The GPS/INS Integrated Navigation Method Based on Adaptive SSR-SCKF Cubature Kalman Filter

Zhe Yue, Baowang Lian and Chengkai Tang

Abstract There are many methods aiming at the nonlinear problem of GPS/INS integrated navigation, such as EKF and UKF, however, these methods have low positioning accuracy and instability. On the study of SCKF and nonlinear model of GPS/INS integrated navigation, aiming at the issues that the state equation of GPS/INS is nonlinear while the measured equation is linear, and the measured noise changes owing to the changing number of visible satellites or multipath. Therefore, this paper promotes the integrated method based on adaptive SSR-SCKF, which uses the spherical simple-radial cubature rule (SSRCR) to set the cubature sampling points. We also provide a linear measured update process on the basis of singular value decomposition (SVD), and it avoids choosing the cubature sampling points. Combining the moving window method, it can adjust the covariance matrix of measurement noise in real-time. The experiment results show that the proposed method has lower computational complexity, while higher estimated accuracy, numerical stability and better adaptive ability to the changing noise than SCKF in the same conditions.

Keywords GPS/INS integrated navigation · SSR-SCKF · Singular value decomposition · Adaptive

1 Introduction

GPS/INS integrated navigation system usually uses the Kalman filter to estimate the system [1], and the estimation method assumes that the equation is linear, however, the GPS/INS integrated navigation system is nonlinear, therefore, the nonlinear

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model can more completely spread the feature of system error [2]. Aiming to the nonlinear model, we usually use the extend kalman filter (EKF) to make the nonlinear equation [3]. However, the linear process results in relatively higher truncation error, and the EKF also require calculating the complex Jacobi matrix. The unscented kalman filter (UKF) uses the unscented transformation (UT) to make Gaussian approximation for the probability density function [3, 4], but if the dimension is more than three, i.e., $n \geq 4$, the suggested tuning parameter will less than zero, i.e., $k = 3 - n < 0$, this phenomenon may make the covariance matrix negative and break the filter. Arasaratnam [5] firstly proposes the cubature kalman filter (CKF), and employs the three-order spherical-radial cubature rule to transform the integral calculation of the Gauss weighted multidimensional nonlinear function in cartesian coordinates to the cubature approximation, and it has higher estimated accuracy than UKF, in addition, the weights of the sampling points of CKF are the same and positive, and the numerical stability is better than UKF. It is said that the CKF has the highest estimated accuracy in the Gauss filters [6]. The superior nonlinear performance of CKF attracts more and more scholars in the field of GPS/INS integrated navigation [7–9]. The existing integrated navigation methods based on CKF usually utilize three order spherical-radial cubature rule to choose the cubature points in the state prediction and measured update process, and the calculation is complex and the numerical stability is not high. Additionally, there is little research on the adaptive CKF about the changing measured noise, owing to the changing number of the visible satellites or the multipath.

Thus we promote the GPS/INS integrated navigation method of the adaptive spherical simplex-radial square-root cubature filter (SSR-SCKF) based on the spherical simplex-radial cubature rule (SSRCR). This method utilizes the SSRCR instead of the three-order spherical-radial cubature rule to achieve higher estimated accuracy. The measured equation of GPS/INS loosely integrated navigation is linear, thus we promote a simple filter based on singular value decomposition (SVD), which reduces the computation complexity, because it does not need to calculate the sampling points in the measured update. Moreover, the SVD can also avoid interruption when the new information covariance matrix cannot be inversed, and it also improves the numerical stability of the method. Meanwhile, in order to coping with the changing measured noise, we use the moving window method to estimate the measured noise covariance matrix in real-time, thus can greatly improve the adaptive ability and estimated accuracy of the changing noise. The experiment shows that, under the same conditions, the method we proposed has greater estimated accuracy, higher numerical stability and lower computational complexity than SCKF. When the measured noise changed, this method has better adaptive ability, and we also prove the superiorities of the method in this paper.

2 The Nonlinear Model of GPS/INS Integrated Navigation

In this paper, we set the east-north-up coordinate as the navigation coordinate of INS and the right-forward-up coordinate as the body coordinate (*b* coordinate). The idea navigation coordinate is called *n* coordinate, and the calculated navigation coordinate is named n' coordinate. The front right coordinate system of the moving object is the carrier coordinate system, i.e., the *b* system, and the geocentric inertial coordinate system is *i* system, and the earth coordinate named *e* coordinate, we also choose the navigation error as the state variables of the filter. The GPS/INS integrated navigation system is loosely, it utilizes the errors of attitude, velocity, positioning, gyroscope drift and constant bias of accelerometer as the 15 error state variables of the filter. The nonlinear error model of the attitude error, velocity error and positioning error are expressed as the follows:

$$\begin{cases} \dot{\phi} = C_{\omega}^{-1}\left(\left(I - C_n^{n'}\right)\hat{\omega}_{in}^n + C_n^{n'}\delta\omega_{in}^n - C_b^{n'}\delta\omega_{ib}^b\right) \\ \delta\dot{v} = \left(I - C_{n'}^n\right)C_b^{n'}\hat{f}_b + C_b^n\delta f_b \\ \qquad -\left(2\hat{\omega}_{ie}^n + \hat{\omega}_{en}^n\right)\times\delta v - \left(2\delta\omega_{ie}^n + \delta\omega_{en}^n\right)\times v \\ \delta\dot{L} = \frac{1}{R_M + h}\delta v_N - \frac{v_N}{(R_M + h)^2}\delta h \\ \delta\dot{\lambda} = \frac{\sec L}{R_N + h}\delta v_E + \frac{v_E \sec L \tan L}{R_N + h}\delta L - \frac{v_E \sec L}{(R_N + h)^2}\delta h \\ \delta\dot{h} = \delta v_U \\ \dot{\varepsilon}^b = 0 \\ \dot{\nabla}^b = 0 \end{cases} \tag{1}$$

$$C_{\omega}^{-1} = \frac{1}{\cos\phi_E}\begin{bmatrix} \cos\phi_N\cos\phi_E & 0 & \sin\phi_N\cos\phi_E \\ \sin\phi_N\sin\phi_E & \cos\phi_E & -\cos\phi_N\sin\phi_E \\ -\sin\phi_N & 0 & \cos\phi_N \end{bmatrix} \tag{2}$$

$$C_n^{n'} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \tag{3}$$

$$\begin{cases} a_{11} = \cos\phi_N\cos\phi_U - \sin\phi_E\sin\phi_N\sin\phi_U \\ a_{12} = \cos\phi_N\sin\phi_U - \sin\phi_E\sin\phi_N\cos\phi_U \\ a_{13} = -\cos\phi_E\sin\phi_N \\ a_{21} = -\cos\phi_E\sin\phi_U \\ a_{22} = \cos\phi_E\cos\phi_U \\ a_{23} = \sin\phi_E \\ a_{31} = \sin\phi_N\cos\phi_U + \sin\phi_E\cos\phi_N\sin\phi_U \\ a_{32} = -\sin\phi_E\cos\phi_N\cos\phi_U + \sin\phi_N\sin\phi_U \\ a_{33} = \cos\phi_E\cos\phi_N \end{cases} \tag{4}$$

$$\begin{cases} \hat{\omega}_{in}^{n} = \omega_{in}^{n} + \delta\omega_{in}^{n} \\ \omega_{in}^{n} = \omega_{ie}^{n} + \omega_{en}^{n} \\ \delta\omega_{in}^{n} = \delta\omega_{ie}^{n} + \delta\omega_{en}^{n} \\ \omega_{ie}^{n} = [0 \quad \omega_{ie}\cos L \quad \omega_{ie}\sin L]^{T} \\ \omega_{en}^{n} = \left[\frac{-v_N}{R_M+h} \quad \frac{v_E}{R_N+h} \quad \frac{v_E \tan L}{R_N+h}\right]^{T} \\ \delta\omega_{ie}^{n} = [0 \quad -\omega_{ie}\sin L \cdot \delta L \quad \omega_{ie}\cos L \cdot \delta L]^{T} \\ \delta\omega_{en}^{n} = \begin{bmatrix} -\frac{\delta v_N}{R_M+h} + \frac{v_N \delta h}{(R_M+h)^2} \\ \frac{\delta v_E}{R_N+h} - \frac{v_E \delta h}{(R_N+h)^2} \\ \frac{\tan L \cdot \delta v_E}{R_N+h} + \frac{v_E \sec^2 L \cdot \delta L}{R_N+h} - \frac{v_E \tan L \cdot \delta h}{(R_N+h)^2} \end{bmatrix} \end{cases} \tag{5}$$

$\phi = [\phi_E \quad \phi_N \quad \phi_U]^T$ denotes the misalignment angles of east, north and up-vertical direction, $\delta v = [\delta v_E \quad \delta v_N \quad \delta v_U]$ indicates the velocity errors of east, north and up-vertical direction, $\delta L, \delta\lambda, \delta h$ represent the latitude, longitude and altitude errors, respectively. $C_n^{n'}$ is the coordinate transformation matrix from the idea navigation coordinate to the calculated navigation coordinate. $\hat{\omega}_{in}^{n}$ means the value which the sum of the earth rotation rate ω_{ie}^{n} and the navigation coordinate rotation rate ω_{en}^{n} in the calculated navigation coordinate. $\delta\omega_{in}^{n}, \delta\omega_{ie}^{n}, \delta\omega_{en}^{n}$ are the calculated errors of $\hat{\omega}_{in}^{n}, \hat{\omega}_{ie}^{n}, \hat{\omega}_{en}^{n}$, respectively. L and h represent the latitude and altitude. R_M and R_N suggest the radius of the meridian and prime vertical of the earth. The gyro error $\delta\omega_{ib}^{b}$ is mainly include the gyro constant bias ε^b and the zero mean Gauss white noise w_g^b. The accelerometer error δf_b is mainly include the accelerometer constant bias ∇^b and the zero mean Gauss white noise w_a^b.

We choose the state vector as Eq. (6):

$$X = [\phi_E \quad \phi_N \quad \phi_U \quad \delta v_E \quad \delta v_N \quad \delta v_U \quad \delta L \quad \delta\lambda \quad \delta h \quad \varepsilon_E^b \quad \varepsilon_N^b \quad \varepsilon_U^b \quad \nabla_E^b \quad \nabla_N^b \quad \nabla_U^b] \tag{6}$$

Then, discretizing Eq. (1) we can obtain the GPS/INS state equation. When take the positioning and velocity errors difference between GPS and INS into consideration, we can get the measurement equation as the follows.

$$Z = HX + V \tag{7}$$

$$Z = [\delta v_E \quad \delta v_N \quad \delta v_U \quad \delta L \quad \delta\lambda \quad \delta h]^T \tag{8}$$

$$H = [0_{6\times3} \quad I_{6\times6} \quad 0_{6\times6}] \tag{9}$$

where V denotes the measured noise.

3 Filtering Method

3.1 The Filtering Method of SCKF

In the filtering method of SCKF, we take the following nonlinear equation into consideration.

$$x_k = f(x_{k-1}) + w_{k-1} \tag{10}$$

$$z_k = h(x_k) + v_k \tag{11}$$

where x_k is the unobservable state vector, z_k is the observable one. $f(\cdot)$ and $h(\cdot)$ represent the known nonlinear state and measured equation. The process noise w_{k-1} and measured noise v_k are white, and they are irrelevant, the mean of them is zero, the covariance matrixes are Q_{k-1} and R_k, respectively. The sampling points and weights calculated by the three-order spherical-radial cubature rule can describe as:

$$\begin{cases} \xi_i = \bar{x} + \sqrt{nP_x}e_i \\ \omega_i = \dfrac{1}{2n} \\ \xi_{i+n} = \bar{x} + \sqrt{nP_x}e_i \\ \omega_{i+n} = \dfrac{1}{2n} \end{cases} i = 1, 2, \ldots, n \tag{12}$$

where e_i is the unit vector of the ith element which is 1. The update steps of the SCKF can be found in [2].

3.2 The Filtering Method of Adaptive SSR-SCKF

In order to get the higher estimated accuracy of the nonlinear equation, this paper uses the SSRCR to choose the sampling points and weights as:

$$S_k^T = qr\left(\left[X'_{k/k-1} - G_k Z_{k/k-1} \;\; G_k S_{R,k}\right]^T\right) \tag{13}$$

where $\left[a_{i,1}, a_{i,2}, \ldots, a_{i,n}\right]^T$, $i = 1, 2, \ldots, n+1$, is the column vector, which is composed with n + 1 vertexes of the n-spherical simplex and can be described as:

$$a_{i,j} = \begin{cases} -\sqrt{\frac{n+1}{n(n-j+2)(n-j+1)}}, & j < i \\ \sqrt{\frac{(n+1)(n-j+1)}{n(n-j+2)}}, & j = i \\ 0, & j > i \end{cases} \tag{14}$$

The initialization module and measured update module are the same as the SCKF except that the sampling rules, it uses the (13) to replace Eq. (12). Aiming at the measured equation of the GPS/INS loosely integrated navigation, we have no necessary to calculate the complex 2n cubature points in the measured update module. In order to improve the numerical stability and avoid the interruption when the new information covariance matrix does not have the inversion, this paper proposes a simple measured update process based on SVD.

A is the square-root matrix of the covariance matrix P. We can uses the SVD to $A_{m\times n}$, when $m<n$, $A = U[D\ \ 0]V^T$, U and V are unitary matrix, D is diagonal matrix. P^{-1} can be described as:

$$\begin{aligned} P^{-1} &= \left(AA^T\right)^{-1} \\ &= U\left(DD^T\right)^{-1}U^T \end{aligned} \tag{15}$$

The inversion of P can be described as (15), and we only need to calculate the inversion of diagonal element to get the inverse matrix of DD^T. Using the SVD can reduce the computation complexity and improve the numerical stability.

The measured update process based on SVD can be described as:

$$[U, D, V] = svd\left(\left[H_k S_{k/k-1}\ \ S_{R,k}\right]\right) \tag{16}$$

$$S_{zz,k/k-1} = UD \tag{17}$$

$$P_{xz,k/k-1} = S_{k/k-1} S_{k/k-1}^T H_k^T \tag{18}$$

$$G_k = P_{xz,k/k-1} P_{zz,k/k-1}^{-1} = P_{xz,k/k-1} U\left(DD^T\right)^{-1} U^T \tag{19}$$

$$\hat{x}_k = \hat{x}_{k/k-1} + G_k\left(z_k - \hat{z}_{k/k-1}\right) \tag{20}$$

$$S_k^T = qr\left(\left[S_{k/k-1} - G_k H_k S_{k/k-1}\ \ G_k S_{R,k}\right]^T\right) \tag{21}$$

The measured noise will change owing to the changing number of visible satellites and multipath, if we use the measured noise covariance matrix which is set by experience, it will result in relatively bigger estimated errors. Thus, in this paper we add the moving window method to the SSR-SCKF to change the measured noise covariance matrix adaptively.

We employ the new measured vector z_k and the $\hat{x}_{k/k-1}$ of the predicted module to define the new measured vector.

$$y_k = z_k - h\left(\hat{x}_{k/k-1}\right) = z_k - H_k \hat{x}_{k/k-1} \tag{22}$$

Therefore, the covariance matrix of the new measured vector is

$$R_{y_k} = R_k + H_k P_{k/k-1} H_k^T = S_k S_k^T + H_k S_{k/k-1} S_{k/k-1}^T H_k^T \tag{23}$$

The R_{y_k} can be received by the mean value in the length of the window with the moving window method.

$$R_{y_k} = \frac{1}{N} \sum_{j=0}^{N} y_{k-j} y_{k-j}^T, \quad k > N \tag{24}$$

The square-root of the measured noise covariance matrix can be received by the moving window method through (23) and (24).

$$S_{R,k} = chol\left(R_{y_k} - H_k S_{k/k-1} S_{k/k-1}^T H_k^T\right) \tag{25}$$

When taking $S_{R,k}$ into the (16) and (21) of the SSR-SCKF, we can obtain the GPS/INS integrated navigation method based on adaptive SSR-SCKF cubature kalman filter.

4 Simulation and Verification

The simulation parameters are set as the follows: the initial position of the body is $(L, \lambda, h) = (34.246°, 108.910°, 380\,\mathrm{m})$, the gyro drift is $0.1°/h$, the random walk of gyro is $0.05°/\sqrt{h}$, the constant bias of the accelerometer is $100\,\mu\mathrm{g}$, the random walk of accelerometer is $50\,\mu\mathrm{g}/\sqrt{\mathrm{Hz}}$, the initial positioning error is $(10, 10, 10)$ m/s, the initial velocity error is $(0.5, 0.5, 0.5)$ m/s, the initial attitude error is $(2°, -2°, 3°)$, the position noise of GPS is $(10, 10, 10)$ m, and the velocity noise is $(0.1, 0.1, 0.1)$ m/s. The flight path of the body is include acceleration, uniform speed, left turn, right turn, climb, descent, deceleration and static. The total time of the simulation is 600 s, the flight path is in Fig. 1. The sample frequency of INS is 100 Hz, and the sample frequency of GPS is 1 Hz.

The positioning and velocity error of SCKF and the method of this paper can be seen in Figs. 2 and 3.

From Figs. 2 and 3, we can conclude that the two methods both have relatively good integrated navigation performance, and the square-root errors are shown in Table 1. From the statistic results of Table 1, it can be seen that the method proposed in this paper is better than SCKF both in the positioning and velocity accuracy. The reason is that the sampling rule of this paper is SSRCR, not the three-order spherical-radial cubature rule as the SCKF. And in the measured update process, SCKF require to calculate the 30 cubature sampling points to make the cubature approximation each time, but the method of this paper do not need to, therefore it reduces the computation complexity.

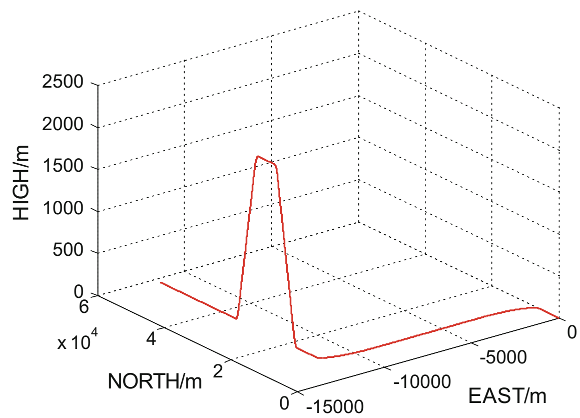

Fig. 1 Flight path

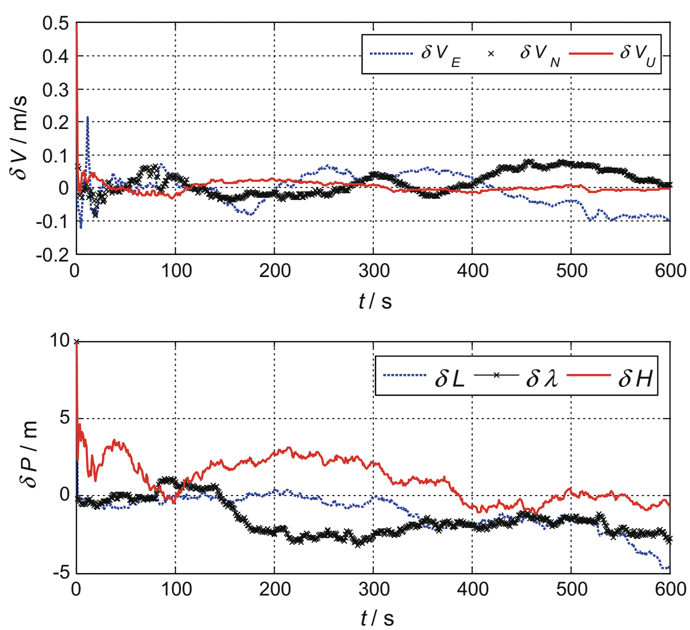

Fig. 2 Position and velocity error of SCKF

The measured noise will change owing to the changing numbers of the visible satellite and multipath, so in order to verify the adaptive ability to the changing measured noise of the method proposed in this paper, we set three different noise in the three stages. The first 200 s the measured noise of position and velocity are $(10, 10, 10)$ m and $(0.1, 0.1, 0.1)$ m/s, respectively. Then in the range of 200–400 s,

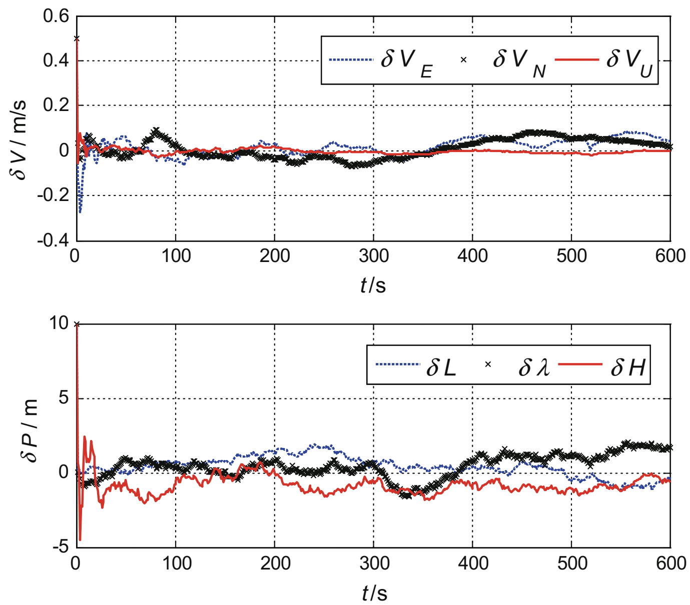

Fig. 3 Position and velocity error of the method of this paper

Table 1 The comparison of root mean square error of SCKF of two methods

	Positioning error (m)	Velocity error (m/s)	The number of sampling points in each update time
SCKF	3.0238	0.0732	60
The method of this paper	1.7307	0.0712	32

the noise becomes the three times to the initial noise, and the last 200 s the noise becomes 2 times to the initial noise. The simulation diagram of the position and velocity errors of SCKF and this paper are in Figs. 4 and 5.

From Fig. 4, we can suggest that, the filtering performance of SCKF is severely affected when the measured noise changes. Through Fig. 5 we can conclude that, under the same conditions, the method of this paper has better adaptive ability. The statistic results of RMS errors are in Table 2. From Table 2, we can also see that when the measured noise changes, the position and velocity errors of SCKF become bigger, because the measured noise does not update correspondingly, The method of this paper can adjust the covariance matrix of the measured noise in real-time, so it can receive the better accuracy and the better anti interference ability.

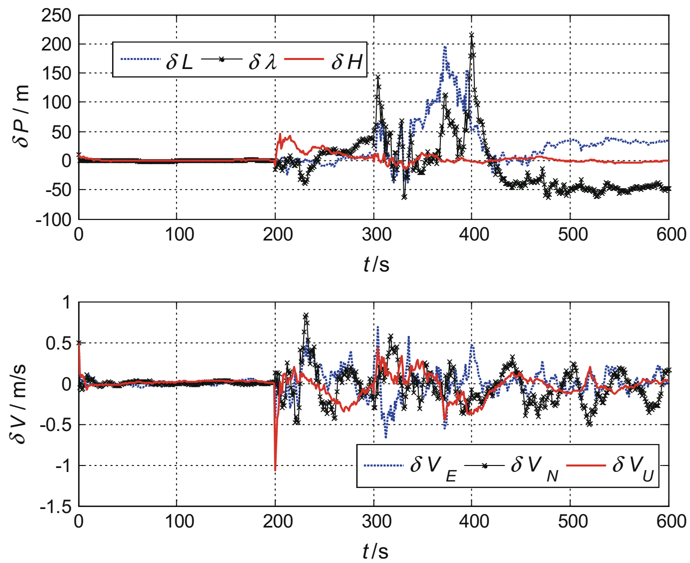

Fig. 4 Position and velocity error of SCKF in different noise

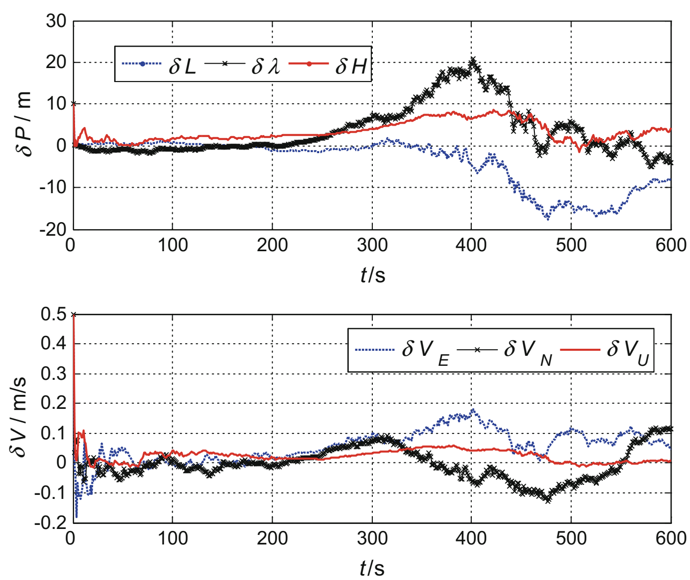

Fig. 5 Position and velocity error of this paper in different noise

Table 2 The comparison of root mean square error of SCKF of two methods in different noise

	Positioning error (m)	Velocity error (m/s)	The number of sampling points in each update time
SCKF	54.4903	0.7578	60
The method of this paper	10.7369	0.1043	32

5 Summary

According to the characteristic that the state equation of GPS/INS integrated navigation is nonlinear and the measured equation is linear, we promote an integrated navigation method based on adaptive SSR-SCKF cubature kalman filter in this paper. Firstly, we use the SSSRCR to choose the sampling points instead of the three-order spherical-radial cubature rule of SCKF to get the higher estimated accuracy. Secondly, aiming at the problem that the measured equation is linear, we propose a new measured update process based on SVD, it does not need to choose the cubature sampling points so that the computation complexity is simple. And the SVD can also improve the numerical stability of the filter. Finally, combing the moving window method, the filter can adjust the covariance matrix of measured noise in real-time, so it can improve the adaptive ability of the method proposed in this paper.

References

1. Johansen TA, Fossen TI (2016) Nonlinear observer for tightly coupled integration of pseudo-range and inertial measurements. In: IEEE transactions on control systems technology, pp 1–8
2. Sun F, Tang L (2012) INS/GPS integrated navigation filter algorithm based on cubature Kalman filter. In: Control and decision, pp 1032–1036
3. Alexander K, Dirk A (2016) Adaptive EKF-based vehicle state estimation with online assessment of local observability. In: IEEE transactions on control system technology, pp 1368–1381
4. Ivan V, Nabil A (2014) Fault diagnosis for MEMS INS using unscented Kalman filter enhanced by Gaussian process adaptation. In: 2014NASA/ESA conference on adaptive hardware and systems, pp 120–126
5. Arasaratnam I, Haykin S (2009) Cubature Kalman filters. IEEE Trans Autom Cont 54(6): 510–518
6. Chandra K, Gu D (2013) Postlethwaite I. Square root cubature information filter. IEEE Sens J 13(2):750–758
7. Wang Z, Ling H, Zhou S, Sun F (2014) The study of square root cubature Kalman smoother and its application on INS/GPS integrated navigation. In: 2014 IEEE international conference on mechatronics and automation, pp 1827–1832
8. Jiang W, Li Y, Chris R (2015) Optimal data fusion algorithm for navigation using triple integration of PPP-GNSS, INS, and terrestrial ranging system. IEEE Sens J: 5634–5644
9. Zhao Y (2015) Cubature + extended hybrid kalman filtering method and its application in PPP/IMU tightly coupled navigation systems. IEEE Sens J 15(12):6973–6985