AMATH 301
Midterm 1: Autumn 2021

DUE: Saturday, 10am PST, October 30, 2021.

NOTE: You get five attempts, you are not allowed to talk to others, it is open book and open notes, you must submit MATLAB or Python code that is executed by gradescope to produce the answers below.

I Let the following be defined:

$$\mathbf{A}=\begin{bmatrix} 8 & 2 & 1 \\ -1 & 5 & 1 \\ 1 & 1 & 4 \end{bmatrix}, \mathbf{B}=\begin{bmatrix} 1 & 1 & 4 \\ -1 & 5 & 1 \\ 8 & 2 & 1 \end{bmatrix},$$

Given the equations $\mathbf{Ax} = \mathbf{b}$ and $\mathbf{Bx} = \mathbf{b}$ where $\mathbf{b} = [1\ 2\ 0]^T$, construct the following iteration algorithms for trying to solve the equations (given an initial guess $\mathbf{x}_1 = [0\ 0\ 0]^T$).

(a) Use Jacobi iterations to compute the first six iterations ($\mathbf{x}_2$ to $\mathbf{x}_7$) for $\mathbf{Ax} = \mathbf{b}$

(b) Use Gauss-Seidel iterations to compute the first six iterations ($\mathbf{x}_2$ to $\mathbf{x}_7$) for $\mathbf{Ax} = \mathbf{b}$

(c) Use Jacobi iterations to compute the first six iterations ($\mathbf{x}_2$ to $\mathbf{x}_7$) for $\mathbf{Bx} = \mathbf{b}$

(d) Use Gauss-Seidel iterations to compute the first six iterations ($\mathbf{x}_2$ to $\mathbf{x}_7$) for $\mathbf{Bx} = \mathbf{b}$

NOTE: $\mathbf{A}$ is strictly diagonal dominant and the iterations will converge, while $\mathbf{B}$ is not and the iterations will diverge.

ANSWERS: Should be 3 rows, 6 columns ($\mathbf{x}_2$ to $\mathbf{x}_7$) written as matrices (a) A1, (b) A2, (c) A3, (d) A4.

II Download **yalefaces.mat** where the individual images are columns of the matrix $\mathbf{X}$, where each image has been downsampled to 32×32 pixels and converted into gray scale with values between 0 and 1. So the matrix is size 1024×2414. (See Homework 3 details)

(a) Compute a 3×3 correlation matrix $\mathbf{C}$ where you will compute the dot product (correlation) between images 37, 532 and 1713 in the matrix $\mathbf{X}$. Thus each element is given by $c_{jk} = \mathbf{x}_j^T \mathbf{x}_k$ where $\mathbf{x}_j$ is the jth column of the matrix. (REMEMBER: In Python the image number is different than the index since python starts to count from zero)

ANSWERS: Save the 3 by 3 matrix $\mathbf{C}$ as A5

(b) Compute the eigenvectors and eigenvalues of $\mathbf{C}$ and sort them from the smallest to largest magnitude of the eigenvalues. (NOTE: You may have to sort them in Python, but it is only a 3 by 3)

ANSWERS: Save the sorted eigenvalues of $\mathbf{C}$ into a 1 by 3 vector A6

ANSWERS: Save the absolute values of the sorted eigenvectors into a 3 by 3 matrix A7

(c) Find the projection of images 37, 532 and 1713 unto the fifth SVD mode of the data matrix $\mathbf{X}$. That is, compute the inner product of each image with the fifth column of $\mathbf{U}$.

ANSWERS: Save a 1 by 3 vector of the absolute value of the projections as A8