Trajectory Optimization with Free Space Constraints

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1 Introduction

Let $x_{max}(t), x_{min}(t)$ and $y_{max}(t), y_{min}(t)$ denote the boundary of the free space in the time interval $[t_0, t_f]$. The following trajectory optimization can be used for computing a smooth trajectory which lies within the characterized free space region.

$$\arg\min w_1(\sum_{i=0}^{i=n} \dddot{x}(t_i) + \dddot{y}(t_i)) + \sum_{i=0}^{i=n} \frac{(\ddot{y}(t_i)\dot{x}(t_i) - \ddot{x}(t_i)\dot{y}(t_i))^2}{(\dot{x}(t_i)^2 + \dot{y}(t_i)^2)^3} \tag{1a}$$

$$x(t_0) = x_0, x(t_f) = x_f, y(t_0) = y_0, y(t_f) = y_f \tag{1b}$$

$$x_{min} \leq x(t_i) \leq x_{max}, y_{min} \leq y(t) \leq y_{max}, \forall t_i \in [t_0, t_f] \tag{1c}$$

$$\dot{x}_{min} \leq \dot{x}(t_i) \leq \dot{x}_{max}, \dot{y}_{min} \leq \dot{y}(t_i) \leq \dot{y}_{max}, \forall t_i \in [t_0, t_f] \tag{1d}$$

$$\ddot{x}_{min} \leq \ddot{x}(t_i) \leq \ddot{x}_{max}, \ddot{y}_{min} \leq \dot{y}(t) \leq \ddot{y}_{max}, \forall t_i \in [t_0, t_f] \tag{1e}$$

1.1 Solving the Optimization

Let us parametrize $x(t)$, $y(t)$ through polynomials

$$x(t) = \sum_{k=0}^{k=n} a_k P_k(t), y(t) = \sum_{k=0}^{k=n} b_k P_k(t) \tag{2}$$

where, $P_k(t)$ are Bernstein polynomial basis defined over the time interval $[t_0, t_f]$ and is known. So the above optimization can be written in terms of parameters a_k, b_k.

Note on constraints 1c: We do not needs to have an analytical equation to describe the boundaries x_{max}, x_{min}, y_{max}, y_{min}. Each of these can be just an array/vector of points which provides a discrete representation of the boundary at different time instants. This in turn will be useful if the geometries of the boundaries are different over time. For example, if we have rectangles of different shapes as boundaries over time.

The optimization can be solved in fmincon. We can look at more efficient techniques later.