* $\phi_{VG}(u;\dots)$ and $\phi_{NIG}(u;\dots)$ are given in the paper.

Then for $t=1$ $\phi_{X_t^{(VG)}}(u,t;\dots)$ and $\phi_{X_t^{(NIG)}}(u,t;\dots)$

$\phi_{VG}(\cdot)$ and $\phi_{NIG}(\cdot)$ respectively.

$\Rightarrow \psi_{X^{(VG)}} = \log \phi_{VG}$, same for NIG

* $\varphi_{CIR}(u,t;\dots,1)$ is the CF of the integrated CIR process Y_t. Where the last parameter $y_0 = 1$.

* Finally we set $u = -i \cdot \psi_X(-i)$ in $\varphi(u,t;\dots,1)$, to calculate $\mathbb{E}[\exp(X_{Y_t})|y_0]$ in time t

* The main log-price process under Lévy Time Change is discretized by the formula given below, for $n=0,1,2,\dots$

$$S_{n\Delta t} = S_0 \frac{e^{(r-q)n\Delta t}}{\mathbb{E}[\exp(X_{Y_{n\Delta t}})|y_0]} \exp(X_{Y_{n\Delta t}})$$

and $X_{Y_{n\Delta t}}$ is available for $n=0,1,2,\dots$

$$
\begin{aligned}
\varphi_{CIR}(u,t;\kappa,\eta,\lambda,y_0) &= E[\exp(\mathrm{i}uY_t)|y_0] \\
&= \frac{\exp(\kappa^2\eta t/\lambda^2)\exp(2y_0\mathrm{i}u/(\kappa+\gamma\coth(\gamma t/2)))}{(\cosh(\gamma t/2)+\kappa\sinh(\gamma t/2)/\gamma)^{2\kappa\eta/\lambda^2}},
\end{aligned}
$$

where

$$
\gamma = \sqrt{\kappa^2 - 2\lambda^2\mathrm{i}u}.
$$

$$
\varphi_{\Gamma-OU}(u;t,\lambda,a,b,y_0) = E[\exp(\mathrm{i}uY_t)|y_0]
$$

$$
= \exp\left(\mathrm{i}uy_0\lambda^{-1}(1-e^{-\lambda t}) + \frac{\lambda a}{\mathrm{i}u-\lambda b}\left(b\log\left(\frac{b}{b-\mathrm{i}u\lambda^{-1}(1-e^{-\lambda t})}\right) - \mathrm{i}ut\right)\right)
$$

$$
\phi_{NIG}(u;\alpha,\beta,\delta) = \exp\left(-\delta\left(\sqrt{\alpha^2-(\beta+\mathrm{i}u)^2} - \sqrt{\alpha^2-\beta^2}\right)\right)
$$

$$
\phi_{VG}(u;C,G,M) = \left(\frac{GM}{GM+(M-G)\mathrm{i}u+u^2}\right)^C
$$