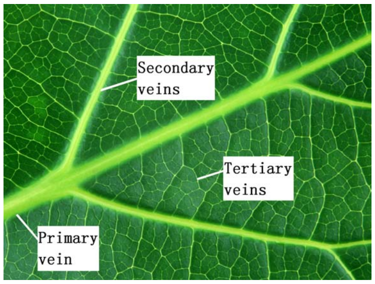


Figure 3. Relevant terminology to describe the shape of leaves

Rodkaew (2002)[5] proposed a modeling method for leaf venations based on particle theory. However, as without combining the generating procedure with biological knowledge, it is difficult to improve the results by incorporating biological knowledge. Rodkaew (2004)[6] proposed an animating system for plant growth with L-systems. But the coupling between vein levels is not very well. In Siggraph 2005, Runions[1] proposed a modeling method based on biologically plausible hypothesis (Figure 4). Runions' method is approximate to the growth process of real leaf

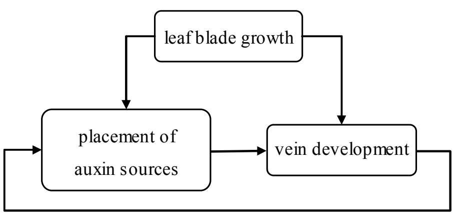


Figure 4. Hypothetical causal relations underlying vein pattern development

venations, resulting in very realistic leaf venations. However, as being not considering the successive growth order, the midvein and lateral veins grow from bottom to top synchronously. So, the average angle (defined in Part 4) between them is very small, and the diameter of midvein is not big enough, which are all inconsistent with the real cases.

It can be observed from the plant leaves' real growth processes, that in the leaf venations most of midvein and blade can almost achieve the same growth rate, while lateral veins and veinlets are differentiated from midvein on both sides. This phenomena was also be found by Candela, when he studied the venation pattern formation process of arabidopsis thaliana vegetative leaves [7] (Figure 5).

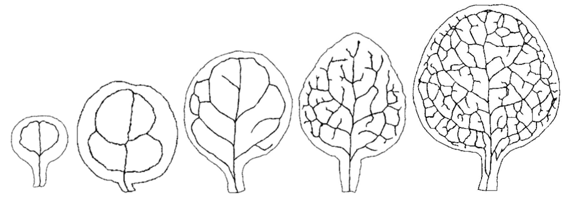


Figure 5. Venation pattern formation process illustrated by Candela

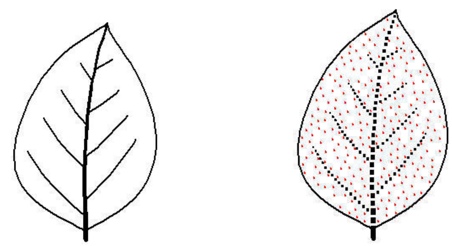
III. IMPROVED METHOD

A. *Foundation of theory*

One can conclude from the statement above, that it is desirable to make the main and lateral veins grow faster. In this paper, L-systems and the weights of main and lateral veins are introduced into Runions' method to achieve this aim.

In 1968, Aristid Lindenmayer introduced a new type of string-rewriting mechanism, subsequently termed L-systems, which were conceived as a mathematical theory of plant development. In 1990, Prusinkiewicz have used L-systems to produce trees, flowers and other plants in details[8].

In improved method, L-systems are used to describe main and lateral veins (Figure 6(a)). Then place auxin sources (black square spots) equal space intervals on the paths of main and lateral veins (Figure 6(b)).



(a) Main and lateral veins drawn by L-systems (b) Place greater auxin paths sources on paths

Figure 6. L-systems are introduced into Runions' method

The weights of black sources are determined by their path widths, while the weights of red sources set as a default smaller value. Because auxin sources on main and lateral vein paths have greater weights, more veins will be attracted to grow towards them, so their growth will be speeded.

As a step of the simulation algorithm, auxin sources' random placement is necessary in every loop. So its improvement in time complexity is very important. In this paper, a new dart-throwing algorithm is employed to substitute that of Runions', which can upgrade time complexity from $O(N^2)$ to $O(N\log N)$.

B. Steps of Improved Method

1) Initial States.

a) *Initial blade contour:* specified by B-spline parametric curves;

b) Initial position of "seed" vein node: determined by the attachment point of the blade to the petiole, which is commonly located at the blade base;

c) Parameters characterizing the interplay between auxin sources and vein development: we use corresponding values of Runions' method[1];

d) Auxin sources set: represented by $S\{s\}$; *venation:* represented by a tree graph $G = <V, E>$; *vein nodes set:* represented by $V\{v\}$; *veins set:* represented by $E\{e\} \subset V \times V$.

2) The Simulation Loop.

a) Use L-systems to generate main and lateral vein paths, then place auxin sources on them, with weights corresponding to the size of vein widths(Figure 6);

b) Place other auxin sources using Daniel's[9] dart-throwing algorithm within existing blade contour;

c) Find "closest vein node" for each source, and record their corresponding relationship;

d) Calculate direction vector $\vec{n}$ to each "closest vein node", which is the average of normalized vectors toward all the sources in S;

$$\vec{n} = \sum_{s(weight) \in S(v)} \frac{s(weight) - v}{\| s(weight) - v \|},$$

where s(weight) denote the auxin sources with weight;

e) A new node v' is created and positioned at a distance D(weight) from v, in the direction $\vec{n}$, then a new vein is created by connecting v and v';

$$Position_{v'} = Position_v + D(weight) \sum_{s \in S(v)} \frac{\vec{n}}{\| \vec{n} \|},$$

$$D(weight) = D * \frac{1}{n} \sum_{i=1}^{n} s_i.weight;$$

f) Test whether there is at least one vein node closer to each source than a threshold distance, and a source is removed when there is at least one vein node arriving it;

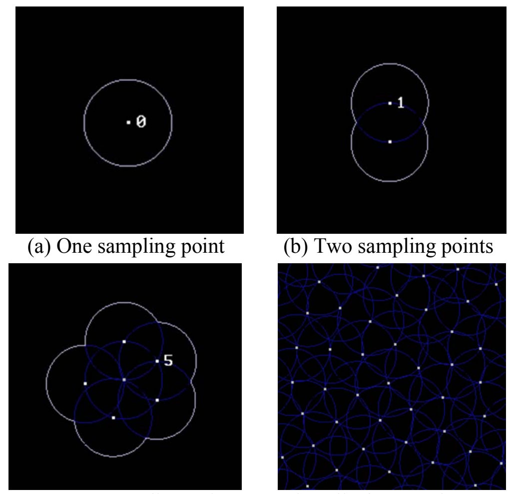
g) Place new sources within expanded blade, and remove the sources that are farther than the threshold distance from any other source or vein node;

h) Go to step *b)*, begin next iteration.

C. New Dart-throwing algorithm

Almost all problems in computer graphics involve sampling. Based on the manipulation of joint unions of scalloped sectors, Daniel[9] proposed a new dart-throwing method for generation of Poisson-disk point sets in O(NlogN) time. We use this algorithm to place auxin sources. Because sampling is done in available regions that have been greatly diminished, this algorithm is faster than that of Runions'.

Figure 7 demonstrates the Daniel's algorithm for placeing point set: (a) whole circle ring around center 0 with radius r is available region; (b) after insertion of another point, the intersection of two circle rings are unavailable region(blue), discard them, then sample only in the available regions(white); (c)continue to discard the unavailable regions(blue) and sample only in available regions(white); (d) a homogeneously distributed point set is obtained after all available regions are used.



(a) One sampling point (b) Two sampling points

(c) Some sampling points (d) Full of rectangle area

Figure 7. Homogeneously distributed point set generated by Daniel's algorithm

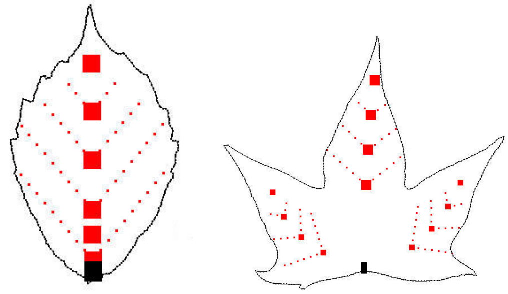
IV. EXPERIMENTAL RESULTS

The algorithms were implemented with Visual C++2003 on Windows XP. All results were completed on a PC with 1.6GHz Pentium4 CPU, 1GB memory and 128M Intel 915GM graphics card.

Main and lateral vein paths drawn by L-systems require O(N) calculations. Let c be the number of candidate auxin sources to be inserted, k the number of existing sources, and n the number of vein nodes. During every simulation loop, inserting new sources using new dart-throwing algorithm requires O(clogc) calculations. The testing whether a candidate source is farther than the threshold from any existing source or vein node requires O(c(k+n)) calculations. Finding the closest vein node for every source s requires O(kn) calculations.

According to the leaf shape classification in Figure 1, we choose two typical leaves (creeper and maple) to verify our improved method.

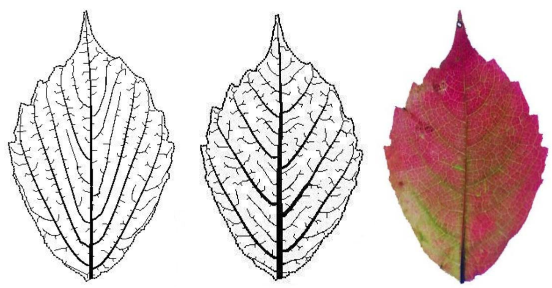
Figure 8 shows how to place auxin sources on main and lateral vein paths of two kinds of leaves.



(a) Creeper (b) Maple

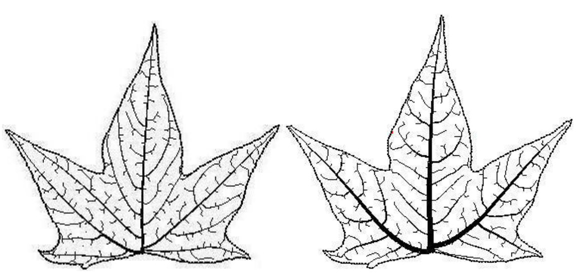
Figure 8. Auxin sources' placement of two kinds of leaves

Figure 9 and figure 10 are the final results generated by the improved method.

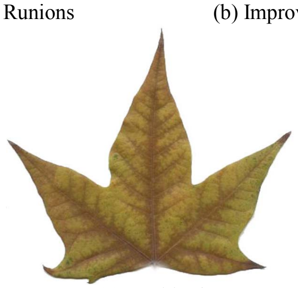


(a) Runions (b) Improved (c) Real leaf

Figure 9. Creeper (entire)



(a) Runions (b) Improved



(c) Real leaf

Figure 10. Maple (digitate)

① For measuring the angle between main and lateral vein, we define an average angle denoted by $\overline{\theta}$, which is calculated by equation $\overline{\theta} = \frac{1}{n}\sum_{i=1}^{n}\theta_i$, and θ_i is the angles between each pair of main and lateral veins (Figure 11).

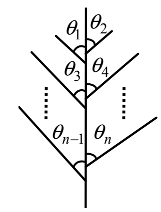


Figure 11. Angles between main and lateral veins

It can be inferred from table Ⅰ that the average angle between main and lateral veins of improved method is bigger than that of Runions', which is more consistent with the real cases.

TABLE I. STATISTICAL TABLE OF AVERAGE ANGLE (TOOLS:PROTRACTOR; UNIT: DEGREE)

Name	Creeper	Maple		
Serial Number	Figure 8	Figure 9		
Leaf Shape	entire	digitate		
		left lobe	middle lobe	right lobe
Runions	23.2	29.4	28.0	28.3
Improved	41.0	48.4	49.6	49.8
Real leaf	49.5	52.3	52.8	51.8

② For measuring diameters of midvein, Murray's law is employed, which states that diameters of vessels before and after a branching point in a ramifying transport system satisfy the formula:

$$d_{parent}^{n} = d_{child1}^{n} + d_{child2}^{n}, \text{ n=3.}$$

The calculations of vein widths start from veinlets, whose width assumed to be the minimum, proceed down to the base of the leaf. However, following MacDonal's[10] observation of different values of this exponent in natural branching systems, the exponent n serves as a parameter in our model, and takes value 2 empirically.

It can be inferred from table Ⅱ that the ultimate diameters of midvein of improved method are bigger than that of Runions', and they are more close to the real cases.

TABLE II. STATISTICAL TABLE OF DIAMETER

Name	**Serial Number**	**Leaf Shape**	**Runions**	**Improved**
creeper	Figure 7	entire	6.945	16.792
maple	Figure 8	digitate	6.214	15.329

③ It can be inferred from table Ⅲ that new dart-throwing algorithm is remarkably faster than that of Runions'.

TABLE III. STATISTICAL TABLE OF CONSUMED TIME WITH TWO METHODS (UNIT: SECOND)

Auxin Sources Total Number	Runions ($O(N^2)$)	Improved (O(NlogN))
500	7.4513	0.3861
700	15.0763	0.6610
1000	32.4589	0.9059
1500	70.5056	1.9055
3000	257.9718	5.6435

As shown in Figure 12, with increase of sources' total number, new dart-throwing algorithm has better stability than that of Runions' in time efficiency.

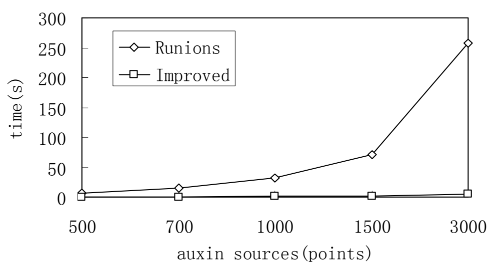


Figure 12. Time efficiency contrast of two algorithms

V. CONCLUSION

An improved method is proposed for modeling of leaf venation pattern which combines Runions' method with L-systems. And a new dart-throwing algorithm is used to place auxin sources in every simulation loop, which effectively improved the drawing speed of whole algorithm. The results verify that improved method can get more realistic venation patterns in shorter time. The resulting patterns are applicable to texture synthesis, such as the incorporation of veins into a detailed geometric model of the leaf, high realistic simulation of light propagation on leaves surface, and the use of fast rendering methods for leaves. Finally, our model can be easily extended to three dimensions.

For further research work, digital image processing method can be used to extract the main and lateral paths directly from the real leaves. The venation generated by this way would be more realistic.

REFERENCES

[1] A. Runions, M. Fuhrer, B. Lane and Federl, "Modeling and visualization of leaf venation patterns," ACM Transactions on Graphics 24, vol. 3, 2005, pp. 702-711.

[2] Dengler, and Kang. "Vascular patterning and leaf shape," Current Opinion in Plant Biology 4,1, 2001, pp. 50-56.

[3] W. W. Judd, C. S. Campbell and E. A. Kellogg, Plant systematics: Aphylogenetic approcach. Sinauer Associates, Sunderland, MA, 1999.

[4] Z. Hejnowicz, J. Romberger, "Growth tensor of plant organs," Journal of Theoretical Biology 110, 1984, pp. 93-114.

[5] Y. Rodkaew, S. Siripant and C. Lursinsap, "An algorithm for generating vein images for realistic modeling of a leaf," Prodeedings of the International Conference on Computational Mathematics and Modeling, 2002, vol. 9.

[6] Y. Rodkaew, "Animating plant growth in L-system by parametric functional symbols," International Journal of Intelligent Systems, 2004, vol. 19 pp. 9-23.

[7] C. Hector, and M. L. Antonio, "Venation pattern formation in arabidopsis thaliana vegetative leaves," Developmental Biology vol. 205, 1999, pp. 205-216.

[8] P. Prusinkiewicz and A. Lindenmayer, "The algorithmic beauty of plants," Springer-Verlag New York, Inc., USA, 1990.

[9] D. Daniel and G. A. Humphreys, "Spatial data structure for fast poisson-disk sample generation," ACM Transactions on Graphics, 25, vol. 3, 2006, pp. 503-508.

[10] N. MacDonald, "Trees and networks in biological models," J. Wiley & Sons, New York, 1983.