Risk-off Shocks and Spillovers in Safe Havens

John Beirne[1] and Eric Sugandi[2]

31 July 2023

Abstract

This paper examines real and financial spillovers to safe haven financial flow destinations due to risk-off shocks in global financial markets. Using country-specific structural vector autoregression (VAR) models over the period 1990 to 2021, we show that dynamics for Japan appear to be different to those of Switzerland and the US in three main ways. First, in response to risk-off episodes, the yen real effective exchange rate (REER) appreciated significantly, with the effect persisting for around 25 days. Second, no significant effects on portfolio flows to Japan are found, in spite of the exchange rate effects, suggesting a rapid adjustment of financial markets to shifts in equilibrium exchange rates. Third, negative real spillovers from risk-off shocks appear to only apply to Japan with exchange rate appreciation exacerbating declines in GDP growth. Our findings have important implications for policymakers in safe haven destinations in managing domestic financial vulnerabilities associated with risk-off episodes.

Keywords: risk-off episodes, safe haven assets, economic policy uncertainty

JEL Classification: F32, F41, F62

[1] Asian Development Bank Institute, Japan; e-mail: jbeirne@adbi.org
[2] Asian Development Bank Institute, Japan; e-mail: esugandi@adbi.org

1. Introduction

During periods of heightened financial stress or for portfolio diversification, global investors place funds in “safe haven” and “hedge” assets. Baur and Lucey (2010) define a “hedge” security as a security that is uncorrelated with stocks or bonds on average, while a “safe haven” security is a security that is uncorrelated with stocks and bonds in a market crash. Hossfeld and MacDonald (2014) refine further this definition, where a safe haven currency is defined as a currency whose effective returns are negatively related to global stock market returns in times of high financial stress, while a hedge currency is a currency whose effective returns are negatively related to global stock market returns on average (i.e., unconditional on the stress-regime).

Many investors have long perceived Japanese financial assets (including Japanese government bonds (JGBs) and stocks) and the Japanese Yen (JPY) as safe-haven instruments. JGBs and the JPY are highly related because investors buy (sell) the JPY when they purchase (sell) JPY-denominated JGBs. Japanese financial assets and the JPY have notable negative correlations with global stock and bond market returns during global financial crises. Foreign investors bought Japanese assets and the JPY during crises, such as the 2001 dot.com crisis and the 2007/08 – 2008 Global Financial Crisis. Some studies point out Japan’s net surplus in its international investment position and monetary policy sovereignty are the main reasons behind Japanese financial assets and the JPY’s safe haven status. While safe havens offer investors a hedge during periods of crisis, domestic currency appreciations can worsen competitiveness, negatively affecting the domestic stock market, with subsequent negative wealth effects. With a focus on Japan, also making comparisons to two other key safe haven destinations, namely Switzerland and the US, this paper aims to empirically examine real and financial spillovers due to risk-off shocks in financial markets.

Country-specific structural vector autoregression (VAR) models are estimated over the period 1990 to 2021 to identify the response of domestic output, policy uncertainty, financial markets, and capital flows in safe havens to risk-off shocks. Overall, we find that Japan responds differently in three main ways. First, in response to risk-off episodes, the yen real effective exchange rate (REER) appreciates significantly, with the effect exhibiting some persistence. Second, no significant effect on portfolio flows to Japan are found, in spite of the appreciating exchange rate effect, suggesting a rapid adjustment of financial markets to shifts in equilibrium exchange rates. Third, negative real spillovers from risk-off shocks appear to only apply to Japan with exchange rate appreciation exacerbating declines in GDP growth. We offer implications for policy makers based on these findings, aimed mainly at addressing risks to domestic financial stability that result from risk-off shocks. The remainder of the paper is organized as follows: Section 2 reviews the related literature on safe haven currencies and assets. Section 3 explores the stylized facts on risk-off episodes in safe haven financial markets. Section 4 describes the data and methodology. Section 5 analyzes the empirical results. Section 6 concludes.

2. Related Literature

This paper contributes to the literature on safe haven financial flows. During periods of heightened financial stress, it is well documented that investors re-balance their assets towards so-called safe haven currencies, which are low-yielding currencies that appreciate in times of higher global financial uncertainty (e.g., Habib and Stracca 2012). Typically, these periods are also characterized by financial fragmentation at the global level (e.g. Beck et al., 2015). Traditionally, the US dollar (USD), the Japanese yen (JPY) and the Swiss franc (CHF) are considered safe haven currencies, with Todorova (2020) noting that global investors tend to place their funds in the JPY and the CHF whenever uncertainty arises in the US stock market or the USD weakens (see also Botman, De Carvalho Filho, and Lam 2013; Ranaldo and Söderlind 2010; De Bock and De Carvalho Filho 2013; Masujima 2017; and Balcilara et al. 2020).

Among the safe haven currencies, Cho and Han (2021) observed that the yen, euro and Swiss franc are more likely to appreciate in periods of high volatility in foreign exchange markets and rising US Treasury bond yields. In addition, it is noted that the decline in US stock returns is related only to yen appreciation. According to Fatum and Yamamoto (2015), the yen appreciates as market uncertainty increases regardless of the prevailing level of uncertainty whereas all other bilateral USD rates display a nonlinear pattern. They find that the JPY is the "safest" of safe haven currencies and that only the JPY has appreciated when market uncertainty increased. The CHF and the USD are the "second safest" and "third safest", respectively. When uncertainty rises, the CHF appreciates significantly against all other currencies but the JPY, while the USD appreciates significantly against all but the JPY and the CHF.

Safe haven currencies tend to be associated with three key factors: low interest rates (carry-trade opportunity); net foreign asset positions; and highly liquid financial markets. Indeed, Habib and Stracca (2012) stress the feature of net foreign asset positions, which (after controlling for carry trade) is the most consistent and robust predictor of a safe haven status, as well as being a key indicator of country risk and external vulnerability. Other important factors characterizing safe havens include their level of financial development and the depth of liquidity in the foreign exchange market. In addition, for currencies that are subject to carry trade, the interest rate spread relative to the US is significant for advanced countries. Confirming that the unwinding of carry trade causes the yen to appreciate against the dollar, Nishigaki (2007) finds that the US stock price has a dominant impact on yen carry trade. The study also indicates that the interest rate differential between Japan and the US and the interest rate adjustment by the Bank of Japan (BOJ) does not influence carry trade activity. Related to this, Imakubo, Kamada, and Kan (2015) discuss the "currency premium" (i.e., the expected excess return on that currency) that determines the demand for safe haven currencies. In their model, the currency premium consists of two disequilibrium factors: (i) the interest rate gap (i.e., the deviation of real interest rates, domestic and foreign, from their equilibrium values; and (ii) the exchange rate misalignment (i.e., the deviation of real exchange rates from their equilibrium values).

Min, McDonald, and Shin (2016) relate a "safe haven" status to negative dynamic conditional correlations (DCCs) between equities and currencies, with currency returns negatively correlated with stock returns in safe haven countries. They also found that stock and foreign exchange volatility indexes increase DCCs for countries without safe assets, while the opposite is found for countries with safe assets, i.e., reducing DCCs. Moreover, higher country-specific risk, as measured by the US Treasury-Euro Dollar (TED) spread and credit default swap (CDS) spread, means higher DCCs, a feature of economies without safe haven status. More recently, Habib, Stracca, and Venditti (2020) noted that inertia (whether the bond behaved as a safe asset in the past) and good institutions foster a safe asset status, while the size of the debt market is also significant, reflecting the special role of the US. There are also differences depending on the level of economic development; the political risk rating and the size of the debt market are found to be important for advanced countries only, while inertia, real GDP, and external sustainability (measured by the current account) are important for emerging markets only.

Hossfeld and MacDonald (2014) identify the safe haven currencies among the G10 countries (i.e., the AUD, CAD, CHF, EUR, JPY, NOK, NZD, SEK, GBP, and USD) by making a distinction between the "safe haven" flows (i.e., capital movements by investors who believe that a particular currency area is a relatively safe place to invest) and the unwinding of currency carry trades (i.e., speculative transactions where investors seek to take advantage of interest rate differentials to generate superior returns) during a crisis. They find that the CHF and USD are safe haven currencies. The JPY appreciation in times of crisis is mainly attributable to the unwinding of the carry trades rather than caused by the "safe haven" inflows. Meanwhile, the EUR does not show any crisis-specific reaction. Grisse and Nitschka (2015) note that the yen is a better hedge than the Swiss franc because of the limited asset market size and liquidity of the Swiss franc. Beckmann and Czudaj (2016) find that the yen is exceptional in two ways: expectation errors decrease with higher uncertainty, especially for monetary policy uncertainty; the yen is the only currency to appreciate in case of higher uncertainty. Rogoff and Tashiro (2015) discuss the "exorbitant" privilege of the JPY due to the country's ability to borrow from abroad at lower rates than other countries, as well as being able to attain a cost advantage in any kind of borrowing or investment instrument in a broader sense. This is similar to the US experience in the post-war period.

Botman, De Carvalho Filho, and Lam (2013) point out the driver of yen risk-off appreciation appears to be unrelated to capital inflows (cross-border transactions) and also not linked to expectations about the relative stance of global monetary policies. Instead, they show that portfolio rebalancing through offshore derivative transactions occur contemporaneously to yen risk-off appreciations. In particular, they find that the JPY on average appreciates against the USD during the risk-off episodes in the global financial market. They find that capital inflows or expectations of the future monetary policy stance cannot explain the safe haven behavior of the JPY. Instead, changes in market participants' risk perceptions trigger derivatives trading, which leads to changes in the spot exchange rate without capital flows. Specifically, the risk-off episodes coincide with the forward hedging and reduced net short positions or a buildup of net long positions in the JPY. On the relationship between monetary policy and safe haven status, Beckmann and Czudaj (2017) argue that monetary policy is a substantial driver for exchange rate expectations and unconventional monetary policy has strong spillover effects across borders, leading to an unexpected safe haven status of the US dollar after 2008 crisis. Moreover, noting that the appreciation of safe haven currency is resilient to the changes in monetary policy, Jäggi, Schlegel, and Zanetti (2019) find that the Japanese yen and Swiss franc respond nonlinearly, depending on the direction of the effect on the exchange rate, with a stronger reaction to surprises generating an appreciation compared to surprises leading to depreciation. Additionally, both currencies also systematically respond to changes in the general market environment.

On Japan, Masujima (2017) suggests that while the yen's strength is driven by its safe haven status, this may slow down the post-crisis recovery via net exports. In addition, large-scale monetary easing as a policy response may mask financial vulnerabilities related to the sustainability of the public finances. However, Iwaisako and Nakata (2017) warn that the impact of yen appreciation on export fluctuations should not be exaggerated, noting that aggregate global demand played a more important role in the trade decline after the 2008 crisis. A more recent study by Belke and Volz (2020) provided insights on how yen appreciation contributed to the structural change of the Japanese economy – the hollowing out of Japanese industry.

Moreover, Dekle (1998) found that the price change induced by exchange rate fluctuation has sizable long-term effect on the Japanese employment market. In other work, Lam and Tokuoka (2013) observe that the yields of the Japanese government bonds (JGBs) have remained low and stable because of steady inflows from the Japanese household and corporate sector, the high domestic ownership of the JGBs, and safe haven inflows. That said, there are risks to the JGB markets, including the decline of private-sector savings (partly due to the aging population) and potential spillovers from global financial distresses (which can push the JGB yields higher). In addition, Horioka, Nomoto, and Terada-Hagiwara (2014) notes that while Japan's excessive government debt has not resulted in high economic costs in the past because of the country's robust domestic savings, it may lead to substantial costs in the future as domestic savings decline (as a result of population aging) and due to the temporary nature of foreign capital inflows to Japanese government securities.

The literature overall highlights the negative impact on the domestic economy due to a safe haven status. Habib, Stracca and Venditti (2020) note that the high demand for safe assets in crisis times can lead to a decline in the natural real interest rate, with adjustment mechanisms disrupted due to exchange rate appreciation and a contraction in global demand emanating. Transitory real appreciation may create hefty adjustment costs to the economy, and subsequently, economic dislocation when exchange rates eventually revert back (e.g., Bussière, Lopez, and Tille 2013). The longer-lasting the real appreciation and surge in capital flows, the greater the potential for vulnerabilities to build up in either private or public sector balance sheets. Moreover, in economies with already low inflation and interest rates close to the zero bound, real appreciations driven by risk-off episodes could feed deflation risks and place downward pressures on aggregate demand.

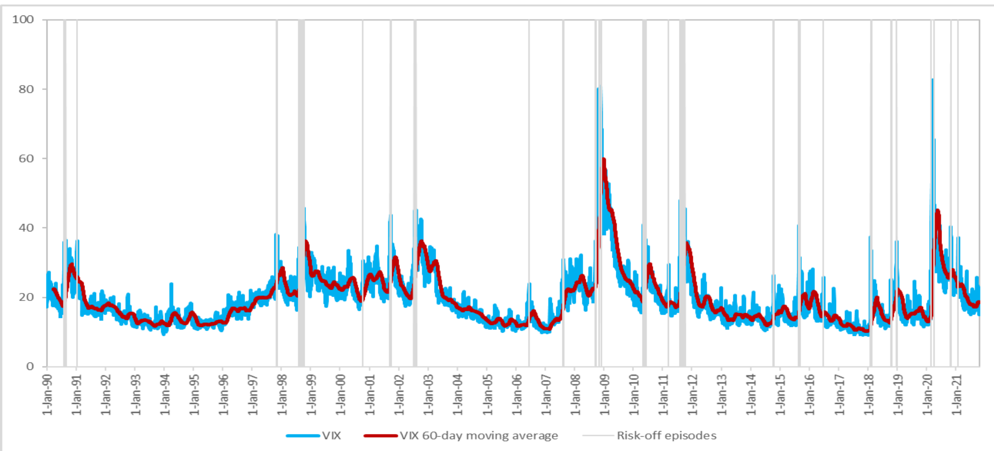
This paper contributes to the prevailing literature by empirically examining real and financial spillovers in safe haven financial flow destinations in response to risk-off shocks. Focusing on responses to risk-off shocks in output, economic policy uncertainty, financial markets, and capital flow dynamics, we aim to uncover similarities and differences across key safe haven destinations in Asia, Europe, and the US. In this way, we can examine whether the conventional characteristics of safe havens apply in a uniform manner.

3. Stylized Facts on Risk-off Episodes in Safe Haven Financial Markets

Prior to discussing the empirical approach and findings, this section presents an overview of the risk-off episode definition and the basic relation between risk-off episodes and developments in financial markets of safe havens. We focus the discussion on Japan, while also making some references to Switzerland and the US. We base our definitions of the risk-off event and episode on a method introduced by De Bock and De Carvalho Filho (2013), which uses the Chicago Board Options Exchange's Volatility Index (i.e., the VIX) as a benchmark. Botman, De Carvalho Filho, and Lam (2013) also use this method. In this method, a risk-off event occurs when the VIX is 10 percentage points higher than its 60-day backward-looking moving average (MA).

Fig. 3.1 displays the VIX movements and the risk-off episodes (i.e., series of risk-off events) from 1990 to 2021, while Table 3.1 lists the initial date of those risk-off episodes. The data sets in those two studies end in 2011, while ours continues until October 2021. We identify six risk-off episodes from 2012 to 2021, including the latest one that started in February 2020 due to the COVID-19 pandemic outbreak.

Fig. 3.1 Risk-off Episodes, 1990–2021



Note: Plotted are fluctuations in the VIX, its 60-day moving average, and our calculated risk-off episodes.

Source: Bloomberg, Authors' calculation.

Table 3.1. Risk-off Episodes Initial Date

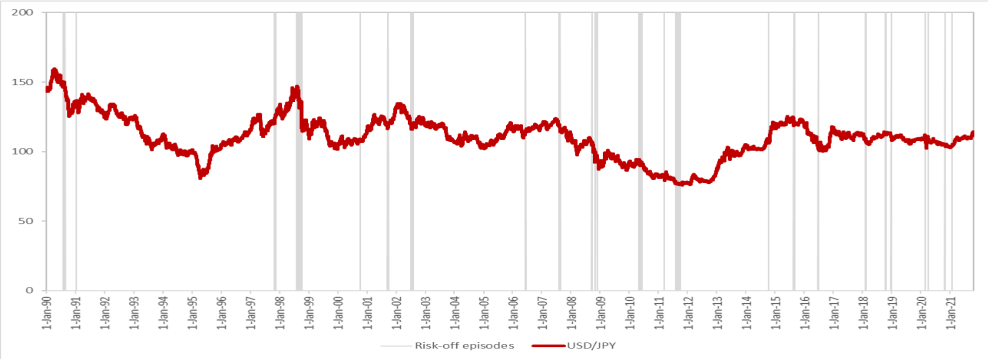
No.	Risk-off event initial date	Our Paper	Botman, De Carvalho Filho, and Lam (2013)	De Bock and de Carvalho Filho (2013)
1	US savings and loans	03-Aug-90	03-Aug-90	---
2	Iraq War	14-Jan-91	14-Jan-91	---
3	Escalation of Asian crisis	29-Oct-97	29-Oct-97	29-Oct-97
4	Concerns on Russian economy	04-Aug-98	04-Aug-98	04-Aug-98
5	Fear of slowing US economy	12-Oct-00	12-Oct-00	12-Oct-00
6	9/11 Attacks	17-Sep-01	11-Sep-01	17-Sep-01
7	Fear of slowing US economy	10-Jul-02	10-Jul-02	10-Jul-02
8	Concerns over rising US inflation	13-Jun-06	---	---
9	BNP Paribas halts withdrawals from three money market mutual funds	09-Aug-07	10-Aug-07	10-Aug-07
10	Disruptions in USD money markets	---	12-Nov-07	12-Nov-07
11	Lehman failure	17-Sep-08	17-Sep-08	17-Sep-08
12	Greek crisis	06-May-10	06-May-10	06-May-10
13	Uncertainty over impact of Japan’s March 11 earthquake	16-Mar-11	16-Mar-11	16-Mar-11
14	Confrontation over US debt ceiling and deterioration of crisis in Euro Area	04-Aug-11	04-Aug-11	04-Aug-11
15	Concerns over deteriorating earning reports (US stock markets fall)	13-Oct-14	---	---
16	Thalys train attack, rising tension in Korean Peninsula	21-Aug-15	---	---
17	UK referendum decision: Brexit	24-Jun-16	---	---
18	Concerns over rising US inflation	05-Feb-18	---	---
19	Concerns over rapidly rising US interest rates	11-Oct-18	---	---
20	Concerns over COVID-19 outbreak	24-Feb-20	---	---

Source: Botman, De Carvalho Filho, and Lam (2013), De Bock and De Carvalho Filho (2013), Bloomberg, Authors’ calculation.

The JPY is both a hedge and a safe haven currency. Global investors buy the JPY both when they seek to hedge their investments during the risk-off episodes. The JPY tends to appreciate against the USD during the risk-off episodes (Fig. 3.2).[3]

[3] The period following the Russian Federation’s invasion of Ukraine on 24 February 2022 coincided with a sharp depreciation of the yen relative to the US dollar, which was driven by widening yield differentials between the US and Japan and rising energy prices worsening Japan’s balance of payments position. Aggressive monetary policy tightening by the US Federal Reserve to combat inflation, in conjunction with higher global risk aversion, meant that the US dollar soared during this period at the global level, including against traditional safe haven currencies such as the yen and Swiss franc. While risk aversion at the global level prevailed, higher relative yields in the US and pressure on the yen are consistent with the assertion of market commentators such as Clynch (2020), who inferred at that time that the return of risk appetite could test further the safe haven status of the yen. Although risk appetite did not resume, higher yields in the US relative to Japan triggered net capital outflows and currency depreciation. The trend halted in November 2022, however, when the US signaled a slowdown in the pace of its monetary tightening.

Fig. 3.2 USD/JPY and Risk-off Episodes



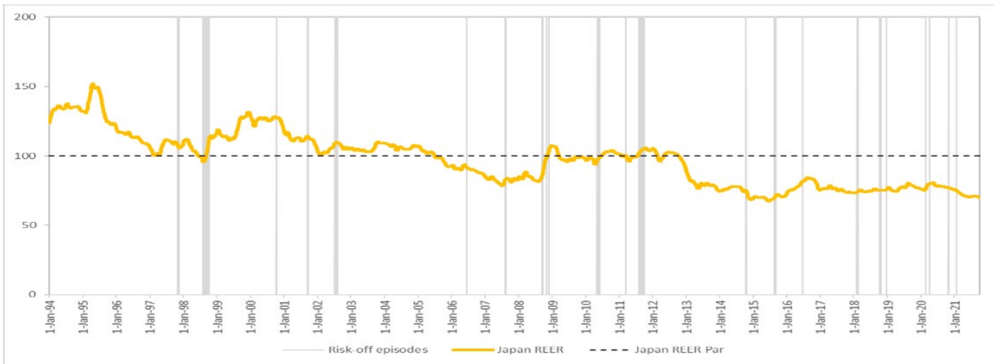
Note: Plotted are the bilateral nominal USD/JPY exchange rate and our calculated risk-off episodes.
Source: Bloomberg, Authors' calculation.

At the start of the global financial crisis (GFC) in 2007, investors tilted portfolios towards the yen in search of safety amid prevailing high-risk aversion. The yen appreciated strongly during this period as a result. This appreciation was bolstered by the Tōhoku earthquake and tsunami of March 2011 as Japanese investors sold assets denominated in foreign currency. At the end of 2012, the announcement by new Japan Prime Minister Shinzo Abe that the Bank of Japan should aggressively increase the money supply to stimulate inflation (part of the so-called "Abenomics" strategy) triggered a yen depreciation. Global risk appetite resumed as investors engaged in yen carry-trade operations, borrowing in the low-yielding yen to fund investment in risky assets abroad. An equity market shock and slowdown in the People's Republic of China (PRC) at the end of 2015 increased global market volatility, leading to yen safe haven flows, marking the end of the depreciating trend that had been in place since the end of 2012. Since 2017, the yen has been relatively stable, even during the COVID-19 crisis, although a mild appreciation was observed in the first stage of the pandemic, which later reverted towards the end of 2020 as uncertainty declined. This has prompted some analysts (e.g., Clynch 2020; Lewis 2020) to suggest that the JPY may no longer be as attractive as a safe haven as it was in the 2000s. Demand for the JPY during the risk-off episodes in recent years has been weaker than in two decades ago. The JPY's trading range has stabilized below 10%, much lower than in the 2000s (Clynch 2020).

Lewis (2020) lists three main reasons behind the underperforming JPY (i.e., it appreciated less than market players' expectations): (1) Japan is suffering from trade deficits; (2) Japanese assets managers continue to purchase foreign assets; and (3) Japanese companies are increasingly investing overseas. Clynch (2020) argues that in the short run, global demand for the JPY can keep the currency's safe haven status despite the massive policy easing by the Bank of Japan (BOJ) during the COVID-19 pandemic. That said, the BOJ's monetary policy easing may pose a threat to the JPY's safe haven status in the long run when the risk appetite is back among global investors.

Fig. 3.3 displays Japan's REER and the risk-off episodes (the REER value below the par value means that the JPY is undervalued, while it is overvalued when above the par). Japan's REER also tends to increase during the risk-off episodes. That said, the JPY has been undervalued since October 2012. It implies that from that month until the end of September 2021 (the last REER observation in our study), the JPY appreciation during the risk-off episodes might not severely harm Japan's competitiveness in international trade.

Fig. 3.3 Japan's REER and Risk-off Episodes



Note: Plotted are Japan's real effective exchange rate and our calculated risk-off episodes.
Source: BIS, Authors' calculation.

Bond markets, stock markets, and capital flow dynamics in safe haven destinations are also subject to sharp fluctuations during periods of heightened financial stress in global markets.[4] For example, the yields of Japanese government bonds (JGBs) tend to fall during the risk-off episodes due to purchases of JGBs by domestic and foreign investors. Although the Japanese government debt and the amount of the debt securities (i.e., JGBs and Japanese government treasury bills—JGTs) has continued to pile up during the COVID-19 pandemic, global investors still perceive Japanese debt securities as safe haven assets because most of these securities are owned by Japanese domestic investors. The yield of the 10-year JGB rose by at most 24 basis points during the COVID-19 pandemic episode from 24 February to 7 April 2020.

Based on Japan's Ministry of Finance (MoF) data as of the end of June 2021, foreign ownership of the Japanese debt securities was recorded at ¥161.8 trillion (about 13% of the total amount of government debt securities). Foreign investors place their funds mostly in the JGTs, where the amount of foreign ownership stood at ¥85.4 trillion (around 51% of the total amount of the JGTs) as of the end of June 2021. While foreign investors place almost the same amount in the JGBs (¥76.4 trillion as of the end of June 2021), the share of foreign ownership of foreign investors was only about 7% of the total amount of JGBs (Ministry of Finance Japan 2021). Japanese debt securities investors have been heavily skewed toward advanced economies that have large, deep, mature debt securities markets, particularly the United States and the European Union countries (Shirai and Sugandi 2019). These countries' government debt securities have higher yields than those of Japan, thus offering a higher rate of return to Japanese investors. The low interest rates in Japan have also induced foreign investors to conduct carry trading, i.e., by borrowing in Japan for investment in higher-yield countries.

While foreign investors have continued to engage in carry trading during the COVID-19 pandemic, the rising yields of long-term JGBs vis- à-vis the yields of other advanced economies—largely due to "unlimited scope and length" of the BOJ quantitative—narrow the premium of Japanese investors' overseas investments (Clynch 2020; Pattanaik 2020). This can discourage Japanese investors from investing abroad and foreign investors from participating in the carry trade.

[4] Please refer to Figs A1.1 to A1.7 in Appendix for details.

Unlike the JGBs (and Japanese debt securities), the Japanese equities are not regarded as safe haven assets, however. The benchmark Nikkei-225 stock market index falls during the risk-off episodes, in line with the movements of major stock indexes in the United States. Foreign investors have a substantial investment in Japanese equities, with a share of around 30% in 2020 (CEIC data). Japan's cross-border equity investment liabilities towards the United States account for more than half of Japan's cross-border equity (International Monetary Fund 2021; Shirai and Sugandi 2019). On capital flows, foreign investors tend to reduce their investment in Japanese equities during the risk-off periods. Meanwhile, the net flows of other investments (i.e., cross-border banking flows) to Japan tend to be, but not always, positive or increasing during the risk-off periods. Japanese investors tend to repatriate their overseas investment to domestic banking accounts during the risk-off episodes.

Overall, it is apparent from the basic analysis of the raw data that risk-off episodes are associated with appreciating exchange rates in safe havens. The extent of these appreciations may not be uniform across the major safe haven destinations, however. In our empirical analysis, we assess the magnitude and duration of impact through which risk-off shocks affect real and financial variables in Japan, Switzerland, and the US.

4. Data and Empirical Methodology

We employ a VAR approach to investigate the main financial channels during the risk-off episodes. While focusing on the Japanese financial markets, we also estimate the VAR models for Switzerland and the United States for comparison. The generic form of our VAR models is specified as follows:

$$Y_t = \sum_{\tau=1..k} Y_{t-\tau} A_\tau + X_t B_t + c_t + \varepsilon_t \tag{1}$$

where Y_t is the vector of endogenous variables; X_t is the matrix of exogenous variables; A_τ and B_t are the coefficient matrixes; c_t is the vector of constants; and ε_t is the vector of error terms. Indexes t and τ are the time indexes, while k is optimum time lag for the VAR model based on the Akaike Information Criterion (AIC).

The endogenous variables in our VAR model for each country based on their ordering are: (1) risk-off events ("Risk-off"); (2) log of world uncertainty index ("WUI"); (3) the spread between the yield of Japan's or Switzerland's 10-year government bond and the 10-year US Treasury bond ("Spread") (only for the Japanese and the Swiss models, not used for the US models); (4) log of real gross domestic product ("RGDP"); (5) log of the real effective exchange rate ("REER"); (6) log of the stock market index ("Stock Index"); (7) net portfolio investment inflows to debt securities as a percentage of nominal GDP ("Debtsec"); (8) net portfolio investment inflows to equity as a percentage of nominal GDP ("Equity"); (9) net other portfolio investment inflows as a percentage of nominal GDP ("Other"); and (10) net inflows of direct investment as a percentage of nominal GDP ("Direct"). The exogenous variables are the time dummy and the seasonal dummies.

For each country, our baseline model is a recursively restricted VAR, with impulse responses generated based on a one-standard deviation structural shock on the Risk-off variable relative to all endogenous variables in each model. A Cholesky identification scheme is used. We also estimate the VAR in unrestricted form. In the unrestricted VAR estimation, each endogenous variable in the model is affected by the lagged values of itself and other endogenous variables. In the restricted estimation, we set the model so that the Risk-off variable is only affected by its own lagged values, while other endogenous variables are affected by each other's lagged values and are affected by the lagged values of the Risk-off variable. To make the restrictions, we set the values of other endogenous variables in the Risk-off equation line of the coefficient matrix A_τ to zeros while keeping the values of the Risk-off variable as it is. Confidence intervals at the 95% level are provided.

We use working days data that span from 14 January 1999 to 31 March 2021 for Japan, 13 March 2007 to 31 March 2021 for Switzerland, and 15 January 2002 to 31 March 2021 for the United States.[5] The WUI and the REER data are converted from monthly to working days frequency using the quadratic interpolation method. Data of the RGDP, Debtsec, Equity, Other, and Direct variables are converted from quarterly to working day frequency also using the interpolation method. Table 4.1 lists the endogenous variables, data, and data sources.

[5] The use of daily data is used given that the risk-off shocks occurred on a specific day. Using lower frequency such as monthly or quarterly dilutes the impact of the shock, which would be spread over a longer time dimension. Interpolating time series to higher frequencies is commonly undertaken in order to address issues related to conducting empirical work that combines financial and macroeconomic time series. In particular, interpolating from lower to higher frequencies is not regarded as being problematic given that the lower frequency variables, such as output, are slower-moving so that any errors due to the interpolation would only marginally affect the impact variables (e.g., Danielsson et al. 2018). Nonetheless, in order to allay concerns, we have also carried out the analysis using weekly and monthly frequencies. We find that the shapes of the baseline daily impulse response functions (IRFs) are generally consistent with those of the weekly and monthly IRFs. For daily compared to monthly IRFs, the wide disparity in the underlying time horizon implies that caution is needed in making comparisons based on IRF shapes, e.g. 40 periods of daily would equate to 2 months (i.e. 20 business days per month), while 40 periods of monthly would equate to over 3 years.

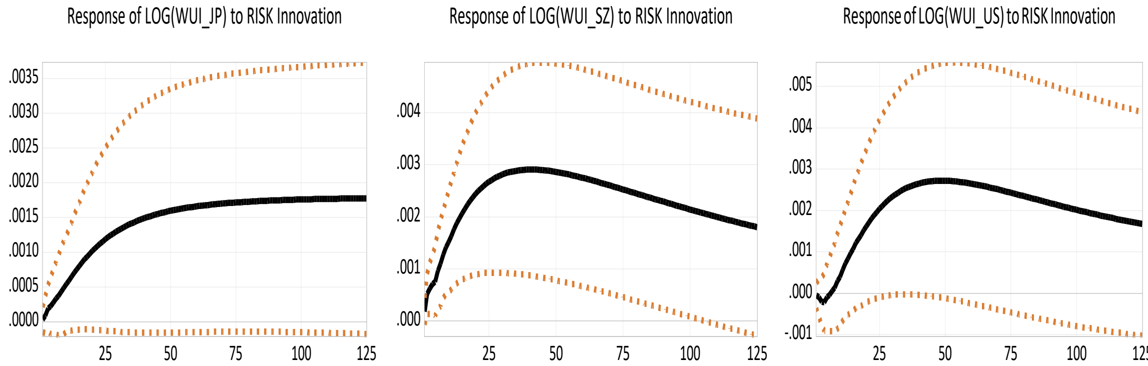
Table 4.1 Variables, Data, and Data Source

Variable	Data	Data Source
Risk-off	Risk-off event (value: 0, 1)	Authors' calculation
Log (WUI)	World Uncertainty Index (WUI) (index unit)	https://worlduncertaintyindex.com/
Spread (vis-à-vis US)	- 10-year Japanese government bond yield (%) - 10-year Switzerland government bond yield (%) - 10-year US Treasury government bond yield (%)	Bloomberg
Log (RGDP)	Constant price GDP	International Monetary Fund (IMF)
Log (REER)	Real effective exchange rate, broad index (2010 = 100, index unit) of: - Japan - Switzerland - United States	Bank for International Settlements (BIS)
Log (Stock_Index)	- Nikkei-225 Index - Swiss Market Index - S&P 500 Index	Bloomberg
Debtsec	- Net portfolio investment inflows to debt securities (USD billion) - GDP at current price (local currency billion) - USD/JPY and CHF/USD exchange rate	- International Monetary Fund (IMF) - CEIC - Bloomberg
Equity	- Net portfolio investment inflows to equity (USD billion) - GDP at current price (local currency billion) - USD/JPY and CHF/USD exchange rate	International Monetary Fund (IMF)
Other	- Net other portfolio investment (USD billion) - GDP at current price (local currency billion) - USD/JPY and CHF/USD exchange rate	- International Monetary Fund (IMF) - CEIC - Bloomberg
Direct	- Net direct investment (USD billion) - GDP at current price (local currency billion) - USD/JPY and CHF/USD exchange rate	- International Monetary Fund (IMF) - CEIC - Bloomberg

5. Empirical results

Figs. 5.1 to 5.7 display the main impulse response function (IRF) results from the restricted VAR models for Japan, Switzerland, and the US.[6] Real and financial spillovers to Japan from risk-off shocks are evident via a number of transmission channels. While risk-off episodes are typically associated with a flight to safe havens, rising uncertainty in markets can also be broad-based, with safe haven destinations also subject to heightened uncertainty. Our impulse responses indicate that risk-off shocks lead to a rise in economic policy uncertainty in the case of Switzerland, while the responses for Japan and the US are not statistically significant (Fig. 5.1).

Fig. 5.1 Economic Policy Uncertainty Responses to Risk-off Shocks in Safe Havens



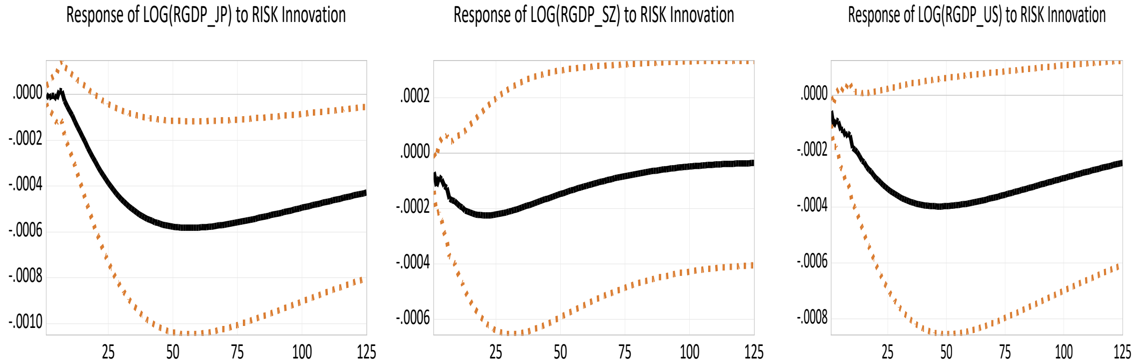
Note: Reported are the IRFs based on equation (1), using a Cholesky identification scheme and block recursive restriction; 95% confidence intervals are provided by the dotted lines; the vertical axis represents percentage points while the horizontal axis refers to the number of days.

The results for Switzerland indicate a significant rise in economic policy uncertainty due to a risk-off shock, which persists until around 100 days, with a peak impact apparent after around 40 days. In the case of Japan, the lack of significant impulse responses could be linked to the much more muted impact of risk-off episodes on the volatility of Japanese financial markets given strong expectations of overseas net portfolio holdings repatriated back to Japan priced in by market participants. Expectations priced in by markets can also help to explain the lack of significance in the US.

During risk-off episodes, it may be intuitive to expect a broad-based decline in global demand from both trade and financial channels. On real spillovers of risk-off shocks to safe havens (Fig. 5.2), there is clear evidence of a significant negative impact on GDP growth in the case of Japan. From around 25 days after the risk-off shock, a statistically significant decline in GDP growth is found, with the negative impact peaking at around 50 days. By contrast, risk-off shocks do not significantly affect the trajectory of output growth overall in Switzerland and the US, apart from some short-lived impacts at low time horizons. The findings may be linked to the stronger interconnectedness of Japan in regional trade and finance vis-à-vis emerging Asian economies that may be vulnerable to risk-off shocks. Negative spillovers to Japan via these channels can emanate due to economic downturns in these economies in crisis times.

[6] The full set of IRFs for both the restricted and unrestricted models are available from the authors upon request. We focus the discussion primarily on statistically significant IRFs at conventional levels.

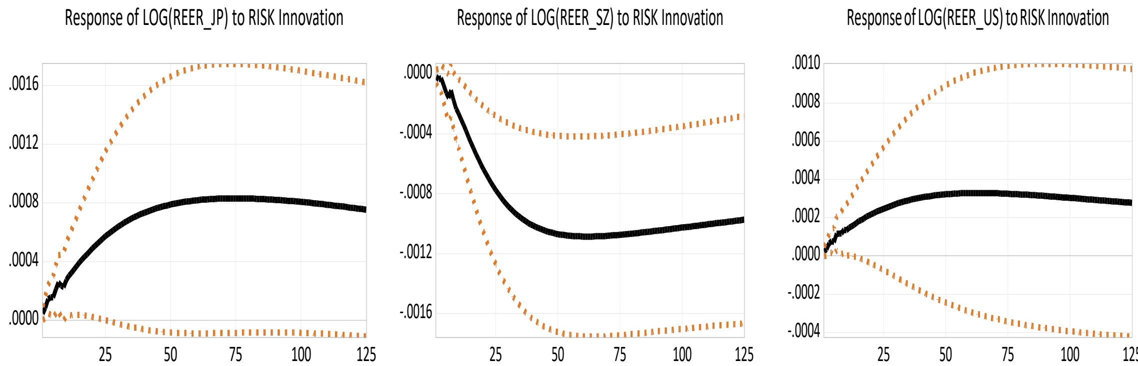
Fig. 5.2 Output Growth Responses to Risk-off Shocks in Safe Havens



Note: Reported are the IRFs based on equation (1), using a Cholesky identification scheme and block recursive restriction; 95% confidence intervals are provided by the dotted lines; the vertical axis represents percentage points while the horizontal axis refers to the number of days.

Financial flows to safe havens due to risk-off episodes can be expected to lead to real exchange rate appreciations, the competitiveness impacts of which can be important contributors to negative real effects over time. The precise effect also depends on the speed of possible policy reaction to counteract appreciating exchange rates, such as intervention by the central bank in foreign exchange markets. Our impulse response analysis shows differences in the responses of REERs for Japan, Switzerland, and the US to risk-off shocks (Fig. 5.3).

Fig. 5.3 REER Responses to Risk-off Shocks in Safe Havens



Note: Reported are the IRFs based on equation (1), using a Cholesky identification scheme and block recursive restriction. 95% confidence intervals are provided by the dotted lines. The vertical axis represents percentage points while the horizontal axis refers to the number of days.

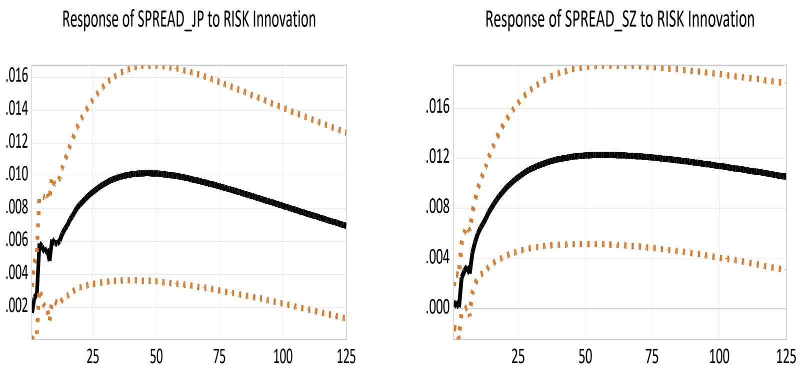
For the Japanese yen, a risk-off shock results in the expected exchange rate appreciation. This appreciating effect of the yen loses significance at around 25 days. For the US, the appreciating effect loses significance almost immediately. By contrast, we find the opposite effect for Switzerland, with the Swiss franc depreciating significantly, which may seem to be counterintuitive.[7] It is important to bear in mind, however, that the Swiss central bank actively and aggressively intervenes in foreign exchange markets to ensure the competitiveness in its export-dependent economy. The Bank of Japan, by contrast, tends not to intervene strongly in foreign exchange markets. This is underpinned by its preference to allow interest

[7] Over longer shock horizons with lower frequency data (such as weekly or monthly), we do not find a significant of the response for the Swiss franc. This can be related to its higher sensitivity to daily frequency induced shocks.

rate yield differentials vis-à-vis the US bear the adjustment, which would keep the USD–JPY bilateral exchange rate in a narrow range. In the case of Japan, the response of GDP growth given an appreciating REER suggests that exchange rate and trade channels are important contributing factors to negative real spillovers. In other words, safe haven financial flows to Japan in periods of heightened global risk aversion dampen economic growth via appreciating exchange rate effects.

On long-term yield differentials relative to the US (Fig. 5.4), also reflecting rising country-specific risk, a significant and positive response of long yield spreads for the Japanese government bond market is found. The effect peaks at around 40 days after the risk-off shock but remains persistent and statistically significant over the estimation duration. A similar effect is found in the case of Swiss bond spreads.

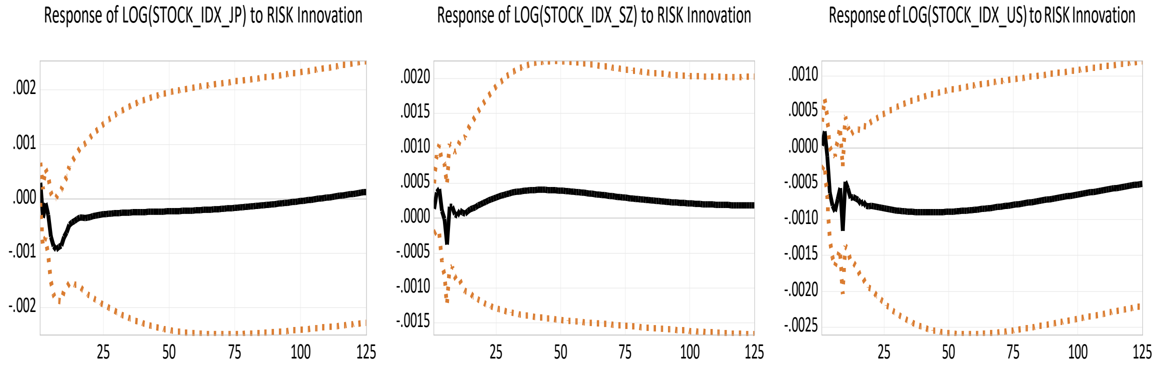
Fig. 5.4 Sovereign Bond Spread Responses to Risk-off Shocks in Safe Havens



Note: Reported are the IRFs based on equation (1), using a Cholesky identification scheme and block recursive restriction; 95% confidence intervals are provided by the dotted lines; the vertical axis represents percentage points while the horizontal axis refers to the number of days.

On sovereign bond spreads, widening spreads underscore the dominance of the US dollar as the main global safe haven currency. During risk-off episodes, flows to US Treasuries imply that even other safe haven destinations for financial flows will face some amplification in sovereign borrowing costs. This also relates to the theoretical literature pointing towards a significant role of liquidity in affecting asset pricing (e.g., Vayanos 2004; Caballero and Krishnamurthy 2008; Brunnermeier and Pederson 2009). For the stock market in safe havens (Fig. 5.5), the response to a risk-off shock is not significant for Japan and Switzerland. For the US, the results are also mostly not significant, apart from short-lived negative significant effects in the days after the shock. Time varying impacts of market volatility on equity risk premia and risk aversion across safe havens are important considerations in this regard (e.g., Bekaert, Engstrom, and Xing 2009).

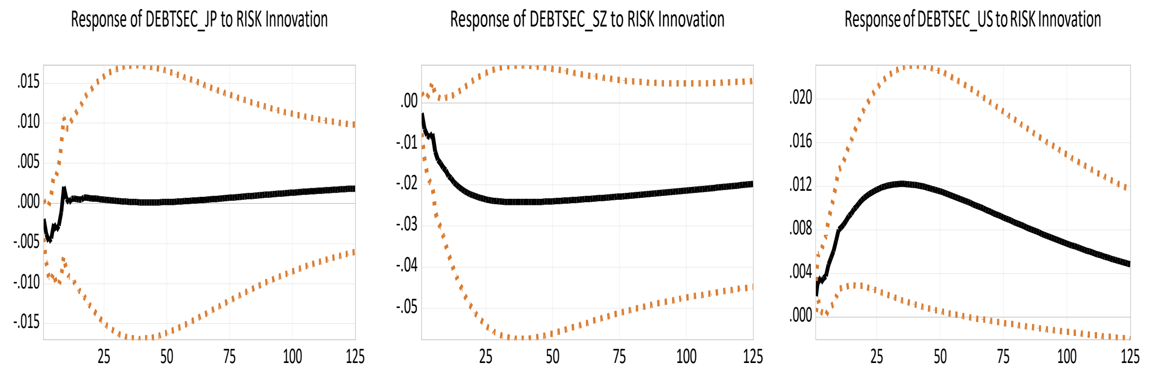
Fig. 5.5 Equity Market Responses to Risk-off Shocks in Safe Havens



Note: Reported are the IRFs based on equation (1), using a Cholesky identification scheme and block recursive restriction; 95% confidence intervals are provided by the dotted lines; the vertical axis represents percentage points while the horizontal axis refers to the number of days.

Turning to capital flows, Figs. 5.6 and 5.7 show the responses in portfolio debt and portfolio equity flows to safe havens. Overall, and contrary to expectations, we do not find a significant effect in the response of portfolio flows to Japan to a risk-off shock. That said, this finding is in line with Botman, De Carvalho Filho, and Lam (2013), who suggest that Japan may be subject to portfolio rebalancing triggered by risk-off episodes via offshore trading in derivatives markets that are not captured in official balance of payments transactions.

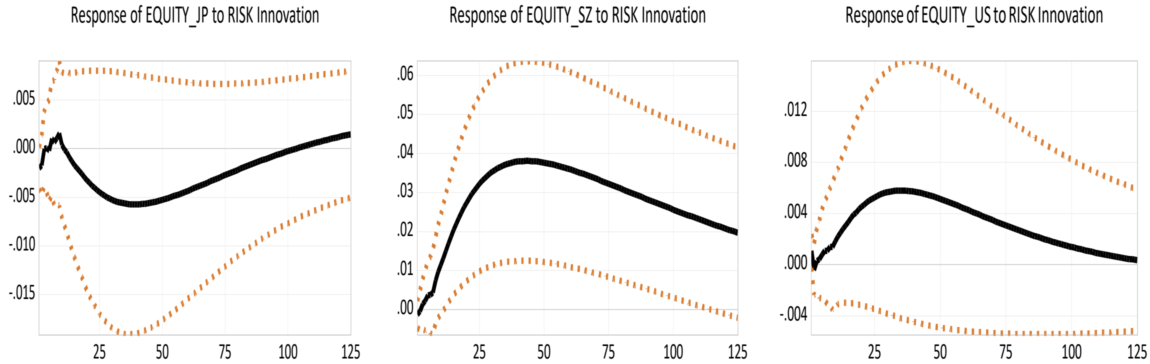
Fig. 5.6 Portfolio Debt Flow Responses to Risk-off Shocks in Safe Havens



Note: Reported are the IRFs based on equation (1), using a Cholesky identification scheme and block recursive restriction; 95% confidence intervals are provided by the dotted lines; the vertical axis represents percentage points while the horizontal axis refers to the number of days.

While no significant effect is found for portfolio debt responses to risk-off in the case of Japan (similar to Switzerland), the response in the US is positive and statistically significant a few days after the shock up until around 50 days. This is in line with conventional expectations that in times of heightened financial stress, US debt securities are the preferred destination for financial flows by investors. As described by Caballero and Krishnamurthy (2009), these flows of primarily nonspeculative capital can lead to excessive leverage by financial institutions as risk-free assets are sold to foreigners during these periods. On portfolio equity responses (Fig. 5.7), we find some evidence of a statistically significance positive response for equity flows to Switzerland after around 10 days, but no significance for Japan or the US.

Fig. 5.7 Portfolio Equity Flow Responses to Risk-off Shocks in Safe Havens



Note: Reported are the IRFs based on equation (1), using a Cholesky identification scheme and block recursive restriction; 95% confidence intervals are provided by the dotted lines; the vertical axis represents percentage points while the horizontal axis refers to the number of days.

Comparing real and financial spillovers from risk-off shocks in Japan to the case of the US and Switzerland suggests an idiosyncrasy to the dynamics at play in Japan. On real spillovers, the effect of risk-off shocks has only a marginal and short-lived significant effect on GDP growth in the US or Switzerland, while a statistically significant negative effect of greater magnitude is found for Japan after around 25 days, remaining persistent thereafter. Japan's protracted lower relative GDP growth may make it more susceptible to shocks. On financial spillovers, the exchange rate effect is specific to Japan, weighing on competitiveness and growth.

6. Conclusions

This paper examines real and financial spillovers of risk-off episodes in safe haven destinations over the period 1990 to 2021. The empirical findings are summarized in Table 6.1.

Table 6.1 Summary of empirical findings

	Japan	Switzerland	US
Economic policy uncertainty	Insignificant	Significant (+)	Insignificant
Output growth	Significant (-)	Insignificant	Insignificant
REER	Significant (+)	Significant (-)	Insignificant
Sovereign bond spread	Significant (+)	Significant (+)	n/a
Equity market	Insignificant	Insignificant	Significant (-)
Portfolio debt flow	Insignificant	Insignificant	Significant (+)
Portfolio equity flow	Insignificant	Significant (+)	Insignificant

We show that dynamics for Japan appear to be different to those of Switzerland and the US in three main ways. First, in response to risk-off episodes, the yen REER appreciates significantly, with the effect exhibiting signs of persistence. Second, no significant effect on portfolio flows to Japan are found, in spite of the appreciating exchange rate effect, suggesting a rapid adjustment of financial markets to shifts in equilibrium exchange rates. Third, negative real spillovers from risk-off shocks appear to only apply to Japan.

Our findings have several implications for policymakers. Effective exchange rate management would appear to be particularly important in the case of Japan, also related to its transmission to the real economy. Policy makers should be vigilant to potential volatility in currency markets and ensure that appropriate countercyclical adjustment mechanisms are available from the suite of fiscal, monetary, and macroprudential policy tools. In addition, persistence identified in bond spreads and portfolio debt securities may contribute to a buildup of financial vulnerabilities and risks to financial stability over the longer term, underscoring the need for careful monitoring by policymakers, in particular by central banks and financial supervisors.

Acknowledgments

We are grateful for comments received on the paper at the Japan Finance Association/PBFJ conference at Kyushu University, Japan on 14–15 March 2022.

References

Balcilara, M., Demirer, R., Gupta, R., and Wohar, M.E. 2020. The Effect of Global and Regional Stock Market Shocks on Safe Haven Assets. *Structural Change and Economic Dynamics* 54: 297–308.

Baur, D., and Lucey, B. 2010. Is Gold a Hedge or a Safe Haven? An Analysis of Stocks, Bonds and Gold. *The Financial Review* 45(2): 217–229.

Beck, R., Beirne, J., Paterno, F., Peeters, J., Ramos-Tallada, J., Rebillard, C., Reinhardt, D., Weissenseel, L., and Wörz, J. 2015. The Side Effects of National Financial Sector Policies – Framing the Debate on Financial Protectionism. ECB Occasional Paper No. 166.

Beckmann, J., and Czudaj, R.2016. Exchange Rate Expectations and Economic Policy Uncertainty. *European Journal of Political Economy* 47: 148–162.

Beckmann, J., and Czudaj, R. 2017. Exchange Rate Expectations Since the Financial Crisis: Performance Evaluation and the Role of Monetary Policy and Safe Haven. *Journal of International Money and Finance* 74: 283–300.

Bekaert, G., Engstrom, E., and Xing, Y. 2009. Risk, Uncertainty, and Asset Prices. *Journal of Financial Economics* 91(1): 59–82.

Belke, A., and Volz, U. 2020. The Yen Exchange Rate and the Hollowing Out of the Japanese Industry. *Open Economies Review* 31: 371–406.

Botman, D., De Carvalho Filho, I., and Lam, W.R. 2013. The Curious Case of the Yen as a Safe Haven Currency: A Forensic Analysis. IMF Working Paper WP/13/228. Washington, DC: International Monetary Fund.

Brunnermeier, M. K., and Pederson, L.H. 2009. Market Liquidity and Funding Liquidity. *Review of Financial Studies* 22: 2201–2238.

Bussière, M., Lopez, C., and Tille, C. 2013. Currency Crises in Reverse: Do Large Real Exchange Rate Appreciations Matter for Growth? MPRA Paper No. 44096.

Caballero, R. J., and Krishnamurthy, A. 2008. Collective Risk Management in a Flight to Quality Episode. *Journal of Finance* 63: 2195–2230.

Caballero, R. J., and Krishnamurthy, A. 2009. Global Imbalances and Financial Fragility. *American Economic Review* 99(2): 584–588.

Cho, D., and Han, H. 2021. The Tail Behavior of Safe Haven Currencies: A Cross-quantilogram Analysis. *Journal of International Financial Markets, Institutions & Money* 70: 101257.

Clynch, H. 2020. The Puzzle of the Japanese Yen's Safe-Haven Status. https://disruptionbanking.com/2021/02/10/the-puzzle-of-the-japanese-yens-safe-haven-status/

Danielsson, J., Valenzuela, M., and Zer, I. 2018. "Learning from History: Volatility and Financial Crises". *The Review of Financial Studies* 31(7): 2774–2805.

De Bock, R., and De Carvalho Filho, I.. 2013.The Behavior of Currencies during Risk-off Episodes. IMF Working Paper 13/08.

Dekle, R. 1998. The Yen and Japanese Manufacturing Employment. *Journal of International Money and Finance* 17:785–801

Fatum, R., and Yamamoto, Y. 2015. Intra-safe Haven Currency Behavior During the Global Financial Crisis. *Journal of International Money and Finance* 66: 49–64.

Grisse, C., and Nitschka, T. 2015. On financial risk and the safe haven characteristics of Swiss franc exchange rates. *Journal of Empirical Finance* 32: 153-164.

Habib, M. H., and Stracca, L. 2012. Getting Beyond Carry Trade: What Makes a Safe Haven Currency? *Journal of International Economics* 87(1): 50–64.

Habib, M. H., Stracca, L., and Venditti, F. 2020. The Fundamentals of Safe Assets. *Journal of International Money and Finance* 102: 102119.

Horioka, C. Y., Nomoto, T., and Terada-Hagiwara, A. 2014. Why Has Japan's Massive Government Debt Not Wreaked Havoc (Yet)? *The Japanese Political Economy* 40(2): 3-23.

Hossfeld, O., and MacDonald, R. 2014. Carry Funding and Safe Haven Currencies: A Threshold Regression Approach. *Journal of International Money and Finance* 59: 185-202.

Imakubo, K., Kamada, K., and Kan, K. 2015. A New Technique for Estimating Currency Premiums. Bank of Japan Working Paper Series No 15-E-9.

International Monetary Fund. 2021. Coordinated Portfolio Investment Survey (CPIS). https://data.imf.org/?sk=B981B4E3-4E58-467E-9B90-9DE0C3367363

Iwaisako, T. and Nakata, H. 2017. Impact of Exchange Rate Shocks on Japanese Exports: Quantitative Assessment Using a Structural VAR Model. *Journal of the Japanese and International Economies* 46:1–16.

Jäggi, A., Schlegel, M., and Zanetti, A.2019. Macroeconomic Surprises, Market Environment, and Safe-haven Currencies. *Swiss Journal of Economics and Statistics* 155:5.

Lam, W.R., and Tokuoka, K. 2013. Assessing the Risks to the Japanese Government Bond (JGB) Market. *Journal of International Commerce, Economic and Policy* 4(1):1-15.

Lewis, Leo. 2020. Why the Japanese Yen Is Not the Haven Asset it Once Was. https://www.ft.com/content/23d721cc-3609-11ea-a6d3-9a26f8c3cba4

Masujima, Y. 2017. Safe Haven Currency and Market Uncertainty: Yen, Renminbi, Dollar, and Alternatives. RIETI Discussion Paper Series 17-E-048. Tokyo: Research Institute of Economy, Trade and Industry.

Min, H-G, McDonald, J.A., and Shin, S-A. 2016. What Makes a Safe Haven? Equity and Currency Returns for Six OECD Countries during the Financial Crisis. *Annals of Economics and Finance* 17-2: 365–402.

Ministry of Finance of Japan. (2021). Japanese Government Bond: Monthly Newsletter of the Ministry of Finance, November 2021.

Nishigaki, H. 2007. Relationship Between the Yen Carry Trade and the Related Financial Variables. *Economics Bulletin* 13(2): 1-7.

Pattanaik, Swaha. 2020. Breakingviews – Japanese Bond Market Shift Spells Stronger Yen. https://www.reuters.com/article/us-japan-bonds-boj-breakingviews-idUSKBN2480FR

Ranaldo, A., and Söderlind, P. 2010. Safe Haven Currencies. *Review of Finance* 14: 385–407.

Rogoff, K. S., and Tashiro, T. 2015. Japan's Exorbitant Privilege. *Journal of the Japanese and International Economies* 35: 43–61.

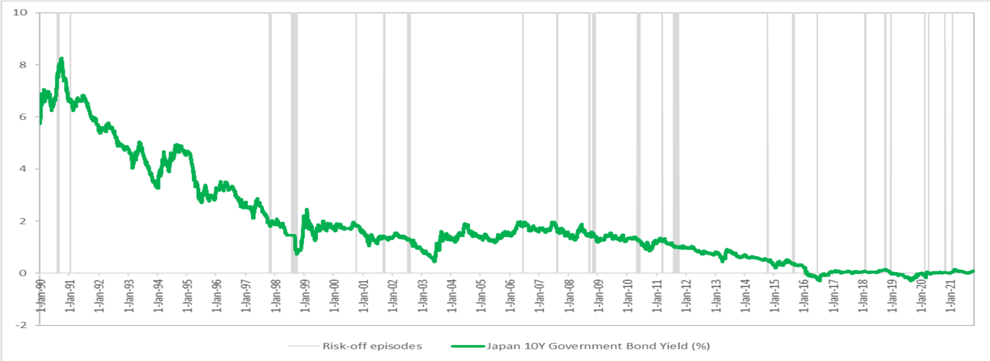
Shirai, S., and E.A. Sugandi 2019. Growing Global Demand for Cash. *International Business Research* 12(12): 74–92.

Todorova, V. 2020. Safe Haven Currencies. *Economic Alternatives* 4: 579–591.

Vayanos, D. 2004. Flight to Quality, Flight to Liquidity, and the Pricing of Risk. NBER Working Paper No. 10327.

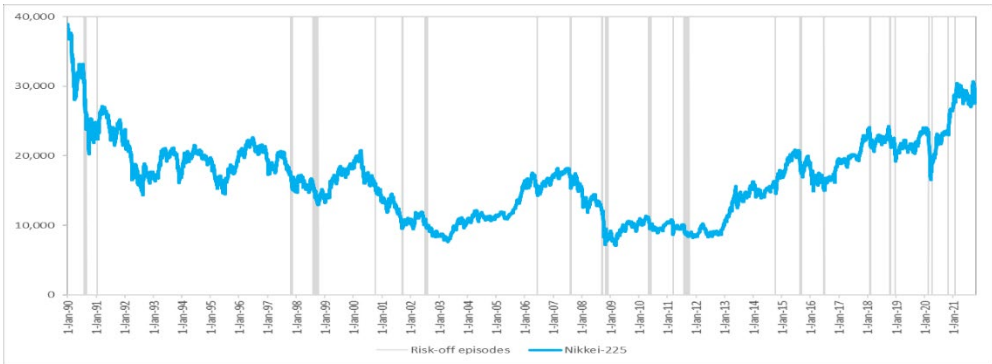
Appendix 1. Financial Markets and Risk-off Events in Japan

Fig. A1.1 The 10-Year JGB Yields (%) and the Risk-off Episodes



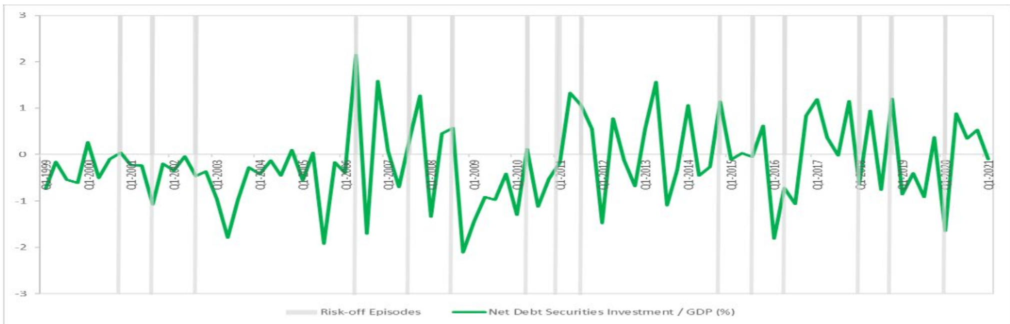
Source: Bloomberg, Authors' calculation.

Fig. A1.2 Nikkei-225 Index and Risk-off Episodes



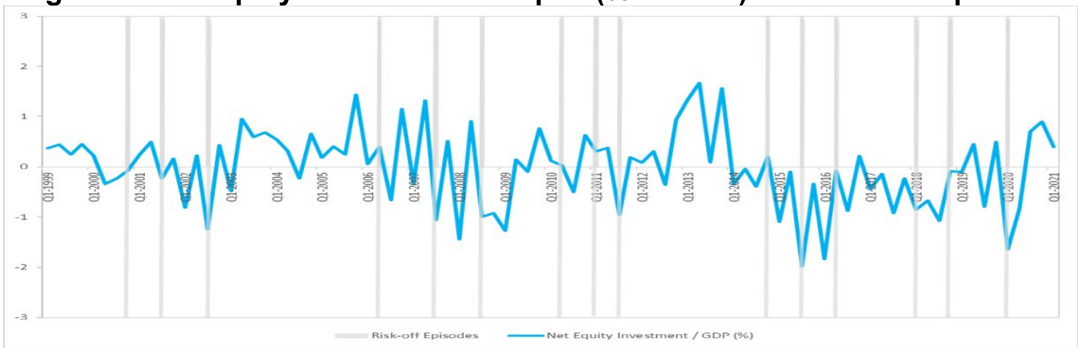
Source: Bloomberg, Authors' calculation.

Fig. A1.3 Net Debt Securities Investment to Japan (% of GDP) and Risk-off Episodes



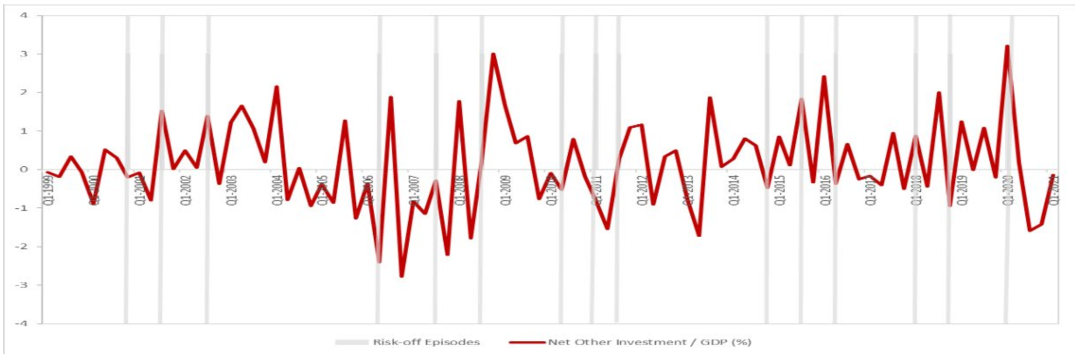
Source: Bloomberg, Authors' calculation.

Fig. A1.4 Net Equity Investment to Japan (% of GDP) and Risk-off Episodes



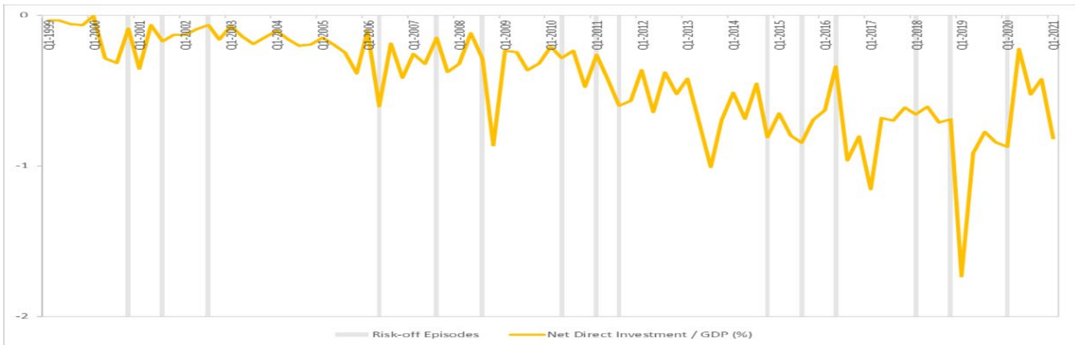
Source: Bloomberg, Authors' calculation.

Fig. A1.5 Net Other Investment to Japan (% of GDP) and Risk-off Episodes



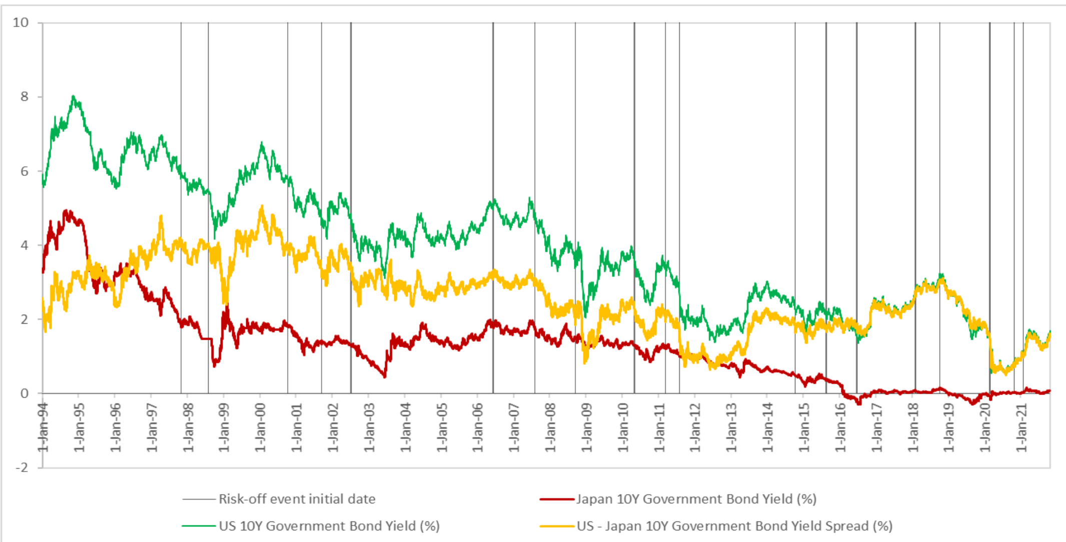
Source: Bloomberg, Authors' calculation.

Fig. A1.6 Net Direct Investment to Japan (% of GDP) and Risk-off Episodes



Source: Bloomberg, Authors' calculation.

Fig. A1.7 Yield Spread Between the 10Y US and Japan Government Bonds



Source: Bloomberg, Authors' calculation.

Could an economy get stuck in a rational pessimism bubble? The case of Japan

Vo Phuong Mai Le*
(Cardiff Business School, Cardiff University)

David Meenagh†
(Cardiff Business School, Cardiff University)

Patrick Minford‡
(Cardiff Business School, Cardiff University, and CEPR)

July 2023

Abstract

Developed economies have experienced slower growth since the 2008 financial crisis, creating fears of "secular stagnation." Rational expectations models have forward-looking bubble solutions, which could cause this; here we investigate the case of Japan. We show that a New Keynesian model with a weak equilibrium growth path driven by pessimism sunspot belief shocks matches economic behaviour. Another possibility is a conventional model where productivity growth has simply slowed down for unknown reasons. Nevertheless, a welfare-optimising approach implies fiscal policy should commit to eliminating the potential sunspot while being prepared to revert to normal policy if inflation rises.

JEL classification: E5; E6; E32

Keywords: Secular stagnation; Pessimism sunspot; Indirect Inference; DSGE model

*Email: LeVP@cardiff.ac.uk

†Corresponding author: Cardiff Business School, Cardiff University, Colum Drive, Cardiff, UK, CF10 3EU. Tel.: +44 (0)29 2087 5198. Email: MeenaghD@cardiff.ac.uk

‡Email: MinfordP@cardiff.ac.uk

1 Introduction

It is a well-known feature of rational expectations models of the future that they imply bubble solutions where expectations of future explosive reactions to current sunspots underpin these current sunspot effects. Thus people can expect a current shock to prices which is underpinned by the expectation it will lead to future price movements; a hyperinflation can occur with such a shock, which is self-validating. Prices rise today, with people expecting future rises which stimulate the demand rise today that creates the price shock — Minford and Peel (2015, chapter 2) set out these mechanics and explain how these bubbles can be eliminated from the model solution by a monetary commitment to stable prices by the central bank.

For output it is usual to assume that it tends to a natural rate equilibrium gradually over time and that it is not affected by expectations of future events; thus typically output does not depend on the future via a forward root in the model, as prices do. This implies that there will be no bubble element in the output solution. However, in recent decades we have observed puzzling stagnation episodes in output in several economies, especially since the financial crisis, leading to a wide range of views suggesting that the modern economy is vulnerable to weak self-perpetuating demand — termed 'secular stagnation'. In particular, Summers (2014) has urged in a series of policy-oriented papers that there should be expansionary fiscal policy to ward off the threat of secular stagnation. He suggests that weak demand can become converted into low long term growth by hysteresis. He postulates a world savings glut as the cause of this demand weakness. Eichengreen (2015) and Backhouse and Boianovsky (2016) review various possible causes of secular stagnation, a theory with a long history much associated with Alvin Hansen (e.g. Hansen, 1954). However, the main problem with applying any of these theories to the current situation is that today's stagnation only appeared after the financial crisis, whereas before growth seemed strong and assured. This link to the crisis is more suggestive of a sudden sunspot cause, triggered by a crisis contraction that destroyed business confidence. Meanwhile, in another series of papers Farmer (e.g. Farmer 2020), together with a wide group of authors reviewed below who focus particularly on asset bubbles, has argued persistently for the presence in the economy of real sunspot equilibria, such as those that can occur in rational expectations models — in this case as real bubbles fed by a forward root in output determination. If permitted by models through such a forward root, such equilibria are usually ruled out by transversality and other terminal conditions, such as general business knowledge of true long run productive potential or a government commitment to force this as the equlibrium via fiscal policy; plainly if no such conditions exist then these sunspot equilibria can occur and achieve the effects postulated by Summers for hysteresis — for which in itself there is not really a clear theoretical cause.

Episodes of persistently weak growth have occurred in many western economies after the financial crisis — including the US and the UK. However, in one economy, Japan, this type of stagnationary behaviour has occurred systematically since the 'bursting of the financial bubble' in 1989, a long period now stretching over more than thirty years. It therefore seems a prime candidate for a sunspot-created secular stagnation. In this paper we ask: did the bursting of a monetary bubble in Japan thirty years ago lead to the start of a real deflationary bubble in output? We investigate this question through a DSGE model of Japan estimated over a sample from 1989 to 2022. In this model, the Bank of Japan (BOJ) eliminates the possibility of a monetary bubble via its well-known commitment to price stability in the form of a low inflation target, which has the side-effect of preventing a higher inflation equilibrium; but the Japanese business sector does not appear to believe in a high output potential equilibrium while the government of Japan (GOJ) refuses to commit via fiscal policy to eliminatiing persistently weak growth, so that there is nothing to prevent a stagnationary output bubble fed by sunspots. At the heart of our model is an output production function in which the current natural rate depends on expectations of future output via their effects on capital and labour supply.

The rest of the paper is organised as follows. Section 2 reviews the literature, Section 3 introduces a DSGE with sunspots, Section 4 discusses the estimation procedure, Section 5 shows the empirical results, Section 6 reviews some policy discussions, and finally, Section 7 concludes.

2 Related literature

Our paper broadly relates to the strand of literature on liquidity traps following Summers (2014). Eggertsson, Mehrotra and Robbins (2019) formalise and quantify the secular stagnation hypothesis by constructing a series of analytic and quantitative overlapping generation models where the full-employment real interest rate is permanently negative and under certain conditions, this leads to secular stagnation, characterised by a chronically binding Zero Lower Bound (ZLB), subpar growth and inflation below target. The persistent fall in interest rates can be driven by slow-moving secular forces, such as a slowdown in population growth, an increase in life expectancy, rising income inequality, a fall in the relative price of investment goods, a slowdown in productivity growth or a deleveraging cycle. These forces alter the relative supply of savings and investment, and subsequently have effects on interest rates. Since this episode can be permanent, policymakers should not wait for a ZLB episode to end itself. They find that an aggressive fiscal policy can eliminate the secular stagnation equilibrium more effectively than a monetary policy of raising the inflation target. This works when the fiscal policy reduces the oversupply of savings and raises the natural rate of interest. Other works have also explored the relationship between demographic changes and secular growth. Gagnon, Johannsen and Lopez-Salido (2016) calibrate an overlapping-generation model with a rich demographic structure to the changes in US population, family composition, life expectancy and labour market activity. They find that these factors' declines were pronounced recently because of demographic declines due to the post-war baby boom and suggest that US real GDP and real interest rates will remain low in the coming decades. Away from the overlapping-generation models, Jones (2021) develops and estimates, using Bayesian estimation, a New Keynesian model with demographic changes, real and monetary shocks and an occasionally binding ZLB on nominal rates for the US. He finds that demographic shocks can generate slow-moving trends in real and nominal interest rates, employment and productivity in the US. This decline in interest rates brings the policy rate rate closer to the ZLB. These papers however do not consider the role of bubbles in the stagnation.

Second, there is work that relates to the branch of the literature that allows for the existence of bubbles and investigates their impact on policies and the economy. This literature recognises the fact that economic bubbles are possible important sources of macroeconomic instability. Beyond the more traditional models of rational bubbles (Samuelson, 1958; Tirole, 1985), the recent literature, such as Farhi and Tirole (2012), Martin and Ventura (2012), Miao and Wang (2012, 2014, 2018), Aoki and Nikolov (2015), Bengui and Phan (2018), and Ikeda and Phan (2019), have extended the analysis to study the role of bubbles in real models with financial frictions. In these models the main idea is that bubbles can help to relax financial constraints and improve investment efficiency, firms' borrowing capacity, investment and output.

To investigate the role of monetary policy and its potential impact on bubbles, nominal rigidities have been embedded into overlapping generations models with rational bubbles (Biswas et al., 2020; Gali, 2014, 2020; Asriyan et al., 2021). This type of model enables analysis of whether central banks should raise interest rates if they are faced with a potential bubble, referred to as a "leaning against the bubble" policy. Gali (2014) studies the interaction between rational bubbles and monetary policy in a two-period overlapping generations model with nominal rigidities. This model produces multiple equilibria and in some equilibria the "leaning against the bubble" policy is counterproductive. The reason is that the bubble component of an asset price has no payoffs to discount and the only equilibrium requirement on its size is that it grows at the rate of interest and thus a monetary tightening would increase the bubble size or fluctuations even though the objective may have been to dampen it. As a result, it concludes that if the average size of the bubble is sufficiently large, the central bank should lower interest rates to stabilise the bubble. This result is controversal and challenges the "leaning against the bubble" idea in the literature. However, there are shortcomings in this model because it only allows the bubble to have redistribution effects, and excludes any possibility of bubble-driven fluctuations, since employment and output are constant in equilibrium and the two-period overlapping generations assumption means that it is difficult to consistently match the bubble behaviour with the data. To analyse the impact of bubble fluctuations on economic activity and reconcile them with the data, Gali (2020) extends the model to incorporate an overlapping generation of finite-lived consumers into a New Keynesian model with stochastic transition to retirement. This model allows for the existence of rational expectations equilibiria with asset price bubbles depending on the retirement incidence. In particular, if the probability of retirement is sufficiently high, beside a bubbles balanced growth path, there exist multiple bubbly balanced growth paths which result in bubble-driven fluctuations. This framework allows for a much more important role for monetary policy. It finds that a "leaning against the bubble" interest rate policy can insulate output and inflation from the effects of the bubble and if aggressive enough, it can rule out bubbles themselves. However, the risk is that if policy does not succeed in eliminating bubble fluctuations, it might increase volatility and the persistence of these fluctuations. The paper also finds that large bubbles are good because expected lifetime utility increases along a deterministic balanced growth path, but their fluctuations are generally not. On the other hand, Allen et al. (2018) challenged the findings of Gali (2014). If they modified the model by Gali (2014) so that asset prices are uniquely determined, then higher rates unambiguously dampen the bubbles, but the problem remains that there is no benefit from leaning against bubbles because the friction that allows bubbles creates dynamic inefficiency in this model, and bubbles ameliorate this inefficiency. Therefore, Allen et al. (2018) propose to modify Gali's model in such a way that the economy is dynamically efficient and they introduce credit and information frictions that interfere with lending. In this environment bubbles arise when agents borrow, and against the interests of their lenders who cannot monitor them, buy risky assets and bid up their price. Asset bubbles reduce welfare by crowding out productive lending and associated activities. In this model, monetary intervention that leans against the wind can dampen a possible bubble to prevent its impact once it bursts, but it would result in a contraction across the economy too. Therefore, the benefits from intervention by monetary policy can outweigh its costs if and only if the default costs are sufficiently high because despite the adverse impact on the economy, higher rates would reduce the amount agents can borrow against bubble assets and reduce the possible default amount if the bubble burst, and make society as a whole better off.

Other studies also find an importance for monetary policy in stabilising economic bubble conditions. Asriyan et al. (2021) investigate the role of monetary policy in a bubbly world using an overlapping generations model with bubbles and money. The model includes financial frictions where entrepreneurs raise funds by issuing debts, which is backed by future output. It also includes two types of unbacked assets, one is referred to as a bubble asset issued by entrepreneurs but only backed by expectations of their future value and another is money issued by a central bank. Savers want to purchase and hold both assets issued by the entrepreneurs and also money, if inflation is low enough. Financial frictions restrict the supply of debts, but given the demand for assets, the entrepreneurs would issue unbacked assets. As long as the expected return to these unbacked assets is sufficiently high, the world is a bubbly one and savers will be willing to hold bubble assets in equilibrium. This framework creates an interaction between unbacked assets, bubbles and money. It finds that central banks' interventions are effective and welfare-enhancing because they expand the net supply of assets available to the private sector and decrease inefficient investment. Biswas et al. (2018) also construct a rational bubbles model combined with financial frictions and downward wage rigidities at the ZLB. The model assumes a stochastic bubble so that in each period the price of the bubbly asset can collapse to the fundamental value with an exogenous probability. The collapse of expansionary bubbles can trigger a collapse of output and cause persistent involuntary unemployment, because net worth falls, leading to contractions in credit and investment. Thus, the demand for labour from firms also contracts and given downward rigid wages, there will be rationing in the labour market, resulting in involuntary unemployment. In turn, this unemployment can lead to an endogenous and protracted recession by eroding the intertemporal allocation of resources. This then further leads to a contraction in capital investment, since entrepreneurs' ability to borrow and invest depends on their net worth. Therefore, the future capital stock will decline, causing further downward pressure on labour demand and wages, thus reducing future capital accumulation. The vicious cycle repeats and only stops when the capital stock has fallen enough below the bubbleless steady-state level. This model thus implies that a "leaning against the bubble" type of macroprudential policy intervention is warranted for excessively large bubbles. It is this model that comes closest to our work in this paper, where entrepreneurs' confidence in future returns drives investment and so current output.

Although the literature on rational bubbles is extensive, many studies, including the above, investigate the issue using theoretical models, with only a few addressing the matter using estimation. Miao et al. (2015) provide the first structural analysis of bubbles using a Bayesian DSGE framework. They construct and estimate a real business cycle model with infinitely lived agents, habit formation, investment adjustment cost and variable capacity utilisation. Firms are subject to idiosyncratic investment efficiency shocks and face endogenous credit constraints. A sentiment shock reflecting households' beliefs about the relative size of the old and the new bubble, drives the fluctuations in the bubble and transmits them through the economy via the credit constraint. The bubble emerges through this feedback loop between the credit constraint and self-fulfilling beliefs. They find that the sentiment shock explains most fluctuations in the stock market and other real variables. It generates the comovement between stock prices and the real economy and can explain the internet bubbles and Great Recession. Ikeda (2013) introduces monetary policy and wage rigidities into this model of Miao et al. (2015). He addresses moderate inflation in asset price booms in a DSGE framework. Based on Bayesian estimation on US data, he finds that nominal wage rigidities and an asset price bubble work together to cause an inefficiently excessive boom, but inflation remains moderate because they relax the financial constraints and apply downward pressure on inflation. Strict inflation targeting fails to effectively address inefficiencies caused by bubbles and also exacerbates them in the short run. He shows that the optimal monetary policy is a monetary tightening to restrain the boom at the cost of greater inflation volatility. The use of Bayesian estimation gives weight to priors that may be rejected by the data and so lead to bias in estimation.

Here we use indirect inference both to estimate and test our model rigorously against the data behaviour; the aim is to compare the reduced form data behaviour generated by the unknown true model with the simulated reduced form behaviour generated by our candidate models. In estimation we search for the paramaters of our candidate model that can most closely match the data behaviour from the true model. Le et al. (2016) and Meenagh et al. (2019) show that in the small samples we encounter here indirect inference exhibits low bias in estimation and high power in testing.

3 A DSGE model with real sunspots

Our model (for more details, see Appendix 1) is a small open DSGE model of Japan with real and nominal rigidities, financial frictions and a central bank that pursues monetary policy either by setting interest rates on short term government bonds or, if that is nullified by a zero lower bound, by asset purchases (QE). It also rules out price and other nominal bubbles, including in asset markets, via terminal conditions. This distances our model from the literature reviewed above. Our focus is on a real output bubble arising through a sunspot of pessimism about future output. This produces self-validating falls in capital investment, labour supply (including via lower births), creating a real bubble solution for output demand and also for output supply potential from a standard production function.

The idea we explore is that Japanese households and firms suffer from random bouts of pessimism about the future fundamental prospects for the economy; these bouts we assume started to happen after the crisis of the end-1980s when the 'financial bubble' was burst by brutally deflationary monetary policy, plunging the economy into deep recession and leaving the banks with weak balance sheets. We model these bouts as sunspots, which affect the expected long term performance of output. These enter the model as the terminal expected value of output. This terminal belief in turn feeds into expected values for earlier consumption, investment and labour supply which are determined by expected output. Expected output in turn reflects these variables; and then goes on to influence expected consumption etc. for still earlier periods. In this way the original bout of pessimism about the long term future cascades back down to current behaviour of the economy.

We solve our DSGE model in repeated bootstrap simulations based on errors retrieved from the model and the data since 1990, the era since the bursting of the nominal 1980s bubble — which we hypothesize caused the situation in which business confidence became prone to bubbles. We then test it by indirect inference against a VECM representation of the Japanese macro data for output, inflation and credit interest rates. Finally, we estimate it by indirect inference — by a search procedure that finds the parameters yielding the closest match of simulated VECM coefficients to those in the data.

The main challenge in this empirical process is finding the bubble simulation solution paths. We do this by postulating a bubble path whose terminal rate of expansion is then imposed on the model to find the self-validating solution consistent with it: consumption and investment demand respond to these output expectations, and in turn they generate output demand that determines output under sticky prices; the model then solves backwards from this future state. The model solves for the optimising capital, labour and innovation strategies of private agents, conditional on this evolving solution path.

In the following subsections, we first write a note on sunspot solutions, we then use a simple example to illustrate the sunspot model of output, and then extend this to the full DSGE model for Japan.

3.1 A note on sunspot solutions in rational expectations models

Rational expectations of future events introduce a forward root into the model solution which is a saddle-path in the case of the usual model where all the forward and backward roots lie within the unit circle. In this solution, the saddle path is the unique stable path, and there is also an infinite number of explosive paths, any one of which can be selected by chance, unless there is some mechanism preventing such a selection.

For example, if current inflation depends on future inflation through the demand for money, the forward root this introduces permits explosive hyperinflationary or hyperdeflationary solutions, that can be selected by agents at random; such a random selection is known as a sunspot, whereby agents spontaneously come to believe an inflation or deflation will happen in a future period, triggering inflation or deflation in the current period, and implying that in later future periods inflation or deflation will increase along the chosen explosive path. Such spontaneous beliefs could be the result of pure sentiment taking hold — potentially triggered by some current event or a rumour of a future event. What the models imply is that once such a sentiment takes hold, it is validated by future events occurring as expected; hence it is a rationally expected solution.

The mathematics of these solutions is relatively simple. We follow the exposition in Minford and Peel (2015, chapter 2). Suppose we take a simple New Classical model of prices (p_t), money (m_t) and output (y_t):

$$m_t = p_t + y_t - \alpha(E_{t-1}p_{t+1} - E_{t-1}p_t)\ (\alpha > 0) \tag{1}$$

$$p_t = E_{t-1}p_t + \delta(y_t - y^*) \tag{2}$$

$$m_t = \overline{m} + \varepsilon_t \tag{3}$$

The general solution for expected prices is:

$$E_{t-1}p_{t+i} = \overline{m} - y^* + A_1(\frac{1+\alpha}{\alpha})^i\ (i \geqslant 0) \tag{4}$$

where $A_1 = [E_{t-1}p_t - (\overline{m} - y^*)]$ from the initial condition. This solution can be conveniently found by solving for expected future prices in terms of lagged prices (the 'backward solution' whereby the forward root, being applied backwards, is inverted and becomes unstable.) The unique stable path is found by setting A_1 to zero. The model's behaviour can be illustrated by a phase diagram where the arrowed line is the saddlepath and the arrows off it indicate the explosive paths.

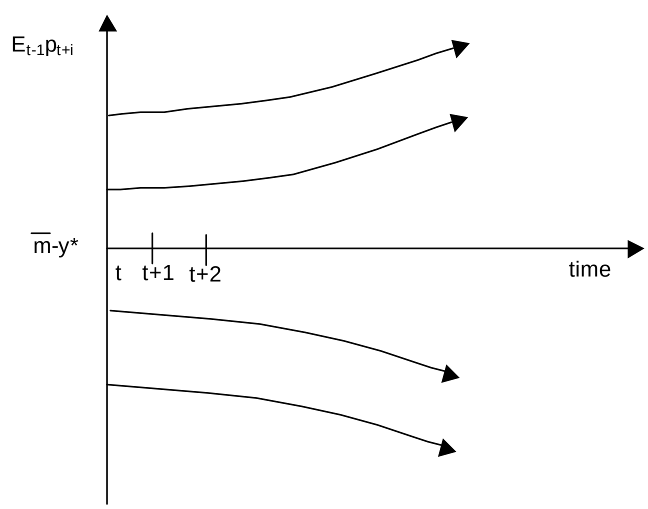


Figure 1: The solution paths for the price level expected at $t-1$, as in equation 4

In models like this with saddlepath stability and an infinity of unstable paths for prices, a natural way to rule out all paths except the saddlepath is for the central bank to commit itself to stopping them via a terminal condition in which future prices away from the stable path would be overidden by money supply movements.

In this paper we are dealing not with unstable price paths but rather unstable output paths. In the model above these do not arise as expected output equals potential output which is an exogenous constant. However, it is possible for output to depend on expected future events through model behaviour. Thus, in a New Keynesian model output depends on demand which in turn depends on expected future events; also potential output depends on capital investment which in turn depends on expected future output, creating a forward root giving rise to a saddle-path solution for output, with an infinity of explosive paths, similar to the above one for prices. These explosive paths could be similarly eliminated by a government commitment, e.g. through fiscal policy, to prevent them.

3.2 A simple illustrative sunspot model of output

To illustrate the workings of a simple model of this type, suppose that natural rate output, y_t^*, depends on capital invested and that this in turn depends on expectations of future y_t^* via the marginal product of capital; thus current y_t^* depends on expected future y_{t+1}^* via a forward root, σ. We can write the equation for natural rate output as:

$$y_t^* = \sigma E_t y_{t+1}^*$$

For the rest of the model, we will assume here the simple New Classical structure as in Section 3.1:

$$m_t = p_t + y_t \tag{5}$$

$$p_t = E_{t-1} p_t + \delta(y_t - y_t^*) \tag{6}$$

$$m_t = \overline{m} + \varepsilon_t \tag{7}$$

To solve this model, we can first solve for y_t^*. This yields a first order difference equation for the future as a function of the previous expected value as:

$$E_t y_{t+i+1}^* = \sigma^{-1} E_t y_{t+i}^*$$

whose general solution is:

$$E_t y_{t+i}^* = A_1 \sigma^{-i} \qquad (i \geq 1)$$

where A_1 is given by the initial value, $E_t y_t^* = A_1$. But this value can be chosen arbitrarily, as nothing in the model ties it down.

We can then find the expected solution for output as $E_t y_{t+i} = \sigma^{-i} E_t y_t^*$ where $E_t y_t^*$ can be chosen at will, and can therefore be treated as a sunspot. It follows that expected prices are $E_t p_{t+i} = E_t m_{t+i} - \sigma^{-i} E_t y_t^*$. This implies that prices can also have a sunspot solution, absent any monetary response. This can be eliminated by setting expected money supply to eliminate the effect of the sunspot on prices, forcing them to a target path, p_{t+i}^*, as follows:$E_t m_{t+i} = p_{t+i}^* + \sigma^{-i} E_t y_t^*$. Once these expected paths are established, actual price and output deviate from them according to the demand shock to the money supply, obtaining: $p_{t+i}^{ue} = \frac{\delta}{1+\delta}\varepsilon_{t+i}$ and $y_{t+i}^{ue} = \frac{1}{1+\delta}\varepsilon_{t+i}$, where the ue superscript denotes 'unexpected.'

It is useful to examine the solution of this very simple model, under plausible assumptions for the sunspot path of the natural rate. Notice that according to this model, output, ignoring iid errors, is equal to expected natural rate output. Due to pessimism about the expected future natural rate output, $E_t y_{t+i}^*$, future output is depressed and continues to diverge from the true potential output path.

In the period after the post-war 'miracle' (when growth was 10%) growth fell to 4.5%; then in the bubble-bursting period from 1990 it fell to about 1%. Since 2000 it has averaged about 0.6%, rising slightly after the financial crisis when fiscal policy became generally stimulative. Most developed countries manage to grow about 2%, which we take to be a reasonable estimate also of the true potential (natural rate) growth in Japan. In the Figure 2 one can see the way GDP has fallen steadily further below this 2% trend path from 1992.[1]

[1] Some argue that growth on a per capita basis should be the policy target; growth per capita, with a declining population, has not slowed so markedly. However, this assumes that population growth does not respond to growth of GDP, which is unlikely. The willingness to have children is likely to respond to the outlook for output and jobs — Kearney and Levine (2020), Black et al. (2013), Autor et al. (2019). The worse the outlook, the more parents must invest in education and other support for their children, to create lifetime opportunities for them. In this model we focus on GDP growth as the key policy outcome.

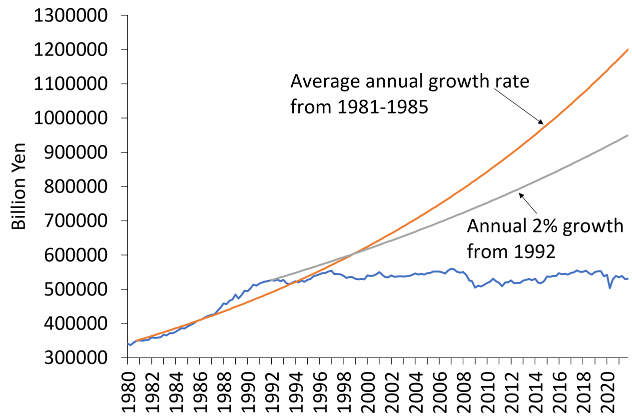


Figure 2: Real GDP 1980-2020 (blue line)

We now show the type of sunspot path this simple model implies given Japanese data. If we take the true potential output path to follow the 2% growth trend, then the sunspot-affected path is actual output; and the sunspot can be derived as the gap betwen this and the 2% growth trend. Using this logic, we get the sunspot series in Figure 3 reflecting this gap between output and the 2% trend.

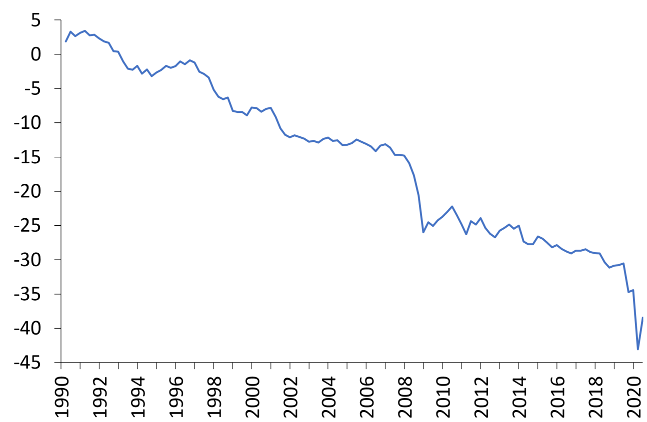


Figure 3: Sunspot Series (output deviations from natural rate)

When we further decompose this into the sequence of sunspot innovations, that is the necessary revision each period compared with the value implied by the previous sunspot, we obtain Figure 4.

It can be seen that up to 2007 the innovations fluctuate moderately around a negative mean, driving expected output steadily downwards. However the financial crisis triggers a large negative shock. This was followed by a long period of further pessimism, culminating in another large negative shock in 2020, the period of Covid, with the post-Covid bounceback, followed by the resumption of further moderate pessimism. With a full set of shocks providing the effect of non-sunspot shocks on output, we would expect these two shocks to be eliminated from the sunspot.

We have shown the type of path this simple model implies given Japanese data. Over the sample period from 1992 the economy has evolved with a succession of negative sunspots, launching the economy on a slowly exploding downward path. We show the contribution of each dated sunspot to the later GDP evolution. It can be seen that the original sunspot around the bursting of the monetary bubble accounts for much of the later evolution. However additional negative and occasionally positive shocks occurred later.

In the subsequent subsections we develop a full model with the investment path set by future expectations

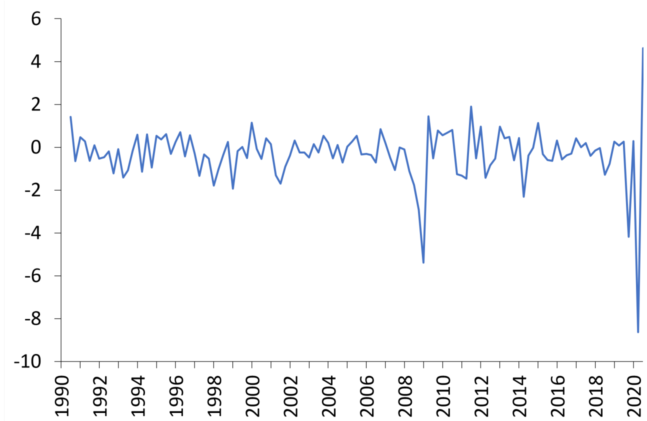


Figure 4: Sunspot Innovations

of the marginal product of capital; and with labour supply responding to expectations of future wages. We generate the expected path of y_t^* according to the natural rate model equation set out above. We then embed this in a New Keynesian model of Japan which comes closest to explaining Japanese data behaviour. The natural rate enters the model via the terminal conditions when output has converged on it. Prior to that date output is demand-determined under the usual New Keynesian assumptions. Hence the effect of the natural rate sunspot solution is to change expectations of output from the terminal date backwards towards the initial period. Unlike in the simple New Classical example developed above, expected output is not equal to the expected natural rate, rather it is impacted by it through the demand effects of expected output.

3.3 Solving the Japanese model with a pessimism bubble

Now we apply this analysis to an estimated DSGE model of Japan. The full model for Japan is based on that of Le et al. (2016a), but with the addition of open economy elements to deal with trade and capital flows — a full account is given in Appendix 1. The Le et al. (2016a) model is developed from the model of Smets and Wouters (2007): it adopts a hybrid price-setting structure, with a fraction of goods markets assumed to be flexprice while the rest set prices for longer durations; similarly with labour markets. Beyond these frictions in labour and goods markets, the model also incorporates financial frictions as proposed by Bernanke et al. (1999) and at the zero lower interest rate bound allows for cheap money collateral to make monetary policy effective via largescale asset purchases, 'Quantitative Easing'. Trade equations enter via an Armington (1969) nested CES utility function of differentiated home and foreign consumption goods, as in Meenagh et al. (2010). On the capital account of the balance of payments uncovered interest parity is assumed between home and foreign bonds.

In this model the central equation driving investment in capital, in log-linearised form, is[2]:

$$E_t r_{t+1}^k - (r_t - E_t \pi_{t+1}) = \chi \left(pk_t + k_t - n_t\right) - \vartheta m_t^0 + \epsilon_t^{prem}$$

In this expression expectations of future output demand determine networth (n) so the expected bubble feeds into n. This in turn depresses investment and capital, reducing current potential output through the production function, and current output via reduced demand. Next we look for the bubble solution for output and its terminal value that permits the model to generate the data path.

3.4 Calculating the sunspot shock in the full model

We take the 2% growth path from 1990 to be illustrative of the unknown true natural rate growth path, as explained above. To obtain our estimate of the actual bubble we argue as follows. Our bubble hypothesis is

[2]Equation 22 in Appendix 1, where r^k is the external financing rate, r is nominal interest rate, π is inflation, pk is price of capital, k is capital stock, n is networth, m^0 is monetary base, and ϵ_t^{prem} is the risk premium shock.

that the economy is driven by the model and a combination of the normal shocks and the sunspot shock to its lower predicted path, which according to the model should get close to matching the economy's average (HP-filtered) actual behaviour, y_t^*. Thus according to this model, $y_t^* = \widehat{y}_t^{normal\ shocks} + \xi_t$, and thus we create the sunspot shock series as $\xi_t = y_t^* - \widehat{y}_t^{normal\ shocks}$. The resultant sunspot shock series is plotted in Figure 5, and its innovations in Figure 6.

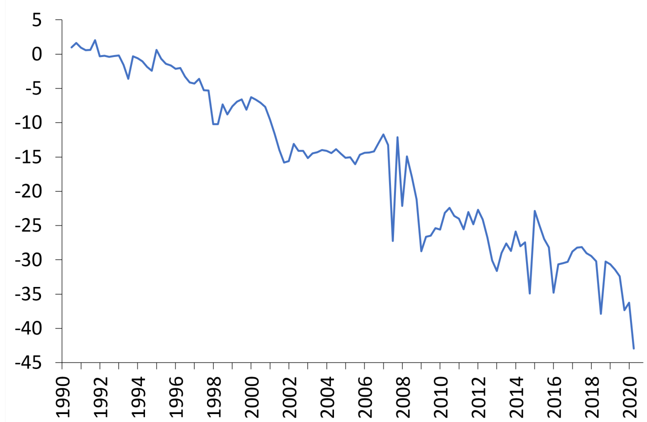


Figure 5: Sunspot Series Generated from Full Model

Figure 6: Sunspot Innovations Generated from Full Model

The sunspot innovations tend to be negative, pushing the economy further and further away from the supposed true potential path. Thus, if our bubble hypothesis is correct, Japanese business is beset with steadily increasing pessimism with no basis in the truth. The policy implications of this situation are stark and challenging; we will revert to them in the final section.

3.5 How financial market sunspots can arise and impact the economy

The analysis above illustrates how sunspots arise from the presence of a forward root in the model, i.e. where a variable depends on its future. The sunspot solution arising from this can be found by solving the model backwards, so making the future depend on the past, inverting this forward root; the solution implies that the future variable value explodes from any arbitrarily chosen initial expectation.

In our literature review, we noted a wide variety of papers in which financial market bubble/sunspots are assumed and cause economy-wide effects. It is useful to give an example of how this could arise in our

Japan model, even though we do not use this financial market mechanism.

We can take a sector of the equity market, such as tech stocks; let the market value of a share be s_t. Their value will be equal to discounted (by discount factor d) future profits, $E_t\pi_{t+1}$, thus $e_t = dE_t\pi_{t+1}$. Now assume that the larger the firm's asset value and production size, the lower are costs and the higher are profits, so that $\pi_t = \theta e_t (\theta \leq 1)$. It follows that $e_t = d\theta E_t e_{t+1}$. The sunspot solution this creates is then $E_t e_{t+i} = (d\theta)^{-i} E_t e_t$.

So far this is simply a 'micro sunspot' for a single sector. However as the sector will enjoy explosively rising value, it will attract investment and become an increasing share of the country's equity portfolio. This in turn will increase the overall net worth of the entrepreneurial sector, raising the return on credit in our central model equation determining investment finaced by credit, as noted above:

$$E_t r_{t+1}^k - (r_t - E_t\pi_{t+1}) = \chi\left(pk_t + k_t - n_t\right) - \vartheta m_t^0 + \epsilon_t^{prem}$$

In this way the financial market sunspot can feed into the economy, fuelling derived sunspot behaviour in the economy. It can be seen from this example that there could be other ways that sunspots could arise at the micro level, and from there feed back into bank credit and investment. All that is needed is some assumed micro process that creates a forward root for asset values and hence for the supply of credit.

However, as noted above, we do not use this type of mechanism here. Instead we assume that general confidence in the future potential of the economy is the channel by which a sunspot takes hold. It is this that distinguishes our work here from the large literature on financial-origin sunspots.

4 Estimation and testing by Indirect Inference

We use the approach of Indirect Inference proposed by Le et al. (2011) to assess the model's ability to match the data. Le et al. (2016b) provide a full description of the procedure. To generate a description of the data against which the theory's performance is indirectly evaluated, the approach uses an auxiliary model that is fully independent of the theoretical one. The estimated parameters of the auxiliary model, or functions of these, might be used to summarise such a description; we name them as the 'data descriptors'. While they are viewed as 'reality,' the theoretical model under consideration is simulated to determine its suggested values. In estimation the parameters of the structural model are chosen so that when the model is simulated, the auxiliary model estimates are similar to those obtained from the real data. The structural model parameters that minimise the distance between a given function of the two sets of estimated coefficients of the auxiliary model are the best.

When evaluating the model's data fit, the structural model is simulated, and the auxiliary model is fitted to each set of simulated data, yielding a sample distribution of the auxiliary model's coefficients. A Wald statistic is computed to see if functions of the auxiliary model's parameters calculated on actual data fall within a confidence interval given by the sampling distribution.

The auxiliary model should be a process that describes how the data evolves under any applicable model. It is well known that the reduced form of a macro model with non-stationary data is a VARMA, in which non-stationary forcing factors are used as conditioning variables to accomplish cointegration (i.e. ensuring that the stochastic trends in the endogenous vector are picked up so that the errors in the VAR are stationary). This in turn can be approximated as a VECM. As an auxiliary model, we utilise a VECM with a temporal trend and the productivity residual inserted as an exogenous non-stationary process, which we re-express as a VAR(1) for the three macro variables of interest (interest rate, output, and inflation). The two exogenous elements have the effect of achieving cointegration. The VAR coefficients on the lagged dependent variables and the VAR error variances are treated as the data descriptors, and the Wald statistic is computed from them. Thus, we are essentially determining whether the observed dynamics and volatility of the selected variables can be explained by the simulated joint distribution of these variables at a given confidence level. The Wald statistic is given by:

$$(\Phi - \overline{\Phi})' \sum\nolimits_{(\Phi\Phi)}^{-1} (\Phi - \overline{\Phi}) \tag{8}$$

where Φ is the vector of VAR estimates of the chosen descriptors yielded in each simulation, with $\overline{\Phi}$ and $\sum_{(\Phi\Phi)}$ representing the corresponding sample means and variance-covariance matrix of these calculated across simulations, respectively.

The joint distribution of the Φ is obtained by bootstrapping the innovations implied by the data and the theoretical model; it is thus a small sample distribution estimate. For small samples, this distribution is usually more accurate than the asymptotic distribution.

This testing procedure is applied to a set of (structural) parameters put forward as the true ones (H_0, the null hypothesis). The test then asks: could these coefficients within this model structure be the true (numerical) model generating the data? We extend our procedure by a further search algorithm, in which we seek other coefficient sets that minimise the Wald test statistic — in doing this we are carrying out indirect estimation.

Thus we calculate the minimum-value Wald statistic using a powerful algorithm based on Simulated Annealing (SA), in which the search takes place over a large range around the initial values, with the search being optimised by random jumps around the space. The benefit of this extended method is that when we ultimately compare model compatibility with data, we use the best possible version of the model.

5 Empirical results

We have found parameters for this model that can match Japanese data (in the form of a VECM) well with a p-value of 0.0994 showing that we do not reject the null hypothesis that the model can generate the data. The key forward root we discover is 0.9929 which gives rise to a moderately exploding solution. What we see is that, similarly to our simple model, the Japanese economy tends to contract systematically towards a very slowly growing 'natural rate' growth path. Besides other normal shocks driving it, there is a sequence of sunspot shocks that push it along this path. These shocks we calculate in a similar way to that in the simple model but with a difference reflecting the greater number of demand channels. There we assumed that gloomy expectations of future potential output drove investment demand: here we assume that these gloomy expectations drive private actions across the whole model. The rational expectations (RE) loop in the model forces the actual output path we observe into equality with the expected output generated by the terminal expected value of potential output, itself determined by the sunspot shock. Thus the RE solution of the model embeds both the effects of normal shocks and those of the sunspot shocks. The economy's demand and supply behaviour responds to this expected path in a way that must replicate the data-based VECM descriptive model to pass the indirect inference test.

The model therefore works as follows. The bubble output expectations feed into entrepreneurial net worth, and depress investment which reduces output through lower demand. Employment falls also via demand. Real wages fall, clearing the labour market. Labour supply also falls, with the fall in income, consumption and real wages. Output potential falls due to lower capital and labour supply. The output gap versus potential is little changed so inflation moves little. Inflation fluctuations come from demand shocks interacting with this output potential.

Table 1 reports the estimated parameters and Wald statistic are reported.

Table 2 shows the auxiliary model estimates on the actual data alongside the mean and 95% confidence bounds from the simulations. Most parameters are within the bounds, but we find that the model under-predicts the output response to lagged output. Similarly for the interest rate response to lagged interest rate. Conversely, the inflation response to lagged inflation is over predicted, by a small margin. The data for interest rate variance is just outside the model's 95% confidence bounds. However, the model fits overall, as shown by the p-value above.

The example simulations (shown in blue) in Figure 7 reveal the prevalence of output, output gap and inflation fluctuations, together with regular switches into the zero lower bound, alongside a slow growth trend. They tend to match the nature of the fluctuations in the actual data (shown in red), so accounting for the good p-value.

	Sunspot model coefficients
Steady-state elasticity of capital adjustment	8.0073
Elasticity of consumption	2.5387
External habit formation	0.4636
Probability of not changing wages	0.4402
Inverse of Frisch elasticity of labour supply (σ_l)	3.3006
Probability of not changing prices	0.8066
Wage indexation	0.1905
Price indexation	0.4457
Elasticity of capital utilisation	0.9435
Share of fixed costs in production (+1)	1.7772
Taylor Rule response to inflation	1.0612
Interest rate smoothing	0.9735
Taylor Rule response to output	0.0060
Taylor Rule response to change in output	0.0126
Share of capital in production	0.3338
Proportion of sticky wages	0.2384
Proportion of sticky prices	0.4806
Elasticity of the premium with respect to leverage	0.0103
Monetary response in crisis time	0.0978
Monetary response in normal time	0.0406
Elasticity of premium with respect to money	0.0494
Bubble forward root	0.9929
Fiscal response	0.6470
Wald	21.1524
Transformed Wald (t-stat)	1.1492
P-Value	0.0994

Table 1: Coefficient Estimates

	Actual	Mean	2.5th Percentile	97.5th Percentile	In/Out
Y_Y	0.8031	0.2770	−0.3191	0.5659	OUT
Y_PI	0.0554	0.0975	−0.4318	0.7039	IN
Y_R	−0.1540	0.0417	−6.5507	4.8900	IN
PI_Y	0.0810	0.0198	−0.0837	0.1066	IN
PI_PI	−0.3040	0.0129	−0.2768	0.3085	OUT
PI_R	0.6767	0.3366	−0.9463	2.6129	IN
R_Y	0.0021	−0.0098	−0.0264	0.0022	IN
R_PI	0.0051	0.0063	−0.0257	0.0385	IN
R_R	0.9320	0.6866	0.1467	0.9308	OUT
Var(Y)	1.5653	2.8773	1.0596	7.8494	IN
Var(PI)	0.2339	0.3869	0.2328	0.5765	IN
Var(R)	0.0009	0.0081	0.0012	0.0177	OUT

Table 2: Auxiliary Model Parameter Bounds

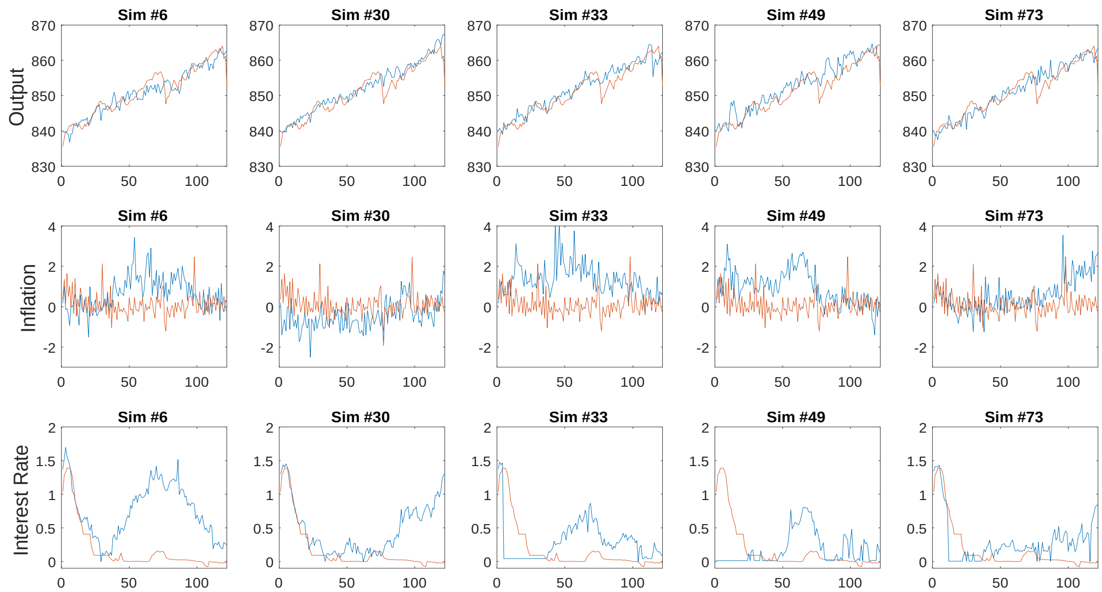


Figure 7: A Selection of Simulations

5.1 The behaviour of the model

The IRF for a negative sunspot shock in Figure 8 shows a persistent and sizeable fall in output, employment and real wages accompanying the slowly exploding sunspot in expected equilibrium output, y^*.

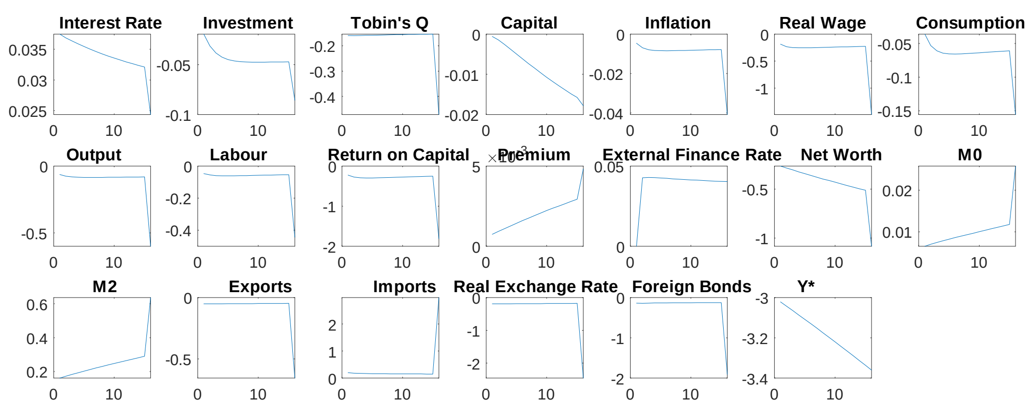


Figure 8: IRFs for a Sunspot Shock

The sunspot enters expected equilibrium output at the terminal date and then induces a fall in consumption, employment and capital investment which in turn depress demand and output. The Taylor rule responds to a rise in $y - y^*$, with output falling less than y^*, by raising interest rates, so also causing real exchange rate appreciation, forcing down net exports which add to the decline in demand and output.

We now turn to the effect of a positive productivity shock (Figure 9). The productivity shock in non-stationary, so has permanent effects. This paradoxically reduces output. However, it does so through a mechanism already well-known in the New Keynesian literature (an early finding is by Gali, 1999): at given demand, fixing output at pre-set prices, a rise in productivity lowers labour demand, hence employment. This in turn depresses consumption and inflation. However, after initially rising due to the drop in inflation, real interest rates are gradually reduced by the Taylor Rule, pushing up consumption and investment, also net exports via real depreciation; output demand therefore gradually recovers, but both capital and employment remain depressed, largely offsetting the productivity effect on y^*.

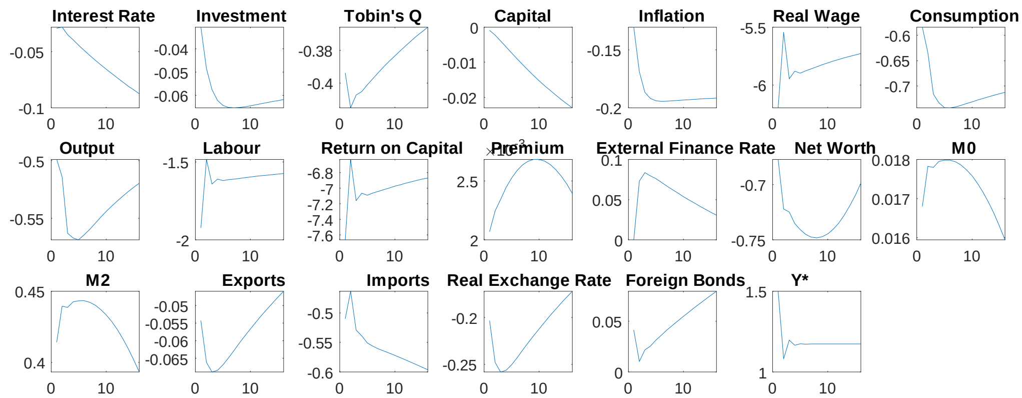


Figure 9: IRFs for a Permanent Productivity Shock

Figure 10 shows the IRF for a positive shock to government spending — a classic demand shock. This has a strong immediate effect on output, but since it raises inflation, the Taylor rule response raises real interest rates which crowd out consumption and investment and (via real appreciation) net export demand. Furthermore the fiscal feedback response offsets some of the initial spending rise. Hence the output expansion dies away fairly rapidly. Similarly, with the other demand shocks to consumption and investment, the output rises are shortlived.

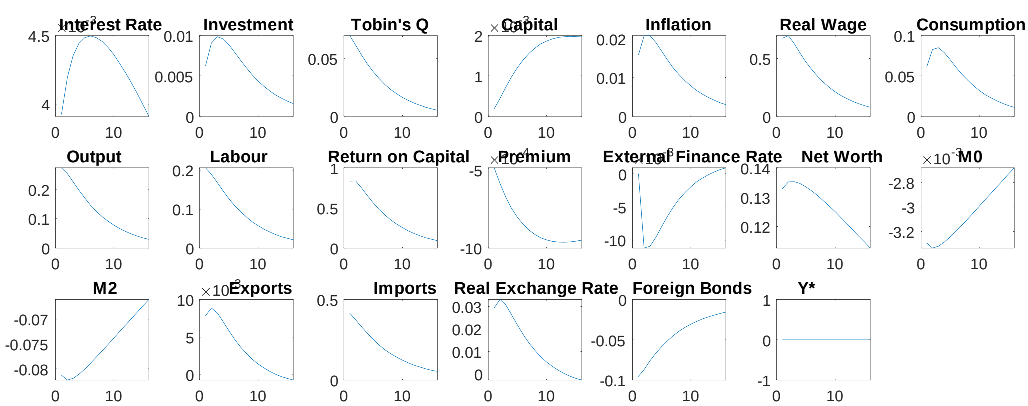


Figure 10: IRFs for a Government Spending Shock

A contractionary monetary shock (Figure 11) has an IRF that similarly reduces demand sharply on impact but that is quickly reversed via the Taylor rule and fiscal feedback responses. Output, employment, home demand and net exports all recover rapidly; the rise in real interest rates and so also the real exchange rate are steadily reversed.

Cost shocks similarly have a sharp short run stimulationary effect on inflation, which via the resulting

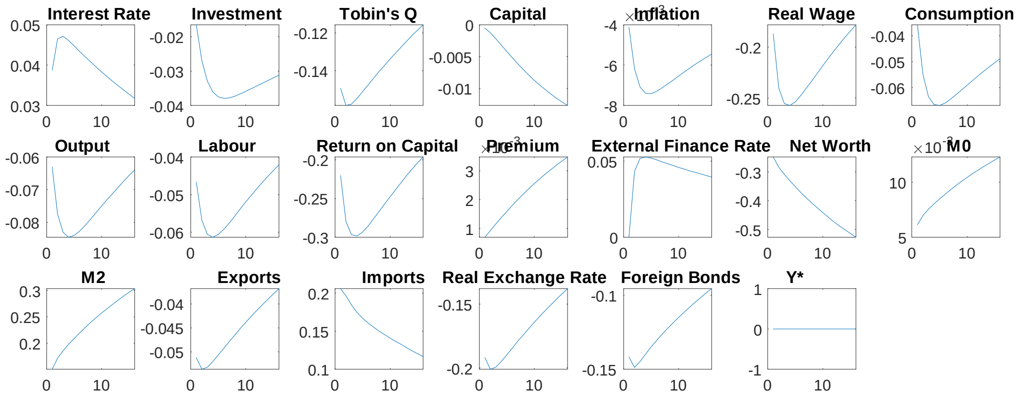


Figure 11: IRFs for a Monetary Policy Shock

fall in the real interest rate also stimulates net export demand and output, before the Taylor rule intervenes to raise real rates and reverse the expansion.

In general, throughout these IRFs we see the combination of the Taylor rule and the fiscal feedback response impart a strongly stabilising impact to the economy.

Overall, it can be seen from the simulations and the IRFs that the model responds to normal shocks in the usual way, since these do not trigger sunspot movements. We can also see that the sunspot shock triggers modestly exploding paths which generate their own inflation/output gap effects, as demand effects fail exactly to match effects on potential output.

5.2 Which shocks cause most variation in the model?

We perform a variance decomposition exercise to see which shocks are the most important. The short-run and long-run analysis are shown in Tables 3 and 4 respectively, where the short-run is 5 periods ahead, and the long-run is 20 periods ahead.

Shock	Interest Rate	Investment	Tobin's Q	Capital	Inflation	Real Wage	Consumption	Output	Labour
Government Spending	0.0017	0.0001	0.0054	0.0000	0.0035	0.0073	0.0088	0.0359	0.0085
Consumer Preference	0.0004	0.0000	0.0001	0.0000	0.0006	0.0065	0.0897	0.0122	0.0026
Investment	0.0011	0.9851	0.0032	0.9994	0.0015	0.0089	0.0084	0.0454	0.0122
Monetary Policy	0.1814	0.0006	0.0430	0.0000	0.0004	0.0011	0.0052	0.0043	0.0010
Productivity	0.1158	0.0019	0.2962	0.0001	0.3107	0.6851	0.7079	0.2051	0.8109
Price Mark-up	0.0218	0.0000	0.0041	0.0000	0.6334	0.0002	0.0006	0.0018	0.0006
Wage Mark-up	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0000	0.0000	0.0000
Labour Supply	0.0020	0.0000	0.0088	0.0000	0.0226	0.1568	0.0002	0.0019	0.0004
Premium	0.0000	0.0007	0.0624	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000
Net Worth	0.0006	0.0013	0.1228	0.0000	0.0014	0.0024	0.0022	0.0118	0.0027
M0	0.0000	0.0010	0.0630	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000
Exports	0.0162	0.0000	0.0064	0.0000	0.0154	0.0933	0.1068	0.4939	0.1173
Imports	0.0052	0.0000	0.0024	0.0000	0.0051	0.0293	0.0323	0.1535	0.0364
Sunspot	0.6539	0.0092	0.3822	0.0004	0.0054	0.0089	0.0379	0.0339	0.0076

Table 3: Short Variance Decomposition

One striking feature of this shock decomposition is the dominant effects on output of the net trade shocks in the short run. In the long run the sunspot shock dominates, with productivity contributing much of the rest. Both shocks are nonstationary which accounts for their relative long run importance. Demand shocks have very limited importance for output, being heaviliy suppressed by fiscal and monetary feedback.

The price mark-up is the overwhelming source of inflation variation in the short run, followed by productivity. Otherwise inflation is largely unaffected by shocks, remaining essentially stable around costs. Productivity also destabilises wages and employment as we see from the IRF, contributing most of their

Shock	Interest Rate	Investment	Tobin's Q	Capital	Inflation	Real Wage	Consumption	Output	Labour
Government Spending	0.0013	0.0002	0.0012	0.0000	0.0017	0.0023	0.0006	0.0095	0.0030
Consumer Preference	0.0001	0.0000	0.0000	0.0000	0.0002	0.0013	0.0040	0.0021	0.0006
Investment	0.0037	0.7205	0.0024	0.9835	0.0049	0.0154	0.0040	0.0216	0.0230
Monetary Policy	0.1189	0.0023	0.0214	0.0004	0.0006	0.0014	0.0010	0.0059	0.0017
Productivity	0.4125	0.0086	0.1743	0.0013	0.4482	0.5643	0.1419	0.1808	0.7484
Price Mark-up	0.0135	0.0004	0.0023	0.0000	0.2086	0.0007	0.0002	0.0033	0.0011
Wage Mark-up	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Labour Supply	0.0075	0.0005	0.0023	0.0000	0.0139	0.0537	0.0004	0.0091	0.0032
Premium	0.0001	0.0017	0.0230	0.0003	0.0000	0.0004	0.0001	0.0021	0.0006
Net Worth	0.0018	0.0050	0.0781	0.0009	0.0020	0.0020	0.0005	0.0086	0.0025
M0	0.0001	0.0041	0.0425	0.0007	0.0001	0.0006	0.0001	0.0023	0.0008
Exports	0.0074	0.0005	0.0013	0.0000	0.0064	0.0237	0.0062	0.1067	0.0343
Imports	0.0045	0.0005	0.0006	0.0000	0.0029	0.0112	0.0029	0.0512	0.0165
Sunspot	0.4284	0.2557	0.6505	0.0128	0.3106	0.3229	0.8382	0.5967	0.1641

Table 4: Long Variance Decomposition

variation. Interest rates and the real exchange rate are mainly disturbed by monetary policy, productivity and the price mark-up, the last two via their effects on costs and so inflation.

We can now consider the effects of the sunspot shock on the longer term trends in the model. We show in Figure 12 the timelines in response to this shock as it evolves, for the main variables: output is depressed by 54% at the terminal date; in other words growth over the sample is depressed by 1.8% p.a.. This in turn depresses investment by 13%, Tobin's Q by 80%, and employment by 39%.

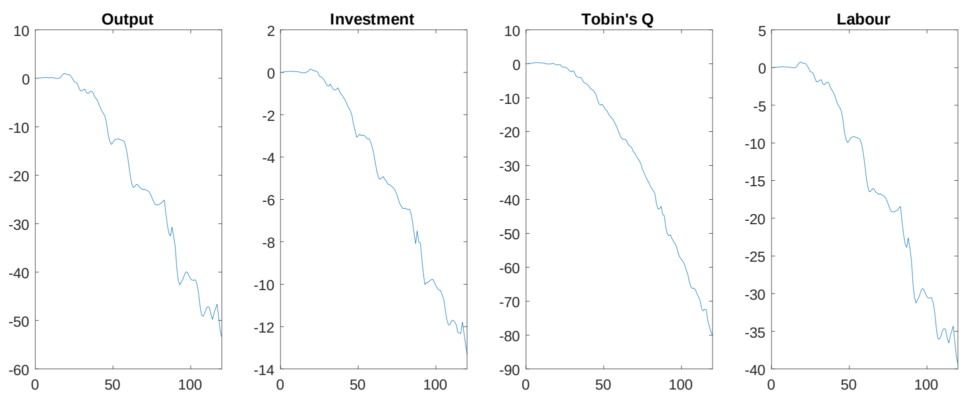


Figure 12: Longrun Effects of the Sunspot Shock

When we accumulate the effects of the sunspot shocks on expected terminal output, we find the picture above of a steadily worsening expected future growth rate. This is how the sunspot undermines Japanese growth. Investment in Japan is linked to these expectations via the marginal product of capital equation with the cost of capital. Employment is depressed by much the same percentage as output, and since consumption falls with output, lowering the demand for leisure, labour supply is reduced at the same time, forcing down real wages strongly.

6 Policy discussion

At the centre of this model fitted to Japanese data is its sunspot solution in which the forward root drives the model downwards in an explosive way towards a very slow growth path. The model generates poor growth outcomes because of this 'pessimism bubble'. It is a key function of government to eliminate such bubbles. It has long been recognised that central banks need to enunciate terminal conditions that eliminate nominal bubbles. Real bubbles like the one here are however rare because usually eliminated by private sector recognition of the true trend in output potential — what Keynes termed 'animal spirits' — so little attention has been given to any government role in eliminating real bubbles. Yet we have shown here that

Japan's weak growth performance can be accounted for by a real bubble not suppressed by private sector beliefs. It follows that in this situation the government needs to step in to suppress it. It cannot do so via monetary means, instructing the central bank, as this only affects nominal outcomes; of course we have observed that the BOJ's policy of targeting higher inflation has, as the model would imply, been ineffective in raising growth. It follows that fiscal policy needs to create the necessary terminal condition, which could take the form of a commitment to raise demand up to the true output potential level. We now show how the model behaves when this commitment is included, eliminating the sunspot shocks. This complements the existing fiscal feedback rule in which fiscal policy responds to the output gap; this helps to create output stability but on its own cannot eliminate the sunspot that creates long term pessimism and undermines long run growth.

In Table 5 we show the variances of output and inflation with and without the sunspot ('without' implying that the government suppresses it via a terminal condition on ystar). It can be seen that suppressing the sunspot reduces the variance of output markedly and makes little difference to the variance of inflation, so that welfare overall improves substantially.

	$var(Y)$	$var(\pi)$	$var(R)$	Welfare Cost
Sunspot	3.2989	0.8154	0.1572	4.2715
No Sunspot	2.6270	0.8532	0.1262	3.6064

Table 5: Effects on Volatility of Sunspot

This suggests that fiscal policy should be committed to suppressing any sunspot. So far the GOJ has expressed no interest in such a policy. This suggests it does not believe there is a sunspot. In the next section we review a model without a sunspot that they implicitly must believe.

6.1 Can we account for Japan's weak growth without the pessimism sunspot?

Japan shifted in the 1990s to a sharply slower growth rate than before. In this paper we have suggested that this came from a sunspot pessimism shock engendered by the economy's collapse when the 'financial bubble' was burst by a brutal monetary squeeze in 1989. What we have shown is that this model cannot be rejected by the behaviour of Japanese data since 1990. However, of course this does not also imply that there is no alternative model of Japan, with no sunspot, that could also match Japanese data behaviour.

In fact we can also match this behaviour with a 'normal model' in which agents correctly expected the trend in output that actually occurred. Under this model Japan would have experienced an exogenous slowdown in productivity growth which in turn produced slowing of capital and labour supply growth. We have also fitted this model to the data and its estimated parameters turn out to be the same as for the sunspot model; the model also matches the data behaviour about as well, with a p-value of 11%, as compared with 10% for the sunspot model — we have put the results in an Appendix 3.

What this reveals is that, though we cannot reject the sunspot model, an equally probable model of Japan's low growth recent history is one in which productivity growth simply slowed down. As this slowdown rolled out over time, expectations of the low productivity continuing into the future rolled out with it, given that productivity is nonstationary, with its innovations following a low order negative AR process.

As noted in the previous section, the GOJ must believe in this no-sunspot model since it has not taken action to eliminate a sunspot. The operative question for policymakers, given that both models are roughly equally probable, is what risks are taken in getting the model wrong. In Table 5 above, we showed that if the sunspot model was the true one, there would be a welfare gain of 32% from suppressing the sunspot. Now we must ask what the loss, if any, would be of doing so if the no-sunspot model is true. In this case fiscal policy would force output to lie above the true ystar value and equal the supposed higher ystar path. It might well be thought that such a policy would be highly inflationary and would be resisted by the BOJ via sharp interest rate increases; since this would be a conflict that cannot be won by either government or Bank, we model fiscal policy as strongly pushing downwards the output difference from its targeted ystar path. Table 6 below shows our simulated welfare results.

For this simulation the no-sunspot model has a ystar trend given by an HP filter of actual output and the fiscal suppression policy has strong fiscal feedback on the log difference of output from this HP ystar augmented to match the 2% growth trend. This feedback rule replaces the one assumed in the sunspot model; it is therefore substantially more aggressive, as besides stabilising output fluctuations, it is forcing output up to the assumed no-sunspot trend. However, our simulation results for this aggressive fiscal policy in the no-sunspot model also reveal a gain in welfare, with the variance of output falling sharply. Furthermore interest rate variance hardly changes either; and the inflation trend is barely affected; it turns out that this aggressive fiscal policy raises average inflation across all simulations by 0.24% p.a., which is pretty small. This might seem highly surprising. But when one considers how little inflation reacted to the huge monetary stimuli applied by the BOJ during our sample period it is less surprising that inflation reacts little to the very large fiscal stimuli implied by our simulated policy, or that in consequence interest rate policy also changes little.

No Sunspot model — HP=y^*	$var(Y)$	$var(\pi)$	$var(R)$	Welfare Cost
No Sunspot base line	2.6270	0.8532	0.1262	3.6064
Fiscal policy	0.1837	0.8160	0.1074	1.1071

Table 6: Effects on Volatility of Sunspot Suppression in No-Sunspot Model

The rather surprising policy conclusion we reach from this analysis is that the GOJ should act to suppress sunspot pessimism on the assumption that the sunspot model is correct; this is because even in the equally probable case that it is incorrect, such an aggressive fiscal policy would still give welfare gains. We may note further that in the event the sunspot model is incorrect, the government will have evidence from the higher inflation rate- even though only slightly higher- that it is indeed incorrect, and it would then abandon its aggressive policy. Equally it may conclude from the evidence of only slightly higher inflation that the sunspot model is correct; in this case it can trumpet its strategy of killing any sunspot, and — this generally understood — can revert to a normal fiscal policy, confident that any sunspot can no longer arise. We would therefore argue that the GOJ would be best advised to embark on such an aggressive fiscal policy.

If we examine the policy rule we estimated the GOJ was following, it reveals that this was a long way from such an aggressive policy. The GOJ has instead weakly stabilised output around a slowly rising potential output trend. It might be argued that this relative restraint was forced on it by fear of insolvency. Yet throughout our sample since the end of the 1990s long term Japanese bond yields have been close to zero, creating little threat to solvency. Debt issues have been held domestically, with Japanese residents unwilling to shift their holdings to foreign stocks. In this paper we assume the model estimated on this sample holds for our policy experiments.

7 Conclusions

In recent decades following the financial crisis the major economies of the OECD have slowed down and lost momentum, leading some economists to argue that 'secular stagnation' has emerged and should be fought by stimulative policies, including fiscal stimulus given that monetary stimulus has been undermined by interest rates hitting the zero lower bound. One possible reason for this loss of momentum since the financial crisis could be a rise in pessimistic beliefs about future potential output growth triggered by a loss of business confidence from the crisis — such a rise could trigger a sunspot equilibrium in a rational expectations New Keynesian model. We examine in this paper whether this theory could account for the slow growth behaviour of the Japanese economy since the crisis of 1989 produced there by the 'bursting of the financial bubble' that had arisen due to the loose money policies of the 1980s. We show in this paper that a New Keynesian model with a weak equilibrium growth path driven by pessimism sunspot belief shocks does match the behaviour of the Japanese economy as represented by a VAR. We also found that an equally probable model of Japan post-1989 remains a conventional one where productivity growth simply slowed down for unknown reasons. Nevertheless, we find that under a welfare-optimising approach Japanese fiscal policy should have committed to eliminating the possible sunspot but stood ready to revert to a normal policy in the face of rising inflation.

References

[1] Allen, F., Barlevy, G. and Gale, D. (2017). On interest rate policy and asset bubbles. Federal Reserve Bank of Chicago Working Paper WP-2017-16..

[2] Aoki, K. and Nikolov, K. (2015): Bubbles, Banks and Financial Stability. Journal of Monetary Economics 74, 33-51.

[3] Armington, P.S. (1969) A Theory of Demand for Products Distinguished by Place of Production. IMF Staff Papers, Vol. 16(1), 159-178.

[4] Asriyan, V., Fornaro, L., Martín A. and Ventura, J. (2021) Monetary Policy for a Bubbly World. Review of Economic Studies 88(3), 1418-1456.

[5] Autor, D., Dorn, D. and Hanson, G. (2019) When Work Disappears: Manufacturing Decline and the Falling Marriage Market Value of Young Men. American Economic Review: Insights, 1(2), 161-78.

[6] Backhouse, R.E., and Boianovsky, M. (2016) Secular Stagnation: The history of a macroeconomic heresy. European Journal of the History of Economic Thought, 23(6), 946-970.

[7] Bengui, J. and Phan, T. (2018) Asset Pledgeability and Endogenously Leveraged Bubbles. Journal of Economic Theory 177, 280-314.

[8] Bernanke, B.S., Gertler, M., and Gilchrist, S. (1999) The financial accelerator in a quantitative business cycle framework. Handbook of Macroeconomics, Vol. 1. Part C. Chapter 21, 1341-1393.

[9] Biswas, S., Hanson, A. and Phan, T. (2020) Bubbly Recessions. American Economic Journal: Macroeconomics 12(4), 33-70.

[10] Black, D., Kolesnikova,N., Sanders, S. and Taylor, L. (2013) Are children normal? The Review of Economics and Statistics, 2013, vol. 95, issue 1, 21-33

[11] Eggertsson, G.B., Mehrotra, N.R., and Robbins, J.A. (2019) A Model of Secular Stagnation: Theory and Quantitative Evaluation. American Economic Review, 11(1), 1-48.

[12] Eichengreen, B. (2015) Secular Stagnation: the Long View. American Economic Review, 105(5), 66-70.

[13] Farhi, E. and Tirole, J. (2011) Bubbly Liquidity. Review of Economic Studies, 79, 678–706

[14] Farmer, R. (2020) The importance of beliefs in shaping macroeconomic outcomes. Oxford Review of Economic Policy 36(3), 675-711

[15] Gagnon, E., Johannsen, B.K., and López-Salido, D. (2021) Understanding the New Normal: The Role of Demographics. IMF Economic Review, 69, 357-390.

[16] Galí, J. (1999) Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations? American Economic Review, 89, 249-271.

[17] Galí, J. (2014) Monetary Policy and Rational Asset Price Bubbles. American Economic Review, 104(3), 721–752.

[18] Galí, J. (2021) Monetary Policy and Bubbles in a New Keynesian Model with Overlapping Generations. American Economic Journal: Macroeconomics, 13(2), 121-67

[19] Ikeda, D. (2013) Monetary Policy and Inflation Dynamics in Asset Price Bubbles. Bank of Japan Working Paper Series 13-E-4, Bank of Japan.

[20] Hansen, A. (1954) Growth or stagnation in the American economy. Review of Economics and Statistics, 36, 409-14.

[21] Ikeda, D. and Phan, T. (2019) Asset bubbles and global imbalances. American Economic Journal: Macroeconomics, 11(3), 209-251.

[22] Jones, C. (2022) Aging, Secular Stagnation and Business Cycles. Review of Economics and Statistics.

[23] Kearney, M.S. and Levine, P. (2020) Half a million fewer children? The coming COVID baby bust. Brookings Report https://www.brookings.edu/research/half-a-million-fewer-children-the-coming-covid-baby-bust/

[24] Le, V. P. M., Meenagh, D., Minford, A. P. L. and Wickens, M. (2011). How much nominal rigidity is there in the US economy? Testing a new Keynesian DSGE model using indirect inference. Journal of Economic Dynamics and Control 35(12), pp. 2078-2104.

[25] Le, V.P.M, Meenagh, D., and Minford, P. (2016a) Monetarism rides again? US monetary policy in a world of quantitative easing. Journal of International Financial Markets, Institutions and Money, 44, 85-102.

[26] Le, V.P.M., Meenagh, D., Minford, P., Wickens, M. and Xu, Y. (2016b) Testing macro models by indirect inference: a survey for users. Open Economies Review, 27 (1), 1-38.

[27] Martín, A. and Ventura J. (2012) Economic Growth with Bubbles. American Economic Review, 102(6), 3033-3058

[28] Meenagh, D., Minford, P., Wickens, M. and Xu, Y. (2019) Testing Macro Models by Indirect Inference: A Survey of recent findings. Open Economies Review, 30(3), 593-620.

[29] Meenagh, D., Minford, P., Nowell, E. and Sofat, P. (2010) Can a real business cycle model without price and wage stickiness explain UK real exchange rate behaviour? Journal of International Money and Finance, 29, 1131-1150.

[30] Miao, J. and Wang, P. (2012). Bubbles and Total Factor Productivity. American Economic Review, 102(3), 82-87.

[31] Miao, J. and Wang, P. (2018). Asset bubbles and credit constraints. American Economic Review, 108(9), 2590-2628.

[32] Miao, J., Wang, P. and Xu Z., (2015) A Bayesian DSGE Model of Stock Market Bubbles and Business Cycles. Quantitative Economics, 6, 599-635.

[33] Minford, P., and Peel, D. (2015) Advanced Macroeconomics: A primer, Second Edition. Elgar, London.

[34] Samuelson, P. A. (1958). An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money. Journal of Political Economy, 66(6), 467-482.

[35] Smets, F., and Wouters, R. (2007) Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. American Economic Review, 97(3), 586-606.

[36] Summers, L.H. (2014) U.S. Economic Prospects: Secular Stagnation, Hysteresis, and the Zero Lower Bound. Business Economics, 49, 65-73.

[37] Tirole, J. (1985). Asset Bubbles and Overlapping Generations. Econometrica, 53(6), 1499-1528.

8 Appendix 1: The DSGE model

8.1 Households

The economy is populated by a continuum of households indexed by $j \in [0,1]$. Each household j consumes a composite consumption $C_{t+s}(j)$, made up of final goods produced domestically $C_t^d(j)$ and imported goods $C_t^f(j)$, supplies labour service $N_{t+s}(j)$, and chooses to hold domestic $(B_t(j))$ and foreign $\left(B_t^f(j)\right)$ bonds to maximise the following present discounted utility function:

$$E_t \sum_{s=0}^{\infty} \beta^s \left[\frac{1}{1-\sigma_c}\left[C_{t+s}(j) - hC_{t+s-1}(j)\right]^{1-\sigma_c}\right] \exp\left(\frac{\sigma_c - 1}{1+\sigma_l} N_{t+s}(j)^{1+\sigma_l}\right)$$

where β is the discount factor, h is the degree of the external consumption habit formation, σ_c is the degree of relative risk aversion and σ_l denotes the inverse of the Frisch elasticity labour supply.

The budget constraint in real term is

$$C_t(j) + \frac{B_t(j)}{\varepsilon_t^b R_t P_t} + \frac{S_t B_t^f(j)}{\varepsilon_t^b R_t^f P_t} + T_t \leq \frac{W_t^h(j) N_t(j)}{P_t} + \frac{B_{t-1}(j)}{P_t} + \frac{S_t B_{t-1}^f(j)}{P_t} + \frac{\Pi_t}{P_t}$$

where R_t and R_t^f are nominal interest rate on deposit and foreign bonds respectively. P_t denotes the domestic price level and S_t is the nominal exchange rate measured as the amount of domestic currecy needed to purchase one unit of foreign currency, so that a rise in S_t means a depreciation in domestic currency. W_t^h is the nominal wage. Π_t is the dividends and T_t is a lump sum tax/transfer. ε_t^b is an financial assets prefence shock and it follows an AR(1) process $\ln \varepsilon_t^b = \rho_b \ln \varepsilon_{t-1}^b + \eta_t^b$.

The optimal conditions on $C_t(j)$ and $L_t(j)$ gives us the standard consumption Euler equation and labour supply, respectively. They replicate those of Smets and Wouters (2007). The optimal conditions in $B_t(j)$ and $B_t^f(j)$ together gives a uncovered interest parity condition

$$\frac{1}{r_t} = \frac{Q_t}{Q_{t+1} r_t^f} \tag{9}$$

where r_t is the real domestic interest rate on domestic bonds, r_t^f is the real foreign interest rate, Q_t is the real exchange rate$\left(Q_t = \frac{S_t P_t^f}{P_t}\right)$, P_t^f is the general foreign price level.

Households choose to consume domestically produced and imported goods to maximise their consumption composite $C_t = [\omega \left(C_t^d\right)^{\frac{\vartheta-1}{\vartheta}} - (1-\omega)\, \varepsilon_t^m \left(C_t^f\right)^{\frac{\vartheta-1}{\vartheta}}]^{\frac{\vartheta}{\vartheta-1}}$, subject to the budget constraint:

$$C_t = p_t^d C_t^d + Q_t C_t^f$$

where ω is the share of domestically produced goods in aggregate consumption, $p_t^d \equiv \frac{P_t^d}{P_t}$ is the relative price of domestic goods, and ϑ is an elasticity of substitution across consumption goods. ε_t^m is the imports demand shock that follows an AR(1) process, $\ln \varepsilon_t^m = \rho_b \ln \varepsilon_{t-1}^m + \eta_t^m,\ \ \eta_t^m \sim N(0,\ \sigma_m)$. The optimisation problem gives the demand for imports

$$IM_t = C_t^f = \left(\frac{(1-\omega)\varepsilon_t^m}{Q_t}\right)^{\vartheta} C_t \tag{10}$$

and the demand for domestically produced goods

$$C_t^d = \left(\frac{\omega}{p_t^d}\right)^{\vartheta} C_t \tag{11}$$

Symmetrically, the foreign demand for domestically produced goods is given as follows:

$$EX_t = \left(\frac{(1-\omega^F)\varepsilon_t^{ex}}{Q_t^f}\right)^{\vartheta^F} C_t^* \tag{12}$$

where ω^F, $\vartheta^F$and ε_t^{m*} is foreign equivalent to ω, ϑ and ε_t^m. C_t^* is the aggregate consumption in foreign country.

The balance of payments is defined as

$$\frac{S_t B_t^f}{R_t^f} - S_t B_{t-1}^f = P_t^d EX_t - S_t IM_t \tag{13}$$

that links the net foreign assets position to the trade balance.

8.2 Labour Market

Following Le, Meenagh and Minford (2016), the labour in production function combines in a fixed proportion labour inputs N_{1t} bought from imperfectly competitive market and labour inputs N_{2t} from perfectly competitive market. The aggregate labour supply N_t is:

$$N_t = N_{1t} + N_{2t}$$

where the share of unionised labour is ω_{NK}^w, so that $N_{1t} = \omega_{NK}^w N_t$ and $N_{2t} = (1 - \omega_{NK}^w) N_t$. These labour inputs enter each household's utility in the same way. Therefore, the aggregate hybrid wage is:

$$W_t = \omega_{NK}^w W_{1t} + (1 - \omega_{NK}^w) W_{2t} \tag{14}$$

W_{2t} where W_{2t} is the wage in a perfectly competitive market and equals to the marginal disutility of work; W_{1t} is the New Keynesian wage following Smets and Wouters (2007). These labour inputs are passed on to labour packer who offers the weighted wage for each unit of aggregate labour to intermediate goods firms. We also assume that household's utility includes these two types of labour in the same way. The wages are determined as below.

8.2.1 New Keynesian wage-setting

As in Smets and Wouters (2007), we assume that households supply a part of their labour services to labour unions, which then differentiate labour services and have the market power to set wages a la Calvo (1983). These differentiated labour services are gathered into labour input and sold on to the labour packer. Labour packer maximises the profit

$$\max_{N_t(j), N_t} W_t N_{1t} - \int_0^1 W_t(l) N_t(l) \, dl$$

where $N_t = \left(\int_0^1 N_t^{\frac{1}{1+\lambda_{w,t}}}(j) \, dj \right)^{1+\lambda_{w,t}}$. This then gives the demand for labour as $N_t(l) = \left(\frac{W_t(l)}{W_t} \right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} N_t$ and the wage cost in this imperfect labour market as $W_t = \left(\int_0^1 W_t^{\frac{1}{\lambda_{w,t}}}(j) \, dj \right)^{\lambda_{w,t}}$. $\lambda_{w,t}$ follows the exogenous ARMA process.

The labour unions purchase labour services from the households at the marginal rate of substitution between leisure and consumption, but then while exercising their market power in setting wages, they are subject to nominal rigidities, i.e. they can readjust wages with a probability $(1 - \zeta_w)$ in each period. For those that cannot adjust wages, $W_t(l)$ will increase at the weighted average of steady state inflation, π_*, and of last period's inflation, π_{t-1}. For those that can adjust, the problem is to choose a wage $\widetilde{W}_t(l)$ that maximises the wage income in all states of nature where the union is stuck with that wage in the future

$$\max_{\widetilde{W}_t(l)} E_t \sum_{s=0}^{\infty} \zeta_w^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+1}} \left[W_{t+s}(l) - W_{t+1}^h \right] N_{t+s}(l)$$

where $N_{t+s}(l) = \left(\frac{W_{t+s}(l)}{W_{t+s}} \right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} N_{t+s}$ and $W_{1t+s}(l) = \widetilde{W}_t(l) \left(\prod_{l=1}^{s} \pi_{t+l-1}^{\iota_w} \pi_*^{1-\iota_w} \right)$ for $s = 1, \ldots \infty$. Ξ_t is the lagrangean multiplier in the household's optimisation problem.

The imperfect labour market's aggregate wage is

$$W_{1t} = \left[(1-\zeta_w) \widetilde{W}_{1t}^{\frac{1}{\lambda_{w,t}}} + \zeta_w \left(\pi_{t+l-1}^{\iota_w} \pi_*^{1-\iota_w} W_{1t-1} \right)^{\frac{1}{\lambda_{w,t}}} \right]^{\lambda_{w,t}} \tag{15}$$

8.2.2 Perfectly competitive market

Households supply the other part of labour services to the competitive labour market, which would be gathered by the labour packer and resold to intermediate goods producers. The perfect market's aggregate wage is given as the marginal rate of substitution between leisure and consumption.

8.3 Goods Market

8.3.1 Final Goods Producers

Final goods producers assemble intermediate goods produced by firms. We follow Le, Meenagh and Minford (2016) in assuming that final goods Y_t is made up of a fixed proportion of intermediate goods sold in imperfectly competitive markets Y_{1t} and those sold in perfectly competitive market Y_{2t}. Hence, final output is given by:

$$Y_t = Y_{1t} + Y_{2t}$$

where ω_{NK}^P is the share of goods from imperfectly competitive markets, so that $Y_{1t} = \omega_{NK}^P Y_t$ and $Y_{1t} = \left(1-\omega_{NK}^P\right) Y_t$. The hydrid average weighted price equation is

$$P_t = \omega_{NK}^P P_{1t} + \left(1-\omega_{NK}^P\right) P_{2t} \tag{16}$$

where P_{1t} follows the New Keynesian price setting as in Smets and Wouters (2007) and P_{2t} is set at marginal costs. The final goods producers combine these two types of intermediate goods as a bundle and sell them at the above weighted average price.

8.3.2 New Keynesian Price setting

Intermediate goods producers/entrpreneurs choose ${K_t}^s(j)$ and $N_t(j)$ to minimise their cost of production

$$\min_{{K_t}^s(j), N_t(j)} R_t^k K_t(j) + W_t N_t(j),$$

subject to the production function

$$Y_t(j) = A_t {K_t}^s(j)^\alpha [\gamma^t N_t(j)]^{1-\alpha} - \Phi \tag{17}$$

where ${K_t}^s(j) = U_t(j) K_{t-1}(j)$ and $N_t(j)$ are the capital and labour input used in production respectively. α is capital share and Φ is fixed cost of producing products. A_t is total factor productivity and follows a nonstationary process. The cost minimisation problem gives an equal capital-labour ratio across firms

$$K_t^s = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t^k} N_t \tag{18}$$

This also gives the same marginal cost for all firms

$$MC_t = \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)} W_t^{1-\alpha} R_t^{ka} (A_t)^{-1} \tag{19}$$

Intermediate goods producers purchase capital and then choose the optimal level of capital utilization for production. The maximizing problem about the optimal degree of capital utilization is shown as:

$$\max_{U_t(j)} \frac{R_t^k U_t(j) K_{t-1}(j)}{P_t} - R_t^k \Upsilon \left[\exp\left(\frac{U_t(j)-1}{\Upsilon} \right) - 1 \right] K_{t-1}(j)$$

where $\Upsilon(U_t(j))$ is the adjustment cost with $\Upsilon(1) = 1$, $\Upsilon'(1) = R_t^k$ and $\frac{\Upsilon'(1)}{\Upsilon''(1)} = \psi$.

Intermediate goods producers sells goods to a part of the goods to final good producers in imperfect goods market and the rest to those in the perfect goods market. In the imperfect goods market, final good producers, would gather the intermediate goods, differentiate them, set prices and sell on. However, they are subject to Calvo pricing rule. Under this rule, the optimal price would be set from the following optimisation problem

$$\max_{\widetilde{P}_t(i)} E_t \sum_{s=0}^{\infty} \zeta_p^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+1}} \left[\widetilde{P}_t(i) \left(\prod_{l=1}^{s} \pi_{t+s-1}^{\iota_p} \pi_*^{1-\iota_p} \right) - MC_{t+s} \right] Y_{t+s}(i)$$

where $Y_{t+s}(i) = Y_{t+s} \left(\frac{P_{t+s}^i}{P_{t+s}} \right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}}$.

The aggregate price for the imperfect competitive goods market is

$$P_t^{\frac{1}{\lambda_{pt}}} = \left(1 - \zeta_p^s\right) \widetilde{P}_t^{\frac{1}{\lambda_{pt}}} + \zeta_p^s \left(\pi_{t-1}^{\iota_p} \pi_*^{1-\iota_p} P_{t-1} \right)^{\frac{1}{\lambda_{pt}}} \tag{20}$$

8.3.3 Perfectly competitive goods market

Intermediate goods producers also sell their products through the perfectly competitive goods market to the final goods producers. These final goods producers resell the goods to households at the price that is equal to the marginal cost of producing the goods.

8.4 Capital Producer

Capital producers are competitive. At the end of each period t, they buy undepreciated capital goods from intermediate good producer at price P_t^K and invest I_t to produce new capital. These capital goods would be sold to intermediate producer, as one of input, to produce products in period $t+1$. The capital producers maximise their profits

$$\sum_{s=0}^{\infty} \beta^s \left[P_t^K K_t - I_t - P_t^K (1-\delta) K_{t-1} \right]$$

subject to the capital evoluation

$$K_t = (1-\delta) K_{t-1} + \varepsilon_t^i \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) \right] I_t$$

where $\left(S'(\bullet) > 0,\ S''(\bullet) > 0, S(1) = S'(1) = 0 \right)$ and ε_t^i is the investment specific shock that affects the efficiency in transforming investment into new capital and follows an AR(1) process.

8.5 Financial Frictions and Quantitative easing

The intermediate goods producers are engaged in loan contracts to finance their capital purchase, choosing the level of capital utilisation and producing intermediate goods. We will look at these activites in turn.

8.5.1 Capital Purchase and Financial Frictions

The model introduces the financial friction a la Bernanke et al (1999). To facilitate this financial friction mechanism, it distinguishes between the intermediate goods producers and the capital goods producers. At period t, intermediate goods producers buy capital K_t from capital producers at price P_t^K and use it in next period production. In the end of period $t+1$, intermediate good producers receive $(1-\delta)P_{t+1}^K$ from reselling undepreciated capital back to capital producers and the marginal product of capital MPK_{t+1} from operating capital goods. The expected rate of return of capital is given by:

$$E_t \left[R_{t+1}^k \right] = E_t \left[\frac{MPK_{t+1} + (1-\delta) P_{t+1}^K}{P_t^K} \right] \tag{21}$$

It states that the expected rate of return on holding a unit of capital from t to $t+1$ consists of the marginal product of capital and the capital gain and it is also equal to the externally finance cost.

In order to purchase capital K_t, intermediate goods producers borrow from financial intermediaries (lenders). There is an asymmetric information problem between borrowers and lenders. The return on capital is sensitive to idiosyncratic risk which is known to intermediate producers but not to lenders, that could cause intermediate goods producers to default their loans and a costly process for lenders to recover funds in this situation. The assumption explains the reason of why external finance is more expensive than internal finance, and this gap between external and internal finance rates is derived from an optimal contract between borrowers and lenders. This gap between external and internal finance ($prem_t$) depends inversely on the share of the intermediate goods producers' capital investment ($pk_t + k_t$) that is financed by their own net worth (n_t). However, to facililate a role of money and effective quantitative easing under the zero lower bound of nominal interest rate, the model follows Le et al. (2016), where financial intermediaries also require intermediate good producers to provide money collateral upfront, which is easier to recover in case of default. The idea of money in the model works as follows. The central bank issues M0 in exchange for short-term bonds held by households. Once households have received this M0, they place it in banks as deposits and obtain the risk-free interest rate. Firms wish to acquire as much M0 as possible from banks to use as collateral for their borrowing. The M0 will appear on firms' balance sheet as the most liquid collateral pledged to banks in the case of bankcruptcy. The central bank can influence the credit market and its credit premium by varying the supply of M0. The loglinearised equation for the credit premium is as below

$$prem_t = E_t r^k_{t+1} - (r_t - E_t \pi_{t+1}) = \chi\,(pk_t + k_t - n_t) - \vartheta m^0_t + \epsilon^{prem}_t \tag{22}$$

where $\vartheta > 0$ denotes the credit easing effect of M0 on the loans. The evolution of networth, in turn, is determined as

$$n_t = \theta n_{t-1} + \frac{K}{N}\left(r^k_t - E_{t-1} r^k_t\right) + E_{t-1} r^k_t + \epsilon^n_t \tag{23}$$

where θ is the survival rate of firms and $\frac{K}{N}$ is the steady-state ratio of capital to networth. Networth depends on past net worth of surviving firms plus their total return on capital minus the expected return (which is paid out in borrowing costs to the bank) on the externally financed part of their capital stock.

The detailed derivation for the credit premium is presented as follows. Suppose the quantity of capital produced by capital producer is exactly same as quantity needed by intermediate firms. In the end of period t, the entrepreneur who manages intermediate firms purchases K_{t+1} amount of capital at a unit price of P^K_t . The entrepreneur has net worth NW_t, which is not sufficient to finance the expenditure on capital goods. So that he must sign in a loan contract with banks. The loan rate is Z_{t+1}. Banks require the c proportion of net worth as collateral and a φ proportion of collateral is used up in liquidating collateral. The amount of borrowing is given by

$$B_{t+1} = P^K_t K_{t+1} - (1-c)\,\mathrm{NW}_t$$

The capital is homogeneous and the gross return on capital of entrepreneur is ωR^k_{t+1}, where R^k_{t+1} is the ex post aggregate return on capital and ω is idiosyncratic disturbance to firm's return. ω is an identically independent distribution (i.i.d.) random variable and it follows the cumulative distribution function (c.d.f.) $F(\omega)$. $\omega \in (0,\ \infty)$ and $E\,(\omega) = 1$. $f(\omega)$ is the pdf of ω . At time $t+1$, entrepreneurs choose either pay off the loan or default. If the entrepreneur stays in business, he repays $Z_{t+1}B_{t+1}$ to the banks and receives $\omega R^k_{t+1} P^K_t K_{t+1} + c\mathrm{NW}_t$. If the entrepreneur defaults, the situation is just the opposite. Thus, the threshold level of $\overline{\omega}$ is defined by:

$$\overline{\omega} R^k_{t+1} P^K_t K_{t+1} + c\mathrm{NW}_t = Z_{t+1} B_{t+1}$$

Denote $L_t = \frac{P^K_{t\ t} K_{t+1}}{\mathrm{NW}_t}$ as leverage, the loan rate Z_{t+1} can be written as:

$$Z_{t+1} = \frac{\overline{\omega} R^k_{t+1}\ L_t + c}{L_t - (1-c)}$$

The higher threshold value of $\overline{\omega}_{t+1}$, the higher leverage and higher loan rates. When $\omega > \overline{\omega}$, the entrepreneur repays the promised repayment to the banks and receives the net revenue $\omega R^k_{t+1} P^K_t K_{t+1} +$

$c\mathrm{NW}_t - Z_{t+1}B_{t+1}$. When $\omega < \overline{\omega}$, return on capital is smaller than the opportunity cost which borrows from bank, the entrepreneur chooses to default. In this case, the entrepreneur gets nothing, banks receive the collateral and $(1-\mu)$ of the gross return on capital where μ is monitor cost.

The optimal debt contract maximizes entrepreneurial welfare subject to banks' feasibility constraint. We first discuss the lender's expected return. Banks are competitive, which ensures that, banks make zero profits in equilibrium. Banks' expected return of lending equals to its opportunity cost of those funds. The opportunity cost is $R_{t+1}B_{t+1}$, where R_{t+1} is riskless nominal interest rate. Thus, banks' zero profit condition is given by:

$$[1-F(\overline{\omega})]\,Z_{t+1}B_{t+1} + (1-\mu)\int_0^{\overline{\omega}} \omega R_{t+1}^k P_t^K K_{t+1} \partial F(\omega) + F(\overline{\omega})(1-\varphi)\,c\mathrm{NW}_t = R_{t+1}B_{t+1}$$

where $F(\overline{\omega})$ is the probability of default. On LHS, the first term is the repayment to the bank when the entrepreneur operates the firm well; the second term is the expected capital return when entrepreneur chooses to default; the third term is the collateral after liquidation under bankruptcy. Assume that $\Gamma(\overline{\omega})$ is the share of entrepreneurial expected capital return accrued to banks, $\Gamma(\overline{\omega}) = \overline{\omega}[1-F(\overline{\omega})] + G(\overline{\omega})$; and $G(\overline{\omega}) = \int_0^{\overline{\omega}} \omega \partial F(\omega)$, $\Gamma'(\overline{\omega}) = 1 - F(\overline{\omega})$, $\Gamma''(\overline{\omega}) = -f(\overline{\omega})$, the banks' zero profit condition is rewritten as:

$$[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})]\,R_{t+1}^k P_t^K K_{t+1} + (1-\varphi F(\overline{\omega}))\,c\mathrm{NW}_t = R_{t+1}\left(P_t^K K_{t+1} - (1-c)\,\mathrm{NW}_t\right),$$

which results in the banks' leverage offer curve as:

$$L_t\left(=\frac{P_t^K K_{t+1}}{NW_t}\right) = \frac{R_{t+1} - c\,[R_{t+1} - 1 + \varphi F(\overline{\omega})]}{R_{t+1} - (\Gamma(\overline{\omega}) - \mu G(\overline{\omega}))R_{t+1}^k} = \frac{R_{t+1} - c\,[R_{t+1} - 1 + \varphi F(\overline{\omega})]}{R_{t+1} - \Theta(\overline{\omega})\,R_{t+1}^k} \tag{24}$$

where it is an increasing and convex curve with respect to $\overline{\omega}$.

The entrepreneur can only make a profit if he does not breach the contract, i.e. drawing $\omega > \overline{\omega}$. The expected entrepreneurial earning from getting a loan is:

$$\omega R_{t+1}^k P_t^K K_{t+1} dF(\omega) - [1-F(\overline{\omega})]\,Z_{t+1}B_{t+1} + [1-F(\overline{\omega})]\,c\mathrm{NW}_t$$

The first term is the expected return on capital, the second term is the expected repayment to banks and the third term is the collateral required in the contract. It can be rewritten as

$$\left[\int_{\overline{\omega}}^{\infty} \omega dF(\omega) - [1-F(\overline{\omega})]\,\overline{\omega}\right] R_{t+1}^k P_t^K K_{t+1} = (1-\Gamma(\overline{\omega}))\,R_{t+1}^k P_t^K K_{t+1}$$

The terms of collateral have been eliminated, which mean that the firm's expected return is unaffected by the amount of collateral. Using the definition of leverage above, we can write the firm's expected return as:

$$(1-\Gamma(\overline{\omega}))\,R_{t+1}^k L_t$$

Hence the formal contracting problem for the entrepreneur is shown as:

$$\max_{L_t,\overline{\omega}} (1-\Gamma(\overline{\omega}))\,R_{t+1}^k L_t$$

s.t.

$$L_t = \frac{R_{t+1} - c\,[R_{t+1} - 1 + \varphi F(\overline{\omega})]}{R_{t+1} - \Theta(\overline{\omega})\,R_{t+1}^k}$$

The FOC is:

$$[R_{t+1} - c\,(R_{t+1} - 1 + \varphi F(\overline{\omega}))]\left[R_{t+1} - \Omega' R_{t+1}^k\right] = \left[\frac{-c\varphi F'(\overline{\omega})\,(1-\Gamma(\overline{\omega}))}{\Gamma'(\overline{\omega})}\right]\left[R_{t+1} - \Theta(\overline{\omega})\,R_{t+1}^k\right]$$

where $\Omega^{'}=\frac{\Theta^{'}(\overline{\omega})}{\Gamma^{'}(\overline{\omega})}+\left[1-\frac{\Theta^{'}(\overline{\omega})}{\Gamma^{'}(\overline{\omega})}\right]\Theta(\overline{\omega})\approx 1$. Therefore, the entrepreneur's optimal choice is

$$L_t\left[R_{t+1}-\Omega^{'}R^k_{t+1}\right]=\frac{-c\varphi F^{'}(\overline{\omega})(1-\Gamma(\overline{\omega}))}{\Gamma^{'}(\overline{\omega})} \tag{25}$$

There are now two equations () and () in $(\overline{\omega},L)$ space. We investigate the comparative static properties of changes around the equilibrium by taking the total differentiation of these two equations in $\partial L,\partial\overline{\omega},\partial\varphi$ and ∂R^k. We evaluate the derivatives at an equilibirum where $\varphi=0$. The total differentiations are expressed respectively as

$$\left[R-\Omega^{'}R^k\right]\partial L_t-L\Omega^{'}\partial R^k=\frac{-cF^{'}(\overline{\omega})(1-\Gamma(\overline{\omega}))}{\Gamma^{'}(\overline{\omega})}\partial\varphi \tag{26}$$

$$\partial L=L\left[\frac{\Theta(\overline{\omega})}{R-\Theta(\overline{\omega})R^k}\right]\partial R^k_{t+1}+\left[\frac{-cF(\overline{\omega})}{R-\Theta(\overline{\omega})R^k}\right]\partial\varphi+L\left[\frac{\Theta^{'}(\overline{\omega})R^k}{R-\Theta(\overline{\omega})R^k}\right]\partial\overline{\omega} \tag{27}$$

Since in the rest of the DSGE model, L is determined by capital and networth, and φ is determined by the provision of M0 as an alternative illiquid collateral, we consider them as exogenous to the financial sector analysis here, and then just need to solve for R^k and $\overline{\omega}$. Our interest lies in finding out the effect of φ on the equilibirum value of R^k and $\overline{\omega}$. These two elements are internal to the bank contract decision and unobservable in the public domain but in turn from these we can solve for the observable cost of the bank credit, Z, from the bankcruptcy threshold as $Z=\frac{R^k\overline{\omega}L+c}{L-1+c}$. We note from eq() that

$$\frac{\partial R^k}{\partial\varphi}=\frac{cF^{'}(\overline{\omega})(1-\Gamma(\overline{\omega}))}{L\Omega^{\prime}\Gamma^{\prime}(\overline{\omega})}>0$$

and from eq() that

$$\frac{\partial\overline{\omega}}{\partial\varphi}=\frac{cF(\overline{\omega})}{L\Theta^{\prime}(\overline{\omega})R^k}\left[1-\frac{F^{'}(\overline{\omega})\Theta(\overline{\omega})(1-\Gamma(\overline{\omega}))}{F(\overline{\omega})\Omega^{\prime}\Gamma^{\prime}(\overline{\omega})}\right]>0.$$

The latter expression is positive, proven numerically in Le et al (2016). These two conditions means that

$$\frac{\partial Z}{\partial\varphi}=\frac{c}{L-1+c}\left[\frac{F(\overline{\omega})}{\Omega^{\prime}(\overline{\omega})}\left(1-\frac{F^{'}(\overline{\omega})\Theta(\overline{\omega})(1-\Gamma(\overline{\omega}))}{F(\overline{\omega})\Omega^{\prime}\Gamma^{\prime}(\overline{\omega})}\right)+\left(\frac{\overline{\omega}F^{'}(\overline{\omega})(1-\Gamma(\overline{\omega}))}{\Theta^{\prime}(\overline{\omega})(1-\Gamma(\overline{\omega}))}\right)\right]>0$$

Since φ is reduced by M0 injections, we can say that a rise in M0 will reduce the required return on capital and also the credit premium.

8.6 Monetary Policy

Following Le et al. (2016) ,the model assumes that the central bank would conduct monetary policy via the short-term interest rate according to the Taylor rule formulation normally. However, under the zero lower bound of normal interest rate, the central bank would resort to using Quantitative easing policy to regulate the economy. The model facilitates the switch between these two states endogenously.

When the nominal interest rate is above zero percent, the central bank set interest rate according to the following Taylor rule

$$r_t=\rho r_{t-1}+(1-\rho)\left(r_p\pi_t+r_y(y_t-y^*_t)\right)+r_{\Delta y}\Delta\left(y_t-y^*_t\right)+\varepsilon^r_t r>0 \tag{28}$$

where ρ measures the degree of interest rate smoothing. r_p, r_y and $r_{\Delta y}$ represents Taylor's rule responses to inflation, output and change in output, respectively. The monetary policy shock follows an AR(1) process $\varepsilon^r_t=\rho_r\varepsilon^r_{t-1}+\eta^r_t$. The supply of M0 is set as

$$m^0_t-m^0_{t-1}=\vartheta_{m2}\left(m^2_t-m^2_{t-1}\right)+\epsilon^{m0}_t \tag{29}$$

to accommodate the broad money supply M2, which is determined by the firms balance sheet quantities

$$m_t^2 = \left(1 - \frac{M0}{M2} + \frac{N}{M2}\right) k_t + \frac{M0}{M2} m_t^0 - \frac{N}{M2} n_t, \tag{30}$$

where $\epsilon_t^{m0} = \rho_{m0}\epsilon_{t-1}^{m0} + \eta_t^{m0}$, $\frac{M0}{M2}$ and $\frac{N}{M2}$ are the steady state ratios of M0 and networth to M2, respectively. The money supply is defined as $M2 = credit + bank\ deposit$,where credit is equal to capital expenditure and collateral in excess of networth, resulting in $M2 = M0 + (K - N + collateral)$. Equation 30) is therefore derived given that collateral is a fixed proportion of money.

When the nominal interest rate reaches 0%, the interest rate Taylor rule ceased to operate, the central bank has to use unconventional monetary policies such as quantitative easing. In this model, we assume that the central bank issue M0, which are transfered to households who then deposit with banks who lend M0 to intermediate producers who want to hold it as much as for the collateral purposes to reduce the cost of borrowing. In turn, the central bank uses the quantitative easing to response to the aggregate demand which depends on the credit premium. The monetary policy under the zero lower bound ($r_t = 0$) is characterised by

$$m_t^0 - m_{t-1}^0 = \vartheta_{prem}\left(prem_t - prem^*\right) + \epsilon_t^{m0} \tag{31}$$

where ϑ_{prem} is the elasticity of M0 to the credit premium.

8.7 Closing the Model

As set in Smet and Wouters(2007), government spending relate to the steady state output path $\varepsilon_g = \frac{G_t}{Y\gamma^t}$, it follows the process that

$$\varepsilon_t^g = \left(1 - \rho_g\right)\varepsilon^g + \rho_g\varepsilon_{t-1}^g + \rho_{ga}\varepsilon_t^a - \rho_{ga}\varepsilon_{t-1}^a + \eta_t^g,\ \eta_t^g \sim (0,\ \sigma_g)$$

where $0 < \rho_g < 1$ and the government spending is affected by the productivity process. The government spending comes from collecting lump sum taxes T_t and issues B_t. The budget constraint is shown as:

$$P_t G_t + B_{t-1} = T_t + \frac{B_t}{R_t}$$

By integrating the behaviours among households, firms, entrepreneurs, central bank and government, the good market clearing condition in log-linearized form is:

$$y_t = \frac{C}{Y}c_t + \frac{I}{Y}i_t + \frac{K}{Y}MPK^*\frac{1-\psi}{\psi}mpk_t + \frac{C^e}{Y}c_t^e + \frac{EX}{Y}ex_t - \frac{IM}{Y}im_t + \varepsilon_t^g \tag{32}$$

where c_t^e is consumption of the bankcrupt firms, but in log it is equal to the networth variable.

9 Appendix 2: Terminal conditions

At the terminal date, two main conditions must be satisfied:

- UIP, such that q, the real exchange rate, is constant, equilibrating the current account, so that $ex = q.im$ at T. $lnex = w.lnWorldGDP + \sigma_x \ln q. \ln im = m. \ln y^* - \sigma_m \ln q$. Hence $lnex = w.lnWorldGDP + (\sigma_x + \sigma_m - 1)\ln q^* - m. \ln y^*$

  so that $\ln q^* = [+m. \ln y^* - w.lnWorldGDP]/(\sigma_x + \sigma_m - 1)$

- consumption must be given by market-clearing:$y^* = c^* + I^* + G^*$; hence $\ln c^* = \frac{c}{y}[\ln y^* - \frac{i}{y}\ln(I) - \frac{g}{y}\ln G]$

Hence Y^* determines q^* and C^*. These then feed into the exchange rate, exports, imports at $T-1$ etc; and into C at $T-1$ etc via Euler equation.

To obtain I^* we note that at T $(r_F + \delta)K^* = (1-\alpha)y^*$ so that $I^* = \Delta K^* = [(1-\alpha)/(r_F + \delta)]\Delta y^*$

To find labour market outcomes note that $\ln Ls = \frac{1}{\sigma_l}(\ln w^* - \ln c); \ln L_D = \ln\alpha + \ln y^* - \ln w^*$; hence $\ln w^* = (\frac{1}{1+1/\sigma_l})[\ln\alpha + \ln y^* + 1/\sigma_l \ln c^*]$

Note that via Cobb-Douglas: $y^* = [K^{*1-\alpha} L^{*\alpha}] A^*$ and $y = wL + rK$.

The Taylor Rule is assumed to be satisfied with inflation at its target and the real interest rate equal to the world level.

10 Appendix 3: Empirical results from model without sunspot

	Sunspot model coefficients
Steady-state elasticity of capital adjustment	8.0073
Elasticity of consumption	2.5387
External habit formation	0.4636
Probability of not changing wages	0.4402
Inverse of Frisch elasticity of labour supply (σ_l)	3.3006
Probability of not changing prices	0.8066
Wage indexation	0.1905
Price indexation	0.4457
Elasticity of capital utilisation	0.9435
Share of fixed costs in production (+1)	1.7772
Taylor Rule response to inflation	1.0612
Interest rate smoothing	0.9735
Taylor Rule response to output	0.0060
Taylor Rule response to change in output	0.0126
Share of capital in production	0.3338
Proportion of sticky wages	0.2384
Proportion of sticky prices	0.4806
Elasticity of the premium with respect to leverage	0.0103
Monetary response in crisis time	0.0978
Monetary response in normal time	0.0406
Elasticity of premium with respect to money	0.0494
Bubble forward root	0.9929
Fiscal response	0.6470
Wald	19.5040
Transformed Wald (t-stat)	0.9898
P-Value	0.1134

Table 7: Coefficient Estimates for Model with No Sunspot

	Actual	Mean	2.5th Percentile	97.5th Percentile	In/Out
Y_Y	0.8031	0.3062	−0.3038	0.5697	OUT
Y_PI	0.0554	0.0819	−0.3580	0.5623	IN
Y_R	−0.1540	2.0262	−15.1630	24.7305	IN
PI_Y	0.0810	0.0325	−0.0697	0.1299	IN
PI_PI	−0.3040	0.0385	−0.2288	0.3424	OUT
PI_R	0.6767	0.2716	−4.0278	5.0695	IN
R_Y	0.0021	−0.0038	−0.0122	0.0032	IN
R_PI	0.0051	−0.0026	−0.0181	0.0133	IN
R_R	0.9320	0.8115	0.0533	0.9932	IN
Var(Y)	1.5653	2.1700	1.0108	7.7991	IN
Var(PI)	0.2339	0.3870	0.2397	0.5710	OUT
Var(R)	0.0009	0.0025	0.0000	0.0108	IN

Table 8: Auxiliary Model Parameter Bounds for Model with No Sunspot

A BRIEF HISTORY OF JAPANESE MONETARY POLICY IN TWENTY-FIRST CENTURY

Abstract

Tanweer Akram[1]
Citibank

VERSION 2.0
September 27, 2023

[1]***Corresponding author***. **Address:** Citibank, 6400 Las Colinas Blvd., Irving, TX 75039, USA. **Phone:** +1 703 405 8656. **Email:** tanweer.akram@gmail.com **ORCID**: https://orcid.org/0000-0002-6646-0902

A BRIEF HISTORY OF JAPANESE MONETARY POLICY IN TWENTY-FIRST CENTURY

ABSTRACT

The Bank of Japan (BOJ) has been a pioneer in innovative, interesting, and experimental monetary policy measures in its attempt to combat low inflation and deflation since the beginning of the twenty-first century. This paper will review the history of the BOJ's monetary policy in the twenty-first century and highlight the strategies that policymakers pursued in the context of the major macroeconomic conditions in Japan and the rest of the world. The source materials for the historical analysis will be monetary policy statements, minutes of monetary policy meetings, outlook reports, semi-annual reports on currency and monetary control, speeches by policymakers and other relevant documents available in English, and Japanese and global macroeconomic data. An historical analysis of the BOJ's monetary policy measures in the light of macroeconomic conditions in Japan and overseas enables a comprehensive and balanced assessment of both the innovations and the limitations of Japan's monetary policy in the twenty-first century.

KEY WORDS: Japanese monetary policy; Bank of Japan; unconventional monetary policy; deflation

JEL CLASSIFICATIONS: E40; E62; F30; N15

Policy Mix and the Sustainability of Public Debts in the Euro Area

Francois Langot* & Hugo Minnella†‡

August 2023

Abstract

This paper highlights the specifics of a monetary union, such as the Euro area, regarding the choices of monetary and fiscal policies. We show that only one policy can be active, while others fiscal and monetary policies should be passive. However, as the dynamics of public debts are specific to the choices made by each government, the stability requires coordination of fiscal policies, particularly in the case of a liquidity trap situation. Thus, a fiscal union, taking the form of a common debt as the one issued during the Covid19 crisis, guarantees the dynamic stability of the area, notwithstanding the monetary policy, chosen or constrained—thus improving institutional robustness of the European Union.

Keywords: Euro area, Public debt, Taylor rule, Fiscal rule, Fiscal theory of price level, NK model

JEL Classification: E32, E52, E62, F41, H63

*Le Mans University (Gains-TEPP & IRA) & Institut Universitaire de France & Paris School of Economics & Cepremap & IZA. Contact: `flangot@univ-lemans.fr`. Adress: Faculté de Droit et des Sciences Economiques, Le Mans Université, Avenue Olivier Messiaen, 72085 Le Mans cedex 9, France.

†Le Mans University (Gains-TEPP & IRA) & Cepremap. Contact: `hugo.minnella@cepremap.org`

‡This work owes much to the discussions with S. Adjemian, P. Beaudry, D. Bonam, T. Brand, D. Cohen, D. Debortoli, A. Ferriere, J. Gali, N. Groshenny, J-O. Hairault, S. Kankanamge, F. Karame, M. Lemoine, V. Nispi Landi, P. Martin, N. Traum, F. Tripier, S. Villemot and R. Wouters. We remain responsible for errors and omissions that may persist.

1 Introduction

When major crises occur asking for large stimulus, the choice of policy mix for ensuring the sustainability of public debts is at the center of macroeconomic analysis. The European Union (EU) requires specific analysis. Indeed, unlike a federal system like the United States (US), the decision-making centers of monetary and fiscal policies are not both centralized and, therefore, do not limit the process of political coordination between two actors. Therefore, the European policy mix also requires coordination of fiscal decisions, involving multiple decision-makers.

In this paper, we determine which set of rules, that is, which monetary-fiscal policies, make it possible to guarantee the stability of a monetary union such as the EU, and therefore the sustainability of public debts. We focus on equilibria where financial markets are complete (the "ideal" monetary union, where the financial integration would be achieved), or in incomplete.[1] Even these equilibria seem to be close to the closed economy equilibrium, it allows us to concentrate ourself only on the coordination between monetary and fiscal policies where the European specificity is restricted to the existence of several public debts.[2] Not surprisingly, but without our prior knowledge, we show that this set of rules is not unique, and it strongly depends on the effective margins of monetary policy. More specifically, we show that a fiscal union, that would take the form of a common debt, is necessary for European stability to overcome the limits of monetary policy when interest rates can no longer be lowered.

As this has been done since Leeper (1991), our paper considers that both monetary and fiscal authorities can only choose between two types of policies, active or passive. In a closed economy, the traditional policy mix associating an "active" monetary policy (application of the Taylor principle) and a "passive" fiscal policy (fiscal rule with a debt brake) ensures the dynamic stability of the New-Keynesian (hereafter, NK) model. However, another policy mix ensures equilibrium stability: it is based on a combination of a "passive" monetary policy (failure to respect the Taylor principle) and an "active" fiscal policy

[1]The integration of financial markets has long been considered to be a crucial condition to foster the adoption of a common currency (see Ingram (1969) and Mundell (1973)). Nevertheless, this view can be moderated by Auray and Eyquem (2014)'s results that shown that the welfare gains from complete markets induced by the possibility of risk sharing can be dominated by welfare costs induced by nominal rigidities when a unique central bank can not close all gaps and replicate the flexible prices equilibrium.

[2]We also focus on rules ensuring the existence of a single path around the steady state (a prerequisite for economic stabilization), leaving aside the analysis of multipliers that are specific to each of the policy mix regimes.

(fiscal rule without debt brake). On this last equilibrium, the Fiscal Theory of the Price Level (hereafter, FTPL) applies.[3] When considering the case of the EU, a cursory analysis could lead to the conclusion that European policy makers have the same arbitrations than those in the US because in open economies with a monetary union and complete financial markets the NK model has the same dynamic characteristics for the inflation and output gap than a closed economy. We show that this comfortable transposition of a policy mix analysis in a closed economy to the European case is not trivial. Indeed, given that each government of the Euro area maintains its fiscal independence, there are multiple public debt dynamics, a priori not controlled by the same debt brake.[4] By taking into account these differences in public debt dynamics across the EU members, we show that an active monetary policy is a coordination device because it reduces the political decision of each government to a binary choice: to fight or not the explosiveness of its public debt, the stability of inflation and output gap being insured by the ECB. When this active monetary policy is possible and thus implemented to apply European treaties, the governments of all countries of the area voluntary control the stability of their public debts. These passive fiscal policies are then in accordance with the European treaties that include the "1/20th rule", a fiscal rule which requires that 1/20th of "excess debt" is eliminated each year (see Fiscal Compact of 2012). A free rider behavior is excluded because it will lead to explosive dynamics of all the area. By contrast, when the ECB policy does not/can not respect the Taylor principle (passive monetary policy), we show that only one country of the Euro area must implement an active fiscal policy. This result is due to the integration of inflation rates as well as output gaps: with unique inflation rate dynamic in the area, only a single public debt is needed to anchor the price level. Indeed, as there is a common price level in a currency union, it only exists one degree of freedom to be pinned down by one monetary policy and two fiscal policies, thus implying that only one of these policies must be active and the others two have to be passive to implement a unique equilibrium.[5] But, this principle leaves unclear which country of the area will be free to escape the European treaty. This lack of coordination device advo-

[3]Leeper (1991), Sims (1994), Woodford (1998), and Cochrane (2001) developed the FTPL, where some intuitions are in Sargent and Wallace (1981) and Aiyagari and Gertler (1985). See Bassetto (2002) for an analysis of a game between market players and the government, where the equilibrium solution provides the foundations of the FTPL. See Bassetto (2008) for an overview of the FTPL or Christiano and Fitzgerald (2000), Leeper and Leith (2016), and Cochrane (2019) for surveys on FTPL. Bassetto and Sargent (2020) analyze the impact of several interactions between monetary and fiscal policies on the stability of equilibirum.

[4]This unique rule is lacking in the current treaties. Indeed, the European treaties provide some "general principles" for the fiscal policy, but not an explicit fiscal rule that is uniform for each state.

[5]This extension of the FTPL in a monetary union, has been already discussed in Bergin (2000).

cates in favor of a fiscal union for coordinating fiscal policies: this can be done by issuing bonds on behalf of the EU. This adds a new public debt in the Euro area that should be managed with a fiscal policy ensuring the dynamic stability of the Euro area whatever the monetary policy is.[6] We show that these results does not change qualitatively when financial markets are incomplete.

Therefore, we provide support to the way chosen to finance the "Next Generation EU" initiative.[7] Indeed, by allowing the European Commission to issue bonds on the financial markets on behalf of the EU, it introduces in the Euro area a new tool that ensures its stability when monetary policy can not react to inflation changes. With respect to design of new rules largely debated in the EU[8], we show that the "fiscal brakes" are crucial for the public debt sustainability of each country: they are necessary when the European Central Bank (ECB) controls inflation through nominal interest rate, but also when this last tool can not be used and is replaced by a common debt.[9]

The impractical implementation of the Taylor principle has been experimented when the nominal interest rate has been close or equal to zero in the past few years.[10] Policy choices

[6] This view goes in the same direction of Sims (2012) who wrote *"a solution would be to fill in the institutional gaps in the original euro framework. At a minimum, this would require a new institution with at least some taxing power, able to issue debt and to buy, or not buy, the debt of euro area governments. Such an institution would of course have to be subject to democratic control"* (p.217). We complement the arguments of Farhi and Werning (2017) and Berger, Dell'Ariccia and Obstfeld (2018) showing that a fiscal union has a greater ability to stabilize asymmetric shocks. In our paper, we focus on the ability of the fiscal union, that takes the form of a common debt, to guarantee the dynamic stability of the area, whatever may be the nature of the shocks. We also complement the arguments of Aguiar et al. (2015) that show why existence of a fiscal externality can rationalizes the imposition of debt ceilings in a monetary union, but above all, show that a high-debt country is less vulnerable to rollover crises in sovereign debt markets, and have higher welfare, when it belongs to a union with an intermediate mix of high- and low-debt members, than one where all other members are low-debt. We do not also discuss the issue on the use of the EU commission debt which can be devoted to green investments, as discussed e.g. by Garicano, (2022) or Darvas (2022).

[7] President von der Leyen announced on May 27, 2020 that the bulk of the recovery measures proposed by the EU will be powered by the "Next Generation EU" with financial firepower of €750 billion. The financing will be possible by borrowing up to €750 billion on behalf of the EU, through the issuance of bonds. See the speech of President von der Leyen, `https://ec.europa.eu/commission/presscorner/detail/en/speech_20_941`.

[8] See among other Blanchard (2023), Blanchard, Leandro and Zettelmeyer (2021), D'Amico, Giavazzi, Guerrieri, Lorenzoni and Weymuller (2022) or Martin, Pisani-Ferry and Ragot (2021) for discussions on the fiscal reforms for the EU.

[9] As it is shown in Bonam and Hobijn (2021), what matters is a credible commitment to apply fiscal breaks in the future, even if fiscal breaks are not strictly respected at each time.

[10] See Blanchard (2023) for a discussion of past and future interest rates.

for the management of the recent crises have been, therefore, implemented in a context where monetary policy is highly constrained. In a liquidity trap, or at the Zero-Lower-Bond (hereafter, ZLB), we show that a fiscal union, or a common debt, is a very useful tool to ensure the dynamic stability of the Euro area. Indeed, at the ZLB, dynamic stability must be ensured without using an active monetary policy. The situation is, therefore, close to an equilibrium where only an active fiscal policy, implemented by a unique European government (the European Council), would ensure the dynamic stability of the Euro area. Indeed, the multiplicity of public debts then raises the problem of the designation of the state which will need to have an active fiscal policy, thus allowing to anchor the price level in the Euro area. This suggests that a fiscal union, and thus a common debt, will solve this coordination problem. Of course, the liquidity trap (or ZLB) can only be perceived as transitory by agents. In this case, we show that what matters for the stability of the equilibrium are the agents' expectations on the policies implemented after this episode of liquidity trap. To this end, we follow Christiano, Eichenbaum, and Rebelo (2011)[11] by assuming a stochastic duration for this ZLB episode. We show that if agents expect that the monetary policy will become active after the ZLB period, then a stable equilibrium of the Euro area will be possible if each European country respects its commitment to the European Treaty by choosing a passive fiscal policy. Conversely, if agents expect the monetary policy to remain passive after the ZLB period, then a stable equilibrium of the Euro area will be possible only if one of the European countries does not respect its commitment to the European Treaty. This last result supports a fiscal union unifying the multiple debt dynamics of the Euro area, and thus making it possible to resolve the problem of the designation of the only state that can avoid the fiscal constraints of the European Treaty.

The remainder of the paper is structured as follows. Section 2 presents the arithmetics of public debt dynamics at partial equilibrium. In section 3, we present a two-country model of the Euro area with complete markets (CM) in which the countries are asymmetric because fiscal rules are country-specific and shocks have local realizations. We describe the dynamic properties of inflation, output gap, and public debts conditionally to the policy choices of the central bank and different governments. In Section 4, we show that our results regarding fiscal-monetary interactions apply to a more general framework where markets are incomplete (IM). Section 5 extends the analysis to episodes when the monetary policy is constrained by a zero lower bound. The link between the results of this paper and previous literature is discussed in section 6. Finally, section 7 concludes.

[11]These authors have based their analysis on the previous works of Eggertsson and Woodford (2003), Christiano (2004), and Eggertsson (2004)

2 The Public Debt Dynamics

Lets us denote $\mathcal{B}_t$ the nominal payoff during period t of the government bonds of the Home country bought during period $t-1$ and $1/R_t$ the nominal price at t of an asset that pays 1€ during period $t+1$. Using P_t as the consumer price index (CPI) in the Home country, the government's budget constraint is given by:

$$\frac{1}{R_t}\frac{\mathcal{B}_t}{P_t} + \frac{\mathcal{D}_t}{P_t} = \frac{\mathcal{B}_{t-1}}{P_t} \quad \Leftrightarrow \quad \frac{1}{R_t}\mathcal{B}_t^r + \mathcal{D}_t^r = \frac{1}{\Pi_t}\mathcal{B}_{t-1}^r,$$

with $\Pi_t = \frac{P_t}{P_{t-1}}$ and where $\mathcal{D}_t^r$ are the real net transfers, namely, $\mathcal{D}_t^r = Tax_t - Tr_t$, with Tax_t a real lump-sum tax and Tr_t a real lump-sum transfer. We deflate the nominal debt ($\mathcal{B}_t$) by the CPI to define the real debt $\mathcal{B}_t^r$. Assume that it exists an exogenous growth characterized by the real growth rate g. If we focus on a balanced growth path, then the stationarized real variables are $B_t^r = \frac{\mathcal{B}_t^r}{(1+g)^t}$ and $D_t^r = \frac{\mathcal{D}_t^r}{(1+g)^t}$. This leads to:

$$\frac{1}{(1+r_t)}B_t^r + D_t^r = \frac{1}{(1+\pi_t)(1+g)}B_{t-1}^r, \quad \text{where} \begin{cases} R_t &= 1+r_t \\ \Pi_t &= 1+\pi_t. \end{cases}$$

Existence of a Steady State Public Debt. At the steady state, the government's budget constraint is given by:

$$D^r = \left(\frac{1}{(1+\pi)(1+g)} - \frac{1}{1+r}\right)B^r \approx \frac{r-\pi-g}{1+r+\pi+g}B^r \quad \Rightarrow \quad \frac{D^r}{B^r} = \Psi(r,\pi,g).$$

This relation defines a rule between D^r and B^r, given $\{r,\pi,g\}$, that must be satisfied. This rule doesn't say anything on the levels of D^r and B^r, only on the ratio $\frac{D^r}{B^r}$. Hence, if they are both expressed as a fraction of GDP, this rule is not able to set the Debt-to-GDP ratio and the Deficit-to-GDP ratio, but only the ratio of these two ratios.[12]

> *"The 60% debt ratio target was always arbitrary.... there is no such thing as a universal threshold over which debt becomes unsustainable,... the relevant debt level depends on many factors, in particular the real interest rate on debt."* Blanchard (2023), p. 75.

[12] This steady state restriction says that with a nominal interest rate on bonds at 3% (2%), an inflation rate at 2%, the real growth rate of the economy must be equal to 6.5% (5.5%) in order to satisfy $\frac{D^r}{B^r} = \frac{-3\% \text{ of the GDP}}{60\% \text{ of the GDP}}$, where values are those of the Maastricht treaty. With a nominal interest rate on bonds at 3%, an inflation rate at 2% and the real growth rate of the economy equal to 2%, the Deficit-to-Debt ratio must be closed to 0.01, thus compatible e.g. with the Deficit-to-GDP ratio of 2% and a Debt-to-GDP ratio of 200%.

In the following, we assume that this rule is satisfied, without any *a priori* on the level of B^r.

Public Debt Dynamics Around its Steady State. The linearized government's budget constraint around the steady state is given by

$$\widehat{b}_t^r \;=\; \beta^{-1}(\widehat{b}_{t-1}^r - \widehat{\pi}_t) + \widehat{r}_t + (1-\beta^{-1})\widehat{d}_t^r$$

where $\beta = \widetilde{\beta}(1+g)^{\sigma} = \frac{1+\pi}{1+r}$ from the Euler equation on consumption (see below), with σ the intertemporal elasticity of substitution for consumption. With $\beta < 1$ and $\{\widehat{\pi}_t, \widehat{r}_t, \widehat{d}_t^r\}$ some stationary processes independent from $\widehat{b}_t^r$, the debt explodes. In order to avoid debt explosion, the government can adopt a "fiscal rule" such that[13]:

$$D_t^r \;=\; D^r + \gamma\left(\frac{1}{R_{t-1}}\frac{B_{t-1}}{P_{t-1}} - \frac{B^r}{R}\right) + \varepsilon_t^F \quad \Rightarrow \quad \widehat{d}_t^r = \gamma\frac{1}{\beta^{-1}-1}(\widehat{b}_{t-1}^r - \widehat{r}_{t-1}) + \varepsilon_t^F$$

where ε_t^F is the discretionary component of the public deficit (stationary), D^r the structural (constant) deficit and $\gamma\left(\frac{1}{R_{t-1}}\frac{B_{t-1}}{P_{t-1}} - \frac{B^r}{R}\right)$ the deficit's component that brakes the debt's explosion. The linearized budget constraint of the government becomes:

$$\widehat{b}_t^r \;=\; (\beta^{-1}-\gamma)\widehat{b}_{t-1}^r - \beta^{-1}\widehat{\pi}_t + \widehat{r}_t + \gamma\widehat{r}_{t-1} + (1-\beta^{-1})\varepsilon_t^F.$$

The public debt does not explode iff $\beta^{-1} - \gamma < 1 \Leftrightarrow \gamma > \beta^{-1} - 1$, when $\{\widehat{\pi}_t, \widehat{r}_t\}$ are some stationary processes independent from $\widehat{b}_t^r$. This "fiscal rule" ($\gamma > 0$) can be viewed as an approximation of the "1/20th rule", which requires that 1/20th of "excess debt" is eliminated each year (see Fiscal Compact of 2012).[14] This basic analysis puts forward the idea that the debt brake, such the "1/20th rule", is a crucial tool to avoid public debts explosion.

If the stationarity (assumed previously) is a property that must share all processes of a non-explosive equilibrium, the independence of aggregates, such as $\{\widehat{\pi}_t, \widehat{r}_t\}$, with respect to public debt is not trivial. Hence, if the stability of the economy depends on the properties of the complete dynamic of the general equilibrium, one must go beyond this partial analysis based uniquely on the government's budget constraint in order to design policies insuring public debts sustainability.

[13]This type of fiscal rule was first discussed in Bohn (1998).

[14]With a gap between nominal interest rate (r) and inflation (π) of $\{-1; 0; 1; 2; 3\}$%, the Debt-to-GDP ratio initially equal to 120% comeback to 90% (a half of the gap to 60%) in $\{13; 15; 18; 24; 35\}$ years with $\gamma = 1/20$. Hence, the European Treaty ensures debt stability. But, with $\gamma = 1/40$, the debt explodes when $r - \pi = 3\%$ and the half of way to comeback to the targeted Debt-to-GDP ratio is done in $\{13; 15; 18; 24\}$ years for the other values for $r - \pi$.

3 Public Debt Sustainability in a Monetary Union

The Euro area is modeled as an economy comprising two "countries". The first one is "a real country", whereas the second one is "the rest of the Euro area", that is, aggregation of all the other Euro area countries.[15] The Euro area is populated by a continuum of infinitely lived households indexed by $j \in [0,1]$. The population in segment $j \in [0,m)$ belongs to country H (Home country) and the population in segment $j \in [m,1]$ belongs to country F (Foreign country). Therefore, m is the relative size of country H in the Euro area. The small Home country case can be derived by taking the limit of the two-country model as $m \to 0$. As in Gali's (2008) textbook, we assume that financial markets are complete, hence assuming that the financial integration promoted when monetary union was built has been realized. This simplify the analysis and allows us to concentrate ourself on the multiplicity of public debts dynamics in interaction with a unique monetary policy[16].

Home bias and country size. The home bias in Home (Foreign) households' preferences is crucial because it determines the gap between CPI and PPI inflation rates. Therefore, we assume that the share of imported goods denoted α (α^*) should decrease with the relative size of country H (F) and with the degree of home bias in Home (Foreign) households' preferences. A tractable way to formalize these ideas is to define $\alpha = \overline{\alpha}(1-m)$ ($\alpha^* = \overline{\alpha}^* m$), where the exogenous parameter $\overline{\alpha}$ ($\overline{\alpha}^*$) is inversely related to the degree of home bias in H (F) households' preferences. In the following, we assume symmetric preferences, namely $\overline{\alpha} = \overline{\alpha}^*$.

3.1 New Keynesian Equilibrium Dynamics

Lets us denoted $\widehat{\pi}_{H,t}$ and $\widehat{\pi}^*_{F,t}$ the inflation rates based on the Production Price Index (PPI) and $\widehat{y}_t$ and $\widehat{y}^*_t$ are the output gaps in the Home and Foreign countries, respectively.[17] The deviation of the nominal interest rate from its natural value is denoted by $\widehat{r}_t$. The log-deviation of the term-of-trade is denoted $\widehat{s}_t$. The complete set of equations describing the

[15] This presentation of a multiple countries economy in two blocks simplify the exposition, but results can be generalized to the N countries case. For example, if the model is extended to three countries, the result that only one of the two countries must have an active fiscal policy can be extended by saying that only one of the three must have an active fiscal policy.

[16] We show in section 4 that our results regarding the monetary-fiscal interactions are also valid when we consider a more general framework with incomplete markets

[17] These gaps are expressed in log-deviations from the steady state.

equilibrium dynamics are the following:[18]

The New Phillips curves (NP curves)

$$\widehat{\pi}_{H,t} = \beta\mathbb{E}_t\left[\widehat{\pi}_{H,t+1}\right] + \kappa\widehat{y}_t - \lambda(\alpha m + \omega - 1)\widehat{s}_t + \lambda t_t \tag{1}$$

$$\widehat{\pi}^*_{F,t} = \beta\mathbb{E}_t\left[\widehat{\pi}^*_{F,t+1}\right] + \kappa\widehat{y}^*_t + \lambda(\omega^* - \alpha m)\widehat{s}_t + \lambda t^*_t \tag{2}$$

where κ and λ are the elasticities of the inflation to output gap and to marginal costs respectively (see appendix B), ω and ω^* are the weights of Home and Foreign countries respectively. The term-of-trade $\widehat{s}_t$ enters in the Phillips curves because households' labor supply is based on nominal wages deflated by the CPI while firms' labor demand is based on wages deflated by the PPI. Firms pay taxes based on their wage bill in each country, with the tax rates being t_t and t^*_t.

The Euler Equations (IS curves)

$$\widehat{y}_t = \mathbb{E}_t[\widehat{y}_{t+1}] - \frac{1}{\sigma}\left(\widehat{r}_t - \mathbb{E}_t[\widehat{\pi}_{H,t+1}]\right) + \frac{1-\overline{\alpha}m-\omega}{\sigma}\mathbb{E}_t\Delta\widehat{s}_{t+1} \tag{3}$$

$$\widehat{y}^*_t = \mathbb{E}_t[\widehat{y}^*_{t+1}] - \frac{1}{\sigma}\left(\widehat{r}_t - \mathbb{E}_t[\widehat{\pi}^*_{F,t+1}]\right) + \frac{\omega^*-\overline{\alpha}m}{\sigma}\mathbb{E}_t\Delta\widehat{s}_{t+1} \tag{4}$$

where σ is the intertemporal elasticity of consumption and $\Delta x_t \equiv x_t - x_{t-1}$, $\forall x$.

The Terms-Of-Trade (TOT)

$$\widehat{s}_t - \widehat{s}_{t-1} = \widehat{\pi}^*_{F,t} - \widehat{\pi}_{H,t} \tag{5}$$

$$\widehat{s}_t = \frac{\sigma}{\omega+\omega^*}(\widehat{y}_t - \widehat{y}^*_t) \tag{6}$$

These relations are crucial to reduce the dynamics of inflation and output gap of each country to a single joint dynamic of inflation and output gap for the monetary union. This comes from the assumptions of preferences symmetry.

The Taylor Rule

$$\widehat{r}_t = \alpha_\pi\left(m\widehat{\pi}_{H,t} + (1-m)\widehat{\pi}^*_{F,t}\right) + \alpha_y\left(m\widehat{y}_t + (1-m)\widehat{y}^*_t\right) + \varepsilon^M_t \tag{7}$$

$$\varepsilon^M_t = \rho^M\varepsilon^M_{t-1} + e_t \tag{8}$$

where $\{\alpha_y, \alpha_\pi\}$ are the sensitivities of the nominal interest rate to output and inflation gaps respectively. The discretionary component of the monetary policy is ε^M_t and we assume that $|\rho^M| < 1$, $e_t \sim iid$.

[18] The definitions of all the reduced form parameters as functions of the structural model's parameters are given in Appendix A. The complete description of the model is provided in Appendix B and a summary of the equations leading to the final reduced form is given in Appendix C.

Public Debt and the Fiscal Rules

$$
\begin{aligned}
\widehat{b}_t^r &= \beta^{-1}(\widehat{b}_{t-1}^r - \widehat{\pi}_{H,t} - \bar{\alpha}(1-m)\Delta\widehat{s}_t) + \widehat{r}_t - (\beta^{-1}-1)\widehat{d}_t^r & (9)\\
\widehat{d}_t^r &= \gamma\frac{1}{\beta^{-1}-1}(\widehat{b}_{t-1}^r - \widehat{r}_{t-1}) + \varepsilon_t^F & (10)\\
\varepsilon_t^F &= \rho\varepsilon_{t-1}^F + \nu_t & (11)\\
\widehat{b}_t^{r*} &= \beta^{-1}(\widehat{b}_{t-1}^{r*} - \widehat{\pi}_{F,t}^* + \bar{\alpha}m\Delta\widehat{s}_t) + \widehat{r}_t - (\beta^{-1}-1)\widehat{d}_t^{r*} & (12)\\
\widehat{d}_t^{r*} &= \gamma^*\frac{1}{\beta^{-1}-1}(\widehat{b}_{t-1}^{r*} - \widehat{r}_{t-1}) + \varepsilon_t^{F*} & (13)\\
\varepsilon_t^{F*} &= \rho^*\varepsilon_{t-1}^{F*} + \nu_t^* & (14)
\end{aligned}
$$

where the discretionary component of fiscal policies are $\{\varepsilon_t^F, \varepsilon_t^{F*}\}$ and we assume that $|\rho| < 1$, $|\rho^*| < 1$, $\nu_t \sim iid$, $\nu_t^* \sim iid$.

After integrating Equations (10) in (9) and (13) in (12) and the Taylor rule (7) in the IS curves (Equations (3) and (4)) and in the government budget constraints (Equations (9) and (12)), we obtain a system of eight equations, where the seven unknowns are $\{\widehat{\pi}_{H,t}, \widehat{\pi}_{F,t}^*, \widehat{y}_t, \widehat{y}_t^*, \widehat{s}_t, \widehat{b}_t^r, \widehat{b}_t^{r*}\}$ and five exogenous variables $\{\varepsilon_t^M, \varepsilon_t^F, \varepsilon_t^{F*}, t_t, t_t^*\}$.[19] In this system, there are four jump variables $\{\widehat{\pi}_{H,t}, \widehat{\pi}_{F,t}^*, \widehat{y}_t, \widehat{y}_t^*\}$, two predetermined variables $\{\widehat{b}_t^r, \widehat{b}_t^{r*}\}$, one static relation between $\widehat{s}_t$ and $\{\widehat{y}_t, \widehat{y}_t^*\}$ and a definition that links $\widehat{s}_t$ with $\{\widehat{\pi}_{H,t}, \widehat{\pi}_{F,t}^*\}$. The static relationship emphasizes that the two IS curves are not independent equations, whereas the definition of the TOT provides the link between the country specific inflation rates.

Note that with this linearized version of the model, the conditions under which a unique stable path exists can be determined in the deterministic version of the model. Therefore, for simplicity, we abstract in the following for the stochastic part of the model and consider that the exogenous variables $\{\varepsilon_t^M, \varepsilon_t^F, \varepsilon_t^{F*}, t_t, t_t^*\}$ are deterministic processes.

3.2 The Terms-Of-Trade Dynamic

In this section, we show that the dynamic of the terms-of-trade (TOT) can be solved independently of the rest of the model.

Proposition 1. *The TOT dynamic do not depend on the rules of monetary and/or fiscal policies (α_π, α_y, γ, and γ^*) as well as on their shocks (ε_t^M, ε_t^F, and ε^{F*}). The TOT dynamic only depends on fiscal gaps in labor costs ($t_t^* - t_t$).*

[19] The dynamics of these exogenous variables are described by Equations (8), (11), (14) and for any processes for $\{t_t, t_t^*\}$.

Proof. The difference between the two Phillips curves (Equations (1) and (2)) leads to:

$$\widehat{\pi}^*_{F,t} - \widehat{\pi}_{H,t} \quad = \quad \beta\left(\widehat{\pi}^*_{F,t+1} - \widehat{\pi}_{H,t+1}\right) + \kappa(\widehat{y}^*_t - \widehat{y}_t) + \lambda(\omega^* + \omega - 1)\widehat{s}_t + \lambda(t^*_t - t_t)$$

Given the definitions of ω and ω^*[20], we deduce, using Equation (6), the equilibrium dynamic of the TOT as follows:

$$0 \quad = \quad \widehat{s}_{t+1} - b\widehat{s}_t + \frac{1}{\beta}\widehat{s}_{t-1} + \frac{\lambda}{\beta}(t^*_t - t_t), \tag{15}$$

where $b = \left(1 + \beta + \frac{\kappa + (\kappa - \sigma\lambda)\overline{\alpha}(2-\overline{\alpha})(\sigma\eta - 1)}{\sigma}\right)/\beta$. The solution is such that:

$$(1 - r_1 L)(1 - r_2 L)\widehat{s}_{t+1} = \frac{\lambda}{\beta}(t^*_t - t_t) \Rightarrow \widehat{s}_{t+1} = r_1\widehat{s}_t - \lambda r_1 \sum_{j=0}^{\infty}\left(\frac{1}{r_2}\right)^j (t^*_{t+1+j} - t_{t+1+j}),$$

where $r_i = \frac{b \pm \sqrt{b^2 - 4/\beta}}{2}$, for $i = 1, 2$ and s.t. $0 < r_1 < 1 < r_2$ (See Appendix D). □

This result emphasizes the first property of an open economy dynamics: the TOT introduces backward and forward-looking components, independent of monetary and fiscal policies. The predetermined component of the TOT comes from the sticky prices, coupled with the fixed exchange rate assumption (Currency union). The TOT fluctuations are driven by tax rate shocks (t_t, t^*_t). This result is used by Farhi et al. (2014) to derive the optimal sequence of these taxes that would allow the equilibrium of a fixed exchange rate open economy to reach the equilibrium of an economy with flexible nominal exchange rates, better suited for damping business cycle shocks.

Corollary 1. *If $t_t, t^*_t = 0$, then $\widehat{s}_t = 0$, $\forall t$. We deduce that $\widehat{y}_t = \widehat{y}^*_t$ and $\widehat{\pi}_{H,t} = \widehat{\pi}^*_{F,t}$, $\forall t$.*

Proof. $\widehat{s}_t = 0\ \forall t$: obvious using equation (15) with $t_t, t^*_t = 0\ \forall t$ and if the initial condition for s_t is its equilibrium value. Using Equation (6), we deduce that $\widehat{y}_t = \widehat{y}^*_t$ and using Equation (5), we deduce $\widehat{\pi}_{H,t} = \widehat{\pi}^*_{F,t}$. □

Without any shocks on tax rates $(t_t, t^*_t = 0\ \forall t)$, the TOT dynamic is deterministic with a trivial solution $\widehat{s}_t = 0$, $\forall t$. Preferences symmetry imply that the equilibrium IS-Phillips equations of one of the two economies are completely redundant in the reduced form equilibrium. The redundancy of these conditions associated to the previous result on TOT dynamic lead to the fact that the model is isomorphic to the closed economy version of the model to which we add a foreign public debt dynamic.[21] Therefore, the study of the Euro area stability can be reduced to the analysis of the 4 dimensions system $\{\widehat{\pi}_{H,t}, \widehat{y}_t, \widehat{b}^r_t, \widehat{b}^{r*}_t\}$.

[20]See Appendix A for details on the parameters.

[21]Indeed, in a Monetary Union the nominal exchange rate is 1, $\forall t$, implying that $\widehat{\pi}_{F,t} = \widehat{\pi}^*_{F,t}$

3.3 A Limit Case: A Small Open Economy

The case of a small open economy (SOE) is obtained when $m \to 0$. Hence, the monetary policy, given by $\widehat{r}_t = \alpha_\pi \pi^*_{F,t} + \alpha_y \widehat{y}^*_t + \varepsilon^M_t$, does not depend on Home aggregates. The Home aggregates have then an autonomous dynamic, with the Foreign variables and the interest rate being exogenous. For simplicity, we thus assume that $\widehat{\pi}^*_{F,t} = \widehat{y}^*_t = t^*_t = 0$. This leads to $\widehat{s}_t = -\widehat{p}_{H,t}$. The equilibrium dynamic of a SOE is given by Equation (15), where $\widehat{s}_t$ is simply replaced by $-\widehat{p}_{H,t}$. Given the equilibrium path for $\widehat{p}_{H,t}$, we deduce the output gap using Equation (6), the equation being evaluated for $m \to 0$. This shows that the dynamic of the Home output gap is given by a Home price dynamic (here, the opposite of the TOT) and an inflation dynamic (the simple difference between the current and the past price levels). As for the TOT, the equilibrium price dynamic is deduced from Equation (15), where $\widehat{s}_t = -\widehat{p}_{H,t}$. Its solution is $\widehat{p}_{H,t+1} = r_1 \widehat{p}_{H,t} - \lambda r_1 \sum_{j=0}^{\infty} \left(\frac{1}{r_2}\right)^j t_{t+j}$, where the roots are $r_i = \frac{b \pm \sqrt{b^2 - 4/\beta}}{2}$, for $i = 1, 2$ and s.t. $0 < r_1 < 1 < r_2$. This is the result presented in Erceg and Linde (2012). It is also a corollary of Proposition 1: in a SOE, the equilibrium dynamic of Home price (opposite of the TOT in a SOE) does not depend on monetary and fiscal policies (rules and shocks).

Proposition 2. *In a small open economy of the Euro area, a debt brake, of a minimal size $\gamma > 1/\beta - 1$, is required to ensure the existence of a saddle path.*

Proof. See Appendix E. □

Given that the path of price gives both output and inflation dynamics, the equilibrium of a SOE is defined only by the price and public debt dynamics. The price and public debt are independent processes in a SOE because the price dynamic of the Home country does not depend on fiscal policy. The stability of the debt can only be ensured by the implementation of a fiscal brake.

From a policy perspective, this result implies that if each SOE of the Euro area adopts a fiscal rule that respect $1/\beta - 1 < \gamma$, then the aggregate public debt of the Euro area will have a sufficient debt break to ensure its stability. Therefore, the individual fiscal policies of each country imply the stability of the Euro aggregates. This is a credible strategy if all countries expect that the ECB respects its mandate which is to stabilize inflation via the implementation of the Taylor principle (active monetary policy). Indeed, given that inflation and output gaps abroad are taken as exogenous and stationary at the level of each SOE, the ECB must apply the European treaties by respecting the Taylor principle in order to stabilize the aggregate output-inflation dynamics of the Euro area.[22] If each SOE of the Euro area does not implement a debt brake, there is no chance for the ECB to correct for this instability.[23]

3.4 General Case: the Euro Area Stability

In the first subsection, we analyze a specific case where it is assumed that all governments of the Euro area adopt the same fiscal rule. This case corresponds to the one where all the governments behave as a representative one, obviously a hypothetical scenario in the current Euro area. We show that two equilibria are possible, as in a closed economy: first, the ECB has an active monetary policy (Taylor principle) and all governments have a passive fiscal policy (implementation of a fiscal brake); and second, the ECB has a passive monetary policy and all governments have an active fiscal policy (no debt brake). In the second subsection, we analyze the case where governments have different fiscal policies, implying that the equilibrium cannot be summarized by aggregates determined by a representative decision maker.

3.4.1 When All Governments Adopt the Same Fiscal Rule

Assume that fiscal brakes are identical across countries.[24] Note that only the components of fiscal policies that account for the debt brake (γ and γ^*) must be homogeneous across countries for the dynamic stability of the Euro aggregates.[25] The components of fiscal policies that account for discretionary decisions (ρ and ρ^*) can be heterogeneous across countries. Thus, we have $\gamma = \gamma^* \equiv \gamma_u$, but $\rho \neq \rho^*$. The Euro aggregates are defined as follows: inflation is $\widehat{\pi}_t^u = m\widehat{\pi}_{H,t} + (1-m)\widehat{\pi}_{F,t}$, output gap is $\widehat{y}_t^u = m\widehat{y}_t + (1-m)\widehat{y}_t^*$, and debt is $\widehat{b}_t^{ru} = m\widehat{b}_t^r + (1-m)\widehat{b}_t^{r*}$. By directly analyzing the aggregates of the Euro area,

[22] For analyzing the SOE, it is assumed that $\widehat{\pi}_{F,t}^* = \widehat{y}_t^* = 0$ for simplicity, but what matters is the stationarity of both $\widehat{\pi}_{F,t}^*$ and $\widehat{y}_t^*$. This stationarity is ensured if it exists a unique equilibrium path for these two variables representing inflation and the output gap of the Euro area.

[23] This is true because we consider here only one SOE for the Euro area, for which stability restrictions does not depend on the monetary policy.

[24] In this section, it is assumed for simplicity that $t_t = t_t^* = 0$, $\forall t$. These exogenous variables simply add shocks to the aggregate Phillips curve of the Euro area, which is not useful at this stage.

[25] Aggregation is also possible under the assumption that the structural parameters that govern nominal rigidities (Phillips curves) are the same across countries (assuming that all Europeans share the same preferences). This assumption is not rejected by the data (see Martin and Philippon (2017)).

the monetary policy described by the following Taylor rule (Equation (7))

$$\widehat{r}_t = \alpha_\pi \pi^u_t + \alpha_y \widehat{y}^u_t + \varepsilon^M_t \tag{16}$$

links the interest rate to inflation and output gap, without distinguishing countries by their weight on these aggregates. By aggregating the two Phillips curves (Equations (1) and (2))

$$\widehat{\pi}^u_t = \beta \widehat{\pi}^u_{t+1} + \kappa \widehat{y}^u_t,$$

the two IS curves (Equations (3) and (4))

$$\widehat{y}^u_t = \widehat{y}^u_{t+1} - \frac{1}{\sigma}\left(\widehat{r}_t - \widehat{\pi}^u_{t+1}\right),$$

the two debt dynamics (Equations 9) and (12) as well as the surpluses (Equations (10) and (13))

$$\begin{aligned}\frac{1}{\beta^{-1}-\gamma_u}\widehat{b}^{ru}_{t+1} &= \widehat{b}^{ru}_t - \widehat{\pi}^u_{t+1} - \frac{\beta^{-1}-1}{\beta^{-1}-\gamma^u}\left(m\varepsilon^F_{t+1} + (1-m)m\varepsilon^{F*}_{t+1}\right) \\ &+ \frac{1}{\beta^{-1}-\gamma_u}\left(\alpha_\pi \widehat{\pi}^u_{t+1} + \alpha_y \widehat{y}^u_{t+1}\right) + \frac{\gamma_u}{\beta^{-1}-\gamma_u}\left(\alpha_\pi \widehat{\pi}^u_t + \alpha_y \widehat{y}_t\right),\end{aligned}$$

and using the Taylor rule (Equation (16)), the equilibrium paths for $\{\widehat{\pi}^u_t, \widehat{y}^u_t, \widehat{b}^{ru}_t\}$ can be solved.[26]

Proposition 3. *When all governments adopt the same fiscal rules, (i) if the Taylor principle applies, then the stability of the Euro aggregates is ensured if a "passive" fiscal policy is implemented in each European state; (ii) if the Taylor principle does not apply, it can also exist a stable equilibrium if fiscal policies are "active" in each country.*

Proof. Using $\varepsilon^F_{t+1} = \rho\varepsilon^F_t$ and $\varepsilon^{F*}_{t+1} = \rho^*\varepsilon^{F*}_t$, the Philips and IS curves as well as the debt dynamics of the Euro area aggregates lead to:

$$\begin{bmatrix} \widehat{\pi}^u_t \\ \widehat{y}^u_t \\ \widehat{b}^u_t \end{bmatrix} = \begin{bmatrix} A & & 0 \\ a_1 & a_2 & a_3 \end{bmatrix} \begin{bmatrix} \widehat{\pi}^u_{t+1} \\ \widehat{y}^u_{t+1} \\ \widehat{b}^u_{t+1} \end{bmatrix} + \Omega \begin{bmatrix} -\kappa & 0 & 0 \\ -1 & 0 & 0 \\ b_1 & m\rho b_2 & (1-m)\rho^* b_2 \end{bmatrix} \begin{bmatrix} \varepsilon^M_t \\ \varepsilon^F_t \\ \varepsilon^{F*}_t \end{bmatrix}$$

[26] Another way to obtain this dynamic system is to assume that Europeans have no home bias $\overline{\alpha} = 1$. In this case, the risk-sharing condition leads to $\widehat{c}_t = \widehat{c}^*_t$. Assuming for simplicity that $m = 1/2$, this implies that $\widehat{y}_t = \widehat{y}^*_t = \frac{\widehat{y}_t + \widehat{y}^*_t}{2} \equiv \widehat{y}^u_t$, $\widehat{\pi}_{H,t} = \widehat{\pi}^*_{F,t} = \frac{\widehat{\pi}_{H,t} + \widehat{\pi}^*_{F,t}}{2} \equiv \widehat{\pi}^u_t$, $\widehat{b}^{r,u}_t \equiv \frac{\widehat{b}^r_t + \widehat{b}^{r*}_t}{2}$ and $\widehat{d}^{r,u}_t \equiv \frac{\widehat{d}^r_t + \widehat{d}^{r*}_t}{2}$ are solutions of the same dynamic system.

$$\text{where} \begin{cases} \Omega = \frac{1}{\sigma+\alpha_y+\kappa\alpha_\pi} & a_1 = -\frac{\beta}{1-\gamma_u\beta}\left(\alpha_y - \beta^{-1} + \gamma_u\sigma(\alpha_\pi\beta - 1)\Omega\right) \\ a_2 = -\frac{\beta}{1-\gamma_u\beta}\left(\alpha_y + \gamma_u\sigma(\alpha_y + \alpha_\pi\kappa)\Omega\right) & a_3 = \frac{\beta}{1-\gamma_u\beta} \\ b_1 = -\gamma_u\frac{\sigma\beta}{1-\gamma_u\beta} & b_2 = (1-\beta)\frac{\sigma+\alpha_\pi\kappa+\alpha_y}{1-\gamma_u\beta} \end{cases}$$

This system is block-diagonal with an upper block defined by:

$$\begin{bmatrix} \widehat{y}_t^u \\ \widehat{\pi}_t^u \end{bmatrix} = A \begin{bmatrix} \widehat{y}_{t+1}^u \\ \widehat{\pi}_{t+1}^u \end{bmatrix} + B\varepsilon_t^M$$

$$\text{where} \quad A = \Omega \begin{bmatrix} \kappa + \beta(\sigma+\alpha_y) & \kappa\sigma \\ 1-\beta\alpha_\pi & \sigma \end{bmatrix}, \quad B = -\Omega \begin{bmatrix} \kappa \\ 1 \end{bmatrix}$$

can be solved first. The two eigenvalues of A are positive, and the characteristic polynomial is such that $\mathbf{P}_A(1) = 1 - \mathrm{tr}(A) + \det(A) = \frac{\kappa(\alpha_\pi - 1)+(1-\beta)\alpha_y}{\sigma+\alpha_y+\kappa\alpha_\pi}$, which indicates that both roots are lower than 1 if $\kappa(\alpha_\pi - 1) + (1-\beta)\alpha_y > 0$. This is the Taylor principle.

This strategy for monetary policy enforces *all* governments of the Euro area to implement a strict fiscal rule, that is, to choose a parameter $\gamma = \gamma^* \equiv \gamma^u$ such that public debt does not explode. A simple restriction is $\gamma^u > \beta^{-1} - 1$, which represents the "strictness" of the fiscal rule. □

This analysis reveals that the stability of the Euro aggregates, and in particular the public debt, does not depend on the nature of shocks, but on the coordination of the policy rules.[27] Moreover, because the TOT dynamic which captures the open relations can be solved independently, the reduced-form equilibrium conditions do not depend on parameters linked to the open-economy framework, namely the country size m, the home-bias preferences $\overline{\alpha}$ and the elasticity between Home and Foreign goods η.

This framework, where all the countries have exactly the same fiscal policy, can be interpreted as a **Federal state** where the central government coordinates fiscal policies which are thus unique. Therefore, we return to the same economy studied by Leeper, Traum, and Walker (2017) in the case of the US modeled as a closed economy. For the Euro area, this simplified model with a representative country is close to the one used by Woodford (1996) in discussing the impact of the Maastricht treaty on maintaining price stability in Europe. **If** the Taylor principle is satisfied, namely $\kappa(\alpha_\pi - 1) + (1-\beta)\alpha_y > 0$, **then** a passive fiscal rule **in each country** is necessary to ensure that public debt remains non-explosive. Indeed, with $\{\widehat{\pi}_t^u, \widehat{y}_t^u\}$ (the stationary solutions of the dynamic system for the aggregate economy when the Taylor principle is respected), debt is sustainable if

[27] Note that this analysis, based on the aggregates of the Euro area, does not imply that the output gap and inflation of each country are the same and equal to the ones of the Euro area.

$\gamma > \beta^{-1} - 1$. This fiscal brake ($\gamma > \beta^{-1} - 1$) represents the "strictness" of the fiscal rule. The commitment of each government of the Euro area is crucial to ensure economic stability. **By contrast**, if the ECB does not respect the Taylor principle, then all countries of the Euro area (in fact, the representative government of the Euro area) must abandon fiscal rules incorporating a fiscal brake. This is an easy strategy for a federal state but less easy for the Euro area where European treaties maintain fiscal independence. Note that this problem is not a theoretical curiosity: it can be concrete when an economy enters a liquidity trap, such as the period when the ZLB constraint is perceived as very persistent (see section 5).

3.4.2 When Fiscal Rules Are Different Across Countries

We now break the simplistic view of unique IS and Phillips curves in Europe. Each country has a specific fiscal policy (rule and shock). The complete dynamics of the two-country model are the solution of the following system (using Equations (1), (3), (9), and (12)):

$$
\begin{aligned}
\widehat{\pi}_{H,t} &= \beta\widehat{\pi}_{H,t+1} + \kappa\widehat{y}_t - \lambda(\bar{\alpha}m + \omega - 1)\widehat{s}_t + \lambda t_t \\
\widehat{y}_t &= \widehat{y}_{t+1} - \frac{1}{\sigma}\left(\widehat{r}_t - \widehat{\pi}_{H,t+1}\right) + \frac{1-\bar{\alpha}m-\omega}{\sigma}(\widehat{s}_{t+1} - \widehat{s}_t) \\
\frac{\beta}{1-\beta\gamma}\widehat{b}^r_{t+1} &= \widehat{b}^r_t - \frac{\bar{\alpha}(1-m)}{1-\beta\gamma}\Delta\widehat{s}_{t+1} - \frac{1}{1-\beta\gamma}\widehat{\pi}_{H,t+1} + \frac{\beta}{1-\beta\gamma}\widehat{r}_{t+1} + \frac{\gamma\beta}{1-\beta\gamma}\widehat{r}_t - \frac{1-\beta}{1-\beta\gamma}\varepsilon^F_{t+1} \\
\frac{\beta}{1-\gamma^*\beta}\widehat{b}^{r*}_{t+1} &= \widehat{b}^{r*}_t + \frac{\bar{\alpha}m-1}{1-\gamma^*\beta}\Delta\widehat{s}_{t+1} - \frac{1}{1-\gamma^*\beta}\widehat{\pi}_{H,t+1} + \frac{\beta}{1-\beta\gamma^*}\widehat{r}_{t+1} + \frac{\gamma^*\beta}{1-\beta\gamma^*}\widehat{r}_t - \frac{1-\beta}{1-\beta\gamma^*}\varepsilon^{F*}_{t+1}
\end{aligned}
$$

where the Taylor rule, given by:

$$
\widehat{r}_t = \alpha_\pi\widehat{\pi}_{H,t} + \alpha_y\widehat{y}_t + (1-m)\left(\alpha_\pi - \alpha_y\frac{\omega+\omega^*}{\sigma}\right)\widehat{s}_t - \alpha_\pi(1-m)\widehat{s}_{t-1} + \varepsilon^M_t \quad (17)
$$

shows that despite the asymmetric weight of each country, the sensitivity of the Taylor rule to inflation and output gap does not depend on the weight of each country. This result is a direct implication of the TOT equation (6).

After integrating Equation (17) in the IS and public debt equations, the dynamics evolves according to $Z_t = \widehat{M}Z_{t+1} + \widehat{D}S_t + \widehat{P}\varepsilon_t$, where $Z_t = [\widehat{\pi}^H_t, \widehat{y}_t, \widehat{b}^r_t, \widehat{b}^{r*}_t]'$, $S_{t+1} = [\widehat{s}_{t+1}, \widehat{s}_t, \widehat{s}_{t-1}]'$, and $\varepsilon_t = [\varepsilon^M_t, \varepsilon^F_t, \varepsilon^{F*}_t]'$. For a given sequence of $\{\widehat{s}_t\}_{t=0}^{\infty}$, this system has two jump variables $\{\widehat{\pi}^H_t, \widehat{y}_t\}$ and two predetermined variables $\{\widehat{b}^r_t, \widehat{b}^{r*}_t\}$. Its stability is then ensured if $\widehat{M}$ has two eigenvalues inside and two outside the unit circle. Using the solution $\widehat{s}_t = 0$ if $t_t = t^*_t = 0\ \forall t$ (see Proposition 1), $\widehat{y}^*_t$ is deduced from $\widehat{y}^*_t = \widehat{y}_t - \frac{\omega+\omega^*}{\sigma}\widehat{s}_t$, and $\widehat{\pi}^*_{F,t}$ from $\widehat{\pi}^*_{F,t} \equiv \widehat{\pi}_{H,t}$.[28]

[28]See appendix F for the complete description of this system.

Proposition 4. *If the Taylor principle is respected, the stability of the Euro area is ensured if all the countries have a passive fiscal policy, that is, if all the countries implement a fiscal brake satisfying* $\gamma, \gamma^* > \beta^{-1} - 1$.

If the Taylor principle is not respected, the stability of the Euro area is ensured if only one country implement an active fiscal policy, all others must respect their commitment to implement a fiscal brake: $\gamma > \beta^{-1} - 1$ *&* $\gamma^* < \beta^{-1} - 1$*, or* $\gamma < \beta^{-1} - 1$ *&* $\gamma^* > \beta^{-1} - 1$.

Proof. Given that $\widehat{M}$ is triangular (See appendix F), its eigenvalues are those of the matrix A and $\left\{\frac{1}{\beta^{-1}-\gamma}, \frac{1}{\beta^{-1}-\gamma^*}\right\}$. In Proposition 3, it is shown that the two eigenvalues of A are inside the unit circle if $\kappa(\alpha_\pi - 1) + (1-\beta)\alpha_y > 0$ (Taylor principle).

If $\kappa(\alpha_\pi - 1) + (1-\beta)\alpha_y > 0$, then the two eigenvalues of A are inside the unit circle. Therefore, the two last eigenvalues of $\widehat{M}$ must be outside the unit circle: $\frac{1}{\beta^{-1}-\gamma} > 1$ and $\frac{1}{\beta^{-1}-\gamma^*} > 1$, that is, $\gamma, \gamma^* > \beta^{-1} - 1$.

If $\kappa(\alpha_\pi - 1) + (1-\beta)\alpha_y < 0$, then one eigenvalue of A is inside the unit circle, the other being outside the unit circle. Therefore, only one of the two last eigenvalues of $\widehat{M}$ must be outside the unit circle. This implies that one set of the following restrictions must be satisfied: *i)* $\frac{1}{\beta^{-1}-\gamma} > 1$ and $\frac{1}{\beta^{-1}-\gamma^*} < 1$, that is, $\gamma > \beta^{-1} - 1$ and $\gamma^* < \beta^{-1} - 1$, or *ii)* $\frac{1}{\beta^{-1}-\gamma} < 1$ and $\frac{1}{\beta^{-1}-\gamma^*} > 1$, that is, $\gamma < \beta^{-1} - 1$ and $\gamma^* > \beta^{-1} - 1$. □

If the Taylor principle is respected, that is, when the ECB implements an "active" monetary policy rule, **all** countries of the Euro area must implement a fiscal brake. This shows that these two policies are complementary to ensure the stability of the area, and thus the sustainability of public debts. The active monetary policy implemented by the ECB can be viewed as a coordination device guiding all governments to respect the European treaties.

If the Taylor principle is not respected, only one country is constrained to adopt a passive fiscal policy. The other country must have an active fiscal policy. Therefore, when the monetary policy of the ECB does not respect the Taylor principle, **one** of the two countries of the Euro area must adopt an active fiscal policy, that is, it must not implement the fiscal brake of the Fiscal Stability Treaty. In this case, the fiscal theory of the price level applies, showing that a non-unified fiscal policy provides the freedom to choose which country can renounce to its European treaty commitments. This degree of freedom can also lead to a coordination problem, unforeseen by the European treaties.

Instability of the Euro area: a common debt as a stabilizing device. The two regimes of Proposition 4 suggest that a *fiscal union*, that takes the form of a common debt, would be the best solution to avoid any destabilization of the Euro area linked to the non-compliance of one of the member states to the Fiscal Stability Treaty[29].

The issue of common debt leads “simply” to an additional public debt dynamic, managed by the European Council. Therefore, the dynamic system of the Euro area would be composed by five equations: the NP and the IS curves, the public debts of each of the two countries and the common public debt of the EU. In this context, even if the monetary policy is not “active”, the stability of the Euro area can be ensured by an “active” fiscal policy for the common EU budget, the stability of the two other public debts being ensured by the implementation of debt brakes (“passive” fiscal policy inside each country). By contrast, if the monetary policy can be “active”, then the three fiscal policies must be “passive”. However, the risk of instability for the Euro area seems lower when the monetary policy is active: if one country deviates, it knows that it could lead to global instability in the area, its gains then being able to exist only in the very short term. Moreover, if this country is small, Proposition 2 suggests that it is not rational for a SOE to find grounds for non-compliance with treaties. Obviously, this reasoning also applies to the governance of the European common debt. Hence, when the monetary policy is active, nobody seems to be tempted to give up its commitment to European treaties. An active monetary policy also acts as a coordination device.

This optimism about the Euro system is undermined in situations where the ECB would choose not to have an active monetary policy. In this case, one of the members must renounce to these European commitments, but no one knows which among the states. This indeterminacy of equilibrium presents a clear problem of stability in the Euro area. The critical situation in the Euro area would then be the moment when the ECB would be forced not to have an active monetary policy at a time where the issue of bonds on behalf of the EU would not be possible.

4 Incomplete markets

In this section, we show that our results are robust even when financial markets are incomplete. For the sack of simplicity, we will consider a framework where incomplete markets take the form of financial autarky where households can only hold national public

[29]However, this solution will require some changes in the European legal framework as article 310(1) of TFEU imposes that Union’s annual budget should be balanced, i.e excluding active fiscal policy at the level of the union.

debt[30]. However, the conclusions regarding the monetary-fiscal interactions that ensure a unique saddle-path of the dynamics also generalize to a more realistic framework where financial trade is allowed[31].

The previous framework with CM made the hypothesis that perfect international risk-sharing through state contingent assets was available for households in each country. This conditions materialized in the model through the IRS (or Backus-Smith (1993)) condition:

$$\widehat{c}_t(s) = \frac{1}{\sigma}\widehat{q}_t(s) + \widehat{c}_t^*(s) \quad \forall t,\ \forall s \tag{18}$$

However, when markets are incomplete and financial trade forbidden, there is no possibility for a country to be net debtor or net creditor internationally. It implies that even if households can trade goods, a country cannot accumulate a trade balance surplus or deficit and imports should be equal to exports at each period no matter the shocks, i.e $\widehat{tb}_t = 0$ and $\widehat{tb}_t^* = 0$, $\forall t$.

4.1 New Keynesian Equilibrium Dynamics

The equilibrium conditions of the two-country model with incomplete markets and financial autarky can be summarized by the following conditions:

The New Phillips curves (NP curves)

$$\widehat{\pi}_{H,t} = \beta\mathbb{E}_t\left[\widehat{\pi}_{H,t+1}\right] + \kappa\widehat{y}_t + \lambda\bar{\alpha}(1-m)(1-\sigma)\widehat{s}_t + \lambda t_t \tag{19}$$

$$\widehat{\pi}_{F,t}^* = \beta\mathbb{E}_t\left[\widehat{\pi}_{F,t+1}^*\right] + \kappa\widehat{y}_t^* - \lambda\bar{\alpha}m(2\eta\sigma+\sigma+1)\widehat{s}_t + \lambda t_t^* \tag{20}$$

The Euler Equations (IS curves)

$$\widehat{y}_t = \mathbb{E}_t[\widehat{y}_{t+1}] - \frac{1}{\sigma}\left(\widehat{r}_t - \mathbb{E}_t[\widehat{\pi}_{H,t+1}]\right) + \bar{\alpha}(1-m)(\sigma^{-1}-1)\mathbb{E}_t\Delta\widehat{s}_{t+1} \tag{21}$$

$$\widehat{y}_t^* = \mathbb{E}_t[\widehat{y}_{t+1}^*] - \frac{1}{\sigma}\left(\widehat{rz}_t - \mathbb{E}_t[\widehat{\pi}_{F,t+1}^*]\right) - \bar{\alpha}m(2\eta-1+\sigma^{-1})\mathbb{E}_t\Delta\widehat{s}_{t+1} \tag{22}$$

where $\widehat{rz}_t$ is the interest rate in the foreign economy.

The Terms-Of-Trade (TOT)

$$\widehat{s}_t - \widehat{s}_{t-1} = \widehat{\pi}_{F,t}^* - \widehat{\pi}_{H,t} \tag{23}$$

$$\widehat{s}_t = \frac{(\widehat{y}_t - \widehat{y}_t^*)}{(2\eta - 1 - \bar{\alpha}\eta + \bar{\alpha} - 2\bar{\alpha}m\eta)} \tag{24}$$

[30]The new budget constraints with regards to the IM framework with financial autarky can be found in Appendix G.1

[31]The description of the model and numerical simulations can be found in Appendix G.2.

Interest rate differential

$$\widehat{r}_t - \widehat{rz}_t = (2\eta\sigma - \sigma - \bar{\alpha}\eta\sigma + \bar{\alpha} - 1)(\widehat{s}_{t+1} - \widehat{s}_t) \quad (25)$$

In the two-country model with IM and financial autarky, there will be a gap between interest rates on public debt in the two countries $\widehat{r}_t - \widehat{rz}_t$. Indeed, by combining the two IS curves (21 and 22) with the equilibrium condition that links output gap in the two regions 24, we get $\widehat{\pi}^*_{F,t+1} - \widehat{\pi}_{H,t+1} = (\widehat{s}_{t+1} - \widehat{s}_t)(2\eta\sigma - \sigma - \bar{\alpha}\eta\sigma + \bar{\alpha}) + (\widehat{rz}_t - \widehat{r}_t)$, and by using the definition of the TOT (Equation 23) that should hold $\forall t$, we deduce (25). This relation highlights that $\widehat{r}_t \neq \widehat{rz}_t$ if $\widehat{s}_t \neq 0$. Interest rate differentials between the two countries is a *sine qua non* condition in a model with a monetary union and incomplete markets, even when there are no risk-premia. This is due to the incompatibility of having at the same time a fixed nominal exchange rate $e_t = 1$ and a peg interest rate for one of the two countries $\widehat{rz}_t = \widehat{r}_t$, $\forall t$.[32] As long as there exists shocks affecting the TOT, interest rate on public debt in the two countries may follow different paths. This result was absent in the specific context of CM where the presence of Arrow-Debreu securities implied an equalization of rate of return no matter the shocks. This condition imply that until financial integration would have not been achieved in the EU, there must be an interest rate differential between EZ countries interest rates.

Public Debt

$$\widehat{b}^r_t = \widehat{r}_t + \widehat{b}^r_{t-1}(\beta^{-1} - \gamma) + \gamma\widehat{r}_{t-1} - \frac{1}{\beta}\widehat{\pi}_{H,t} - \frac{\bar{\alpha}(1-m)}{\beta}\Delta\widehat{s}_t + (1-\beta^{-1})\varepsilon^F_t \quad (26)$$

$$\widehat{b}^{r*}_t = \widehat{rz}_t + \widehat{b}^{r*}_{t-1}(\beta^{-1} - \gamma^*) + \gamma^*\widehat{rz}_{t-1} - \frac{1}{\beta}\widehat{\pi}^*_{F,t} + \frac{\bar{\alpha}m}{\beta}\Delta\widehat{s}_t + (1-\beta^{-1})\varepsilon^{F*}_t \quad (27)$$

where the dynamics of ε^F_t and ε^{F*}_t are given by (11) (14).

With a monetary policy given by (7) and (8), these equilibrium conditions are equivalent to the ones defined in the equilibrium with CM, except that the parameters that are linked to the TOT are different. The new specification regarding financial market will materialize through different adjustments in TOT. As a consequence, following a shock, the different adjustments in TOT will imply different responses for output and inflation gaps in the two countries. Finally, incomplete markets will imply a different rate of return on public debt in the two countries.

[32] Note that having an interest peg in one of the two countries *de facto* imply that the Taylor principle is not respected in that country.

4.2 The Terms-Of-Trade Dynamic

Proposition 5. *As in the CM framework, the TOT dynamic do not depend on the rules of monetary and/or fiscal policies (α_π, α_y, γ, and γ^*) as well as on their shocks (ε_t^M, ε_t^F, and ε^{F*}). The TOT dynamic only depends on fiscal gaps in labor costs and can be solved independently of the rest of the model.*

Proof. The difference between the two Phillips curves (Equations (19) and (20)) leads to:

$$\widehat{\pi}^*_{F,t} - \widehat{\pi}_{H,t} \;=\; \beta\left(\widehat{\pi}^*_{F,t+1} - \widehat{\pi}_{H,t+1}\right) + \kappa(\widehat{y}^*_t - \widehat{y}_t) - \lambda\Upsilon\widehat{s}_t + \lambda(t^*_t - t_t)$$

where $\Upsilon = (\bar{\alpha}m(2\eta+\sigma+1) + \bar{\alpha}(1-m)(1-\sigma))$. By using the the definition of the TOT 23 and the equilibrium condition that links home and foreign output 24 we deduce the equilibrium dynamic of the TOT:

$$0 \;=\; \widehat{s}_{t-1} - \chi\widehat{s}_t + \beta\widehat{s}_{t+1} + \lambda(t^*_t - t_t) \tag{28}$$

where $\chi = [(1+\beta) + \bar{\alpha}(1-m)(1-\sigma)\lambda + \kappa(2\eta - 1 - \bar{\alpha}\eta + \bar{\alpha} - 2\bar{\alpha}m\eta) + \lambda\bar{\alpha}m(2\eta\sigma + \sigma + 1)]$. The way to solve this recurrence equation is the same than in Proposition 1. □

Corollary 2. *As in the CM framework, If $t_t, t^*_t = 0$, then $\widehat{s}_t = 0$, $\forall t$. We deduce that $\widehat{y}_t = \widehat{y}^*_t$ and $\widehat{\pi}_{H,t} = \widehat{\pi}^*_{F,t}$, $\forall t$.*

Proof. $\widehat{s}_t = 0$ $\forall t$: obvious using equation (28) with $t_t, t^*_t = 0$ $\forall t$ and if the initial condition for s_t is its equilibrium value. Using Equation (24), we deduce that $\widehat{y}_t = \widehat{y}^*_t$ and using Equation (23), we deduce $\widehat{\pi}_{H,t} = \widehat{\pi}^*_{F,t}$. □

Without any shocks on tax rates ($t_t, t^*_t = 0$, $\forall t$) , the TOT dynamic is deterministic with a trivial solution $\widehat{s}_t = 0$, $\forall t$. As with CM, preferences symmetry imply that the equilibrium IS-Phillips curves of one of the two economies are completely redundant. Therefore, the study of the Euro area stability with IM can be reduced to the analysis of the 4 dimensions system $\{\widehat{\pi}_{H,t}, \widehat{y}_t, \widehat{b}^r_t, \widehat{b}^{r*}_t\}$.

From Proposition 5, we deduce that $\widehat{r}_t \neq \widehat{rz}_t$ if $\widehat{s}_t \neq 0$ and it will be the case only when country-specific tax rate shocks (t_t, t^*_t) are different. On the other hand, interest rate in the two region will follow the same path ($\widehat{r}_t = \widehat{rz}_t$, $\forall t$) for shocks that do not affect the TOT, namely, fiscal and monetary shocks ($\varepsilon^F_t, \varepsilon^{F*}, \varepsilon^M_t$).

4.3 The Euro Area Stability

As in the CM framework, the size of the previous equilibrium system can be reduced in order to study its stability. The complete dynamics of the two-country model with incomplete markets and financial autarky can be summarized by the following equations:

$$
\begin{aligned}
\widehat{y}_t &= \mathbb{E}_t[\widehat{y}_{t+1}] - \frac{1}{\sigma}\left(\widehat{r}_t - \mathbb{E}_t[\widehat{\pi}_{H,t+1}]\right) + \bar{\alpha}(1-m)(\sigma^{-1}-1)\mathbb{E}_t\Delta\widehat{s}_{t+1} \\
\widehat{\pi}_{H,t} &= \beta\mathbb{E}_t\left[\widehat{\pi}_{H,t+1}\right] + \kappa\widehat{y}_t + \lambda\bar{\alpha}(1-m)(1-\sigma)\widehat{s}_t + \lambda t_t \\
\widehat{b}^r_t &= \widehat{r}_t + \widehat{b}^r_{t-1}(\beta^{-1}-\gamma) + \gamma\widehat{r}_{t-1} - \frac{1}{\beta}\widehat{\pi}_{H,t} - \frac{\bar{\alpha}(1-m)}{\beta}\Delta\widehat{s}_t + (1-\beta^{-1})\varepsilon^F_t \\
\widehat{r}_t - \widehat{rz}_t &= (2\eta\sigma - \sigma - \bar{\alpha}\eta\sigma + \bar{\alpha} - 1)\Delta\widehat{s}_{t+1} \\
\widehat{b}^{r*}_t &= \widehat{rz}_t + \widehat{b}^{r*}_{t-1}(\beta^{-1}-\gamma^*) + \gamma^*\widehat{rz}_{t-1} - \frac{1}{\beta}\widehat{\pi}_{H,t} + \frac{\bar{\alpha}m-1}{\beta}\Delta\widehat{s}_t + (1-\beta^{-1})\varepsilon^{F*}_t \\
\widehat{r}_t &= \alpha_\pi\widehat{\pi}_{H,t} + \alpha_y\widehat{y}_t + (1-m)\Big[\alpha_\pi - (2\eta - 1 - \bar{\alpha}\eta + \bar{\alpha} - 2\bar{\alpha}m\eta)\Big]\widehat{s}_t - \alpha_\pi(1-m)\widehat{s}_{t-1} + \varepsilon^M_t
\end{aligned}
$$

This system is almost equivalent to the system with CM. However, we have an additional condition due to interest rate differentials that as mentioned in the previous section, will exist each time the economies face shocks affecting the TOT. Under the hypothesis that tax rate shocks are equal to zero ($t_t = t^*_t = 0$)[33], the dynamic stability of the two-country model with incomplete markets and financial autarky is exactly the same than with CM. It resumes to the four equations system that is composed of the Philipps curve and the IS curve of the home economy and the two debt dynamics of home and foreign. As a consequence, the conditions regarding the monetary-fiscal interactions that ensure the uniqueness of the saddle path are equivalent in the two framework and are not affected by the hypotheses regarding financial markets.

Proposition 6. *In a two-country model of the EZ with incomplete markets and financial autarky, the policy interactions that ensure the uniqueness of the saddle-path are:*
If the Taylor principle is respected, the stability of the Euro area is ensured if all the countries have a passive fiscal policy, that is, if all the countries implement a fiscal brake satisfying $\gamma, \gamma^* > \beta^{-1} - 1$.
If the Taylor principle is not respected, the stability of the Euro area is ensured if only one country implement an active fiscal policy, all others must respect their commitment to implement a fiscal brake: $\gamma > \beta^{-1} - 1$ *&* $\gamma^* < \beta^{-1} - 1$, *or* $\gamma < \beta^{-1} - 1$ *&* $\gamma^* > \beta^{-1} - 1$.

These conditions are equivalent to the ones with CM (see Proposition 4). Thus, this underlines their robustness.

[33] As in the previous framework with CM, this hypothesis is useful to simplify the exposition and relaxing it do not affect the conditions that ensure the uniqueness of the saddle-path.

5 Zero-Lower-Bound: A Risk Arguing in Favor of a Fiscal Union

When the Euro area is caught in a liquidity trap, it could be not possible for an active monetary policy to coordinate the governments of the Euro area on an equilibrium where they all choose a passive fiscal policy (Proposition 4). This section presents the options for a policy mix in this context by highlighting that these strategies mainly depend on agents' expectations.

5.1 A highly persistent Zero-Lower-Bound

If the ZLB is expected to be highly persistent and can be approximated by a permanent state, a passive monetary policy will be favored in the agents' expectations. Therefore, Proposition 4 shows that one country of the Euro area must leave the Fiscal Stability Treaty in order to stabilize the Euro area. However, which country will do this? If each country considers itself as one among the many SOEs in the Euro area, then none of them will be interested in choosing an active fiscal policy (abandonment of the fiscal brake of the European treaties) because this strategy destabilizes a SOE (see proposition 2). However, in the case of larger economies of the Euro area, such as Germany, France, Italy or Spain, it is possible that these governments do not consider their countries as a SOE, and therefore rely on their partners to ensure the stability of the Euro area, while "enjoying" an active fiscal policy. However, without a coordination system, each country can consider itself as one that can have an active fiscal policy, thereby leading to instability in the Euro area.

Proposition 7. *When the ZLB is perceived to be permanent, a fiscal union stabilizes the Euro area by selecting a unique active fiscal policy at the federal state level.*

Proof. Using proposition 4, we know that if the monetary policy is passive (the Taylor principle cannot be respected), an active fiscal policy must be implemented because one eigenvalue of A is outside the unit circle.

Given that the monetary policy is constrained to be passive at the ZLB, a fiscal union that would take the form of a unification of all public debts, can ensure that $\gamma = \gamma^* \equiv \gamma^u$ with $\gamma^u < \beta^{-1} - 1$. With only one real-debt dynamic (fiscal union), the dynamics would have two eigenvalues inside the unit circle and one outside.

Another way, consist of issuing bonds on behalf of the EU, that will manage them by implementing an active fiscal policy. At the same time, all the states must respect their European commitments by implementing a passive fiscal policy, as in the case where monetary policy is active. □

Proposition 7 shows that a fiscal union is a solution to ensure the stability of the Euro area when deprived of an active monetary policy. The policy of this fiscal union would then be to select the equilibrium where the Fiscal Theory of Price Level (FTPL) would apply. The non-Ricardian properties of this equilibrium would, therefore, allow fiscal transfer policies to revive the Euro area economy. The budgetary tools for regulating economic activities would, therefore, be greatly appreciated during periods when the interest rate is constant, that is, locked in at zero. It seems easier to issue new public debt on behalf of the EU than to unify the debts of the member states. This is the Franco-German proposal (Macron-Merkel), presented by the President of the European Commission on May 27, 2020.

Note that even in an extreme context of permanence of the ZLB constraint, policies other than those leading to an equilibrium where the FTPL applies are possible. Specifically, it is possible to envisage rules inducing fiscal devaluations in response to the business cycle and then allowing to stabilize the Euro area around a "Ricardian" equilibrium. On this equilibrium, the fiscal policy of each state must remain passive (a fiscal brake exists) as each of them is committed to it by signing the European treaties. In this equilibrium, employment subsidies in each state depend on economic activities (inflation and output gaps), and then induce fluctuations in the real exchange rate. This policy rule replaces a stabilization by the nominal interest rate.[34] This selection of a stable equilibrium via adjustments to the TOT is a specificity of open economies that opted for a monetary union.

5.2 Transitory Zero-Lower-Bound

The assumption of permanence of the ZLB constraint is certainly too extreme. Nonetheless, it allows us to see how essential anticipation is for the design of a policy mix: by considering that the ZLB constraint will be permanent, then the stability of the economy

[34] Appendix H creates restrictions on the rules such that $t_t = \nu_y \widehat{y}_t + \nu_\pi \widehat{\pi}_{H,t}$, allowing to have stable dynamics of inflation and the output gap in the absence of the Taylor rule (passive monetary policy or ZLB). Given this stability in the inflation-output gap dynamics, the fiscal policy must then be passive to ensure the dynamic stability of the Euro area.

can only be assessed around this stationary state where the monetary policy is doomed to be silent. In a more realistic case where the ZLB period is finite in expectation, the dynamics of the Euro area depend on agents' expectations of the monetary policy that the ECB will likely lead, once the economy has left the ZLB. To model a ZLB constraint that is only transitory, a stochastic ending of this regime à la Christiano, Eichenbaum, and Rebelo (2011) is considered.

Proposition 8. *If the ZLB is only a transitory period, what matters is the expected monetary policy at the end of the liquidity trap:*

- *If the monetary policy is expected to be active after the period when the ZLB is binding, then* ***all*** *governments must choose a passive fiscal policy. The fiscal union is not necessary to coordinate fiscal policies to achieve stability.*
- *If the monetary policy is expected to be passive after the period when the ZLB is binding, then one of the government must choose an active fiscal policy, or a common public debt must ensure this stabilizing task.*

Proof. See Appendix I. □

A fiscal union that takes the form of bond issues on behalf of the EU makes it possible to guarantee the stability of the Euro area whatever maybe the expectations of agents. This will be necessary only if the agents expect that an active monetary policy will no longer be possible in the future, even when the economy will leave the ZLB regime. When the agents expect that an active monetary policy will revive, a fiscal union is not necessary; however, cannot destabilize the dynamics if the European council adopts a passive fiscal policy, as all the member states. Therefore, a fiscal union can be viewed as a guarantee, ensuring, whatever maybe agents' expectations and the constraints on monetary policy, the non-explosiveness of the Euro area dynamics.

6 Discussion

Our paper relates to several strands of the literature.

First, it offers complementary arguments in favor of a fiscal union to those presented in Farhi and Werning (2017) and Berger, Dell'Ariccia, and Obstfeld (2018). These authors show that the loss of adjustment margin induced by the fixed exchange rate of a currency union, can be compensated by transfers between countries, or organized by a fiscal union. They also show that a fiscal union can discipline moral hazard problems induced by this more efficient risk sharing. Hence, such studies focus on the higher ability of a fiscal union to stabilize asymmetric shocks. We complement these arguments by highlighting the ability of a fiscal union to guarantee the dynamic stability of the area, irrespective of the nature of shocks. This is shown by considering explicitly the public debt dynamics of each country of the area, a dimension too often overlooked in business cycle analysis.

Second, our analysis provides answers to the stability problems of the Euro area raised by Woodford (1996) and Sims (1999):[35]

> "*Even if one government is fiscally responsible and keeps its real primary deficit at some sustainable constant level, variations in the budget deficit of the other government will result in price level instability for that government as well. Thus, there is a clear reason for a government concerned to maintain stable prices to care about fiscal policies of the other governments with which it shares a common currency.*" Woodford (1996), p. 30

> "*What is the weakness of the EMU system in other circumstances—the fiscal free rider problem—would work toward resolving the difficulty. Even one country that is sufficiently fiscally expansive, despite the Maastricht rules, could undo the liquidity trap, with the resultant reversal of deflation benefiting all members of the EMU. On the other hand, if the logical foundations of the need for fiscal coordination are not understood, the need to break the Maastricht rules in this situation could undermine adherence to them more generally.*" Sims, (1999), p. 425

Following these remarks, we show that the construction of European institutions cannot be founded on a single vision of price stability, based on the effectiveness of Taylor's principle. If, the policy mix of the Euro area is open to a wider range of practices than the conventional ones, exposed in Gali's (2008) textbook, the FTPL, presented by Leeper (1991), Sims (1994), Woodford (1998), and Cochrane (2001), can help to design new and more robust rules.[36] In this paper, we show that a fiscal union is not necessary when the monetary policy can be active. In this case, no country has the incentive to

[35] See also Dornbusch (1996) and Feldstein (1997) for criticisms of the European institutions.

[36] As emphasized by Woodford (1998), the study by D'Autume and Michel (1987) is important in the literature on the fiscal theory of the price level: "*It thus appears to be usually an admissible policy for the government to distribute to the public subsidies financed by an ever-increasing monetary debt. The*

deviate from the implementation of the fiscal brake suggested by the Treaties. But a fiscal union can be a necessity when monetary policy cannot be active. In this case where the FTPL applies, the Euro area's inflation is determined by the nominal public debt dynamic of one country of the area that must implement an active fiscal policy, whereas the others must respect their fiscal commitments (passive fiscal policy by implementing a fiscal brake). The degree of freedom concerning the choice of the country that must have an active fiscal policy, advocates in favor of a fiscal union that trivially solves this coordination problem. Therefore, our paper complements Bergin (2000) who analyzed the implications of the FTPL in an endowment open economy. Bergin (2000) shows that if an unifying central bank (the ECB) controls the issue of new money through open market bond purchases, and transfers its interest income back to the national governments via lump sum transfers, then a unique equilibrium can exist when monetary policy follows an interest rate peg (a passive monetary policy) only if one country's fiscal policy is passive while the other country's fiscal policy is active. Our results generalize Bergin's ones by showing that this policy mix can be one solution for the Euro area stability, among all the other combinations presented in our paper, in particular to issue bonds on behalf of the EU.[37]

Third, since the 2008 global financial crisis and for a period that will last certainly for another few years after the COVID-19 crisis, we also contribute to the analysis of the Euro area's policy mix under the ZLB constraint, as it has already been investigated by Farhi and Werning (2016). These authors focus on the fiscal multiplier size during a liquidity trap period and highlight the limits of conventional stabilizers in this context.[38] Our study suggests that other fiscal multipliers can be evaluated in the context where individuals expect that monetary policy will never be active in the future. But, beyond the size of the multipliers, our results suggest that issuing bonds on the financial markets on behalf of the EU could also reinforce the dynamic stability of the Euro area, and thus ensure the robustness of the recovery measures.

Fourth, the sequence of the Ukrainian crisis after that of Covid-19 has pushed up the debts

---

reason, of course, is that the government does not pay any interest on this special kind of debt" (p. 1351). "*If the government freely chooses the paths of its instruments without caring about its intertemporal budget constraint, the choice of the initial price level must ensure that [...] the real value of initial debt are such that the government budget constraint is satisfied ex post*" (p. 1363).

[37] Dupor (2000) also analyzes the implications of the FTPL in the framework of an open economy, but in flexible exchange rates system, which does not apply directly to the current European experience.

[38] Indeed, exchange rate adjustments greatly reduce the impact of conventional stimuli, as the fiscal multipliers are less than one in an open economy in a currency union and greater than one in the context of a closed economy with a liquidity trap.

of European countries well beyond what the treaties authorize, leading to the temporary suspension of EU budget rules. This new context has carried out to reform treaties in 2023.

> *"For each Member State with a government deficit above 3% of GDP or public debt above 60% of GDP, the Commission will issue a country-specific "technical trajectory". This trajectory will seek to ensure that debt is put on a plausibly downward path or stays at prudent levels, and that the deficit remains or is brought and maintained below 3% of GDP in the medium term.... The ratio of public debt to GDP will have to be lower at the end of the period covered by the plan than at the start of that period; and a minimum fiscal adjustment of 0.5% of GDP per year as a benchmark will have to be implemented so long as the deficit remains above 3% of GDP. "* European Commission (2023)

Therefore, new rules keep the targets of 3% deficit, 60% debt, but replace the "1/20th rule" by the "0.5% rule" accompanied by the commitment of "lower at the end of the plan's period" Debt-to-GDP ratio. This could be interpreted as the non-uniqueness of γ in the future, but a γ country-specific (the case analyzed in section 3.4.2). This flexibility of γ could also manage its state-dependency with respect to its threshold insuring public debts sustainability.[39]

> *"The green and digital transitions, the strengthening of economic and social resilience and the need to bolster Europe's security capacity will require large and sustained public investment in the years to come.... The positive interaction between reforms (of fiscal rules) and investment is already showing its benefits under NextGenerationEU's* ***Recovery and Resilience Facility****."* European Commission (2023)

where "Recovery and Resilience Facility" plan allows the Commission to raise funds for investments in order to address European challenges, thus validating the principle of a common debt managed by the European Commission, also advocated in this paper.

But all these analyzes are limited to studying the stability of equilibria in the context of models with representative agents. Kaplan et al (2023) have analyzed how the interactions between monetary and fiscal policies can be affected by the presence of heterogeneous

[39] As explained in section 2, the γ-threshold depends on $\{r, \pi, g\}$ which are stochastic variables. See Blanchard, Leandro and Zettelmeyer (2021) for a detailed discussion on the "stochastic" nature of public debt management.

agents in a HANK model. They shed light on the fact that some policy-mix equilibria, unstable in a representative agent model, would be consistent with the uniqueness of the saddle-path when agents are heterogeneous. In that perspective, it seems paramount for us to extend our paper's analyses to a case where agents are heterogeneous as in Kaplan et al (2023) but with the specificities of the Euro Area, in particular the multiplicity of the fiscal policies.

7 Conclusion

In this paper, we show that, in a monetary union, the dynamic stability of open economies, which are all in charge of their public debt, depends on the combination of the choice of fiscal and monetary policy rules. If an active monetary policy can be implemented, or if agents expect its implementation after experiencing a period of a liquidity trap situation, then we show that each country will choose to have a passive fiscal policy (debt brake). Indeed, if the central bank is able to ensure the dynamic stability of the inflation and output gap by applying the Taylor principle, then each government will voluntarily choose to implement a fiscal brake so that its debt does not explode. The fiscal union is not rationally necessary in this environment, with all governments choosing the same option, which is to implement a fiscal brake already present in the European treaties.

Conversely, when monetary policy is constrained, as it has been the case for the past few years, it becomes necessary to adapt the current EU institutions. Thus, we show that a fiscal union, that would take the form of bond issues on behalf of the EU, and thus define a public debt common to the countries of the Euro area, makes it possible to overcome the coordination problem induced by the need to manage differently a single public debt among those of the area. At the ZLB, it becomes necessary for an actor to have an active fiscal policy to guarantee the stability of inflation on a balance where the fiscal theory of the price level applies. This actor could be the European Council, as it will be the case for the management of the "Next Generation EU" initiative financed by a common debt.

Consequently, these results provide the basis for the new European budgetary rules (European Commission (2023)): a deficit reduction plan must be put in place when the public debt becomes excessive, and the issuance of a common debts can allow better coordination.

References

Aiyagari, S. and Gertler, M., 1985, The backing of government bonds and monetarism, *Journal of Monetary Economics*, 16, pp. 19-44.

Aguiar, M., Amador, M., Farhi, E. and Gopinath, G., 2015. Coordination and Crisis in Monetary Unions, *The Quarterly Journal of Economics*, 130, pp. 1727-1779.

Auray, S. and Eyquem,A., 2014, Welfare Reversals in a Monetary Union. *American Economic Journal : Macroeconomics*, 6, pp. 246-290.

D'Amico, L., Giavazzi, F., Guerrieri, V., Lorenzoni, G. and Weymuller, C.H., 2022, Revising the European Fiscal Framework, Mimeo

Backus, D.K. and Smith, G.W., 1993. Consumption and Real Exchange Rates in Dynamic Economies with Non-Traded Goods. *Journal of International Economics*, 35, pp. 297-316.

Bassetto, M., 2002. A Game-Theoretic View of the Fiscal Theory of the Price Level. *Econometrica*, 70, pp. 2167-2195.

Bassetto, M., 2008. Fiscal Theory of the Price Level. *The New Palgrave Dictionary of Economics*, edited by Steven N. Durlauf and Lawrence E. Blume.

Bassetto, M. and Sargent, T. , 2020. Shotgun Wedding: Fiscal and Monetary Policy, *Annual Review of Economics*, Annual Reviews, 12, pp 659-690.

Bergin, P., 2000, Fiscal Solvency and Price Level Determination in a Monetary Union, *Journal of Monetary Economics*, 45, pp. 37-53.

Blanchard, O., 2023. Fiscal Policy under Low Interest Rates. *MIT Press.*

Blanchard, O. and Kahn, C., 1980. The Solution of Linear Difference Models under Rational Expectations. *Econometrica*, 48, pp. 1305-1311.

Blanchard, O., Leandro, A. and Zettelmeyer, J., 2021. Redesigning EU Fiscal Rules: from Rules to Standards. *Economic Policy*, 36, pp. 195-236.

Bohn, H., 1998. The Behavior of U.S. Public Debt and Deficits. *The Quarterly Journal of Economics*, 113, pp. 949-963.

Bonam, D. and Hobijn, B., 2021, Generalized Stability of Monetary Unions under Regime Switching in Monetary and Fiscal Policies. *Journal of Money, Credit and Banking*, 53, pp.:73-94.

Calvo, G., 1983. Staggered Prices in a Utility Maximizing Framework. *Journal of Monetary Economics*, 12, pp. 383-398.

Christiano, L., 2004. The Zero-Bound, Zero-Inflation Targeting, and Output Collapse. *Manuscript*, Northwestern University

Christiano, L., Eichenbaum, M. and Rebelo, S. , 2011, When Is the Government Spending Multiplier Large? *Journal of Political Economy*, 119, pp. 78-121

Christiano, L. and Fitzgerald, T., 2000, Understanding the Fiscal Theory of the Price Level, *Federal Reserve Bank of Cleveland, Economic Review*, 36, pp. 01-38,

Cochrane, J. 2001, Long Term Debt and Optimal Policy in the Fiscal Theory of the Price Level, *Econometrica*, 69, pp. 69-116.

Cochrane, J. 2019, The Fiscal Theory of the Price Level, *Manuscript*, University of Chicago.

d'Autume, A. and Michel, P., 1987. Transversality Conditions, Budget Constraints and the Determinacy of a Perfect Foresight Equilibrium in a Monetary Growth Model, *European Economic Review*, 31, pp. 1343-1367.

Darvas, Z. 2022. A European Climate Fund or a Green Golden Rule: not as Different as They Seem. Bruegel

Dornbusch, R., 1996, Euro Fantasies, *Foreign Affairs*, 75, pp. 110-124.

Dupor, B., 2000, Exchange Rates and the Fiscal Theory of the Price Level, *Journal of Monetary Economics*, 45, pp. 613-630

Eggertsson, G., 2004, Monetary and Fiscal Coordination in a Liquidity Trap. *PhD dissertation:* Optimal Monetary and Fiscal Policy in the Liquidity Trap, chapter 3, Princeton University.

Eggertsson, G. and Woodford, M., 2003. The Zero Interest-Rate Boundand Optimal Monetary Policy. *Brookings Papers of Econonmic Activity*, 1, pp. 139-211.

Erceg, C. and Linde, J., 2012, Fiscal Consolidation in an Open Economy, *American Economic Review*, 102, pp. 186-191

European Commission, 2023. On the Effective Coordination of Economic Policies and Multilateral Budgetary Surveillance and Repealing Council Regulation, Regulation of the European Parliament and of the Council, EC No 1466/97

Farhi, E. and Gopinath, G. and Itskhoki, O., 2014, Fiscal Devaluations, *Review of Economic Studies*, 81, pp. 725-760

Farhi, E and Werning, I., 2016, Fiscal Multipliers: Liquidity Traps and Currency Unions, *Handbook of Macroeconomics*, Edited by Taylor, J. and Uhlig, H., chapter 31, Volume 2, pp. 2417-2492.

Farhi, E and Werning, I., 2017, Fiscal Union, *American Economic Review*, 107, pp. 3788-3834

Feldstein, M., 1997, The Political Economy of the European Economic and Monetary Union: Political Sources of an Economic Liability, *Journal of Economic Perspectives*, 11, pp. 23-42

Gali, J. and Monacelli, T., 2008. Optimal Monetary and Fiscal Policy in a Currency Union, *Journal of International Economics*, 76, pp. 116-132.

Gali, J., 2008. Monetary Policy, Inflation, and the Business Cycle, *Princeton University Press*, Princeton, NJ.

Garicano, L., 2022. Combining environmental and fiscal sustainability: A new climate facility, an expenditure rule, and an independent fiscal agency. VoxEU CEPR

Ingram, J., 1969. Comment: The Optimum Currency Problem. *Monetary Problems in International Economy*, Chicago University Press, edited by R. Mundell and A. Swoboda, pp. 95-100.

Kaplan, G., Nikolakoudis, G. and Violante, G., 2023, Price Level and Inflation Dynamics in Heterogeneous Agent Economies, *NBER Working Paper* 31433

Leeper, E., 1991. Equilibria Under "Active" and "Passive" Monetary and Fiscal Policies, *Journal of Monetary Economics*, 27, pp. 129-147.

Leeper, E. and Leith, C., 2016. Understanding Inflation as a Joint Monetary-Fiscal Phenomenon, *Handbook of Macroeconomics*, Volume 2, pp. 2305-2415.

Leeper, E., Traum, N. and Walker, T., 2017. Clearing Up the Fiscal Multiplier Morass, *American Economic Review*, 107, pp. 2409-2454

Martin, P. and Philippon, T., 2017. Inspecting the Mechanism: Leverage and the Great Recession in the Eurozone, *American Economic Review*, 107, pp. 1904-1937.

Martin, P., Pisani-Ferry, J. and Ragot, X., 2021, Reforming the European Fiscal Framework, CAE Worhing paper, 63

Mundell, R., 1973. Uncommon Arguments for Common Currencies. *The Economics of Common Currencies*, Oxford, edited by H. Johnson and A. Swoboda, pp. 14-32.

Sargent, T. and Wallace, N., 1981, Some Unpleasant Monetarist Arithmetic. *Federal Reserve Bank of Minneapolis Quarterly Review*, 5, pp. 1-17.

Schmitt-Grohé, S. and Uribe, M., 2003, Closing Small Open Economy Models. *Journal of international Economics*, 61, pp. 163-185.

Sims, C., 1994. A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy. *Economic Theory* 4, pp. 381-399.

Sims, C., 1999, The Precarious Fiscal Foundations of EMU, *De Economist*, 147, pp. 415-436.

Sims, C., 2012. Gaps in the institutional structure of the euro area, *Financial Stability Review*, Public Debt, Monetary Policy and Financial Stability, Banque de France, pp. 217-224.

Sims, C., 2013. Paper Money, *American Economic Review*, 103, pp. 563-584.

Taylor, J.B., 1999. An Historical Analysis of Monetary Policy Rules, *Monetary policy rules*, University of Chicago Press, Chicago, pp. 319-341.

Woodford, M., 1996. Control of the Public Debt: a Requirement for Price Stability, *NBER Working Paper*, 5684.

Woodford, M., 1998. Public Debt and the Price Level, *Manuscript*, Princeton University.

Appendix

A From Structural to Reduced Form Parameters

Parameter		Interpretation
β		Subjective discount factor
σ		Intertemporal elasticity of substitution for consumption
φ		Intertemporal elasticity of substitution for employment
$\overline{\alpha}$		Home Bias
m		Size of the Home country
α	$\overline{\alpha}(1-m)$	Trade openness in Home country
α^*	$\overline{\alpha}m$	Trade openness in the Foreign country
η		Elasticity of substitution between Home and Foreign baskets
ε		Elasticity of substitution between Home (Foreign) goods
ρ_a		Persistence of technological shocks
θ		Probability to not reset price for a firm
γ, γ^*		Fiscal brake in Home (Foreign) country
ρ, ρ^*		Persistence of fiscal transfer in Home (Foreign) country
α_π		Elasticity of the interest rate to inflation (Taylor rule)
α_y		Elasticity of the interest rate to output gap (Taylor rule)
Parameters for the reduced form		
λ	$\frac{1-\theta}{\theta}(1-\theta\beta)$	Elasticity of the inflation to marginal costs
κ	$\lambda(\sigma+\varphi)$	Elasticity of the inflation to output gap in a closed economy
ω	$1-\alpha^*$ $+\alpha(2-\overline{\alpha})(\sigma\eta-1)$	Weight of the Home country
ω^*	α^* $+\alpha^*(2-\overline{\alpha})(\sigma\eta-1)$	Weight of the Foreign country

Table 1: Model parameters

B Individual Behaviors and Equilibrium

B.1 Households

The quantities and prices in country F are denoted by an asterisk and those in country H are without asterisks. The representative households' preferences are given by

$$\mathcal{U}^j = \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{1}{1-\sigma}(C_t^j)^{1-\sigma}-\frac{1}{1+\varphi}(N_t^j)^{1+\varphi}\right)\right] \tag{29}$$

$$\mathcal{U}^{j*} = \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{1}{1-\sigma}(C_t^{j*})^{1-\sigma}-\frac{1}{1+\varphi}(N_t^{j*})^{1+\varphi}\right)\right], \tag{30}$$

where $\beta \in (0,1)$ is the subjective discount factor. $N_t = \int_0^m N_t^j dj = mN_t^j$ and $N_t^* = \int_m^1 N_t^{j*}(i)di = (1-m)N_t^{j*}$ are the total hours worked in countries H and F, respectively. C_t^j and C_t^{j*} denote the consumption index of a household in countries H and F, and are defined as follows:

$$C_t^j = \left((1-\alpha)^{-\frac{1}{\eta}}(C_{H,t}^j)^{\frac{\eta-1}{\eta}}+\alpha^{-\frac{1}{\eta}}(C_{F,t}^j)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{31}$$

$$C_t^{j*} = \left((1-\alpha^*)^{-\frac{1}{\eta}}(C_{F,t}^{j*})^{\frac{\eta-1}{\eta}}+(\alpha^*)^{-\frac{1}{\eta}}(C_{H,t}^{j*})^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}, \tag{32}$$

where the index of the consumption of goods produced in country H, and the index of the consumption of goods produced in country F are

$$C_{H,t}^j = \left(\left(\frac{1}{m}\right)^{\frac{1}{\varepsilon}}\int_0^m C_t^j(h)^{\frac{\varepsilon-1}{\varepsilon}}dh\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{33}$$

$$C_{F,t}^{j*} = \left(\left(\frac{1}{1-m}\right)^{\frac{1}{\varepsilon}}\int_m^1 C_t^{j*}(f)^{\frac{\varepsilon-1}{\varepsilon}}df\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{34}$$

For simplicity, assume that the number of goods produced in each "country" is equal to the number of people living in each "country" (one skill-one good). The elasticity of substitution between the differentiated goods produced in each country satisfies $\varepsilon > 1$. The trade openness of each "country" is measured by $\alpha, \alpha^* \in [0,1]$. The elasticity of substitution between domestic and foreign goods from the viewpoint of the domestic consumer is given by $\eta \in (0,1)$.

The maximizations of (29) and (30) are subject to sequences of budget constraints of the following form:

$$
\begin{aligned}
& \int_0^m P_t(h)C_t^j(h)dh + \int_0^m P_t(f)C_t^j(f)df + \mathbb{E}_t[Q_{t,t+1}A_t^j] + \frac{1}{R_t}B_t^j \\
= \; & A_{t-1}^j + B_{t-1}^j + W_t N_t^j + T_t^j \qquad (35) \\
& \int_m^1 P_t^*(h)C_t^{j*}(h)dh + \int_m^1 P_t^*(f)C_t^{j*}(f)df + \mathbb{E}_t[Q_{t,t+1}A_t^{j*}] + \frac{1}{R_t}B_t^{j*} \\
= \; & A_{t-1}^{j*} + B_{t-1}^{j*} + W_t^* N_t^* + T_t^{j*}, \qquad (36)
\end{aligned}
$$

where $P_t(h)$ and $P_t^*(f)$ are the prices of goods produced in country H and country F, respectively. A_t denotes the nominal payoff during period $t+1$ of the portfolio (Arrow securities) bought during period t and $Q_{t,t+1}$ the prices of these contingent assets (i.e., the price in t to have 1 € in period $t+1$). B_t denotes the nominal payoff during period t of the government bonds bought during period $t-1$ and $1/R_t$ the nominal price at t of an asset that pays 1 € during period $t+1$. The asset price ($Q_{t,t+1}$) and bond price $1/R_t$ are the same in both countries because all exchanges take place in the Euro area. W_t and T_t are the nominal wage and lump-sum transfers (taxes)[40], respectively. The transfers include the sum of dividends redistributed by the firm to households and the lump-sum tax net of government transfers.

Within each category of goods, the optimal choices are given by

$$
C_t^j(h) = \frac{1}{m}\left(\frac{P_t(h)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t}^j \;, \qquad C_t^j(f) = \frac{1}{1-m}\left(\frac{P_t(f)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t}^j \qquad (37)
$$

$$
C_t^{j*}(h) = \frac{1}{m}\left(\frac{P_t^*(h)}{P_{H,t}^*}\right)^{-\varepsilon} C_{H,t}^{j*} \;, \qquad C_t^{j*}(f) = \frac{1}{1-m}\left(\frac{P_t^*(f)}{P_{F,t}^*}\right)^{-\varepsilon} C_{F,t}^{j*}, \qquad (38)
$$

where $P_{H,t} = \left(\frac{1}{m}\int_0^m P_t(h)^{1-\varepsilon}dh\right)^{\frac{1}{1-\varepsilon}}$ and $P_{F,t}^* = \left(\frac{1}{1-m}\int_m^1 P_t(f)^{1-\varepsilon}df\right)^{\frac{1}{1-\varepsilon}}$ denote the PPI. The price indices of imported goods, $P_{H,t}^*$ and $P_{F,t}$, are defined analogously to $P_{H,t}$ and $P_{F,t}^*$. The optimal allocations of expenditure between domestic and imported goods are

$$
C_{H,t}^j = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t^j \;, \qquad C_{F,t}^j = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t^j \qquad (39)
$$

$$
C_{H,t}^{j*} = \alpha^*\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta} C_t^{j*} \;, \qquad C_{F,t}^{j*} = (1-\alpha^*)\left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\eta} C_t^{j*}, \qquad (40)
$$

where the CPIs are given by

$$
\begin{aligned}
P_t &= \left((1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right)^{\frac{1}{1-\eta}} \qquad (41) \\
P_t^* &= \left((1-\alpha^*)(P_{F,t}^*)^{1-\eta} + \alpha^*(P_{H,t}^*)^{1-\eta}\right)^{\frac{1}{1-\eta}} \qquad (42)
\end{aligned}
$$

[40] T_t comprises lump-sum transfers and taxes from the government and profits from the firms.

Equations (37) and (38) imply that the total expenditure in country H is the sum of $\int_0^m P_t(h)C_t^j(h)dh = P_{H,t}C_{H,t}^j$ and $\int_0^m P_t(f)C_t^j(f)df = P_{F,t}C_{F,t}^j$. Similarly, the total expenditure in country F is the sum of $\int_m^1 P_t^*(h)C_t^{j*}(h)dh = P_{H,t}^*C_{H,t}^{j*}$ and $\int_m^1 P_t^*(f)C_t^{j*}(f)df = P_{F,t}^*C_{F,t}^{j*}$. Then, from Equations (39) and (40), the following relations hold: $P_{H,t}C_{H,t}^j + P_{F,t}C_{F,t}^j = P_tC_t^j$ and $P_{H,t}^*C_{H,t}^{j*} + P_{F,t}^*C_{F,t}^{j*} = P_t^*C_t^{j*}$. These expressions can be used to rewrite Equations (35) and (36) as:

$$P_tC_t^j + \mathbb{E}_t[Q_{t,t+1}A_t^j] + \frac{1}{R_t}B_t^j = A_{t-1}^j + B_{t-1}^j + W_tN_t^j + T_t \tag{43}$$

$$P_t^*C_t^{j*} + \mathbb{E}_t[Q_{t,t+1}A_t^{j*}] + \frac{1}{R_t}B_t^{j*} = A_{t-1}^{j*} + B_{t-1}^{j*} + W_t^*N_t^{j*} + T_t^{j*} \tag{44}$$

Using the definition of the aggregates $C_t = mC_t^j$, $C_t^* = (1-m)C_t^{j*}$, $N_t = mN_t^j$, and $N_t^* = (1-m)N_t^{j*}$, the first-order conditions of the maximizations of (29) subject to (43), and the maximizations of (30) subject to (94) are:

$$\beta\frac{C_{t+1}^{-\sigma}P_t}{C_t^{-\sigma}P_{t+1}} = Q(z_{t+1}|z_t) \quad \forall z \tag{45}$$

$$\beta\mathbb{E}_t\left[\frac{C_{t+1}^{-\sigma}P_t}{C_t^{-\sigma}P_{t+1}}\right] = \frac{1}{R_t} \tag{46}$$

$$m^{-(\sigma+\varphi)}C_t^{\sigma}N_t^{\varphi} = \frac{W_t}{P_t} \tag{47}$$

$$\beta\frac{(C_{t+1}^*)^{-\sigma}P_t^*}{(C_t^*)^{-\sigma}P_{t+1}^*} = Q(z_{t+1}|z_t) \quad \forall z \tag{48}$$

$$\beta\mathbb{E}_t\left[\frac{(C_{t+1}^*)^{-\sigma}P_t^*}{(C_t^*)^{-\sigma}P_{t+1}^*}\right] = \frac{1}{R_t} \tag{49}$$

$$(1-m)^{-(\sigma+\varphi)}(C_t^*)^{\sigma}(N_t^*)^{\varphi} = \frac{W_t^*}{P_t^*}, \tag{50}$$

where $Q_{t,t+1}$ is the vector of $Q(z_{t+1}|z_t)$. Equations (45) and (48) imply that:

$$\beta\mathbb{E}_t\left[\frac{C_{t+1}^{-\sigma}P_t}{C_t^{-\sigma}P_{t+1}}\right] = \mathbb{E}_t\left[Q(z_{t+1}|z_t)\right] \equiv Q_t \tag{51}$$

$$\beta\mathbb{E}_t\left[\frac{(C_{t+1}^*)^{-\sigma}P_t^*}{(C_t^*)^{-\sigma}P_{t+1}^*}\right] = \mathbb{E}_t\left[Q(z_{t+1}|z_t)\right] \equiv Q_t. \tag{52}$$

The short-term nominal interest rate, R_t, which is also the ECB's policy instrument, is linked to the nominal bond price, namely $1/R_t$. The no-arbitrage condition leads to $1/R_t = Q_t$.

Log-linearizing around the steady state for the symmetric case (i.e., where $\overline{\alpha} = \overline{\alpha}^*$), Equations (51) and (52), and using the expression for R_t, we find that Home and Foreign aggregate consumptions are:

$$\widehat{c}_t = \mathbb{E}_t[\widehat{c}_{t+1}] - \frac{1}{\sigma}\left(r_t - \mathbb{E}_t[\pi_{t+1}] + \log(\beta)\right) \tag{53}$$

$$\widehat{c}^*_t = \mathbb{E}_t[\widehat{c}^*_{t+1}] - \frac{1}{\sigma}\left(r_t - \mathbb{E}_t[\pi^*_{t+1}] + \log(\beta)\right), \tag{54}$$

where $\widehat{x}_t = \log(X_t/X)$, $\forall x_t$ with X the steady-state value of X_t and $x_t = \log(X_t)$. Therefore, $\pi_t = p_t - p_{t-1}$ ($\pi^*_t = p^*_t - p^*_{t-1}$) is the Home (Foreign) CPI inflation.

B.2 PPI and CPI inflation rates, real exchange rate, TOT, and international risk sharing

To determine the link between CPI and PPI inflation rates, we consider the price of goods produced in country F in terms of the price of goods produced in country H. The TOT are defined by $\mathcal{S}_t = \frac{P_{F,t}}{P_{H,t}}$. The log-linearized CPI inflation rate is

$$\widehat{\pi}_t = \widehat{\pi}_{H,t} + \overline{\alpha}(1-m)(\widehat{s}_t - \widehat{s}_{t-1}) \quad ; \quad \widehat{\pi}^*_t = \widehat{\pi}^*_{F,t} - \overline{\alpha}m(\widehat{s}_t - \widehat{s}_{t-1}), \tag{55}$$

where $\widehat{\pi}_{H,t} = \widehat{p}_{H,t} - \widehat{p}_{H,t-1}$ and $\widehat{\pi}^*_{F,t} = \widehat{p}^*_{F,t} - \widehat{p}^*_{F,t-1}$ denote the PPI inflation in countries H and F, respectively. Equation (55) shows that CPI inflation depends on PPI inflation and changes in the TOT. The effect of a change in the Home country's TOT on the gap between the Home CPI and PPI increases with the weight of the imported (Foreign) goods in Home households' preferences, given by $\overline{\alpha}(1-m) = \alpha$, which decreases with the relative size (m) of country H and with the degree of home bias, inversely related to $\overline{\alpha}$. Therefore, in the specific case of a small country H, when m is close to 0, $\widehat{p}^*_t = \widehat{p}^*_{F,t}$ ($\widehat{\pi}^*_t = \widehat{\pi}^*_{F,t}$); **however**, we maintain a gap between the CPI and PPI in country H ($\widehat{\pi}_t = \widehat{\pi}_{H,t} + \overline{\alpha}(\widehat{s}_t - \widehat{s}_{t-1})$ because $\alpha = \overline{\alpha}(1-m)$ and $m \to 0$).

We assume that the law of one price (LOP) always holds for individual goods, both for import and export prices. This implies that $P_t(f) = \mathcal{E}_t P^*_t(f)$, $\forall f$ and $P_t(h) = \mathcal{E}_t P^*_t(h)$, $\forall h$, where $\mathcal{E}_t$ denotes the price of country F's currency in terms of country H's currency. Substituting these conditions into the price indices for the two countries yields $P_{F,t} = \mathcal{E}_t P^*_{F,t}$ and $P_{H,t} = \mathcal{E}_t P^*_{H,t}$. In the Euro area, we have $\mathcal{E}_t = 1$, therefore, $P_{F,t} = P^*_{F,t}$ and $P_{H,t} = P^*_{H,t}$. However, due to the home bias introduced in the preferences over consumption bundles ($\alpha \neq \frac{1}{2}$), purchasing power parity (PPP) does not hold in general. Hence, $P_t \neq \mathcal{E}_t P^*_t$.

The real exchange rate, denoted by $\mathcal{Q}_t = \frac{\mathcal{E}_t P^*_t}{P_t}$, reduces to $\mathcal{Q}_t = \frac{P^*_t}{P_t}$ in the Euro area, where $\mathcal{E}_t = 1$. Hence, we have[41]:

$$\mathcal{Q}_t = \frac{\left((1-\alpha^*)\mathcal{S}_t^{1-\eta} + \alpha^*\right)^{\frac{1}{1-\eta}}}{\left((1-\alpha) + \alpha\mathcal{S}_t^{1-\eta}\right)^{\frac{1}{1-\eta}}} \quad \Rightarrow \quad \widehat{q}_t = \widehat{s}_t(1-\overline{\alpha}), \tag{56}$$

where we use the properties that for any m, the steady-state solution is such that $\mathcal{S} = 1$, when preferences are identical across countries $\overline{\alpha} = \overline{\alpha}^*$. Hence, home bias is the only source of the violation of PPP.[42] The real exchange rate's volatility increases with the degree of Home bias and volatility in the TOT. Although the LOP holds for all goods individually, the real exchange rate $\widehat{q}_t$ is directly related to the TOT $\widehat{s}_t$, which fluctuates over time in response to shocks in both countries, because the Foreign (Home) preference places a higher weight on Foreign (Home) goods than the Home (Foreign) preference does.

Turning to the financial market and uncovered interest parity (UIP) conditions, we have in a monetary union (the nominal exchange rate is equal to one, $\mathcal{E}_t = 1$):

$$\left.\begin{array}{rcl} \beta\frac{(C_{t+1}^j)^{-\sigma}}{(C_t^j)^{-\sigma}} & = & \frac{P_{t+1}}{P_t}Q(z_{t+1}|z_t) \\ \beta\frac{(C_{t+1}^{j*})^{-\sigma}}{(C_t^{j*}(i))^{-\sigma}} & = & \frac{\mathcal{Q}_{t+1}}{\mathcal{Q}_t}\frac{P_{t+1}}{P_t}Q(z_{t+1}|z_t) \end{array}\right\} \Rightarrow \quad C_t = \frac{m}{1-m}\vartheta\mathcal{Q}_t^{\frac{1}{\sigma}}C_t^*, \tag{57}$$

where $\vartheta = \mathcal{Q}_t^{-\frac{1}{\sigma}}\frac{C_0}{C_0^*}$ and with $C_t = \int_0^m C_t^j dj = mC_t$ and $C_t^{j*} = \int_m^1 C_t^{j*}dj = (1-m)C_t^*$. Hence, home bias allows for a variable gap between Home and Foreign households' consumption growth rates, even if the international financial market structure is complete. The ratio between the Home and Foreign aggregate consumption levels collapses to zero as the Home country becomes a small economy.

Following a general procedure in the literature, we assume the same initial conditions for Home and Foreign households, so that $\vartheta = 1$. Log-linearizing (57) around the steady state for the symmetric case ($\overline{\alpha} = \overline{\alpha}^*$), we obtain:

$$\widehat{c}_t = \frac{1}{\sigma}\widehat{q}_t + \widehat{c}_t^* \tag{58}$$

[41] We use the definitions of price indexes:

$$\begin{array}{rclcl} P_t & = & \left((1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right)^{\frac{1}{1-\eta}} & \Rightarrow & \frac{P_t}{P_{H,t}} = \left((1-\alpha) + \alpha\mathcal{S}_t^{1-\eta}\right)^{\frac{1}{1-\eta}} \\ P_t^* & = & \left((1-\alpha^*)P_{F,t}^{1-\eta} + \alpha^* P_{H,t}^{1-\eta}\right)^{\frac{1}{1-\eta}} & \Rightarrow & \frac{P_t^*}{P_{H,t}} = \left((1-\alpha^*)\mathcal{S}_t^{1-\eta} + \alpha^*\right)^{\frac{1}{1-\eta}} \end{array}$$

With $\mathcal{Q}_t = \frac{P_t^*}{P_t}$, this leads to

$$\widehat{q}_t = (1-\alpha^*-\alpha)\,\widehat{s}_t = (1-\overline{\alpha}m - \overline{\alpha}(1-m))\,\widehat{s}_t = (1-\overline{\alpha})\,\widehat{s}_t$$

[42] $\widehat{q}_t = 0$ in every period when there is no home bias (i.e., when $\overline{\alpha} = 1$).

showing that risk sharing implies a perfect international correlation between consumption levels.

B.3 Firms

There is a continuum of firms indexed by $i \in [0,1]$. Firms on the interval $[0,m)$ are located in country H, while firms on the interval $[m,1]$ are located in country F. Each monopolistic competitive firm sets the relative price of its differentiated goods, faced an isoelastic and downward-sloping demand curve and is subject to a technological constraint. Firms use only a homogeneous type of labor for production and there is no investment. The labor market is competitive.

B.3.1 Cost minimization

All Home firms operate with an identical constant returns-to-scale technology:

$$Y_t(i) = A_t L_t(i),$$

where $Y_t(i)$ is the Home firm i's output and $L_t(i)$ is the Home firm i's labor demand. A_t is the Home's total factor productivity shifter. Its log-linearized counterpart follows an $AR(1)$ process:

$$\widehat{a}_t = \log(A_t/A) = \rho\widehat{a}_{t-1} + \epsilon_{a,t},$$

where $0 < \rho < 1$ and $\epsilon_{a,t}$ are i.i.d Gaussian shocks. All Foreign firms operate with a similar technology, but these processes may face different persistence ($\rho \neq \rho^*$). The shocks $\epsilon_{a,t}$ and $\epsilon^*_{a,t}$ may be correlated. Technology constraints imply that the Home and Foreign ith firms' labor demands are given respectively by:

$$\begin{aligned} L_t(i) &= \frac{Y_t(i)}{A_t} \\ L^*_t(i) &= \frac{Y^*_t(i)}{A^*_t} \end{aligned}$$

so that the Home and Foreign nominal marginal costs are given by $MC^n_t = (1+t_t)W_t/A_t$ and $MC^{n*}_t = (1+t^*_t)W^*_t/A^*_t$, while the Home and Foreign real marginal costs are defined as:

$$MC_t = \frac{MC^n_t}{P_{H,t}} = \frac{(1+t_t)W_t}{P_{H,t}A_t} \tag{59}$$

$$MC^*_t = \frac{MC^{n*}_t}{P^*_{F,t}} = \frac{(1+t^*_t)W^*_t}{P^*_{F,t}A^*_t}, \tag{60}$$

where t_t and t^*_t are payroll tax rates in Home and Foreign economies.

B.3.2 Price setting

Firms set prices in a staggered way, as in Calvo (1983): every period, a measure of $1-\theta$ randomly selected firms set a new price, with an individual firm's probability of readjusting at each period being independent of the time elapsed since it last reset its price. A Home firm i adjusting its price in period t sets a new price $\overline{P}_{H,t}$ to maximize the present value of its stream of expected future profits:

$$\max_{\overline{P}_{H,t}(i)} \sum_{\tau=0}^{\infty} \theta^{\tau} \mathbb{E}_t \left[Q_{t,t+\tau} (\overline{P}_{H,t}(i) - MC^n_{t+s}) Y_{t+s}(i) \right]$$

subject to the demand constraint:

$$Y_{t+s}(i) = \left(\frac{\overline{P}_{H,t}}{P_{H,t+s}} \right)^{-\varepsilon} C_{t+s}.$$

A firm cannot decide on its optimal price in each period. To simplify the notation, we omit the index i for the firm. Following Calvo (1983), on each date, the firm receives a signal advising it whether it can revise its price $P_{H,t}$ in an optimal manner. There is a probability θ that the firm cannot revise its price in a given period. When a firm receives a positive signal (with probability $1-\theta$), it chooses price $P_{H,t}$ that maximizes its profit discerning that during the next periods it may be unable to choose its price optimally. Let $\widetilde{\mathcal{V}}_t$ be the value of a firm that receives a positive signal in period t and $\mathcal{V}_t(P_{H,t-1})$ the value of a firm that receives a negative signal. As a firm that receives a negative signal simply follows the *ad hoc* pricing rule $P_{H,t} = P_{H,t-1}$, its value at time t depends only on $P_{H,t-1}$. Denoting $\Pi(\overline{P}_{H,t}) = (\overline{P}_{H,t} - MC^n_t) Y_t$, the value of a firm that receives a positive signal in period t is:

$$\widetilde{\mathcal{V}}_t = \max_{\overline{P}_{H,t}} \left\{ \Pi(\overline{P}_{H,t}) + \beta \mathbb{E}_t \left[Q_{t+1} \left((1-\theta) \widetilde{\mathcal{V}}_{t+1} + \theta \mathcal{V}_{t+1}(\overline{P}_{H,t}) \right) \right] \right\} \tag{61}$$

The value of a firm that cannot re-optimize is:

$$\mathcal{V}_t(P_{H,t-1}) = \Pi(P_{H,t-1}) + \beta \mathbb{E}_t \left[Q_{t+1} \left((1-\theta) \widetilde{\mathcal{V}}_{t+1} + \theta \mathcal{V}_{t+1}(P_{H,t-1}) \right) \right] \tag{62}$$

The first-order and envelope conditions associated with (61)–(62) determine the dynamics of inflation. These are given by:

$$\Pi'_t(\overline{P}_{H,t}) + \beta\theta \mathbb{E}_t \left[Q_{t+1} \mathcal{V}'_{t+1}(\overline{P}_{H,t}) \right] = 0 \tag{63a}$$

$$\mathcal{V}'_t(P_{H,t-1}) = \Pi'_t(P_{H,t-1}) + \beta\theta \mathbb{E}_t \left[Q_{t+1} \mathcal{V}'_{t+1}(P_{H,t-1}) \right] \tag{63b}$$

Iterating forward on these first-order conditions, we can obtain:

$$
\begin{aligned}
\mathbb{E}_t \left[\sum_{j=0}^{\infty} \theta^j Q_{t+j} Y_{t+j} \left(\overline{P}_{H,t} - \frac{\varepsilon}{\varepsilon - 1} MC^n_{t+j} \right) \right] &= 0 \\
\mathbb{E}_t \left[\sum_{j=0}^{\infty} \theta^j Q_{t+j} Y_{t+j} \left(\frac{\overline{P}_{H,t}}{P_{H,t-1}} - \frac{\varepsilon}{\varepsilon - 1} MC_{t+j} \frac{P_{H,t+j}}{P_{H,t-1}} \right) \right] &= 0 \qquad (64)
\end{aligned}
$$

Log-linearizing this equation around the zero-inflation steady state and given that $Q_{t+j} = \beta^j$ must hold at the steady state (see (45) and (48)), we obtain:

$$
\widehat{\overline{p}}_{H,t} - \widehat{p}_{t-1} = (1 - \beta\theta) \sum_{j=0}^{\infty} (\beta\theta)^j \mathbb{E}_t \left[\widetilde{mc}_{t+j} + (\widehat{p}_{t+j} - \widehat{p}_{t-1}) \right],
$$

where $\widetilde{mc}_t$ is the deviation of the real marginal cost from its steady-state level. It is equal to $\widetilde{mc}_t = t_t + \widehat{w}_t - \widehat{p}_{H,t} - \widehat{a}_t$.

$$
\widehat{\pi}_{H,t} = \beta \mathbb{E}_t \left[\widehat{\pi}_{H,t+1} \right] + \lambda \widetilde{mc}_t,
$$

where $\lambda = \frac{1-\theta}{\theta}(1 - \theta\beta)$.

Proceeding in the same way with the Foreign country, we obtain:

$$
\widehat{\pi}^*_{F,t} = \beta \mathbb{E}_t \left[\widehat{\pi}^*_{F,t+1} \right] + \lambda \widetilde{mc}^*_t,
$$

where $\widetilde{mc}^*_t$ is the deviation of the real marginal cost from its steady-state level. The Home and Foreign real marginal costs under flexible prices are given by $mc_t = mc^*_t = -\log\left(\frac{\varepsilon}{\varepsilon-1}\right)$, where $\log\left(\frac{\varepsilon}{\varepsilon-1}\right)$ is the gross markup (in log).

B.4 Government budget constraints

Each agent j in country H (or F) can only invest in its own country public debts. We assume for simplicity that a lump sum transfer (Tr_t and Tr^*_t) allows each government to redistribute the revenues of the payroll taxes paid only in sticky price economies: this implies that $Tr_t = t_t W_t N_t$ and $Tr^*_t = t^*_t W^*_t N^*_t$.

The budget constraint (after receiving and redistributing the payroll taxes) of the state in the Home country is given by:

$$
\frac{1}{R_t} \frac{B_t}{P_t} + D^r_t = \frac{B_{t-1}}{P_t} \quad \Leftrightarrow \quad \frac{1}{R_t} B^r_t + D^r_t = \frac{1}{\pi_t} B^r_{t-1},
$$

where D^r_t are the real net transfers from the government to households, namely, $D^r_t = Tax_t - Tr_t$, with Tax_t a real lump-sum tax and Tr_t a real lump-sum transfer. We deflate the nominal debt (B_t) by the CPI to define the real debt B_t^r. By the forward iteration of this budgetary constraint, the price level P_t depends on the sequence of the choice of the state ($\{D_{t+i}^r\}_{i=0}^{\infty}$), via the valuation of the debt:

$$\frac{B_{t-1}}{P_t} = \mathbb{E}_t \sum_{i=0}^{\infty} \left(\prod_{j=1}^{i} \frac{\pi_{t+j}}{R_{t+j-1}} \right) D_{t+i}^r$$

Given an initial condition for the nominal debt B_{t-1}, this equation shows that the general price level depends on the sequence of net spending by the state of the Home country ($\{D_{t+i}^r\}_{i=0}^{\infty}$): this is one of the basis of the price tax theory.

The linearized versions of the budget constraints of each state (Home and Foreign) are given by:

$$\widehat{b}_t^r = \beta^{-1}(\widehat{b}_{t-1}^r - \widehat{\pi}_t) + \widehat{r}_t + (1-\beta^{-1})\widehat{d}_t^r \tag{65}$$

$$\widehat{b}_t^{r*} = \beta^{-1}(\widehat{b}_{t-1}^{r*} - \widehat{\pi}_t^*) + \widehat{r}_t + (1-\beta^{-1})\widehat{d}_t^{r*} \tag{66}$$

As for the Taylor rule, one can model the government's choices using a fiscal rule. For example, we can assume that the surplus of the Home government $\widehat{d}_t$ follows the following rule:

$$D_t^r = D^r + \gamma \left(\frac{1}{R_{t-1}} \frac{B_{t-1}}{P_{t-1}} - \frac{B^r}{R} \right) + \varepsilon_t^F$$

$$\Rightarrow \quad \widehat{d}_t^r = \gamma \frac{1}{\beta^{-1}-1} (\widehat{b}_{t-1}^r - \widehat{r}_{t-1}) + \varepsilon_t^F \tag{67}$$

This last equation represents the fiscal rule. The same rule exists in the Foreign country. Introducing (67) into the budget constraints of the government in the Home (Foreign) country (Equations (65) and (66)), and using the CPI definition 55 we obtain:

$$\widehat{b}_t^r = (\beta^{-1} - \gamma)\widehat{b}_{t-1}^r - \beta^{-1}\widehat{\pi}_{H,t} + \widehat{r}_t + \gamma \widehat{r}_{t-1} - \frac{\bar{\alpha}(1-m)}{\beta}\Delta\widehat{s}_t + (1-\beta^{-1})\varepsilon_t^F \tag{68}$$

$$\widehat{b}_t^{r*} = (\beta^{-1} - \gamma^*)\widehat{b}_{t-1}^{r*} - \beta^{-1}\widehat{\pi}_{F,t}^* + \widehat{r}_t + \gamma \widehat{r}_{t-1} + \frac{\bar{\alpha}m}{\beta}\Delta\widehat{s}_t + (1-\beta^{-1})\varepsilon_t^{F*} \tag{69}$$

These budgetary constraints depend on the ECB's policy, that is, directly through the interest rate and indirectly through inflation and TOT.

B.5 Equilibrium

The log-linearization around the steady state of the equilibrium equations of the goods market around the steady state symmetric ($\overline{\alpha} = \overline{\alpha}^*$), using the UIP condition (Equation

58), leads to

$$\widehat{y}_t = \widehat{c}_t + \frac{\omega + \overline{\alpha} - 1}{\sigma}\widehat{s}_t \tag{70}$$

$$\widehat{y}_t^* = \widehat{c}_t^* - \frac{\omega^*}{\sigma}\widehat{s}_t, \tag{71}$$

where $\omega = 1 - \overline{\alpha}m + (1-m)\overline{\alpha}(2-\overline{\alpha})(\sigma\eta - 1) > 0$ and $\omega^* = \overline{\alpha}m + m\overline{\alpha}(2-\overline{\alpha})(\sigma\eta - 1) > 0$. Using Equations (55), (70), and (71), Equations (53) and (54) become:

$$\begin{aligned}
\widehat{y}_t &= \mathbb{E}_t[\widehat{y}_{t+1}] - \frac{1}{\sigma}\left(r_t - \mathbb{E}_t[\widehat{\pi}_{H,t+1}] + \log(\beta)\right) + \frac{1 - \overline{\alpha}m - \omega}{\sigma}\Delta\mathbb{E}_t\widehat{s}_{t+1} \\
\widehat{y}_t^* &= \mathbb{E}_t[\widehat{y}_{t+1}^*] - \frac{1}{\sigma}\left(r_t - \mathbb{E}_t[\widehat{\pi}_{F,t+1}^*] + \log(\beta)\right) - \frac{\overline{\alpha}m - \omega^*}{\sigma}\mathbb{E}_t\Delta\widehat{s}_{t+1}
\end{aligned}$$

Using Equations (58) and (56), we obtain:

$$\widehat{s}_t = \frac{\sigma}{\omega + \omega^*}\left(\widehat{y}_t - \widehat{y}_t^*\right) \tag{72}$$

B.5.1 Flexible price equilibrium

In the flexible price economy, it is assumed that $t_t = t_t^* = 0$. Home and Foreign firms optimize their prices in each period. Hence, these prices are equal to a mark-up over the marginal cost. We find that the Home and Foreign real marginal costs are given by:

$$mc_t = mc_t^* = -\log\left(\frac{\varepsilon}{\varepsilon - 1}\right),$$

where $mc_t = mc_t^n - p_{H,t}$ and $mc_t^* = mc_t^n - p_{F,t}^*$. Hence, we deduce the TOT

$$\overline{s}_t = \frac{1+\varphi}{1 + \frac{\varphi}{\sigma}(\omega + \omega^*)}(\widehat{a}_t - \widehat{a}_t^*),$$

and the natural level of the Home and Foreign outputs:

$$\begin{aligned}
\overline{y}_t &= \log m - \frac{1}{\varphi + \sigma}\log\left(\frac{\varepsilon}{\varepsilon - 1}\right) + (1 - \Theta)\frac{1+\varphi}{\varphi + \sigma}\widehat{a}_t + \Theta\frac{1+\varphi}{\varphi+\sigma}\widehat{a}_t^* \\
\overline{y}_t^* &= \log(1-m) - \frac{1}{\varphi + \sigma}\log\left(\frac{\varepsilon}{\varepsilon - 1}\right) + \Theta^*\frac{1+\varphi}{\varphi + \sigma}\widehat{a}_t + (1 - \Theta^*)\frac{1+\varphi}{\varphi+\sigma}\widehat{a}_t^*,
\end{aligned}$$

with $\Theta = \frac{\sigma(1-\overline{\alpha}m-\omega)}{\sigma+\varphi(\omega+\omega^*)}$ and $\Theta^* = \frac{\sigma(\overline{\alpha}m-\omega^*)}{\sigma+\varphi(\omega+\omega^*)}$. Using the $AR(1)$ processes for $\widehat{a}_t$ and $\widehat{a}_t{}^*$, we obtain:

$$\mathbb{E}_t\Delta\overline{s}_{t+1} = \frac{1+\varphi}{1 + \frac{\varphi}{\sigma}(\omega + \omega^*)}[(\rho - 1)\widehat{a}_t - (\rho^* - 1)\widehat{a}_t^*]$$

This leads us to deduce the natural expected interest rate ($r_t^e \equiv \overline{r}_t - \mathbb{E}_t\overline{\pi}_{H,t}$ and $r_t^{e*} \equiv \overline{r}_t - \mathbb{E}_t\overline{\pi}^*_{F,t}$):

$$\begin{aligned} r_t^e &= \frac{(1-\Gamma)\sigma(1+\varphi)(\rho-1)}{\varphi+\sigma}\widehat{a}_t + \frac{\Gamma\sigma(1+\varphi)(\rho^*-1)}{\varphi+\sigma}\widehat{a}_t^* - \log\beta \quad (73) \\ r_t^{e*} &= \frac{\Gamma^*\sigma(1+\varphi)(\rho-1)}{\varphi+\sigma}\widehat{a}_t + \frac{(1-\Gamma^*)\sigma(1+\varphi)(\rho^*-1)}{\varphi+\sigma}\widehat{a}_t^* - \log\beta, \quad (74) \end{aligned}$$

where $\Gamma = \frac{\varphi(\overline{\alpha}m+\omega-1)}{\sigma+\varphi(\omega+\omega^*)}$ and $\Gamma^* = \frac{\varphi(\omega-\overline{\alpha}m)}{\sigma+\varphi(\omega+\omega^*)}$.

B.5.2 Equilibrium with nominal rigidities

As in Gali and Monacelli (2005), we assume that the deviations of the dispersion of the Home and Foreign firms' output induced by price rigidities around the steady state symmetric are of second order, so that up to a first-order approximation, we can set them to zero. Therefore, using the labor market equilibrium, we have:

$$\begin{aligned} mc_t &= (\sigma+\varphi)(y_t - \log m) - (\overline{\alpha}m+\omega-1)s_t - (1+\varphi)a_t + t_t \\ mc_t^* &= (\sigma+\varphi)(y_t^* - \log(1-m)) - (\overline{\alpha}m-\omega^*)s_t - (1+\varphi)a_t^* + t_t^* \end{aligned}$$

Assuming that t_t and t_t^* are different from zero only in the sticky price economies, we obtain:

$$\begin{aligned} \widehat{\pi}_{H,t} &= \beta\mathbb{E}_t\left[\widehat{\pi}_{H,t+1}\right] + \kappa\widehat{y}_t - \lambda(\alpha m+\omega-1)\widehat{s}_t + \lambda t_t \quad (75) \\ \widehat{\pi}^*_{F,t} &= \beta\mathbb{E}_t\left[\widehat{\pi}^*_{F,t+1}\right] + \kappa\widehat{y}_t^* + \lambda(\omega^*-\alpha m)\widehat{s}_t + \lambda t_t^*, \quad (76) \end{aligned}$$

which are the Home and Foreign New Phillips curves, where $\kappa = \lambda(\varphi+\sigma)$. The IS curves are given by:

$$\begin{aligned} \widehat{y}_t &= \mathbb{E}_t[\widehat{y}_{t+1}] - \frac{1}{\sigma}\left(r_t - \mathbb{E}_t[\widehat{\pi}_{H,t+1}] - \overline{r}_t^e\right) + \frac{1-\overline{\alpha}m-\omega}{\sigma}\mathbb{E}_t\Delta\widehat{s}_{t+1} \quad (77) \\ \widehat{y}_t^* &= \mathbb{E}_t[\widehat{y}_{t+1}^*] - \frac{1}{\sigma}\left(r_t - \mathbb{E}_t[\widehat{\pi}^*_{F,t+1}] - \overline{r}_t^{e*}\right) + \frac{\omega^*-\overline{\alpha}m}{\sigma}\mathbb{E}_t\Delta\widehat{s}_{t+1} \quad (78) \end{aligned}$$

Therefore, the equilibrium dynamics of this two-country NK model with asymmetric countries is defined by:

- the New Phillips curves (Equations (75) and (76)),
- the IS curves (Equations (77) and (78)), and
- the natural expected interest rates (Equations (73) and (74)),

- The Taylor rule of the ECB:

$$\widehat{r}_t = \alpha_\pi \widehat{\pi}_t^u + \alpha_y \widehat{y}_t^u + \epsilon_{r,t}, \tag{79}$$

where $\widehat{\pi}_t^u = m\widehat{\pi}_{H,t} + (1-m)\widehat{\pi}_{F,t}^*$ and $\widehat{y}_t^u = m\widehat{y}_t + (1-m)\widehat{y}_t^*$. The distinction between CPI inflation and PPI inflation, while meaningful at the level of each country, disappears for the ECB. Formally, given that $p_t = p_{H,t} + \overline{\alpha}(1-n)s_t$ and $p_t^* = p_{F,t}^* - \overline{\alpha}ns_t$, we have $p_t^u = np_t + (1-n)p_t^* = np_{H,t} + (1-n)p_{F,t}^*$ (see Equation (55)).

- Gap in the TOT is given by:

$$\widehat{s}_t = \frac{\sigma}{\omega + \omega^*}(\widehat{y}_t - \widehat{y}_t^*) \tag{80}$$

- Public debt of each country given by Equations (65) and (66) and the fiscal rules in Equation (67).

For simplicity, we assume in the following that $\widehat{a}_t = \widehat{a}_t^* = 0$, $\forall t$, leading to $r^e = r^{e*} = -\log(\beta)$. This does not change the properties of the dynamic system. The dynamic system is

$$Z_t = A^{-1}B\ \mathbb{E}_t Z_{t+1} + A^{-1}C\ e_t$$

C Summary of the Model's Equations

This appendix provides the equations used to define the model equilibrium. The individual behaviors allowing us to derive these reduced forms are explained in Appendix B. Appendix A provides a complete description of the model parameters.

C.1 Households

Consumption demand

$$\widehat{c}_t = \mathbb{E}_t[\widehat{c}_{t+1}] - \frac{1}{\sigma}\left(\widehat{r}_t - \mathbb{E}_t[\widehat{\pi}_{t+1}]\right) \tag{81}$$

$$\widehat{c}^*_t = \mathbb{E}_t[\widehat{c}^*_{t+1}] - \frac{1}{\sigma}\left(\widehat{r}_t - \mathbb{E}_t[\widehat{\pi}^*_{t+1}]\right) \tag{82}$$

Labor supply

$$\sigma\widehat{c}_t + \varphi\widehat{n}_t = \widehat{w}_t - \widehat{p}_t \tag{83}$$

$$\sigma\widehat{c}^*_t + \varphi\widehat{n}^*_t = \widehat{w}^*_t - \widehat{p}^*_t \tag{84}$$

C.2 Prices, exchange rates, and international risk sharing

Terms-of-Trade

$$\mathcal{S}_t = \frac{P_{F,t}}{P_{H,t}} \quad \Rightarrow \quad \widehat{s}_t = \widehat{p}_{F,t} - \widehat{p}_{H,t}$$

Price indexes

$$P_t = \left((1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right)^{\frac{1}{1-\eta}} \quad \Rightarrow \quad \frac{P_t}{P_{H,t}} = \left((1-\alpha) + \alpha\mathcal{S}_t^{1-\eta}\right)^{\frac{1}{1-\eta}}$$

$$P^*_t = \left((1-\alpha^*)(P^*_{F,t})^{1-\eta} + \alpha^*(P^*_{H,t})^{1-\eta}\right)^{\frac{1}{1-\eta}} \quad \Rightarrow \quad \frac{P^*_t}{P^*_{H,t}} = \left((1-\alpha^*)\mathcal{S}_t^{1-\eta} + \alpha^*\right)^{\frac{1}{1-\eta}}$$

$$\Rightarrow \begin{cases} \widehat{p}_t - \widehat{p}_{H,t} = \alpha\widehat{s}_t = \overline{\alpha}(1-m)\widehat{s}_t \\ \widehat{p}^*_t - \widehat{p}^*_{H,t} = (1-\alpha^*)\widehat{s}_t = (1-\overline{\alpha}m)\widehat{s}_t \end{cases}$$

Nominal exchange rate

$$P_{F,t} = \mathcal{E}_t P^*_{F,t} \quad \Rightarrow \quad \widehat{p}_{F,t} = \widehat{e}_t + \widehat{p}^*_{F,t}$$

Real exchange rate

$$\mathcal{Q}_t = \frac{\mathcal{E}_t P_t^*}{P_t} \quad \Rightarrow \quad \begin{aligned} \widehat{q}_t &= \widehat{e}_t + \widehat{p}_t^* - \widehat{p}_t \\ \widehat{q}_t &= \widehat{e}_t + \widehat{p}_{H,t}^* + (1-\alpha^*)\widehat{s}_t - \widehat{p}_{H,t} - \alpha\widehat{s}_t \\ \widehat{q}_t &= \widehat{e}_t + (1-\alpha^*-\alpha)\widehat{s}_t + \widehat{p}_{H,t}^* - \widehat{p}_{H,t} \end{aligned}$$

In the Euro Area, we have $\widehat{e}_t = 0$, $\forall t$ and thus:

$$\widehat{q}_t \;=\; (1-\alpha^*-\alpha)\,\widehat{s}_t = (1-\overline{\alpha}m - \overline{\alpha}(1-m))\,\widehat{s}_t = (1-\overline{\alpha})\,\widehat{s}_t$$

CPI Inflation

$$\begin{aligned} \widehat{\pi}_t &= \widehat{\pi}_{H,t} + \overline{\alpha}(1-m)(\widehat{s}_t - \widehat{s}_{t-1}) \\ \widehat{\pi}_t^* &= \widehat{\pi}_{F,t}^* - \overline{\alpha}m(\widehat{s}_t - \widehat{s}_{t-1}) \end{aligned}$$

Financial markets

$$\left.\begin{aligned} \beta\frac{(C_{t+1}^j)^\sigma}{(C_t^j)^\sigma} &= \frac{P_{t+1}}{P_t}Q(z_{t+1}|z_t) \\ \beta\frac{(C_{t+1}^{j*})^\sigma}{(C_t^{j*}(i))^\sigma} &= \frac{\mathcal{Q}_{t+1}}{\mathcal{Q}_t}\frac{P_{t+1}}{P_t}Q(z_{t+1}|z_t) \end{aligned}\right\} \Rightarrow \quad C_t = \frac{m}{1-m}\vartheta\mathcal{Q}_t^{\frac{1}{\sigma}}C_t^*, \tag{85}$$

where $\vartheta = \mathcal{Q}_t^{-\frac{1}{\sigma}}\frac{C_0}{C_0^*}$ and with $C_t = \int_0^m C_t^j dj = mC_t^j$ and $C_t^* = \int_m^1 C_t^{j*}dj = (1-m)C_t^{j*}$. This leads to:

$$\widehat{c}_t = \frac{1}{\sigma}\widehat{q}_t + \widehat{c}_t^* \tag{86}$$

C.3 Price-setting rule

The Phillips curves are:

$$\widehat{\pi}_{H,t} \;=\; \beta\mathbb{E}_t\left[\widehat{\pi}_{H,t+1}\right] + \kappa\widehat{y}_t - \lambda(\alpha m + \omega - 1)\widehat{s}_t \tag{87}$$

$$\widehat{\pi}_{F,t}^* \;=\; \beta\mathbb{E}_t\left[\widehat{\pi}_{F,t+1}^*\right] + \kappa\widehat{y}_t^* + \lambda(\omega^* - \alpha m)\widehat{s}_t, \tag{88}$$

which are the Home and Foreign New Phillips curves, where $\kappa = \lambda(\varphi+\sigma)$.

C.4 Resource constraints

$$\begin{aligned} Y_t &= C_{H,t} + C_{H,t}^* = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}C_t + \alpha^*\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta}C_t^* \\ Y_t^* &= C_{F,t} + C_{F,t}^* = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta}C_t + (1-\alpha^*)\left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\eta}C_t^* \end{aligned}$$

Using (85) $\Leftrightarrow \frac{1-m}{m}\frac{1}{\vartheta}\mathcal{Q}_t^{-\frac{1}{\sigma}}C_t = C_t^*$, we have:

$$Y_t \quad = \quad \left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} + \alpha^*\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta}\frac{1-m}{m}\frac{1}{\vartheta}\mathcal{Q}_t^{-\frac{1}{\sigma}}\right]C_t,$$

where

$$\begin{aligned}
\mathcal{Q}_t &= \frac{\mathcal{E}_t P_t^*}{P_t} = \frac{\left((1-\alpha^*)\mathcal{S}_t^{1-\eta} + \alpha^*\right)^{\frac{1}{1-\eta}}}{\left((1-\alpha) + \alpha\mathcal{S}_t^{1-\eta}\right)^{\frac{1}{1-\eta}}} \\
\frac{P_{H,t}}{P_t} &= \left((1-\alpha) + \alpha\mathcal{S}_t^{1-\eta}\right)^{-\frac{1}{1-\eta}} \\
\frac{P_{H,t}^*}{P_t^*} &= \left((1-\alpha^*)\mathcal{S}_t^{1-\eta} + \alpha^*\right)^{-\frac{1}{1-\eta}}
\end{aligned}$$

This shows that $Y_t = \mathcal{Y}(\mathcal{S}_t, C_t)$. Log-linearizing these equations lead to:

$$\widehat{y}_t = \widehat{c}_t + \frac{\omega + \overline{\alpha} - 1}{\sigma}\widehat{s}_t \tag{89}$$

$$\widehat{y}_t^* = \widehat{c}_t^* - \frac{\omega^*}{\sigma}\widehat{s}_t, \tag{90}$$

where $\omega = 1 - \overline{\alpha}m + (1-m)\overline{\alpha}(2-\overline{\alpha})(\sigma\eta - 1) > 0$ and $\omega^* = \overline{\alpha}m + m\overline{\alpha}(2-\overline{\alpha})(\sigma\eta-1) > 0$. Remark that (89) and (90) can be rewritten as follows:

$$\widehat{y}_t = \widehat{c}_t + \frac{\omega + \overline{\alpha} - 1}{\sigma}(\widehat{p}_{F,t} - \widehat{p}_{H,t}) \tag{91}$$

$$\widehat{y}_t^* = \widehat{c}_t^* - \frac{\omega^*}{\sigma}(\widehat{p}_{F,t} - \widehat{p}_{H,t}) \tag{92}$$

Therefore, the IRS condition (86) can be rewritten as follows:

$$\widehat{y}_t - \widehat{y}_t^* = \frac{\omega + \omega^*}{\sigma}(\widehat{p}_{F,t} - \widehat{p}_{H,t}) = \frac{\omega + \omega^*}{\sigma}\widehat{s}_t$$

D Roots of the Terms-Of Trade Dynamic Equation

The following equation gives the deterministic part of the TOT dynamic:[43]

$$0 = \widehat{s}_{t+1} - \underbrace{\left(\frac{1+\beta+\frac{\kappa+(\kappa-\sigma\lambda)\overline{\alpha}(2-\overline{\alpha})(\sigma\eta-1)}{\sigma}}{\beta} \right)}_{=b} \widehat{s}_t + \frac{1}{\beta}\widehat{s}_{t-1}$$

The solution is such that $(1-r_1L)(1-r_2L)\widehat{s}_t = 0$, where the roots are:

$$r_i = \frac{b \pm \sqrt{b^2-4/\beta}}{2} \quad i=1,2 \quad \text{where } \; 0<r_1<1<r_2$$

First, remark that:

$$b = \frac{1+\beta+\frac{\kappa+(\kappa-\sigma\lambda)\alpha(2-\alpha)(\sigma\eta-1)}{\sigma}}{\beta} = \frac{1+\beta}{\beta} + \frac{\lambda(\varphi+\sigma)+\lambda\varphi\alpha(2-\alpha)(\sigma\eta-1)}{\beta\sigma} \equiv \frac{1+\beta}{\beta} + \Gamma,$$

where $\Gamma > 0$ when $\eta\sigma > 1$. Therefore, assuming $\eta\sigma > 1$, we have $|r_1| < 1$ and $|r_2| > 1$ iff:

$$\begin{array}{cc} |r_1| < 1 & |r_2| > 1 \\ \dfrac{b-\sqrt{b^2-4/\beta}}{2} < 1 & \dfrac{b-\sqrt{b^2+4/\beta}}{2} > 1 \\ \dfrac{1+\beta}{\beta} < \dfrac{1+\beta}{\beta} + \Gamma & \dfrac{1+\beta}{\beta} < \dfrac{1+\beta}{\beta} + \Gamma \end{array}$$

These two inequalities lead to the same restriction, which is always satisfied. Thus, given that $0 < r_1 < 1 < r_2$ when $\eta\sigma > 1$, the TOT equation has a unique saddle path.

[43] See Appendix A for a description of the model parameters.

E Equilibrium Dynamics of a SOE

The system describing the equilibrium is:[44]

$$
\begin{aligned}
\widehat{p}_{H,t+1} &= b\widehat{p}_{H,t} - \frac{1}{\beta}\widehat{p}_{H,t-1} \\
\widehat{b}^r_{t-1} &= \frac{\beta}{1-\beta\gamma}\widehat{b}^r_t + \frac{(1-\bar{\alpha}(1-m))}{1-\beta\gamma}(\widehat{p}_{H,t} - \widehat{p}_{H,t-1}) + \frac{\beta^{-1}-1}{\beta^{-1}-\gamma}\varepsilon^F_t
\end{aligned}
$$

This system can be rewritten as follows:

$$
\begin{bmatrix} 0 & 1 & 0 \\ -\frac{1}{\beta} & b & 0 \\ \frac{(1-\bar{\alpha}(1-m))}{1-\beta\gamma} & \frac{(\bar{\alpha}(1-m)-1)}{1-\beta\gamma} & 1 \end{bmatrix}
\begin{bmatrix} \widehat{p}_{H,t-1} \\ \widehat{p}_{H,t} \\ \widehat{b}_{t-1} \end{bmatrix}
=
\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{\beta}{1-\beta\gamma} \end{bmatrix}
\begin{bmatrix} \widehat{p}_{H,t} \\ \widehat{p}_{H,t+1} \\ \widehat{b}_t \end{bmatrix}
+
\begin{bmatrix} 0 \\ 0 \\ \frac{\beta^{-1}-1}{\beta^{-1}-\gamma} \end{bmatrix}\varepsilon^F_t
$$

$$
\Rightarrow \begin{bmatrix} \widehat{p}_{H,t-1} \\ \widehat{p}_{H,t} \\ \widehat{b}_{t-1} \end{bmatrix}
=
\begin{bmatrix} b\beta & -\beta & 0 \\ 1 & 0 & 0 \\ \varpi & \delta & \frac{\beta}{1-\beta\gamma} \end{bmatrix}
\begin{bmatrix} \widehat{p}_{H,t} \\ \widehat{p}_{H,t+1} \\ \widehat{b}_t \end{bmatrix}
+
\begin{bmatrix} 0 \\ 0 \\ \frac{\beta^{-1}-1}{\beta^{-1}-\gamma} \end{bmatrix}\varepsilon^F_t
$$

where:

$$
\varpi = \frac{(1-\bar{\alpha}-b\beta+\bar{\alpha}m+\bar{\alpha}b\beta-\bar{\alpha}b\beta m)}{1-\beta\gamma},\ \delta = \frac{(\beta-\bar{\alpha}\beta+\bar{\alpha}\beta m)}{1-\beta\gamma}
$$

The eigenvalues of its deterministic dynamics are:

$$
\lambda_{1,2} = \beta\frac{b\pm\sqrt{b^2-4/\beta}}{2} < 1, \quad \lambda_3 = \frac{\beta}{1-\beta\gamma} \lesseqgtr 1
$$

We have $\lambda_3 > 1$ if $1/\beta - 1 < \gamma$. This restriction gives the minimum size of the debt brake.

[44]See Appendix A for a description of the model parameters.

F Dynamic System for the Two-Country Model

The dynamic system for the two-country model when each country has a specific fiscal policy is:[45]

$$
\begin{aligned}
\widehat{\pi}_{H,t} - \kappa \widehat{y}_t &= \beta \widehat{\pi}_{H,t+1} - \lambda(\bar{\alpha} m + \omega - 1)\widehat{s}_t \\
\widehat{y}_t \left(1 + \frac{\alpha_y}{\sigma}\right) + \frac{\alpha_\pi}{\sigma}\pi_{H,t} &= \widehat{y}_{t+1} + \frac{1}{\sigma}\widehat{\pi}_{H,t+1} \\
&\quad + \frac{1 - \bar{\alpha} m - \omega}{\sigma}\widehat{s}_{t+1} - \frac{\Lambda}{\sigma}\widehat{s}_t + \frac{\alpha_\pi(1-m)}{\sigma}\widehat{s}_{t-1} - \frac{1}{\sigma}\varepsilon_t^M \\
\frac{\beta}{1-\beta\gamma}\widehat{b}_{t+1}^r &= \widehat{b}_t^r - \frac{1}{1-\beta\gamma}\widehat{\pi}_{H,t+1} - \frac{1-\beta}{1-\beta\gamma}\varepsilon_{t+1}^F + \frac{\beta}{1-\beta\gamma}\varepsilon_{t+1}^M + \frac{\beta\gamma}{1-\beta\gamma}\varepsilon_t^M \\
&\quad + \frac{\beta}{1-\beta\gamma}\left(\alpha_\pi \widehat{\pi}_{H,t+1} + \alpha_y \widehat{y}_{t+1}\right) + \frac{\beta\gamma}{1-\beta\gamma}\left(\alpha_\pi \widehat{\pi}_{H,t} + \alpha_y \widehat{y}_t\right) \\
&\quad + \frac{(1-m)\zeta}{1-\beta\gamma}\widehat{s}_{t+1} + \frac{\Upsilon}{1-\beta\gamma}\widehat{s}_t - \frac{\beta\gamma}{1-\beta\gamma}\alpha_\pi(1-m)\widehat{s}_{t-1} \\
\frac{\beta}{1-\beta\gamma^*}\widehat{b}_{t+1}^{r*} &= \widehat{b}_t^{r*} - \frac{1}{1-\beta\gamma^*}\widehat{\pi}_{H,t+1} - \frac{1-\beta}{1-\beta\gamma}\varepsilon_{t+1}^{F*} + \frac{\beta}{1-\beta\gamma^*}\varepsilon_{t+1}^M + \frac{\beta\gamma^*}{1-\beta\gamma^*}\varepsilon_t^M \\
&\quad + \frac{\beta}{1-\beta\gamma^*}\left(\alpha_\pi \widehat{\pi}_{H,t+1} + \alpha_y \widehat{y}_{t+1}\right) + \frac{\beta\gamma^*}{1-\beta\gamma^*}\left(\alpha_\pi \widehat{\pi}_{H,t} + \alpha_y \widehat{y}_t\right) \\
&\quad + \frac{\Phi}{1-\beta\gamma^*}\widehat{s}_{t+1} + \frac{\Xi}{1-\beta\gamma}\widehat{s}_t - \frac{\beta\gamma^*}{1-\beta\gamma^*}\alpha_\pi(1-m)\widehat{s}_{t-1}
\end{aligned}
$$

where:

$$
\begin{aligned}
\Lambda &= \left[(1-m)\left(\alpha_\pi - \alpha_y \frac{\omega + \omega^*}{\sigma}\right) + 1 - \bar{\alpha} m - \omega\right] \\
\zeta &= \left[\beta\left(\alpha_\pi - \alpha_y \frac{\omega + \omega^*}{\sigma}\right) - \bar{\alpha}\right] \\
\Upsilon &= \left[\bar{\alpha} - \beta\alpha_\pi(1-m) + \beta\gamma\left(\alpha_\pi - \alpha_y \frac{\omega + \omega^*}{\sigma}\right)\right] \\
\Phi &= \left[(\bar{\alpha} m - 1) + \beta(1-m)\left(\alpha_\pi - \alpha_y \frac{\omega + \omega^*}{\sigma}\right)\right] \\
\Xi &= \left[(1 - \bar{\alpha} m) - \beta\alpha_\pi(1-m) + \beta\gamma^*(1-m)\left(\alpha_\pi - \alpha_y \frac{\omega + \omega^*}{\sigma}\right)\right]
\end{aligned}
$$

Given that:

$$
\begin{aligned}
\widehat{\pi}_{F,t+1}^* &= \widehat{s}_{t+1} - \widehat{s}_t + \widehat{\pi}_{H,t+1} \\
\widehat{r}_t &= \alpha_\pi \widehat{\pi}_{H,t} + \alpha_y \widehat{y}_t + (1-m)\left(\alpha_\pi - \alpha_y \frac{\omega + \omega^*}{\sigma}\right)\widehat{s}_t - \alpha_\pi(1-m)\widehat{s}_{t-1} + \varepsilon_t^M,
\end{aligned}
$$

[45] See Appendix A for a description of the model parameters.

we deduce:

$$\begin{bmatrix} \widehat{\pi}_{H,t} \\ \widehat{y}_t \\ \widehat{b}_t^r \\ \widehat{b}_t^{r*} \end{bmatrix} = \begin{bmatrix} & A & & 0 & 0 \\ m_1 & m_2 & m_3 & 0 \\ m_1^* & m_2^* & 0 & m_3^* \end{bmatrix} \begin{bmatrix} \widehat{\pi}_{H,t+1} \\ \widehat{y}_{t+1} \\ \widehat{b}_{t+1}^r \\ \widehat{b}_{t+1}^{r*} \end{bmatrix} + N \begin{bmatrix} \widehat{s}_{t+1} \\ \widehat{s}_t \\ \widehat{s}_{t-1} \end{bmatrix} + P \begin{bmatrix} \varepsilon_{t+1}^M \\ \varepsilon_t^M \\ \varepsilon_{t+1}^F \\ \varepsilon_t^F \\ \varepsilon_{t+1}^{F*} \\ \varepsilon_t^{F*} \end{bmatrix}$$

where $m_3 = \frac{1}{\beta^{-1}-\gamma}$, $m_3^* = \frac{1}{\beta^{-1}-\gamma^*}$ and

$$\begin{aligned}
m_1 &= [\tfrac{\alpha_y\alpha_\pi\beta-\sigma-\alpha_\pi\kappa-\alpha_y+\alpha_y\beta\gamma+\alpha_\pi\beta\sigma+\alpha_\pi^2\beta\kappa+\alpha_\pi\beta^2\gamma\sigma+\alpha_\pi\beta\gamma\kappa}{(\beta\gamma-1)(\alpha_y+\sigma+\alpha_\pi\kappa)}] \\
m_2 &= [\tfrac{\beta(\alpha_y\sigma+\alpha_y^2+\alpha_y\alpha_\pi\kappa+\alpha_y\gamma\sigma+\alpha_\pi\gamma\kappa\sigma)}{(\beta\gamma-1)(\alpha_y+\sigma+\alpha_\pi\kappa)}] \\
m_1^* &= [\tfrac{\alpha_y\alpha_\pi\beta-\sigma-\alpha_\pi\kappa-\alpha_y+\alpha_y\beta\gamma^*+\alpha_\pi\beta\sigma+\alpha_\pi^2\beta\kappa+\alpha_\pi\beta^2\gamma^*\sigma+\alpha_\pi\beta\gamma^*\kappa}{(\beta\gamma^*-1)(\alpha_y+\sigma+\alpha_\pi\kappa)}] \\
m_2^* &= [\tfrac{\beta(\alpha_y\sigma+\alpha_y^2+\alpha_y\alpha_\pi\kappa+\alpha_y\gamma^*\sigma+\alpha_\pi\gamma^*\kappa\sigma)}{(\beta\gamma^*-1)(\alpha_y+\sigma+\alpha_\pi\kappa)}]
\end{aligned}$$

G Incomplete markets

G.1 Incomplete markets with financial autarky

In the two-country model with incomplete markets and financial autarky, the households new budget constraints where state-contingent assets are not available:

$$P_t C_t^j + \frac{1}{R_t} B_t^j = B_{t-1}^j + W_t N_t^j + T_t \tag{93}$$

$$P_t^* C_t^{j*} + \frac{1}{R_{zt}} B_t^{j*} = B_{t-1}^{j*} + W_t^* N_t^{j*} + T_t^{j*} \tag{94}$$

The only financial assets available are their national public debt.

G.2 Incomplete markets with financial trade

In this section, we will provide numerical simulations to show that our results apply also when we consider a more general framework when financial trade is allowed across countries. When trade in assets is available and markets incomplete, Schmitt-Grohé and Uribe (2003) have demonstrated that for a small open-economy (SOE), the model is non-stationary (i.e the steady-state depends on initial conditions). Therefore, there is a need to introduce stationarity inducing devices. Without such mechanisms, the model will exhibit a unit-root. This feature is obviously also present in a two-country model of the EZ and thus, we need to introduce stationarity inducing mechanisms in order to assess the stability properties of the model. In that perspective, we will introduce portfoflio adjustment costs (PAC) to foreign asset.

G.2.1 The model

With foreign debt and PAC, the representative households in each region j and j^* want to maximize the following flows of utilities:

$$\mathcal{U}^j = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\sigma} (C_t^j)^{1-\sigma} - \frac{1}{1+\varphi} (N_t^j)^{1+\varphi} \right) \right] \tag{95}$$

$$\mathcal{U}^{j*} = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\sigma} (C_t^{j*})^{1-\sigma} - \frac{1}{1+\varphi} (N_t^{j*})^{1+\varphi} \right) \right] \tag{96}$$

under the following budget constraints:

$$P_t C_t^j + \frac{B_t^j}{R_t} + \frac{B_{h,t}^j}{R_t^h} + \frac{B_{f,t}^j}{R_t^f} = B_{t-1}^j + B_{h,t-1}^j + B_{f,t-1}^j + W_t N_t^j + T_t^j - \frac{\kappa}{2}\Big(B_{f,t}^j - \bar{B_f^j}\Big)^2 \tag{97}$$

$$P_t^* C_t^{j*} + \frac{B_t^{j*}}{R_{zt}} + \frac{B_{h,t}^{j*}}{R_t^h} + \frac{B_{f,t}^{j*}}{R_t^f} = B_{t-1}^{j*} + B_{h,t-1}^{j*} + B_{f,t-1}^{j*} + W_t^* N_t^{j*} + T_t^{j*} - \frac{\kappa^*}{2}\Big(B_{h,t}^{j*} - \bar{B_h^{j*}}\Big)^2 \tag{98}$$

In addition to their national public debt, households can now trade private assets internationally. Households in country H can hold their private domestic bonds $B_{h,t}^j$ that pay interest rate R_t^h but they can also hold foreign private bonds $B_{f,t}^j$ that pay interest rate R_t^f. Similarly, households in country F can hold their domestic private debt $B_{f,t}^{j*}$ remunerated at rate R_t^f but also foreign private debt $B_{h,t}^{j*}$ that pays R_t^h. Both households face quadratic adjustment costs each time they change their holding of foreign debt $\{B_{f,t}^j, B_{h,t}^{j*}\}$ relative to their steady-state target $\{\bar{B_f^j}, \bar{B_h^{j*}}\}$ where κ and κ^* measure the strength of these costs. Without this hypothesis, the model exhibits a unit-root similar to the one in Schmitt-Grohé and Uribe (2003) in the case of a SOE. The FOCs associated to this new problem are given by:

$$\beta \mathbb{E}_t \Big[\frac{C_t^{j,\sigma}}{C_{t+1}^{j,\sigma}} \frac{1}{\pi_{t+1}}\Big] = \frac{1}{R_t} \tag{99}$$

$$\beta \mathbb{E}_t \Big[\frac{C_t^{j,\sigma}}{C_{j,t+1}^{j,\sigma}} \frac{1}{\pi_{t+1}}\Big] = \frac{1}{R_t^h} \tag{100}$$

$$\beta \mathbb{E}_t \Big[\frac{C_t^{j,\sigma}}{C_{j,t+1}^{j,\sigma}} \frac{rer_{t+1}}{rer_t} \frac{1}{\pi_{t+1}^*}\Big] = \frac{1}{R_t^f} + \kappa(b_{f,t} - \bar{b_f}) \tag{101}$$

$$\beta \mathbb{E}_t \Big[\frac{C_t^{j^*,\sigma}}{C_{t+1}^{j^*,\sigma}} \frac{1}{\pi_{t+1}^*}\Big] = \frac{1}{R_{zt}} \tag{102}$$

$$\beta \mathbb{E}_t \Big[\frac{C_t^{j^*,\sigma}}{C_{t+1}^{j^*,\sigma}} \frac{1}{\pi_{t+1}^*}\Big] = \frac{1}{R_t^f} \tag{103}$$

$$\beta \mathbb{E}_t \Big[\frac{C_t^{j^*,\sigma}}{C_{t+1}^{j^*,\sigma}} \frac{rer_t}{rer_{t+1}} \frac{1}{\pi_{t+1}}\Big] = \frac{1}{R_t^h} + \kappa^*(b_{h,t}^* - \bar{b_h^*}) \tag{104}$$

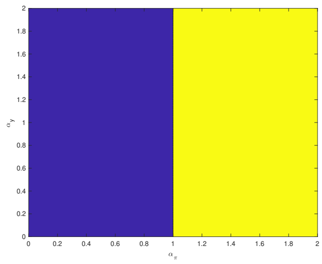
where $\{b_{f,t}, b_{h,t}^*\}$ are real value of private debt assets.

The introduction of private debt adds two new market clearing conditions where in equilibrium, net interhouseholds lending should be zero.

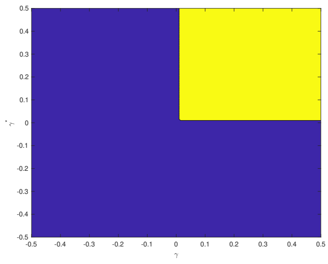
$$m b_{h,t} + (1-m) b_{h,t}^* = 0 \tag{105}$$

$$m b_{f,t} + (1-m) b_{f,t}^* = 0 \tag{106}$$

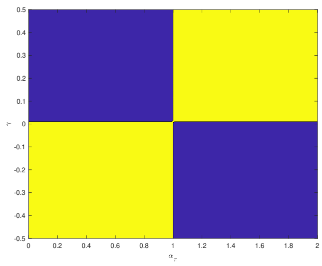
G.2.2 Numerical analysis



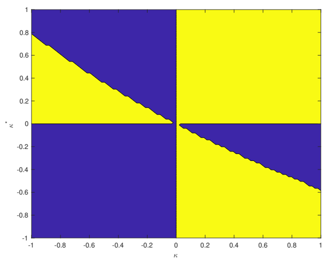
(a) Varying α_π, α_y



(b) Varying γ, γ^*



(c) Varying α_π, γ



(d) Varying κ, κ^*

Figure 1: **Stability of the Euro Area with IM**. Each panel ((a)-(d)) represent the regions where eigenvalues ensure the uniqueness of the saddle-path (yellow) or not (purple).

Panels (a), (b) and (c) of the Figure 1 highlights that the policy-mix regimes that ensures the uniqueness of the saddle-path (yellow surfaces) when financial trade is allowed are similar to the cases with CM and with IM and financial autarky.[46] Finally, under the constraint that adjustment costs parameters $\{\kappa, \kappa^*\} \geq 0$, the model is determinate for strictly positive values of these parameters (see Panel (d) of the Figure 1).

[46]The same graphic of the one of panel (c) is obtained for varying values for $\{\alpha_\pi, \gamma^*\}$.

H Fiscal Devaluation and Stability of the Euro Area

When the monetary policy is constrained to be passive ($\widehat{r}_t = 0$), we show in this appendix that it is possible to stabilize inflation and output via the introduction of policy rules on payroll taxes. This policy allows governments to respect their European commitments by implementing a budgetary brake.

We assume the rates of the payroll tax change as follows:

$$\begin{aligned} t_t &= \nu_y \widehat{y}_t + \nu_\pi \widehat{\pi}_{H,t} \\ t_t^* &= \nu_y^* \widehat{y}_t^* + \nu_\pi^* \widehat{\pi}_{F,t} \end{aligned}$$

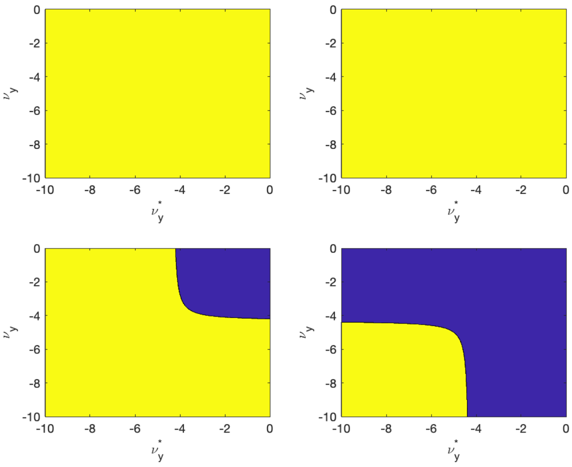
Therefore, the dynamics of the inflation and output gap are given by, knowing the TOT dynamic:

$$\begin{aligned} (1-\lambda\nu_\pi)\widehat{\pi}_{H,t} &= \beta\widehat{\pi}_{H,t+1} + (\kappa + \lambda\nu_y)\widehat{y}_t - \lambda(\alpha m + \omega - 1)\widehat{s}_t \\ \left(1 + \frac{\alpha_y}{\sigma}\right)\widehat{y}_t &= \widehat{y}_{t+1} + \frac{1}{\sigma}\widehat{\pi}_{H,t+1} - \frac{\alpha_\pi}{\sigma}\widehat{\pi}_{H,t} + \frac{1-\overline{\alpha}m - \omega}{\sigma}\widehat{s}_{t+1} + \frac{\alpha_\pi(1-m)}{\sigma}\widehat{s}_{t-1} \\ &\quad -\frac{1}{\sigma}\left[1 - \overline{\alpha}m - \omega + (1-m)\left(\alpha_\pi - \alpha_y\frac{\omega+\omega^*}{\sigma}\right)\right]\widehat{s}_t - \frac{1}{\sigma}\varepsilon_t^M \\ \widehat{s}_{t+1} &= \left(b - \frac{\lambda}{\beta}\left(\nu_\pi^* - \nu_y^*\frac{\omega+\omega^*}{\sigma}\right)\right)\widehat{s}_t - \frac{1}{\beta}\left(1-\nu_\pi^*\right)\widehat{s}_{t-1} \\ &\quad -\frac{\lambda}{\beta}\left[(\nu_\pi^* - \nu_\pi)\widehat{\pi}_{H,t} + (\nu_y^* - \nu_y)\widehat{y}_t\right] \end{aligned}$$

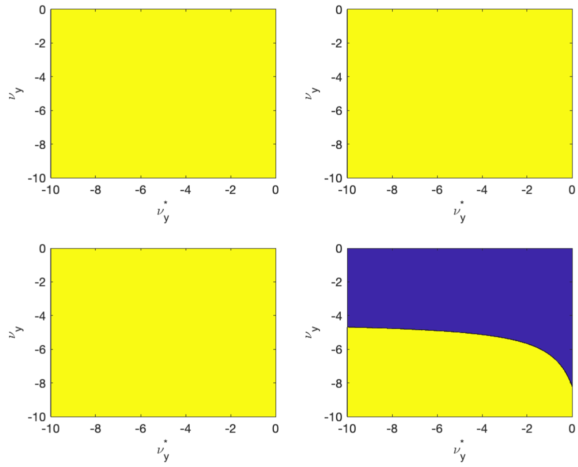
This system can be rewritten as $Z_t = MZ_{t+1} + R\varepsilon_t^M$, where $Z_t = [\widehat{\pi}_{H,t}, \widehat{y}_t, \widehat{s}_t, \widehat{s}_{t-1}]$. This dynamic system will be stable if and only if three of the four eigenvalues of M are inside the unit circle. These three dimensions correspond to the two jump variables that are inflation and the output gap and to the "forward" component of the real exchange rate. The last eigenvalue must be outside the unit circle because it corresponds to the "backward" component of the real exchange rate.

The eigenvalues of the matrix M, for different values of the parameters $\{\nu_y, \nu_\pi, \nu_y^*, \nu_\pi^*\}$, are presented in Figure 1.[47] This figure only reports cases where the tax rates are countercyclical, that is, cases where governments implement a fiscal devaluation when the country's economic activity is poor. This figure shows two results. The first is that it is more likely to obtain a stable dynamic system when the tax rates depend simultaneously on inflation and the output gap. Indeed, panel (a) of Figure 1 shows that two eigenvalues can be simultaneously greater than one, if $\nu_\pi = \nu_\pi^* = 0$, while in panel (b), where

[47] For this numerical illustration, the model parameters are the following: $\beta = 0.98$, $m = 0.5$, $\sigma = 2$, $\alpha = 0.5$, $\eta = 2$, $\theta = 0.8$, $\varphi = 3$.



(a) Varying ν_y, ν^*_y and $\nu_\pi = \nu^*_\pi = 0$



(b) Varying ν_y, ν^*_y and $\nu_\pi = \nu^*_\pi = -0.5$

Figure 2: **Stability of the Euro Area with Fiscal Devaluation Policies**. The four graphics on each panel ((a) and (b)) represent the regions, where an eigenvalue is larger (purple) or lower (yellow) than one. In panel (a), $\nu_\pi = \nu^*_\pi = 0$, whereas in panel (b) $\nu_\pi = \nu^*_\pi = 1.5$. In these two cases, we have $\nu_y \in [-10, 0]$ and $\nu^*_y \in [-10, 0]$, respectively.

both $\nu_\pi < 0$ and $\nu^*_\pi < 0$, only one eigenvalue can be greater than unity. However, even in this case, the uniqueness of the dynamic path is not always guaranteed. The stable path is unique only when ν_y is not less than a threshold value, here around -4 (see panel (b) of the Figure 1). For all the parameters allowing this "automatic" fiscal devaluation policy to guarantee dynamic stability, it would then be interesting to assess its impact on welfare, compared to the case where stabilization is obtained via an active fiscal policy implemented on one of the public debt of one of the two countries. This study is left to future research.

I Model solution at the ZLB

As in Christiano, Eichenbaum and Rebelo (2011)[48], we assume that the economy hits the ZLB after a "shock" on the discount factor. The discount factor follows a two-state Markov process, where the low value is an absorbing state. At the time of shock, the discount factor is at its high value and is sufficient to ensure that the zero bound binds. Thereafter, the discount factor can return to its low value with a probability of $1-p$.

A) If the monetary policy is expected to be active after the ZLB, the model is solved by the agents by taking $\{\widehat{\pi}_{H,t}, \widehat{y}_t\}$ as the two jump variables of the economy. This is the solution provided by Christiano, Eichenbaum and Rebelo (2011). Given the paths of $\{\widehat{\pi}_{H,t}, \widehat{y}_t\}$ that solve this system with stochastic regime switches, it is then necessary to impose stability conditions on the real debt dynamics.

B) If the monetary policy is expected to be passive after the ZLB, the model is solved by the agents by taking $\{\widehat{\pi}_{H,t}, \widehat{b}^r_t\}$ as the two jump variables of the economy. After solving for $\{\widehat{\pi}_{H,t}, \widehat{b}^r_t\}$, we deduce the dynamics of the Home output gap followed by the Foreign variables.

First, we show how to solve the model in this case when the ZLB is not binding.[49] Introducing the Taylor rule in the debt dynamics, we obtain:

$$\begin{aligned}\widehat{b}^r_{t-1} &= \beta\widehat{b}^r_t + (1-\beta\alpha_\pi)\widehat{\pi}_{H,t} + (1-\beta)\widehat{d}^r_t - \beta\varepsilon^M_t \\ \sum_{i=0}^{\infty}\beta^i\widehat{\pi}_{H,t+i} &= \frac{1}{1-\beta\alpha_\pi}\left[\widehat{b}^r_{t-1} - (1-\beta)\sum_{i=0}^{\infty}\beta^i\widehat{d}^r_{t+i} + \beta\sum_{i=0}^{\infty}\beta^i\varepsilon^M_{t+i}\right], \end{aligned} \tag{107}$$

where the right-hand side provides the expected value of government revenues, while the left-hand side is the expected value of inflation. The expected value of inflation (left-hand side) is the solution of the inflation dynamic, which is given by:

$$\widehat{\pi}_{H,t+1} = \lambda_1\widehat{\pi}_{H,t} + \frac{\sigma^{-1}\kappa}{\beta\lambda_2}\varepsilon^M_t\sum_{j=0}^{\infty}\left(\frac{\rho_M}{\lambda_2}\right)^j = \lambda_1\widehat{\pi}_{H,t} + \frac{\sigma^{-1}\kappa}{\beta(\lambda_2-\rho_M)}\varepsilon^M_t,$$

where $|\lambda_1| < 1 < |\lambda_2|$ are the eigenvalues of A and using $\varepsilon_{t+1} = \rho_M\varepsilon_t$. The present value of the expected inflation is then given by:

$$L^{-j}\widehat{\pi}_{H,t} = \lambda_1^j\widehat{\pi}_{H,t} + \left(\lambda_1^{j-1} + \lambda_1^{j-2}L^{-1} + ... + \lambda_1 L^{-(j-2)} + L^{-(j-1)}\right)\frac{\sigma^{-1}\kappa}{\beta(\lambda_2-\rho_M)}\varepsilon^M_t$$

[48] These authors have based their analysis on the previous works of Eggertsson and Woodford (2003), Christiano (2004), and Eggertsson (2004)

[49] In the following, we always assume that $\alpha_y = 0$ without loss of generality, but with significant gains for the exposal simplicity.

Given that $L^{-i}\varepsilon_t = \rho_M \varepsilon_t$, the expected value of inflation is thus:

$$\sum_{i=0}^{\infty} \beta^i \widehat{\pi}_{H,t+i} = \sum_{i=0}^{\infty} \beta^i L^{-i} \widehat{\pi}_{H,t} \quad = \quad \frac{1}{1-\beta\lambda_1}\widehat{\pi}_{H,t} + \frac{\sigma^{-1}\kappa}{\beta(\lambda_2-\rho_M)}\frac{1}{1-\beta\lambda_1\rho_M}\varepsilon_t^M$$

Integrating this result in Equation (107), the equilibrium is defined by:

$$\begin{aligned}
& \frac{1}{1-\beta\lambda_1}\widehat{\pi}_{H,t} + \frac{\sigma^{-1}\kappa}{\beta(\lambda_2-\rho_M)}\frac{1}{1-\beta\lambda_1\rho_M}\varepsilon_t^M \\
= \quad & \frac{1}{1-\beta\alpha_\pi}\left[\widehat{b}_{t-1}^r - (1-\beta)\sum_{i=0}^{\infty}\beta^i \widehat{d}_{t+i}^r + \beta\sum_{i=0}^{\infty}\beta^i\varepsilon_{t+i}^M\right] \\
= \quad & \frac{1}{1-\beta\alpha_\pi}\left[\widehat{b}_{t-1}^r - \frac{1-\beta}{1-\beta L^{-1}}\widehat{d}_t^r + \frac{\beta}{1-\beta\rho_M}\varepsilon_t^M\right] \\
\widehat{\pi}_{H,t} \quad = \quad & \frac{1-\beta\lambda_1}{1-\beta\alpha_\pi}\left[\widehat{b}_{t-1}^r - \frac{1-\beta}{1-\beta L^{-1}}\widehat{d}_t^r\right] \\
& +(1-\beta\lambda_1)\left(\frac{\beta}{(1-\beta\alpha_\pi)(1-\beta\rho_M)} - \frac{\sigma^{-1}\kappa}{\beta(1-\beta\lambda_1\rho_M)(\lambda_2-\rho_M)}\right)\varepsilon_t^M
\end{aligned}$$

If the fiscal policy is active, as for instance,$\gamma = 0$, then $\widehat{d}_t^r = \varepsilon_t^F$ and thus:

$$\begin{aligned}
\widehat{\pi}_{H,t} \quad = \quad & \frac{1-\beta\lambda_1}{1-\beta\alpha_\pi}\left[\widehat{b}_{t-1}^r - \frac{1-\beta}{1-\beta\rho_F}\varepsilon_t^F\right] \\
& +(1-\beta\lambda_1)\left(\frac{\beta}{(1-\beta\alpha_\pi)(1-\beta\rho_M)} - \frac{\sigma^{-1}\kappa}{\beta(1-\beta\lambda_1\rho_M)(\lambda_2-\rho_M)}\right)\varepsilon_t^M
\end{aligned}$$

We deduce the equilibrium path of the output gap using the Phillips curve:

$$\widehat{y}_t \quad = \quad \frac{1}{\kappa}(1-\beta L^{-1})\widehat{\pi}_{H,t},$$

where $t_t = t_t^* = 0$ leading to $\widehat{s}_t = 0$, $\forall t$, for simplicity. The dynamics of Foreign aggregates y_t^* and $\widehat{\pi}_{F,t}^*$ are deduced from Equations (6) and $\widehat{\pi}_{F,t}^* \equiv \widehat{\pi}_{H,t}$, given that $t_t = t_t^* = 0$ implying $\widehat{s}_t = 0$, $\forall t$. Therefore, γ^* must be larger than $\beta^{-1} - 1$ (fiscal brake) in order to ensure the stability of the Foreign real debt, given the stationarity of the others aggregates $\{\widehat{y}, \widehat{y}_t^*, \widehat{\pi}_{H,t}, \widehat{\pi}_{F,t}^*, \widehat{b}_t^r\}$.

Now, assuming that the ZLB binds at the initial period. The debt dynamic is then:

$$
\begin{aligned}
\widehat{b}^r_{t-1} &= \beta\widetilde{b}^r_t + \widetilde{\pi}_{H,t} + (1-\beta)\widehat{d}^r_t \\
\widehat{b}^r_{t-1} &= \beta\left[\begin{array}{l} \beta p\widetilde{b}^r_{t+1} + \beta(1-p)\widehat{b}^r_{t+1} + p\widetilde{\pi}_{H,t+1} + (1-\beta\alpha_\pi)(1-p)\widehat{\pi}_{H,t+1} \\ +(1-\beta)p\widetilde{d}^r_{t+1} + (1-\beta)(1-p)\widehat{d}^r_{t+1} - (1-p)\beta\varepsilon^M_{t+1} \end{array}\right] \\
&\quad +\widetilde{\pi}_{H,t} + (1-\beta)\widehat{d}^r_t \\
&= \beta^2 p\widetilde{b}^r_{t+1} + \beta^2(1-p)\widehat{b}^r_{t+1} + \sum_{i=0}^{1}(\beta p)^i\widetilde{\pi}_{H,t+i} + (1-\beta)\sum_{i=0}^{1}(1-p)^i\widehat{d}^r_{t+i} \\
&\quad +(1-\beta)p\widetilde{d}^r_{t+1} + (1-p)\left[(1-\beta\alpha_\pi)\widehat{\pi}_{H,t+1} - \beta\varepsilon^M_{t+1}\right] \\
&= \beta^2 p\left[\begin{array}{l} \beta p\widetilde{b}^r_{t+2} + \beta(1-p)\widehat{b}^r_{t+2} + p\widetilde{\pi}_{H,t+2} + (1-\beta\alpha_\pi)(1-p)\widehat{\pi}_{H,t+2} \\ +(1-\beta)p\widetilde{d}^r_{t+2} + (1-\beta)(1-p)\widehat{d}^r_{t+2} - (1-p)\beta\varepsilon^M_{t+2} \end{array}\right] \\
&\quad +\beta^2(1-p)\left[\beta\widehat{b}^r_{t+2} + (1-\beta\alpha_\pi)\widehat{\pi}_{H,t+2} + (1-\beta)\widehat{d}^r_{t+2} - \beta\varepsilon^M_{t+2}\right] \\
&\quad +\sum_{i=0}^{1}(\beta p)^i\widetilde{\pi}_{H,t+i} + (1-\beta)\sum_{i=0}^{1}(1-p)^i\widehat{d}^r_{t+i} \\
&\quad +(1-\beta)p\widetilde{d}^r_{t+1} + (1-p)\left[(1-\beta\alpha_\pi)\widehat{\pi}_{H,t+1} - \beta\varepsilon^M_{t+1}\right] \\
&= \sum_{i=0}^{\infty}(\beta p)^i\widetilde{\pi}_{H,t+i} + (1-\beta)\beta p\sum_{i=0}^{\infty}(\beta p)^i\widetilde{d}_{t+1+i} + (1-\beta)\widehat{d}_{t+i} \\
&\quad +(1-p)\beta^2\sum_{i=0}^{\infty}(\beta p)^i\widehat{b}^r_{t+1+i}
\end{aligned}
$$

This gives the value of expected inflation:

$$
\sum_{i=0}^{\infty}(\beta p)^i\widetilde{\pi}_{H,t+i} = \widehat{b}^r_{t-1} - \frac{(1-\beta)\beta p}{1-\beta pL^{-1}}\widetilde{d}^r_{t+1} + (1-\beta)\widehat{d}^r_t - \frac{(1-p)\beta^2}{1-\beta pL^{-1}}\widehat{b}^r_{t+1}
$$

After the shock, the dynamics of inflation must also consider the stochastic end of the ZLB:

$$
\begin{aligned}
&\text{No ZLB:} && \widehat{\pi}_{H,t+2} - \frac{1+\beta+\sigma^{-1}\kappa}{\beta}\widehat{\pi}_{H,t+1} + \frac{1+\alpha_\pi\sigma^{-1}\kappa}{\beta}\widehat{\pi}_{H,t} = -\frac{\sigma^{-1}\kappa}{\beta}\varepsilon^M_t \\
&\text{with ZLB:} && p^2\widetilde{\pi}_{H,t+2} + (p(1-p) + (1-p)^2)\widehat{\pi}_{H,t+2} \\
& && -\frac{1+\beta+\sigma^{-1}\kappa}{\beta}p\widetilde{\pi}_{H,t+1} - \frac{1+\beta+\sigma^{-1}\kappa}{\beta}(1-p)\widehat{\pi}_{H,t+1} + \frac{1}{\beta}\widetilde{\pi}_{H,t} = -\frac{\sigma^{-1}\kappa}{\beta}\varepsilon^M_t \\
& \Leftrightarrow && \widetilde{\pi}_{H,t+2} - \frac{1+\beta+\sigma^{-1}\kappa}{\beta p}\widetilde{\pi}_{H,t+1} + \frac{1}{\beta p^2}\widetilde{\pi}_{H,t} \\
& && = \underbrace{\frac{1-p}{p^2}\left[\frac{1+\alpha_\pi\sigma^{-1}\kappa}{\beta}\widehat{\pi}_{H,t} + \frac{\sigma^{-1}\kappa}{\beta}\varepsilon^M_t\right]}_{x_t}
\end{aligned}
$$

The two roots of the second-order differential equation for inflation are $\lambda_i = \frac{1}{2}\frac{1}{p\beta}(a \pm \sqrt{a^2 - 4\beta})$ with $a = 1 + \beta + \sigma^{-1}$, with $|\lambda_1| < 1 < |\lambda_2|$, for $p \in]0.5; 1[$. Using $(L^{-1} - \lambda_1)(L^{-1} - \lambda_2)\widetilde{\pi}_t = x_t$, we deduce the solution of the inflation dynamic when the ZLB binds:

$$\widetilde{\pi}_{H,t+1} = \lambda_1 \widetilde{\pi}_{H,t} - \lambda_1 \beta p \sum_{i=0}^{\infty} \lambda_2^{-i} x_{t+i}$$

The expected value of inflation is then:

$$\sum_{i=0}^{\infty} (\beta p)^i \widetilde{\pi}_{H,t+i} = \frac{1}{1 - \beta p \lambda_1} \widetilde{\pi}_{H,t} - \frac{\lambda_1 \beta p}{1 - \beta p \lambda_1 L^{-1}} x_t$$

After equalizing the two expressions of the expected inflation, we obtain:

$$\frac{1}{1 - \beta p \lambda_1} \widetilde{\pi}_{H,t} - \frac{\lambda_1 \beta p}{1 - \beta p \lambda_1 L^{-1}} x_t \quad = \quad \widehat{b}^r_{t-1} - \frac{(1-\beta)\beta p}{1 - \beta p L^{-1}} \widetilde{d}^r_{t+1} + (1-\beta)\widehat{d}^r_t - \frac{(1-p)\beta^2}{1 - \beta p L^{-1}} \widehat{b}^r_{t+1}$$

When the fiscal policy is active, for instance, $\gamma = 0$, the equilibrium inflation is:

$$\widetilde{\pi}_{H,t} \quad = \quad (1 - \beta p \lambda_1) \left[\widehat{b}^r_{t-1} - \frac{(1-\beta)(2\beta p \rho - 1)}{1 - \beta p \rho} \varepsilon^F_t - \frac{(1-p)\beta^2}{1 - \beta p L^{-1}} \widehat{b}^r_{t+1} + \frac{\lambda_1 \beta p}{1 - \beta p \lambda_1 L^{-1}} x_t \right],$$

where the sign of the impact of a redistributive shock ε^F_t depends on the inequality $\beta p \rho \lesseqgtr 1/2$. As for the previous case, the equilibrium path of the output gap is deduced from the Phillips curve:

$$\widehat{y}_t \quad = \quad \frac{1}{\kappa}(1 - \beta L^{-1})\widehat{\pi}_{H,t},$$

where $t_t = t^*_t = 0$ leading to $\widehat{s}_t = 0$, $\forall t$, for simplicity. The dynamics of Foreign aggregates y^*_t and $\widehat{\pi}^*_{F,t}$ are deduced from Equations (6) and $\widehat{\pi}^*_{F,t} \equiv \widehat{\pi}_{H,t}$, given that $t_t = t^*_t = 0$ implying $\widehat{s}_t = 0$, $\forall t$.

The transactions for carbon pricing in Japan

Abstract

Although the climate change problem is a planet wide problem, we don't have the right type institution. Therefore, we have much difficulty to find a solution for the problem through political process, such as the negotiations among governments or the initiatives of international organizations. In Japan, there is further difficulty on the governments, where the chargeable organizations are not good integrated. First, the central government has two control towers, which are the Ministry of Environment and the Ministry of Economy, Trade, and Industry. Besides, from the perspectives of individual local governments, which have responsibilities to implement the environment administration, they don't have enough physical and personal resources and abilities.

In contrast, we can obtain the most efficient solution when we rely on the price mechanism by carbon pricing. However, we need arrangement of institutional frameworks by governments following the Law and Economics theorem to find the market-based solution. As for carbon pricing, the ideas of carbon tax or emission trade system were proposed as the framework.

We found fundamental challenge there, namely, the legal and accounting formalization of the transaction. In the case of emission trade system, we have additional challenges, such as the pattern of intermediary service for the transactions and how to keep the integrity of the market. EU-ETS has cleared the challenges with the core concept of 'the regulatory auction'. In addition, we can see the movement of the private initiatives to establish voluntary carbon markets, such as Task Force for Scaling Voluntary Carbon Market or London Stock Exchange, which also want to clear the challenges. In Japan, the situation is worse. It has a structural problem that liquidities are generally quite short, as we can see the poor metabolism of the enterprises and employments. Besides, people want to have the formal incorporation in legal and accounting institutes for de novo assets such as carbon credits. Under these circumstances, Japan has had mal tradition to depend too much on governments in establishing a new way of business.

There are other challenges for the market regulation. Generally, it is a premise for the price discovery function of a market that incentives of various participants, such as issuers, investors, intermediary firms, gate keepers and so on, are coordinated in an order. The global standard in overseas markets basically leave the coordination of incentives on economic rationalities of market participants, and would take an exceptional measure only after the real harmful effects had occurred. It should be in similar fashion for carbon pricing. However, in Japan, there is a strong opinion that the competent authority must prepare regulation to prevent the expected harm effects. The related authorities fear very much to be criticized. Therefore, the compartmentalized regulations become a hazard for the transaction of de novo assets like carbon credits. In addition, carbon pricing in Japan cannot escape from the critics such as 'The transactions of carbon credits are just money game.', 'The carbon offset statements based on the carbon credits purchased with money are just escapes from the obligation of public institutions of society.'

As the Japan's central government must demonstrate the leadership to achieve its own NDC, particularly antidoting the risks for private enterprises, it implemented the carbon market experiment, based on the supplementary budget for fiscal year 2022. However, this experiment did not achieve enough results to establish the marketplace for carbon pricing. The basic reason for the poor performances is the framework as a governmental budget project itself. Meanwhile, Tokyo Metropolitan Government and Saitama Prefecture are implementing their own carbon market experiments, but the transaction volumes do not have been enough to discover the fair carbon prices.

In the next stage, at the very beginning, we must improve the market structure reflecting the poor performances of the experiment in 2023, but it might be difficult for the governmental budget project. Therefore, private initiatives have keys to get successful carbon pricing in Japan. In the second place, we might utilize DLT (Distributed Ledger Technology). Utilization of security tokens for carbon credits bring us many economic advantages, and it might give us a breakthrough to reform the current rigid regulation system.

1 The character of climate change problem and the role of international public sectors

(1) The character of climate change problem and the role of individual government

① Roles of governments in the climate change problem

The climate change problem is the largest challenge for the current mankind. Although this is a planet wide problem, we mankind don't have the right type institution. We must find a solution for the problem through the international political process.

In this process, NDC (Nationally Determined Commitment) under the framework of joint communiqué such as Paris Agreement has been the clue. Japanese government has also made its own NDC, which is 'carbon neutral by 2050'. However, we must find common rules to solve the climate change problem via negotiations among individual governments, such as COP27 (United Nations Framework Convention on Climate Change). Things there does not have gone well, as we can see every year.

② Difficulty in the negotiations among governments

It is not easy for the negotiations among governments to adjust the differences in recognitions and measures regarding the global climate change problem.

First, there are obdurate opinions to deny the causal relationship of the GHG (Greenhouse effect Gas) emission such as CO_2 (Carbon Dioxide) by mankind with the planet wide climate change problem. Although it is a scientific theme if the GHG emission by mankind brings the problem, the industries which are currently emitting much GHG might not substantially change their negative viewpoints.

Second, there is no concrete consensus who must bear the burden to reduce GHG emissions. There are various opinions among individual countries including the one to claim that industrial countries must reduce their emissions to a larger extent, while developing countries which want to raise their people's life standards can commit the milder targets. Besides, in domestic opinions within an individual country, there is an assertion that rich people who already emitted much GHG must reduce their emissions with severity, while poor people would be damaged harder by the climate change problem. Regarding these differences of opinions, some insisted that solution must be based on the social fairness. However, it is quite difficult to find the objective and decisive answer to this kind of value judgement questions. [1]

Moreover, there is no standard concerning how to reduce the CO_2 emission. For example, forestry offset project types are preferred because they contribute to both of CO_2 reductions and development aids. We have another example of building nuclear power plants. They are encouraged by industry people because they do not emit CO_2 and relatively less expensive if we do not count the future social cost to decommit the reactors. However, both methodologies are criticized by many people including the academia from various reasons, such as ineffectiveness of CO_2 absorption, higher risk and so forth. There is also a country such as Germany denied the use of nuclear energy at all. It is not easy to adjust the difference regarding the methodologies to reduce CO_2 emission.

In addition, individual government has a dilemma concerning carbon leakage. This is a problem caused by the competitive disadvantage of the industry whose products are under heavier regulations on emission reduction than foreign potential competitors. The individual government wants to set its own measure to offset the problem. For example, European Union, which has a large market with many member countries, has established the rule to distinguish the green projects from brown ones, at the same time, has published the attitude to employ the CBAM (Carbon Border Adjustment Measure). [2]On the other hand, Japanese government, for example, committed its own NDC but has shown passive and ad hoc responses to the global carbon leakage problem.

Moreover, even if the negotiations went well, it would not be clear if the agreement would improve the welfare level of mankind.

---

1 See [1]
2 See [25]

③ Initiatives of international organizations

Under these circumstances, an international organization sometime takes an initiative to accelerate the coordination between an industry country and a developing country. CDM（Clean Development Mechanism）is an example where an emission reduction project was made through a cooperation between an industry country and a developing country can be permitted to issue a carbon credit. This scheme is based on the article 6 of the Paris Agreement. It can be recognized as an offset within their own NDCs, and allowed to be transacted among the related entities. However, they are not popular because of bureaucratic works of the international organization. For example, it takes around 2 years before the permission to issue a carbon credit. Although it is an understandable examination process to avoid slanders as green washing, it is criticized by industry people for lacking necessary flexibly as a private economic activity.

(2) Situation in Japan

① Central government

In Japan, there are further difficulty on the governments, where the chargeable organizations are not good integrated.

First, the central government has two control towers, which are the Ministry of Environment and the Ministry of Economy, Trade, and Industry. They have had quite different stances to the climate change problem. The Ministry of Environment has had the responsibility for the negotiations with other countries, and has been the competent authority for local governments to implement the environmental administration practically. On the other hand, the Ministry of Economy, Trade, and Industry has responsibility for overall industry policy of Japan, and in charge of an individual industry supervision just like the Ministry of Agriculture and Fishing, the Ministry of Land and Transport and so on. The ministries have many traditional wisdoms. For example, as for a public certificate organization for carbon credits, the management of organization is decided following the negotiation between the Ministry of Environment and the Ministry of Economy, Trade, and Industry, but the individual certification of a carbon credit should be approved by the competent ministry for the industry. This is a well-organized system in the government but quite sticky. Agile improvements of the certification system of carbon credits are very difficult in Japan.

The cabinet, partly aiming to remove the harmful effects of compartmentalized administration, established 'GX (Green Transformation) Implementation Conference' chaired by the Prime Minister supported by the Minister for Economy, Trade, and Industry and the Chief Secretary of Cabinet Office. However, the headquarter for this conference is not a permanently installed organization to integrate the whole central government, but just a secretariate for temporal conferences to examine the necessary measures to implement GX. Accordingly, 'The fundamental attitude for the promotion of GX' place the concept of 'Economic growth-oriented carbon pricing' as the mean to raise huge money necessary for the emission reduction investments under the cooperation of public and private sectors, and encourages the usage of nuclear power. Following the concept of 'Economic growth-oriented carbon pricing', the attitude decides to take measures towards three targets such as bold assistances to the investments with GX economy transition bond, initiatives for the early investment for GX by carbon pricing and the usage of new financial methods. However, the attitude will introduce 'the regulatory auction' only for the electricity industry from as late as 2033. Honestly speaking, this attitude is falling behind the global trend and inward facing.

② Local governments

Local governments implement the environmental administration guided by the Ministry of Environment. From the perspectives of local governments, they don't have enough physical and personal resources to carry the responsibilities to implement the whole environment administrations practically. For example, according to the 'Environment White Paper', 'Prefectures and oordinance designated cities have 6,441 persons for the public pollution matters (excluding industrial wastes and sewers) and 2,105 persons for the protection of ecosystems. 223 out of about 3ooo municipalities have their own divisions specialized for the environment administrations.' [3]

In addition, most of chargeable employees in local governments are interested in the local ecosystems or prevention of public pollution, not in the planet wide problem. Besides, there is a natural limit for those who are interested in the climate change problem to plan the marketplace for carbon pricing which might be beyond their chargeable borders.

3 See [4]

2 The climate change problem and price mechanism

(1) Carbon pricing

① The function of price mechanism

As written above, the climate change problem is very difficult for the public sectors to find a solution through the political process. In contrast, carbon pricing, which relies on the price mechanism, can provide the more efficient solutions because the price information work as a common signal for all entities in the economic society.

First, we can obtain the decisive answer based on market voting regarding the causal relationship of GHG emissions by mankind with the climate change problem. Besides, we can find the right combination of burdens and methods to reduce GHG emission. Moreover, according to the 'Coase Theorem', which is the orthodox of the Law and Economics, externality caused by a public pollution, such as the climate change problem caused by GHG emission, could be solved optimally through internalization of the external cost into the market mechanism among the related entities.

However, the Law and Economics also requires the government to develop the institutional framework of the market.

As for carbon pricing, the idea of carbon tax or emission trade system were proposed as such institutional developments. There are many challenges in the proposals.

First, enterprises might regard carbon pricing just as the increase of cost, which bring about the additional credit risk of their businesses.[4] Therefore, the industry enterprises, particularly those who emit much CO2 currently, might outwardly make advertisements of their efforts to reduce CO2 emissions but inwardly do not want to develop the institutional framework for carbon pricing. The governments are under the political pressure by those enterprises, so that they do not have much incentives to develop the framework proactively.[5]

Second, the chargeable authority must decide the elements of the framework proposals such as carbon tax rate or the penalties to emission regulation as the premise of carbon pricing. In the past system plannings such as the land holding tax or regulation on the industrial wastes, the government relied on the existing similar systems such as the financial products taxation or the penalties for the other violations of the industrial regulation. The relationship between system planning and transactions are just like 'egg and chicken'. We have similar challenges concerning the subjects of carbon taxations or emission regulations.[6]

To prevent the political pressure and obtain the fundamental information to design the institutional frameworks, we should depend on the transactions of the assets underpinned with carbon pricing.

② Fundamental challenges to carbon pricing

In the current institution, we found fundamental challenge. It is the legal and accounting formalization of the transactions of de novo asset.

In the case of emission trade system[7], we have additional challenges such as the pattern of intermediary service for the transactions of assets and how to keep the integrity of the market.[8]

---

[4] See [19]

[5] According to [29], many of carbon taxes or emission trade systems were introduced as the financial resources for subsidies to emission reduction investments. Besides, the beginning of EU-ETS was started with gratis allocations to the heavy industries in Europe.

[6] The policy planner needs such substantial information. However, in financial markets, carbon prices were not reflected in the price of the existing financial instruments such as stocks and bonds. See [12]

[7] The proposal of carbon tax has its own additional challenges starting from the implement cost of taxation bureaus.

[8] The definition of 'carbon pricing' is the exchange rate between assets underpinned with carbon emission and currency. This theme has fundamental questions to be resolved, such as how to consider the relationships with standard financial instruments like deposits, stocks, bonds and so on, or the relationships with metabolisms of issuers, most of them are enterprises. However, in this article, I leave from these fundamental questions, and employ the definitions of carbon credit prices, following the usual definitions by private initiatives. See [12]

EU-ETS has cleared these fundamental challenges by the core concept of 'the regulatory auction'. It employed the structure of a cap & trade market backed by the coercive regulations of member countries, the transactions were bit and ordered over the counter trade of member banks at the beginning (the introduction of the regulated exchange such as European Energy Exchange to list carbon credits was made only recently), and EU commission by itself has issued the emission allowances of the regulations by member countries as the subjects of the transactions. Based on the premises, both of spots of European emission allowances and their futures are dealt in EU-ETS which become a part of global activities of European commercial banks. [9]

In addition, some of international organizations are claiming that the floor of the carbon price only for major countries such as China. This proposal aims more reduction of CO2 emission based on the existing carbon pricing, but be supposed to put emphasis on the international political process. We can see the considerations on the equity among countries in the assertion. [10]

On the other hand, there are private initiatives to establish the voluntary carbon markets.[11] For example, Task Force for Scaling Voluntary Carbon Market is a private initiative represented by Mark Caney, the former governor of the Bank of England, supported by Mackinsey & Company. It has published the first report in 2021, which describes the elements and future growth of the voluntary carbon market, and published the second report in 2022, which sketched the governance structure to clear the fundamental challenges to the market.[12]Besides, London Stock Exchange has proposed in 2022, to establish the voluntary carbon market where the equities of a fund which consist of the various emission reduction projects of enterprises are transacted.[13]

（2） The function of price mechanism in Japan

In Japan, there is a structural problem that liquidities are generally quite short, as we can see the lack of metabolism in the enterprises and employments. We have an example for this in the financial market., deposit taking institutions like commercial banks have larger weight in the monetary intermediation compared to other western countries. According to the statistics by Bank of Japan, where the share of deposits in total financial intermediation between households and enterprises are more than 200% in Japan, in contrast to 50% of the USA, 90% of Europe. In contrast, the weight of market in the monetary intermediation is quite small.

Besides, we also find the overall tendency in the background to reject the economic activities signalized by prices. For example, the concept 'regulatory auction' is totally denied in Japan's institutions, as we can see in the allocation system of electric waves to telephone companies or boarding gates of an airport to airlines.

Moreover, de novo assets such as carbon credits need the incorporation in the legal and accounting institutes.[14] This

---

[9] See [33],[37],[38]

[10] See [39]

[11] See [27]

[12] See [41]

[13] See [31]

[14] According to my idea, the relationship between real transactions and the authorization concerning de novo assets is just like in 'chicken and egg' situation. Indeed, for example of cash, we use them to settle our transactions every day, without any decisive legal and accounting formalizations. In the similar manner, we need not the decisive formalizations for de novo assets. On the contrary, the demand for the formalizations might be harmful. For example, the establishing law has the formalization only for tangible goods not for intangible services. It might be a hazard for the growth of service industry in Japan. But reality in Japan.is exactly opposite. People want to have the formal incorporation in legal and accounting institutes for de novo assets. As for carbon credits, they do not have legal framework as an asset. It is not a formal asset without the public certification based on Paris agreement, in contrast to the tangible assets like office equipment, so that the purchase of carbon credits is not recorded as investment but as donation. That means the investor must pay corporate tax before the purchase of carbon credits, in contrast to other tangible assets.

is common challenge in the world, but especially in Japan people want to have such an authorization in the formal institutes surely. Accordingly, as for the transaction of them, people prefer bit and offer orders via regulated exchanges to OTC (Over the counter trade) because it is formalized in the written act.

Under these circumstances, Japan has had mal tradition to depend too much on governments, which is not so proactive nowadays as earlier, in establishing a new way of business. I am afraid that Japan's economic society would follow what became a standard in oversees market in the transactions of de novo assets such as carbon credits.

3 Market regulation

(1) Price discovery based on the transactions of assets

① Incentives of interested parties

There is another challenge for the establishment of marketplace.[15]

Generally, it is a premise for the price discovery function of a market that incentives of various participants who で deal assets are coordinated in an order. Those participants in contention are issuers, investors, intermediary firms (exchanges, security houses and so on), gatekeepers (standard setters, validators, accountants and so on). The combination of the incentives should be transformed spontaneously based on economic rationalities of participants, just like the business model for the market of asset backed securities.

It should be in similar fashion for carbon pricing. Assumed participants are similar too, namely, issuers, investors, intermediary firms, gate keepers and so on.

First, issuers are interested in profitability of their investments for emission reduction, and fundraising for their investments. The former depends on the future carbon price, and the relationship of profitability and future price is just like the 'chicken and egg' situation. However, this kind of uncertainty has been always accompanied with issuance of stocks, the analysis based on the existing prices is normal procedure for potential issuers as we see in internal carbon prices of many enterprises. The latter has a timing difference between requirement of the future achievement of emission reduction and the current fund raising for the investment. However, this difference is also usual for the issuance of stocks, can be mitigated by the ingenuity of financial intermediation firms, such as structured finance or insurance on the ex-post achievement of the investments.

Second, broad range of investors should be involved for the transactions. It would be effective for this purpose to remove the range limitation of the investors and to strengthen the incentives to invest in carbon credits by the establishment of the disclosure duties about carbon emission of the investors. The international efforts are made nowadays, particularly Stage3 of GHG protocol is expected to have much effect to incentivize the broad investors.

Besides, the ingenuity of financial intermediary firms is indispensable, such as the set-up of the financial instruments for investments or providing reliable information regarding their future prices. [16]

Moreover, standard setters and the validators should obtain the trust of investors on meeting the requirements for carbon credits, such as additionality, effectiveness, carbon leakage, harmlessness, base-line, and monitoring. As most of standard setters are based on the strictness of their standards, they are supposed to have enough incentives to satisfy the condition as the part of the transaction system. [17]The validators are still not independent from the other entities, but to be expanded from now on.

② Market regulations by overseas authorities

Public sectors in overseas markets basically leave the coordination of incentives on economic rationalities of market participants, and would take exceptional measures to prevent the real harmful effects only when they had occurred.

For example, in the case of the global financial crisis caused by 'Rehman Shock', the measures to strengthen the credibility of central counter parties such as regulated exchanges to shut down the contagions of systemic risk. This measure was presented in the governmental conference such as Philadelphia Summit, and come to the detailed agreement by BIS (Bank for International Settlement) and IOSCO (International Organization of Securities Commissions).[18] In the case of domestic financial crisis such as Great Depression in the USA, American congress has

[15] The discussion in this chapter matters only for the emission trade market, but does not relate with carbon tax.

[16] The overseas carbon markets had already developed this kind of financial services. Many analyses were made regarding the profitability of the services concerning carbon credits transactions. See [18]

[17] In Indonesia or Vietnam there are the movements to establish their own certificate organizations. See [42]

[18] See [10]

decided the measures such as the establishment of SEC (Securities Exchange Commission). In both cases, they did not have denied the incentives combination of market participants.[19]

As for carbon pricing, the regulation on transaction subjects or price information are not the topic for the regulators, but the institutional arrangements on information disclosure to construct the market mechanism or the institutional adjustments for security token are important topics for global system development.[20]

(2) Situation in Japan

① The compartmentalized regulations

On the other hand, in Japan, there is a strong public opinion that the effective regulation should be prepared for the expected harm effects. [21]It is because the competent authority would be severely criticized by mass media when they were not enough prepared the preventions for the harmful effects. The concerned ministries might buck the bug onto each other, when the harm effects really occurred. In this situation, it could happen that the potential competent authority suppresses the transactions of de novo assets. This problem could be same in financial instruments and let the Japanese market fall behind the innovative markets where the technical progress always intensifies the competition among the markets.

As for the regulation on the asset transactions in Japan, the financial assets and other assets are strictly divided. Financial assets are regulated by Financial Service Agency, which is the competent authority of FIEA (Financial Instruments and Exchange Act). Other assets are regulated by other ministries starting from the Ministry of Economics, Trade, and Industry, which oversee CIEA (Commodity Instruments and Exchange Act).

The compartmentalized regulation system is based on the histories of the concerned institutions, such as FIEA comes from the history of the security scandals 100years ago in the USA and CIEA comes from the history of the operator scandals 50years ago in this country. As carbon credits are de novo assets, they have no history.

This way, compartmentalized regulations on existing assets, with a strong public opinion that the effective regulation should be prepared for the expected harm effects, might disturb the transactions of de novo assets like carbon credits.

As the Ministry of Environment has been weak in Japan's central government, it has no power to overcome the hazard by compartmentalized regulations. It was established in 2001, has had the mission for Japan's response to planet wide problems like climate change, so that is has made severe compromises on various issues such as the certification system for carbon credits or guidance to the local governments regarding their implementation of environment administration. Although the Japanese government has published the NDC 'carbon Neutral by 2050' nowadays, it does not have reformed the execution system. Japan's government, for the purpose of relaxation of rigid regulations, established 'Regulatory Sand-Box' system. However, it does not have worked good for the marketplace, where participants freeze before they make necessary investments to introduce the new transaction systems.

② The critics on the transaction

Besides, carbon pricing in Japan cannot escape from the critics such as 'The transactions of carbon credits are just a money game.', 'The carbon offset statements based on the carbon credits purchased with money are just escape from the obligation of public institutions of society.'[22]

---

[19] See [34]

[20] It was similar in Japan during the Edo period, so that the market mechanism like 'Dojima Komeichiba' has developed as the top runner in the world. See [2]

[21] See [26]

[22] Japan's FIEA enumerate financial assets to be regulated by the Act from title securities listed in general institutions such as commercial law, and gives them special legal effects and regulations. Under this act, the market infrastructures are regulated quite strictly. For example, the range of business of financial instruments exchanges are limited to the financial assets and the businesses of them are controlled in every detail. Besides, the transactions of legally designated securities are protected by several legal means such as the surveillance by the SESC (Security Exchange Surveillance

4 Evaluation of governmental market experiments and the future improvement direction in Japan

(1) Evaluation of governmental market experiments in Japan

As Japan's government has made an international commitment, NDC of 'carbon neutral by 2050), it must do something. It wanted to demonstrate the leadership of the public sector, so that it antidotes the risks for private enterprises, and encourages the private entities by showing concrete proposals. The carbon market experiment by the central government was the measure to achieve this purpose, based on the supplementary budget for fiscal year 2022.

However, this experiment did not achieve enough results to contribute to the establishment of marketplace for carbon pricing. First, the price information discovered in the market diversified very much depending the methodologies employed. For example, the price of carbon credits by energy saving is 1600~ 800 yen per reduction of CO2 tons, in contrast to the prices of credits by forestry, which were 10,000~16,000 yen per reduction of CO2 tons. No one would invest in carbon credits with such diversified prices. Market liquidity in the experiment was short, too. This experiment was implemented for 85 business days, but the number of transactions made in the market was only 163. As in other markets, the number of market transactions are normally tens of thousands, the market experiment hardly provided the transactions of carbon credits. Besides, most of the transactions, namely 88%, were made by public entities, which already had issued carbon credits by themselves. [23]

There are many factors for the poor performances of the market experiment. First, the transaction subjects in this experiment were limited to public carbon credits certified by the governmental organization such as J-Credit managed by Mizuho Research & Technology. It did not include the voluntary carbon credits or futures of spot carbon credits. Second, the ways of bit and orders were limited to the regulated exchange, namely Tokyo Stock Exchange, which won the bid of the experiment by the government. It showed a sharp contrast to the EU-ETS, which started the transactions via OTC (Over the counter) orders. In Japan also, financial products except spots of common stocks are traded mainly via OTC orders. They have dominance in the real markets normally, as we can see the broad financial instruments except

---

Committee), assurance on the correctness of disclosed information of issuers by the audit farms, secured transformations of rights by the Japan Securities Deposit Center. On the other hand, FIEA does not limit the range of business of financial instruments operators (security houses), in contrast to Banking Act, but regulates the detailed transactions that they supplied based on criminal penalties.

In contrast, the CIEA does not provide such a closed system, but prohibits the operators of asset transactions to solicitate without invitation from their clients as the means to hold the integrity of the market.

Originally, FIEA was the compound of the Japan's Security Exchange Act and the Fund Exchange Act. The later imitated the former at that time, which was transplanted from the American federal law. The Japan's Security Exchange Act was based on a fundamentally different legal institutions to American. For example, in USA, company laws were set by the states, implementation of the regulations does not depend on criminal penalties. Therefore, FIEA brings many obstacles to the Japan's financial market. For example, it excessively strengthens the compliance of the market participants, and it makes the legal change and the implementation of the regulation excessively rigid.

Besides, the original Security Exchange Act was based on out of dated financial technology in the USA, so that it is deviated from the current financial transactions. For example, the current FIEA charged the detailed and strict regulations on regulated exchanges, such as the regulation on the margin for transactions or financial structures of regulated exchanges, because authorities wanted to shut down the contagion of the market systemic risk among the participants. However, it was assumed that regulated exchanges had the advantages related with the lower technical standard. At that time, hub &spoke transaction systems provided by regulated exchanges had more economical rational than OTC trade. Recently in most transactions of new financial assets, OTC trade has overwhelmingly large share in the market. On the other hand, current regulated exchanges are private enterprises. They depend on non-financial elements of member issuers such as the reputations to recruit the employees, or 'freeze' of market participants to innovate the financial system. Therefore, the same implementation of FIEA as before would bring about the harm effect on the new business of regulated exchanges. Regarding this point, we should also note the basis law to supervise regulated exchange in Europe is Banking Act, in contrast to Japan's FIEA.

[23] See [13]

spots of stocks. Third, devises for the transactions were limited to the choice of Tokyo Stock Exchange as the implementer of the experiment, while there are various methodologies to reduce CO2 emission. Structured finance instruments by the primary market operators, such as the one proposed by London Stock Exchange half year before the experiment, was excluded from the beginning. From the perspectives of investors, those creativities by financial service operators were absolutely needed. [24]

In the background, the framework of the market experiment as a governmental budget project itself had the problem. As this project is a formal governmental business, the Ministry of Economy, Trade, and Industry as a competent authority must ask Tokyo Stock Exchange as a major regulated exchange that was proven to have managed secondary market business. All new attempts like voluntary carbon credits or futures of spot carbon credits, OTC orders, structured finance instruments were excluded from the beginning. They had to follow the ancient regime since 2001.We could not expect the good performances of the market experiment by the central government with the governmental budget.

Meanwhile, Tokyo Metropolitan Government and Saitama Prefecture are implementing their own carbon market experiments. However, the transaction volumes do not have been enough to discover carbon price. This is because that they have originally limitations as written in 1 ⑵ ② and that Tokyo has its own additional difficulty to have few fabrics to emit large volume of CO2 which were regulated by the Ministry of Environment.[25]

(2) The future improvement direction in Japan

At the very beginning of next stage, we must improve the market structure reflecting the poor performances of the last experiment in 2023. Japan's market has little time to compete with overseas markets in the era of innovating information technology and provide effective solution to the planet wide problem. From the viewpoints of financial instrument industry, the future market should have positive participation of primary market operators by providing the structured finance instruments based on the pool of various new technologies for emission reductions. Besides, from the viewpoints of investors, their incentives should be strengthened by deregulation of market participation and broader disclosure standard like scope3 of GHG protocol. Moreover, not only transactions in the regulated exchange but also OTC orders should be introduced. These amendments might be difficult to include in the governmental budget business. [26]Therefore, private initiatives have keys to get successful carbon pricing in Japan. [27]

In the second place, we might consider to utilize DLT (Distributed Ledger Technology) and to make security tokens of carbon credits. Utilization of security tokens for carbon credits bring us many economic advantages, such as round the clock transactions, practically instantaneous settlements, transparency for emission reductions and market prices, cost economics, interoperability and so on. [28]The problems pointed out like becoming triggers to involve green washing are not unique for tokenization but already dealt by standard setters. [29]

Besides, tokenization might give us a breakthrough to reform the current institution. As for carbon credits, which do not have legal categorization yet, the utilization of new technology such as DLT might bring the first step for the

---

[24] As for the evaluation of carbon credits, there an influential opinion that an all-in-one information of issuing firm might be preferred to every information of an individual project. See [30]

[25] See [2] and [14]

[26] The constraints written in (1) of this chapter might be unavoidable to prevent the political risk concerning the budget project under the ancient regime.

[27] See [32]

[28] These advantages are not limited to the transactions to utilize information technology of distributed lagers, but also appliable for all electronic transactions and payments using open network as general. DLT would promote the pros of economic commerce on open network to the extreme, such as the best price competition among suppliers who can rely on crowd computing thoroughly, the best possibility for the selection by demander who can compare the cost price without care about the information transparencies, and the best competition beyond the industries and borders among countries.

[29] In overseas markets, we already found the concrete proposals by financial industry to use DLT for carbon credits and the countermeasures by standard setters. See [28], [34] and [42].

formalization, and the relaxation of compartmentalized regulation.[30]

But at the same time, it is noteworthy for the government to play their roles flexibly realization of the advantages. For example, in Japan, deposits have an overwhelmingly large share in financial instruments, which were strictly distinguished from capital market instruments such as stocks and bonds, and from other general title documents. On the other hand, we should provide cross-sectional incentives to broad range of investors to invest in carbon credits. Regarding this situation in Japan, the government might consider to involve the scope3 of GHG protocol in the regulation for deposit taking institutions. Most of commercial banks are listed companies, so that they are already obliged to disclose the total emission following the scope3. The regulation would not charge additional costs to them. However, if the emission disclosure would be involved in the regulation of all deposit taking institutions, it would give strong incentives to the client companies. Most of those companies are unlisted, so that they don't have any interest to disclose their CO_2 emission volume.[31]

Except this, the government should leave the emission reduction to the market, and other regulations such as Act on Promotion of Global Warming Countermeasures should not be strengthened. In this country, where enterprises cannot do anything what is not written in the law, the government should make this attitude clear in advance.

---

[30] Japan's FIEA is very liberal in the principle, but in the practice, the intervenes by the government is quite dense. For example, in the secondary market, every detail of the business of regulated exchanges was written in their business method manuals, and all changes of their manuals must be permitted by the chargeable officer in advance. The balance of pros and cons to be market infrastructures were maintained so far by the exceptional legal effects for the infrastructure services by those entities. The utilization of DLT might resolve all linkages between exceptional legal effects and dense supervisions.

[31] As most of commercial banks are listed companies, the scope 3 of GHG protocol should be applied to the banks and their clients. However, it might lead to the shift of client borrowers to financial cooperations, which are not listed companies. Therefore, it would be desirable to include the obligations by Banking Act to disclose the information following the scope 3 of GHG protocol.

References

<Japanese>

１. Jusen, Asuka `Green New Deal' (Iwanami-Shinsho, June 2021)

2. Toshihide Arimura 'Current situation and Outlook' of Carbon Pricing -Ex-post Examination of Emission Trade and the possibility in Japan' (Kannkyo Kagaku Zassi 35 (1):1-9, September 2021)

３. Takashi Iida ' Rule and private organizations in markets: trial : An essay on market governance' (Bank of Japan Institute for Monetary Economics Study, July 2021)

4. Ministry of Environment ' White paper on environment, Social sustainability, and Biodiversity'(2023 et)

5. Task Force on Climate-related Financial Disclosures, 'Final Report Recommendations of TCFD, June 2017)

6. Account Standards Board of Japan ' Practical Treatments for current handling of accounting for the emission trade' (Final Revise, 23.June 2009)

7. Nobuyuki Kinoshita

'Kessai kara Kinyuu wo Kangaeru' (Kinyuuzaiseijijoukennkyuukai, April 2015)
'Digital Innovation to Kinyuu System' (Kinyuuzaiseijijoukennkyuukai, August 2020)

8. Ministry of Economy, Trade and Industry

'Interim Summing up'
(Study group regarding the economic solutions or the global realization of carbon neutral' (August,2021)
'Carbon Credit Report' (August,2022)
'An overview of CBAM (Carbon Border Adjustment Measure) and the relationship with WTO'S rule' (March2023)

9. Coase、R.H. 'Enterprises・Market・Law' Japanese Version 'Kigyou・Sijou・Hou' (Translated by Kenichi Miyazawa, Akira Goto, Yosihumi Hujigaki, Toyo Keizai Sinpousha, October 1992)

10. BIS (Bank for International Settlement)& IOSCO (International Organization of Securities Commissions

'Principles for Financial Market Infrastructures'
(The Committee on Payment and Settlement Systems (CPSS) and the International Organization of Securities Commissions, April 2012)

11．The Cabinet Secretariate Office　‘GX implementation Conference’ (June 2022~)

12．Bank of Japan

‘Comparison of Fund cycle in Japan, USA and Europe’ (August 2023)
’The second survey on the market function regarding the climate change’ (June 2023)

13．Japan Exchange Group, ‘the results of the carbon credits market experiment’

(a presentation on the study group regarding the economic solutions or the global realization of carbon neutral’ (March,2023)

14．Masayo Wakabayashi, Wataru Kimura

‘The evaluation on the emission trade market experiment of the Tokyo metropolitan government’ (Denryokukenkyu 65, April 2018)

15．Parchomofsky, Gideon,　Stein, Alex　‘Torts, innovation, and growth’

（10th Chapter of ‘Rules for Growth’

English version: (Compiled by Litan, R.E)
Japanese version　‘Seicho Senryakuron’ (Translated by Nobuyuki Kinoshita, Hirohiko Nakahara, Atsuto Suzuki, NTT Shuppan, February 2016）

<English>

16．Achara V,　Pedersen L.H.

‘Economics with Market Liquidity Risk’（Critical Finance Review, 8（1-2）,111－125,2019）

17．Akerlof G.

‘The Market for Lemons: Quality Uncertainty and the Market Mechanism.’
（Quarterly Journal of Economics 84 (3), 488-500. 1970）

18．Bauer D.Michael, Huber Daniel, Rudebush Glenn,Wilms Ole

‘Where is the Carbon Premium? Global Performance of Green and Brown Stocks’
(Hutchins Center Working Paper#83, January 2023)

19. Bouchet Vincent, Theo Le Guendal,
‘Carbon Risk Sensitivity to Carbon Price’　(Available at SSRN, April 2020)

20. Brown R. James, Martinsson Gustav, Petersen C. Bruce
‘Law, Stock Markets, and Innovation’ (The Journal of Finance, Vol. 68, No. 4, August 2013)

21. Bolton Patrick, Reichelstein Stefan, Kacperczyk Marcin, Leuz Christian, Ormazabal Gaizka, Schoenmaker Dirk
'Mandatory Corporate Carbon Disclosure and to the Path to Net Zero'
（CEPR Policy Insight No. 111.04　October 2021）

22. Benedikt Downar, Jürgen Ernstberger, Stefan Reichelstein,
Sebastian Schwenen & Aleksandar Zaklan
'The Impact of Carbon Disclosure Mandates on Emissions and Financial Operating Performance'
(Review of Accounting Studies 26, Pages1137–1175 (2021))

23. Si Chen, Marbouh Dounia, Moore Sherwood, Stern Kris
'Voluntary Carbon Offsets: An Empirical Market Study' (Open-Source Strategies Inc., December2021)

24．'Eden A., Unger C., Acworth W., Wilkening K., Haug C.
'Benefits of Emissions Trading' (International Carbon Action Partnership, Berlin, July 2016)

25. EU technical expert group on sustainable finance
'Taxonomy: Final Report of the Expert Group on Sustainable Finance'
(Europe Union, March 2020)

26. Hideki Kanda, Cartis Milhaupt
'Re-examining Legal Transplants: The Director's Fiduciary Duty in Japanese Corporate Law'
(Columbia Law, March 2003)

27．The Integrity Council for the Voluntary Carbon Market (IC-VCM)
'Global Benchmark for High-integrity Carbon Credits Aims to mobilize Climate Finance at
Speed and Scale （July 2023）
'The Core Carbon Principle'（July 2022）

28．Franke Laura, Schletz Marco, Salomo Soeren
'Designing a Blockchain Model for the Paris Agreement's Carbon Market Mechanism'
(Sustainability, February 2020)

29.　Lisenmeier Manuel, Mohommad Adil, Schwerhoff Gregor
'Policy Sequencing Toward Carbon Pricing: Empirical Evidence from G20 Economies and other Emitters'　(IMF Working Paper/22/66, April 2022)

30. Mendelsohn Robert O., Litan Robert E., Fleming John
'A Framework to ensure that Voluntary Carbon Markets will truly help combat Climate Change'
(Brooking Institution, September 2021)

31. London Stock Exchange
'The Voluntary Carbon Market'　（October 2022）

32．International Organization of Securities Commissions

‘Working Paper on Voluntary Carbon Credit Market’ (November 2022)

33．Ibikunle, G., Gregorio, A., G.F.Hoepner,A., Rhodes. M,

‘Liquidity and Market Efficiency in the World's Largest Carbon Market’

(The British Accounting Review， Vol.48, Issue 4, 431-447, December 2016)

34. ISDA （International Swaps and Derivatives Association）

‘Role of Derivatives in Carbon Markets’ (September 2021)
‘Legal Implications of Voluntary Carbon Credits’ (December 2021)

35．Kinoshita, N.

’The Future of Japan’s Financial Market’
(Columbia University, Academic Common, WP 372. December 2019)

36．McNish Tyler

‘Carbon Offsets are a Bridge too far in the tradable Property Rights Evolution’

(Harvard Environmental Law Review, Vol. 36, No. 1, 2012）

37．Milunovich G.

‘Market Efficiency and Price Discovery in the EU Carbon Market’
(Applied Financial Economics, Vol. 20, No. 10, 2010)

38. Mizrach, B., Otsubo, Y.
‘The Market Microstructure of the European Climate Exchange’
(Journal of Banking & Finance, Vol. 39, 107-116, February 2014)

39．Parry Ian, Black Simon, Roaf James

‘Proposals for an International Carbon Price among Large Emitters’
(IMF Staff Climate News, June 2021)

40．Ross Samantha

‘The Role of Accounting and Auditing in Addressing Climate Change’

（The Center for American Progress, March 2021）

41. The Taskforce on Scaling Voluntary Carbon Markets
' The Phase I Report'　(January 2021）

’The Phase II Report'（July 2021)

42. World Bank
‘State and Trends of Carbon Pricing 2023’
‘Country Perspectives: Opportunities and Challenges for International Voluntary Carbon Markets in the Context of the Paris Agreement’

Model Aggregation for Improving Forecasts of Canada's Core Inflation Rate

Alexander Murray[1], **Sandi Alakhras**[1], **Josh Brault**[1], **Stenio Fernandes**[1], **Pierre Siklos**[2]

[1] Bank of Canada
[2] Wilfrid Laurier University and Balsillie School of International Affairs

Abstract

We present a model aggregation approach for forecasting Canadian core inflation that is based on a collection of heterogenous forecasting sub-models from a wide variety of sources. These models range in type from structural, and semi-structural, to time series models (single and multiple equations). Each sub-model was chosen for its predictive power and uniqueness relative to the others in the aggregate model. The sub-models' forecasts are combined linearly with time-varying weights that are estimated using ridge regression. Canadian data are used to train and evaluate both the sub-models and the aggregate model. It is found that the aggregate model has greater predictive power than any of the individual models over the evaluation data. One policy implication is that central banks can improve inflation performance by combining models as opposed to combining individual point forecasts.

1. Introduction

Macroeconomic models serve as a key decision-making input for policy makers in conducting forecasting and economic analysis exercises. As former Bank of Canada Deputy-Governor, John Murray, pointed out Murray (2012), the first of five stages in the central bank's preparations for setting the policy rate at its Governing Council meetings is the presentation of staff projections. Models that serve as the basis for forecasts used by central banks have, over the years, become more complex and varied, spurred on by a succession of financial and economic crises experienced by the world economy since 2008. The last two decades have encouraged modelers to relax more assumptions embedded in their models, generated by the widening gap between the realities of what drive inflation and economic activity more generally, and the capacity of existing models to accommodate behaviour unaccounted for by these models. Moreover, older models made it even more difficult to accurately forecast the outlook in part because the forces that drive inflation (e.g., aggregate demand versus aggregate supply) have also changed over time. Unsurprisingly, much of the focus since 2021 has been on how the most recent surge in inflation (both core and headline) did not show up in the outlook provided by most projection models in the quarters leading up

Alexander Murray: amurray@bank-banque-canada.ca; Sandi Alakhras: SAlakhras@bank-banque-canada.ca; Josh Brault: JBrault@bank-banque-canada.ca; Stenio Fernandes: SFernandes@bank-banque-canada.ca; Pierre Siklos: psiklos@wlu.ca
We thank Paul Corrigan, Tolga Ozden, and Tony Chernis for their many helpful comments and feedback, and Karsten Gerdrup (Norges Bank), Reuben Jacob (Reserve Bank of New Zealand), May Rostom (Bank of England), Ingvar Strid (Riksbank), Shunichi Yoneyama (Bank of Japan), Mariana Garcia-Schmidt (Bank of Chile), and Guillermo Escudé (Central Bank of Argentina) for research assistance.

to the global rise inflation. Kryvtsov et al. (2023), relying on Canadian data, conclude that the shocks experienced after 2019 were exceptional and likely triggered by the pandemic. Blanco et al. (2022) focus on inflation surges in history in a data set covering over 50 countries and find these episodes were almost wholly unanticipated. Moreover, its effects dissipate slowly over time (i.e., 3 to 4 years). Eggertsson and Kohn (2023) consider the US experience and also argue that pandemic related and geopolitical shocks were critical ingredients in the sharp rise in inflation beginning in 2021. However, they also add that certain features of the Fed's modified monetary policy strategy delayed the eventual tightening of its stance thereby exacerbating the inflation control problem. Finally, Lane (2022) exhaustively documents the sources of the inflation surge in the eurozone and, as many others have reported, finds multiple culprits that translated into large relative price shocks (e.g., war and pandemic related).

While the Bank of Canada's remit aims for 2 percent in headline CPI inflation central banks are especially interested in core inflation because, as former Governor of the US Federal Reserve, Frederic Mishkin pointed out "…focusing on core inflation can help a central bank from responding too strongly to transitory movements in inflation." Mishkin (2007). This does not detract from the Bank of Canada's responsibility to discuss or emphasize headline inflation in its public communication.

Despite shifts away from treating the financial sector as a veil in models, allowing for the existence of heterogeneous economic agents, relaxing assumptions related to flexibility in price setting behaviour, or treating expectations as being formed rationally Gauss and Gibbs (2018), the complex and interconnected nature of most economies renders prediction, even under normal conditions, an exceptionally challenging task. The inherent difficulty of attempting to predict human decision-making necessitates that assumptions and simplifications are made to suit a particular context or purpose for which the model is designed. For example, in economies where the export of commodities plays a large role, there will typically be more focus in the model on the dynamics surrounding the extraction and value of those commodities. Those dynamics may be completely absent from a model designed for an economy where, for example, the financial sector plays a much larger role.

Globalization adds another layer of model complexity. In response to this complexity one approach is for models to become even more realistic so that as many of the potential sources of shocks are accounted for. However, decision-makers understandably also prize tractability. Generally, they also prefer models that are clear in how their results are derived. This clarity aids not only in policy discussions, but also helps in the communication of policy decisions. While voluminous research finds that judgmental forecasts are hard to beat Ang et al. (2007); Faust and Wright (2013), models offer decision-makers clearer views about the economic forces that drive the outlook. Ideally, the profession should strive to develop models that can provide both superior forecasts and assist with the narrative that guides how the stance of monetary policy is set. Unfortunately, the resulting model is likely to be "…unwieldy and not cost effective." (op.cit., p.23) Faust and Wright (2013). Instead, this paper proposes to harness the predictive power of a large collection of models as a means of generating superior forecasts via model aggregation, in a manner to be defined below, and it provides empirical support for this strategy in the Canadian context. Our focus is on core inflation. A by-product of our approach is that it can offer some insights into links between model elements and forecast performance over time. Moreover, since the aggregate model consists of several heterogeneous models, this presents central banks with an opportunity to communicate to the wider public its awareness that improved forecasts of inflation requires richness in its modelling strategy. However, we leave these extensions for future research.

While model averaging is not unheard of within economics, it is used infrequently Bajari et al. (2015). When it is used, a relatively homogenous set of models is often chosen. The main contribution of this paper is a novel approach to forecasting Canadian inflation, which is based on Dynamic Model Averaging (DMA) using a heterogenous set of sub-models. It is also demonstrated that the aggregate model consistently produces better out-of-sample (OOS) inflation forecasts for Canada and that model heterogeneity is critical to obtaining this conclusion.

The rest of the paper is organized as follows. The next section explains the role and potential for model aggregation and a brief overview of the relevant literature. It attempts to draw succinctly from different approaches to obtaining forecasts. Section 3 briefly describes the universe of models, both structural, semi-structural, and of the time series variety. Their sources, properties and a few illustrations are provided in this section, as well as details about the clustering approach used to arrive at the representative set of models used by the aggregate models. The section concludes by briefly describing the final set of models used in the model aggregation exercise. Section 4 describes model aggregation methodologies. Section 5 discusses the main findings while Section 6 concludes with a summary, policy implications, and suggests avenues for future research.

2. Model Aggregation and Model Forecasts

2.1. Model Aggregation: Key Considerations

Models have a role as a communication device.[1] Hence, there is an advantage in relying on simplicity as a modelling strategy. Of course, there is a trade-off with realism that usually translates into greater complexity. The latter feature might be useful in improving the quality of forecasts. However, model aggregation does not typically seek to find the optimal level along this trade-off. Instead, a large set of existing models of various sizes and complexity is taken as a given and combined to generate improved inflation forecasts. Model aggregation, therefore, should be seen as a form of hedging, that is, as a device to check against benchmark forecasts or individual models that are consulted routinely.

One issue that can be raised is that some models are intended for purely forecasting purposes while others provide projections.[2] Generally, models are constructed to generate predictions, that is, some indication of the economic outlook. The assumptions and the modalities underlying the construction of models intended for a variety of uses are, of course, different. However, all have in common the need to be forward-looking, an essential ingredient in the conduct of monetary policy. Some research has also examined whether central bank projections can influence private sector (viz., professional) forecasts.[3] Population, energy consumption, and climate change effects, in addition to standard economic variables such as inflation and real economic growth, have all been the subject of this line of research.

Although there have been many recent advances in our understanding of the patterns inherent in deriving aggregate economic performance Slobodyan and Wouters (2012), much work remains to be done. Indeed, most forecasting models used by central banks begin with a theoretical rationale inspired by two sets of theoretical developments over the past few decades. The simplest expression of the New Keynesian (NK) approach is built around three equations. Clarida et al. (1999) provides an early survey, while Woodford (2003) provides the seminal theoretical foundation. Paralleling this development are real business cycle models which add micro-foundations and optimizing behaviour assuming rational agents (e.g., see Goodfriend and King (1997)). These advances would inspire the specification of Dynamic Stochastic General Equilibrium (DSGE) models that represent the workhorse models used by central banks for simulation and policy analysis. The DSGE methodology has evolved greatly to incorporate a variety of economic frictions and loosen some of the strong assumptions made in early vintages of these models.[4]

Given the wide variety of econometric forecasting methodologies, some central banks have

[1] Indeed, (García-Schmidt et al., 2020) advocates that the conduct of monetary policy should consist of a model of the central bank's likely future behaviour. Burgess et al. (2013) makes the case for models constrained by rules-like behaviour and such that, despite the many challenges surrounding them, they are easier to communicate.

[2] Projections here refer to scenarios that are viewed as being "central" or "baseline", but often supplemented with judgment. Forecasts, in turn, are meant to provide of what is expected to happen in future. It should be noted that there is a long tradition in the literature in evaluating projections by treating them as if they are forecasts. Inter alia, see Morris and Shin (2005); Smith (1987); Doan et al. (1984); Jain and Sutherland (2018) for illustrations where the two concepts related to prediction are often blended.

[3] This idea stems from Morris and Shin (2005) seminal piece about how greater central bank transparency about the outlook impacts the private sector's desire to generate its own independent judgment about the future.

[4] A challenge with DSGE models is validating them by comparing its impulse response functions with those of a vector autoregression (VAR). While the details, and other criticisms levelled at DSGE models are beyond the scope of this paper (see, however, Morris and Shin (2005) and Smith (1987)) our approach is agnostic on the subject since both kinds of models are included in the universe of models considered.

begun investigating and implementing model aggregation.[5] in order to achieve superior forecasts Araujo and Gaglianone (2023); Chakraborty and Joseph (2017b). Model aggregation is a modelling approach dating back to at least the 1970's (Leamer, 1978) and can take a number of different forms.[6] In it's most basic form, an aggregate model consists of a linear combination of the output of each sub-model with some weight on each sub-model output (Barutçuoglu and Alpaydin, 2003; Prudencio and Ludermir, 2006; Bajari et al., 2015).[7] The principal benefit of model aggregation is that the advantages of varying modelling approaches can be harnessed jointly such that better predictive power is achieved relative to any individual model. An obvious analogy is the oft reported finding that forecast averaging usually outperforms individual forecasts over time (Timmermann, 2006). Of course, the simplest solution is to give each model equal weight. However, as explained in Section 5, this strategy does not necessarily generate the best forecast as the optimal weight allocation for each model may vary over time as different models could provide better value for decision making depending on the state of the economy at the time the forecasts are generated.

While each sub-model serves as an abstraction of the economy, when deployed in combination, they may create a new meta-model that is closer to reality than any of them individually. The primary disadvantage of model aggregation is its complexity. Each sub-model must be individually estimated, and thus there is potentially a much higher computational burden in comparison with a single large model. However, it can be the case that the design or estimation of a single large model is simply intractable, or that there exists a model forecast which relies on confidential or qualitative data[8] In such cases, model aggregation can serve improve upon each of the individual forecasts.

In addition to improving predictive power, some aggregate models can help policy makers to observe which set of models performs best over time. This can be done by examining the weights placed on the individual forecasts made in an aggregated model in order to observe which models are most important in the aggregation. However, any analysis of the aggregated model output essentially requires analysis of each of the included models that contribute most to improving forecasts. This element of aggregation provides an additional benefit since multiple perspectives on the same phenomenon can be provided. For example, one sub-model may imply, say, that variable A is the primary driver of inflation, while another sub-model finds variable B to be the most important factor. The weights in the aggregate model would provide a notion of relative importance between the two perspectives. Therefore, instead of running a horserace to determine which model generates the best forecast, the relative contribution of all available models can be exploited to achieve the desired objective, namely obtaining a superior inflation forecast. This strategy is also motivated by the knowledge that the 'best' model will change over time.

The ability of model aggregation techniques to balance the biases of each of the sub-models to provide a less-biased forecast is well-documented in a variety of contexts (Barutçuoglu and Alpaydin, 2003; Prudencio and Ludermir, 2006; Bajari et al., 2015; Timmermann, 2006). However, in economic modelling, the scale and heterogeneity of the aggregation exercise is typically rather limited. For example, Check and Piger (2021) employs the Bayesian model averaging (BMA) method on a collection of auto-regressive models to form their forecast model. In contrast, the approach presented in this paper initially draws upon a set of 300 models. These include the current models available at several central banks (Corrigan et al., 2021; Gervais and Gosselin, 2014; Hirakata et al., 2019; García-Schmidt et al., 2020; Burgess et al., 2013), and the Macro Model Database (Wieland, 2023). Of this set of 300 models, a smaller set of representative models is selected using a clustering-based approach. Each of the models is algorithmically labelled according to the presence (or lack thereof) of the following features, namely linearity; occasionally binding constraints (e.g., zero or effective lower bound; ELB); existence of a steady state; and the assumption of rational expectations.

As discussed below, this results in 16 possible model classifications. Other model features could

[5]In this paper, model aggregation refers to the construction of a meta-model. That is, a model of models, which is the structure defining the composition of multiple other models.

[6]For example, the use of Bagging in ensemble learning can be seen as a type of model aggregation

[7]This approach is sometimes referred to as akin to "thick modelling" (Granger and Jeon, 2004).

[8]Such as the forecast given by a professional forecaster, which may take concepts like politics, climate, and geography into account which may be difficult to incorporate into an economic model.

have been chosen (e.g., homogeneous or heterogeneous agents, incorporating a finance or housing sector, and so on). However, the number of possible model classifications increases rapidly as the number of classifications rises. Of the 16 classifications, only 6 are non-empty. One representative model is selected from each of the non-empty classifications to construct the set of models to aggregate. Once the models are selected, a dynamic model averaging approach is applied as described in detail in Section 4. DMA constructs the aggregate model as a linear combination of the forecasts of each of the sub-models.[9] The weights on the sub-model forecasts at each time step are allowed to vary, albeit with a penalty parameter on changes between time steps. This is done primarily to allow for regime changes, such as introducing inflation targeting, or other institutional changes, for which one model may be better suited over another. Once forecasts from each sub-model and the aggregate model have been generated, a variety of metrics can be deployed to evaluate predictive power.

While it is shown in Section 5 that model aggregation can improve inflation forecasts, the use of time-varying weights in particular is found to produce the best forecasts. This result stands in contrast with a frequently cited result from the point forecast combination literature (e.g. Timmermann (2006)), where the simple average of point forecasts is often found to outperform individual forecasts. Another advantage of time-varying weights is that it can serve as a new monitoring tool. Indeed, there is the potential for this approach to provide a signal to consumers of forecasts about when to rely more heavily on one type or set of models over others when judging the most plausible forecast. Finally, it should be noted that many models currently employed by central banks and academics to derive inferences from shocks to the economy, or generate forecasts, share a high degree of similarity (e.g., see Binder et al. (2019)). The deliberate reliance on model heterogeneity may also be a useful device to mitigate the impact of groupthink in generating estimates for the economic outlook and especially inflation. That said, in the present context, there are significant data related challenges, as we shall see, when aiming for model heterogeneity when useful models used elsewhere (i.e., outside Canada) are also adapted to the Canadian environment in the interests of increasing the heterogeneity of the universe of available models.

2.2. Forecast Evaluation and Properties

Two strategies are often adopted in forecast evaluation.[10] A researcher can consider how well the model performs in-sample. A more stringent test, as far as policy makers, financial markets, and the public are concerned, is how well a model performs out-of-sample (OOS). Typically, only part of the available data is used for model estimation so that some can be reserved for forecast evaluation.[11] Although both types of tests are provided, OOS forecast evaluation is the most demanding metric that dictates the net benefits of the proposed model aggregation exercise employed in our study.

Forecasting exercises must also confront methodological issues. While purely judgment or sentiment-based predictions sometimes play a role, central bank forecasts are typically primarily based on structural models that are consistent with some underlying economic theory. The current standard at most central banks relies on one (or more) Dynamic Stochastic General Equilibrium

[9] Hauzenberger et al. (2023) also adopt this strategy as well as considering non-linear shrinkage techniques which can, under certain conditions, out-perform linear approaches for US inflation data. However, their application relies on a flexible state space models that, while able to encompass a wide range of models, cannot match the heterogeneity we exploit with our methodology. Model aggregation is relatively less common than the aggregation of time series via techniques such as principal components or factor models have proved very popular recently. Indeed, for the US, Low and Meghir (2017) have developed and regularly update a large data set (https://research.stlouisfed.org/econ/mccracken/fred-databases/) used to extract common factors.

[10] The focus here is on the forecasting process alone. There are separate questions related to the plausible economic interpretation of, say, coefficient estimates of a model. In what follows we are not concerned with this step in the modelling strategy or, rather, we assume that all models considered pass such a test.

[11] A complication that arises is that some of the series in any model can be revised over time. Ideally, one would prefer to use real time data to carry out forecast evaluation. Given the size and complexity of many candidate models in this study, this is not feasible. It may be noted that inflation data does not exhibit significant revisions over time. The same is not true, however, of output or output gaps which are also present in many, but not all, forecasting models considered. As a result, a parallel development is the nowcasting exercise that utilize data available before official data are released. There may be a subjective element in such approaches, but this is also outside the scope of this study.

(DSGE) models built from micro-foundations of individual utility maximizing behaviour. Policy makers are assumed to be credible and subject to a policy constraint (e.g., an inflation objective). A simple example of theory guiding model structure is given by the example where a tightening of monetary policy, via higher policy rates, is expected to reduce inflation over time, usually within 2 years. Expectations are typically also assumed to be anchored which facilitates the reduction in inflation following the introduction of a more hawkish monetary policy (however, see below).

In general, DSGE models are derived from the so-called New Keynesian paradigm which also incorporates elements from the neoclassical synthesis. There is insufficient space to go into the details about the inspiration for such models (however, see e.g.Goodfriend and King (1997); Woodford (2003); Romer (1993); Del Negro and Schorfheide (2013); Christiano et al. (2018)). Suffice it to say that since the seminal works of Smets and Wouters (2007) and Christiano et al. (2018) (also see Dotsey et al. (2011)) the level of sophistication and complexity of these models have grown considerably. The Global Financial Crisis (GFC) led to models that incorporate financial shocks, often via the inclusion of a banking sector, and other financial frictions. Later vintages of DSGE models admit the possibility that agents are heterogeneous instead of relying on a representative agent (e.g., some consumers are more patient than others) and there is growing acceptance of less than rational consumers, at least relative to the rational expectations benchmark.[12] The DSGE approach continues to evolve as modellers seek to incorporate more frictions or forms of behaviour that appear to have significant macroeconomic consequences but were previously not accounted for. Finally, in the case of small open economies such as Canada's, a further distinction is between DSGE models of a closed economy versus open economy models that admit the effects of external shocks.[13]

A potential risk, however, is that parallel developments in DSGE modeling risk reducing model heterogeneity. As Dorich et al. (2017) made clear, model heterogeneity is essential in generating net benefits from model aggregation exercises aimed at improving economic forecasts. While there is no consensus definition of what constitutes sufficient heterogeneity, examples of heterogeneous models are ones that incorporate a housing sector versus ones that do not, models that incorporate a significant role for an important driver of economic activity such as an energy or resource sectors, or ones that admit behaviourally motivated expectations. Other variants can also be imagined.

Next to structural models that are frequently used by central banks and in academic assessments of forecasting performance are semi-structural models. As the name implies, these models retain some basic theoretical precepts (e.g., higher policy rates will eventually reduce inflation, there exists persistence in inflation movements) but relax some of the restrictions incorporated into structural models. As a result, many such models end up being reduced forms where cause and effect is more difficult to establish than in, say, DSGE models. Indeed, identification assumptions are required to estimate the structural parameters (e.g., see Ramey (2016) for a survey). Several identification methods exist, and new ones are always being proposed. However, since our concern in this study is to determine whether any model type, when combined, can improve forecasts, these concerns are secondary. That said, semi-structural models have the limitation that, as Low and Meghir (2017) point out, they are unable to "...define how outcomes relate to preferences and to relevant factors in the economic environment, identifying mechanisms that determine outcomes." (op.cit. p. 33).

A large class of models can be placed under the semi-structural umbrella. Generally, they range from the univariate variety, where the variable of interest, such as inflation, is forecasted exclusively from its own history to the multivariate kind where economic theory guides the forecaster's choice of series that interact with each other econometrically. Models of this variety include autoregressive integrated moving average (ARIMA) models, or some variant (e.g., see Enders (2015)), or vector autoregressive (VAR) and vector error-correction (VECM) models.[14]

The profession has been aware for some time of the challenges to forecasting created when

[12] Often referred to as Full Information Rational Expectations (FIRE).

[13] Many DSGE models are semi-structural so that their attributes are derived exclusively from economic theory combined with features that are obtained from a time series analysis of the data.

[14] In the case of VARs, it should be noted that many variations that have been proposed since Sims (1980) introduced this methodology. Previous surveys of VARs include Hyndman and Khandakar (2008); Chen et al. (2012); Akiba et al. (2019).

shocks are historically large.[15] For example, Del Negro and Schorfheide (2013) evaluates how DSGE type models perform when subjected to a shock such as the financial crisis in 2008-9 since referred to as the Global or Great Financial Crisis (GFC).

In addition to DSGE and statistical time series forecasting models, machine learning (ML) models have established themselves as strong competitors to the classical methods in the domain of timeseries forecasting Ahmed et al. (2010). ML models leverage their ability to learn complex patterns and relationships in the data without relying on a predetermined equation. In the case of time series forecasting, whether in the univariate or multivariate setting, the forecasting problem is typically approached as a supervised regression problem where the data are re-arranged in an input-output form. In order to transform the data in this manner, most implementations follow a time-delay embedding technique that encodes the autoregressive structure of the time series by utilising the lagged values of the target variable (or with the addition of other variables in the case of multivariate series) along with other possible temporal feature engineered embeddings Joseph (2022).

At least two other issues are worth noting when considering the OOS forecasting properties of models. First, whereas it is often the case that a single metric, typically RMSE, is used to determine the superior forecast (e.g., as in Binder et al. (2019)), and we follow this approach for the most part, other metrics are also available.[16]

A second issue worth highlighting is that forecasters are expected to generate forecasts over different horizons. For example, the frequently used convention is that a change in the stance of monetary policy usually is expected to take anywhere from 4 to 6 quarters to complete.[17] Nevertheless, most observers are also keenly interested in forecasts at shorter horizons. This may be particularly true when significant revisions to forecasts are likely. Alternatively, longer horizons might be of interest when multiple shocks imply that the time span to a return to some equilibrium or desired level of inflation is longer than the 8 quarters ahead convention. There exists a forecast horizon beyond which predictions are not more informative than some mean value for the series of interest (e.g., see Galbraith and Tkacz (2007)). In the empirical results reported below we consider several horizons ranging from the one quarter ahead up to 16 quarters ahead horizons.

If model aggregation is one solution to the problem of improving inflation forecasts one must still confront how the set of models being considered are aggregated. A few strategies are available. Granger and Jeon (2004) propose the thick modelling strategy wherein all estimated specifications are retained and the one chosen relies on a metric (e.g., fit or RMSE). Indeed, the theory of forecast combination clearly demonstrates the net benefits of utilizing many forecasts in part because it is highly unlikely that the same forecaster will produce the best forecast over time and for all forecast horizons and time periods. While the use of equally weighted and carefully selected models has been shown to work well Granger and Jeon (2004),[18] it is shown in Section 5 this is not the case when the models are not carefully selected and that time-varying weights ultimately has the best overall RMSE OOS regardless of the set of models included in the aggregate model.[19] It is also debatable how much heterogeneity is admitted when relying on thick modelling. That said, bootstrapping methods can be employed to generate confidence intervals for thick models. McAdam and McNelis (2005) is one application to US, euro area and Japanese data that uses thick modelling to deliver relatively superior inflation forecasts.

---

[15]There is no formal definition of the distinction between a large and a small shock. Nevertheless, one can classify shocks according to their size and use percentiles or the number of standard deviations from the mean to define the size of a shock.

[16]These include: mean absolute prediction error (MAPE), mean absolute error (MAE), and the maximum informative forecast horizon (MIFH), to name the better-known ones. Since forecasts are evaluated the record over a considerable span of time, relying on multiple metrics potentially alerts us to the various sources of forecast errors while, for example, large forecast errors tend to favour using the RMSE. Some metrics (e.g., MAE) are more sensitive to outliers while others (viz. namely MAPE) are less informative when forecasts and outturns are small numbers. Finally, forecast evaluations can also account for the possibility that consumers of forecasts can assign different weights to positive versus negative forecast errors.

[17]See, for example, https://www.bankofcanada.ca/2021/04/understanding-how-monetary-policy-works/.

[18]It is possible that this simple strategy succeeds due to its inherent hedging of the weight allocated to each sub-model OOS.

[19]Although it should be added that Timmermann (2006) also consider alternatives such as weights obtained, for example, via regression methods.

One method of constructing a set of "best" models is through the use of model confidence sets. Model confidence sets (MCS) are reminiscent of the usual statistical problem of constructing a confidence interval. The investigator does not need to know the true model but the approach of Hansen et al. (2011) allows one to leave out models that are uninformative. Bootstrapping is used to reduce the dimensionality of the problem when the number of candidate models is too large. One of the applications considered by Hansen et al. (2011) is the problem of forecasting inflation and an advantage of the technique is that it does not require a benchmark model which aids in reducing the so-called 'data-snooping' problem (e.g., see White (2000)). Hence, MCS is a model selection technique. However, the MCS method potentially shares the same drawback as thick modelling because it limits model heterogeneity. On the other hand, the MCS approach can deal with, for example, models specified in both levels and first differences. Hansen et al. (2011) use MCS to investigate the performance of the Phillips curve but finds that factor models or standard forecast combination methods can generate better forecasts.

The approach adopted in this paper is an unsupervised machine learning technique. A clustering method is also employed to select the smallest set of models that provides a certain degree of heterogeneity. Chakraborty and Joseph (2017a), and references therein, offers a more complete description of clustering techniques while additional details are also provided below. Perricone (2018) is an application to inflation, although the focus is on AR-type models only.

3. Model Heterogeneity: A Summary of Model Types

The models considered below capture the essence of model heterogeneity used to illustrate the potential for model aggregation to improve inflation forecasts. Table 1 provides a summary of model types that are candidates for aggregation. The classification is a simplified one to fix ideas. Since the universe of models considered in this study consists of approximately 300 models it is impractical to list them all here.[20] In the case of structural (i.e., DSGE type) models shown the aim is to highlight that models considered are generally ones used by central banks in Canada and elsewhere. In the case of non-Canadian models, we replace the original data with Canadian data that best matches the variables developed by central banks outside of Canada. The remaining models are time series models that are specified and estimated using a wide variety of methodologies and assumptions about the number of variables considered and the degree of endogeneity and exogeneity of the series included.

A practical issue arises because different models can potentially rely on different information sets. However, this does not invalidate the aggregation methodology.[21] For example, two VARs that are estimated for different lag lengths can be compared even if the effective degrees of freedom are different. Similarly, two VARs with different dimensions (i.e., the number of variables included) can be compared because, a priori, we cannot know whether a selection or all the variables in both models significantly improve forecast performance. Finally, even if a model contains more variables or equations does not imply that every one of its components significantly contribute to generating better forecasts at all horizons. The statistical and machine learning models are listed according to whether they are univariate or multivariate specifications.[22]

In what follows we provide a few essential details about the class of models beginning with DSGE type models which are representative of the class of structural models. This is then followed by brief summaries of select semi-structural and time series models. This class of modelling is well known. Hence, the narratives are very brief.

[20]Provision will be made to provide all necessary data related details and other materials.

[21]Indeed, using diverse information sets among the individual forecasts typically leads to improved aggregate model forecasts due to higher forecast heterogeneity. Furthermore, forecast aggregation may be the only option when it is infeasible to construct a single model that incorporates all of the information sets Timmermann (2006).

[22]Note that both statistical and machine learning models are sometimes referred to as "time series models" when time series data is given as input.

Table 1. Examples of Model Types Used to Generate Projections

Structural (source)	Semi-Structural	Univariate Statistical/ML	Multivariate Statistical/ML
ToTEM (Canada)	FRB-US (USA)	ARIMA	VAR
COMPASS (UK)	LENS (Canada)	UCSV	SVAR
TNT (Chile)	ECB-BASE (Europe)	Regression	TVP-VAR
NZSIM (New Zealand)	Q-JEM (Japan)	Markov Switching	VARMA
MAJA (Sweden)	MARTIN (Australia)	Moving Average	Factor Model
World3 (Club of Rome)		N-BEATS	XGBoost
SAMBA (Brazil)		Auto-regressive	
ARGEM (Argentina)			

Note: ToTEM is Terms of Trade Economic Model; COMPASS is Central Organizing Model for Projection Analysis and Scenario Simulation; TNT is Tradeable and Non-Tradeable Sectors; NZSIM is New Zealand Structural Inflation Model; MAJA is Modell för Allmän Jämvikts Analys (Model for General equilibrium Analysis); ARGEM is Argentina is Argentina Economic Model; SAMBA (Stochastic Analytical Models with a Bayesian Approach); LENS is Large Empirical and Semi-Structural Model; ECB-BASE is the European Central Bank Multi-country Semi-Structural Model; FRB-US is Federal Reserve Board-United States; Q-JEM is quarterly Japan Economic Model; Martin is Macroeconomic Relationships for Targeting Inflation; IMPACT is International Model for Projecting activity. Source refers to the country whose central bank developed the model in question.

3.1. Selected Examples of Structural Models

To conserve space, we briefly describe the individual forecasting performance of a selection of models covering the model types discussed above. OOS forecast performance is evaluated for horizons that range from one to 16 quarters ahead. We begin with a brief graphical description of forecast performance followed by more formal performance metrics below. It is also worth reminding readers that we are interested in forecasts of core inflation.

ToTEM The terms of trade economic model (ToTEM) is a large-scale open economy DSGE model of the Canadian economy. One of its distinctive properties is that it features significantly more firm- and household-level disaggregation than well-known DSGE models, such as those of Christiano et al. (2005) and Smets and Wouters (2007). On the firm side, the model features five distinct sectors producing final goods for consumption, residential investment, business investment, government spending and non- commodity exports. The model also includes a separate commodity-producing sector. This structure helps the model capture the composition of Canadian gross domestic product (GDP), which is important to accurately evaluate monetary policy frameworks that target the level or growth rate of nominal GDP or incorporate some role for the output gap. The firms responsible for producing final goods face nominal rigidities when setting their prices. More specifically, in a given final-good-producing sector, some of the firms re-optimize their prices in a forward-looking but staggered fashion, as in the literature following Smith (2017), while the other firms set their prices using a rule of thumb (RoT) similar to that in Hyndman and Khandakar (2008). Estimations of the sector-specific shares for each of these two pricing types find that the share of RoT price setters is relatively high in some sectors during the time since the introduction of inflation-targeting regime in Canada.

The household block of ToTEM features three prominent household types differing in terms of the financial markets they have access to and their status as savers or borrowers in those markets. On the saver side, the model follows Chen and Guestrin (2016) and Akiba et al. (2019) in assuming that some savers are "restricted" (they can access only long- term debt markets) while others are "unrestricted" (they have access to short- and long-term debt markets). As a result of these two saver types, ToTEM allows short- and long-term interest rates to influence aggregate household spending in distinct ways. Taken together, the two saver types—restricted and unrestricted—account for roughly half of all households in the economy. A single borrower type accounts for most of the remaining households in the economy. Borrowers in ToTEM finance part of their spending using long-term loans secured from saver households. When doing this, borrowers are assumed to face a collateral constraint under which new loans must be backed by some combination of new housing investment and home equity.

ToTEM follows most of the DSGE literature in assuming that workers enjoy some degree of wage-setting power but are subject to nominal rigidities similar to those faced by price setters. Lastly, monetary policy in ToTEM follows a simple rule under which the interest rate is set as a linear function of the previous period's interest rate, the output gap and the deviation of expected inflation over the next four quarters from the central bank's inflation target. ToTEM is estimated by Bayesian methods in a multi-step process. First, the variables are de-trended. Next, the rest-of-world block of the model is estimated. The estimated parameters of the international part of the model are then fed into the domestic part of the model wherein the rest of the parameters are estimated. Like many such models, ToTEM has evolved over time and continues to do so.

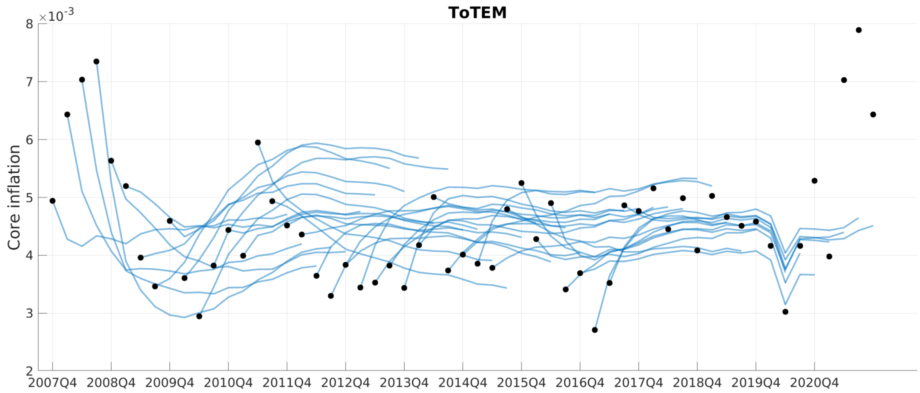


Figure 1. The 16 step ahead forecasts for ToTEM at each quarter from 2007Q4 to 2017Q4.

Figure 1 displays OOS forecasts for ToTEM included in the model aggregation exercise carried out below. The dots are the observed core inflation and each line plots 16 step ahead forecasts using data up to and including the point where it begins. Overall, the forecasts generated by ToTEM appear reasonable, although they display a tendency to overshoot. The wide undershooting of the model at the end of the sample is also clearly visible.

LENS The large empirical and semi-structural (LENS) model is as its name suggests, a semi-structural model driven more by the empirical properties of the data than economic theory. Like ToTEM, it is built to model the Canadian economy. However, while ToTEM is entirely inspired by economic theory, LENS is instead composed of reduced-form equations that are partly informed by theory but not fully micro-founded. As a result, the coefficients in LENS are less likely to be invariant to policy, making it less suitable for counterfactual analyses. An advantage of LENS is that its reduced-form structure makes the model easier to update as it can be estimated by blocks of equations rather than as a full system.

Long-run trends are also part of LENS and respond endogenously to shocks, meaning no ex-ante data detrending is required. LENS has a rich model structure which can be used for forecasting at a disaggregated level. For example, the model provides an explicit decomposition of the exports forecast into non-commodity, energy, and non-energy commodity exports, each of which depends on different foreign demand variables and relevant exchange rate measures.

The LENS forecasts frequently predict a sharp increase at short horizons followed by a gradual decrease, despite the fact that this behaviour seems to be rarely observed in the data.

COMPASS is the main organizing device used by the Monetary Policy Committee of the Bank of England for analyzing, forecasting, and conducting counterfactual experiments. Compared to many other DSGE models employed by central banks, COMPASS is relatively small in terms of

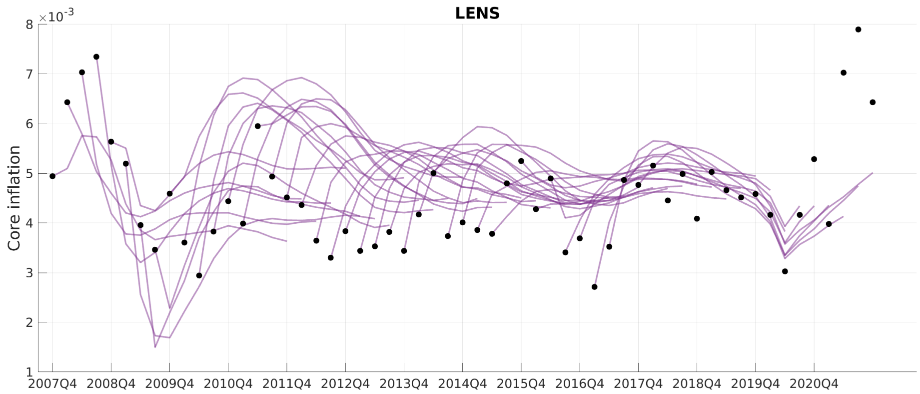


Figure 2. The 16 step ahead forecasts for LENS at each quarter from 2007Q4 to 2017Q4.

number of variables. However, its limited size is by design and COMPASS is complemented by a suite of over 50 other models which are used to assess its outputs and provide valuable input on missing channels. Examples of missing channels include: financial disruptions, energy price shocks, unconventional monetary policy actions, and fiscal policy that is more realistic than the model's highly simplified treatment.

COMPASS is characterized by the interactions among households, firms, the fiscal and monetary authorities, and a rest-of-world block, subject to 18 different types of exogenous shocks. Fiscal policy follows an exogenous AR(1) process and monetary policy is conducted according to a Taylor rule. There are two types of households referred to as unconstrained and RoT consumers. Unconstrained households operate in a utility maximizing way by choosing consumption (both current and future) and savings, subject to their respective budget constraints and external shocks. RoT consumers do not have access to savings and consume their entire income each period. Production in COMPASS does not distinguish between different sectors. Many of the steady state features of the model are calibrated according to national accounts data, while the remaining parameters are estimated using Bayesian methods. The estimation procedure uses 15 observables, primarily consisting of the aggregate expenditure components, prices and wages, and proxies for activity and prices in the rest of the world. According to COMPASS, domestic markup and monetary policy shocks have accounted for significant inflationary pressures after 2010, with imported price shocks playing a more prominent role prior to 2010.

The fact that COMPASS resides at the center of a larger network of models provides several benefits. First, because of its limited size, the model can be re-estimated frequently as new data arrives or are revised. Second, the flexibility of the approach allows for judgement and external forecasts to be included in the model. Third, the network of models can challenge and inform COMPASS outputs. However, the flexibility of this approach can also present some challenges, especially when large gaps between COMPASS's forecasts and policy prescriptions and ones generated by its extended network of models.

Keeping in mind that the forecasts shown in Figure 3 are based on adapting COMPASS to Canadian data, the resulting forecasts in the early portion of the sample, that is, until 2011, show a decline in core inflation at first before rising again. This pattern is reversed during the remaining years of the sample. This may explain why one of its sister models may be needed to produce more accurate forecasts. Note, however, that forecasts are overestimates the year before the pandemic strikes and then, in parallel with the preceding two models, the model is unable to capture the surge in inflation.

TNT Like many DSGE models, the TNT model of the Central Bank of Chile is characterized by an economy with households, firms, monetary and fiscal policy, and a rest-of-world block. However, TNT features much more heterogeneity on the firm side, distinguishing between firms producing in commodities, imports, exports, and non-tradeables. The difference in approaches comes from the

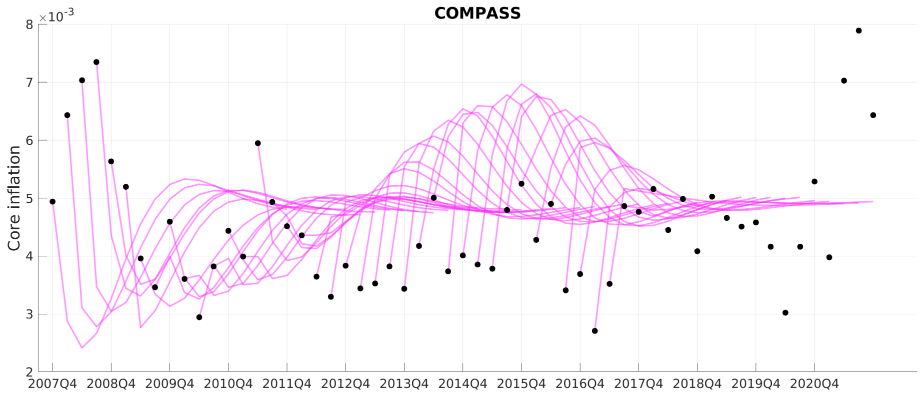


Figure 3. The 16 step ahead forecasts for COMPASS at each quarter from 2007Q4 to 2017Q4.

fact that explaining inflation in Chile requires a much more detailed specification for tradable/non-tradable sectors and the role of exchange rate passthrough to domestic production. This is evident by the fact that 29% and 37% of consumption and investment production in Chile is composed of importable intermediate goods. Fiscal policy in the model follows an exogenous AR(1) process and monetary policy is conducted according to a Taylor rule responding to both total and core inflation.

TNT features 20 exogenous shocks, with shocks originating in both the domestic and foreign blocks of the model. Similar to COMPASS, a number of steady state ratios are calibrated according to national accounts data. The remaining parameters are estimated with Bayesian methods, using 23 observables. The observables include breakdowns of value added according to tradable and non-tradable sectors, prices and wages, and rest-of-world measures such as the world interest rate, output, and inflation (commodity, imports, and consumption goods). According to the unconditional variance decomposition provided by TNT, the major shocks affecting inflation in Chile are world interest rates, uncovered interest rate parity, and world prices.

TNT does not confront the same dilemma as COMPASS regarding differing forecasts/policy prescriptions since it is a singular model. However, the model is subject to a criticism that can be levelled at all DSGE models namely that, if the model is mis-specified, this can lead to forecasts and policy prescriptions which are at odds with optimal policy.

Both COMPASS and TNT build on the medium-scale New Keynesian (NK) DSGE literature, with model economies characterized by sticky prices and wages (e.g. Christiano et al. (2005) and Smets and Wouters (2007)). In this framework the expectations of households and firms play an important role in the evolution of real and nominal activity, and monetary policy can play a stabilizing role through its adjustment of the short-term policy rate. However, monetary policy can only impact real activity in the short-run, and in the long-run real economic activity is determined by supply side factors such as total factor productivity growth. While COMPASS and TNT are broadly characterized as NK-DSGE models, at a more granular level they emphasize very different channels and propagation mechanisms due to the inherent differences between the UK and Chilean economies.

Figure 4 displays forecasts of core Canadian inflation once the model has been adapted to the Canadian environment. The forecasts generated by TNT seem to consistently predict a downturn in core inflation at short horizons followed by a steady increase, despite this behaviour being rarely observed in observed Canadian data used for estimation. Indeed, the model predicts deflation around the time of the GFC. In line with all of the previous examples, the 2021 inflation surge is missed.

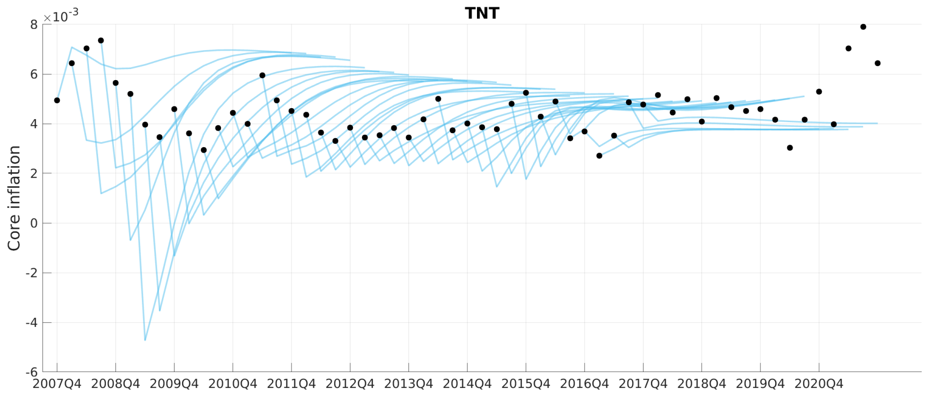


Figure 4. The 16 step ahead forecasts for TNT at each quarter from 2007Q4 to 2017Q4.

3.2. Selected Examples of Semi-Structural and Time Series Models

Naive A set of "naïve" forecasts are also included in the ensemble since it often serves as a baseline for the other forecasting models and is easy to implement. The naïve forecasts simply predict that, whatever the current state, it will not change over the entire forecast horizon. For example, if inflation is at 2.5% at t=0 then the naïve model will predict that inflation will remain at 2.5% from h=1 until h=16 where h is the forecast horizon.

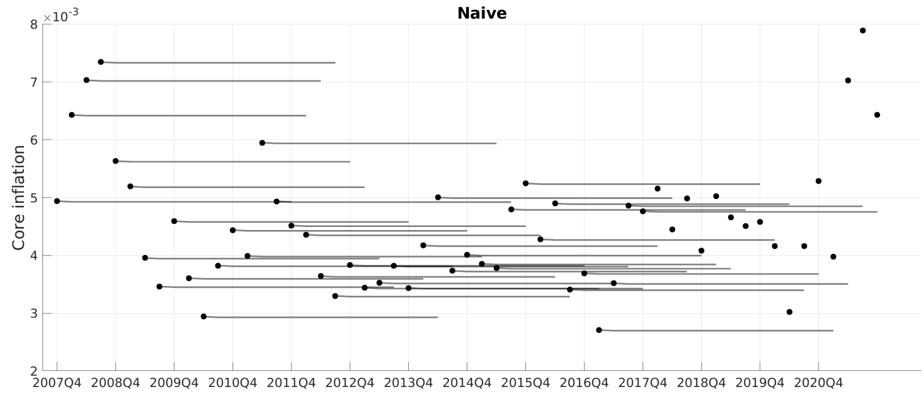


Figure 5. The 16 step ahead forecasts for the naïve model at each quarter from 2007Q4 to 2017Q4.

Figure 5 displays the naïve core inflation forecasts. Unsurprisingly, this forecast does poorly in the COVID era. However, it is also worth noting that the naïve forecast does well when inflation fluctuates in a narrow band as it did between approximately 2012-2015.

ARIMA The autoregressive integrated moving average (ARIMA) model is one of the most widely used traditional statistical time series forecasting models Calvo (1983). ARIMA directly assumes a linear relationship between the future value of a stationary univariate time series' variable and its past observations and errors as per the following equation:

$$y_t = \theta_0 + \psi_1 y_{t-1} + \psi_2 y_{t-2} + \ldots + \psi_p y_{t-p} + \epsilon_t - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t-2} - \ldots - \theta_q \epsilon_{t-q}$$

where y_t is the prediction of the variable y at time t, ϵ_t is the random error at time t, ψ_j ($j \in 1, 2, 3, \ldots, p$) are the autoregressive parameters, and θ_i ($i \in 0, 1, 2, \ldots, q$) are moving average parameters. Due to its relative simplicity in interpretation and ease of implementation, ARIMA has been a cornerstone in time series forecasting research for the past few decades Calvo (1983).

However, its simplicity can prevent it from being able to capture more complex non-linear relationships that often characterize real world data.

Autocorrelation and partial autocorrelation functions are used to estimate an interval of the possible model orders p and q.[23] The Darts (Data Analysis and Regression Time Series) time series forecasting library is chosen as the main software library for the implementation of the ARIMA model. In the case of ARIMA, Darts implements a wrapper model around the pmdarima AutoARIMA model (Gali and Gertler, 1999). In order to find the best model, the pmdarima AUTOARIMA model uses the step-wise selection algorithm proposed by Hyndman and Khandakar in 2008 (see Andres et al. (2004)) to select and identify the optimal model parameters from a given interval. The models are then evaluated and optimized per the user's choice of validation criteria[24] and the model with the best Akaike Information Criterion is returned Gali and Gertler (1999).

Shown in Figure 6 are the 16 step ahead forecasts generated by ARIMA at each quarter from 2007Q4 to 2017Q4. The forecasts are generated using input data ranging from 1987Q1 up to the quarter before the forecast begins. In common with other individual models considered the models perform most poorly during the COVID and recovery periods (i.e., 2020 and 2021).

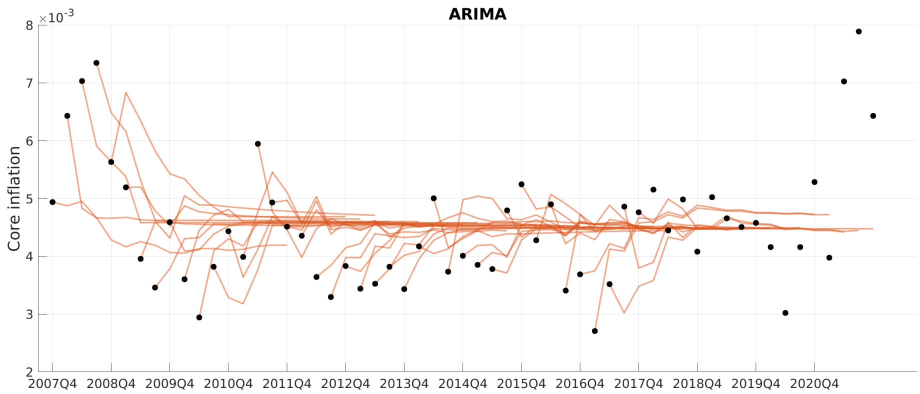


Figure 6. The 16 step ahead forecasts for ARIMA at each quarter from 2007Q4 to 2017Q4.

XGBoost Extreme Gradient Boosting (XGBoost) was initially proposed in 2016 by Chen and Guestrin and is based on the gradient boosting framework (Chen et al., 2012). XGBoost uses a boosting strategy that combines numerous weak prediction models, often decision trees, into a strong ensemble model. It iteratively constructs trees based on the residual data to minimise a loss function, allowing it to capture complicated relationships and patterns in the given data Chen et al. (2012). XGBoost has various advantages, including scalability, missing data handling, and the ability to handle both classification and regression tasks. Nevertheless, if not adequately regularised, XGBoost, like similar boosting algorithms, is prone to overfitting the training data leading to poor predictive performance on out-of-sample data.

The XGBoost model used in this paper is estimated in a multi-step process. First, the model is trained on 8 timeseries which were predetermined to hold particular economic significance.[25] Next, new time series are added one at a time to the estimation dataset. This iterative process continues until a plateau in the model's performance is observed. At each stage, the time series that provided the greatest improvement in OOS RMSE is chosen and incorporated into the final estimation dataset. It should be noted that the hyper parameter tuning library OPTUNA (Zhang, 2003) is used to tune the model in terms of lags, feature lags and maximum tree length. XGBoost-h denotes a version of XGBoost targeting h steps ahead.

The forecasts shown in Figure 7 obtained XGBoost are erratic, which could be due to the

[23] In accordance with the Box-Jenkis methodology Box and Jenkins (1970).

[24] Such as the Akaike Information Criterion (AIC), Corrected Akaike Information Criterion (AICC), Bayesian Information Criterion (BIC), Hannan-Quinn Information Criterion (HQIC) or out-of-bag (OOB)

[25] The time series in question are inflation, foreign exchange rate, output gap, short-term interest rest-of-world (ROW) inflation, ROW commodity prices, ROW output gap, and ROW short-term interest rate.

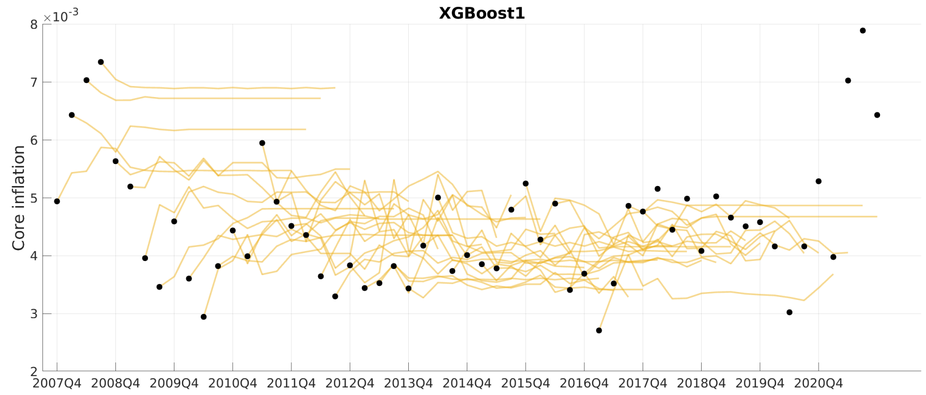


Figure 7. The 16 step ahead forecasts for XGBoost at each quarter from 2007Q4 to 2017Q4.

relatively small amount of input data (51 timeseries over a sample of up to 124 quarters). Improved forecasts are likely possible with increased tuning. In common with the other models examined so far, the post-COVID inflation surge is missed while, once again, model performance is improved when inflation is low and stable.

Vector Autoregressions (VAR) Vector Autoregressive (VAR) models are a class of time series models used in econometrics and statistics to analyze and forecast multiple time series variables simultaneously. Like other time series models, VAR models assume that the variables are stationary, meaning that their statistical properties, such as mean and variance, remain constant over time. VAR models use lagged values of the variables as predictors. The number of lags p is determined by the modeler and can vary depending on the data and the desired model complexity, as per the following equation:

$$y_t = \theta_0 + \psi_1 y_{t-1} + \psi_2 y_{t-2} + \ldots + \psi_p y_{t-p} + \epsilon_t$$

Where p is the number of lags, $y_t \in \mathbb{R}^N$ is the data to be modelled and the parameters to be estimated are $\theta_0 \in \mathbb{R}^N$ and $\psi_i, \forall i \in 1, \ldots, p$. Like ARIMA, this structure requires stationarity and assumes some degree of linearity in the temporal relationships between the variables.

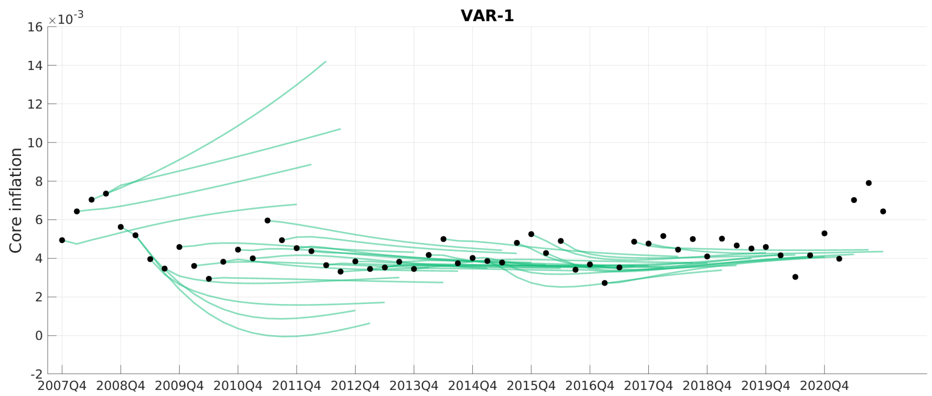


Figure 8. The 16 step ahead forecasts for VAR-1 at each quarter from 2007Q4 to 2017Q4.

The VAR models in this paper are estimated using Ordinary Least Squares (OLS), which assumes Gaussian error terms and the forecasts are shown in Figure 8. VAR-p denotes a VAR model with p lags. While the forecasts which come out of the VAR-1 model are very poor early on, the majority of its forecasts are similar to those of ARIMA and trend towards the sample mean.[26] To estimate the model eight times eries are given as input: quarterly inflation, exchange rate, output gap, interest rate, rest of world (ROW) quarterly inflation, ROW output gap, ROW interest rate, and ROW commodity prices.

4. Model Aggregation

The aggregate model is constructed by selecting weights $w \in \mathbb{R}^2$ for each forecast f_m in the set of M models at each timestep t. This is done by minimizing the squared difference of the weighted sum of forecasts to the univariate timeseries data d_t over the period $t = 1, \ldots, T$. Additionally, changes in the weights between timesteps are penalized with penalty parameter λ. The resulting optimization problem can be represented as follows:

$$\begin{aligned} &\min_w \sum_{t=1}^{T} \left(\left(\sum_{m=1}^{M} w_{t,m} f_m(t) \right) - d_t \right)^2 + \lambda \sum_{t=1}^{T} \sum_{m=1}^{M} (w_{t+1,m} - w_{t,m})^2 \\ &s.t. \sum_{m=1}^{M} w_{t,m} = 1, \qquad \forall t \in 1, \ldots, T \end{aligned} \tag{1}$$

In practice, it is found that a penalty parameter of about 0.1 works best for the given data.[27] The penalty term in the optimization problem serves to reduce fluctuations in the weights while still allowing for some regime shifting, overall improving OOS fit. Similarly, the constraint on the weights also improves OOS fit by forcing the aggregate model to generate forecasts between $[min(f_1(t), \ldots, f_M(t)), max(f_1(t), \ldots, f_M(t))]$ at each timestep t.

While expression 1 can be employed to construct an ensemble model, it can also be solved iteratively using sets of $1, \ldots, N$ step-ahead forecasts. The result is a new ensemble model for each step ahead, with weights which may differ greatly from each other. The ensemble of these ensemble models can then be used to generate a forecast for the period $t = T + 1, \ldots, T + N$. As explained below, different models are weighted differently depending on the number of steps ahead being forecast.

5. Empirical Results

The models are estimated over the 1987Q1-2017Q3 sample and then OOS are generated beginning with 2017Q4 for horizons of up to 16 quarters ahead. The forecasts and outturns discussed below are measured on a quarter-to-quarter basis. It is straightforward to convert the data to annual or annualized rates. Core inflation is defined here as the quarterly rate of change in an average of the trim, median, and common inflation rates published by the Bank of Canada.[28]

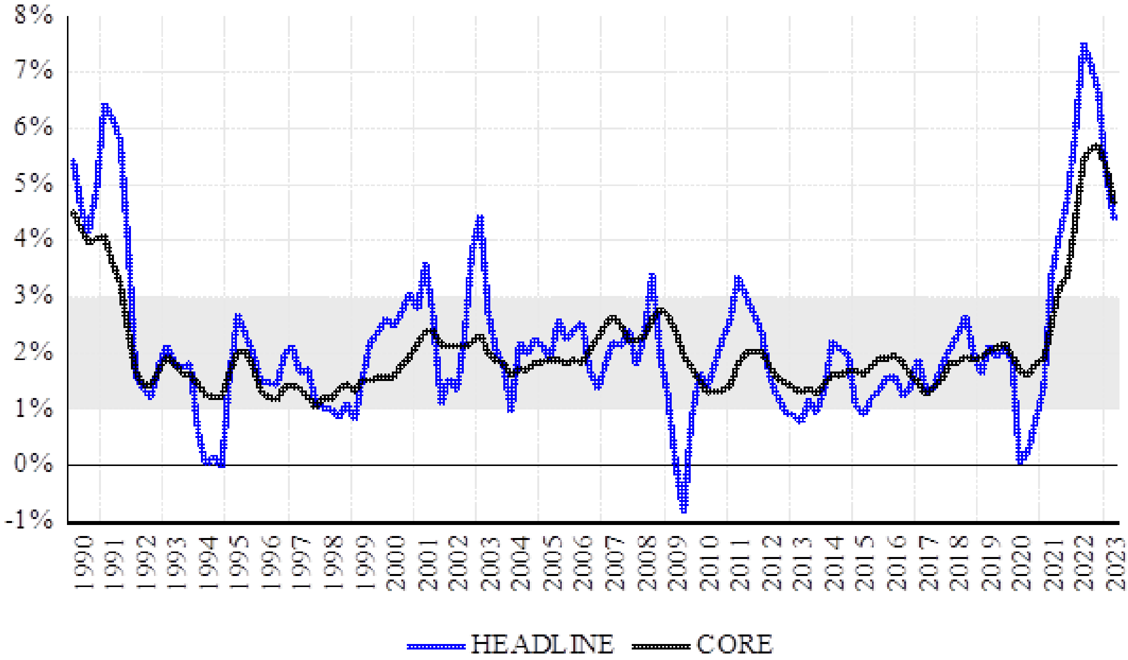
The top portion of Figure 9 plots headline inflation against core inflation while the bottom portion displays the three individual inflation series that make up core inflation. As is well known headline inflation is considerably noisier than core inflation. However, deviations between core and headline inflation are not very persistent. Hence, differences between the two series are stationary.[29] More generally, core inflation is stationary. The bottom figure illustrates that there are different ways of extracting core measures of inflation depending on how one removes the noisiest components on inflation. Since, a priori, there is little theoretical guidance about the ideal measure of core inflation the average of the three proxies shown in the bottom portion of the figure is used.[30]

Shown below in Figure 10 are the weights obtained by solving expression 1 using the $1, 2, \ldots, 16$ step-ahead forecasts generated by the models described in Section 3 given inflation data from

[26] One explanation for this may be because using too few lags can result in autocorrelated errors. Likewise, using too many lags results in over-fitting, and thus reduced out-of-sample predictive power. Selection of an appropriate lag is critical to inference in VARs Lutkepohl (2017).

[27] This corresponds to a penalty parameter that is 2 orders of magnitude larger than the given data.

[28] The CPI data are part of the "key monetary policy variables" used by the Bank of Canada. See https://www.bankofcanada.ca/rates/indicators/key-variables/. In principle, our results can accommodate headline



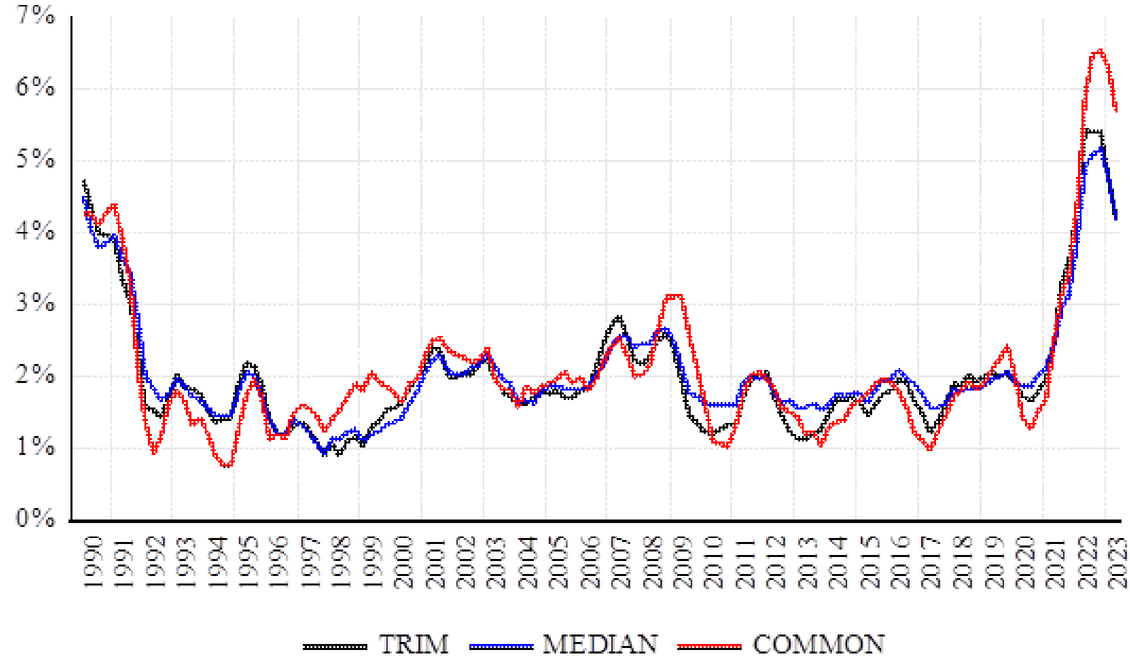


Figure 9. (Top) Headline and core inflation. The latter is the series being forecast. (Bottom) The three core inflation measures averaged to produce core inflation. Details about the definitions of TRIM, MEDIAN, and COMMON inflation can be found in boc (2023). Essentially, all three measures seek to remove the noisy components of headline inflation. Note also that the figures show inflation as the year over year change in CPI. The variable being forecast is the quarterly rate of change which, as seen below, is a noisier series. We show these versions to facilitate the description of a few stylized facts.

inflation. We leave this extension for future research.

[29]Tests (not shown) confirm this to be the case.

[30]A variety of so-called unit root tests, with and without allowance for structural breaks, confirm that inflation is

2007Q4 until 2017Q4. Interestingly, there is a high degree of variation across the forecast horizons, which indicates that model usefulness changes in the short, medium, and longer term.

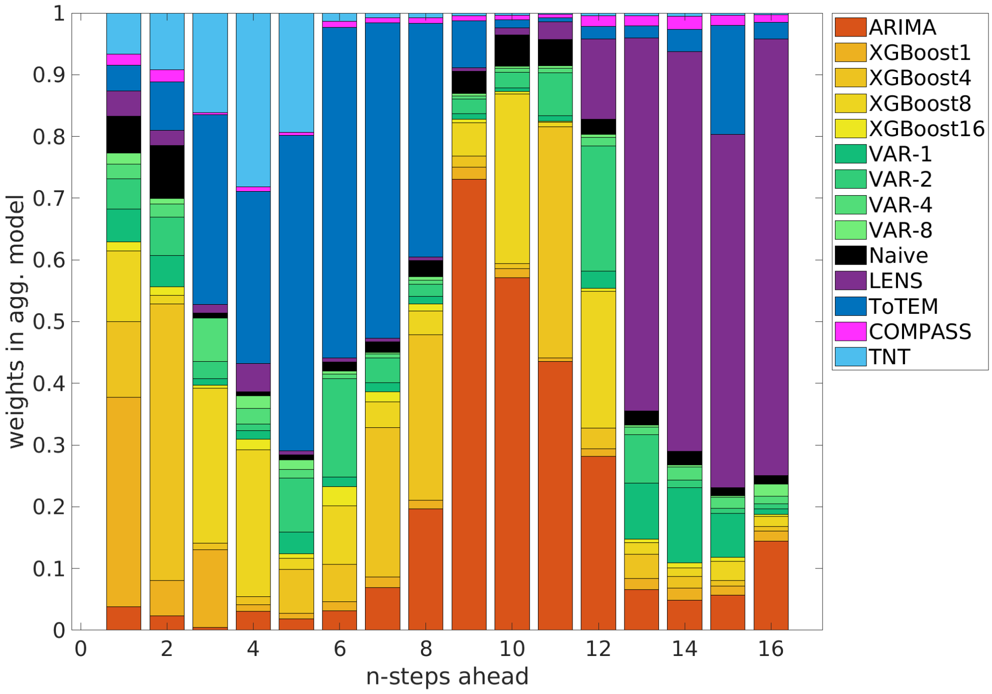


Figure 10. Weights assigned to each forecast in each n-step ahead aggregate model given forecasts from 2007Q4 to 2017Q4.

Unsurprisingly, the naïve forecast is most useful in the very short term since, generally speaking, inflation does not change dramatically each quarter. What is more surprising is that the naïve forecasts retain some weight around 3 years out, which could be an indication of a slight 3-year cycle in Canadian inflation from 2007 to 2017. The somewhat cyclical behaviour of the inflation data in this period seems to have been captured by ARIMA and the XGBoost models, which is why they are weighted so highly overall. Interestingly, despite the VAR models having access to more timeseries during model estimation, they are generally given less weight than ARIMA.

Another interesting observation that can be made from Figure 10 is that the structural models are assigned little weight in both the short-term (1 and 2 steps ahead) and longer-term (9-12 steps ahead). The reason is that their RMSE over the sample period is worse than the other models and the objective function of the aggregate model in expression 1 is effectively seeking to minimize RMSE. While LENS is weighted highly for the very long-term forecasts (13 to 16 steps ahead) in Figure 10, it should be noted that most of the models considered in Section 3 have forecasts which tend towards the sample mean. When the estimation period is changed the other weights stay more, or less, the same while those for 14 to 16 steps ahead can change considerably.

Given the weights shown in Figure 10, the aggregate model's OOS forecast is shown below in Figure 11 along with the individual forecasts and observed values. The forecasts generated by the aggregate model at each point within the sample period are shown in Figure 12. Note that the

stationary over the full sample. A structural break is identified for core inflation in 2021Q1 when inflation begins to show signs of surging, but the assumption of stationarity in inflation is not easily be rejected. For example, the so-called Dickey-Fuller test, with or without a break, rejects the null of a unit root (i.e., non-stationarity) at least at the 10% significance level for headline inflation. The same conclusion holds for core inflation (p-value of 1%). A break is detected in headline inflation very early in the inflation targeting era (1992Q1). Of course, more than one break is possible. More detailed test results are available on request. A variety of unit root tests also support the foregoing conclusions.

aggregate model's forecasts are less volatile than many of the component models.

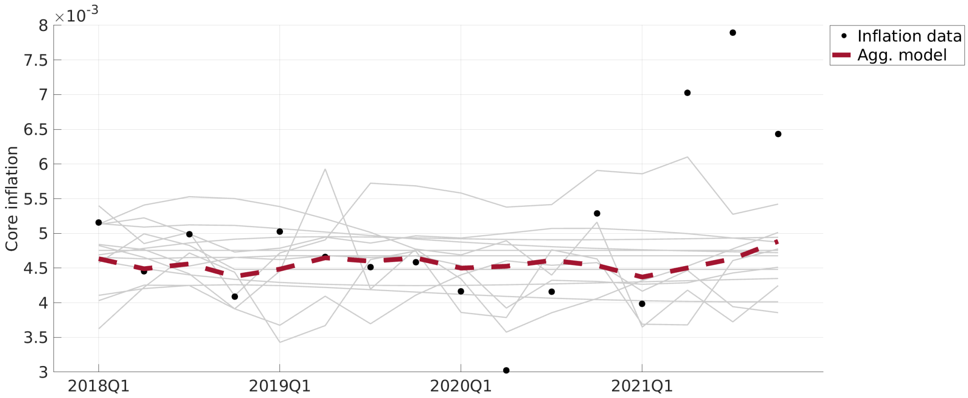


Figure 11. Out-of-sample forecasts for each model given estimation data up to 2017Q4. The aggregate model (dashed red line) uses the weights shown in Figure 8. The black dots represent the observed values for quarterly inflation.

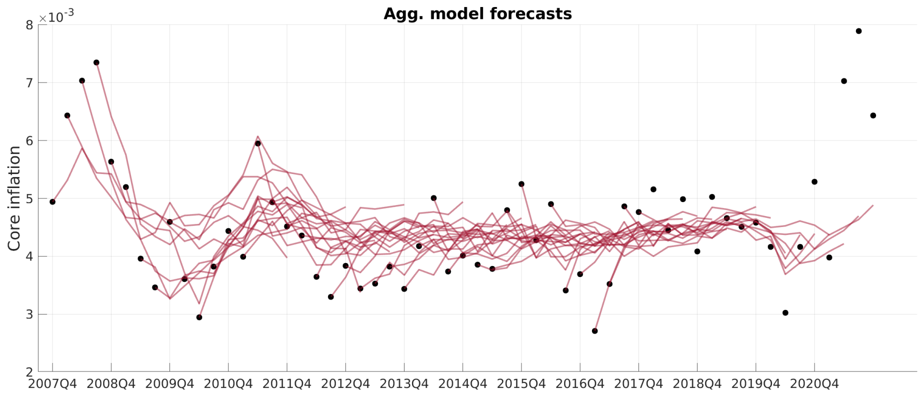


Figure 12. The 16 step ahead forecasts for the aggregate model at each quarter from 2007Q4 to 2017Q4.

Shown below in Table 2 are the average RMSE, MAPE, and MAE values for each model. The average error metrics are calculated by first estimating expression (1) over the period from $2007Q4$ to $2010Q1, \ldots, 2017Q4$ and then calculating the error metrics in the 16 step ahead OOS period starting just after the end point of the estimation sample period.

Table 2. Average OOS error for each model (10^{-3})

Model Name	16 step ahead RMSE	16 step ahead MAPE	16 step ahead MAE	8 step ahead RMSE	8 step ahead MAPE	8 step ahead MAE
Agg. model	0.8224	174.47	0.687	0.7448	168.93	0.6526
ARIMA	0.8068	170.72	0.667	0.7749	173.58	0.6702
XGBoost1	0.8368	164.49	0.681	0.7900	162.48	0.6528
XGBoost4	0.8760	173.61	0.710	0.8268	172.69	0.6870
XGBoost8	0.8791	167.74	0.691	0.7777	157.56	0.6300
XGBoost16	1.1388	235.01	0.936	0.9367	200.09	0.7829
Naive	0.9807	193.62	0.817	0.8928	175.16	0.7549
ToTEM	0.8624	181.95	0.713	0.9187	182.95	0.7739
LENS	1.1468	246.33	0.947	0.9616	207.88	0.8285
COMPASS	1.2229	259.73	0.989	1.4392	317.57	1.2486
TNT	1.8287	405.88	1.597	0.9514	198.39	0.8182
VAR-1	0.9380	173.07	0.762	1.3440	319.11	1.2088
VAR-2	0.9444	176.77	0.763	0.7679	174.73	0.6767
VAR-4	1.0121	215.08	0.857	1.3786	317.02	1.2065
VAR-8	1.8609	402.27	1.597	1.5892	347.92	1.4085

Overall, the error metrics in Table 3 correlate quite closely with one another, indicating that the results are robust to the choice of metric used in evaluating predictive power. While the aggregate model achieves the best predictive power in the 8 step ahead forecasting period, its predictive power is slightly less than that of ARIMA in the 16-step ahead OOS forecast period. This contradicts the hypothesis that the aggregate model would achieve greater predictive power by incorporating more and more diverse perspectives. This then begs the question: "does a subset of forecasts exist that yields the best predictive power for the aggregate model?"

If only ARIMA, XGBoost8, ToTEM, VAR1 and VAR2 are included in the aggregate model then it can achieve an average RMSE of 0.00072895 in the 16 step ahead OOS forecasting period. Of course, averaging over a 16 quarters horizon is not likely to be the objective policy makers have in mind when deliberating the ideal course to set for monetary policy.

As mentioned in Section 2, there are alternative model aggregation approaches to expression (1). One could for example use equal weights, or non-equal but non-time-varying weights. The former is equivalent to the special case of $w_{t,m} = 1/M$ for all t and m, while the latter is the case where $\lambda = \infty$ in expression (1). For the sake of comparison, the average OOS errors of each approach are shown below in Table 3.

Table 3. Average 16 step-ahead OOS error for alternative model aggregation methods (10^{-3})

	RMSE:	MAPE:	MAE:
Expression (1) – all models	0.8224	174.47	0.6871
Const. weights – all models	0.8383	176.61	0.6952
Equal weights – all models	0.8534	180.82	0.7124
Expression (1) – best RMSE	0.7289	142.23	0.5907
Const. weights – best RMSE	0.7309	145.36	0.5958
Equal weights – best RMSE	0.7585	155.06	0.6197

A striking result from Table 3 is that the average OOS predictive power can be drastically increased by limiting the selection of models available to the aggregate model. Specifically, the set of models that leads to the best outcome for the aggregate model is quite heterogenous. It includes a variety of statistical models, an ML model, and a structural model. This seems to indicate that model heterogeneity is key in model aggregation. By breaking down the average OOS RMSE by predictive horizon, as shown in Figure 13, we see that the improvement is primarily coming from improved predictive power in the first five steps in the OOS forecast. This does seem to come at the cost of some predictive power in the longer term time horizons, along with some increased bias (see Figure 14).

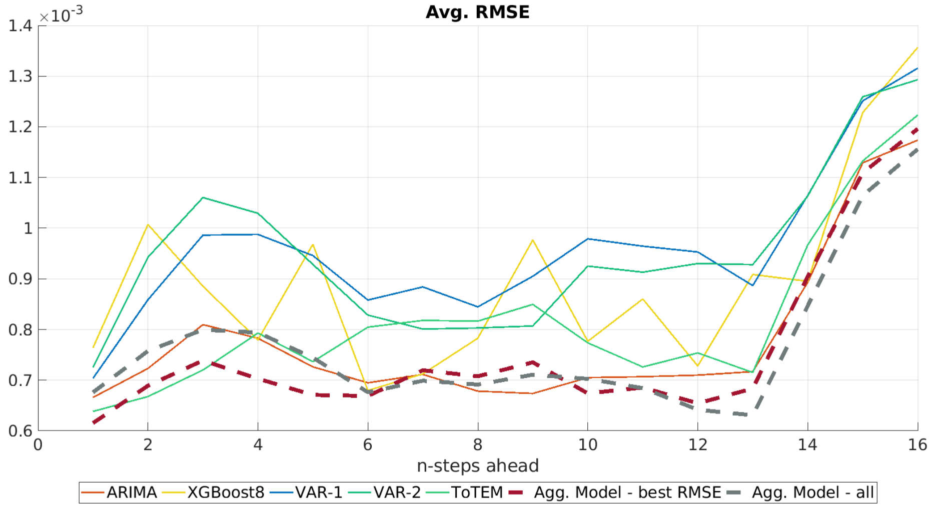


Figure 13. Average OOS RMSE of model forecasts from 2012Q1 to 2017Q4

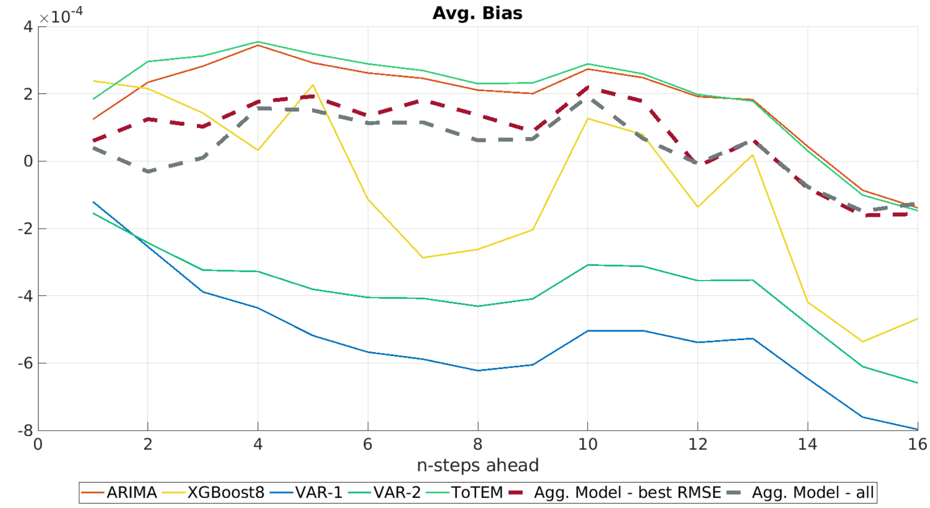


Figure 14. Average OOS bias of model forecasts from 2012Q1 to 2017Q4

6. Conclusions

Skepticism about inflation forecasts has received a lot of attention recently. While completely unexpected shocks are unlikely to ever be captured in existing models there exist strategies that have yet to be exploited that offer the promise of improved forecasts. This paper considers aggregating forecasts from a variety of model types ranging from time series to structural models. That said, a restriction that needs to be imposed is to select a set of models with diverse forecast biases (i.e., errors) and which may be better or worse at a variety of forecast horizons. The aim is not to replace any specific model. Rather, the methodology employed in this paper exploits the underlying heterogeneity of existing models. After all, experience has shown that some models outperform others at various times for a variety of reasons including the underlying information set, the complexity of the model, or the dynamics imposed on the specification employed.

The main finding is that model aggregation can improve forecasts of inflation. However, significant forecast improvement requires that models are weighted in a time-varying manner. This result stands in contrast with the related literature that investigates how best to combine point forecasts and often concludes that unweighted averaging does just as well as more complex forecast combinations. Moreover, there are significant differences in forecast performance at different forecast horizons. Second, model heterogeneity is found to be an important device to mitigate the impact of groupthink in generating estimates for the outlook on inflation. This is also a critical finding given the similarity of many models that are currently employed by central banks and academics to derive inferences from shocks to the economy or in producing forecasts.

Nevertheless, several extensions and improvements remain to be implemented. While the results presented in this study examine core inflation forecast performance, policy makers also require headline inflation forecasts. After all, the public is more likely to focus on overall CPI behaviour than on its counterparts stripped of its most volatile elements. Furthermore, a particular methodology is applied to aggregate the models. An additional robustness test calls for repeating the estimates of this study relying on other ways of aggregating models. The aggregation process focuses on certain characteristics that are considered key in influencing forecasts. However, other characteristics such as how far backward-looking or forward-looking the various models are, the vintage of the model and the data, the nature of price stickiness in the structural and semi-structural models, which sectors of the economy are excluded (e.g., commodities, finance, housing), or model scale, could be considered. A Bayesian approach to aggregation is another possibility that should be attempted wherein confidence intervals around aggregated forecasts could be constructed. Additionally, the model aggregation approach could yield interesting results when applied to other variables on interest, such as real output or unemployment. Finally, a positive side-effect from this exercise is that the set of models included in the aggregated model may, someday, provide clues about model elements that contribute to ensuring forecast errors are minimized over time. These extensions are left for future research.

References

Consumer price index, 2023. URL https://www.bankofcanada.ca/rates/price-indexes/cpi/.

N. Ahmed, A. Atiya, N. Gayar, and H. El-Shishiny. An Empirical Comparison of Machine Learning Models for TIme Series Forecasting. *Economic Reviews*, 29:594–621, 2010.

T. Akiba, S. Sano, T. Yanase, T. Ohta, and M. Koyama. Optuna: A Next-Generation Hyperparameter Optimization Framework. *Proceedings of the 25th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining*, 2019.

J. Andres, J. D. López-Salido, and E. Nelson. Tobin's imperfect asset substitution in optimizing general equilibrium. *Journal of Money, Credit and Banking*, pages 665–690, 2004.

A. Ang, G. Bekaert, and M. Wei. Do macro variables, asset markets, or surveys forecast inflation better? *Journal of Monetary Economics*, 54(4):1163–1212, 2007.

G. S. Araujo and W. P. Gaglianone. Machine learning methods for inflation forecasting in brazil: New contenders versus classical models. *Latin American Journal of Central Banking*, 4(2): 100087, 2023.

P. Bajari, D. Nekipelov, S. P. Ryan, and M. Yang. Demand estimation with machine learning and model combination. *National Bureau of Economic Research*, 2015.

Z. Barutçuoglu and E. Alpaydin. A comparison of model aggregation methods for regression. *Artificial Neural Networks and Neural Information Processing—ICANN/ICONIP 2003*, pages 76–83, 2003.

M. Binder, P. Lieberknecht, J. Quintana, and V. Wieland. Model Uncertainty in Macroeconomics: On the Implications of Financial Frictions. *[Online]. Available: https://macromodelbase.com. [Accessed 05 01 2023].*, pages 679–778, 2019.

A. Blanco, P. Ottonello, and T. Ranosova. The dynamics of large inflation surges. Technical report, National Bureau of Economic Research, 2022.

G. Box and G. Jenkins. *Time Series Analysis, Forecasting and Control*. Holden-Day, 1970.

S. Burgess, E. Fernandez-Corugedo, C. Groth, R. Harrison, F. Monti, K. Theodoridis, M. Waldron, et al. The Bank of England's forecasting platform: COMPASS, MAPS, EASE and the suite of models. *working document*, (471):13, 2013.

G. A. Calvo. Staggered prices in a utility-maximizing framework. *Journal of monetary Economics*, 12(3):383–398, 1983.

C. Chakraborty and A. Joseph. Machine Learning at Central Banks. *Bank of England working paper*, (674), 2017a.

C. Chakraborty and A. Joseph. Machine learning at central banks. 2017b.

A. Check and J. Piger. Structural breaks in us macroeconomic time series: A bayesian model averaging approach. *Journal of Money, Credit and Banking*, 53(8):1999–2036, 2021.

H. Chen, V. Curdia, and F. A. The macroeconomic effects of large-scale asset purchase programmes. *The economic journal*, 122(564):289–315, 2012.

T. Chen and C. Guestrin. XGBoost: A Scalable Tree Boosting System. *Proceedings of the 22nd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, 2016.

L. Christiano, M. Eichenbaum, and M. Trabandt. On DSGE Models. *Journal of Economic Perspectives*, 32:113–140, 2018.

L. J. Christiano, M. Eichenbaum, and C. L. Evans. Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of political Economy*, 113(1):1–45, 2005.

R. Clarida, J. Gali, and M. Gertler. The science of monetary policy: a new keynesian perspective. *Journal of economic literature*, 37(4):1661–1707, 1999.

P. Corrigan, H. Desgagnés, J. Dorich, V. Lepetyuk, W. Miyamoto, and Y. Zhang. Totem iii: The bank of canada's main dsge model for projection and policy analysis. Technical report, Bank of Canada, 2021.

M. Del Negro and F. Schorfheide. DSGE Model-Based Forecasting. *Handbook of Economic Forecasting*, 2:57–140, 2013.

T. Doan, R. Litterman, and C. Sims. Forecasting and conditional projection using realistic prior distributions. *Econometric Reviews*, 3(1):131–144, 1984.

J. Dorich, O. Kryvtsov, V. Traclet, and J. Witmer. Workshop Summary: Central Bank Models: The Next Generation. *30 January 2017. [Online]. Available: https://www.bankofcanada.ca/2016/11/central-bank-models-next-generation/*, 2017.

M. Dotsey, M. Del Negro, A. Sbordone, and K. Sill. System DSGE Project Documentation. *Federal Reserve Board memo, 9 June 2011. [Online]. Available: https://www.federalreserve.gov/monetarypolicy/files/FOMC20110609memo02.pdf*, 2011.

G. B. Eggertsson and D. Kohn. The inflation surge of the 2020s: The role of monetary policy. *Presentation at Hutchins Center, Brookings Institution*, 23, 2023.

W. Enders. *Applied Econometric Time Series, 4th Edition*. John Wiley and Sons, 2015.

J. Faust and J. Wright. Forecasting inflation. *Handbook of Economic Forecasting*, 2(A):4–51, 2013.

G. Galbraith and G. Tkacz. How Far Can Forecasting Models Forecast? *Bank of Canada working paper*, 2007.

J. Gali and M. Gertler. Inflation dynamics: A structural econometric analysis. *Journal of monetary Economics*, 44(2):195–222, 1999.

M. García-Schmidt, J. García-Cicco, et al. *A TNT DSGE model for chile: explaining the ERPT*. Banco Central de Chile, 2020.

E. Gauss and C. Gibbs. Expectations and the empirical fit of dsge models. *Forth-coming in Studies in Nonlinear Dynamics and Econometrics*, 2018. URL http://https://christopherggibbs.weebly.com/uploads/3/8/2/6/38260553/exp_and_dsge_2018.pdf.

O. Gervais and M.-A. Gosselin. Analyzing and forecasting the canadian economy through the lens model. Technical report, Bank of Canada, 2014.

M. Goodfriend and R. G. King. The new neoclassical synthesis and the role of monetary policy. *NBER macroeconomics annual*, 12:231–283, 1997.

C. W. Granger and Y. Jeon. Thick modeling. *Economic Modelling*, 21(2):323–343, 2004.

P. Hansen, A. Lunde, and J. Nason. The Model Confidence Set. *Econometrica*, 79:453–497, 2011.

N. Hauzenberger, F. Huber, and K. Klieber. Real-time inflation forecasting using non-linear dimension reduction techniques. *International Journal of Forecasting*, 39:901–921, 2023.

N. Hirakata, K. Kan, A. Kanafuji, Y. Kido, Y. Kishaba, T. Murakoshi, T. Shinohara, et al. The quarterly japanese economic model (Q-JEM): 2019 version. 2019.

R. Hyndman and Y. Khandakar. Automatic Time Series Forecasting: The forecast package for R. *Journal of Statistical Software*, 27:1–22, 2008.

M. Jain and C. Sutherland. How do central bank projections and forward guidance influence provate-sector forecasts, 2018.

M. Joseph. *Modern time series forecasting with python explore inustry-ready time series forecasting using modern machine learning and deep learning.* 2022.

O. Kryvtsov, J. J. MacGee, and L. Uzeda. The 2021–22 surge in inflation. Technical report, Bank of Canada, 2023.

P. Lane. Inflation diagnostics, 2022. URL https://www.ecb.europa.eu/press/blog/date/2022/html/ecb.blog221125~d34babdf3e.en.html.

E. E. Leamer. *Specification searches: Ad hoc inference with nonexperimental data*, volume 53. John Wiley & Sons Incorporated, 1978.

H. Low and C. Meghir. The use of structural models in econometrics. *Journal of Economic Perspectives*, 31(2):33–58, 2017.

H. Lutkepohl. Estimation of Structural Vector Autoregressive Models. *Communications for Statistical Applications and Methods*, 24:421–441, 2017.

P. McAdam and P. McNelis. Forecasting Inflation with Thick Models and Neural Networks. *Economic Modelling*, 22:848–867, 2005.

F. Mishkin. Headline versus core inflation in the conduct of monetary policy, October 2007. URL https://www.federalreserve.gov/newsevents/speech/mishkin20071020a.htm. speech given at the Business Cycles, International Transmission and Macroeconomic Policies Conference, Montreal, Canada.

S. Morris and H. Shin. Central bank transparency and the signal value of prices. *Brookings Papers on Economic Activity*, 36(2):1–66, 2005.

J. Murray. Monetary policy decision-making at the bank of canada. *Remarks at the Mortgage Brokers Association of B.C.*, 2012.

C. Perricone. Clustering Macroeconomic Variables. *Structural Change and Economic Dynamics*, 44:23–33, 2018.

R. Prudencio and T. Ludermir. A machine learning approach to define weights for linear combination of forecasts. *International Conference on Artificial Neural Networks*, pages 274–283, 2006.

V. Ramey. Macroeconomic shocks and their propagation. *J.B. Taylor and H. Uhlig (Eds.), Handbook of Macroeconomics*, 2A:71–162, 2016.

D. Romer. The New Keynesian Synthesis. *Journal of Economic Perspectives*, 7:5–22, 1993.

C. Sims. Macroeconomics and Reality. *Econometrica*, 48:1–48, 1980.

S. Slobodyan and R. Wouters. Learning in a medium-scale dsge model with expectations based on small forecasting models. *American Economic Journal: Macroeconomics*, 4(2):65–101, 2012.

F. Smets and R. Wouters. Shocks and frictions in us business cycles: A bayesian dsge approach. *American economic review*, 97(3):586–606, 2007.

S. Smith. Tests of forecast accuracy and bias for country population projections. *Journal of the American Statistical Association*, 82:991–1003, 1987.

T. Smith. pmdarima: ARIMA estimators for Python. *[Online]. Available: http://www.alkaline-ml.com/pmdarima*, 2017.

A. Timmermann. Forecast combinations. *Handbook of economic forecasting*, pages 135–196, 2006.

H. White. A Reality Check for Data Snooping. *Econometrica*, 68:1097–1126, 2000.

V. Wieland. Macroeconomic model database. *[Online]. Available: https://macromodelbase.com. [Accessed 05 01 2023].*, 2023.

M. Woodford. Interest and prices. 2003.

G. Zhang. Time series forecasting using a hybrid ARIMA and neural network model. *Neurocomputing*, pages 159–175, 2003.

Japan’s Strategic Development Cooperation

Shu Fukuya

Abstract:

Japanese development cooperation policy through the Japan Bank for International Cooperation (JBIC) has historically been rooted in the ideals of mercantilism, setting it apart from the conventional development finance approaches of Western countries. Donors from Europe and the United States were traditionally perceived as non-commercial, concessional entities without business interests, distinct from commercial lenders. In the 1970s, Japan’s mercantilist development finance policy faced significant criticism from Western societies, prompting a shift towards reassessment and more concessional lending.

The 1950s, when Japan’s development cooperation policy began, overlapped with war reparations initiated to support the strategic goals of the United States, playing a crucial role in strengthening the economies of Southeast Asia. The expansion of Japan’s development assistance can be seen as a reactionary response, shaped by war reparations aligned with American strategic interests. The evolution of Japan’s development cooperation policy through JBIC, a key player in Japanese development finance, has been influenced by legal reforms and domestic and international factors. Understanding the reactive nature of Japan’s approach requires insight into the historical changes accompanying JBIC’s law revisions.

This paper aims to explore Japan’s government-led development cooperation policy, focusing specifically on JBIC, an aspect that has not received much attention in academic research. By reflecting on the history of JBIC’s support, evaluating its alignment with the Indo-Pacific vision and national security strategy, the paper discusses an appropriate approach to Japan’s distinctive style of development cooperation.

Sixth Annual Conference of the Japan Economy Network
Hitotsubashi University, Tokyo, 21-22 March 2024
Paper Proposal

Transnationalism and Foreign Workers in Japan: Between State Policies and Market Forces

David Chiavacci, University of Zurich

Japan is for over two decades a new immigration country with a continuous and significant inflow of immigrants, which allows us to analyze the long-term effects of new immigration. This paper discusses comparatively the socio-economic integration of the three major groups of new foreign workers in Japan: Chinese students, Chinese interns, and Nikkeijin (Japanese emigrants and their descendants). Our analysis reveals as main result a striking degree of structural diversity. Patterns of integration and exclusion differ fundamentally between the three major groups. Whilst the Chinese entering as students are successful overall, achieving middle class positions, the Chinese entering as interns are fully marginalized and have no chance of becoming part of Japan's mainstream society. The Nikkeijin are an ambivalent case. On the one hand, they have decent incomes. On the other hand, they are not integrated into Japan's social security system and are highly vulnerable to unemployment as the economic crisis in the late 2000s has exemplified. In view of the educational record of their children, they might even constitute a new ethnic underclass in the making.

Transnationalism is a crucial structural factor for Chinese immigrants but has completely different effects on the two groups. Chinese entering as students are empowered by transnationalism and have successfully used the opportunities granted them through transnationalism. Through their agency they have had a structural effect on transnational links by significantly enlarging business relations between China and Japan. Chinese interns, however, are structurally embedded into a transnational total institution which marginalizes them. The functioning of the foreign intern system depends on their transnational mind-set. Transnationalism is, in the case of Nikkeijin not such an important factor but has a negative impact. Empirical studies show that transnational behavior, such as travelling back and forth between Japan and South America, has a negative influence on the socio-economic integration of first-generation immigrants and on the educational attainments of the second generation. Whilst the odds of non-enrolment in school doubles for Brazilian school children who five years earlier lived outside of Japan, this factor has no influence on the enrolment of Chinese school children.

Overall, the socio-economic integration of new immigrants in Japan is a mixed bag and shows a complex interplay between state policies and market forces. Among the three major groups of immigrants, one is successfully integrating (Chinese entering as students), one is fully marginalized (Chinese entering as interns), and one has achieved a short-term stable position but might become an ethnic underclass in the long-term (Nikkeijin). Based on this conclusion, the paper will also discuss the potential future impact of recent changes in immigration policy (2019 immigration law reform) and immigration flows (sharp incerase of Vietnamese interns) as well as ongoing discussion on the reform of the foreign intern system on the socio-economic integration of foreign workers in Japan.

Investment, Liquidity Traps and Real Interest Rates: A Comparison between Japan and the U.K.

Wenti Du (Akita International University) & Eric Pentecost (Loughborough University)

Extended Abstract

The Japanese and U.K. economies have experienced similar problems in restoring growth after a period of financial repression and liquidity trap (Japan since the end of 1998 and early 1999, while the U.K. between the end of 2008 and the end of 2021). Although there has been a great deal written about these economic episodes (see, for example, Bernanke, 2002; Krugman, 1998; and Uedo, 2012), there is still little agreement about how to get the economies growing again. In some recent papers, for example, there seem to be directly opposing views as to the role of the rate of interest and its relationship with investment.

One view is that interest rates are high when investment is high - because as investment increases, presumably for given rates of savings, the rate of interest must rise (George, 2023). On the other hand, the more traditional view is that as interest rates fall, investment increases (Hicks, 1937) - as the cost of borrowing declines- and so when interest rates are low, investment should be high. This, of course, assumes that business expectations are ignored and the marginal efficiency of capital remains unchanged. This has led to QE and negative real interest rates in both Japan and the U.K., although at least as far as the U.K. is concerned, according to the Bank of England, QE, apart from the initial tranche, has been largely ineffective at stimulating output.

In this paper, we attempt to test empirically the two hypothesised relationships between real interest rates and investment for both Japan and the U.K. when the economies are in a liquidity trap situation. We will use the real 10-year government bond yields for Japan, the U.S. and the U.K. and gross fixed capital formation for Japan and the U.K. One potential innovative aspect of this test is to use the U.S. real interest rate as a world interest rate. In this case, if the capital markets are, in fact, global, we may find that the interest rate differential between the U.S. and the U.K. and the U.S. and Japan has some explanatory power. We may also include measures of economic policy uncertainty as controls in our investment equations to serve as a rough test of Keynes's idea about expectations.

Moreover, as Keynes believed that volatile private investment was a primary cause of recessions, a second adjacent hypothesis - is that there is a relationship between real interest rates and the business cycle. On the one hand, Kalecki (1971) argued that interest rates are driven by the cycle, in that they rise in a boom and decline in recessions. On the other hand, if yield-curve inversion is a good predictor of recessions due to the effect that short-term interest rate increases have on real variables like consumption and investment and, thus, on long-term interest rates (see George, 2023), then real interest rates may drive the cycle. We plan to investigate the strength and direction of causality between real interest rates and investment in the case of Japan and the U.K. in liquidity trap conditions.

References

Bernanke, B.S. (2002). "Deflation: Making Sure 'It' Doesn't Happen Here." Remarks before the National Economists Club, November 21, Washington, DC. Available at: http://www.federalreserve.gov/boarddocs/speeches/2002/20021121/default.htm (accessed December 31, 2015).

George, P. (2023). "Why is yield-curve inversion such a good predictor of recession?" Real-World Economics Review 103, 12–127.

Hicks, J.R. (1937). "Mr. Keynes and the 'Classics': A Suggested Interpretation." Econometrica 5(2): 147–59.

Kalecki, M. (1971). Selected Essays on the Dynamics of the Capitalist Economy, 1933–1970. Cambridge, UK: Cambridge University Press.

Keynes, J.M. (1936). General Theory of Employment, Interest, and Money. New York, NY: Macmillan.

Krugman, P. (1998b). "It's Baaack! Japan's Slump and the Return of the Liquidity Trap." Brookings Paper on Economic Activity 2: 137–87.

Uedo, K. (2012). "Deleveraging and Monetary Policy: Japan Since the 1990s and the United States Since 2007." Journal of Economic Perspectives 26(3): 177–202.

Gender equality in Japan: the progress of womenomics according to axial inequality index

Rebrey S.M. PhD, Associate professor

MGIMO-University

Japan launched womenomics in 2013 first in order to increase female labor force participation rate and thus to stimulate economic growth, but then to achieve gender equality and women empowerment.

Present research marks a decade of womenomics and follows its progress using axial institutions index. Axial institutions are family and labor market, education and science, state and corporate governance. The index measures both the gender gaps in opportunities and the gender gaps in the access to resources, unlike other gender indexes that focus only on the former.

The analysis of womenomics' progress shows that the gender equality was advanced significantly in labor market, that is explained by targeting female labor force participation in the first place. However, the progress is seen not only in quantitative dimension. The number of women included in lifetime employment increases whether the gender wage gap decreases.

Family institution shows more equal distribution of unpaid domestic labor between spouses: in 2011 gender gap in time allocated on unpaid domestic labor was and 3,3867 and in 2021 it decreased to 2,9873. It means that the shift in society towards egalitarian values have advanced.

Education provides full gender equality in school enrollment but struggles to create gender friendly environment where girls could thrive. It is particularly severe in STEM.

Science and governance have very little or no progress at all, as women remain poorly represented o the decision-making positions, including academia. It means, that despite the fact that the womenomics were initiated by the governance, it is still not ready to embrace more egalitarian approach in decision making, but views womenomics as an instrument to spur economic growth.

Simultaneous Recruitment of New Graduates (SRoNG) – What It Is, Why It Persists, and How It Is Changing

Stefan Heeb (Kanagawa University)

This paper provides a systematic account of simultaneous recruitment of new graduates (SRoNG), the predominant hiring practice of big companies in Japan. The authors characterize SRoNG as macro-level coordination in the process of new graduate recruitment between companies and universities, along with homogeneous hiring conditions offered by individual firms, consisting of same initial salaries, initial internal vocational training and absence of job specification.

The paper further provides a historical account of SRoNG's emergence, tracing it back to early-modern government structures, education expansion, wage control efforts, and the system's expansion and establishment as a predominant institution.

Finally the paper focuses on SRoNG's more recent empirical evolution. The main business association, Keidanren, called to abandon the practice in the wake of intensified competition with foreign multinationals in the recruitment of new graduates. While there are noteworthy ongoing changes, the authors submit that SRoNG is unlikely to suffer more than a partial and minor abandonment.

Japan's Great Stagnation; The Failure of Abenomics

Hiroaki Richard Watanabe, Professor, Ritsumeikan University

Long abstract

The Japanese economy has been stagnant since the collapse of the bubble economy in the early 1990s. Japan's GDP growth has remained low, and real wages have hardly grown most of the time. Even after the implementation of Abenomics, especially its experimental, expansionary monetary policy, the situation remains the same. After more than three decades of economic stagnation, Japan will be surpassed by Germany in GDP and fall to the fourth largest economy in the world in 2023, according to the IMF forecast (Sakakibara 2023).

Scholars have examined the causes of Japan's economic stagnation in several policy areas, such as monetary policy, fiscal policy and structural reform (Katz 1998, Maswood 2002, Blomstrom et al. 2003, Callen and Ostry 2003, Hutchison and Westermann 2006, Garside 2012, Wakatabe 2015, Yoshino and Taghizadeh-Hesary 2017, Park et al. 2018). This article conducts a critical review of the relevant literature and assesses the validity of several explanations of the causes of Japan's economic stagnation. This article shows its understanding of the causes and identifies the economic consequences of Abenomics. It examines Japan's economic stagnation by analyzing the incentives of the relevant political and economic actors to promote certain specific policies and their problems. After a few years since the end of the Abe administration and with the prospect of certain changes in the Bank of Japan's monetary policy, now is a good time to assess the effectiveness of Abenomics as an economic policy to overcome stagnation and its (unintended) consequences.

As for monetary policy, Japanese government's recent policies – Abenomics and the similar policies of the following administrations – have relied on expansionary monetary policy or monetary easing, both qualitative and quantitative, to overcome deflation and stimulate economic growth. Some economic scholars, especially those called "reflationists", proposed monetary easing and the setting of inflation targets to create an inflationary expectation and overcome economic stagnation (Krugman 1998, Posen 1998, Bernanke, et al. 1999, Harada 2003, Iwata 2003, Ito and Mishkin 2005, Hamada and Okada 2009, Wakatabe 2015).

These scholars overestimated the effectiveness of monetary policy to achieve economic growth without understanding the essential role of structural reform in achieving economic growth through higher productivity and efficiency. In hindsight, monetary easing did not contribute to significant economic growth. Also, the BOJ's 2% inflation target was achieved only after the price increases of imported products such as raw materials caused by such incidents as the Russian invasion of Ukraine and disruptions to supply chains. It was not

inflation based on the increase in business investment and consumer spending, which the LDP government and the BOJ influenced by the reflationist idea hoped to achieve. Instead, monetary easing has had a negative impact on consumers' economic welfare and their spending by depreciating yen and raising prices of imported products, especially those of daily necessities. This had a negative impact on economic growth.

Although Abenomics was useful for increasing share prices and promoting export with depreciated yen at the beginning, it is also responsible for the currently high level of inflation and has proved to be detrimental to consumer spending and economic growth. The BOJ's monetary policy based on extremely low interest rates widened the gaps between Japanese interest rates and those of other countries and contributed to the historic low exchange rates of Japanese yen. The easy monetary policy has also enabled so-called "zombie" companies, which suffer from low levels of efficiency, productivity and competitiveness, to survive. The current estimate is that around 13 percent of Japanese companies are zombie companies (Lewis 2023). Due to the existence of many such companies, the Japanese economy has not been able to achieve sufficient economic growth and continues stagnating. Real wages have not grown either, despite the record amount of profits by some large Japanese companies. Indeed, real wages have decreased for 18 consecutive months as of September 2023 (Reuters 2023). The reflationists' assumption that (real) wages would increase with larger business profits based on the "trickle-down" theory has proved to be wrong.

As for fiscal policy, the LDP government has engaged in a large amount of public works to stimulate the economy, although the government spending for public works has been in a decreasing trend, especially during the Koizumi administration aimed at reducing public works in a neoliberal fashion. One of the main resources for the LDP to maintain political power is "interests group" politics, which has often been seen in the close and symbiotic relationship between the LDP and its supportive interest groups based on the quid pro quo – LDP's protection of and subsidies to interest groups and the votes (and donations) from interest groups to the LDP in return (bureaucrats are also part of the interest group politics).

Interest group politics was also seen in the second arrow of Abenomics aimed at economic growth through fiscal expansion. While the creation of jobs based on public works is a quintessential Keynesian policy and may be useful for stimulating economic growth to some extent, especially at the time of economic crises, the LDP government's reliance on public works for economic stimulation has wasted a huge amount of taxpayers' money and contributed to the worst situation in the world of the accumulated debt of the Japanese government – currently higher than 260 percent of GDP. Many companies engaging in public works are small- and medium sized enterprises that lack productivity and efficiency such as those in the construction sector. In this way, LDP's reliance on public works for both economic stimulation and its maintenance of power has contributed to the survival of non-competitive companies with low productivity. This further contributed to the lackluster performance of the Japanese economy on average in terms of GDP growth. The effectiveness of fiscal policy in achieving economic growth, especially the one through often unnecessary public works a la the LDP interest group politics, has proved to be limited after the creation of a large number of public works for decades.

As for structural reform, the LDP government has shown its intention to implement structural reform since the 1990s, but reforms were implemented insufficiently in many cases. As mentioned above, one of the power resources for the LDP (and bureaucrats) to maintain power has been interest group politics. This has restricted the degree of reforms aimed at enhancing labor productivity and economic efficiency. For example, the delay in the reform of financial supervision based on the "convoy" system enabled financial institutions lacking productivity, efficiency and competitiveness to survive longer than necessary (Amyx 2004). This exacerbated the problem of non-performing loans and contributed to economic stagnation in the 1990s.

The failure to reform regular employment in the form of easier dismissal may have deterred interfirm labor mobility from declining to promising industries although easier dismissal has no guarantee to increase employment or promote labor mobility, as seen in several European countries such as Italy that relaxed dismissal regulation (Cirillo et al, 2016). The content of Japanese labor market reform was mostly deregulation of non-regular employment, but this contributed to the increase in low-paid workers in precarious conditions (Watanabe 2014). The introduction of the "equal pay for equal work" principle in the Work-style Reform in 2018 was an appropriate measure in this context, but this principle has several loopholes in its implementation as it was introduced as a policy measure to promote economic growth rather that a measure to improve working conditions and enhance worker protection (Watanabe 2020). The increase in such workers who lack spending power is likely to have contributed to lower growth. The neoliberal deregulation of non-regular employment may be considered a bad example of structural reform as it has contributed to economic stagnation.

However, the above case does not negate the overall necessity of structural reform in certain industrial sectors. Many Japanese companies in the service sector suffer from low labor productivity and business inefficiency due to government regulation, anti-competitive business customs, a lack of digitalization, inefficient human resource management, inflexible business style, and so on. While structural reform implemented by the government may not solve all these problems, there are several things the government can and should do to promote economic growth, such as reducing government regulations to protect inefficient businesses. This is an essential measure to enhance productivity of Japanese businesses and promote economic growth. However, we probably cannot expect much from the governing party that depends on interest group politics.

While the implementation of appropriate structural reforms aimed at enhancing labor productivity and business efficiency is essential to overcome economic stagnation and promote economic growth, the over-reliance on macroeconomic policy (monetary and fiscal policy) in the ways proposed by reflationists of Abenomics has proved to be not only useless but also detrimental. The implementation of fiscal policy mostly based on the creation of public works is not as effective as it used to be in the past and should not be relied easily, given the increasing amount of social welfare spending despite the already very high level of the government debt-to-GDP ratio. The implementation of expansionary monetary policy could not achieve the 2% inflation target in the way the Abe administration and reflationists hoped – a virtuous cycle of the increase in corporate profits, the increase in workers' salaries and the increase in consumer spending. Instead, the 2% inflation target was achieved only because of the price hikes caused by external factors such as the disruption to global supply chains.

Abenomics failed to raise real wages that have stagnated since the collapse of the bubble economy, partly due to the lack of a mechanism in which a sufficient portion of corporate profits can be converted to workers' salaries. Instead, the expansionary monetary policy of Abenomics contributed to the significant depreciation of Japanese yen and has had a negative impact on consumer spending, thus contributing to economic stagnation. This monetary easing also contributed to the survival of inefficient businesses lacking productivity and competitiveness. This has also made it more difficult for Japan to overcome economic stagnation. The Japanese economy needs structural reform to achieve growth. Easy monetary policy and fiscal policy based on public works are convenient policy tools for the LDP government, but their usefulness for economic growth has proved to be limited. Instead, they are more likely to enable economic stagnation to continue.

References

Amyx, J. 2004. *Japan's Financial Crisis: Institutional Rigidity and Reluctant Change*. Princeton: Princeton University Press.

Bernanke, B., T. Laubach, F, Mishkin and A. Posen. 1999. *Inflation Targeting: Lessons from the International Experience*. Princeton: Princeton University Press.

Blomstrom, M., J. Corbett, F. Hayashi, and A. Kashyap, eds. 2003. *Structural Impediments to Growth in Japan*. Chicago: Chicago University Press.

Callen T. and J.D. Ostry, eds. 2003. *Japan's Lost Decade: Policies for Economic Revival*. Washington, D.C.: International Monetary Fund.

Cirillo, V., M. Fana and D, Guarascio. 2016. "Did Italy need more labour flexibility?", *Intereconomics: Review of European Economic Policy*, 51, 2: 79-86.

Garside, W.R. 2012. *Japan's Great Stagnation: Forging Ahead, Falling Behind*. Cheltenham, UK: Edward Elgar.

Hamada, K and Y. Okada. 2009. "Monetary and International Factors behind Japan's Lost Decades", *Journal of the Japanese and International Economies*, 23: 200-219.

Harada, Y. 2003. *Nihon no "Daiteitai" ga Owaru Hi*. Tokyo: Nihon Hyoronsha.

Hutchison, M.M. and F.E. Westermann, eds. 2006. *Japan's Great Stagnation: Financial and Monetary Policy Lessons for Advanced Economies*. Cambridge, MA: MIT Press.

Ito, T. and F.S. Mishkin. 2005. "Monetary Policy in Japan: Problems and Solutions", in T. Ito, H. Patrick and D.E. Weinstein, eds. *Reviving Japan's Economy*. Cambridge, MA: MIT Press.

Iwata, K., ed. 2003. *Mazu Defure wo Tomeyo*. Tokyo: Nihon Keizai Shimbunsha.

Katz, R. 1998. *Japan: The System that Soured: The Rise and Fall of the Japanese Economic Miracle*. Armonk, NY: M.E. Sharpe.

Krugman 1998. "It's Baack! Japans' Slump and the Return of the Liquidity Trap". *Brookings Papers on Economic Activity*, 2: 137-187.

Lewis, L. 2023. "Japan must start culling its zombie companies soon", *Financial Review*, January 19.

Maswood, J.S. 2002. *Japan in Crisis*. New York: Palgrave Macmillan.

Park, G., N.S. Katada, G. Chiozza and Y. Kojo. 2018. *Taming Japan's Deflation: The Debate over Unconventional Monetary Policy*. Ithaca, NY: Cornell University Press.

Posen, A. 1998. *Restoring Japan's Economic Growth*. Washington, D.C.: Institute for International Economics.

Reuters. 2023. "Japan's wages, consumer spending extend declines in test for BOJ policy", November 8.

Sakakibara, K. 2023. "IMF: Japanese economy to drop to 4th in world, trail Germany", *Asahi Shimbun*, October 25.

Wakatabe, M. 2015. *Japan's Great Stagnation and Abenomics: Lessons for the World*. Armonk, NY: Palgrave Macmillan.

Watanabe, H.R. 2014. *Labour Market Deregulation in Japan and Italy: Worker Protection under Neoliberal Globalisation*. London: Routledge.

_____. 2020. *The Japanese Economy*. New Castle, UK: Agenda Publishing.

Yoshino, N. and F. Taghizadeh-Hesary, eds. 2017. *Japan's Lost Decade: Lessons for Asian Economies*. Tokyo: Asian Development Bank Institute.

Women's economy.

Tokyo's new housing patterns and urban development.

Keywords: *Womenomics*, housing, gender, urban development.

Marcela Riva de MONTI (PhD candidate)
Universidad Politécnica de Madrid

In Japan, women's access to housing is materialised under the traditional patriarchal system with the male head of household. Women are seen as dependent on him. As the traditional saying goes, "女 わ 3回 に 家 なし - Onna wa sankai ni ie nashi" (women do not own a house during the three stages of their lives, neither as a daughter, nor as a wife, nor as a mother)[1]. These gendered social norms translate into gendered institutional structures that limit women's choices and access to housing (Hirayama, Y., & Izuhara, M. 2018).
The structural changes faced by post-growth societies explain the shift away from this conventional family model. The transformation of the family system based on traditional marriage is both a cause and a consequence of these societies, which are characterised by an ageing population and demographic decline on the one hand, and a delay in the formation of family units and low fertility rates on the other.

In this context, so-called *womenomics*[2] policies are based on the conceptualisation of economic growth as dependent on increased female labour force participation. This in turn implies that the female economy becomes an important factor in the consumer market. With this conceptual framework, under the government of Shinzo Abe (2012-2020), policies are being implemented to address the consequences of the "lost decade": the economic slowdown, the loss of relevance of the Japanese economy in the international arena, and the challenges of a shrinking population, by promoting the inclusion of women in the workforce at all levels and implementing neoliberal policies that include the liberalisation of government policies on access to housing.
In order to defend this argument, the report on *womenomics* analyses the growing weight of women in the consumption of luxury goods and, more importantly, in the single person housing market. This is a consequence of the increasing tendency of women to remain unmarried. With women's access to mortgage credit, their investment in real estate products is encouraged, forming a specific segmentation of the market (Aveline-Dubach, N. 2014)

As a result of the application of these policies, the theory of the man as the main economic support of the family unit is undermined, motivating the formation of diverse family units, including professional and single women with purchasing power, who prefer small units close to the workplace rather than the suburban units representative of the patriarchal system. As women enter the full-time labour market, family dynamics change and so do their spatial needs. Working women (married or not) prefer to live close to their place of work. This new demand for housing cannot be met by the traditional model of the women-housewife, where the preference is for housing in the periphery, where family life takes precedence, but rather by greater density within the nuclear city.

This growing tendency of women to remain single and escape patriarchal control consolidates a space of freedom inhabited mainly by elite female empowerment. As neo-liberal political trends have particularly influenced the redefinition of labour and housing systems, these policies result in individuals and families becoming responsible for many aspects of their lives, including access to housing and childcare, representing another dimension of widening social inequalities.

At the same time, the city of Tokyo is undergoing a process of transforming its productive base into a service-based economy, leaving the industrial base open to development. The process of

[1] Housing in Post-growth Society. Pag 122
[2] Goldman Sachs Japan Portfolio Strategy, © Goldman, Sachs & Co., Agosto, 1999

reorganisation of government agencies makes centrally located public land available to the market for intervention. The regulatory framework, at both national and local level, is equipped with the necessary instruments to accompany this regeneration process, which aims to make Tokyo a world reference city by encouraging foreign investment in the real estate market, incorporating many of its premises.

Based on these social, economic and regulatory vectors, this paper examines the urban policies implemented to respond to this new paradigm and the changes that are evident in the construction of the city.

Bibliography

Aveline-Dubach, N. (2014). New patterns of property investment in “post-bubble” Tokyo: The shift from land to real estate as a financial asset. *Globalization and New Intra-Urban Dynamics in Asian Cities,* , 265-294.

Elysia, J., Juniari, N. P., & Devi, N. L. P. C. (2023). Womenomics: Gender inclusivity as a growth strategy in japan. *Transformasi Global, 10*(1), 47-58.

Hirayama, Y., & Izuhara, M. (2018). *Housing in post-growth society: Japan on the edge of social transition* Routledge.

Koh, K., Yoshiharu, T., & Ryue, N. (2010). *Tokyo metabolizing* Toto.

Matsui, K. (2019). Is womenomics working? *Women Shaping Global Economic Governance,* , 151.

Yui, Y. (2020). Changes in essential facilities of housing estates in an aging society: The failure of city planning in japan. *The Rise in Vacant Housing in Post-Growth Japan: Housing Market, Urban Policy, and Revitalizing Aging Cities,* , 33-42.

STANFORD UNIVERSITY

"Here and There:" Energy Crisis and Environmental Concern in Contemporary Performance Arts

Vivien Jiaqian Zhu
ズー・ジヤチエン
朱嘉倩

Sixth Annual JEN Conference
The Japan Economy Network (JEN)
Department of Economics of SOAS, University of London
Graduate School of Economics, Institute of Economic Research at Hitotsubashi University
German Institute for Japanese Studies (DIJ)

November 21, 2023

Topic: Energy & Environment

Introduction

In 2011 Japan, on a Friday afternoon, a catastrophic earthquake struck the Fukushima prefecture. Then the tsunami caused the reactors of the Fukushima Daiichi nuclear plants to melt and leak out.[1] This unexpected disaster called attention to crisis management, environmental concern of nuclear radiation, and geopolitics in the aftermath of accidents.[2] In the wake of the Tōhoku earthquake and tsunami, artists attempt to explore ways to present the motif of 3/11 beyond geographic confinement—beyond Japan. With a consideration that not everyone is able

---

[1] For an anecdotal and insider view of the Fukushima Daiichi site and the Tokyo Electric Power Company, or TEPCO, see Margaret Steen, "Fukushima Points to Need for Sharing Crisis-Management Talent | Stanford Graduate School of Business," *Insights by Stanford Business*, last modified February 07, 2012, https://www.gsb.stanford.edu/insights/fukushima-points-need-sharing-crisis-management-talent.

[2] For press release of the TEPCO Internal Investigation Committee Final Report, see "Release of the Fukushima Nuclear Accidents Investigation Report," *TEPCO Releases ・Announcements*, June 20th, 2012, https://www.tepco.co.jp/en/press/corp-com/release/2012/1205638_1870.html. See also "Safety Measures Implementation at Kashiwazaki-Kariwa Nuclear Power Station," TEPCO, accessed November 21, 2023, https://www.tepco.co.jp/en/nu/kk-np/safety/index-e.html.
For a video of the current state of Fukuashima Daiichi nuclear plant, see "The current situation at Fukushima Daiichi NPS" –From 3.11 toward the future- (ver,Jan.2017), TEPCO Releases ・Announcements | Photo & Video Library - Videos, January 16, 2017, video, 9:09, https://www.tepco.co.jp/en/news/library/archive-e.html?video_uuid=o6iw41m6&catid=61795. For the decommissioning roadmap, see "東京電力ホールディングス(株) 福島第一原子力発電所の廃止措置等に向けた 中長期ロードマップ（案）| 平成 29 年 9 月 26 日 | 廃炉・汚染水対策関係閣僚等会議," September 26, 2017, https://www.tepco.co.jp/nu/fukushima-np/roadmap/2017/images2/t170926_04-j.pdf.
For an interview with TEPCO's Agile Energy X, see Andy Binns, "Corporate Explorers to Watch: Kenji Tateiwa, CEO Agile Energy X," last modified March 1, 2023, https://changelogic.com/blog/corporate-explorers-to-watch-kenji-tateiwa-ceo-agile-energy-x/. For the new company Agile Energy X, Inc., see "Formation of Agile Energy X, Inc., a New Company on a Mission to Accelerate the Introduction of Renewable Energy By Flexibly creating power demand, Agile Energy X will create digital and environmental value while mitigating grid congestion," *TEPCO Power Grid*, September 21, 2022, https://www.tepco.co.jp/en/pg/about/newsroom/press/archives/2022/20220921_01.html; "Agile Energy X, TRIPLE-1, TEPCO Power Grid to implement distributed computing that hybridizes renewable energy and cutting-edge semiconductors," *TRIPLE-1 NEWS*, accessed November 21, 2023, https://triple-1.com/en/news/release2022-12_2/.
Special thanks to Kenji Tateiwa 立岩健二 (tateiwa.kenji@nifty.com), Founder/CEO of Agile Energy X (https://agileenergyx.co.jp/en/), an alumni of Stanford Graduate School of Business - Stanford MBA Class of 2004, for source informations.

to have access to the nuclear zone, how do artists respond to 3/11 without direct physical proximity to Fukushima? How do overseas Japanese artists represent what took place back in their homeland—Japan? How do people outside Japan gain the access to experience and respond to 3/11? In an interview with regard to his performance art *Does this soup taste ambivalent?* (2014) (fig. 1), a contemporary Japanese artist Ei Arakawa touched upon this representational dilemma of "here and there," and attempted to mediate a geographic gap between New York (where the artist works) and Fukushima (where the artist comes from), and intended to bring his family (his brother and mother) to Frieze London to reach a geographic balance between Fukushima and the international world.[3]

Ei Arakawa the New York based artist has a strong personal connection to his hometown Fukushima. After the 3/11 disaster, given his dislocation in New York and his distance from Fukushima, Ei Arakawa decided to borrow the agency of the performance practice and cooperated with his brother Tomoo Arakawa as UNITED BROTHERS to participate in the artistic representation of Fukushima. In this conceptual performance project on 3/11 in 2014 Frieze Art Fair, Ei and Tomoo Arakawa aimed to "give people outside the Japan to experience the anxiety and dilemma over the safety of food in Fukushima in a direct corporeal way."[4] Besides UNITED BROTHERS, their mother bought ingredients from a neighbourhood sixty kilometres south of the power plant and flew to London to help cook soup in Green Tea Gallery

---

[3] Peter Yeung, "UNITED BROTHER Serve Soup From Fukushima at Frieze London," *Whitewall* (blog), October 15, 2014, http://www.whitewallmag.com/art/united-brothers-serve-soup-from-fukushima-at-frieze-london.

[4] Jake Wallis Simons, "Would you eat Fukushima soup? The all-consuming world of Frieze Art Fair," *CNN Style* (Blog), October 16, 2014 (10:31 AM EDT), http://www.cnn.com/2014/10/16/travel/frieze-art-fair-storify/.

in London.[5] Their performance reviewed the difficulty of how to present what happen in Japan to people in London.

The performance drew on food, one of our basic necessities, from Fukushima—site of 2011 nuclear disaster—to emblematically stand for areas haunted by radioactivity in the wake of 3/11. The piece provoked a psychological dilemma for London people to decide whether or not to try the soup; an action of eating the soup further invited and incorporated the audience's experience and response into the performance project. Audience's proximity to the soup, in some cases ingestion, associates people in London with post-3/11 radioactive exposures in Japan. The multi-sensory nature of the soup incorporated heat, smell, taste and viewing creation into cooking. Compared to photographs documenting disaster areas, does an incorporation of food make it more effective than two-dimensional photographs to present a concern of the risk of post-3/11 nuclear radiations in Japan? Does an involvement of eating food make it more accessible for the audience in London to experience what happens in Japan despite geographical distance?

The significance of the soup manifested through the UNITED BROTHERS' performance project stands out and further leads me to consider an incorporation of food in other art representations. The scene that UNITED BROTHERS together with their mother made soup in the gallery resonates with Rirkrit Tiravanija's (1961-) 1992 exhibition entitled *Untitled (Free)* (fig. 2) in New York, in which Tiravanija served rice and Thai curry for free in the gallery. Given that UNITED BROTHERS draws on Japanese soup to allude to the 3/11 disaster in Japan and that Rirkrit Tiravanija draws on Thai curry to suggest his Thai heritage, this ethnical and cultural

[5] Grace Wang, "DOES THIS SOUP TASTE AMBIVALENT?," *Opening Ceremony Art Talks* (Blog), October 21, 2014 (4:00 PM),http://blog.openingceremony.com/entry.asp?pid=10508.

reference indicated through food also reminds me of another socially-engaged art practice *Conflict Kitchen* (2010-) (fig. 3), a takeout restaurant in Pittsburg only serving food from countries from which the United States in currently in conflict with.

Bringing comparative examples of *Conflict Kitchen* and Rirkrit Tiravanija's *Untitled (Free/Still)* with *Does This Soup Taste Ambivalent?*, I attempt to extend a geographical boundary between "here and there" to those invisible yet crucial boundaries with regard to human relations. Other figurative moments of "here and there" can suggest boundaries between artists and spectators/participants, between social environment and human beings and between various ethical groups. An examination of an incorporation of food in art performances not only embodies repeated moments of "here and there," but also shows that an incorporation of the food enables performance arts to perform the representational difficulty of *boundaries*, to provide a corporeal medium to raise concerns on safety, and to take advantage of its *taste* to unite people from heterogeneous backgrounds. Repeated in art practices and social media, an incorporation of food is able to produce a new micro-social and micro-political narrative to reflect on the condition of food safety, human safety and community across time and space.

Does This Soup Taste Ambivalent?

In 2014 London, an artistic/sibling duo Ei and Tomoo Arakawa working under the name UNITED BROTHERS presented a live performance work *Does This Soup Taste Ambivalent?* at the Frieze Art Gallery. UNITED BROTHERS, with the help of their mother, made soup with ingredients from Fukushima, offered free soup to Frieze visitors at 1 pm every day. The soup was made up of dried radish and dried shiitake mushroom, and mixed with UK carrots and other vegetables.[6] Bright colors of the ingredients—green, yellow and red—left a visually pleasing impression on the audience and mediated the disturbing implication of nuclear radiation through a mobilization of audience's multiple senses—through the heat, smell, taste and viewing creation of the soup for instance.

In terms of artistic representations of 3/11, photographers like Handa Yasushi and Lieko Shiga went inside the nuclear zone to give people outside Fukushima or outside Japan a visual access to the disastrous areas. Rather than offer a visual re-presentation of 3/11 to an overseas audience, *Does This Soup Taste Ambivalent?* allows London people to have empathy with Fukushima people through haptic perceptions and active engagements into the performance, which can hardly be achieved by looking at documentary photos. London people were able to psychologically experience a choice people from Fukushima have to make every day—whether or not to consume potentially radioactive food. Their proximity to the soup also enabled them to physically perceive the potential risk of radioactivity. The soup made with ingredients from Fukushima served as a synecdoche to emblematically stand for toxicity and potential radioactivity from Japan, suggesting a presence of Fukushima—the "there"—through material identity to people in London—the "here."

Drawing on the agency of food, UNITED BROTHERS' conceptual performance more effectively dealt with a representational difficulty of "here and there" and further provoked a psychologic dilemma for London people. *Does This Soup Taste Ambivalent?* not only targeted at people who responded with "yes" and "no," but also embraced people with an "ambivalent" attitude towards whether or not to eat the soup. For instance, the Frieze art fair's co-founder

[6] Sherman Sam, "A conversation with Ei Arakawa," *OCULA* (Blog), October 19, 2014, https://ocula.com/magazine/conversations/ei-arakawa/.

Matthew Slotover admitted that he was skeptical about eating the soup.[7] The media referred to conceptual performance in the headlines as "nuclear soup"[8] or "Fukushima soup"[9] in a deceptively simple way. As for people outside Japan, anything related Fukushima can become a media sensation corresponding to a social presumption of potential radioactivity. According to Ei Arakawa, the PR company hired by Frieze also took advantage of this sensational effect of Fukushima to advertise Frieze art fair per se.[10]

Nevertheless, local NPO organisation examined the vegetables and detected no radioactive elements—Cesium-134 and Cesium-137—in vegetables. Main ingredients of the soup—shiitake mushroom and daikon radish—were also certificated safe by the Japanese Farmers' Association with no radioactivity detected. Ei Arakawa clarified that "I don't want people to be anxious. I wanted to present the choice whether you eat or not because it's good to have discussion, and activate discourse about the stress of the Fukushima disaster."[11]

Furthermore, UNITED BROTHERS' mother's presence to cook the soup provisionally transformed the gallery into a gendered and domestic space, which may evoke a sense of comfort

---

[7] Grace Wang, "DOES THIS SOUP TASTE AMBIVALENT?," *Opening Ceremony Art Talks* (Blog), October 21, 2014 (4:00 PM),http://blog.openingceremony.com/entry.asp?pid=10508.

[8] Jessica Holtaway, "What is the micro-political significance of Friezr Art Fair's 'Nuclear Soup'?," *Jessica Holtaway* (Blog), October 22, 2014 (7:15 AM), https://jessicaholtaway.wordpress.com/2014/10/22/what-is-the-micro-political-significance-of-frieze-art-fairs-nuclear-soup/.

[9] Jake Wallis Simons, "Would you eat Fukushima soup? The all-consuming world of Frieze Art Fair," *CNN Style* (Blog), October 16, 2014 (10:31 AM EDT), http://www.cnn.com/2014/10/16/travel/frieze-art-fair-storify/.

[10] Sherman Sam, "A conversation with Ei Arakawa," *OCULA* (Blog), October 19, 2014, https://ocula.com/magazine/conversations/ei-arakawa/.

[11] Peter Yeung, "UNITED BROTHER Serve Soup From Fukushima at Frieze London," *Whitewall* (blog), October 15, 2014, http://www.whitewallmag.com/art/united-brothers-serve-soup-from-fukushima-at-frieze-london.

at home. Between 30 minutes to 2 hours, about 80 to 100 soups were gone every day.[12] As for their mother, this piece meant more of an extension of cooking and offering free food than a conceptual performance. With cooking as part of the live performance, a gesture of sharing free food with fairgoers transformed spectators into participants and blurred the demarcation between the artist and the audience. An action of eating the soup further invited the audience's corporeal experience and response into the performance project. Audience's active response and physical perception kept constituting the performance practice and expanded the dynamic body of the live performance. As a consequence, cooking food in the gallery not only alters a correlation among artists, spectators and the performance per se, but also forms a new community that gathers people who are concerned about 3/11 and free food together.

Food Safety

Frieze Fairs' have an enduring connection with food: *Grizedale arts* bought in an experimental dining experience for visitors in 2012 and *FOOD 1971/2013* in 2013 was an art piece/restaurant where artists were invited to perform/cook on a stage.[13] Following this tradition, *Does This Soup Taste Ambivalent?* also draws on food to reflect on 3/11 in Frieze London. Daikon radish, the main ingredient of the soup, is a main food in Fukushima prefecture. In the wake of the 3/11 disaster, thousands of residents in Fukushima were ravaged with permanent ailments particularly, especially after consuming certain food and water (including Daikon

---

[12] Sherman Sam, "A conversation with Ei Arakawa," *OCULA* (Blog), October 19, 2014, https://ocula.com/magazine/conversations/ei-arakawa/.

[13] Harriet Thorpe, "Frieze London 2014," *Studio International* (Blog), November 7, 2014, http://www.studiointernational.com/index.php/frieze-art-fair-london-2014-review.

radish).[14] Building upon this social context, UNITED BROTHERS incorporated daikon radish—a metonym of Japanese everyday scene and Fukushima prefecture—into ingredients of the soup and emphasized the disturbing origin of Fukushima to provoke London people's doubt about food safety near the nuclear area. It is a physical proximity to the soup that enables many London people to build up a relation with post-3/11 radioactive exposures in Japan and have a corporeal reflection on food safety. They were able to psychologically experience a choice people from Fukushima have to make every day and to physically perceive the potential radioactivity through haptic engagements with the soup. Ingestion of the soup into one's own body challenges a psychological concern of being exposed to nuclear radiation.

Certificated safe by the Japanese Farmers' Association, the soup in *Does This Soup Taste Ambivalent?* attempted to create an opening-ending platform to involve active discourses. With a consideration of audience's haptic perceptions, the soup as an agency to represent 3/11 also points out the indispensable role of food as daily necessities and its close association with human body. The corporeal empathy London audience experienced through *Does This Soup Taste Ambivalent?* not only reminds them of nuclear radioactivity in Japan, but also draws their attention to the safety of their own body. In addition to a reflection on physiological safety provoked by the risk of nuclear radiation, Rirkrit Tiravanija's 1992 exhibition entitled *Untitled* and a socially-engaged art practice *Conflict Kitchen* offers insights into how to deal with invisible boundaries such as an interaction among people as well as a correlation between a community/individual and a social environment, and how to achieve a more comfortable/suitable

[14] Peter Yeung, "UNITED BROTHER Serve Soup From Fukushima at Frieze London," *Whitewall* (blog), October 15, 2014, http://www.whitewallmag.com/art/united-brothers-serve-soup-from-fukushima-at-frieze-london.

relation between the human body and external social context. Just as an evocation of homelike comfort produced by the mother's presence to cook soup mediates a relation between viewers and an environment of the museum, the "safe" state of a human body in a social context of museum or various ethnical groups might also be fulfilled through the multi-sensory nature of food per se.

Food and Community

Much of UNITED BROTHERS' *Does This Soup Taste Ambivalent?*'s idea of cooking and serving food in the gallery as well as an experiment on human interactions alludes to Rirkrit Tiravanija's 1992 exhibition entitled *Untitled (Free)* at 303 Gallery in New York, where Tiravanija served rice and Thai curry for free. As a pioneer of the Relational Aesthetics movement, the New York and Chiang Mai based Thai artist Rirkrit Tiravanija deconstructs the concept of art into its relational components, focuses on human interactions and often goes beyond performance to create socially-engaged conceptual works that blur the boundaries of art in novel ways and bridge division between public and private.[15] In *Untitled (Free)*, Rirkrit Tiravanija emptied out the office of 303 Gallery and installed a makeshift kitchen with fridge, hot plates, rice steamers, tables, and stools.[16] As Rirkrit Tiravanija were cooking Thai curry, museum-goers not only watched him cooking, but also dropped in and served him- or herself freely. Similar to the viewers' active participation in *Does This Soup Taste Ambivalent?*, viewers of *Untitled (Free)* also became within the performance or part of the art-making process as they ate curry and talked with friends or new acquaintances.

---

[15] "Rirkrit Tiravanija," *Artsy*, accessed December 17, 2016, https://www.artsy.net/artist/rirkrit-tiravanija.

[16] Jerry Saltz, "Conspicuous Consumption," *New York Art* (Blog), October 23, 2007, http://nymag.com/arts/art/reviews/31511/.

Centering on an experiment of relational aesthetics, both pieces incorporate a process of cooking and serving food as the main part of the live performance, breaks the boundary between viewer and artist/artwork and invite the viewer to interact with art in a more sociable way. Through ingesting food, these viewers not only shift from passive viewing to active participation, but also become an essential ingredient in the performance. Within such a cooking-as-art performance, food as an agency produces experimental relations among people and performs the function of art as sustenance, healing and communion.

In addition to an engagement into relational aesthetics, both pieces carry on a strong autobiographical element manifested through food. Rirkrit Tiravanija shows his Thai heritage through Thai curry and UNITED BROTHERS refers to their hometown through food from Fukushima. Given that both Thai curry and Japanese soup are foreign to American and European audience, an incorporation and exoticization of *foreign* food not only gives the performance art a cultural and ethnical implication of the imagined "there," but also performs a cultural and racial integration through a moment of various people eating one type of food together. Thus, an incorporation of food enables performance art to attract viewers, mobilize human interactions and create a sense of community within an art-as-cooking environment. The artist's choice of what kind of food to choose also enables the performance to raise concerns about one particular culture or ethnical groups.

Take another socially-engaged art practice *Conflict Kitchen* based in Pittsburgh, Pennsylvania for instance. *Conflict Kitchen*, a take out restaurant established in 2010, only serves cuisine from regions which the United States is in conflict with, including Iran, Afghanistan, Cuba, North Korea, Palestine and Venezuela. The restaurant offers a basic menu of traditional meals from one "conflict" country and replaces it with a new menu from a different place every five months: Kubideh from Iran, bolani from Afghanistan, arepas from Venezuela, and bibimbop from North Korea.[17] Framed by a colorful facade, the take out window often creates a platform for public dialogue. The food line presents a space for hungry Pittsburgers to engage with people and places the US media consistently distorts and misrepresents. The takeout counter is staffed by chefs and public artists to who are trained to facilitate conversations about the featured country for each time; each food wrapper is printed with personal profiles of people who live in the country being celebrated as well as articles on the country's food, art, religion, culture and government.[18]

Conflict Kitchen recognises the indispensable and effective role of food in human interactions, and further draws on food as a transmission instrument for cross-cultural dialogue and political discussion. Food in this socially-engaged art practice serves as a synecdoche to represent a country, an ethnical group or a political inclination, and as a site for cultural exchange among people from heterogeneous backgrounds. Through tasting a new meal, interacting with fellow customers and reading statements on the food wrapper, people from different social, geographical and political contexts are able to express their points of view, to hear other voices and to become part of the ensuing conversation. Therefore, *Conflict Kitchen* breaks down exclusive boundaries set up by one particular ethnical background, but rather produces an open micro-political platform for a community of people who share the same political tendency or the same *taste*. The psychological choice provoked by UNITED BROTHERS becomes a political

[17] Chelsea Haines, "Jon Rubin: Conflict Kitchen," *Guernica* (Blog), January 16, 2015, https://www.guernicamag.com/jon-rubin-conflict-kitchen/.

[18] "Case Study: Conflict Kitchen," *Beautiful Trouble*, accessed December 17, 2016, http://beautifultrouble.org/case/conflict-kitchen/.

choice in *Conflict Kitchen* of whether or not to eat food from a country which the United States is currently in conflict with. Some people do believe it is a political act to eat from *Conflict Kitchen* three days a week because they recognize that they are financially supporting the premise of the project.[19] Believing that the quickest way to a person's heart is through their stomach, *Conflict Kitchen* seeks to build a cross-cultural understanding, form a more inclusive community and promote peace globally by introducing U.S. people to the food and culture of those "conflict" places.[20]

Conclusion

An examination of UNITED BROTHERS' *Does This Soup Taste Ambivalent?*, Rirkrit Tiravanija's *Untitled (Free)* and *Conflict Kitchen* reviews attempts to mediate a representational gap between "here and there," between the artists and the viewer and between different ethnical groups. Besides London, UNITED BROTHERS also performed the process of cooking soup with Fukushima ingredients in Los Angeles. Besides the 1992 exhibition, Rirkrit Tiravanija re-created *Untitled (Free)* in 1995, 2007 and 2011. Besides purchasing food from *Conflict Kitchen*, people write and read the statements on the wrapper, take pictures of them with the food, and post those pictures on Facebook or Twitter to claim their political tendency. Through a process of self-reproduction and an engagement with mass media, these socially-engaged art practices gradually fabricate a micro-social and micro-political narrative to give thought to the notion of safety and community across time and space, and transform the role of food as a tool in

---

[19] Chelsea Haines, "Jon Rubin: Conflict Kitchen," *Guernica* (Blog), January 16, 2015, https://www.guernicamag.com/jon-rubin-conflict-kitchen/.

[20] "Case Study: Conflict Kitchen," *Beautiful Trouble*, accessed December 17, 2016, http://beautifultrouble.org/case/conflict-kitchen/.

performance into a part of the performance. An incorporation of food keeps constituting to the performance as a main ingredient and further enlivens the performance into a dynamic body.

Bibliography

"Agile Energy X, TRIPLE-1, TEPCO Power Grid to implement distributed computing that hybridizes renewable energy and cutting-edge semiconductors." *TRIPLE-1 NEWS*, Accessed November 21, 2023. https://triple-1.com/en/news/release2022-12_2/.

Binns, Andy. "Corporate Explorers to Watch: Kenji Tateiwa, CEO Agile Energy X." Last modified March 1, 2023. https://changelogic.com/blog/corporate-explorers-to-watch-kenji-tateiwa-ceo-agile-energy-x/.

"Case Study: Conflict Kitchen." *Beautiful Trouble*, accessed December 17, 2016, http://beautifultrouble.org/case/conflict-kitchen/.

"Formation of Agile Energy X, Inc., a New Company on a Mission to Accelerate the Introduction of Renewable Energy By Flexibly creating power demand, Agile Energy X will create digital and environmental value while mitigating grid congestion." *TEPCO Power Grid.* September 21, 2022. https://www.tepco.co.jp/en/pg/about/newsroom/press/archives/2022/20220921_01.html;

Haines, Chelsea. "Jon Rubin: Conflict Kitchen," *Guernica* (Blog), January 16, 2015, https://www.guernicamag.com/jon-rubin-conflict-kitchen/.

Holtaway, Jessica. "What is the micro-political significance of Friezr Art Fair's 'Nuclear Soup'?," *Jessica Holtaway* (Blog), October 22, 2014 (7:15 AM), https://jessicaholtaway.wordpress.com/2014/10/22/what-is-the-micro-political-significance-of-frieze-art-fairs-nuclear-soup/.

"Release of the Fukushima Nuclear Accidents Investigation Report." *TEPCO Releases ・ Announcements.* June 20th, 2012. https://www.tepco.co.jp/en/press/corp-com/release/2012/1205638_1870.html.

"Rirkrit Tiravanija." *Artsy*, accessed December 17, 2016, https://www.artsy.net/artist/rirkrit-tiravanija.

"Safety Measures Implementation at Kashiwazaki-Kariwa Nuclear Power Station." TEPCO. Accessed November 21, 2023. https://www.tepco.co.jp/en/nu/kk-np/safety/index-e.html.

Saltz, Jerry. "Conspicuous Consumption," *New York Art* (Blog), October 23, 2007, http://nymag.com/arts/art/reviews/31511/.

Sam, Sherman. "A conversation with Ei Arakawa," *OCULA* (Blog), October 19, 2014, https://ocula.com/magazine/conversations/ei-arakawa/.

Simons, Jake Wallis. "Would you eat Fukushima soup? The all-consuming world of Frieze Art Fair," *CNN Style* (Blog), October 16, 2014 (10:31 AM EDT), http://www.cnn.com/2014/10/16/travel/frieze-art-fair-storify/.

Steen, Margaret. "Fukushima Points to Need for Sharing Crisis-Management Talent | Stanford Graduate School of Business." *Insights by Stanford Business*. Last modified February 07, 2012. https://www.gsb.stanford.edu/insights/fukushima-points-need-sharing-crisis-management-talent.

"The current situation at Fukushima Daiichi NPS" –From 3.11 toward the future- (ver,Jan.2017). TEPCO Releases ・Announcements | Photo & Video Library - Videos. January 16, 2017. Video, 9:09.https://www.tepco.co.jp/en/news/library/archive-e.html?video_uuid=o6iw41m6&catid=61795.

Thorpe, Harriet. "Frieze London 2014," *Studio International* (Blog), November 7, 2014, http://www.studiointernational.com/index.php/frieze-art-fair-london-2014-review.

"東京電力ホールディングス(株) 福島第一原子力発電所の廃止措置等に向けた 中長期ロードマップ（案）| 平成 29 年 9 月 26 日 | 廃炉・汚染水対策関係閣僚等会議," September 26, 2017, https://www.tepco.co.jp/nu/fukushima-np/roadmap/2017/images2/t170926_04-j.pdf.

Wang, Grace. "DOES THIS SOUP TASTE AMBIVALENT?," *Opening Ceremony Art Talks* (Blog), October 21, 2014 (4:00 PM),http://blog.openingceremony.com/entry.asp?pid=10508.

Yeung, Peter. "UNITED BROTHER Serve Soup From Fukushima at Frieze London," *Whitewall* (blog), October 15, 2014, http://www.whitewallmag.com/art/united-brothers-serve-soup-from-fukushima-at-frieze-london.

Image

Fig. 1 UNITED BROTHERS, *Does This Soup Taste Ambivalent?*, 2014. A live performance at Frieze London. Photo courtesy of United Brothers.

Fig. 2 Rirkrit Tiravanija. *Untitled (Free)*. 1992. Refrigerator, table, chairs, wood, drywall, food and other materials; dimensions variable.

Fig. 3 Jon Rubin and Dawn Weleski, *Conflict Kitchen*, 2010-present. A take out restaurant that only serves cuisine from regions with which the United States is in conflict.

Understanding Japanese Inflation as a Joint Monetary-Fiscal Phenomenon*

Nobuhiro Abe[†], Takuji Fueki[‡], and Sohei Kaihatsu[§]

November 2023

Abstract

This article addresses what roles monetary and fiscal policy stances play on the behavior of inflation in Japan. A Markov switching dynamic stochastic general equilibrium model with changes in monetary/fiscal policy interaction is estimated. Focusing in particular on the prolonged deflation observed in Japan, we find that Japan's policy regime was characterized by the combination of passive monetary and passive fiscal policy. We quantitatively clarify that this combination played a substantial role in propagating negative demand shocks, leading to prolonged deflation. But, after the introduction of unconventional monetary policy, the monetary policy then switched from a passive regime to an active one. We show that the policy regime change has helped to push up inflation. We also argue that the optimistic beliefs on the passive fiscal policy regime play a role in resolving the public debt valuation puzzle in Japan.

JEL classification: E52, E62, C32

Keywords: Unconventional monetary policy, Inflation, Markov-switching DSGE

*The authors are grateful to Kosuke Aoki, Andrea Ferrero, Naohisa Hirakata, Hibiki Ichiue, Ryo Jinnai, Takushi Kurozumi, Kevin Lansing, Andrew Levin, Ichiro Muto, Toshitaka Sekine, Shigenori Shiratsuka, Nao Sudo, Tomohiro Sugo, Yosuke Uno, Shingo Watanabe, Fan Dora Xia, Lee Ohanian, and seminar participants at the Bank of Japan, the Bank for International Settlements, the 2020 World Congress of the Econometric Society, Kobe University for helpful comments. We also thank Hibiki Ichiue and Yoichi Ueno for providing the data on shadow interest rates. Any remaining errors are the sole responsibility of the authors. The views expressed in this paper are those of the authors and do not necessarily reflect the official views of the Bank of Japan.

†Bank of Japan (E-mail: nobuhiro.abe@boj.or.jp)

‡Kagawa University (E-mail: fueki.takuji@kagawa-u.ac.jp)

§Bank of Japan (E-mail: souhei.kaihatsu@boj.or.jp)

1 Introduction

In response to the 2008/2009 global financial crisis, we have seen dramatic changes in monetary and fiscal policies and increases in uncertainty surrounding policy conduct. For instance, to overcome the limitations of conventional monetary policy due to the zero lower bound (ZLB) on nominal interest rates, central banks in advanced economies introduced unconventional monetary policies such as quantitative easing and forward guidance on future policies. Meanwhile, fiscal positions have started to deteriorate through an expansion of fiscal expenditure to stimulate the economy. Worsening fiscal positions have in turn put great uncertainty and restrictions on further fiscal expansions, which has brought the risks linked to fiscal imbalances to the forefront of policy concerns.

These developments have led to a renewed interest in the sensitivity of the economy to changes in the monetary/fiscal policy mix. One of the first studies on this topic is that by Leeper (1991). This has been followed by a growing number of studies estimating Markov Switching Dynamic Stochastic General Equilibrium (MS-DSGE) models that allow for changes in the mix of monetary and fiscal policies (Bianchi and Ilut (2017); Bianchi and Melosi (2017)). Such models make it possible to identify changes in monetary-fiscal policy combination and quantify the impact of such changes on the behavior of economic outcomes.

However, few studies have assessed whether the introduction of unconventional monetary policy has enabled central banks to actively respond to the inflation rate. In addition, research on how changes in the monetary/fiscal policy mix have shaped economic outcomes after the global financial crisis has been scarce. The aim of this paper is to fill this gap.

To this end, we construct and estimate a new Keynesian model allowing for changes in the monetary and fiscal policy mix, using quarterly data for Japan for the period from 1982:2Q to 2017:1Q. Our focus is on Japan because it has a long history of implementing unconventional monetary policies. Thus, Japan provides a great laboratory to examine whether the unconventional policies pursued by central banks have helped actively respond to inflation rate. After estimating our model, we then quantitatively investigate the impact of changes in monetary-fiscal policy combination on the evolution of Japan's economy. More specifically, we address the following questions: Has the introduction of the BoJ's unconventional policy enabled to actively respond to inflation rate? What is the role played by the policy mix in the behavior of inflation? And has the unconventional policy pursued by the Bank of Japan helped to push up inflation?

The MS-DSGE model that we construct builds on the influential contribution of Bianchi and Ilut (2017). They develop a recurrent MS-DSGE model with monetary and fiscal policy interaction and estimate the model for the U.S. economy focusing on the period from 1954 to 2009, which does not include any periods in which the ZLB was binding. They consider three monetary/fiscal policy regimes. The first consists of active monetary and passive fiscal policy (AM/PF), where both the interest rate response to inflation and the tax response to debt are strong.[1] The second consists of passive monetary and active fiscal policy (PM/AF), where both responses are weak. Finally, the third consists of active monetary and active fiscal policy (AM/AF).

This paper differs from Bianchi and Ilut (2017) in two major respects. First, we estimate our model using the shadow rate as an aggregate that captures both conventional and unconventional monetary policy. Specifically, following Wu and Zhang (2019), we assume the monetary authority sets the shadow nominal interest rate in response to inflation. The shadow rate essentially corresponds to the policy rate to reflect the effects of conventional monetary policy when the ZLB is not binding. However, it can be below zero when the ZLB is binding to reflect the effects of unconventional monetary policy. Hence, the shadow rate is informative in order to evaluate the response of monetary policy to inflation even when the ZLB is binding and unconventional monetary policy is implemented. In addition, using the shadow rate allows us to reduce the computational burden of handling the ZLB, because the shadow rate can be below zero and does not have a ZLB.

Second, inspired by Chen et al. (2019), we consider a richer set of policy regimes and transitions across regimes. That is, we also consider the PM/PF regime, which Bianchi and Ilut (2017) do not consider in detail.[2] The PM/PF regime consists of passive monetary and passive fiscal policy, where the response to inflation of the monetary authority is weak and the response to debt of the fiscal authority is strong. This richer set of policy regimes allows

[1] We use the terminology of Leeper (1991) in specifying policies as active or passive. We provide a detailed explanation further below.

[2] In addition, we do not restrict the transition across regimes. For example, it is not necessary to transition through the AM/AF regime when moving from the PM/AF regime to the AM/PF regime as in Bianchi and Ilut (2017).

us to identify the evolution of monetary and fiscal policies without restrictions.[3]

Our main findings are twofold:

- From 1998 to 2013, a policy mix in Japan was characterized by a PM/PF regime: During the period from 1998 to 2013, the ZLB substantially constrained the Bank of Japans ability to mitigate the impact of negative demand shocks on the economy, leading to deflation. In addition, agents' perceptions mattered: once the PM/PF regime was in place, it was expected to last, and agents believed that the negative impact of demand shocks was unlikely to be mitigated. Consequently, the PM/PF policy mix played a substantial role in the propagation of negative demand shocks and contributed to the prolonged deflation in Japan. Our results indicate that if monetary policy had been more active (i.e., unconventional policies had been implemented more aggressively), the economy would have escaped from deflation.

- We find that since the introduction of quantitative and qualitative easing (QQE) in 2013:2Q, an active monetary policy regime has been in place. This finding suggests that the introduction of QQE enables the Bank of Japan to actively respond to the inflation rate and thus has helped to push up inflation. [4]

CONTACT WITH THE LITERATURE Our article is closely connected to two strands of the literature. First, our article builds on the recent literature on quantitatively studying interactions between fiscal and monetary policies. Much of the recent work has argued that the monetary/fiscal policy interaction is a key to understanding inflation dynamics in the United States (Bianchi (2016); Bianchi and Ilut (2017); Bianchi and Melosi (2016); Davig and Leeper (2006); Bhattarai et al. (2016); Chen et al. (2019)). Among others, the most closely related to this paper is Bianchi and Ilut (2017). They estimate a model allowing for changes in the stance of monetary and fiscal policy rules over time using the US data. They then stress that a combination of passive monetary policy and active fiscal policy led to higher inflation from 1965 to 1982. They also find an active monetary/passive fiscal decreased

[3] Bianchi and Ilut (2017) show that when the PM/PF regime is introduced into the model, the performance of the model in terms of the marginal likelihood deteriorates. However, their observation period does not include any periods in which the ZLB was binding. Different from their study, our analysis includes periods in which the ZLB was binding.

[4] We also apply our method to the United States using the shadow federal funds rate estimated by Ichiue and Ueno (2018). The results are shown in Appendix D.

inflation sharply in the 1980s. However, they focus on the period when the conventional monetary policy had been implemented. We push the approach by Bianchi and Ilut (2017) further to include the period when unconventional policies have been introduced. More specifically, what sets this paper apart from them is twofold. First, this paper assesses the stance of unconventional policies such as quantitative easing and forward guidance. Second, we apply to Japan and examine the role of monetary and fiscal interactions in escaping from pro-longed low inflation.

Second, our findings provide new insights into the literature on testing the impacts of unconventional monetary policies. There is a still growing literature studying the role played by the unconventional monetary policy in tackling low inflation in the aftermath of the Great Recession theoretically and empirically (Gertler and Karadi (2011); Gertler and Karadi (2013); Sims and Wu (2021); Hohbergera et al. (2019); Ikeda et al. (2022)). However, few studies assess how actively central banks has responded to inflation rate after the introduction of unconventional monetary policies. In addition, how the policy mix has shaped the macroeconomic dynamics has not been examined. The contribution of this paper to this literature is to fill this gap, quantifying the macroeconomic consequences of policy interactions after the introduction of unconventional policies in Japan.

Structure The remainder of the paper is organized as follows. Section 2 describes the model and the policy mix structure. Section 3 presents the solution method for the MS-DSGE model and the empirical strategy. Section 4 shows the estimation results and the model dynamics. Section 5 draws on counterfactual simulations to discuss the role of the policy mix in accounting for inflation dynamics. Section 6 presents concluding remarks.

2 Model

We follow the standard New Keynesian model with fiscal block and independent Markov switching processes developed by Bianchi and Ilut (2017). The model allows for recurrent changes in the policy mix (ξ_t^{sp}) and volatility regimes (ξ_t^{vo}) and contains a wide range of fiscal variables such as government debt, government purchases, tax revenue, and government expenditure.

For our analysis, we add two features to the approach developed by Bianchi and Ilut

(2017). First, the monetary authority is assumed to set the shadow interest rate, which can be interpreted as an aggregate that captures the overall effect of unconventional monetary policies. Second, we consider a richer set of policy regimes in that we also allow for a passive monetary/passive fiscal policy regime (PM/PF).

The economy consists of a continuum of identical households, a continuum of monopolistically competitive intermediate goods producing firms, perfectly competitive final goods producing firms, a government that engages in fiscal policy, and a central bank that controls monetary policy.

2.1 HOUSEHOLDS Households maximize the following lifetime utility function separable into consumption C_t and hours worked h_t:

$$\mathbb{E}_0\left[\sum_{s=0}^{\infty}\beta^s e^{d_s}\left\{\log(C_s-\Phi C^A_{s-1})-h_s\right\}\right] \tag{1}$$

where C^A_t stands for the average level of consumption and Φ denotes the degree of external habit formation. We assume that demand (preference) shock, d_s, follows an autoregressive process: $d_t=\rho_d d_{t-1}+\sigma_{d,\xi^{vo}_t}\epsilon_{d,t}$ where $\epsilon_{d,t}\sim N(0,1)$. ξ^{vo}_t denotes the volatility regime which is in place at time t. We allow for two volatility regimes, Low and High, which is discussed in detail below.

Households' budget constraint is given by

$$P_tC_t+P^s_tB^s_t+P^m_tB^m_t=P_tW_th_t+B^s_{t-1}+(1+\rho P^m_t)B^m_{t-1}+P_tD_t-T_t+TR_t \tag{2}$$

where P_t is the general price level, W_t is the real wage, D_t represents firmsreal profits, T_t is a lump-sum tax, and TR_t stands for transfers. In this economy, we consider short-term and long-term government bonds with different maturities (B^s_t and B^m_t). Short-term bonds mature in one period, while long-term bonds are a combination of government bonds with different maturities longer than one term. While the net supply of one period government bonds B^s_t is zero, that of long-term government bonds B^m_t is non-zero. Accordingly, B^s_t has a price of P^s_t, which is equal to R^{-1}_t, while B^m_t has a price of P^m_t, which has the payment structure $\rho^{T-(t+1)}$ for $T>t$ and $0<\rho<1$. Then, the value of long-term bonds issued in period t in future period $t+i$ can be computed as $P^{m-i}_{t+i}=\rho^i P^m_{t+i}$.

2.2 Firms Final good producers aggregate intermediate goods, which are indexed by $j \in [0,1]$:

$$Y_t = \left(\int_0^1 y_t(j)^{1-\upsilon_t} dj \right)^{\frac{1}{1-\upsilon_t}} \tag{3}$$

Firms take input prices $p_t(j)$ and output prices P_t as given. The demand for inputs is given by the profit maximization condition:

$$y_t(j) = \left(\frac{p_t(j)}{P_t} \right)^{-\frac{1}{\upsilon_t}} Y_t \tag{4}$$

where $\frac{1}{\upsilon_t}$ is the time-varying elasticity of substitution across differentiated intermediate input goods. Under the assumption of free entry into the final good market, profits are zero in equilibrium, and the price of the aggregate good is given by:

$$P_t = \left(\int_0^1 p_t(j)^{\frac{\upsilon_t - 1}{\upsilon_t}} dj \right)^{\frac{\upsilon_t}{\upsilon_t - 1}} \tag{5}$$

We define gross inflation rate at time t as $\Pi_t = P_t/P_{t-1}$. Intermediate goods firm, denoted with the subscript j, produce their products only with labor, $y_t(j) = A_t h_t^{1-\alpha}(j)$. A_t is an exogenous productivity process given by $\ln(A_t/A_{t-1}) = \gamma + a_t$, where $a_t = \rho_a a_{t-1} + \sigma_{a,\xi_t^{vo}} \epsilon_{a,t}$ and $\epsilon_{a,t} \sim N(0,1)$. Intermediate goods producing firms face the following quadratic adjustment costs:

$$AC_t(j) = \frac{\varphi}{2} \left(\frac{p_t(j)}{p_{t-1}(j)} - \Pi_{t-1}^{\zeta} \Pi^{1-\zeta} \right)^2 \frac{p_t(j)}{P_t} y_t(j) \tag{6}$$

where φ governs the price stickiness in the economy, Π is the steady state of inflation rate, Π_t, and the parameter ζ denotes the degree of indexation to lagged inflation. The elasticity of substitution can be converted into the markup, which can be characterized by $\Theta_t = \frac{1}{1-\upsilon_t}$. As in the conventional New Keynesian model with lagging inflation term, the mark-up can be rescaled to $\mu_t \left(\equiv \frac{\kappa}{1+\zeta\beta} \log(\frac{\Theta_t}{\Theta}) \right)$ with the slope of the Phillips curve, $\kappa \left(\equiv \frac{1-\upsilon}{\upsilon \phi \Pi^2} \right)$. We assume the rescaled markup follows an autoregressive process, $\mu_t = \rho_\mu \mu_{t-1} + \sigma_{\mu,\xi_t^{vo}} \epsilon_{\mu,t}$, where $\epsilon_{\mu,t} \sim N(0,1)$. Firm j chooses its labor input $h_t(j)$ and the price $p_t(j)$ to maximize the present value of future profits:

$$\mathbb{E}_t \left[\sum_{s=t}^{\infty} q_s \left(\frac{p_s(j)}{P_s} y_s(j) - W_s h_s(j) - AC_s(j) \right) \right] \tag{7}$$

where q_s is the real stochastic discount factor of households, who own the intermediate goods firms.

2.3 Government We assume the government's debt burden solely comes from the long-term bond, and the interest payment for the short-term bond is unnecessary since the net supply of one-period debt, B_t^s, is assumed to be zero in the economy. Thus, the flow budget constraint of the government is:

$$P_t^m B_t^m = B_{t-1}^m (1 + \rho P_t^m) - T_t + P_t G_t + TR_t + TP_t \tag{8}$$

where $P_t^m B_t^m$ is the market value of debt, T_t represents government tax revenues, $P_t G_t$ is government goods purchases, and TR_t denotes government transfers. The term TP_t reflects measurement errors and is used to fill the gap between the market value of debt at the end of the last quarter and the sum of debt and the primary balance in the quarter. We assume this gap includes changes in the maturity structure, which we cannot rigorously take into account here.

We convert all fiscal variables in terms of the nominal GDP when considering the equilibrium condition for long-term debt. Dividing the both side of equation (8) by the nominal GDP, $P_t Y_t$, we have:

$$b_t^m = (b_{t-1}^m R_{t-1,t}^m)/(\Pi_t Y_t / Y_{t-1}) - \tau_t + g_t + tr_t + tp_t, \tag{9}$$

where lower case letter denotes fiscal variables with respect to the nominal GDP, for example, $b_t^m \equiv \frac{P_t^m B_t^m}{P_t Y_t}$. The return on the long-term bond is, $R_{t-1,t}^m = \frac{1+\rho P_t^m}{P_{t-1}^m}$ and we assume the exogenous process for the term premium is, $tp_t = \rho_{tp} tp_{t-1} + \sigma_{tp,\xi_t^{vo}} \epsilon_{tp,t}$ and $\epsilon_{tp,t} \sim N(0,1)$. Linearized government expenditure, $E_t = P_t G_t + TR_t$, as a fraction of GDP, $\tilde{e}_t$, is divided into a short-term component, $\tilde{e}_t^S$, and a long-term component, $\tilde{e}_t^L$:

$$\tilde{e}_t^L = \rho_{e^L} \tilde{e}_{t-1}^L + \sigma_{e^L,\xi_t^{vo}} \epsilon_{e^L,t} \tag{10}$$

$$\tilde{e}_t^S = \rho_{e^S}\tilde{e}_{t-1}^S + (1-\rho_{e^S})\phi_y(\hat{y}_t - \hat{y}_t^*) + \sigma_{e^S,\xi_t^{vo}}\epsilon_{e^S,t} \tag{11}$$

where $\epsilon_{e^L,t}{\sim}N(0,1)$ and $\epsilon_{e^S,t}{\sim}N(0,1)$. We assume that the long-term component is exogenous and represents major government programs such as the pension system. As such, it does not react to business cycle fluctuations. Instead, it is the short-term component that reacts to business cycle fluctuations and hence the output gap, $\hat{y}_t - \hat{y}_t^*$. Next, we define the ratio of government transfers in total government expenditure as $\chi_t{\equiv}TR_t/E_t$ and assume that

$$\tilde{\chi}_t = \rho_\chi\tilde{\chi}_{t-1} + (1-\rho_\chi)\iota_y(\hat{y}_t - \hat{y}_t^*) + \sigma_{\chi,\xi_t^{vo}}\epsilon_{\chi,t}, \tag{12}$$

where $\epsilon_{\chi,t}{\sim}N(0,1)$. Below, we refer to $\epsilon_{\chi,t}$ as transfer shocks.

Lastly, the market clearing condition for final goods is,

$$Y_t = G_t + C_t. \tag{13}$$

2.4 Monetary and Fiscal Rules, and Recurrent Regime Change

As noted above, this model allows for independent changes in the conduct of monetary and fiscal policy. In what follows, we first show the principal monetary and fiscal policy rules and then provide details of the regime switching parameters.

Monetary policy The monetary policy authority sets the interest rate and follows a feedback rule, which Wu and Zhang (2019) call the shadow rate Taylor rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_{R,\xi_t^{sp}}} \left[\left(\frac{\Pi_t}{\Pi}\right)^{\psi_{\pi,\xi_t^{sp}}} \left(\frac{Y_t}{Y_t^*}\right)^{\psi_{y,\xi_t^{sp}}}\right]^{(1-\rho_{R,\xi_t^{sp}})} e^{\sigma_{R,\xi_t^{vo}}\epsilon_{R,t}}. \tag{14}$$

R_t is the nominal interest rate, but it can potentially go negative, as described below. R and Π denote the steady state level of nominal interest and inflation rates. The shock to monetary policy rule follows i.i.d. process, $\epsilon_{R,t}{\sim}N(0,1)$. Note that the monetary policy response parameters $\rho_{R,\xi_t^{sp}}$, $\psi_{\pi,\xi_t^{sp}}$, and $\psi_{y,\xi_t^{sp}}$ could be switched. ξ_t^{sp} represents the monetary/fiscal policy regime which is in place at t. There are four monetary/fiscal policy regimes, which is also discussed in detail below.

In a departure from Bianchi and Ilut's (2017) setup, where R_t represents the policy interest rate, in our model R_t represents the shadow interest rate, which can be negative when the ZLB is binding and unconventional policy is implemented. In other words, R_t essentially corresponds to the policy rate (the call rate in Japan's case) when the ZLB is not binding but can be below zero when the ZLB is binding and in this case reflects the effects of unconventional monetary policy. Thus, like Wu and Zhang (2019), we can incorporate both conventional and unconventional policy by using the shadow rate.

Fiscal policy The fiscal authority collects lump-sum taxes from households according to the following rule:

$$\tilde{\tau}_t = \rho_{\tau,\xi_t^{sp}}\tilde{\tau}_{t-1} + (1-\rho_{\tau,\xi_t^{sp}})[\delta_{b,\xi_t^{sp}}\tilde{b}^m_{t-1} + \delta_e\tilde{e}_t + \delta_y(\hat{y}_t - \hat{y}_t^*)] + \sigma_{\tau,\xi_t^{vo}}\epsilon_{\tau,t} \tag{15}$$

where $\tilde{\tau}_t$ and $\tilde{b}^m$ is the deviation of the tax-to-GDP ratio and debt-to-GDP ratio from its own steady state, and $\epsilon_{\tau,t}{\sim}N(0,1)$. Taxes respond not only to the debt $\delta_{b,\xi_t^{sp}}$ but also to government expenditure δ_e and the GDP gap δ_y. Note that, like the monetary policy response parameters, the fiscal policy response parameters ρ_τ, ξ_t^{sp}, and $\delta_{b,\xi_t^{sp}}$ could be switched.[5]

Policy mix Following Leeper (1991), the general criteria on parameter values regarding the existence and uniqueness of a solution to the model in the absence of policy regime switches are as follows:

- Regime I: Passive monetary and active fiscal policy; i.e., $\psi_\pi < 1$ and $\delta_b < \beta^{-1} - 1$.
- Regime II: Passive monetary and passive fiscal policy; i.e., $\psi_\pi < 1$ and $\delta_b > \beta^{-1} - 1$.
- Regime III: Active monetary and active fiscal policy; i.e., $\psi_\pi > 1$ and $\delta_b < \beta^{-1} - 1$.
- Regime IV: Active monetary and passive fiscal policy; i.e., $\psi_\pi > 1$ and $\delta_b > \beta^{-1} - 1$.

Let us consider these regimes in more detail. In Regime I, the passive monetary and active fiscal (PM/AF) regime, monetary policy does not satisfy the Taylor principle. In addition, the fiscal authority weakly responds to debt. In other words, the tax response

[5] We assume that the inflation target and the target for the government debt-to-GDP ratio are constant over time. As highlighted by Bianchi (2013), it is difficult to distinguish a change in a particular target from a change in the policy response parameters, unless we have some additional information to identify a change in a particular target. Hence, following studies such as Bianchi and Ilut (2017), we assume that what changes is the response with which policy makers try to achieve the target (goal), not the target itself, to reduce the difficulties of identifying policy regimes.

rate to debt is smaller than the steady state quarterly real interest rate $\beta^{-1} - 1$. Therefore, taxes are not sufficiently adjusted to keep the government debt burden on a stationary path. Next, in Regime II, both fiscal and monetary policy are passive (PM/PF) and the economy is subject to multiple equilibria. This is because the level of debt does not directly affect inflation dynamics (Bhattarai et al. (2014)). In Regime III, both fiscal and monetary policy are active (AM/AF), and no stationary equilibrium exists, which means the transversality condition is violated, as both fiscal and monetary authorities try to determine the price level without paying attention to the debt level. Finally, in Regime IV, monetary policy is active and fiscal policy is passive (active monetary and passive fiscal policy, AM/PF). The monetary policy dominates the fiscal policy when the price level is determined. In other words, the Taylor rule determined the inflation rate. As for fiscal policy, since the fiscal authority firmly adjusts taxes to keep debt stable, the stance of fiscal policy does not matter when determining the price level.[6]

However, the above arrangement is only valid if economic agents are unaware of regime changes and assume that the current regime will last forever. When recursive changes between regimes are taken into account, as in this paper, the argument of uniqueness of equilibrium is not particularly valid. In our model, monetary and fiscal policy regimes change freely and independently of each other. Therefore, our model allows different combinations of active and passive monetary and fiscal policies at different points in time. As outlined above, the PM/PF regime leaves the equilibrium undetermined in the absence of policy regime switches. In addition, an AM/AF regime means that either no equilibrium exists or, if it does exist, the equilibrium is explosive and non-stationary. However, when regime changes are modeled as independent Markov switching processes as in our model, the PM/PF regime does not necessarily leave the equilibrium indeterminate, as is mentioned by Chen et al. (2019). Similarly, the AM/AF regime does not necessarily violate the transversality condition, because agents correctly assign a positive probability to returning to a regime that prevents the government debt from accumulating in an explosive manner.

[6]This is the regime usually assumed in the standard new Keynesian literature. It is also helpful to mention the relationship between these regimes and Woodford (1995) terms Ricardian and non-Ricardian. Sometimes economists refer to the AM/PF regime as Ricardian and the PM/AF regime as non-Ricardian. It should be noted, however, that these one-to-one correspondences are only possible when assuming an economy with no regime changes and that when there are regime changes, for example, even AM/PF may behave in a non-Ricardian manner.

Transition matrices In addition to the four policy regimes outlined above (PM/AF, PM/PF, AM/AF, and AM/PF), we allow two volatility regimes, Low and High. Here, we describe the transition matrices governing the probability of transition from one regime to another. In our model, agents are aware of the possibility of a switch to another regime or staying in the current regime and thus take the possibility of regime changes into account when forming expectations and making decisions. Therefore, the transition probability matrices play an important role in determining the economic outcomes. Specifically, the transition probability matrices look as follows.

$$H^{sp} = \begin{bmatrix} H^{sp}_{11} & H^{sp}_{12} & H^{sp}_{13} & H^{sp}_{14} \\ H^{sp}_{21} & H^{sp}_{22} & H^{sp}_{23} & H^{sp}_{24} \\ H^{sp}_{31} & H^{sp}_{32} & H^{sp}_{33} & H^{sp}_{34} \\ H^{sp}_{41} & H^{sp}_{42} & H^{sp}_{43} & H^{sp}_{44} \end{bmatrix} \tag{16}$$

$$H^{vo} = \begin{bmatrix} H^{vo}_{11} & H^{vo}_{12} \\ H^{vo}_{21} & H^{vo}_{22} \end{bmatrix} \tag{17}$$

where for H^{sp} subscripts 1, 2, 3, and 4 correspond to the PM/AF, PM/PF, AM/AF, and AM/PF regime, respectively. For the stochastic volatilities, H^{vo}, subscripts 1 and 2 correspond to the Low and High volatility regime, respectively.[7]

3 Estimation

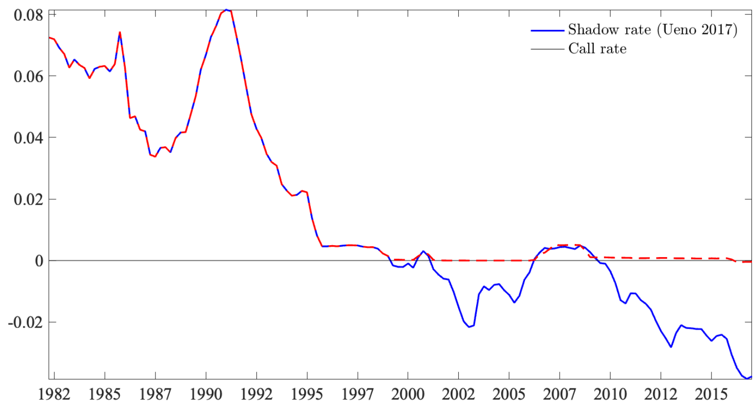
We employ the maximum likelihood (ML) method to estimate the structural parameters of the model. Parameters that are not estimated are calibrated in the standard fashion.

3.1 Data We estimate the model using quarterly data for Japan for the period from 1982:2Q to 2017:1Q. It thus includes the period when ZLB was binding, which was from 1999:2Q onward.[8] The data consist of 140 time series, including real GDP growth (Δy_t), the consumer price index (all items less food and energy, π_t), the interest rate (the call rate up to 1991:Q1 and then the shadow rate, R_t), the debt-to-GDP ratio (B_t/GDP_t), the ratio

[7]As highlighted by Sims and Zha (2006) and Cogley and Sargent (2006), accounting for the stochastic volatility of exogenous shocks is essential when trying to identify policy regime shifts.

[8]We follow an official statement of the Bank of Japan to identify the date when nominal interest rates hit the ZLB.

Figure 1: Interest Rates in Japan



SOURCE: Bank of Japan; Ueno (2017).

of government purchases to GDP (G_t/GDP_t), the government expenditure-to-GDP ratio (E_t/GDP_t), and the government tax revenue-to-GDP ratio (T_t/GDP_t). Our data source and detail are listed in Appendix B.

SHADOW RATE For our analysis, we make use of the shadow rate calculated by Ueno (2017) to grasp the response of monetary policy to inflation even during the period when unconventional policy has been implemented in Japan.[9] The shadow rate used in this paper is shown in Figure 1. The shadow rate essentially corresponds to the call rate when the ZLB is not binding but goes below zero when the ZLB is binding, which reflects the effects of unconventional monetary policy. This shows that the shadow rate can be interpreted as an aggregate that captures both conventional and unconventional monetary policy. Hence, the shadow rate is informative to evaluate changes in the response of monetary policy to inflation even when the ZLB is binding and unconventional monetary policy is implemented.[10]

In addition, using the shadow rate has another advantage. The ZLB on nominal interest rates explicitly introduces nonlinearity with respect to nominal interest rates into the model, making it computationally burdensome to deal with the ZLB. However, the shadow rate does not have a ZLB and can go below zero, which reflects the effects of unconventional monetary

[9] Admittedly, the estimation results for the shadow rate are highly dependent on the theoretical assumptions underlying the estimation. Therefore, while we use the shadow rate calculated by Ueno (2017) as our benchmark, we also check the robustness of our results using the shadow rate estimated below Krippner (2020). Doing so indicates that our results presented below are robust.

[10] Wu and Xia (2016), for example, argue that their estimated shadow federal funds rate can be used to illustrate the effects of unconventional monetary policy on the macroeconomy. Wu and Zhang (2019) propose a comprehensive framework to evaluate both conventional and unconventional monetary policies using the shadow federal funds rate.

policy. Hence, using the shadow rate as an aggregate that captures both conventional and unconventional monetary policy, we can reduce the computational burden in dealing with this nonlinearity.[11]

3.2 Model Solution We follow Bianchi and Ilut (2017) to solve the model. When a solution exists, we can obtain the solution using the following regime-switching vector auto-regression:

$$S_t = T(\xi_t^{sp}, \theta^{sp}, H^{sp})S_{t-1} + R(\xi_t^{sp}, \theta^{sp}, H^{sp})Q(\xi_t^{vo}, \theta^{vo}, H^{vo})\varepsilon_t \tag{18}$$

where S_t represents the vector of variables in our model and $\varepsilon_t \sim N(0, I)$. θ^{sp} is a vector of the structural parameters which could be switched, and θ^{vo} is a vector of the volatility parameters. $T(\xi_t^{sp}, \theta^{sp}, H^{sp})$ and $R(\xi_t^{sp}, \theta^{sp}, H^{sp})$ take four sets of values corresponding to the four policy regimes. In addition, $Q(\xi_t^{vo}, \theta^{vo}, H^{vo})$ take two sets of values corresponding to the two volatility regimes. Note that $T(\xi_t^{sp}, \theta^{sp}, H^{sp})$, $R(\xi_t^{sp}, \theta^{sp}, H^{sp})$, and $Q(\xi_t^{vo}, \theta^{vo}, H^{vo})$ are characterized by the structural parameters (θ^{sp}) and the volatility parameters (θ^{vo}). Moreover, note that the probabilities of moving across regimes (H^{sp} and H^{vo}) are also included. Hence, the possibility of regime change also matters when agents form expectations and make decisions, which means that agents' beliefs likely play a significant role in determining the law of motion of the economy.

3.3 Model Estimation We conduct the maximum likelihood estimation of the non-calibrated structural parameters of the model following Davig and Leeper (2006). We combine the law of motion with a system of observation equations as follows:

$$X_t = D + ZS_t \tag{19}$$

where X_t represents observable variables and D is a constant. S_t means the vector of variables in our model. Z maps S_t into the observables.

[11] While central banks cannot directly set a negative shadow rate in practice, they can achieve one through unconventional monetary policy tools. Wu and Zhang (2019), for example, show that the impact of unconventional policies such as quantitative easing can be mapped in the standard New Keynesian model similar to changes in the shadow rate. We implicitly assume the same mapping equivalence in this paper. The estimated shadow rate can be interpreted as an aggregate that captures both conventional and unconventional monetary policy and is informative in order to identify the response of monetary policy to inflation even when the ZLB binds.

More specifically, the law of motion of variables is linked to the data, following the measurement equations specified as

$$\begin{bmatrix} \Delta y_t \\ \pi_t \\ R_t \\ B_t/GDP_t \\ G_t/GDP_t \\ T_t/GDP_t \\ E_t/GDP_t \end{bmatrix} = \begin{bmatrix} 100 \times \gamma_{ss} \\ 100 \times \pi_{ss} \\ 400 \times R_{ss} \\ 100 \times b^m_{ss} \\ 100 \times g_{ss} \\ 100 \times \tau_{ss} \\ 100 \times e_{ss} \end{bmatrix} + \begin{bmatrix} 100 \times (\hat{y}_t - \hat{y}_{t-1} + a_t) \\ 100 \times \tilde{\pi}_t \\ 400 \times \tilde{R}_t \\ 100 \times \tilde{b}^m_t \\ 100 \times \tilde{g}_t \\ 100 \times \tilde{\tau}_t \\ 100 \times \tilde{e}^S_t + 100 \times \tilde{e}^L_t \end{bmatrix}$$

where γ_{ss}, π_{ss}, R_{ss}, b^m_{ss}, g_{ss}, e_{ss}, and τ_{ss} represent the steady state values of output growth, inflation, the interest rate, the debt-to-GDP ratio, the ratio of government purchases to GDP, the government expenditure-to-GDP ratio, and the government tax revenue-to-GDP ratio respectively.

Given the system of stochastic difference equations describing the equilibrium dynamics of the model, it is straightforward to numerically evaluate the likelihood function of the data given the vector of estimated parameters, which we denote by $L(Y|\Theta)$, where Y is the data sample and Θ is the vector of parameters to be estimated. The likelihood $L(Y|\Theta)$ is computed using the modified Kalman filter described in Kim and Nelson (1999). We use the minimization algorithm "csminwel" by C. Sims to find the estimates.

4 Estimation Results

This section presents our estimation results. Specifically, Section 4.1 reports the parameter estimates obtained using our regime switching DSGE framework. Section 4.2 then presents the identified regimes. Section 4.3 shows the impulse response functions. Finally, Section 4.4 provides a historical decomposition of inflation dynamics into the contribution of various shocks.

4.1 Parameter Estimates Before discussing the estimated parameters, Table 1 shows the parameter values which we directly take from the literature. The values are in line with those used by Bianchi and Ilut (2017), Fueki et al. (2016), and Sugo and Ueda (2008). We

Table 1: Calibrated Parameters

Description	Parameter	Value
Capital share	α	0.3300
Discount rate	β	0.9985
Persistence of long term-expenditure	ρ_{e^L}	0.9900
S.D. of long term-expenditure	$100\sigma_{e^L}$	0.1000
Parameter for maturity structure	ρ	0.9583

set the income share of capital in total output, α, to 0.33, which implies a labor income share of 0.67. The discount factor β is 0.9985, which is almost conventional value in the literature. The parameter for determining the maturity structure of government bonds is calibrated to match the average maturity of Japanese government bonds from 1982 to 2016, which is about six years.

In the following, we show our estimates for the structural parameters. In the main text, we consider two sets of parameters to be noted: the policy parameters, and the transition matrix governing the probability of transition from one policy regime to another. Our estimates of the other structural parameters are in line with other studies, which are presented in Appendix C.

The parameter estimates monetary and fiscal policy are presented in Table 2. Note that they allow for recurrent regimes changes in our model, where monetary and fiscal policy can change independently of each other. Turning to monetary policy behavior, there are two regimes: a regime in which monetary policy responds strongly to inflation (active policy), and a regime in which it does not respond strongly enough to achieve the inflation target (passive policy). In other words, the shadow rate reacts stronger to both inflation ($\psi_{\pi,AM} = 2.150$) and the output gap ($\psi_{y,AM} = 0.569$) under the active regime than under the passive regime ($\psi_{\pi,PM} = 0.389$, $\psi_{y,PM} = 0.246$). Similarly, there are two fiscal policy regimes: one in which tax policy is active and responds strongly to debt, and one in which tax policy is passive and responds weakly to debt. In the passive regime, the tax response rate to debt is larger ($\delta_{b,PF} = 0.028$) than in the active regime. Note that under the active fiscal policy regime, we restrict tax response by fiscal authority to zero, as in Bianchi and Ilut (2017).

Table 3 provides the transition probabilities. Reviewing the properties of the estimated transition matrix, it should be noted that there are also important differences in the duration

Table 2: Estimated Policy Parameters

Description	Parameter	Value
Monetary policy response to inflation (PM)	$\psi_{\pi,PM}$	0.389
Monetary policy response to inflation (AM)	$\psi_{\pi,AM}$	2.150
Monetary policy response to output (PM)	$\psi_{y,PM}$	0.246
Monetary policy response to output (AM)	$\psi_{y,AM}$	0.569
Monetary policy rule smoothing (PM)	$\rho_{R,PM}$	0.843
Monetary policy rule smoothing (AM)	$\rho_{R,AM}$	0.936
Fiscal policy response to debt (AF)	$\delta_{b,AF}$	*0.000*
Fiscal policy response to debt (PF)	$\delta_{b,PF}$	0.028
Fiscal policy rule smoothing (AF)	$\rho_{\tau,AF}$	0.926
Fiscal policy rule smoothing (PF)	$\rho_{\tau,PF}$	0.992

NOTE: It should be noted, again, that we do not estimate the parameter for fiscal policy response to debt under the Active Fiscal regime, $\delta_{b,AF}$. We restrict tax response by fiscal authority to zero following Bianchi and Ilut (2017).

of the regimes. Specifically, note that the two polar cases of the AM/PF regime and the PM/AF regime are persistent, while the AM/AF regime is the most transient regime, as in Bianchi and Ilut (2017). Moreover, the estimation results have two implications. First, when the AM/PF regime is in place, there is a positive probability of movement to the PM regime in the next period. In other words, even if the active monetary policy regime is in place, agents might not believe they will be in the AM/PF regime in the future and they expect the Bank of Japan to move to a passive policy with some positive probability. Second, and more interestingly, the PM/PF regime is also persistent. This indicates that, when the PM/PF regime is in place, the probability of going to the AM regime is low and agents therefore are less likely to expect that AM regime will be in place in the future.

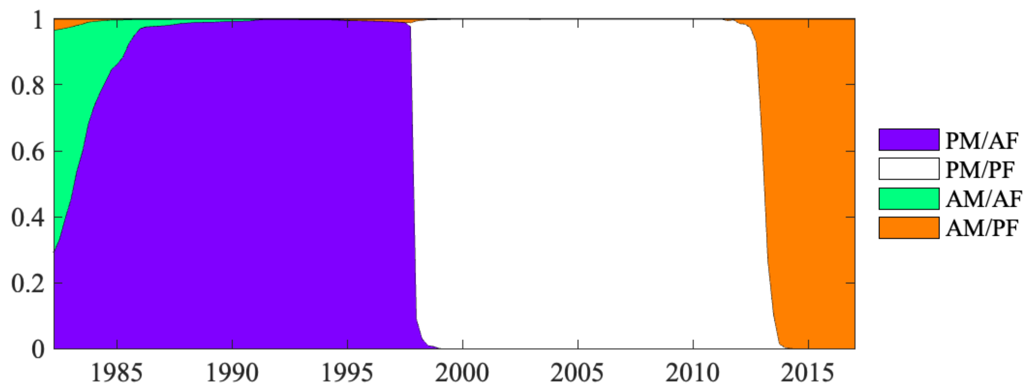
4.2 IDENTIFIED REGIMES Figure 2 presents the smoothed probabilities assigned to the four policy regimes. Throughout the first half of the observation period, fiscal policy was active. Fiscal policy then switched from an active regime to a passive regime in 1998. On the other hand, monetary policy remained passive. This may reflect the fact that the ZLB was binding and that the lack of unconventional monetary policy tools did not allow the Bank to strongly respond to falling inflation. Hence, the late 1990s to 2013 were a period in which both policies were passive.

Since the ZLB did not allow the Bank of Japan to strongly respond to inflation, it was not until the beginning of 2013 that monetary policy became active. The switch may reflect the

Table 3: Estimated Parameters for Markov Probability

Description	Parameter	Value
from PM/AF to PM/AF	H_{11}^{sp}	0.998
from PM/PF to PM/PF	H_{22}^{sp}	0.974
from AM/AF to AM/AF	H_{33}^{sp}	0.925
from AM/PF to AM/PF	H_{44}^{sp}	0.974
from PM/AF to PM/PF	H_{21}^{sp}	0.002
from PM/AF to AM/AF	H_{31}^{sp}	0.000
from PM/PF to PM/AF	H_{12}^{sp}	0.000
from PM/PF to AM/AF	H_{32}^{sp}	0.000
from AM/AF to PM/AF	H_{13}^{sp}	0.075
from AM/AF to PM/PF	H_{23}^{sp}	0.000
from AM/PF to PM/AF	H_{14}^{sp}	0.002
from AM/PF to PM/PF	H_{24}^{sp}	0.021

Figure 2: Probability of Policy Regimes



Note: This figure shows the smoothed probabilities given to the four policy regimes. AM/PF: Active monetary policy/passive fiscal policy regime, AM/AF: Active monetary policy/active fiscal policy regime, PM/PF: Passive monetary policy/passive fiscal policy regime, PM/AF: Passive monetary policy/active fiscal policy regime.

introduction of QQE. On the other hand, fiscal policy remained passive and accommodated the switch in the conduct of monetary policy. Therefore, our estimation results suggest that the policy mix in Japan has become the AM/PF regime after the introduction of QQE.

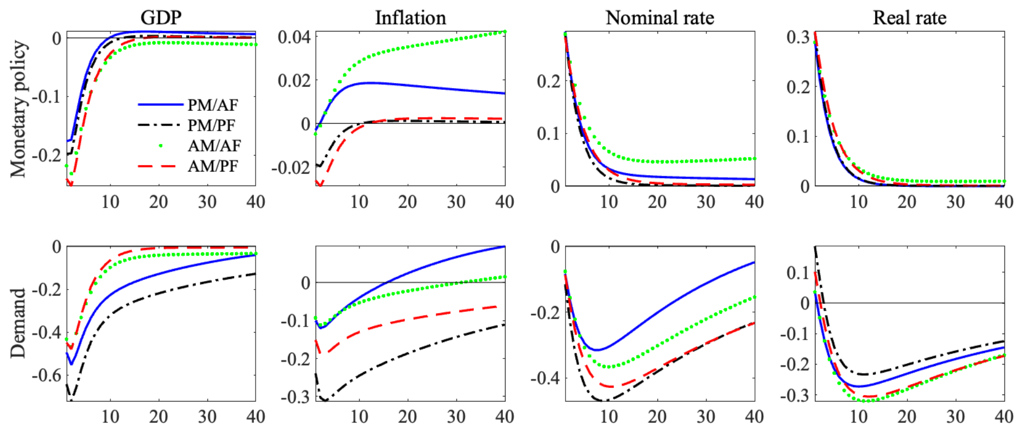
4.3 Impulse Response Functions In order to improve our understanding of differences in the propagation of shocks across the different regimes, it is useful to look at impulse response functions. When computing the impulse response functions, we assume that no regime shifts occur and a particular regime is in place over the entire horizon. However, when making decisions, agents take into account the possibility that regime shifts may occur in the future based on the estimated transition probabilities. The dynamics will therefore differ from those that would be obtained if we used a DSGE model without regime shifts, as highlighted by Sims (2016).

The first row of Figure 3 reports the impulse responses to an exogenous monetary tightening. The solid blue line, the dot-dash black line, the dotted green line, and the dashed red line respectively represent the PM/AF, PM/PF, AM/AF, and AM/PF regimes. The initial shock is equal to a one standard deviation monetary policy shock under the low volatility regime. The responses under the three regimes also considered by Bianchi and Ilut (2017) i.e., the PM/AF, AM/AF, and AM/PF regimes - are all consistent with their results.

To start with, note that the policy regime matters: the responses differ substantially both qualitatively and in size across the policy regimes. In particular, the ability of a central bank to control inflation is substantially reduced under the AF regimes (that is, the AM/AF regime and the PM/AF regime). The mechanism through which a central bank loses the ability to control inflation under the AF regimes is as follows. Monetary tightening increases the real cost of debt and the recession associated with monetary tightening increases the debt burden. Under the AF regimes, agents expect that there will be no offsetting increase in current or future tax obligations despite the fact that the government debt burden will increase. Households will therefore assume that the sustainability of government debt is not assured and try to substitute out of government debt into goods. Households' consumption path therefore shifts upward. Higher demand for goods will push up the price level. On the other hand, under the AM/PF regime inflation is well anchored. With respect to the PM/PF regime, the responses to monetary tightening look qualitatively similar to those under the AM/PF regime given our estimated parameters, although the responses across the two regimes slightly differ in size.

Turning to the second row of Figure 3, this shows the responses to a negative demand shock. The initial shock is equal to a one standard deviation demand shock under the low volatility regime. Looking at the responses of the real economy and inflation, we observe a slowdown in the real economy and a decline in inflation immediately after a shock in response to a negative demand shock in all regimes. In addition, when fiscal policy is active, inflation turns positive at the end of the simulation period. These results are consistent with Bianchi and Ilut (2017). The reason is that the decline in real activity will lead to an inflationary

Figure 3: Impulse Responses



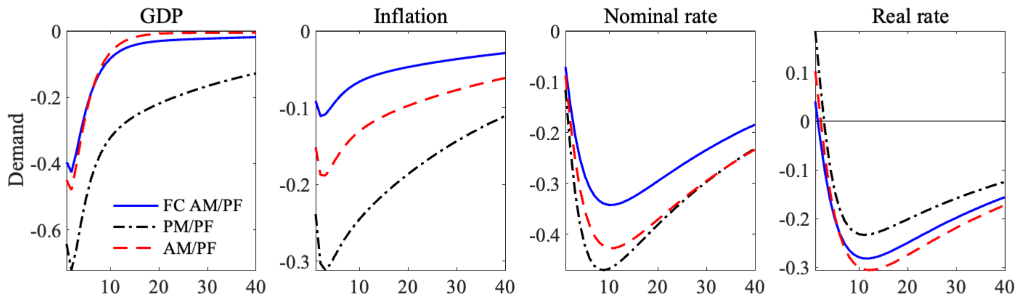
NOTE: The first row shows the impulse response functions to an exogenous monetary tightening, while the second row shows those to a demand shock. The horizontal axes show the number of quarters after the shock. The solid blue line, the dot-dash black line, the dotted green line, and the dashed red line represent the PM/AF regime, the PM/PF regime, the AM/AF regime, and the AM/PF regime, respectively. The initial shock is equal to a one standard deviation shock under the low volatility regime.

increase in the fiscal burden only when agents expect that a non-AF regime will prevail for many periods.

In addition, what is worth noting is that the propagation of a negative demand shock is strongest under the PM/PF regime. Specifically, there is a strong and persistent decline in the inflation rate. However, under the PM/PF regime, there is no strong monetary policy response to stimulate the economy. As a result, monetary policy provides little stabilization in response to the negative impact of a demand shock on inflation.

ROLE OF AGENTS' BELIEF It should be highlighted that the impulse responses to a negative demand shock depend on agents' beliefs about being in or moving to the AM/PF regime in the future. In our model, agents take the possibility of regime changes into account when forming expectations and making decisions. Therefore, changes in the probability of staying in or a switch to the AM/PF regime play a role in determining economic outcomes. To examine the role played by agents beliefs in stabilizing the economy in the wake of a negative demand shock, we investigate the response to a negative demand shock under the AM/PF regime. Specifically, we examine what happens if the transition probabilities from the AM/PF regime to another are changed from the estimated probabilities. In this counterfactual scenario, we set the probability of staying in the AM/PF regime to one, which implies agents believe that the AM/PF regime will be in place forever. We refer

Figure 4: Influence of Agents' Beliefs on Impulse Responses



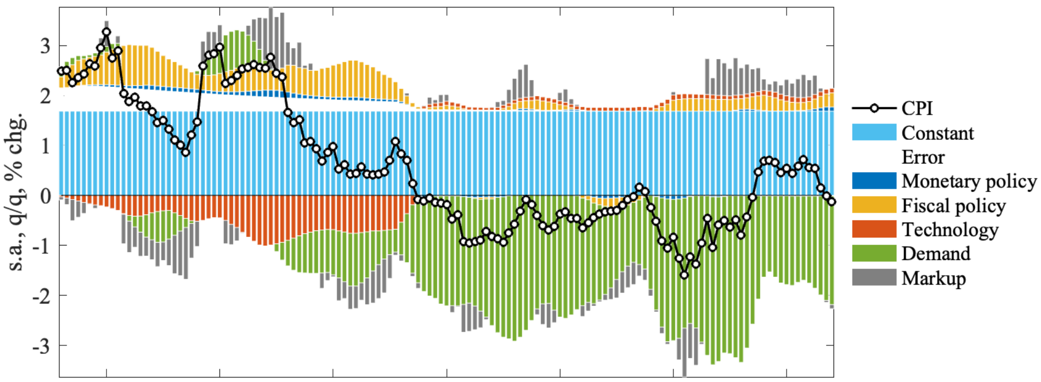
NOTE: "FC AM/PF" stands for the fully credible AM/PF regime. We assume that the AM/PF policy mix is the only possible regime, and agents regard the regime which is in place as lasting forever. The solid blue line, the dot-dash black line, and the dashed red line represent the fully credible AM/PF regime, the PM/PF regime, and the AM/PF regime, respectively. The horizontal axes show the number of quarters after the shock. The vertical difference between the solid blue line and the dashed red line represents the effect of agents' beliefs about being in the AM/PF regime. The initial shock is equal to a one standard deviation shock under the low volatility regime.

to this counterfactual scenario as the fully credible AM/PF regime as in Bianchi and Ilut (2017). Note that the differences between the responses under the estimated transition probabilities and those under the counterfactual scenario (the fully credible AM/PF regime) should quantitatively reflect the role played by agents beliefs in the propagation of a negative demand shock.

The results are shown in Figure 4. The solid blue line, the dot-dash black line, and the dashed red line respectively represent the fully credible AM/PF regime, the PM/PF regime, and the AM/PF regime. The vertical difference between the solid blue line and the dashed red line represents the impact of agents' belief about being in the AM/PF regime, in which inflation is anchored. The results imply that negative demand shocks have a much smaller impact on inflation and output under the fully credible AM/PF regime scenario. The reason is that agents believe that the central bank will actively mitigate the negative impact of demand shocks in the future. This result illustrates the important role that agents' belief about being in an active monetary policy regime plays in stabilizing the economy in the wake of a negative demand shock.

4.4 Historical Decomposition What factors have played a role in determining inflation dynamics in Japan during the observation period? To answer this question, Figure 5 presents a historical decomposition of CPI inflation (less food and energy) in Japan. Specifically, inflation dynamics are broken down into the contribution of monetary policy, fiscal

Figure 5: Historical Decomposition of Inflation



NOTE: This figure shows the historical decomposition of changes in the CPI (less food and energy, four-quarter backward moving average). Changes in the CPI are broken down into the contribution of monetary policy, fiscal policy, demand, mark-ups, and technology shocks. Fiscal policy shocks consist of changes in transfers, the tax-to-GDP ratio, short-term expenditure, long-term expenditure, and term premiums.

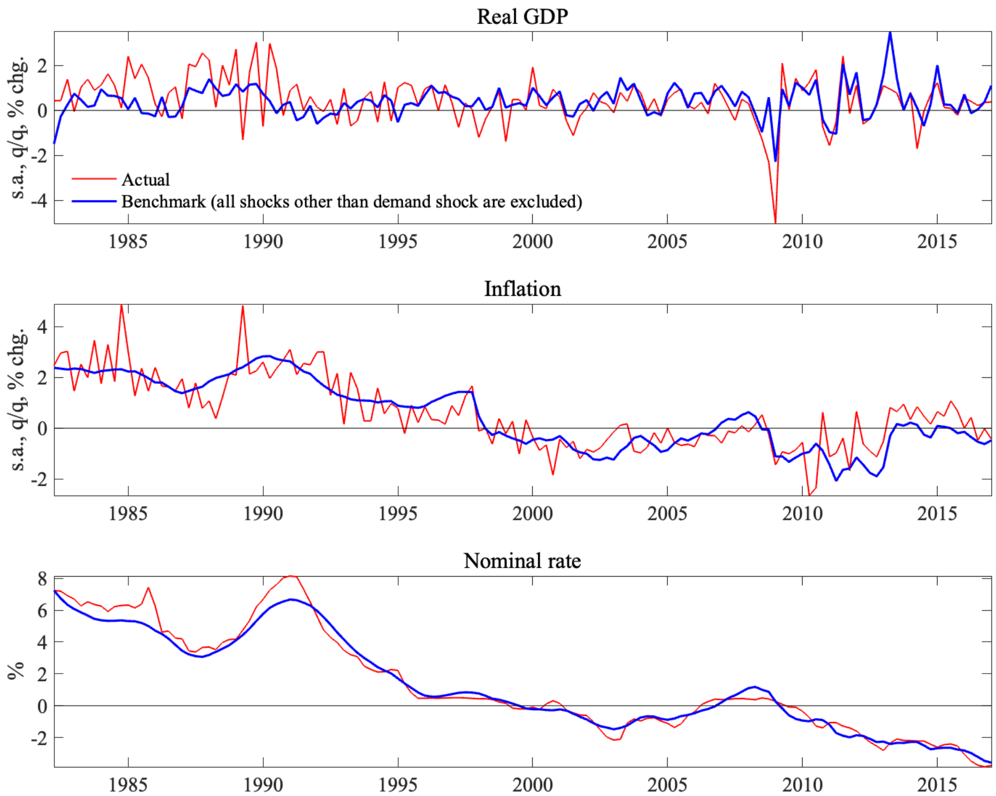
policy, demand, mark-ups, and technology shocks. Fiscal policy shocks consist of changes in transfers, the tax-to-GDP ratio, short-term expenditure, long-term expenditure, and term premiums. The results in Figure 5 clearly show that demand shocks have played a substantial role in driving inflation dynamics in Japan. As seen in the impulse responses to demand shocks, this is because negative demand shocks were propagated strongly and persistently under the PM/PF policy mix. To highlight the importance of demand shocks in accounting for the evolution of Japans economy, Figure 6 plots the paths of real GDP growth and CPI inflation (less food and energy) that would be obtained if all shocks other than demand shocks were excluded. Their paths track the data well.

5 Counterfactual Simulations

In this section, we examine the role of the policy regime in the evolution of Japanese economy, in particular inflation dynamics, through the lens of our model.

As shown above, the key factor responsible for the prolonged deflation is negative demand shocks. The issue therefore is how policymakers can reduce the negative impact of demand shocks in order to escape from the prolonged deflation. We therefore explore whether changes in the policy mix would help to reduce the negative impact of demand shocks on inflation or not. To address this question, we examine two types of simulations. First, we investi-

Figure 6: Historical Decomposition (demand shock)



NOTE: The dotted line represents actual observations. We set all shocks other than demand shocks to zero to find the contribution of demand shocks to the benchmark results (solid red line). The lower panel shows the four-quarter backward moving average.

gate what would have happened under alternative policy mix scenarios. This is a standard counterfactual simulation. We use the structural shocks extracted from the estimates to simulate the economy but assume alternative (counterfactual) changes in the monetary and fiscal policy mix.

Second, and more interestingly, we ask what would have happened if the probabilities of transition from one policy regime to another had been different from the estimated probabilities. As in the first type of simulation, we simulate the economy using the structural shocks extracted from the estimates. Bianchi and Ilut (2017) call this latter type of simulation "beliefs counterfactual" simulation. This exercise allows us to study how changes in agents perceptions about the policy mix change the impact of a negative demand shock.

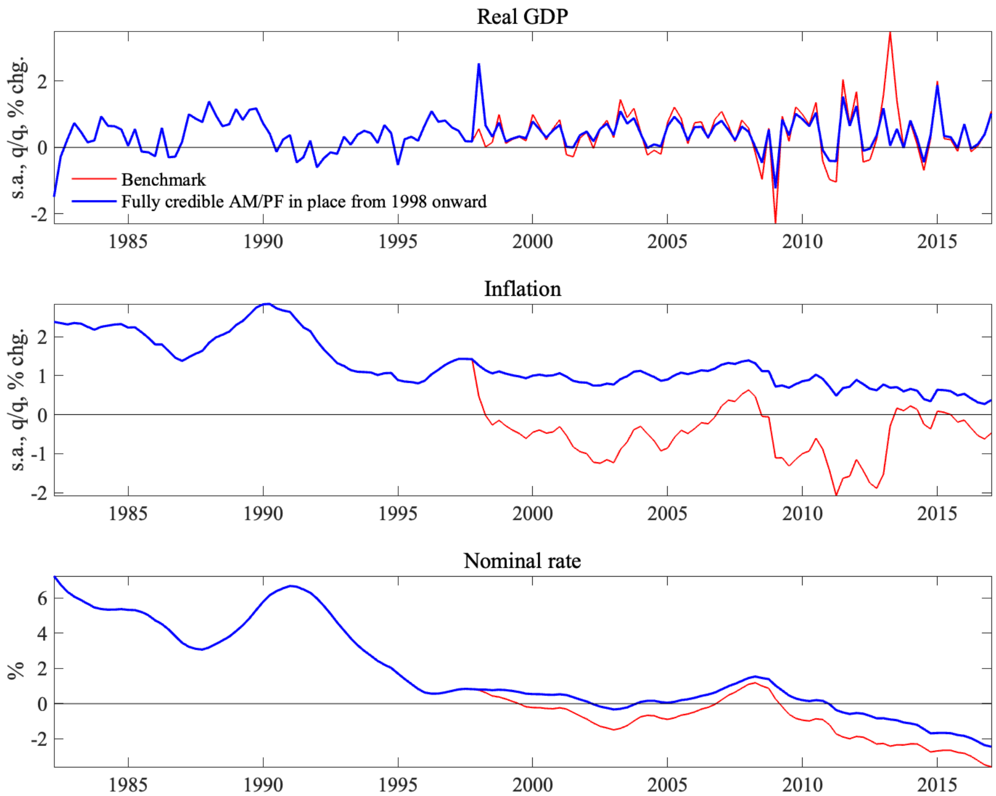
5.1 Inflation Dynamics during the 2000s Had the Policy Mix been Different

Focusing on the period from the late 1990s to 2013, we simulate the path of Japan's economy assuming the same sequence of demand shocks. However, we assume that the policy mix, and in particular the monetary policy response, as well as agents' beliefs with regard to policy regime changes had been different. Specifically, we assume that monetary policy had been active. Thus, we assume that the AM/PF regime instead of the PM/PF regime was in place since 1998. In addition, to see more clearly the impact of a change in monetary policy regime from passive to active, we assume that economic agents perceive the AM/PF regime as the only regime. That is, agents fully believe that the policy regime is AM/PF. In practical terms, this implies that the Bank of Japan had started with unconventional monetary policy in 1998 (rather than in 2001, as it actually did), and had pursued it more aggressively than it actually did. Moreover, agents believed that the Bank would conduct active monetary policy forever.

The results of this simulation are presented in Figure 7. The dashed blue line shows the paths of real GDP growth, CPI inflation (less food and energy), and interest rates had the fully credible AM/PF regime been in place from 1998 onward. The results clearly indicate that inflation would have been much higher had the AM/PF regime been in place. There are two reasons. The first is the active response of monetary policy: the Bank of Japan would have tried to mitigate the effects of negative demand shocks. That is, the Bank would have tried to lower the shadow rate to stimulate the economy by implementing unconventional policy. The second reason is agents' beliefs about being in the AM/PF regime, which are also key in mitigating the impact of negative demand shocks, as discussed in the impulse response analysis. These findings imply that if the monetary policy response had been different, the deflation of the 2000s would not have occurred.

5.2 Revisiting QQE Next, we try to illustrate the impact of the changes in the policy mix brought about by the introduction of QQE (from 2013:2Q). As above, we conduct our simulation assuming the same sequence of demand shocks; this time, however, we assumed that monetary policy did not switch to an active policy regime and examine the path of the economy in the absence of QQE. That is, we assume that the PM/PF regime before 2013:Q2 remained in place. Figure 8 compares the course of the economy in the benchmark

Figure 7: Counterfactual Simulation (1)

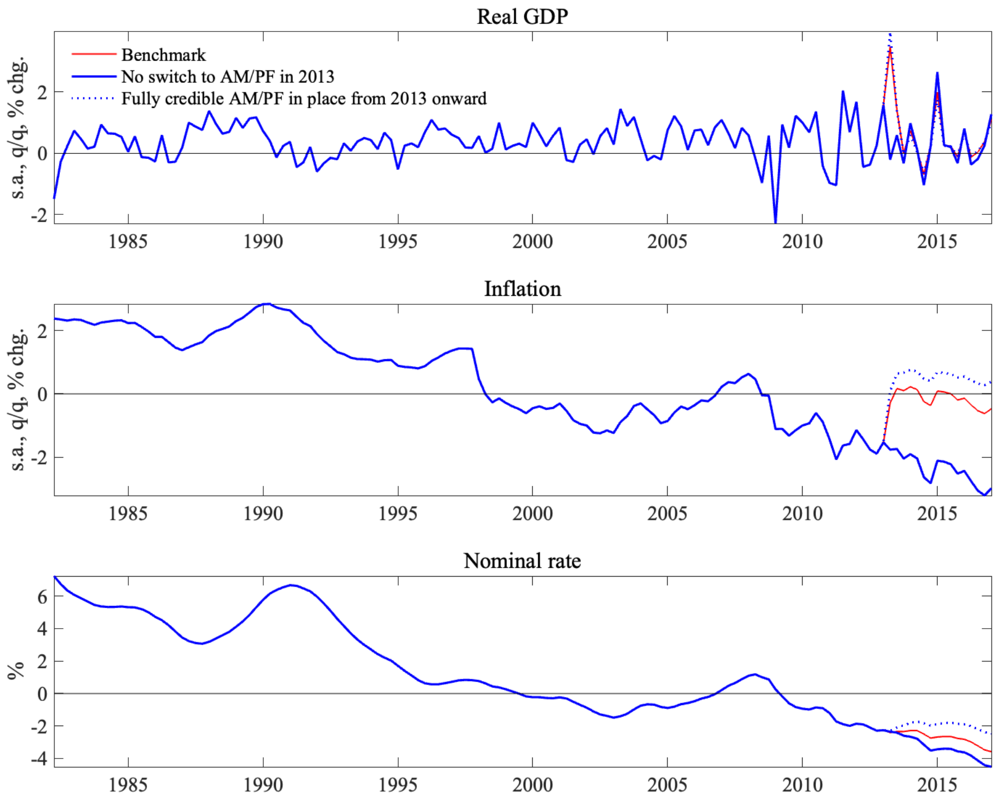


NOTE: We consider the following scenario relative to the benchmark result in which all shocks other than demand shocks are excluded (solid red line): Fully credible AM/PF in place from 1998 onward (dashed blue line): From 1998 onward, we simulate the path of Japans economy assuming the AM/PF policy mix is the only possible one and agents regard the regime as lasting forever. As in the benchmark, we exclude all shocks other than demand shocks.

model (represented by the solid red line) and the counterfactual scenario assuming that the PM/PF regime had remained in place (represented by the solid blue line). As can be seen, the inflation rate would have been lower had the PM/PF policy mix remained in place. This clearly indicates that the introduction of QQE has helped to push up inflation.

Finally, we look at the beliefs counterfactual simulation. Specifically, we ask whether inflation would have been higher had the probability of being in the AM/PF regime been one. This scenario implies that agents fully believe that the AM/PF regime will last forever. In Figure 8, the path of the economy in the benchmark model (represented by the solid red line) is compared with the course of the economy in the beliefs counterfactual scenario in which agents regard the AM/AF regime as the only possible regime (represented by the

Figure 8: Counterfactual Simulation (2)



NOTE: We consider the following scenarios relative to the benchmark result in which all shocks other than demand shocks are excluded (solid red line): No switch to AM/PF in 2013 (solid blue line): The PM/PF regime remained in place. Fully credible AM/PF in place from 2013 onward (dotted blue line): From 2013 onward, we simulate the path of Japans economy assuming the AM/PF policy mix is the only possible one and agents regard the regime as lasting forever. As in the benchmark, we exclude all shocks other than demand shocks.

dotted blue line). Figure 8 clearly indicates that inflation would have increased by more than it did in the benchmark case, indicating that agents' beliefs about being the in AM/PF regime play an important role. In other words, inflation would be higher if agents more strongly believed that they were in an AM/PF regime. This implies that the Bank of Japan could have pushed up inflation if it had managed to convince agents that it would conduct active monetary policy forever.

6 Conclusion

This paper investigated the role played by monetary-fiscal policy interaction in the behavior of inflation in Japan, focusing in particular on the prolonged deflation observed in Japan. We found that Japan's policy regime during the period was characterized by the combination of passive monetary (PM) and passive fiscal (PF) policy and that this played a substantial role in propagating negative demand shocks, leading to prolonged deflation. In addition, we found that the extent to which agents believe that the AM/PF regime remains in place is also key in stabilizing the economy in the wake of negative demand shocks and, in particular, in pushing up inflation.

However, an important issue remains for future analysis. This paper assumes that the probabilities of transition from one policy regime to another are time-invariant. That is, it is assumed that agents' beliefs about the policy do not change over time. However, it may be reasonable to assume that agents' beliefs evolve based on events. For instance, observing that the Bank of Japan has started to respond strongly to inflation, agents may have become more confident that the AM regime will remain in place. Agents may therefore have updated their beliefs and started to assign a higher probability of being in the AM regime in the future. Therefore, an interesting task for the future is to endogenize how agents update their beliefs regarding the policy response and to examine the macroeconomic consequences of dynamic changes in agents' beliefs, as examined by Bianchi and Melosi (2013) and Bianchi and Melosi (2016).

References

Bhattarai, Saroj, Jae Won Lee, and Woong Yong Park, "Inflation Dynamics: The Role of Public Debt and Policy Regimes," *Journal of Monetary Economics*, 2014, *67*, 93–108.

_ , _ , and _ , "Policy regimes, policy shifts, and US business cycles," *Review of Economics and Statistics*, 2016, *98* (5), 968–983.

Bianchi, Francesco, "Regime Switches, Agents' Beliefs, and Post-World War II U.S. Macroeconomic Dynamics," *Review of Economic Studies*, 2013, *80* (2), 463–490.

_ , "Methods for measuring expectations and uncertainty in Markov-switching models," *Journal of Econometrics*, 2016, *190* (1), 79–99.

_ and Cosmin Ilut, "Monetary/Fiscal policy mix and agents' beliefs," *Review of Economic Dynamics*, 2017, *26*, 113–139.

_ and Leonardo Melosi, "Dormant Shocks and Fiscal Virtue," *NBER Macroeconomics Annual*, 2013, *28*, 1–46.

_ and _ , "Modeling the Evolution of Expectations and Uncertainty in General Equilibrium," *International Economic Review*, 2016, *57* (2), 717–756.

_ and _ , "Escaping the Great Recession," *American Economic Review*, 2017, *107* (4), 1030–1058.

Chen, Xiaoshan, Eric M. Leeper, and Campbell Leith, "US Monetary and Fiscal Policies - Conflict or Cooperation?," 2019. Mimeo.

Cogley, Timothy and Thomas J. Sargent, "Drifts and Volatilities: Monetary Policies and Outcomes in the Post WWII U.S.," *Review of Economic Dynamics*, 2006, *8*, 262–302.

Davig, Troy and Eric M. Leeper, "Fluctuating Macro Policies and the Fiscal Theory," *NBER Macroeconomics Annual*, 2006, *21*, 247–315.

Fueki, Takuji, Ichiro Fukunaga, Hibiki Ichiue, and Toyoichiro Shirota, "Measuring Potential Growth with an Estimated DSGE Model of Japan's Economy," *International Journal of Central Banking*, 2016, *12*, 1–32.

Gertler, Mark and Peter Karadi, "A Model of Unconventional Monetary Policy," *Journal of Monetary Economics*, 2011, *58*, 17–34.

_ and _ , "QE1 vs. 2 vs, 3: A Framework for Analyzing Long Term Asset Purchases as an Monetary Policy Tool," *International Journal of Central Banking*, 2013.

Hohbergera, Stefan, Romanos Priftisb, and Lukas Vogelc, "The macroeconomic effects of quantitative easing in the euro area: Evidence from an estimated DSGE model," *Journal of Economic Dynamics & Control*, 2019, *108*, 103756.

Ichiue, Hibiki and Yoichi Ueno, "A Survey-based Shadow Rate and Unconventional Monetary Policy Effects," 2018. IMES Discussion Paper Series.

Ikeda, Daisuke, Shangshang Li, Sophocles Mavroeidis, and Francesco Zanetti, "Testing the Effectiveness of Unconventional Monetary Policy in Japan and the United States," 2022. Working Paper.

Kim, Chang-Jin and Charles R. Nelson, *State-Space Models with Regime Switching*, Cambridge, MA: MIT Press, 1999.

Krippner, Leo, "Documentation for measures of monetary policy," 2020. Mimeo.

Leeper, Eric M., "Equilibria under 'active' and 'passive' monetary and fiscal policies," *Journal of Monetary Economics*, 1991, *27*, 129–147.

Lubik, Thomas A and Wing Leong Teo, "Deep habits in the new keynesian phillips curve," *Journal of Money, Credit and Banking*, 2014, *46* (1), 79–114.

Sims, Christopher A., "Fiscal policy, monetary policy and central bank independence," 2016. Kansas Citi Fed Jackson Hole Conference.

_ and Tao Zha, "Were There Regime Switches in U.S. Monetary Policy?," *American Economic Review*, 2006, *96* (1), 54–81.

Sims, Eric and Jing Cynthia Wu, "Evaluating Central Banks Tool Kit: Past, Present, and Future," *Journal of Monetary Economics*, 2021, *118*, 135–160.

Sugo, Tomohiro and Kozo Ueda, "Estimating a dynamic stochastic general equilibrium model for Japan," *Journal of the Japanese and International Economies*, 2008, *22*, 476–502.

Ueno, Yoichi, "Term Structure Models with Negative Interest Rates," 2017. IMES Discussion Paper Series.

Woodford, Michael, "Price-level determinacy without control of a monetary aggregate," in "Carnegie-Rochester conference series on public policy," Vol. 43 Elsevier 1995, pp. 1–46.

Wu, Jing Cynthia and Fan Dora Xia, "Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound," *Journal of Money, Credit and Banking*, 2016, *48* (2).

_ **and Ji Zhang**, "A Shadow Rate New Keynesian Model," *Journal of Economic Dynamics & Control*, 2019, *107*.

A Model details

In this section, we solve out the optimization problem of households and intermediate goods producing firms to derive the linearized equilibrium system. As preparation, we should remember that we detrend some economic variables, such as GDP and consumption, with the technological process, A_t. Also, we should note that most of the fiscal variables are evaluated with respect to the nominal GDP, but the government expenditure is specially defined as $g_t \equiv \frac{1}{1-G_t/Y_t}$. It is useful to derive the linearized equilibrium condition for final goods market, $Y_t = C_t + G_t$, for the sake of later use. We obtain the linearized market clearing condition for final goods as $\widehat{c}_t - \widehat{y}_t + \widetilde{g}_t = 0$ with the following procedure:

$$\frac{C_t}{A_t}\frac{A_t}{Y_t} + \frac{G_t}{Y_t} = 1 \Leftrightarrow \frac{c_t}{y_t} - \frac{1}{g_t} = 0 \Leftrightarrow \widehat{y}_t = \widehat{c}_t + \widetilde{g}_t, \tag{A.1}$$

where $\widetilde{g}_t = \ln(g_t/g)$.

A.1 IS-curve

We start with deriving the IS curve in this economy. We can obtain the following Euler equation by solving the optimality of households for choice of consumption and saving in the short-term bond,

$$1 = \mathbb{E}_t \beta \frac{R_t}{\Pi_{t+1}} \frac{\exp(d_{t+1})}{\exp(d_t)} \frac{C_t - \Phi C_{t-1}}{C_{t+1} - \Phi C_t}. \tag{A.2}$$

Precisely speaking, obtaining this Euler equation is not simple as explained. C_t is a consumption bundle of differentiated goods aggregated with a conventional Dixit-Stiglitz (CES) aggregator, and C^A_{t-1} is the average level of consumption in this economy, which is identical to the level of consumption of each household ex-post. If you are interested in the complete derivation with the external habit model, see Lubik and Teo (2014). Detrending with the technological process, we obtain,

$$\mathbb{E}_t \exp(\gamma + a_{t+1})c_{t+1} - \Phi c_t = \mathbb{E}_t \beta \frac{R_t}{\Pi_{t+1}} \frac{\exp(d_{t+1})}{\exp(d_t)} \left[c_t - \frac{\Phi c_{t-1}}{\exp(\gamma + a_t)} \right]. \tag{A.3}$$

Log-linearizing around the steady state yields the IS-curve in terms of consumption,

$$\Gamma \mathbb{E}_t \widehat{c}_{t+1} + \Gamma \mathbb{E}_t a_{t+1} - \Phi \widehat{c}_t = [\Gamma \widehat{c}_t - \Phi \widehat{c}_{t-1} + \Phi a_t] + (\Gamma - \Phi) \left[\widehat{R}_t - \mathbb{E}_t \widetilde{\pi}_{t+1} - d_t + \mathbb{E}_t d_{t+1} \right]. \tag{A.4}$$

Remembering the final goods market clearing condition, $\widehat{c}_t - \widehat{y}_t + \widetilde{g}_t = 0$, and rearranging the above equation, we have,

$$
\begin{aligned}
(\Gamma + \Phi)\left(\widehat{y}_t - \widetilde{g}_t\right) &= \Gamma\left(\mathbb{E}_t\widehat{y}_{t+1} - \mathbb{E}_t\widetilde{g}_{t+1} + \mathbb{E}_t a_{t+1}\right) + \Phi\left(\widehat{y}_{t-1} - \widetilde{g}_{t-1} - a_t\right) \\
&\quad - (\Gamma - \Phi)\left[\widehat{R}_t - \mathbb{E}_t\widetilde{\pi}_{t+1} - d_t + \mathbb{E}_t d_{t+1}\right] \\
\Leftrightarrow (\Gamma + \Phi)\,\widehat{y}_t &= (\Gamma + \Phi)\,\widetilde{g}_t + \Gamma\left(\mathbb{E}_t\widehat{y}_{t+1} - \mathbb{E}_t\widetilde{g}_{t+1} + \rho_a a_t\right) + \Phi\left(\widehat{y}_{t-1} - \widetilde{g}_{t-1} - a_t\right) \\
&\quad - (\Gamma - \Phi)\left[\widehat{R}_t - \mathbb{E}_t\widetilde{\pi}_{t+1} - d_t + \rho_d d_t\right],
\end{aligned}
\tag{A.5}
$$

where we use our assumption on the productivity and the demand shock, which are autoregressive processes.

A.2 Flexible Price Output In the case that there is no nominal rigidity, the output should be identical to the potential output. From the households' optimality toward labor supply, we have,

$$W_t = C_t - \Phi C_{t-1}. \tag{A.6}$$

In the equilibrium, we have the condition that the marginal cost of labor is the real wage by taking first order condition of labor demand of intermediate goods producing firms:

$$W_t = A_t(1-\alpha)h_t^{-\alpha}. \tag{A.7}$$

Combining those two conditions with respect to the real wage, we have:

$$C_t - \Phi C_{t-1} = A_t(1-\alpha)h_t^{-\alpha} = A_t(1-\alpha)\left(\frac{Y_t}{A_t}\right)^{-\frac{\alpha}{1-\alpha}}, \tag{A.8}$$

where we use the production function, $Y_t = A_t h_t^{1-\alpha}$, to the last equality. Detrending with the technological process, we obtain,

$$c_t - \frac{\Phi c_{t-1}}{\exp(\gamma + a_t)} = (1-\alpha)\left(y_t\right)^{-\frac{\alpha}{1-\alpha}}. \tag{A.9}$$

Now notice that $(1-\Phi\Gamma^{-1}) = (1-\alpha)\frac{\overline{y}}{\overline{c}}\left(\frac{1}{\overline{y}}\right)^{\frac{1}{1-\alpha}}$ at the steady state. Then we take a deviation from the steady state on both side of the equation, which yields,

$$\begin{aligned} &\frac{\alpha}{1-\alpha}\left(1-\Phi\Gamma^{-1}\right)\widehat{y}_t^* + \widehat{c}_t = \Phi\Gamma^{-1}\left(\widehat{c}_{t-1} - a_t\right) \\ \Leftrightarrow &\left[\frac{1}{1-\Phi\Gamma^{-1}} + \frac{\alpha}{1-\alpha}\right]\widehat{y}_t^* = \frac{1}{1-\Phi\Gamma^{-1}}\widetilde{g}_t + \frac{\Phi\Gamma^{-1}}{1-\Phi\Gamma^{-1}}\left(\widehat{y}_{t-1}^* - \widetilde{g}_{t-1} - a_t\right). \end{aligned} \tag{A.10}$$

A.3 Phillips curve The Phillips curve in the economy is derived from the optimization problem of the intermediate goods firms. Firm j chooses its price $p_t(j)$ to maximize the present value of future profits:

$$\mathbb{E}_t\left[\sum_{s=t}^{\infty} q_s\left(\frac{p_s(j)}{P_s}y_s(j) - W_s h_s(j) - AC_s(j)\right)\right], \tag{A.11}$$

where q_s is the real stochastic discount factor of households who own the intermediate goods firms. The first order condition with respect to Firm j is:

$$\begin{aligned} [p_s]: \quad &\left(1-\frac{1}{v_s}\right)\left[1-\frac{\varphi}{2}\left(\Pi_s - \Pi_{s-1}^{\zeta}\Pi^{1-\zeta}\right)^2\right] - \varphi\left(\Pi_s - \Pi_{s-1}^{\zeta}\Pi^{1-\zeta}\right)\Pi_s \\ &+ \varphi\frac{q_{s+1}}{q_s}\frac{Y_{s+1}}{Y_s}\left(\Pi_{s+1} - \Pi_s^{\zeta}\Pi^{1-\zeta}\right)\Pi_{s+1} + \frac{1}{v_s}\frac{W_s}{(1-\alpha)Y_s}\left(\frac{Y_s}{A_s}\right)^{\frac{1}{1-\alpha}} = 0, \end{aligned} \tag{A.12}$$

where $\frac{W_s}{(1-\alpha)Y_s}\left(\frac{Y_s}{A_s}\right)^{\frac{1}{1-\alpha}}$ is the marginal cost that is identical across symmetric intermediate good firms. Taking the first order approximation around the steady state, we have:

$$\begin{aligned} (1+\zeta\beta)\varphi\Pi^2\left(\widetilde{\pi}_t - \widetilde{\mu}_t\right) = &\frac{1-v}{v}\left[\frac{\alpha}{1-\alpha}\left(\widehat{y}_t - \widehat{y}_t^*\right) - \left(\widehat{y}_{t-1} - \widehat{y}_{t-1}^*\right) + \frac{\left(\widehat{y}_t - \widehat{y}_t^*\right) - \left(\widehat{y}_{t-1} - \widehat{y}_{t-1}^*\right)}{1-\Phi\Gamma^{-1}}\right] \\ &+ \varphi\Pi^2\zeta\widetilde{\pi}_{t-1} + \varphi\Pi^2\beta E_t\left[\widetilde{\pi}_{t+1}\right] \end{aligned} \tag{A.13}$$

Define $\kappa \equiv \frac{1-v}{v\varphi\Pi^2}$ and divide both sides of equation by $(1+\zeta\beta)$, then we simplify the equation:

$$\begin{aligned} \widetilde{\pi}_t = &\frac{\kappa\left(1-\Phi\Gamma^{-1}\right)^{-1}}{1+\zeta\beta}\left(\left[1+\frac{\alpha}{1-\alpha}\left(1-\Phi\Gamma^{-1}\right)\right]\left(\widehat{y}_t - \widehat{y}_t^*\right) - \Phi\Gamma^{-1}\left(\widehat{y}_{t-1} - \widehat{y}_{t-1}^*\right)\right) \\ &+ \frac{\zeta}{1+\zeta\beta}\widetilde{\pi}_{t-1} + \frac{\beta}{1+\zeta\beta}E_t\left[\widetilde{\pi}_{t+1}\right] + \widetilde{\mu}_t. \end{aligned}$$

Substituting the flexible price output (potential output), which we derive in the previous subsection, we have:

$$\widetilde{\pi}_t = \frac{\kappa\left(1-\Phi\Gamma^{-1}\right)^{-1}}{1+\zeta\beta}\left(\left[1+\frac{\alpha}{1-\alpha}\left(1-\Phi\Gamma^{-1}\right)\right]\widehat{y}_t - \widetilde{g}_t - \Phi\Gamma^{-1}\left(\widehat{y}_{t-1} - \widetilde{g}_{t-1} - a_t\right)\right)$$
$$+ \frac{\zeta}{1+\zeta\beta}\widetilde{\pi}_{t-1} + \frac{\beta}{1+\zeta\beta}E_t\left[\widetilde{\pi}_{t+1}\right] + \widetilde{\mu}_t.$$

A.4 GOVERNMENT

$$\chi_t \equiv P_t G_t / E_t \tag{A.14}$$

$$\widetilde{\chi}_t = \frac{1}{g-1}\widetilde{g}_t - e^{-1}\widetilde{e}_t \tag{A.15}$$

In what follows, $\widehat{x}_t \equiv \log\left(\left(X_t/A_t\right)/\left(X/A\right)\right)$ represents the percentage deviation of a detrended variable from its steady state. For all the variables normalized with respect to GDP (debt, expenditure, and taxes) $\widetilde{x}_t$ denotes a linear deviation ($\widetilde{x}_t = X_t - X$), while for all the other variables $\widetilde{x}_t$ denotes a percentage deviation ($\widetilde{x}_t = \log\left(X_t/X\right)$). This distinction avoids having the percentage change of a percentage.

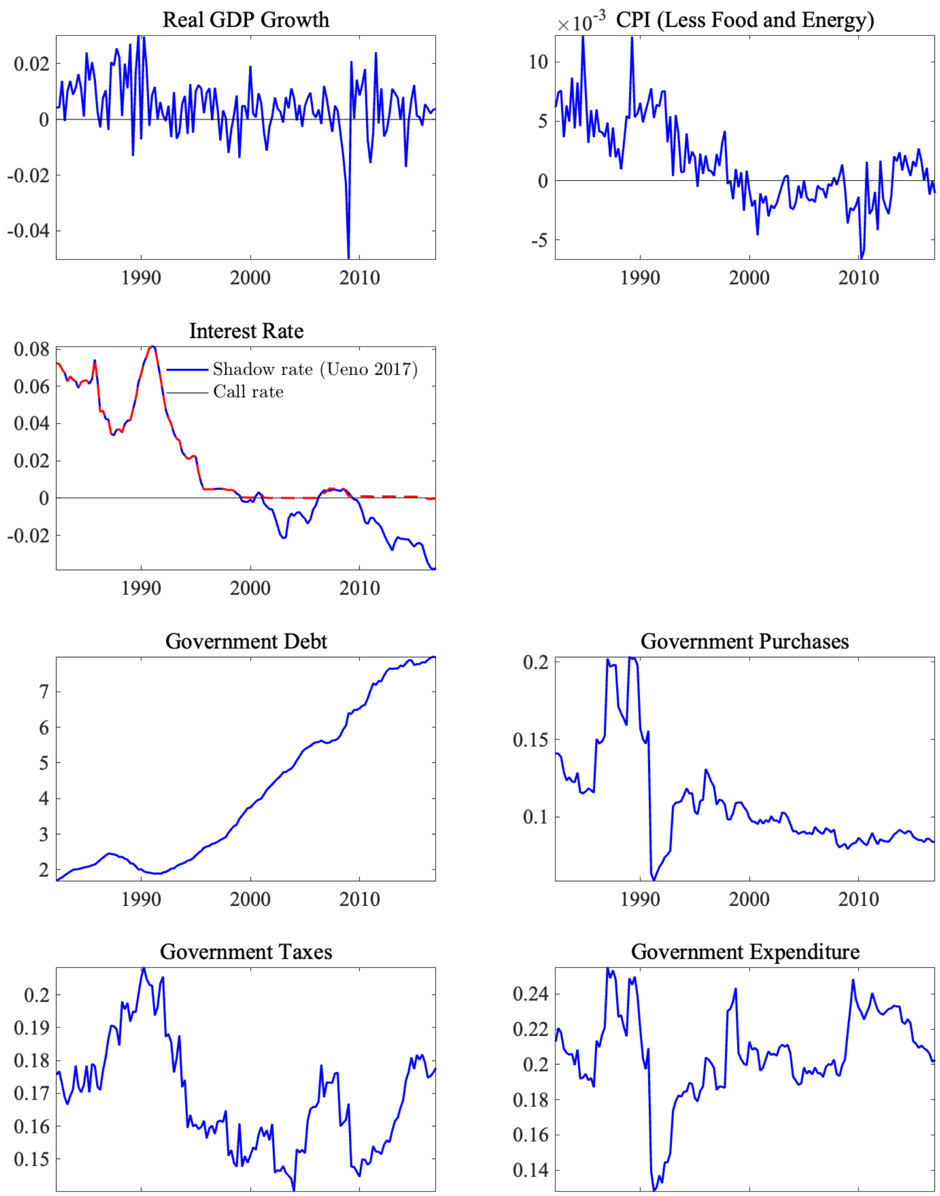
B Data Sources

Our dataset covers the following seven observable variables: real output growth (y_t), the rate of consumer price inflation (π_t), the nominal rate of interest (R_t), the debt-to-GDP ratio (B_t/GDP_t), the ratio of government purchases to GDP (G_t/GDP_t), the government expenditure-to-GDP ratio (E_t/GDP_t), and the ratio of government tax revenues to GDP (T_t/GDP_t). Details of the data sources are provided below. In Figure B.1, we show developments in variables used in the analysis.

Table B.1: Data Sources

Variables	Sources
Real GDP Growth	Cabinet Office, "National Accounts"
Consumer Price Index (less food and energy)	Ministry of Internal Affairs and Communications, "Consumer Price Index"
Interest Rate	Bank of Japan "Call Rate" from 1982/2Q to 1999/1Q Ueno (2017) "Shadow Rate" from 1999/2Q to 2017/1Q
Government Debt	Cabinet Office, "National Accounts" Closing Balance Sheet Account Liabilities
Government Purchases	Cabinet Office, "National Accounts" Income and Outlay Accounts - General Government Collective Consumption Expenditure Consumption of Fixed Capital (Less) Closing Balance Sheet Account - General Government Net Purchases of Non-produced assets (Natural resources) Gross Capital Formation - General Government Gross fixed capital formation Changes in inventories
Government Expenditure	Sum of 'Government Purchase' and 'Government Transfer'
Government Purchase	see the above
Government Transfer	Cabinet Office, "National Accounts" Income and Outlay Accounts - General Government Individual consumption expenditure Social benefits other than social transfers in kind, payable Other current transfers, payable less receivable Net social contributions, receivable (less) Subsidies, payable Capital Account - General Government Capital transfers, payable less receivable
Government Taxes	Cabinet Office, "National Accounts" Income and Outlay Accounts - General Government Taxes on production and imports, receivable Property income receivable less payable Current taxes on income, wealth, etc., receivable

Figure B.1: Developments in Variables Used in the Analysis



NOTE: The panels for real GDP growth and changes in the CPI (less food and energy) show four-quarter backward moving averages.

C Other Results

Table C.1: Other Estimated Structural Parameters

Description	Parameter	Value
Fiscal policy response to output	δ_y	1.461
Fiscal policy response to expenditure	δ_e	0.639
Transfer response to output	ι_y	0.135
Short-term expenditure response to output	ϕ_y	-0.472
Degree of lagging in inflation	ζ	0.327
Degree of external habit	Φ	0.292
Slope of Phillips curve	κ	0.003
Transfer smoothing	ρ_χ	0.990
Persistence of neutral technology shocks	ρ_a	0.535
Persistence of preference shocks	ρ_d	0.976
Short-term expenditure smoothing	ρ_{e^S}	0.921
Persistence of mark-up shocks	ρ_μ	0.033
Persistence of term premium shocks	ρ_{tp}	0.837
Inflation rate	100π	0.421
Neutral technological change	$100ln(\gamma)$	0.450
Debt-to-GDP ratio (annualized)	b^m	0.524
Purchase to GDP ratio	g	0.093
Tax-to-GDP ratio	τ	0.195
S.D. of monetary shocks (Low)	$100\sigma_{R,1}$	0.080
S.D. of monetary shocks (High)	$100\sigma_{R,2}$	0.080
S.D. of transfer shocks (Low)	$100\sigma_{\chi,1}$	2.823
S.D. of transfer shocks (High)	$100\sigma_{\chi,2}$	27.808
S.D. of neutral technology shocks (Low)	$100\sigma_{a,1}$	0.715
S.D. of neutral technology shocks (High)	$100\sigma_{a,2}$	2.073
S.D. of tax shocks (Low)	$100\sigma_{\tau,1}$	0.541
S.D. of tax shocks (High)	$100\sigma_{\tau,2}$	0.831
S.D. of preference shocks (Low)	$100\sigma_{d,1}$	4.269
S.D. of preference shocks (High)	$100\sigma_{d,2}$	8.840
S.D. of short-term expenditure (Low)	$100\sigma_{e^S,1}$	0.472
S.D. of short-term expenditure (High)	$100\sigma_{e^S,2}$	2.796
S.D. of term premium shocks (Low)	$100\sigma_{tp,1}$	1.221
S.D. of term premium shocks (High)	$100\sigma_{tp,2}$	3.512
S.D. of mark-up shocks (Low)	$100\sigma_{\mu,1}$	0.129
S.D. of mark-up shocks (High)	$100\sigma_{\mu,2}$	0.077

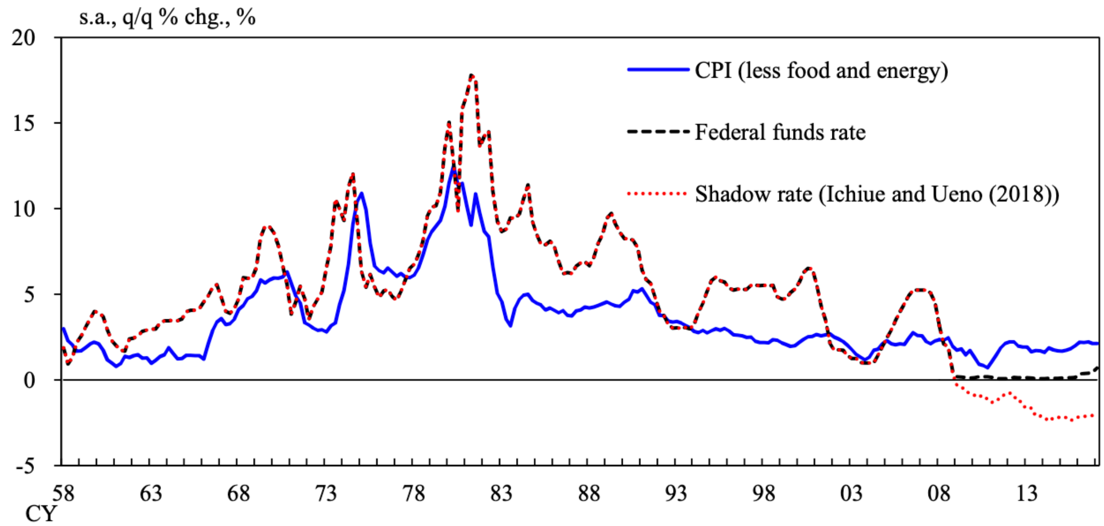
Table C.2: Estimated Parameters for Markov Probability

Description	Parameter	Value
from Low volatility to Low volatility	H_{11}^{vo}	0.88
from High volatility to High volatility	H_{22}^{vo}	0.18

D Application to the U.S.

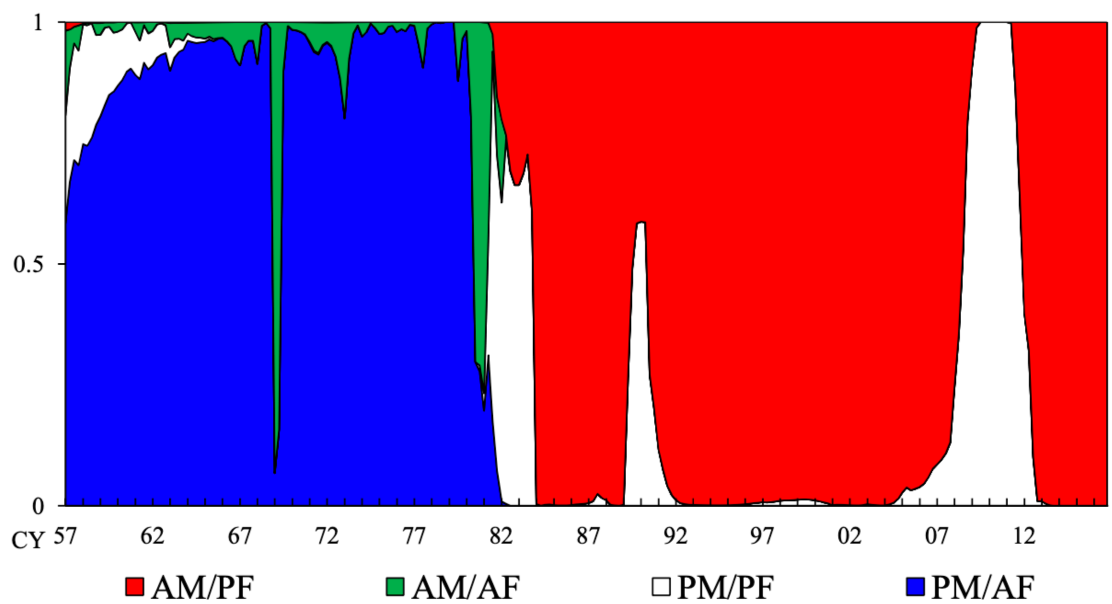
We also apply our approach to the United States. In a departure from the Bianchi and Ilut (2017) study, we try to capture changes in the monetary-fiscal policy interaction even during the period when the ZLB is binding and unconventional monetary policy is implemented. To estimate our model, we used the same seven observable variables as for Japan. It should be noted, however, that we use the shadow federal funds rate estimated by Ichiue and Ueno (2018). In addition, our analysis covers the period from 1957:2Q to 2017:1Q, while the observation period of Bianchi and Ilut (2017) is from 1954:4Q to 2009:3Q. Figure D.1 shows the main observation variables, which are different from an original work in Bianchi and Ilut (2017). We can see that the shadow rate of Ichiue and Ueno (2018) goes below zero during the unconventional monetary policy regime. The estimation results are shown in Figure D.2 and show that the regimes identified are almost the same and consistent with those identified by Bianchi and Ilut (2017) from 1954:4Q to 2009:3Q. However, for the period after the global financial crisis, which they do not cover, we find that the policy regime was also PM/PF, as in Japan. Therefore, it would be interesting to examine the role of the PM/PF regime in the evolution of the U.S. economy after the financial crisis, focusing in particular on the behavior of inflation. We leave this analysis to future work.

Figure D.1: Inflation and Interest Rates (U.S.)



NOTE: CPI (less food and energy) represents the four-quarter backward moving average of the rate of change in the CPI.

Figure D.2: Probability of Policy Regimes (U.S.)



NOTE: This figure shows the smoothed probabilities given to the four policy regimes. AM/PF: Active monetary policy/passive fiscal policy regime, AM/AF: Active monetary policy/active fiscal policy regime, PM/PF: Passive monetary policy/passive fiscal policy regime, PM/AF: Passive monetary policy/active fiscal policy regime.