Bayesian Approach to Multisensor Data Fusion with Pre- and Post-Filtering

Waleed A. Abdulhafiz
Low and Medium Voltage Division
Siemens, Cairo, Egypt
Email: waleed.ashraf@siemens.com

Alaa Khamis, *IEEE Senior Member*
Engineering Science Dept., Faculty of Petroleum and Mining Engineering, Suez University, Suez, Egypt
Email: akhamis@pami.uwaterloo.ca

***Abstract*—Data provided by sensors is always affected by some level of uncertainty or lack of certainty in the measurements. Combining data from several sources using multisensor data fusion algorithms exploits the data redundancy to reduce this uncertainty. This paper proposes an approach to multisensor data fusion that relies on combining a modified Bayesian fusion algorithm with Kalman filtering. Three different approaches namely: Pre-Filtering, Post-Filtering and Pre-Post-Filtering are described based on how filtering is applied to the sensor data, to the fused data or both. A case study of estimating the position of a mobile robot using optical encoder and Hall-effect sensor is presented. Experimental study shows that combining fusion with filtering helps in handling the problem of uncertainty and inconsistency of the data in both centralized and decentralized data fusion architectures.**

***Index Terms*—Multisensor data fusion, Bayesian approach, Kalman filtering, mobile robot positioning**

I. Introduction

Multisensor data fusion is a multi-disciplinary research area borrowing ideas from many diverse fields such as signal processing, information theory, statistical estimation and inference, and artificial intelligence. This is indeed reflected in the variety of the techniques reported in the literature [1].

Many definitions for data fusion exist in the literature. Joint Directors of Laboratories (JDL) [2] defines data fusion as a "multi-level, multi-faceted process handling the automatic detection, association, correlation, estimation, and combination of data and information from several sources." Klein [3] generalizes this definition, stating that data can be provided either by a single source or by multiple sources. Both definitions are general and can be applied in different fields including remote sensing. In [4], the authors present a review and discussion of many data fusion definitions. Based on the identified strengths and weaknesses of previous work, a principled definition of data fusion is proposed as the study of efficient methods for automatically or semi-automatically transforming data from different sources and different points in time into a representation that provides effective support for human or automated decision making.

Data fusion finds wide application in many areas of autonomous systems. Autonomous systems must be able to perceive the physical world and physically interact with it through computer-controlled mechanical devices. A critical problem of autonomous systems is the imperfection aspects of the data that the system is processing for situation awareness. These imperfection aspects [1] include uncertainty, imprecision, incompleteness, inconsistency and ambiguity of the data that may results in wrong beliefs about system state and/or environment state. These wrong beliefs can lead consequently to wrong decisions. To handle this problem, multisensor data fusion techniques are used for the dynamic integration of the multi-thread flow of data provided by a homogenous or heterogeneous network of sensors into a coherent picture of the situation.

This paper discusses how multisensor data fusion can help in handling the problem of uncertainty and inconsistency as common imperfection aspects of the data in autonomous systems. The paper proposes an approach to multisensor data fusion that relies on combining a modified Bayesian fusion algorithm with Kalman filtering [5]. Three different approaches namely: Pre-Filtering, Post-Filtering and Pre-Post-Filtering are described based on how filtering is applied to the sensor data, to the fused data or both. These approaches have been applied experimentally to handle the problem of data uncertainty and inconsistency in a mobile robot as an example of an autonomous system.

The remainder of this paper is organized as follows: Section II reviews the most commonly used multisensor data fusion techniques followed by describing Bayesian approaches in section III. The proposed approach is presented in section IV. A case study of position estimation of a mobile robot is discussed in section V to show the efficacy of the proposed approaches. Finally conclusion and future work are summarized in section VI.

II. Multisensor Data Fusion Approaches

Different multisensor data fusion techniques have been proposed with different characteristics, capabilities and limitations. A data-centric taxonomy is discussed in [1] to show how these techniques differ in their ability to handle different imperfection aspects of the data. This section summarizes the most commonly used approaches to multisensor data fusion.

A. Probabilistic Fusion

Probabilistic methods rely on the probability distribution/density functions to express data uncertainty. At the core of these methods lies the Bayes estimator, which enables fusion

978-1-4673-5200-0/13/$31.00 ©2013 IEEE

of pieces of data, hence the name ”Bayesian fusion” [6]. More details are provided in the next section.

B. Evidential Belief Reasoning

Dempster-Shafer theory introduces the notion of assigning beliefs and plausibilities to possible measurement hypotheses along with the required combination rule to fuse them. It can be considered as a generalization to the Bayesian theory that deals with probability mass functions. Unlike the Bayesian Inference, the Dempster-Shafer theory allows each source to contribute information in different levels of detail [6].

C. Fusion and fuzzy reasoning

Fuzzy set theory is another theoretical reasoning scheme for dealing with imperfect data. Due to being a powerful theory to represent vague data, fuzzy set theory is particularly useful to represent and fuse vague data produced by human experts in a linguistic fashion [6].

D. Possibility Theory

Possibility theory is based on fuzzy set theory, but was mainly designed to represent incomplete rather than vague data. Possibility theory’s treatment of imperfect data is similar in spirit to probability and D-S evidence theory with a different quantification approach [7].

E. Rough Set-based Fusion

Rough set is a theory of imperfect data developed by Pawlak [8] to represent imprecise data, ignoring uncertainty at different granularity levels. Rough set theory enables dealing with data granularity.

F. Random Set Theoretic Fusion

The principles of random sets theory were first proposed to study integral geometry in 1970s [9]. The most notable work on promoting random finite set theory (RFS) as a unified fusion framework has been done by Mahler in [10]. Compared to other alternative approaches of dealing with data imperfection, RFS theory appears to provide the highest level of flexibility in dealing with complex data while still operating within the popular and well-studied framework of Bayesian inference. RFS is a very attractive solution to fusion of complex soft/hard data that is supplied in disparate forms and may have several imperfection aspects [11].

G. Hybrid Fusion Approaches

The main idea behind development of hybrid fusion algorithms is that different fusion methods such as fuzzy reasoning, D-S evidence theory, and probabilistic fusion should not be competing, as they approach data fusion from different (possibly complementary) perspectives [1].

III. Bayesian Approach for Data Fusion

Bayesian inference is a statistical data fusion algorithm based on Bayes’ theorem of conditional or a posteriori probability to estimate an n-dimensional state vector X, after the observation or measurement Z has been made.

A. Simplified Bayesian Approach (SB)

Assuming a state-space representaion, the Bayes estimator provides a method for computing the posterior (conditional) probability distribution/density of the hypothetical state x_k at time k given the set of measurements $Z_k = \{z_1; ...; z_k\}$ (up to time k) and the prior distribution as following,

$$p(x_k|Z^k) = \frac{p(z_k|x_k)p(x_k|Z^{k-1})}{p(Z^k|Z^{k-1})} \tag{1}$$

Where

- $p(z_k|x_k)$ is called the likelihood function and is based on the given sensor measurement model
- $p(x_k|Z^{k-1})$ is called the prior distribution and incorporates the given transition model of the system
- The denominator is a merely a normalizing term to ensure that the probability density function integrates to one

The probabilistic information contained in Z about X is described by the probability density function $p(Z|X)$, which is a sensor dependent objective function based on observation. The likelihood function relates the extent to which the a posteriori probability is subject to change, and is evaluated either via offline experiments or by utilizing the available information about the problem. If the information about the state X is made available independently before any observation is made, then likelihood function can be improved to provide more accurate results. Such a priori information about X can be encapsulated as the prior probability and is regarded as subjective because it is not based on observed data. The information supplied by a sensor is usually modeled as a mean about a true value, with uncertainty due to noise represented by a variance that depends on both the measured quantities themselves and the operational parameters of the sensor. A probabilistic sensor model is particularly useful because it facilitates a determination of the statistical characteristics of the data obtained. This probabilistic model captures the probability distribution of measurement by the sensor z when the state of the measured quantity x is known. This distribution is extremely sensor specific and can be experimentally determined. Gaussian distribution is one of the most commonly used distributions to represent the sensor uncertainties and is given by the following equation:

$$p(Z = z_j|X = x) = \frac{1}{\sigma_j\sqrt{2\pi}}\exp\{\frac{-(x - z_j)^2}{2\sigma_j^2}\} \tag{2}$$

where j represents the sensors. Thus if there are two sensors that are modeled using (2), then from Bayes’ Theorem the fused mean of the two sensors is given by the Maximum a posteriori (MAP) esimate:

$$x_f = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}z_1 + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}z_2 \tag{3}$$

where σ_1 is the standard deviation of sensor 1 and σ_2 is the standard deviation of sensor 2. The fused variance given by [12]:

$$\sigma_f^2 = \frac{1}{\sigma_1^{-2} + \sigma_2^{-2}} \tag{4}$$

B. Modified Bayesian Approach (MB)

Sensors often provide data which is spurious due to sensor failure or due to some ambiguity in the environment. The simplified Bayesian approach described previously does not handle the spurious data efficiently. The approach yields the same weighted mean value as given by (3), regardless the data coming from any of the sensors is bad or not. The posterior distribution always has a smaller variance than either of individual distributions being multiplied. The simplified Bayesian does not have a mechanism to identify if data from certain sensor is spurious and thus it might lead to inaccurate estimation. In [12], a modified Bayesian approach has been proposed which considers measurement inconsistency.

$$p(X=x|Z=z_1,z_2) \propto \frac{1}{\sigma_1\sqrt{2\pi}}\exp\{\frac{-(x-z_1)^2}{2\sigma_1^2 f}\} \times \frac{1}{\sigma_2\sqrt{2\pi}}\exp\{\frac{-(x-z_2)^2}{2\sigma_2^2 f}\} \tag{5}$$

The modification observed in (5) causes an increase in the variance of the individual distribution by a factor given by

$$f = \frac{m^2}{m^2-(z_1-z_2)^2} \tag{6}$$

The parameter m is the maximum expected difference between the sensor readings. Larger difference in the sensor measurements causes the variance to increase by a bigger factor. The MAP estimate of state x remains unchanged but the variance of the fused posterior distribution changes. Thus depending on the squared difference in measurements from the two sensors, the variance of the posterior distribution may increase or decrease as compared to the individual Gaussian distributions that represent the sensor models. This concluded as in [12] that the modified Bayesian was highly effective in identifying inconsistency in sensor data and thus reflecting the true state of the measurements.

The difference between the simplified and the modified Bayesian can be seen in Figures 1 and 2. In this example, there are two sensors where sensor 1 has a standard deviation of 2 and sensor 2 has a standard deviation of 4. In the first case shown in Figure 1, the two sensors are in agreement. It can be seen that fused posterior distribution obtained from the proposed strategy has a lower value of variance than that of the each of the distributions being multiplied indicating that fusion leads to a decrease in posterior uncertainty.

In the second case in Figure 2, the two sensors are in disagreement. The fused posterior distribution obtained from the modified Bayesian has a larger variance as compared to the variance of both sensors. However, the fused variance due to the simplified Bayesian was the same as the fused variance in Figure 1. This concluded as in [12] that the modified Bayesian was highly effective in identifying inconsistency in sensor data and thus reflecting the true state of the measurements.

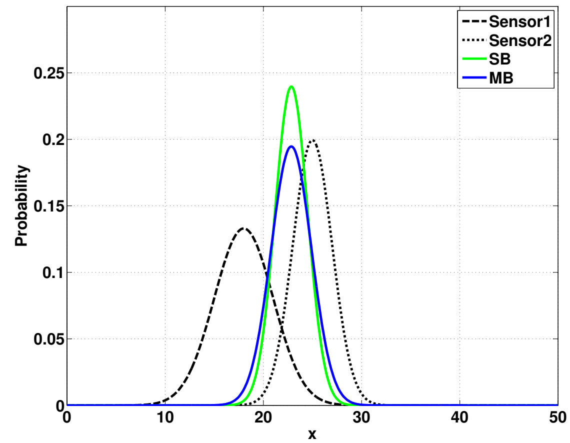

Fig. 1. Two Sensors in Agreement

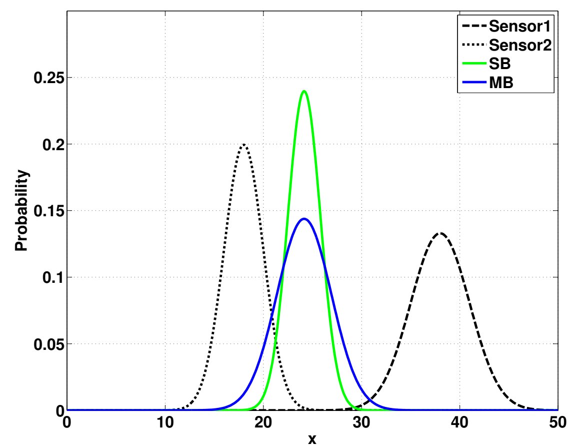

Fig. 2. Two Sensors in Disagreement

IV. Proposed Approach

The proposed approach to multisensor data fusion in this paper relies on combining a modified Bayesian fusion algorithm described in the previous section with Kalman filtering. Three different techniques namely: Pre-Filtering, Post-Filtering and Pre-Post-Filtering are described in the following subsections based on how filtering is applied to the sensor data, to the fused data or both.

A. Modified Bayesian Fusion with Pre-Filtering (F-MB)

The first proposed technique is to add Kalman filters before the fusion node. As shown in Algorithm 1, Kalman filter is added to every sensor to filter out the noise from the sensor measurements. The filtered measurements are then fused together using modified Bayesian to get a single result that represents the state at a particular instant of time.

B. Modified Bayesian Fusion with Post-Filtering (MB-F)

The second proposed technique is to add Kalman filter after the fusion node which fuses the measurements using modified Bayesian to produce x_{int}. Kalman filtering is then applied to the fused state x_{int} in order to filter out the noise, as shown in Algorithm 2. The output of the Kalman filter represents the

Algorithm 1: The Pre-Filtering Algorithm (F-MB)

Input : σ_1, σ_2, $z_1(k)$, $z_2(k)$, $x_1(k-1)$, $x_2(k-1)$, $P_1(k-1)$, $P_2(k-1)$
Output: $x_f(k)$, $\sigma_f^2(k)$
begin
  $\xi \leftarrow \sigma_1/\sigma_2$;
  for $j \leftarrow 1$ **to** 2 **do**
    $(x_j(k), P_j(k)) \leftarrow$ **Call** Kalman Filter Algorithm
  $x_f(k) \leftarrow x_1(k)/(1+\xi^2) + x_2(k)/(1+\xi^{-2})$;
  Calculate f as in (6);
  $\sigma_f^2(k) \leftarrow (\sigma_1^{-2} f^{-1} + \sigma_2^{-2} f^{-1})^{-1}$;

state x_f at a particular instant of time as well as the variance of the estimated fused state P_f.

Algorithm 2: The Post-Filtering Algorithm (MB-F)

Input : σ_1, σ_2, $z_1(k)$, $z_2(k)$, $x_f(k-1)$, $P_f(k-1)$
Output: $x_f(k)$, $P_f(k)$
begin
  $\xi \leftarrow \sigma_1/\sigma_2$;
  $x_{int}(k) \leftarrow z_1(k)/(1+\xi^2) + z_2(k)/(1+\xi^{-2})$;
  Calculate f as in (6);
  $\sigma_{int}^2(k) \leftarrow (\sigma_1^{-2} f^{-1} + \sigma_2^{-2} f^{-1})^{-1}$;
  $(x_f(k), P_f(k)) \leftarrow$ **Call** Kalman Filter Algorithm;

C. Modified Bayesian Fusion with Pre- and Post-Filtering (F-MB-F)

In this technique, Kalman filter is applied before and after the fusion node. The algorithm of this technique is the integration of Algorithms 1 and 2, as shown in Algorithm 3.

Algorithm 3: The Pre- and Post-Filtering Algorithm (F-MB-F)

Input : σ_1, σ_2, $z_1(k)$, $z_2(k)$, $x_1(k-1)$, $x_2(k-1)$, $x_f(k-1)$, $P_1(k-1)$, $P_2(k-1)$, $P_f(k-1)$
Output: $x_f(k)$, $P_f(k)$
begin
  $\xi \leftarrow \sigma_1/\sigma_2$;
  for $j \leftarrow 1$ **to** 2 **do**
    $x_j(k) \leftarrow$ **Call** Kalman Filter Algorithm;
  $x_{int}(k) \leftarrow x_1(k)/(1+\xi^2) + x_2(k)/(1+\xi^{-2})$;
  Calculate f as in (6);
  $\sigma_{int}^2(k) \leftarrow (\sigma_1^{-2} f^{-1} + \sigma_2^{-2} f^{-1})^{-1}$;
  $(x_f(k), P_f(k)) \leftarrow$ **Call** Kalman Filter Algorithm;

V. Case Study: Mobile Robot Local Positioning

In this section, a case study of position estimation of a mobile robot is presented to proof the efficacy of the proposed approach. Mobile robot positioning provides answer for the question: "Where the robot is?". Positioning techniques solutions can be roughly categorized into relative position measurements (dead reckoning) and absolute position measurements. In the former, the robot position is estimated by applying to a previously determined position the course and distance traveled since. In the later, the absolute position of the robot is computed from measuring the direction of incidence of three or more actively transmitted beacons or using artificial or natural landmarks or using model matching where features from a sensor-based map and the world model map are matched to estimate the absolute location of the robot [13].

A. Experiment Setup

Odometry is a relative positioning method that uses encoders to measure wheel rotation and/or steering orientation of the robot. The right wheel's motor is equipped with a Hall-effect sensor while the left wheel's motor is equipped with an optical encoder. The microcontroller aligns the data and adjusts the resolutions so that both sensors can have the same resolution and the data could be compared with each other. The distance moved by each wheel is calculated, given the sampling time of the data and the constant velocity of the robot. The microcontroller then sends this data to Matlab to perform the proposed algorithms in order to estimate the position of the robot given the uncertain and inconsistent data collected from the sensors.

Several experiments were carried out in order to model the uncertainty of each sensor which was represented in the form of a white Gaussian noise with a standard deviation of 2.378 cm and 2.260 cm for the optical encoder and Hall-effect sensor respectively. In addition, it was specified to have the robot moving with a constant linear velocity of 7.8 cm/sec and a disturbance of approximately 0.493 cm/sec. The sampling time is 0.5 sec and the robot should move in a straight line for 20 sec.

B. Evaluation Metrics

The performance of the algorithms was evaluated based on four criteria:

- **CPU Running Time:** This represents the total processing time of the algorithm to estimate the position of the robot throughout the travelling time. It is desired to minimize this running time.
- **Residual sum of squares (RSS):** This represents the summation of the squared difference between the theoretical position of the robot and the estimated state at each time sample. The smaller the RSS, the accurate the algorithm will be because this means that the estimated position of the robot is getting closer to the theoretical position. This is given by:

$$RSS = \sum_{i=1}^{n} (x_{theoretical,i} - x_{estimated,i})^2 \tag{7}$$

- **Variance (P):** This represents the variance of the estimated position of the robot. The variance will reflect the performance of the filters in each algorithm.

- **Coefficient of Correlation:** This is a measure of association that shows how the state estimate of each technique is related to the theoretical state. The coefficient of correlation will always lie between -1 and +1. For example, a correlation close to +1 indicates the two data are very strongly positively correlated [14].
- **Criterion Function (CF):** A computational decision-making method was used to calculate a criterion function that is a numerical estimate of the utility associated with each of the three proposed techniques. A weighting function w (from 0 to 1) will be defined for each criterion (time, RSS, variance), depending on its importance. The three weights should sum up to 1. The cost value c (calculated from the experiments) of each technique is obtained and finally, CF is calculated as the weighted sum of the utility for each technique as follows:

$$CF = w_1 \times \frac{c_1}{c_{1max}} + w_2 \times \frac{c_2}{c_{2max}} + w_3 \times \frac{c_3}{c_{3max}} \quad (8)$$

where w_1, w_2 and w_3 are the weights of the time, RSS and variance respectively. These weights are adjusted according to the application and desire of the user. In this case study, it was assumed that $w_1 = 0.3$, $w_2 = 0.3$ and $w_3 = 0.4$. The values c_1, c_2 and c_3 are the values obtained from the experiments for the time, RSS and variance respectively. The values c_{1max}, c_{2max} and c_{3max} represent the maximum value achieved in each of the criteria: time, RSS and variance respectively. The objective is to minimize this function such that the algorithm will produce accurate estimates in a short time with a minimum variance.

C. Results and Discussion

The raw data collected from the sensors were used to carry out 5000 iterations in order to simulate the results of the proposed algorithms using Matlab. Figure 3 shows the errors that are produced due to the noisy uncertain measurements obtained from the sensors. In addition, the figure shows the errors that are produced due to estimating the position of the robot using the proposed three algorithms. The error represents the difference between the theoretical state and the output state of each algorithm, at a particular sample time. The measurement errors are high compared to the estimation errors. The first interpretation of this result is that the proposed techniques provide better estimates than relying on the measurements directly.

Table I summarizes the average results of 5000 runs for the three evaluation metrics described previously. The minimal value in each criteria has been bolded. Although MB takes the minimal execution time yet it has the maximum RSS value compared to other techniques and this is clear in the CF shown in Table I. Figure 4 shows the estimated variance of each technique. It is clear that the variance of the MB-F and F-MB-F converged earlier to lower values than MB and F-MB. This proves the efficiency of the kalman filters in MB-F and F-MB-F. Table I shows that MB-F has a smaller variance than F-MB-F. Moreover, the proposed techniques estimates were all

TABLE I
COMPUTATIONAL DECISION-MAKING CHART FOR THE FUSION TECHNIQUES USING 2 SENSORS

Fusion Techniques	Criteria c_1, c_2, c_3 respectively: Time (s)	RSS (cm^2)	P (cm^2)	Criterion Function
MB	**0.001**	36.729	3.068	0.724
F-MB	0.011	10.885	3.079	0.692
MB-F	0.006	15.713	**0.399**	**0.292**
F-MB-F	0.017	**7.154**	0.405	0.464
c_{max}	0.017	36.729	3.079	

positively and strongly correlated to the theoretical values with a correlation coefficient of +0.99.

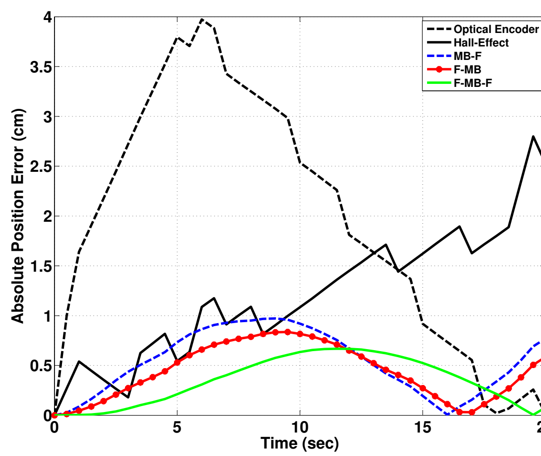

Fig. 3. Measurements and Estimated Errors

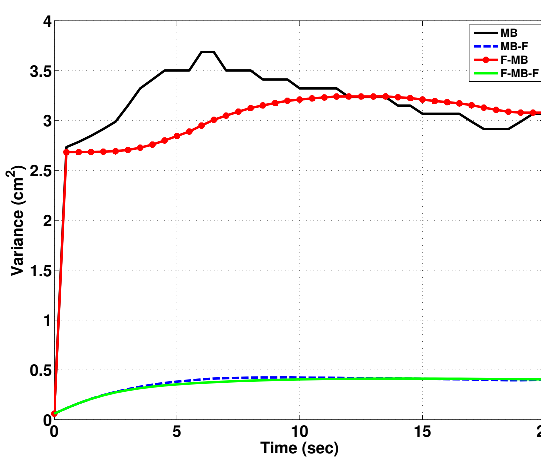

Fig. 4. Estimated Variance

The same experiments were repeated but using three sensors [15]. Readings from the sensors were fused using centralized and decentralized architectures. In centralized fusion, the data

TABLE II
COMPUTATIONAL DECISION-MAKING CHART FOR THE FUSION TECHNIQUES USING 3 SENSORS

Fusion	Time (s)		RSS (cm^2)		P (cm^2)		Criterion Function	
Techniques	Centralized	Decentralized	Centralized	Decentralized	Centralized	Decentralized	Centralized	Decentralized
MB	0.056	**0.029**	76.076	**70.288**	2.607	**2.038**	0.917	**0.701**
F-MB	0.072	**0.046**	31.338	**28.764**	2.555	**2.034**	0.795	**0.603**
MB-F	0.063	**0.034**	50.838	**49.956**	0.363	**0.322**	0.499	**0.378**
F-MB-F	0.077	**0.051**	25.108	**23.666**	0.364	**0.322**	0.455	**0.339**
c_{max}	0.077		76.076		2.607			

from all sensors are fused simultaneously. Alternatively, the data can be fused sequentially and this is called decentralized fusion. The decentralized fusion is more robust in terms of individual component failure and is more efficient in using communication resources as compared to conventional schemes [16].

As shown in Table II the decentralized fusion generally outperforms the centralized in terms of running time, RSS and estimated variance. However, the shortest running time was the MB of the decentralized system while the longest time was the centralized F-MB-F. This was the same case when fusing using two sensors, the direct fusion was faster than F-MB-F. Regarding to RSS, the F-MB-F was the most accurate with minimal error in both cases of the centralized and decentralized while the maximum error occured in the MB. This is a reasonable result because the presence of the Kalman filters, produces estimates with less noise than the measurements and close to the accuracy of the theoretical states.

Moreover, the variance of the estimated state was the least in MB-F and F-MB-F with a noticeable difference between the variance of these techniques and the variance of F-MB and MB. Therefore, this proves that the filtering process is working efficiently in the cases of post-filtering as well the pre-post filtering.

Overall, the F-MB-F had the least CF and then follows it the MB-F with a very small difference. It can be concluded that both techniques will produce reliable results, however, it would be recommended to use MB-F in applications where time is an important factor and to use the F-MB-F if high accuracy is required.

VI. CONCLUSION

Three techniques for multisensor data fusion have been discussed in this paper. These techniques combine a modified Bayesian data fusion algorithm with pre- and post-data filtering in order to handle the uncertainty and inconsistency problems of sensory data. The proposed techniques have been applied for a mobile robot position estimation using odometry sensors. The experimental results proved that combining fusion with filtering improves residual sum of squares and variance of the fused result. The paper also proved that MB-F will still be efficient for any n number of sensors and particularly in the decentralized fusion architecture. Future research is possible in numerous directions. More powerful filtering techniques such as particle filters can be used instead of Kalman filtering to allow for fusion of data with nonlinear dynamics. Increasing problem dimensionality will be considered as well.

REFERENCES

[1] B. Khaleghi, A. Khamis, F. O. Karray, and S. N. Razavi, "Multisensor data fusion: A review of the state-of-the-art," in *Information Fusion*, 2011.

[2] U.S. Department of Defense, Data Fusion Subpanel of the Joint Directors of Laboratories, Technical Panel for C3., *Data Fusion lexicon*, 1991.

[3] L. A. Klein, "Sensor and data fusion concepts and applications.," in *Society of Photo-Optical Instrumentation Engineers (SPIE)*, 1993.

[4] H. Bostrm, S. Andler, M. Brohede, R. Johansson, A. Karlsson, J. V. Laere, L. Niklasson, M. Nilsson, A. Persson, and T. Ziemke, "On the definition of information fusion as a field of research.," tech. rep., Informatics Research Centre, University of Skvde, 2007.

[5] G. Welch and G. Bishop, *An Introduction to the Kalman Filter*. Department of Computer Science, University of North Carolina at Chapel Hill, July 2006.

[6] H. Durrant-Whyte and T. Henderson, *Sensor Fusion*, ch. 25, pp. 585–608. Springer, 2008.

[7] J. Llinas, C. Bowman, G. Rogova, A. Steinberg, E. Waltz, and F. White, "Revisiting the jdl data fusion model ii," in *Proc. of the International Conference on Information Fusion*, pp. 1218–1230, 2004.

[8] Z. Pawlak, *Rough Sets: Theoretical Aspects of Reasoning about Data*. Kluwer Academic Publishers, Norwell, MA, 1992.

[9] D. Kendall, *Foundations of a theory of random sets, in: Stochastic Geometry*. J. Wiley, 1974.

[10] R. Mahler, *Statistical Multisource-Multitarget Information Fusion*. Artech House, Boston, MA, 2007.

[11] B. Khaleghi, A. Khamis, and F. Karray, "Random finite set theoretic based data fusion with application for target tracking," in *IEEE 2010 International Conference on Multisensor Fusion and Integration for Intelligent Systems*, 2010.

[12] D. Garg, M. Kumar, and R. Zachery, "A generalized approach for inconsistency detection in data fusion from multiple sensors," in *American Control Conference*, pp. 2078–2083, June 2006.

[13] A. Khamis, *Lecture 3 - Mobile Robot Positioning. MCTR1002: Autonomous Systems.* Mechatronics Engineering Department, German University in Cairo, 2011-2012.

[14] G. Keller and B. Warrack, *Statistics for Management and Economics*. Duxbury Press, 4 ed., 1997.

[15] W. Abdulhafiz, "Bayesian approach to multisensor data fusion with pre- and post-filtering," Master's thesis, Mehcatronics Engineering Department, German University in Cairo, July 2012.

[16] M. Kumar, D. P. Garg, and R. A. Zachery, "A method for judicious fusion of inconsistent multiple sensor data," *IEEE Sensors Journal*, vol. 7, pp. 723–733, May 2007.