```
In[ ]:= x1 = r1 * Cos[θ1[t]];
        y1 = r1 * Sin[θ1[t]];
        x2 = L1 * Cos[θ1[t]] + r2 * Cos[θ1[t] + θ2[t]];
        y2 = L1 * Sin[θ1[t]] + r2 * Sin[θ1[t] + θ2[t]] ;

In[ ]:= x1dot = D[x1, t];

In[ ]:= y1dot = D[y1, t];
        x2dot = D[x2, t];
        y2dot = D[y2, t];

In[ ]:= P = m1 * g * y1 + m2 * g * y2;

In[ ]:= K = 0.5 * (m1 * (x1dot ^ 2 + y1dot ^ 2) + m2 * (x2dot ^ 2 + y2dot ^ 2)) +
           0.5 * ((I1 * (D[θ1[t], {t, 1}]) ^ 2) + (I2 * (D[θ2[t], {t, 1}]) ^ 2));

In[ ]:= L = K - P;

In[ ]:= term1 = D[Grad[L, {D[θ1[t], t], D[θ2[t], t]} ], t];(*d/dt(dL/dqdot)*)

In[ ]:= term2 = Grad[L, {θ1[t], θ2[t]}]; (*dL/dq*)

In[ ]:= Tau = term1 - term2 ;

In[ ]:= tau1 = TrigReduce [Tau⟦1⟧]

        g L1 m2 Cos[θ1[t]] + g m1 r1 Cos[θ1[t]] + g m2 r2 Cos[θ1[t] + θ2[t]] - 2.` L1 m2 r2 Sin[θ2[t]] θ1′[t] θ2′[t] -
           1.` L1 m2 r2 Sin[θ2[t]] θ2′[t]^2 + 1.` I1 θ1″[t] + 1.` L1^2 m2 θ1″[t] + 1.` m1 r1^2 θ1″[t] +
           1.` m2 r2^2 θ1″[t] + 2.` L1 m2 r2 Cos[θ2[t]] θ1″[t] + 1.` m2 r2^2 θ2″[t] + M =
          np.array ([[m1 * r1 ** 2 + m2 * r2 ** 2 + m2 * L1 ** 2 + 2 * m2 * L1 * r2 * np.cos (l_theta2 _[i]), m2 * L1 *
              r2 * np.cos (l_theta2 _[i])], [m2 * r2 ** 2 + m2 * L1 * r2 * np.cos (l_theta2 _[i]), m2 * r2 ** 2]])
        C = np.array ([[- m2 * L1 * r2 * l_theta2 _ddot[i] * np.sin (l_theta2 _[i]),
             - m2 * L1 * r2 * l_theta1 _ddot[i] * np.sin (l_theta2 _[i]) -
              m2 * L1 * r2 * l_theta2 _ddot[i] * np.sin (l_theta2 _[i])], [
             m2 * L1 * r2 * l_theta1 _ddot[i] * np.sin (l_theta1 _[i]), 0]])
        G = np.array ([(m1 * L1 + m2 * L1) * g * np.cos (l_theta1 _[i]) + m2 * r2 * g * np.cos
               (l_theta2 _[i] + l_theta1 _[i]), m2 * r2 * g * np.cos (l_theta2 _[i] + l_theta1 _[i])])
        1.`
         L1
         m2
         r2
         Cos[
          θ2[
           t]] θ2″[t]

In[ ]:= tau2 = TrigReduce [Tau⟦2⟧]

Out[ ]= g m2 r2 Cos[θ1[t] + θ2[t]] + 1. L1 m2 r2 Sin[θ2[t]] θ1′[t]^2 +
         1. m2 r2^2 θ1″[t] + 1. L1 m2 r2 Cos[θ2[t]] θ1″[t] + 1. I2 θ2″[t] + 1. m2 r2^2 θ2″[t]
```

In[ ]:=
```
poly = a*t^5+ b*t^4+c*t^3+d*t^2+e*t+f
```
Out[ ]=
$$f + e\,t + d\,t^2 + c\,t^3 + b\,t^4 + a\,t^5$$

In[ ]:=
```
polyVel = D[poly, t]
```
Out[ ]=
$$e + 2\,d\,t + 3\,c\,t^2 + 4\,b\,t^3 + 5\,a\,t^4$$

In[ ]:=
```
polyAcc = D[poly, {t, 2}]
```
Out[ ]=
$$2\,d + 6\,c\,t + 12\,b\,t^2 + 20\,a\,t^3$$

In[ ]:=
```
(*Solve[{Diff1==0,Diff2==0},{p,q}]*)
```

In[ ]:=
```
tau2
```
Out[ ]=
$$g\,m2\,r2\,\mathrm{Cos}[\theta1[t] + \theta2[t]] + 1.\,L1\,m2\,r2\,\mathrm{Sin}[\theta2[t]]\,\theta1'[t]^2 + 1.\,m2\,r2^2\,\theta1''[t] + 1.\,L1\,m2\,r2\,\mathrm{Cos}[\theta2[t]]\,\theta1''[t] + 1.\,I2\,\theta2''[t] + 1.\,m2\,r2^2\,\theta2''[t]$$

In[ ]:=
```
M = MatrixForm[
  {{I1+m1*r1^2+m2*r2^2+m2*L1^2+2 m2*L1*r2*Cos[θ2], m2*r2^2+m2*L1*r2*Cos[θ2]},
   {m2*r2^2+m2*L1*r2*Cos[θ2], m2*r2^2+I2}}]
```
Out[ ]//MatrixForm=
$$\begin{pmatrix} I1 + L1^2\,m2 + m1\,r1^2 + m2\,r2^2 + 2\,L1\,m2\,r2\,\mathrm{Cos}[\theta2] & m2\,r2^2 + L1\,m2\,r2\,\mathrm{Cos}[\theta2] \\ m2\,r2^2 + L1\,m2\,r2\,\mathrm{Cos}[\theta2] & I2 + m2\,r2^2 \end{pmatrix}$$

In[ ]:=
```
C1 =
 MatrixForm[{{-m2*2*L1*r2*D[θ2[t], {t, 1}]*Sin[θ2], -m2*L1*r2*D[θ2[t], {t, 1}]*Sin[θ2]},
   {m2*L1*r2*D[θ1[t], {t, 1}]*Sin[θ2], 0}}]
```
Out[ ]//MatrixForm=
$$\begin{pmatrix} -2\,L1\,m2\,r2\,\mathrm{Sin}[\theta2]\,\theta2'[t] & -L1\,m2\,r2\,\mathrm{Sin}[\theta2]\,\theta2'[t] \\ L1\,m2\,r2\,\mathrm{Sin}[\theta2]\,\theta1'[t] & 0 \end{pmatrix}$$

In[ ]:=
```
G = Simplify[
  MatrixForm[{{g *L1 *m2 *Cos[θ1[t]]+g *m1 *r1* Cos[θ1[t]]+g *m2 *r2 *Cos[θ1[t]+θ2[t]]},
    {g *m2* r2* Cos[θ1[t]+θ2[t]]}}]]
```
Out[ ]//MatrixForm=
$$\begin{pmatrix} g\,((L1\,m2 + m1\,r1)\,\mathrm{Cos}[\theta1[t]] + m2\,r2\,\mathrm{Cos}[\theta1[t] + \theta2[t]]) \\ g\,m2\,r2\,\mathrm{Cos}[\theta1[t] + \theta2[t]] \end{pmatrix}$$

In[ ]:=
```
MInv = Inverse[{{m11, m12}, {m21, m22}}]
```
Out[ ]=
$$\left\{\left\{\frac{m22}{-m12\,m21 + m11\,m22}, -\frac{m12}{-m12\,m21 + m11\,m22}\right\}, \left\{-\frac{m21}{-m12\,m21 + m11\,m22}, \frac{m11}{-m12\,m21 + m11\,m22}\right\}\right\}$$

In[ ]:=
```
m11 = M[[1, 1, 1]]
```
Out[ ]=
$$I1 + L1^2\,m2 + m1\,r1^2 + m2\,r2^2 + 2\,L1\,m2\,r2\,\mathrm{Cos}[\theta2]$$

In[ ]:=
```
m12 = M[[1, 1, 2]]
```
Out[ ]=
$$m2\,r2^2 + L1\,m2\,r2\,\mathrm{Cos}[\theta2]$$

```
In[ ]:= m21 = M[[1, 2, 1]]
Out[ ]= m2 r2^2 + L1 m2 r2 Cos[θ2]

In[ ]:= m22 = M[[1, 2, 2]]
Out[ ]= I2 + m2 r2^2

(*Dot[MInv ,{{tau11},{tau21}}], u=theta1dot ,v=theta2dot *)
```

Obtaining Differential Equations

```
In[ ]:= Solve[{k1 * D[u[t], t] + k2 * D[v[t], t] - k3 * u * v - k4 * v ^ 2 + k51 - th1 == 0 ,
           k5 * D[u[t], t] + k6 * D[v[t], t] - k7 * u ^ 2 + k8 - th2 == 0 }, {D[u[t], t], D[v[t], t]}]
Out[ ]= {{Derivative [1][u][t] →
           -(((- k51) * k6 + k2 * k8 + k6 * th1 - k2 * th2 - k2 * k7 * u ^ 2 + k3 * k6 * u * v + k4 * k6 * v ^ 2) /
              (k2 * k5 - k1 * k6)),
         Derivative [1][v][t] → -(((- k5) * k51 + k1 * k8 + k5 * th1 - k1 * th2 -
                 k1 * k7 * u ^ 2 + k3 * k5 * u * v + k4 * k5 * v ^ 2) / ((- k2) * k5 + k1 * k6))}}

In[ ]:= Collect[tau1 , {D[θ1[t], {t, 2}], D[θ2[t], t], D[θ1[t], t], D[θ2[t], t]}]
Out[ ]= g L1 m2 Cos[θ1[t]] + g m1 r1 Cos[θ1[t]] + g m2 r2 Cos[θ1[t] + θ2[t]] -
         2. L1 m2 r2 Sin[θ2[t]] θ1′[t] θ2′[t] - 1. L1 m2 r2 Sin[θ2[t]] θ2′[t]^2 +
         (1. I1 + 1. L1^2 m2 + 1. m1 r1^2 + 1. m2 r2^2 + 2. L1 m2 r2 Cos[θ2[t]]) θ1″[t] +
         1. m2 r2^2 θ2″[t] + 1. L1 m2 r2 Cos[θ2[t]] θ2″[t]

In[ ]:= Collect[tau2 , {D[θ1[t], {t, 2}], D[θ2[t], t], D[θ1[t], t], D[θ2[t], t]}]
Out[ ]= g m2 r2 Cos[θ1[t] + θ2[t]] + 1. L1 m2 r2 Sin[θ2[t]] θ1′[t]^2 +
         (1. m2 r2^2 + 1. L1 m2 r2 Cos[θ2[t]]) θ1″[t] + 1. I2 θ2″[t] + 1. m2 r2^2 θ2″[t]

In[ ]:= Solve[{k1 * D[u[t], t] + k2 * D[v[t], t] - k3 * u * v - k4 * v ^ 2 - th1 == 0 ,
            k5 * D[u[t], t] + k6 * D[v[t], t] - k7 * u * v - th2 == 0 }, {u , v}];
```