18.01 Single Variable Calculus
Fall 2006

18.01 Problem Set 1

Due Friday 9/15/06, 1:55 pm

18.01/18.01A Supplementary Notes, Exercises and Solutions are for sale. This is where to find the exercises labeled 1A, 1B, etc. You will need these for the first day's homework.

Part I consists of exercises given in the Notes and solved in section S of the Notes. It will be graded quickly, checking that all is there and the solutions not copied.
Part II consists of problems for which solutions are not given; it is worth more points. Some of these problems are longer multi-part exercises posed here because they do not fit conveniently into an exam or short-answer format. See the guidelines below for what collaboration is acceptable, and follow them.

To encourage you to keep up with the lectures, both Part I and Part II tell you for each problem on which day you will have the needed background for it.

Part I (30 points)

Notation: 2.1 = Section 2.1 of the Simmons book; Notes G = section G of the Notes;
1A-3 = Exercise 1A-3 in Section E (Exercises) of the Notes (solved in section S)
2.4/13; 81/4 = in Simmons, respectively, section 2.4 Problem 13; page 81 Problem 4

Recitation 0. Wed. Sept. 6 Graphing functions.
Read: Notes G, sections 1-4 Work: 1A-1a, 2a, 3abe, 6a, 7a

Lecture 1. Thurs., Sept. 7 Derivative; slope, velocity, rate of change.
Read: 2.1-2.4 Work: 1C-3abe, 4ab (use 3), 5, 6 (trace axes onto your answer sheet)
Work: 1B-2, 1C-1a (start from the definition of derivative)

Lecture 2. Fri. Sept. 8 Limits and continuity; some trigonometric limits
Read: 2.5 (bottom p.70-73; concentrate on examples, skip the $\epsilon - \delta$ def'n)
Read: 2.6 to p. 75; learn def'n (1) and proof "differentiable $\Longrightarrow$ continuous" at the end.
Read: Notes C Work: 1D-1bcefg, 4a; 1C-2, 1D-3ade, 6a, 8a (hint: "diff $\Longrightarrow$ cont.")

Lecture 3. Tues. Sept. 12 Differentiation formulas: products and quotients;
Derivatives of trigonometric functions.
In the exercises, an *antiderivative* of $f(x)$ is any $F(x)$ for which $F'(x) = f(x)$.
Read: 3.1, 3.2, 3.4 Work: 1E-1ac, 2b, 3, 4a, 5a; 1J-1e, 2

Lecture 4. Thurs. Sept. 14 Chain rule; higher derivatives.
Read: 3.3, 3.6 Work: 1F-1ab, 2, 6, 7bd; 1J- 1abm 1G-1b, 5ab

Lecture 5. Fri. Sept. 15 Implicit differentiation; inverse functions.
Read: 3.5, Notes G section 5 Work: given on Problem Set 2.

Part II (40 points)

Directions and Rules: Collaboration on problem sets is encouraged, **but**

a) Attempt each part of each problem yourself. Read each portion of the problem before asking for help. If you don't understand what is being asked, ask for help interpreting the problem and then make an honest attempt to solve it.

b) Write up each problem independently. On both Part I and II exercises you are expected to write the answer in your own words.

c) **Write on your problem set whom you consulted and the sources you used.** If you fail to do so, you may be charged with plagiarism and subject to serious penalties.

d) It is illegal to consult materials from previous semesters.

0. (not until due date; 3 points)

Write the names of all the people you consulted or with whom you collaborated and the resources you used, or say "none" or "no consultation". This includes visits outside recitation to your recitation instructor. If you don't know a name, you must nevertheless identify the person, as in, "tutor in Room 2-106," or "the student next to me in recitation." Optional: note which of these people or resources, if any, were particularly helpful to you.

This "Problem 0" will be assigned with every problem set. Its purpose is to make sure that you acknowledge (to yourself as well as others) what kind of help you require and to encourage you to pay attention to how you learn best (with a tutor, in a group, alone). It will help us by letting us know what resources you use.

1. (Wed, 3 pts) Express $(x-1)/(x+1)$ as the sum of an even and an odd function. (Simplify as much as possible.)

2. (Thurs, 6 pts: 3 + 3) Sensitivity of measurement: Suppose f is a function of x. If $x = x_0 + \Delta x$, then we define $\Delta f = f(x) - f(x_0)$ and $\Delta f/\Delta x$ measures how much changes in x affect the value of f.

The planet Quirk is flat. GPS satellites hover over Quirk at an altitude of $20,000$ km (unlike Earth where the satellites circle twice a day). See how accurately you can estimate the distance L from the point directly below the satellite to a point on the planet surface knowing the distance h from the satellite to the point on the surface in two cases. (The letter h is for hypotenuse.)

a) Use a calculator to compute $\Delta L/\Delta h$ for $h = h_0 \pm \Delta h = 25,000 \pm \Delta h$, and $\Delta h = 1,\ 10^{-1},\ 10^{-2}$. Write an estimate for L in the form

$$|L - L_0| = |\Delta L| \leq C|\Delta h|$$

choosing the simplest round number C that works for all three cases.

b) Do the same for $h = 20,001 \pm \Delta h$, $\Delta h = 1,\ 10^{-1},\ 10^{-2}$. Is the value of L estimated more or less accurately than in part (a)? We will revisit this problem more systematically using calculus.

3. (Thurs, 4pts) On the planet Quirk, a cell phone tower is a 100-foot pole on top of a green mound 1000 feet tall whose outline is described by the parabolic equation $y = 1000 - x^2$. An ant climbs up the mound starting from ground level ($y = 0$). At what height y does the ant begin to see the tower?

4. (Thurs, 6 pts) 3.1/21 (parabolic mirrors)

5. (Thurs, 4pts: 2 + 2)

a) A water cooler is leaking so that its volume at time t in minutes is $(10 - t)^2/5$ liters. Find the average rate at which water drains during the first 5 minutes.

b) At what rate is the water flowing out 5 minutes after the tank begins to drain.

6. Friday (8 pts: 1 + 1 + 1 + 1 + 1 + 1 + 2) 2.5/19d (put $u = 1/x$), 19f, 19g, 20c, 20g (show work); 22a (needs a calculator), 22b (see the proof on page 73).

7. Tuesday (6 pts: 2 + 4)

a) If u, v and w are differentiable functions, find the formula for the derivative of their product, $D(uvw)$.

b) Generalize your work in part (a) by guessing the formula for $D(u_1 u_2 \cdots u_n)$—the derivative of the product of n differentiable functions.

Then prove your formula by mathematical induction (i.e., prove its truth for the product of $n+1$ functions, assuming its truth for the product of n functions).