AMATH 301
Homework 3: Autumn 2021

DUE: Midnight on Monday, October 25

Download the file from the website link (the DATA link near the CODES link): **yalefaces.mat**. This file has a total of 39 different faces with about 65 lighting scenes for each face (2414 faces in all). The individual images are columns of the matrix $\mathbf{X}$, where each image has been downsampled to 32×32 pixels and converted into gray scale with values between 0 and 1. So the matrix is size 1024×2414. To important the file in MATLAB, use

```
load yalefaces.mat
```

For Python, use the following

```
import numpy as np
from scipy.io import loadmat
results=loadmat('yalefaces.mat')
X=results['X']
```

(a) Compute a 100×100 correlation matrix $\mathbf{C}$ where you will compute the dot product (correlation) between the first 100 images in the matrix $\mathbf{X}$. Thus each element is given by $c_{jk} = \mathbf{x}_j^T \mathbf{x}_k$ where $\mathbf{x}_j$ is the jth column of the matrix.
ANSWERS: Save the matrix $\mathbf{C}$ as A1

(b) From the correlation matrix for part (a), which two images are most highly correlated? (This will be A2). Which are most uncorrelated? (This should be A3).
ANSWERS: The row vector A2 should list two numbers: the first image number (j) and the second image number (k).
ANSWERS: The row vector A3 should list two numbers: the first image number (j) and the second image number (k).

(c) Repeat part (a) but now compute the 10×10 correlation matrix between images $[1, 313, 512, 5, 2400, 113, 1024, 87, 314, 2005]$.
(Just for clarification, the first image is labeled as one, not zero like python might do)
ANSWERS: Save the matrix as A4

(d) Create the matrix $\mathbf{Y} = \mathbf{X}\mathbf{X}^T$ and find the first six eigenvectors with the largest magnitude eigenvalue.
ANSWERS: Save the absolute value of the first six eigenvectors (6 columns and 1024 rows) as A5

(e) SVD the matrix $\mathbf{X}$ and find the first six principal component directions.
ANSWERS: Save the first six SVD modes (6 columns and 1024 rows) as A6

(f) Compare the first eigenvector $\mathbf{v}_1$ from (d) with the first SVD mode $\mathbf{u}_1$ from (e) and compute the norm of their difference.
ANSWERS: Save the norm difference $\||\mathbf{v}_1| - |\mathbf{u}_1|\|$ as A7

(g) Compute the percentage of variance captured by each of the first 6 SVD modes.
ANSWERS: Save the results as a row vector with six components in A8