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Nonholonomic Mobile Manipulators: Kinematics, Velocities and Redundancies

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Abstract. We consider nonholonomic mobile manipulators built from an n_a joint robotic arm and a nonholonomic mobile platform with two independently driven wheels. Actually, there is no efficient kinematic formalism for these systems which are generally characterized by their high number of actuators. So, kinematic modelling is presented with particular emphasis on redundancy. Whereas kinematic redundancy is well known in the holonomic case, it is pointed out that it is necessary to define velocity redundancy in the case of nonholonomic systems. Reduced velocity kinematics based on quasi-velocities are shown to provide an efficient formalism. Two examples of mobile manipulators are presented. Finally, reduced velocity kinematics and velocity redundancy are shown to be adequate tools in order to realize operational task while optimizing criteria such as manipulability.

Key words: mobile manipulator, nonholonomy, kinematics, velocities, redundancies.

1. Introduction

Mobile manipulators is now a widespread term to refer to robots that combine capabilities of *locomotion* and *manipulation*. When these systems are devoted to indoor tasks, they are often equipped with wheels. The arrangement of the wheels and their actuation device determine the *holonomic* or *nonholonomic* nature of this locomotion system (Campion et al., 1996). Whereas some wheeled mobile manipulators, built from an omnidirectional platform are holonomic (Khatib et al., 1996), many of them are not (Seraji, 1998; Yamamoto and Yun, 1995).

The tasks assigned to these systems are often translated in terms of End Effector (EE) evolution, either in point-to-point or in continuous path. Very often, the *redundancy*, and thus the *versatility*, of these systems is emphasized. Although this concept is well known for robotic arms (see, e.g., (Yoshikawa, 1990; Nakamura, 1991)), it is quite different in the case of nonholonomic systems. Actually, according to the task at hand for the EE, point to point or continuous path, the

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spaces to consider, and thus the redundancy concepts to use, are different. We are then led to define in this paper two concepts of redundancy: *kinematic* and *velocity* redundancy. We show both redundancies are defined by the same algebraic criterion applied to two different matrices, the first one being the Jacobian of the system whereas the second one takes into account implicitly the nonholonomic constraint and is based on *reduced velocity kinematics* by using quasi-velocities that define the *mobility control* of the system. Singular configurations are also studied. These results are applied to two mobile manipulators. The first one is chosen for an easy application of the concepts. The second one exhibits kinematic redundancy, but does not allow to follow any path in its operational workspace. Finally, operational motion is considered for the first example of mobile manipulator. Manipulability is defined for the whole system and is used in a pseudo-inversion control scheme.

2. Modelling

We consider a mobile manipulator composed of a HILARE-like mobile platform (Giralt et al., 1984) on which is mounted an n_a-joint rigid robotic arm. The mobile platform is equipped with two independent driven wheels and possibly some castors and it moves on a plane surface.* Figure 1 shows an example of such a mobile manipulator: the $\mathrm{H_2BIS}$ mobile platform on which is mounted the GT6A 6-revolute-joints robotic arm. Indices $._p$ denote the parameters relative to the mobile platform, and indices $._a$ those relative to the arm.

We note $\mathcal{R} = (O, \vec{x}, \vec{y}, \vec{z})$ a fixed frame and $\mathcal{R}' = (O', \vec{x}', \vec{y}', \vec{z}')$ a mobile frame linked to the platform where O' is the middle point of the driven wheels axle. We define also a frame $\mathcal{R}_{n_a} = (O_{n_a}, \vec{x}_{n_a}, \vec{y}_{n_a}, \vec{z}_{n_a})$ linked to the EE, and a

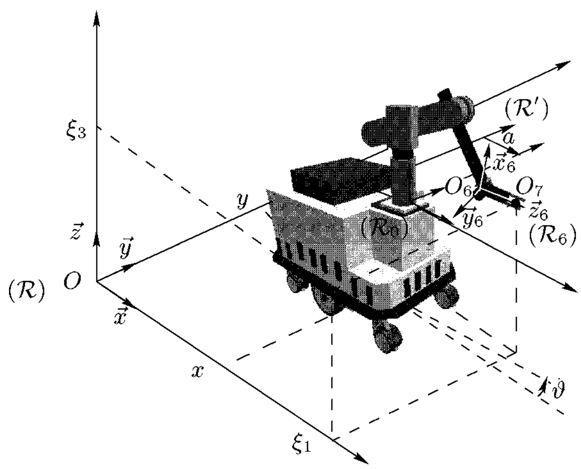

Figure 1. $\mathrm{H_2BIS}$ mobile platform equipped with GT6A robotic arm (with $n_a = 6$).

* The methods and notions presented here applies also to car-like platforms equipped with steering wheels. For details, see (Bayle, 2001).

point O_{n_a+1} which is the EE center (see Figure 1, for an example). In a classical way, the *configuration* of a mechanical system is known when, relative to a given frame, the position of all of its points is known. It is defined by a vector $\boldsymbol{q}$ of M independent coordinates, called *generalized coordinates*. Configuration is then naturally defined over a M-dimensional manifold $\mathcal{N}$ and M is called here *mobility index* (some authors, e.g., (Hunt, 1978), call this quantity simply *mobility* but it will be useful in the sequel to distinguish mobility *index* and mobility *degree*).

If we leave out of account the angular value at the wheels, to define the platform configuration is equivalent to define the configuration of a rectangle on a plane. Its generalized coordinates are then three in number: two for the position and one for the orientation. Let $\boldsymbol{q}_p = [q_{p1} \quad q_{p2} \quad q_{p3}]^{\mathrm{T}} = [x \quad y \quad \vartheta]^{\mathrm{T}}$, where x and y are the abscissa and the ordinate of the point O' in $\mathcal{R}$ and ϑ is the angle made by the vectors $\vec{x}$ and $\vec{x}'$. Whether the platform is holonomic or not, all the triples (x, y, ϑ) are reachable (see Remark in Section 4.1).

The robotic arm is mounted on the mobile platform. It is made of n_a rotoïd or prismatic joints. Its generalized coordinates correspond to the characteristic quantities (angular value for rotoïd joints, translation length for prismatic ones) of the joints and are n_a in number: $\boldsymbol{q}_a = [q_{a_1} \quad q_{a_2} \quad \ldots \quad q_{n_a}]^{\mathrm{T}}$. Finally, for the mobile manipulator, $M = 3 + n_a$ and its configuration is defined by:

$$\begin{aligned}\boldsymbol{q} &= [q_1 \quad q_2 \quad \ldots \quad q_M]^{\mathrm{T}} \\ &= [x \quad y \quad \vartheta \quad q_{a_1} \quad q_{a_2} \quad \ldots \quad q_{n_a}]^{\mathrm{T}} = \left[\boldsymbol{q}_p^{\mathrm{T}} \quad \boldsymbol{q}_a^{\mathrm{T}}\right]^{\mathrm{T}}.\end{aligned}$$

In other respects, the robotic task is naturally defined in terms of *location* of the EE that characterizes the *position* of the point O_{n_a+1} and the *orientation* of the frame $\mathcal{R}_{n_a}$. Whatever the mechanical system may be, the EE location can then be defined by means of a vector $\boldsymbol{\xi}$ of m independent coordinates called *operational coordinates* (see (Khatib, 1986)):

$$\boldsymbol{\xi} = [\xi_1 \quad \xi_2 \quad \ldots \quad \xi_m]^{\mathrm{T}}.$$

The location $\boldsymbol{\xi}$ is then naturally defined over a m-dimensional manifold $\mathcal{M}$ with $0 < m \leqslant 6$.

Remarks.

– It is important to note that contrary to the generalized coordinates, which are *global* variables, the operational coordinates – such as they are defined here – are generally only *local* variables, defined over a particular *map* of the manifold $\mathcal{M}$.

– In the most general case where $m = 6$, we choose the operational coordinates such that ξ_1, ξ_2, ξ_3 are Cartesian coordinates of the point O_{n_a+1} in the frame $\mathcal{R}$, and (ξ_4, ξ_5, ξ_6) are a set of angular parameters, that describe the orientation of the frame $\mathcal{R}_{n_a}$ relative to the frame $\mathcal{R}$. It is then well known that the variables ξ_4, ξ_5 and ξ_6 are only defined on a map of the manifold SO(3) which is not isomorphic to $\mathbb{R}^3$. When the problem is in the plane or needs a lower number of operational coordinates, it is always possible to use locally a subset of this choice of coordinates.

3. Kinematics

The study of kinematics allows to write the relations between the mobile manipulator configuration and the EE location. In particular, it allows to determine the configuration(s) corresponding to a desired EE location, in the framework of a point to point task, for example. *Direct Kinematics* (DK) expresses the EE location as a function of the mobile manipulator configuration – or operational coordinates as functions of generalized ones:

$$\begin{aligned} \boldsymbol{f}\colon \ \mathcal{N} &\longrightarrow \mathcal{M} \\ \boldsymbol{q} &\longmapsto \boldsymbol{\xi} = \boldsymbol{f}(\boldsymbol{q}), \end{aligned}$$

i.e. $\xi_i = f_i(q_j) \quad 1 \leqslant i \leqslant m,\ 1 \leqslant j \leqslant M.$

Inverse Kinematics (IK) allows to compute – when it exists – a configuration of the mobile manipulator $\boldsymbol{q}$ that leads to an imposed EE location $\boldsymbol{\xi}$. IK writes:

$$\begin{aligned} \boldsymbol{f}^{-1}\colon \ \boldsymbol{f}(\mathcal{N}) &\longrightarrow \mathcal{N} \\ \boldsymbol{\xi} &\longmapsto \boldsymbol{q} = \boldsymbol{f}^{-1}(\boldsymbol{\xi}), \end{aligned}$$

where $\boldsymbol{f}^{-1}$ is *one* reciprocal function of $\boldsymbol{f}$.

The Jacobian matrix $J(\boldsymbol{q})$ of the function $\boldsymbol{f}$ has a major significance in order to define the notions to come. This $m \times M$ matrix is such that:

$$J(\boldsymbol{q}) = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{q}}, \quad \text{i.e.} \quad J_{ij}(\boldsymbol{q}) = \frac{\partial f_i}{\partial q_j}, \quad 1 \leqslant i \leqslant m,\ 1 \leqslant j \leqslant M,$$

and represents the linear map between the tangent spaces:

$$J(\boldsymbol{q})\colon \quad T_{\boldsymbol{q}}\mathcal{N} \to T_{\boldsymbol{\xi}}\mathcal{M},$$

with $\boldsymbol{\xi} = \boldsymbol{f}(\boldsymbol{q})$. Thus: rank $J(\boldsymbol{q})$ + dim Null $J(\boldsymbol{q}) = M$. Let us note $d(\boldsymbol{q}) =$ rank $J(\boldsymbol{q})$ in the configuration $\boldsymbol{q}$ and let us call it *local degree of freedom of the* EE. This corresponds to the fact that the EE is constrained by the mechanical system supporting it and undergoes a holonomic constraint of class $c(\boldsymbol{q}) = 6 - d(\boldsymbol{q})$. In fact, locally, $d(\boldsymbol{q})$ independent operational coordinates are necessary and sufficient to describe the EE location.

Let:

$$D = \max_{\boldsymbol{q} \in \mathcal{N}} d(\boldsymbol{q}).$$

D is the *(global) degree of freedom of the* EE. From the preceding relations, we verify that $D \leqslant M$. When $D = M$, an equal number of independent parameters describes the EE location on one side, the mobile manipulator configuration on the other. In this case, a finite number of configurations corresponds *almost everywhere* (a.e.) to the same EE location. On the contrary, when $D < M$, an infinity of configurations corresponds a.e. to the same EE location. With these notations, we will denote mobile manipulator *kinematic redundancy*, the usual redundancy such that it is defined for holonomic systems and thus for robotic arms.

KINEMATIC REDUNDANCY. A mobile manipulator, holonomic or not, is kinematically redundant – with degree R – when the degree of freedom D of its EE is strictly lower than its mobility index M. In this case, $R = M - D$ and a.e. for a given EE location there is a R-dimensional set of corresponding configurations.

This is simply the ordinary definition of redundancy for robotic arms (see (Gorla et al., 1984)). Generally, mobile manipulators are kinematically redundant. The configurations $\boldsymbol{q}$ such that $d(\boldsymbol{q}) < D$ are called *Kinematically Singular Configurations* (KSC). The singularity order of the KSC will be equal to, by definition, $D - d(\boldsymbol{q})$.

Remark. The notion of degree of freedom of the EE may be used in order to determine the necessary number of operational coordinates. In fact, if a first choice of m_1 operational coordinates is such that the maximal value of the rank of the Jacobian matrix is strictly lower than m_1, then these coordinates are not independent and there exists another set of coordinates in number $m_2 = D < m_1$ that describes the whole set of admissible locations.

Here, it is worth noting that the Jacobian matrix $J(\boldsymbol{q})$ does not gather all the constraints acting on the operational velocities since the nonholonomic constraint is not taken into account in the definition of $J(\boldsymbol{q})$ (the nonholonomic constraint is not integrable and cannot be written as a mere configuration dependent constraint).

In other words, $\dot{\boldsymbol{\xi}} = J(\boldsymbol{q})\dot{\boldsymbol{q}}$ expresses only a part of the constraints acting on $\dot{\boldsymbol{\xi}}$. In the same way, KSC do not reveal specific cases concerning the admissibility of operational velocities but are to be connected to particular cases in the computation of solutions of IK.

4. Velocity Kinematics

When we are interested in the operational $\dot{\boldsymbol{\xi}}$ and generalized $\dot{\boldsymbol{q}}$ velocities that are associated to, it is natural to use the notion of *mobility degree* of the mechanical system and to define notions such as redundancy from it. In fact, whereas the *mobility index* indicates the dimension of the space the generalized coordinates are defined on, the *mobility degree* indicates the space dimension of the *admissible* generalized velocities.

In this way – in a similar manner to the definition of kinematic redundancy which has been established from a choice of generalized coordinates that forms a basis of the generalized space – the *velocity redundancy* will be defined from a choice of *quasi-velocities* that form a basis of the admissible subset of the tangent space to the generalized space.

4.1. NONHOLONOMIC CONSTRAINT AND QUASI-VELOCITIES

The mobile platform evolution on the plane is constrained by the rolling without slipping condition:

$$\dot{x}\sin\vartheta - \dot{y}\cos\vartheta = 0,$$

i.e. $G_p(\boldsymbol{q}_p)\dot{\boldsymbol{q}}_p = 0$, with $G_p(\boldsymbol{q}_p) = [\sin\vartheta \quad -\cos\vartheta \quad 0]$.

This relation is not integrable or *nonholonomic*. It expresses the dependency, linear for a given angle ϑ, between the generalized velocities $\dot{q}_1$ and $\dot{q}_2$. Thus, we can choose a 2-dimensional distribution* that defines in each configuration a basis for the *admissible* generalized velocities. In this way, the platform is characterized by a mobility index $M_p = 3$ and a *mobility degree* (Campion et al., 1996) that gives here the number of independent admissible generalized velocities $\delta_p = M_p - 1 = 2$ since the generalized velocities verify *one* nonholonomic constraint.
Thus, admissible generalized velocities write:

$$\dot{\boldsymbol{q}}_p = \boldsymbol{g}_1(\boldsymbol{q}_p)\eta_{p_1} + \boldsymbol{g}_2(\boldsymbol{q}_p)\eta_{p_2},$$

where $\boldsymbol{g}_1, \boldsymbol{g}_2$ are two 3-dimensional vector fields and η_{p_1}, η_{p_2} two independent parameters – also called *quasi-velocities* (Neimark and Fufaev, 1972). Such a system is called a *Caplygin system* (Neimark and Fufaev, 1972). For these systems, there exists a natural choice concerning η_{p_1} and η_{p_2} and the corresponding distribution $(\boldsymbol{g}_1, \boldsymbol{g}_2)$. In fact, it can be shown that one of the two parameters η_{p_1}, η_{p_2} can naturally be chosen as the derivative of the generalized coordinate appearing in the nonholonomic constraint whereas the second one must be the derivative of a quantity which is not a coordinate. In this way, we obtain $\eta_{p_1} = \dot{\sigma} = v$ and $\eta_{p_2} = \dot{\vartheta} = \omega$, where σ is the curvilinear abscissa of the point O' along its path. Finally, we have: $\boldsymbol{g}_1 = [\cos\vartheta \quad \sin\vartheta \quad 0]^{\mathrm{T}}$ and $\boldsymbol{g}_2 = [0 \quad 0 \quad 1]^{\mathrm{T}}$. The vector of quasi-velocities $\boldsymbol{\eta}_p = [v \quad \omega]^{\mathrm{T}}$ is the *mobility control vector*. Then:

$$\dot{\boldsymbol{q}}_p = S_p(\boldsymbol{q}_p)\boldsymbol{\eta}_p \quad \text{and} \quad \boldsymbol{\eta}_p = S_p^{\mathrm{T}}(\boldsymbol{q}_p)\dot{\boldsymbol{q}}_p$$

with:

$$S_p^{\mathrm{T}}(\boldsymbol{q}_p) = \begin{bmatrix} \cos\vartheta & \sin\vartheta & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

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* Basically, a (smooth) distribution

$$\Delta = span\{f_1, \ldots, f_d\} = (f_1, \ldots, f_2)$$

is the assignment – to each point x of an open set U of $\mathbb{R}^n$ – of the subspace spanned by the values at x of some smooth vector fields $f_1(x), \ldots, f_d(x)$ defined on U. Pointwise, a distribution is a vector space, a subspace of $\mathbb{R}^n$ (Isidori, 1989).

These quasi-velocities are the time derivatives of the quasi-coordinates σ et ϑ. In this way, we go back to the well known fact that there exists a one-to-one relation between the velocities of both wheels on one side, and the angular and linear velocities of the platform on the other.

Remark. Since $(\boldsymbol{g}_1, \boldsymbol{g}_2, [\boldsymbol{g}_1, \boldsymbol{g}_2])$ spans* a 3-dimensional space, the platform is controllable (Nijmeijer et al., 1990), and can reach any of its configurations (x, y, ϑ).

4.2. VELOCITY KINEMATICS OF THE MOBILE MANIPULATOR

Generalized and operational velocities $\dot{\boldsymbol{q}}$ and $\dot{\boldsymbol{\xi}}$ are related by $\dot{\boldsymbol{\xi}} = J(\boldsymbol{q})\dot{\boldsymbol{q}}$. However, the rolling without slipping condition, that was written $G_p(\boldsymbol{q}_p)\dot{\boldsymbol{q}}_p = 0$ for the platform, can be rewritten for the mobile manipulator as:

$$G(\boldsymbol{q})\dot{\boldsymbol{q}} = 0 \quad \text{with } G(\boldsymbol{q}) = [G_p(\boldsymbol{q}_p) \quad 0].$$

This equation imposes a nonholonomic constraint on the generalized velocities $\dot{\boldsymbol{q}}$ of the mobile manipulator. In this way, for every configuration $\boldsymbol{q}$, $\dot{\boldsymbol{q}}$ belongs to the δ-dimensional submanifold $\Pi_{\boldsymbol{q}}$ of the tangent space $T_{\boldsymbol{q}}\mathcal{N}$, where $\delta = M - 1$ is the *mobility degree* of the mobile manipulator:

$$\dot{\boldsymbol{q}} \in \Pi_{\boldsymbol{q}} \subset T_{\boldsymbol{q}}\mathcal{N},$$

and there exists a linear map $\overline{J}(\boldsymbol{q})$ such that:

$$\overline{J}(\boldsymbol{q})\colon\ \Pi_{\boldsymbol{q}} \to T_{\boldsymbol{\xi}}\mathcal{M}.$$

The matrix of this linear map, which can be also denoted by $\overline{J}(\boldsymbol{q})$, is of dimension $m \times \delta$ and *is not a Jacobian matrix*. A δ-dimensional vector $\boldsymbol{\eta}$ of the following form:

$$\boldsymbol{\eta} = [\boldsymbol{\eta}_p^{\mathrm{T}} \quad \dot{\boldsymbol{q}}_a^{\mathrm{T}}]^{\mathrm{T}}$$

can be chosen in $\Pi_{\boldsymbol{q}}$ to express this linear map. It is called the *mobility control vector* of the mobile manipulator.

Finally, the constraints acting on operational and generalized velocities are:

$$\begin{bmatrix} 0 \\ \dot{\boldsymbol{\xi}} \end{bmatrix} = \begin{bmatrix} G(\boldsymbol{q}) \\ J(\boldsymbol{q}) \end{bmatrix} \dot{\boldsymbol{q}}. \tag{1}$$

If we notice that:

$$\boldsymbol{\eta} = S^{\mathrm{T}}(\boldsymbol{q})\dot{\boldsymbol{q}} \quad \text{and} \quad \dot{\boldsymbol{q}} = S(\boldsymbol{q})\boldsymbol{\eta}$$

* $[\boldsymbol{g}_1, \boldsymbol{g}_2]$ is the Lie bracket of $\boldsymbol{g}_1$ and $\boldsymbol{g}_2$.

with:

$$S(\boldsymbol{q}) = \begin{bmatrix} S_p(\boldsymbol{q}_p) & 0 \\ 0 & I_{n_a} \end{bmatrix},$$

in which I_{n_a} is the n_a-order identity matrix, we can write *Reduced Direct Velocity Kinematics* (RDVK):

$$\dot{\boldsymbol{\xi}} = \overline{J}(\boldsymbol{q})\boldsymbol{\eta}, \tag{2}$$

with $\overline{J}(\boldsymbol{q}) = J(\boldsymbol{q})S(\boldsymbol{q})$.
Then:

$$\text{rank } \overline{J}(\boldsymbol{q}) + \dim \text{Null } \overline{J}(\boldsymbol{q}) = \delta. \tag{3}$$

Now, let:

$$\bar{d}(\boldsymbol{q}) = \text{rank } \overline{J}(\boldsymbol{q}),$$

and:

$$\overline{D} = \max_{\boldsymbol{q}\in\mathcal{N}} \bar{d}(\boldsymbol{q}). \tag{4}$$

From relations (3) and (4), we verify: $\overline{D} \leqslant \delta$. When $\overline{D} = \delta$, an equal number of independent parameters describes the EE velocity $\dot{\boldsymbol{\xi}}$ on one side, the mobile manipulator generalized velocity $\dot{\boldsymbol{q}}$ on the other. In this case, a finite number of generalized velocities $\dot{\boldsymbol{q}}$ corresponds a.e. to the same EE velocity $\dot{\boldsymbol{\xi}}$. On the opposite, when $\overline{D} < \delta$, an infinity of generalized velocities $\dot{\boldsymbol{q}}$ corresponds a.e. to the same operational velocity $\dot{\boldsymbol{\xi}}$. This leads us to the definition of *velocity redundancy*.

VELOCITY REDUNDANCY. A mobile manipulator is velocity redundant – with degree $\overline{R}$ – when $\overline{D}$ is strictly lower than its mobility degree δ. In this case, $\overline{R} =$

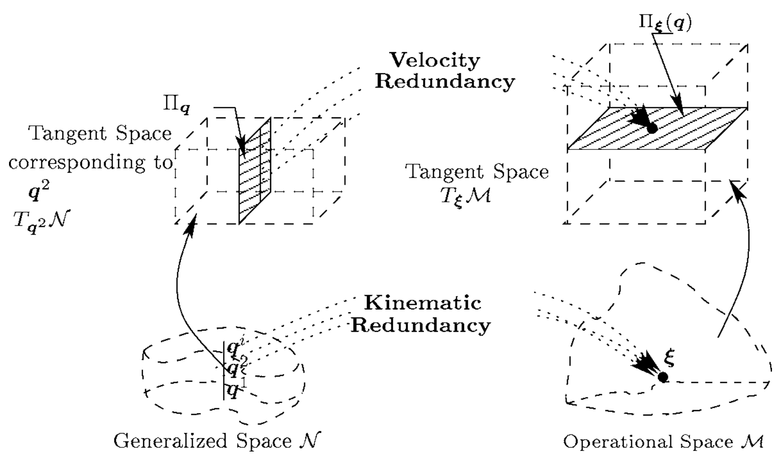

Figure 2. Spaces and redundancies (case $\overline{D} < D$).

$\delta - \overline{D}$ and for a given EE velocity there is a.e. a $\overline{R}$-dimensional set of generalized velocities $\dot{\boldsymbol{q}}$.

This definition is useless for holonomic systems such as ordinary robotic arms. In fact, in this case, mobility degree and mobility index are identical and both redundancies illustrate the same notion.

Remarks.

- *Velocity Singular Configurations* (VSC) are the configurations $\boldsymbol{q}$ such that $\bar{d}(\boldsymbol{q}) < \overline{D}$. In these configurations, the dimension of $T_{\boldsymbol{\xi}}\mathcal{M}$ decreases and $\dot{\boldsymbol{\xi}}$ undergoes one or several additional constraints. We will say that these configurations are of singularity order equal to, by definition, $\overline{D} - \bar{d}(\boldsymbol{q})$.
- We always have: $\overline{D} \leqslant D$. When $\overline{D} = D$, the whole tangent space $T_{\boldsymbol{\xi}}\mathcal{M}$ is admissible and the velocities $\dot{\boldsymbol{\xi}}$ can be chosen in a space with dimension equal to that of $\mathcal{M}$. When, $\overline{D} < D$, only an hyperplane $\Pi_{\boldsymbol{\xi}}(\boldsymbol{q})$ of $T_{\boldsymbol{\xi}}\mathcal{M}$ is admissible in $\boldsymbol{q}$. In this latter case, the system may be velocity redundant but such that no generalized velocity $\dot{\boldsymbol{q}}$ allows to realize a given operational velocity $\dot{\boldsymbol{\xi}}$.

Figure 2 is an illustration of the different redundancies introduced in this section.

4.3. VELOCITY KINEMATICS INVERSION

For a given $\dot{\boldsymbol{\xi}}$ Velocity Kinematics Inversion expresses, when there exists, the solution(s) $\dot{\boldsymbol{q}}$ which verifies relation (1). Using the notions of quasi-velocities and mobility degree, it is judicious to decompose this inversion into two steps:

- *First Step:* computation of *Reduced Inverse Velocity Kinematics* (RIVK) that consists in inverting RDVK (2) in order to obtain $\boldsymbol{\eta}$. This inversion problem has at least one solution if and only if RDVK is *consistent*, that is:

  $$\bar{d}(\boldsymbol{q}) = \operatorname{rank}\left[\overline{J}(\boldsymbol{q}) \mid \dot{\boldsymbol{\xi}}\right].$$

  If this condition is verified, all RIVK write as:

  $$\boldsymbol{\eta} = \overline{J}^{\#}(\boldsymbol{q})\dot{\boldsymbol{\xi}} + \left(I_{\delta} - \overline{J}^{\#}(\boldsymbol{q})\overline{J}(\boldsymbol{q})\right)\boldsymbol{z},$$

  where $\overline{J}^{\#}(\boldsymbol{q})$ is an arbitrary generalized inverse* of $\overline{J}(\boldsymbol{q})$, i.e., such that $\overline{J}(\boldsymbol{q})\overline{J}^{\#}(\boldsymbol{q})\overline{J}(\boldsymbol{q}) = \overline{J}(\boldsymbol{q})$ and $\boldsymbol{z}$ is an arbitrary δ-order column matrix.
- *Second Step:* computation of $\dot{\boldsymbol{q}}$ from $\boldsymbol{\eta}$:

  $$\dot{\boldsymbol{q}} = S(\boldsymbol{q})\boldsymbol{\eta}.$$

* It is always possible to choose $\overline{J}^{+}$ the pseudo-inverse – or Moore–Penrose inverse – of $\overline{J}(\boldsymbol{q})$.

5. Modelling Examples

5.1. FIRST EXAMPLE: A PLANAR MOBILE MANIPULATOR WITH A MOBILITY INDEX M EQUAL TO 5

In this section we study the mobile manipulator composed of a HILARE-like mobile platform on which is mounted an horizontal double pendulum robotic arm (i.e., such that both rotation axes are vertical) (Yamamoto, 1994; Seraji, 1995) (see Figure 3). Its generalized coordinates are $\boldsymbol{q} = [q_1 \quad q_2 \quad q_3 \quad q_4 \quad q_5]^{\mathrm{T}} = [x \quad y \quad \vartheta \quad q_{a1} \quad q_{a2}]^{\mathrm{T}}$ and thus $M = 5$. The EE operational coordinates in $\mathcal{R}$ are $\boldsymbol{\xi} = [\xi_1 \quad \xi_2 \quad \xi_3]^{\mathrm{T}}$ $(m = 3)$; the first two Cartesian coordinates of the EE center in $\mathcal{R}$ characterize the EE position whereas its orientation is given by the angle ξ_3 measured between the axis $(O, \vec{x})$ and the main direction of the EE.

DK writes:

$$\begin{cases} \xi_1 = q_1 + l_1 C_{34} + l_2 C_{345}, \\ \xi_2 = q_2 + l_1 S_{34} + l_2 S_{345}, \\ \xi_3 = q_3 + q_4 + q_5, \end{cases}$$

and the Jacobian matrix as follows:

$$J(\boldsymbol{q}) = \begin{bmatrix} 1 & 0 & -(l_1 S_{34} + l_2 S_{345}) & -(l_1 S_{34} + l_2 S_{345}) & -l_2 S_{345} \\ 0 & 1 & l_1 C_{34} + l_2 C_{345} & l_1 C_{34} + l_2 C_{345} & l_2 C_{345} \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix},$$

where $C_{34} = \cos(q_3 + q_4)$, $S_{34} = \sin(q_3 + q_4)$, $C_{345} = \cos(q_3 + q_4 + q_5)$, $S_{345} = \sin(q_3 + q_4 + q_5)$.

Then, $d(\boldsymbol{q}) = \operatorname{rank} J(\boldsymbol{q}) = 3$, $\forall \boldsymbol{q} \in \mathcal{N}$ and $D = \max_{\boldsymbol{q} \in \mathcal{N}} d(\boldsymbol{q}) = 3$. Consequently, the system is kinematically redundant with degree $R = 2$, and there is no KSC.

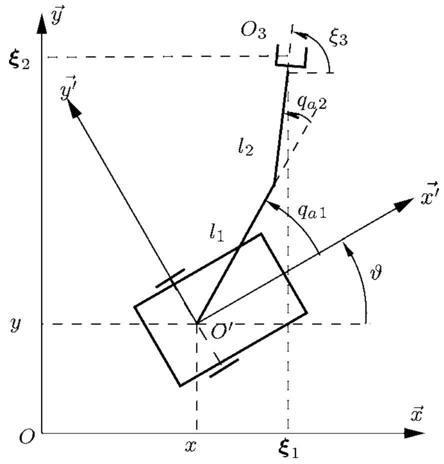

Figure 3. A planar mobile manipulator.

Concerning IK, we cannot separate the action of q_3 from that of q_4 in so far as only the sum $q_3 + q_4$ appears in DK. Thus, there is already a simple infinity of solutions when solving IK since for each value of q_3 one can find a corresponding q_4. In fact, there is a double infinity of solutions since one of the members of the pair (q_1, q_2) can be chosen arbitrarily, the other one having to satisfy the constraint:

$$(\xi_1 - l_2 \cos \xi_3 - q_1)^2 + (\xi_2 - l_2 \sin \xi_3 - q_2)^2 = {l_1}^2.$$

Once q_3 and one of the members of the pair (q_1, q_2) have been chosen, the other member is deduced from the previous relation and (q_4, q_5) verifies:

$$\begin{cases} q_4 = \operatorname{arctan2}(\xi_2 - l_2 \sin \xi_3 - q_2,\ \xi_1 - l_2 \cos \xi_3 - q_1) - q_3, \\ q_5 = \xi_3 - \operatorname{arctan2}(\xi_2 - l_2 \sin \xi_3 - q_2,\ \xi_1 - l_2 \cos \xi_3 - q_1). \end{cases}$$

Thus, in the platform generalized space, the set of configurations (x, y, ϑ) that are solutions of IK, for a given EE location $\boldsymbol{\xi} = [\xi_1 \quad \xi_2 \quad \xi_3]^{\mathrm{T}}$ in $\mathcal{R}$, is the cylinder $\mathcal{S}(\boldsymbol{\xi})$ defined by:

$$\mathcal{S}(\boldsymbol{\xi}):\ (\xi_1 - l_2 \cos \xi_3 - x)^2 + (\xi_2 - l_2 \sin \xi_3 - y)^2 = {l_1}^2.$$

Thus, all points of the cylinder with axis $(x = \xi_1 - l_2 \cos \xi_3, y = \xi_2 - l_2 \sin \xi_3)$ and radius l_1 in the space (x, y, ϑ) are such that it exists a pair (q_4, q_5) such that the EE location in $\mathcal{R}$ is (ξ_1, ξ_2, ξ_3). Finally, the set of configurations corresponding to a given EE location is verified to be a 2-dimensional manifold.

Concerning velocities, RDVK expresses $\dot{\boldsymbol{\xi}}$ as function of $\boldsymbol{\eta} = [v \quad \omega \quad \dot{q}_4 \quad \dot{q}_5]^{\mathrm{T}}$. It writes: $\dot{\boldsymbol{\xi}} = \overline{J}(\boldsymbol{q}_a, \vartheta)\boldsymbol{\eta}$ with:

$$\overline{J}(\boldsymbol{q}_a, \vartheta) = \begin{bmatrix} C_3 & -(l_1 S_{34} + l_2 S_{345}) & -(l_1 S_{34} + l_2 S_{345}) & -l_2 S_{345} \\ S_3 & l_1 C_{34} + l_2 C_{345} & l_1 C_{34} + l_2 C_{345} & l_2 C_{345} \\ 0 & 1 & 1 & 1 \end{bmatrix},$$

where $C_3 = \cos q_3$, $S_3 = \sin q_3$. The rank of $\overline{J}(\boldsymbol{q}_a, \vartheta)$ is equal to 3 a.e., i.e., when $q_4 \neq (2k+1)\pi/2$. This rank decreases to 2 when $q_4 = (2k+1)\pi/2$. Thus $\overline{D} = 3$ and for $q_4 = (2k+1)\pi/2$, $\bar{d} = 2$. Notice that $\delta = 4$.

Remarks.

- The velocity redundancy degree $\overline{R}$ is equal to 1. In the space tangent to the configuration space, the set of points that are associated by RIVK to the same operational velocity is of dimension 1.
- VSC are such that $q_4 = (2k+1)\pi/2$ and are of singularity order equal to 1.

5.2. SECOND EXAMPLE: A SYSTEM WITH A MOBILITY INDEX M EQUAL TO 6

We now study a mobile manipulator built from a HILARE-like mobile platform on which is mounted an RRP arm (see Figure 4).

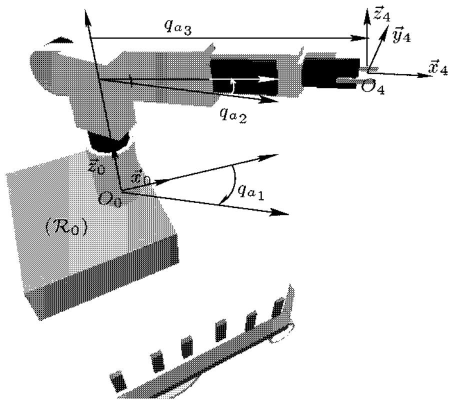

Figure 4. A mobile manipulator with a RRP arm.

Its generalized coordinates are:

$$\boldsymbol{q} = [q_1 \quad q_2 \quad q_3 \quad q_4 \quad q_5 \quad q_6]^{\mathrm{T}} = [x \quad y \quad \vartheta \quad q_{a1} \quad q_{a2} \quad q_{a3}]^{\mathrm{T}}$$

and thus $M = 6$. The EE operational coordinates in $\mathcal{R}$ are:

$$\boldsymbol{\xi} = [\xi_1 \quad \xi_2 \quad \xi_3 \quad \xi_4 \quad \xi_5]^{\mathrm{T}};$$

the first three coordinates are the Cartesian coordinates of the EE center (point O_4) in $\mathcal{R}$ and the last two ones are angles defining the orientation of frame $\mathcal{R}_4$ in $\mathcal{R}$ (see Figure 4).

DK writes:

$$\begin{cases} \xi_1 = q_1 + q_6 C_5 C_{34}, \\ \xi_2 = q_2 + q_6 C_5 S_{34}, \\ \xi_3 = q_6 S_5, \\ \xi_4 = q_3 + q_4, \\ \xi_5 = q_5, \end{cases}$$

and the Jacobian matrix as follows:

$$J(\boldsymbol{q}) = \begin{bmatrix} 1 & 0 & -q_6 C_5 S_{34} & -q_6 C_5 S_{34} & -q_6 S_5 C_{34} & C_5 C_{34} \\ 0 & 1 & q_6 C_5 C_{34} & q_6 C_5 C_{34} & -q_6 S_5 S_{34} & C_5 S_{34} \\ 0 & 0 & 0 & 0 & q_6 C_5 & S_5 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

with $C_5 = \cos q_5$ and $S_5 = \sin q_5$.

Then, $d(\boldsymbol{q}) = \operatorname{rank} J(\boldsymbol{q}) \in \{4, 5\}, \forall \boldsymbol{q} \in \mathcal{N}$ and $D = \max_{\boldsymbol{q} \in \mathcal{N}} d(\boldsymbol{q}) = 5$. Consequently, the EE location varies on a 5-dimensional manifold, the system is kinematically redundant with degree $R = 1$, and KSC are of singularity order equal to 1 and such that $q_5 = k\pi$ since the rank of $J(\boldsymbol{q})$ is 4 in these configurations.

By solving IK, it is easy to show that there is an infinity of pairs (q_3, q_4) that are solutions and that locally in the KSC there is a double infinity of solutions.

Concerning velocity kinematics, RDVK expresses $\dot{\boldsymbol{\xi}}$ as a function of $\boldsymbol{\eta} = [v \quad \omega \quad \dot{q}_4 \quad \dot{q}_5 \quad \dot{q}_6]^{\mathrm{T}}$. It writes $\dot{\boldsymbol{\xi}} = \overline{J}(\boldsymbol{q}_a, \vartheta)\boldsymbol{\eta}$ with:

$$\overline{J}(\boldsymbol{q}_a, \vartheta) = \begin{bmatrix} C_3 & -q_6 C_5 S_{34} & -q_6 C_5 S_{34} & -q_6 S_5 C_{34} & C_5 C_{34} \\ S_3 & q_6 C_5 C_{34} & q_6 C_5 C_{34} & -q_6 S_5 S_{34} & C_5 S_{34} \\ 0 & 0 & 0 & q_6 C_5 & S_5 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

Now, we have a square system but it is not invertible. In fact, the maximal rank $\overline{D}$ of $\overline{J}(\boldsymbol{q}_a, \vartheta)$ is equal to 4 since $\bar{d} = 4$ when $q_4 \neq k_4\pi$ or $q_5 \neq k_5\pi$ and $\bar{d} = 3$ when $q_4 = k_4\pi$ and $q_5 = k_5\pi$:

- The velocity redundancy degree $\overline{R}$ is equal to 1 since $\delta = 5$. In the tangent space to the generalized space, the set of points that are associated by the RIVK to the same operational velocity is of dimension 1.
- However, not all the operational velocities $\dot{\boldsymbol{\xi}}$ are admissible since $\overline{D} < D$. When operational velocities verify the consistency relation:

  $$S_3 S_5 \dot{\xi}_1 - C_3 S_5 \dot{\xi}_2 + S_4 C_5 \dot{\xi}_3 + C_4 C_5 S_5 q_6 \dot{\xi}_4 - S_4 q_6 \dot{\xi}_5 = 0$$

  there exists a simple infinity of generalized velocities $\dot{\boldsymbol{q}}$.
- VSC are such that $q_4 = k_4\pi$ and $q_5 = k_5\pi$: they are of singularity order equal to 1.

Among the different problems that arise in mobile manipulation, we consider in the sequel the *operational motion planning problem* and we show how reduced velocity kinematics, velocity redundancy and the associated notions of manipulability allow to solve it.

6. Operational Motion Planning Problem

We consider the operational motion planning problem when the mobile manipulator is velocity redundant i.e. when the dimension δ of its control of mobility $\boldsymbol{\eta}$ is greater than the maximum value $\overline{D}$ of the rank of $\overline{J}$.

Here, we propose a pseudo-inversion scheme to solve the velocity redundancy and obtain a particular generalized motion for the mobile manipulator when the end-effector motion is given. From a velocity kinematics point of view we can explain this coordination strategy as follows. The main idea is to use velocity redundancy in order to decrease/increase the value of an adequate function and eventually to minimize/maximize its value. Thus, it is based on a gradient descent method. Among other geometric or kinematic natural choices (see (Bayle et al., 2000) for singularity avoidance), we propose here to use different manipulability measures and the resulting strategy is applied to the first mobile manipulator example.

For a given operational motion $\boldsymbol{\xi}^*(t)$, the problem is to find the mobility control $\boldsymbol{\eta}(t)$ such that $\dot{\boldsymbol{\xi}}^*(t) = \overline{J}(t)\boldsymbol{\eta}(t)$, in order to asymptotically stabilize the operational error $\boldsymbol{e}(t) = \boldsymbol{\xi}^*(t) - \boldsymbol{\xi}(t)$. The matrix $\overline{J}(t)$ is $m \times \delta$ with $m \leqslant \delta$ and we suppose that its rank is m. Then the previous linear system is *consistent* and all its *exact* solutions are given by:

$$\boldsymbol{\eta}(t) = \overline{J}^+(t)\dot{\boldsymbol{\xi}}^*(t) + \big(I_\delta - \overline{J}^+(t)\overline{J}(t)\big)\boldsymbol{g}(t),$$

in which $\overline{J}^+(t)$ is the pseudo-inverse of $\overline{J}(t)$ and $\boldsymbol{g}(t)$ any δ-dimensional vector. The solution obtained is the one that minimizes the Euclidian norm $||\boldsymbol{\eta} - \boldsymbol{g}||$.

In fact, in order to asymptotically stabilize the error $\boldsymbol{e}(t)$, one can choose:

$$\boldsymbol{\eta}(t) = \overline{J}^+(t)\big(\dot{\boldsymbol{\xi}}^*(t) + W\big(\boldsymbol{\xi}^*(t) - \boldsymbol{\xi}(t)\big)\big) + \big(I_\delta - \overline{J}^+(t)\overline{J}(t)\big)\boldsymbol{g}(t), \tag{5}$$

in which W is a m-order definite positive matrix.

Actually, since $\overline{J}^+(t)$ is a right-inverse of $\overline{J}(t)$, the previous control leads to the asymptotical stability of the transient error $\boldsymbol{e}(t)$, due to the equation:

$$\dot{\boldsymbol{e}}(t) + W\boldsymbol{e}(t) = 0,$$

By using the expression of $\boldsymbol{\eta}$ given in (5), $\dot{\boldsymbol{q}}(t) = S(t)\boldsymbol{\eta}(t)$ writes:

$$\dot{\boldsymbol{q}}(t) = S(t)\overline{J}^+(t)\big(\dot{\boldsymbol{\xi}}^*(t) + W\big(\boldsymbol{\xi}^*(t) - \boldsymbol{\xi}(t)\big)\big) + S(t)\big(I_\delta - \overline{J}^+(t)\overline{J}(t)\big)\boldsymbol{g}(t). \tag{6}$$

In this equation the first term is due to the input and the second one is the *internal motion*. We now use velocity redundancy to propose a coordination strategy for the internal motion based on a gradient descent method. In general, let $\mathcal{P}$ be a scalar function depending on the mobile manipulator configuration $\boldsymbol{q}(t)$. We can write:

$$\begin{aligned}\dot{\mathcal{P}}(t) &= \nabla^{\mathrm{T}}\mathcal{P}(\boldsymbol{q}(t))\dot{\boldsymbol{q}}(t)\\ &= \nabla^{\mathrm{T}}\mathcal{P}(\boldsymbol{q}(t))S(t)\big(I_\delta - \overline{J}^+(t)\overline{J}(t)\big)\boldsymbol{g}(t),\end{aligned}$$

for the internal motion where $\nabla\mathcal{P}(\boldsymbol{q}(t))$ is the gradient of the function $\mathcal{P}(\boldsymbol{q}(t))$. In order to decrease $\mathcal{P}(t)$, that is $\dot{\mathcal{P}}(t) \leqslant 0$, we propose the choice:

$$\boldsymbol{g}(t) = -k\big((\nabla^{\mathrm{T}}\mathcal{P})H\big)^{\mathrm{T}},$$

where k is a positive scalar and $H(t) = S(t)(I_\delta - \overline{J}^+(t)\overline{J}(t))$. Indeed, with this choice:

$$\dot{\mathcal{P}}(t) = -k\big((\nabla^{\mathrm{T}}\mathcal{P})HH^{\mathrm{T}}(\nabla\mathcal{P})\big)$$

and then $\dot{\mathcal{P}}(t) \leqslant 0$.

Finally, the mobility control is:

$$\boldsymbol{\eta}(t) = \overline{J}^+(t)\big(\dot{\boldsymbol{\xi}}^*(t) + W\big(\boldsymbol{\xi}^*(t) - \boldsymbol{\xi}(t)\big)\big) - k(\nabla^{\mathrm{T}}\mathcal{P})HH^{\mathrm{T}}(\nabla\mathcal{P}).$$

This approach can be applied to various functions $\mathcal{P}$. In the next paragraph, we demonstrate how it can be applied by taking $\mathcal{P}$ as manipulability measures.

7. Using Mobile Manipulator Manipulability Measures

7.1. MANIPULABILITY

Manipulability theory has been introduced by Yoshikawa for holonomic robotic arms (Yoshikawa, 1990). Basically, it describes the set of operational velocities $\dot{\boldsymbol{\xi}}_a$ realizable by the robotic arm in a given configuration when generalized velocities $\dot{\boldsymbol{q}}_a$ are bounded in a unit ball $||\dot{\boldsymbol{q}}_a|| \leqslant 1$ where $||\cdot||$ stands for some Riemmanian norm in order to normalize from velocity bounds, inertia matrix, etc. So manipulability aims to answer in different metrics the question: for a given configuration, what is the ability of the system to provide operational velocities in any directions? When considering the simplest case of an Euclidean norm, this answer relies essentially on the Singular Value Decomposition (SVD) of the Jacobian matrix $J_a(\boldsymbol{q}_a)$ that maps generalized velocities $\dot{\boldsymbol{q}}_a$ to operational velocities $\dot{\boldsymbol{\xi}}_a$ of the robotic arm EE. The resulting set of operational velocities is an ellipsoid whose axes dimensions are given by the singular values $\sigma_1, \sigma_2, \ldots, \sigma_{m_a}$. Different algebraic measures have been proposed to characterize this ellipsoid. They are often called *manipulability measures* and give a scalar information. The more usual manipulability measure is $w_a = \sigma_1\sigma_2 \cdots \sigma_{m_a}$ which is proportional to the ellipsoid volume. It thus gives a quantitative information on the manipulability.

In this paper, we will also use a manipulability measure* extending the notion of *eccentricity* of the ellipse (Bayle et al., 2001):

$$w_{a_5} = \sqrt{1 - \frac{\sigma_{m_a}^2}{\sigma_1^2}}.$$

7.2. THE MANIPULABILITY OF MOBILE MANIPULATORS

The first contribution, to our knowledge, that dealt with manipulability in mobile manipulation is devoted to the manipulability of the sole arm (Yamamoto, 1994). Depending on the tasks at hand, there is an interest in considering the ability of generating velocities at the end-effector by acting on the sole arm or by acting on the whole system. Here, we develop an analysis of the whole mobile manipulator manipulability.

Manipulability can be defined for nonholonomic mobile manipulators from the definition of the RDVK:

$$\dot{\boldsymbol{\xi}} = \bar{J}\boldsymbol{\eta},$$

* We call this measure w_{a_5} as Yoshikawa defines four other measures.

since this model describes the instantaneous velocities of the EE for given controls of mobility. That way, we are looking for the realizable EE velocities such that the corresponding control of mobility $\boldsymbol{\eta}$ verifies $||\boldsymbol{\eta}|| \leqslant 1$.

It is generally not possible to separate analytically the effects of the platform and of the robotic arm on manipulability. So they will be visualized through several numeric simulations.

Remarks.

- The norm we considered has been obtained from the maximum generalized velocities in our study, but for sake of simplicity we do not mention the normalization in the writings.
- The *mobile manipulator* manipulability measure has been defined in a way similar to that of the arm. Yet, depending on the application we may have to consider the whole system manipulability or the robotic arm manipulability (for instance, when the mobile manipulator is not used in a coordinated fashion strategy). If the user wants to keep the platform motionless to manipulate with the arm alone, it would be convenient to reach the operating site in a good configuration for the arm, from a manipulation point of view. Also, both manipulability definitions can be useful for the same task.
- The analytical expression of the manipulability is complex even for a simple mobile manipulator. It may not be helpful to design the $\mathcal{P}$ function. Rather it would be more interesting to consider functions of manipulability with minimum corresponding to optimal configurations, such as $(-w)$ or w_5, and to compute their numerical gradient.

In the sequel, we report the results obtained for two different tasks. For both of them, only the *position* of the end-effector is imposed. From case to case, we will choose:

- w_5 manipulability measure, whose value decreases with anisotropy of EE admissible velocities;
- Yoshikawa's manipulability measure with opposite sign $(-w)$ whose value decreases when the system moves away from singular configurations.

First task. Figures 5 illustrates the tracking of an operational motion the associated path of which being elliptic. Here, the global manipulability of the mobile manipulator is considered through the choice of w_5 measure.

After a transient phase, the EE follows its imposed motion. It is worth noting that during the most part of the motion, the manipulability ellipse is very similar to a circle: the w_5 manipulability measure with respect to the subset of the considered operational coordinates – position coordinates in the plane – is minimized.

Second task. It is interesting to leave the user free to choose the relative weighting between the arm manipulability and the mobile manipulator manipulability measures. In fact, depending on the task at hand, we may need to use both kind of

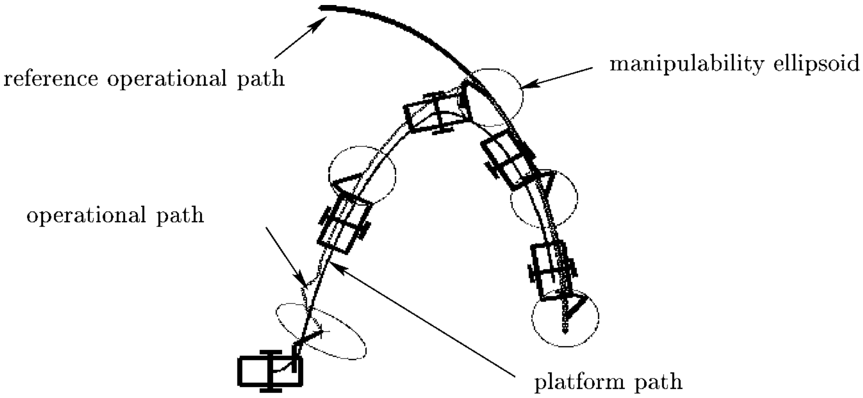

Figure 5. Tracking an operational motion with elliptic path.

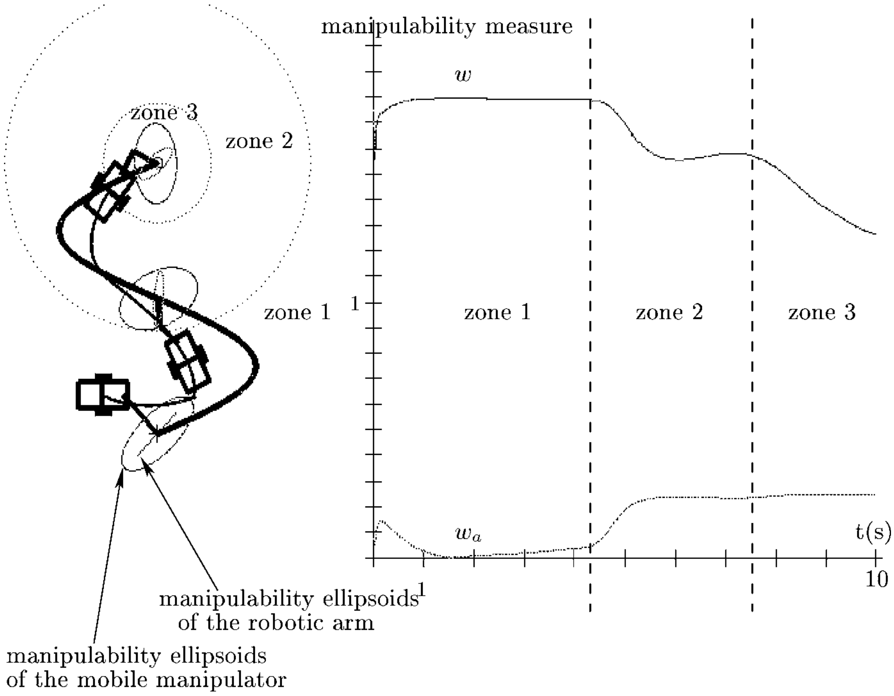

Figure 6. Convex combination of manipulability measures.

measures. Let us take the example of a mobile manipulator that must first realize a coordinated operational task and then manipulate in a narrow zone. In this latter zone, moving the platform may be unsuitable and it is interesting to manipulate only with the arm: in this case, it will be interesting to consider the sole arm manipulability. Influence of both tasks can be taken into account by using a function that writes:

$$\mathcal{P} = \alpha(\boldsymbol{\xi}^*)\widetilde{\mathcal{P}}_{p+a} + \big(1 - \alpha(\boldsymbol{\xi}^*)\big)\widetilde{\mathcal{P}}_a. \tag{7}$$

It is a convex combination of the arm manipulability measure $\widetilde{\mathcal{P}}_a$ and of that of the mobile manipulator $\widetilde{\mathcal{P}}_{p+a}$ where the function $\alpha(\boldsymbol{\xi}^*) \in [0\ 1]$ is a cubic polynomial in order to verify the 4 boundary conditions. This function allows us to adapt the criterion to the mobile manipulator configuration or to the EE location. *Thus, we do not use a multi-criteria function, but a transition from a criterion to another one.* Such a choice is illustrated by Figure 6.

The mobile manipulator moves in free space, from a control mode where its manipulability is taken into account to another control mode where the arm manipulability is taken into account. It can be remarked that the arm manipulability may be poor whereas the whole system keeps a good measure of manipulability (see Figure 6, zone 1). This underlines that the choice of the manipulability measure to be used is task-dependent.

8. Conclusion

This paper has introduced notions of kinematic and velocity redundancies concerning nonholonomic robotic systems. Kinematic redundancy characterizes the dimension of the space of solutions in inverse kinematics, whereas velocity redundancy characterizes the dimension of the space of solutions in velocity kinematics inversion. The latter notion is expressed in a quasi-velocities based formalism. In this way, kinematic and velocity redundancies are obtained by a rank computation of two matrices; the first one is the classical Jacobian matrix whereas the other one implicitly takes into account the nonholonomic constraint.

These notions are illustrated by two nonholonomic mobile manipulators built from a HILARE-like platform. They are also totally adapted to the definition of notions such that *self motion manifold* or *manipulability* in the nonholonomic case and allow to realize velocity control schemes with a reduced set of independent coordinates. In this frame, manipulability based control schemes have been proved useful for solving velocity redundancy when operational path or motion is imposed.

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