AMATH 301
Homework 1: Autumn 2021

DUE: Midnight on Thursday, October 7, 2021.

I The following expressions all result in zero:

$$1000 - \sum_{i=1}^{10000} 0.1\,, \qquad 10000 - \sum_{i=1}^{100000} 0.1\,, \qquad 100000 - \sum_{i=1}^{1000000} 0.1$$

Write an algorithm to compute each of the above repeated subtractions and compare the answer to the exact answer of zero (i.e. calculate the Absolute Error).

ANSWERS: Should be written out as A1–A3

II Let the following be defined:

$$\mathbf{A}=\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}, \mathbf{B}=\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \mathbf{C}=\begin{bmatrix} 2 & 0 & -3 \\ 0 & 0 & -1 \end{bmatrix}, \mathbf{D}=\begin{bmatrix} 1 & 2 \\ 2 & 3 \\ -1 & 0 \end{bmatrix} \mathbf{x}=\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \mathbf{y}=\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{z}=\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix},$$

Calculate the following:

(a) $\mathbf{A}+\mathbf{B}$, (b) $3\mathbf{x} - 4\mathbf{y}$, (c) $\mathbf{A}\mathbf{x}$, (d) $\mathbf{B}(\mathbf{x}\text{-}\mathbf{y})$, (e) $\mathbf{D}\ \mathbf{x}$, (f) $\mathbf{D}\ \mathbf{y} + \mathbf{z}$, (g) $\mathbf{A}\mathbf{B}$, (h) $\mathbf{B}\mathbf{C}$, (i) $\mathbf{C}\mathbf{D}$

ANSWERS: Should be written out as A4–A12

III Consider the logistic equation

$$x_{n+1} = \rho x_n(1 - x_n)$$

which was first developed to model the growth and decay of a population of some species. Iterate the equation for the following values of ρ with $x_1 = 0.5$:

$$\rho = 0.8, 1.5, 2.8, 3.2, 3.5, 3.65$$

Iterate the equation for each ρ value and calculate six column vectors (one for each ρ value) of length 50 which contains $x(1)$ to $x(50)$ (or in python $x(0)$ to $x(49)$).

ANSWERS: Should be written out as A13–A18