ECON 470: Optimization in Julia **Fall 2023**

October 12th, 2023 - Assignment 2

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1. (insert youtube link)
2. (insert photos)
3. $c = (4, 0); b = (0, 3); a = b + c = (4, 3)$
   - (i) $||a|| = 5, ||b|| = 3, ||c|| = 4$
   - (ii) $a^T b = 4 * 0 + 3 * 3 = 9$
   - (iii) $||\hat{b}|| = \frac{|a^T b|}{||a||} = \frac{9}{5}$
   - (iv) $cos\theta = \frac{a^T b}{||a||*||b||} = \frac{3}{5}$
   - (v) $\frac{|b^T a|}{||a||*||b||}$ represents the orthogonal projection of a onto b, while $\frac{a^T b}{||a||}$ is the orthogonal projection of b onto a
4. $c_1 = \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}, ..., c_k = \begin{bmatrix} a_k \\ b_k \end{bmatrix}$
   - (a) Assume $c_1, ..., c_k$ is not linearly independent. Therefore
     $\beta_1 c_1 + ... + \beta_k c_k = 0$
     has a solution where some $\beta_1, ..., \beta_k \neq 0$. We can expand the expression above to show:
     $\begin{bmatrix} \beta_1 a_1 \\ \beta_1 b_1 \end{bmatrix} + ... + \begin{bmatrix} \beta_k a_k \\ \beta_k b_k \end{bmatrix} = 0$
     $=> \beta_1 a_1 + ... + \beta_k a_k = 0$
     We know that $a_1, ..., a_k$ is linearly independent, which means that the only solution to the equation above is $\beta_1, ..., \beta_k = 0$. This results in a contradiction with our earlier conclusion that some $\beta_1, ..., \beta_k \neq 0$.
     Therefore, via proof by contradiction, $c_1, ..., c_k$ must be linearly independent if $a_1, ..., a_k$ is linearly independent.
   - (b) Suppose $b_1, ..., b_k$ is linearly independent. Using the proof showed in (a), we know that $c_1, ..., c_k$ must also be linearly independent. This means we know that the linear dependence of $a_1, ..., a_k$ does not mean that $c1, ..., c_k$ is linearly dependent.
5. $b^T(a - \gamma b) = 0$
   $b^T - b^T \gamma b = 0$
   $b^T a = \gamma b^T b$
   $\gamma ||b||^2 = b^T a$
   $\gamma = \frac{b^T a}{||b||^2}$