AMATH 301
Homework 6: Autumn 2021

DUE: Midnight on Wednesday, November 24

Consider Fourier mode expansions on the domain $x \in [0, 2\pi]$ (divide the domain into 200 points) which take the following forms:

(a) Construct the sine basis $\psi_n = c_n \sin(nx/2)$ where c_n is a normalization constant in order to make an orthonormal basis. Specifically, construct a matrix A1 with 10 columns (and 200 rows) of orthonormal vectors corresponding to $n = 1, 2, \cdots 10$. Compute the correlation matrix A2=A1^{T}A1. (check to see that A2 looks like the identity).

ANSWER: Should be written out as matrices A1 and A2 respectively.

(b) Construct the cosine basis $\phi_n = c_n \cos(nx/2)$ where c_n is a normalization constant in order to make an orthonormal basis. Specifically, construct a matrix A3 with 10 columns (and 200 rows) of orthonormal vectors corresponding to $n = 0, 1, \cdots 9$. Compute the correlation matrix A4=A3^{T}A3 (check to see that A4 looks like the identity).

ANSWER: Should be written out as matrices A3 and A4 respectively.

(c) Compute the correlation matrix between the sine and cosine basis A5=A1^{T}A3 (check to see that A5 does not look like the identity).

ANSWER: Should be written out as matrix A5.

(d) Using the sine basis A1, approximate the function $f(x) = \cos(x)$. Specifically, compute the norm of the error between the function and your approximation

$$E(n) = \text{norm}\left(\cos(x) - \sum_{j=1}^{n} a_n \psi_n\right)$$

for $n = 1, 2, \cdots n$. Note that the ψ_n are the columns of A1. So you just need to compute the projection of each mode onto the function $f(x)$, i.e. the a_n is the dot product of $f(x)$ with ψ_n.

ANSWER: Save the 1 by 10 vector **E** as A6.