【計量ファイナンスA】

15. High-frequency SV モデル

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Introduction

Estimation of time-varying volatility

1. GARCH and Stochastic Volatility (SV) models
    - First, we must model the dynamics of volatility.
    - Then, volatility is estimated by estimating the model.
    - Volatility estimates depend on the model.
2. Realized volatility (RV)
    - Volatility is estimated using intraday high-frequency returns.
    - We need not model the dynamics of daily volatility.

Introduction

- We like to beat RV by modeling the dynamics of intraday high-frequency volatility.
- It is not straightforward to model the dynamics of intraday high-frequency volatility.
  (1) Intraday seasonality
  (2) Announcement effects
- We also take account of
  (3) Negative correlation between return and volatility
  (4) Jumps in return and volatility
  (5) Fat-tail return distribution

Introduction

This research

- We extend daily SV models to intraday high-frequency SV models.
- develop a Bayesian method using MCMC for the analysis of intraday SV models.
- apply them to 5-min returns of Nikkei 225 stock index.
- show
  (1) Intraday SV models fit the data better than intraday GARCH-type models.
  (2) Intraday SV models perform better than daily RV models such as HAR and realized EGARCH models in one-day-ahead volatility forecasting.

Intraday SV models

Intraday high-frequency return

$$y_t = V_t \varepsilon_t + J_t Z_t^y, \quad \varepsilon_t \sim N(0,1) \text{ or standardized } t(\nu)$$

- V_t = total volatility
- $J_t Z_t^y$ = jump component

t distribution

$$\varepsilon_t = \sqrt{\lambda_t} z_t, \quad \frac{\nu - 2}{\lambda_t} \sim \chi^2(\nu), \quad z_t \sim N(0,1)$$

- We assume $\nu > 2$ for a finite variance.

Intraday SV models

Total volatility

$$V_t = X_t S_t A_t \quad \text{or} \quad h_t = x_t + s_t + a_t$$

- X_t = SV part
- S_t = intraday seasonality
- A_t = announcement effects
- $h_t = \log(V_t^2)$, $x_t = \log(X_t^2)$, $s_t = \log(S_t^2)$, $a_t = \log(A_t^2)$

SV part

$$x_{t+1} = \mu + \phi(x_t - \mu) + J_t Z_t^v + \eta_t$$

$$\begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} \sim N(\mathbf{0}, \boldsymbol{S}), \quad \boldsymbol{S} = \begin{pmatrix} 1 & \rho\sigma \\ \rho\sigma & \sigma^2 \end{pmatrix}$$

Intraday SV models

Jump components

- Jump components $(J_t Z_t^y, J_t Z_t^v)$ are assumed to be coincident in returns and the volatility with the common jump indicator variable, $J_t \in \{0, 1\}$.
- Jump occurs with the probability $\Pr[J_t = 1] = \kappa$.
- We assume

$$Z_t^y \sim N(\mu_y, \sigma_y^2), \quad Z_t^v \sim N(\mu_v, \sigma_v^2)$$

Intraday SV models

Intraday seasonality

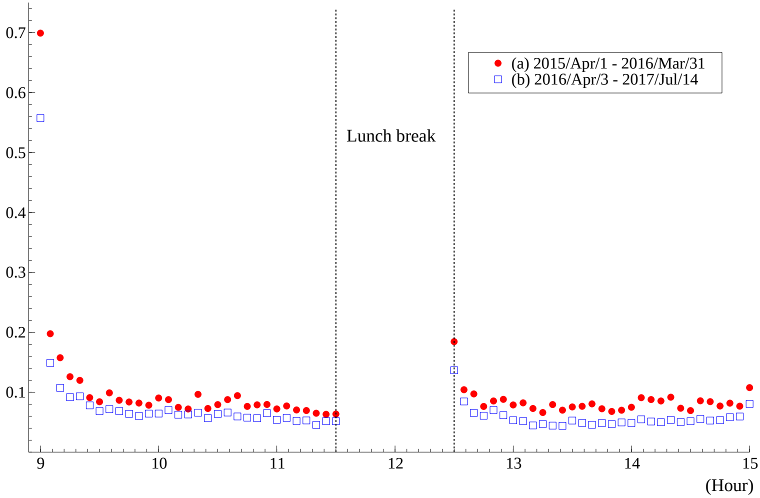
- Trigonometric function: Harada and Watanabe (2009), Neely (2011), Tsuchida, Watanabe and Yoshiba (2016)
- Cubic smoothing spline: Stroud and Johaness (2014)
- Random walk: This paper

Intraday SV models

- K = number of intraday returns during a day
- N = number of days in the sample periods
- $T = K \times N$: total sample size
- Tokyo stock exchange is open for 9:00-11:30 and for 12:30-15:00.
- We use 5-min returns when the market is open.
- We also use overnight (15:00-9:00) and lunch-time (11:30-12:30) returns.
- Then, $K = 62$.

Intraday SV models

Average intraday Nikkei 225 absolute 5-min returns



Intraday SV models

- β_k =intraday seasonality at intraday period k $(k = 1, \ldots, K)$.
- H_{tk} = intraday-period indicator

$$H_{tk} = \begin{cases} 1 & \text{if time } t \text{ corresponds to intraday period } k \\ 0 & \text{otherwise} \end{cases}$$

- Then, s_t is represented by

$$s_t = \sum_{k=1}^{K} H_{tk}\beta_k, \quad t = 1, \ldots, T$$

Intraday SV models

- We assume that β_k follows the following random walk.

$$\beta_{k+1} = \beta_k + w_k, \quad w_k \sim N(0, c_k v_\beta^2), \quad k = 1, \dots, K-1$$
$$\beta_1 \sim N(0, 100)$$

- We set $c_k = 100$ for $k = 1, 31, 32, 61$ and $c_k = 1$ for other k.

- Identifying restriction: $\frac{1}{K} \sum_{k=1}^{K} \beta_k = 0$

Intraday SV models

Announcement effects

$$
\begin{aligned}
a_t &= \sum_{j=1}^{J} \sum_{\ell=1}^{L} I_{jt\ell} \alpha_{j\ell} \\
\alpha_{j,\ell+1} &= \psi_j \alpha_{j\ell} + \zeta_{j\ell}, \quad \zeta_{j\ell} \sim N(0, v_{\alpha j}^2), \quad |\psi_j| < 1
\end{aligned}
$$

- $J = 4$ (GDP, IP, CPI, MPM)
- $t_j^* =$ announcement time of the jth variable
- $I_{jt\ell} =$ announcement-period indicator:

$$
I_{jt\ell} = \begin{cases} 1 & \text{if } t \in \{t_j^*, t_j^* + 1, \ldots, t_j^* + L - 1\} \\ 0 & \text{otherwise} \end{cases}
$$

- $L = 18$ (i.e., 90 minutes)

Intraday SV models

Parameters

1. Parameters for SV: $\boldsymbol{\theta}_X = (\phi, \sigma, \rho, \mu)$
2. Jump parameters: $\boldsymbol{\theta}_J = (\kappa, \mu_y, \sigma_y, \mu_v, \sigma_v)$
3. Parameters for intraday seasonality and announcement effects: $\boldsymbol{v} = (v_\beta, v_{\alpha 1}, \ldots, v_{\alpha J})$, $\boldsymbol{\psi} = (\psi_1, \ldots, \psi_J)$
4. The degree of freedom for t distribution: ν

Intraday SV models

Latent variables

1. SV process: $\boldsymbol{x} = (x_1, \ldots, x_T)$
2. Jump components:
   $\boldsymbol{J} = (J_1, \ldots, J_{T-1})$
   $\boldsymbol{Z}^{\varphi} = (Z_1^{\varphi}, \ldots, Z_{T-1}^{\varphi})$ for $\varphi = y, v$
3. Intraday seasonality and announcement effects:
   $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_K)$
   $\boldsymbol{\alpha} = \{\boldsymbol{\alpha}_1, \ldots, \boldsymbol{\alpha}_J\}$ where $\boldsymbol{\alpha}_j = (\alpha_{j1}, \ldots, \alpha_{jL})$
4. Latent variables for t distribution: $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_T)$

Bayesian analysis and computation

Estimation

- We develop a Bayesian estimation using MCMC.
- Parameters: $\boldsymbol{\theta} = \{\boldsymbol{\theta}_X, \boldsymbol{\theta}_J, \boldsymbol{v}, \psi, \nu\}$
- Latent variables: $\boldsymbol{\Theta} = \{\boldsymbol{x}, \boldsymbol{J}, \boldsymbol{Z}^y, \boldsymbol{Z}^v, \boldsymbol{\beta}, \boldsymbol{\alpha}, \boldsymbol{\lambda}\}$
- We sample them from the joint posterior density $\pi(\boldsymbol{\theta}, \boldsymbol{\Theta}|\boldsymbol{y})$ using the Gibbs sampler.

Bayesian analysis and computation

1. Parameters for SV: straightforward
2. Jump parameters: straightforward
3. Parameters for the intraday seasonal and announcement effects: straightforward
4. The degree of freedom for the Student-t distribution: Watanabe (2001)
5. SV: Omori and Watanabe (2008)
6. Jump components: straightforward
7. Intraday seasonal and announcement effects: Watanabe and Omori (2004)
8. Latent variables for t distribution: Watanabe (2001)

Empirical analysis

Data

- 5-min intraday returns of Nikkei 225 stock index.
- Total sample period: April 1, 2015–July 14, 2017
- Estimation: April 1, 2015–March 31,2016
- Forecasting: April 1, 2016–July 14, 2017

Priors

$$
\begin{aligned}
&(\phi+1)/2 \sim B(20, 1.5),\ \sigma^2 \sim IG(40, 0.2),\ (\rho+1)/2 \sim B(1,1),\\
&\mu \sim N(-5, 4),\ \nu \sim G(16, 0.8)I[\nu > 2],\ \kappa \sim B(1, 500),\\
&\mu_y \sim N(0,1),\ \mu_v \sim N(1,1),\ \sigma_i^2 \sim IG(20, 4)\ (i = y, v),\\
&v_\beta^2 \sim IG(10, 1),\ v_{\alpha,j}^2 \sim IG(10,1),\ (\psi_j + 1)/2 \sim B(20, 1.5)
\end{aligned}
$$

Empirical analysis

1. **SV:** normal distribution, no jumps
2. **SVt:** Student *t*-distribution, no jumps
3. **SVJ:** normal distribution, with jumps in return and volatility
4. **SVJt:** Student *t*-distribution, with jumps in return and volatility

Empirical analysis

Posterior estimates of the selected parameters for the high-frequency SV models (2015/Apr – 2016/Mar)

	SV	SVt	SVJ	SVJt
	0.9898 (0.0015)	0.9935 (0.0011)	0.9918 (0.0011)	0.9935 (0.0011)
ϕ	[0.9868, 0.9926]	[0.9911, 0.9956]	[0.9894, 0.9941]	[0.9913, 0.9955]
	5.0	16.5	12.6	20.1
	0.1558 (0.0070)	0.1213 (0.0069)	0.1082 (0.0068)	0.1090 (0.0068)
σ	[0.1411, 0.1694]	[0.1094, 0.1358]	[0.0956, 0.1224]	[0.0954, 0.1221]
	15.5	44.5	36.1	53.2
	-0.2279 (0.0301)	-0.2711 (0.0348)	-0.2989 (0.0384)	-0.2917 (0.0405)
ρ	[-0.2872, -0.1685]	[-0.3392, -0.2008]	[-0.3673, -0.2197]	[-0.3701, -0.2123]
	10.7	9.4	16.1	23.3

- The first row: posterior mean and standard deviation in parentheses.
- The second row: 95% credible interval in square brackets.
- The third row: inefficiency factor.

Empirical analysis

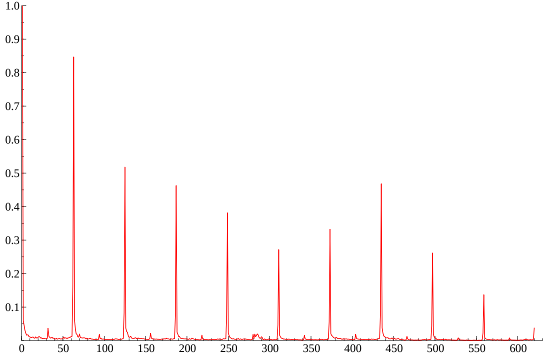
	SV	SVt	SVJ	SVJt
	-4.6803 (0.1228)	-4.6400 (0.1542)	-5.4439 (0.1762)	-4.9691 (0.1757)
μ	[-4.9192, -4.4400]	[-4.9405, -4.3284]	[-5.8133, -5.1185]	[-5.3179, -4.6423]
	0.5	1.0	22.0	30.3
	0.2325 (0.0201)	0.2328 (0.0212)	0.2430 (0.0217)	0.2317 (0.0209)
v_β	[0.1967, 0.2745]	[0.1946, 0.2806]	[0.2046, 0.2880]	[0.1944, 0.2764]
	0.9	0.9	1.4	1.0
	0.3241 (0.0521)	0.3225 (0.0522)	0.3240 (0.0512)	0.3222 (0.0515)
$v_{\alpha 1}$	[0.2417, 0.4470]	[0.2404, 0.4467]	[0.2423, 0.4414]	[0.2385, 0.4395]
	0.9	0.6	1.1	1.7
	0.7074 (0.1414)	0.7056 (0.1389)	0.7082 (0.1401)	0.7071 (0.1436)
ψ_1	[0.4223, 0.9430]	[0.4138, 0.9518]	[0.4231, 0.9563]	[0.4150, 0.9512]
	0.8	0.7	0.7	0.4
		10.889 (0.9258)		12.851 (1.3137)
ν		[9.2183, 12.784]		[10.759, 15.874]
		43.1		57.2

Empirical analysis

	SVJ	SVJt
κ	0.0091 (0.0018) [0.0057, 0.0130] 24.5	0.0017 (0.0008) [0.0005, 0.0034] 69.8
μ_y	-0.0134 (0.0407) [-0.0970, 0.0632] 9.4	-0.1258 (0.1187) [-0.3744, 0.1048] 31.7
μ_v	0.6391 (0.1198) [0.4256, 0.8921] 15.4	1.2841 (0.2916) [0.7553, 1.8865] 47.3
σ_y	0.3301 (0.0255) [0.2853, 0.3821] 17.7	0.4155 (0.0410) [0.3445, 0.5039] 10.0
σ_v	0.4898 (0.0540) [0.3927, 0.6068] 17.7	0.4753 (0.0562) [0.3787, 0.5988] 7.3

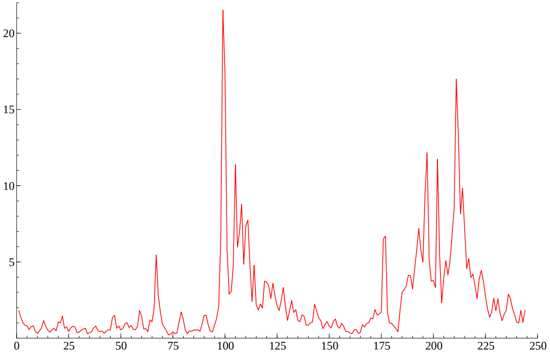
Empirical analysis

Posterior mean of total volatility V_t^2 (5-min in the first 10 days)



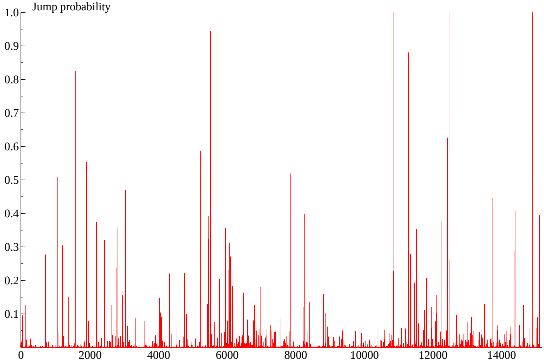
Empirical analysis

Posterior mean of total volatility V_t^2 (daily)



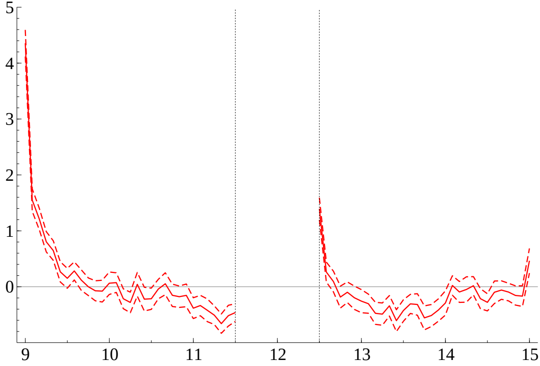
Empirical analysis

Posterior probability of jump (5-min)



Empirical analysis

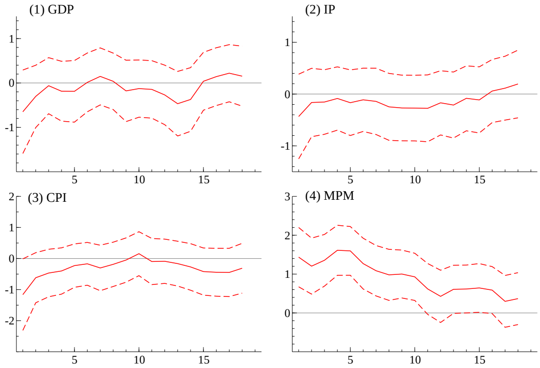
Posterior mean of parameters $(\beta_1, \ldots, \beta_K)$ for the intraday seasonality



- Posterior means (solid) and 95% credible intervals (dashed).

Empirical analysis

Announcement effects $\alpha_{j\ell}$ for GDP, industrial production (IP), consumer price index (CPI) and the Bank of Japan's monetary policy meeting (MPM)



- Posterior means (solid) and 95% credible intervals (dashed).

Empirical analysis

- This result is consistent with

  (1) Harada and Watanabe (2009): Yen/Dollar exchange rate

  (2) Tsuchida, Watanabe and Yoshiba (2016): Japanese government bond futures

Model comparison

BIC for intraday SV and GARCH models
(2015/Apr/1 – 2016/Mar/31)

Model	BIC	Ranking
SV	-30090	3
SVJ	-29723	4
SVt	-31016	1
SVJt	-30327	2
GARCH	-26275	9
GJR	-26341	8
EGARCH	-26264	10
GARCHt	-27315	7
GJRt	-27379	5
EGARCHt	-27371	6

Model comparison

Models for daily RV

- HAR model (Corsi 2009)

$$
\begin{aligned}
& \log \mathrm{RV}_t \\
&= \quad c + b_d \log \mathrm{RV}_{t-1} + b_w \log \mathrm{RV}^{(w)}_{t-1} + b_m \log \mathrm{RV}^{(m)}_{t-1} \\
& \quad\quad + u_t
\end{aligned}
$$

where

$$
\mathrm{RV}^{(w)}_{t-1} = \frac{1}{5}\sum_{i=1}^{5} \mathrm{RV}_{t-i}, \quad \mathrm{RV}^{(m)}_{t-1} = \frac{1}{22}\sum_{i=1}^{22} \mathrm{RV}_{t-i}
$$

Model comparison

- Realized EGARCH model (Hansen and Huang 2016)

$$
\begin{aligned}
R_t &= \sigma_t z_t, \quad z_t \sim N(0,1) \\
\log \sigma_{\mathrm{t}}^2 &= \omega + \beta \ln(\sigma_{t-1}^2) + \tau_1 z_t + \tau_2 (z_t^2 - 1) + \gamma v_{t-1} \\
\log \mathrm{RV}_t &= \xi + \log \sigma_{\mathrm{t}}^2 + \delta_1 z_t + \delta_2 (z_t^2 - 1) + v_t \\
& \quad v_t \sim N(0, \sigma_v^2)
\end{aligned}
$$

- Realized stochastic volatility model (Takahashi, Omori and Watanabe 2009, Takahashi, Watanabe and Omori 2016)

Model comparison

Out-of-sample forecast performance

- We compare the predictive ability of one-day-ahead volatility of our model with those of the HAR and realized EGARCH models.
- Following Patton (2011), we use the following loss functions.

$$
\begin{aligned}
\mathrm{MSE} &= \frac{1}{N_1}\sum_{\tau=N_0+1}^{N_0+N_1}\left(\widehat{\sigma}_\tau^2-\sigma_\tau^2\right)^2 \\
\mathrm{QLIKE} &= \frac{1}{N_1}\sum_{\tau=N_0+1}^{N_0+N_1}\left(\frac{\sigma_\tau^2}{\widehat{\sigma}_\tau^2}+\log\widehat{\sigma}_\tau^2\right)
\end{aligned}
$$

where σ_τ^2 and $\widehat{\sigma}_\tau^2$ denote the true volatility and the volatility forecast for day τ, respectively.

Model comparison

- Since the true volatility is unobserved, we must use the proxy.
- Patton (2011) shows that the MSE and QLIKE are robust loss functions in the sense that they lead to the same ranking as the one when the true volatility is used if the proxy is the unbiased estimator of the true volatility.
- We calculate the realized kernel (Barndorff-Nielsen et al 2008, 2009) using 1-min returns when the market is open.
- We convert it to the volatility estimates for 24 hours including non-trading hours such as lunch-time and overnight following Hansen and Lunde (2005).

Model comparison

One-day ahead volatility forecasting performance (2016/Apr/1 – 2017/July/14)

Model	MSE	QLIKE
SV	10.85 [3]	1.381 [4]
SVJ	10.80 [2]	1.352 [3]
SVt	11.04 [4]	1.402 [5]
SVJt	10.32 [1]	1.305 [1]
HAR	11.34 [5]	1.309 [2]
REGARCH	11.60 [6]	1.417 [6]

- Ranking is in brackets.

Conclusion

1. We extend daily SV models to intraday high-frequency SV models.
2. develop an MCMC Bayesian method for the analysis of intraday high-frequency SV models.
3. apply them to 5-min returns of Nikkei 225 stock index.
4. show
   (1) Intraday SV models fit the data better than intraday GARCH-type models.
   (2) Intraday SV models perform better than daily RV models such as HAR and realized EGARCH models in one-day-ahead volatility forecasting.