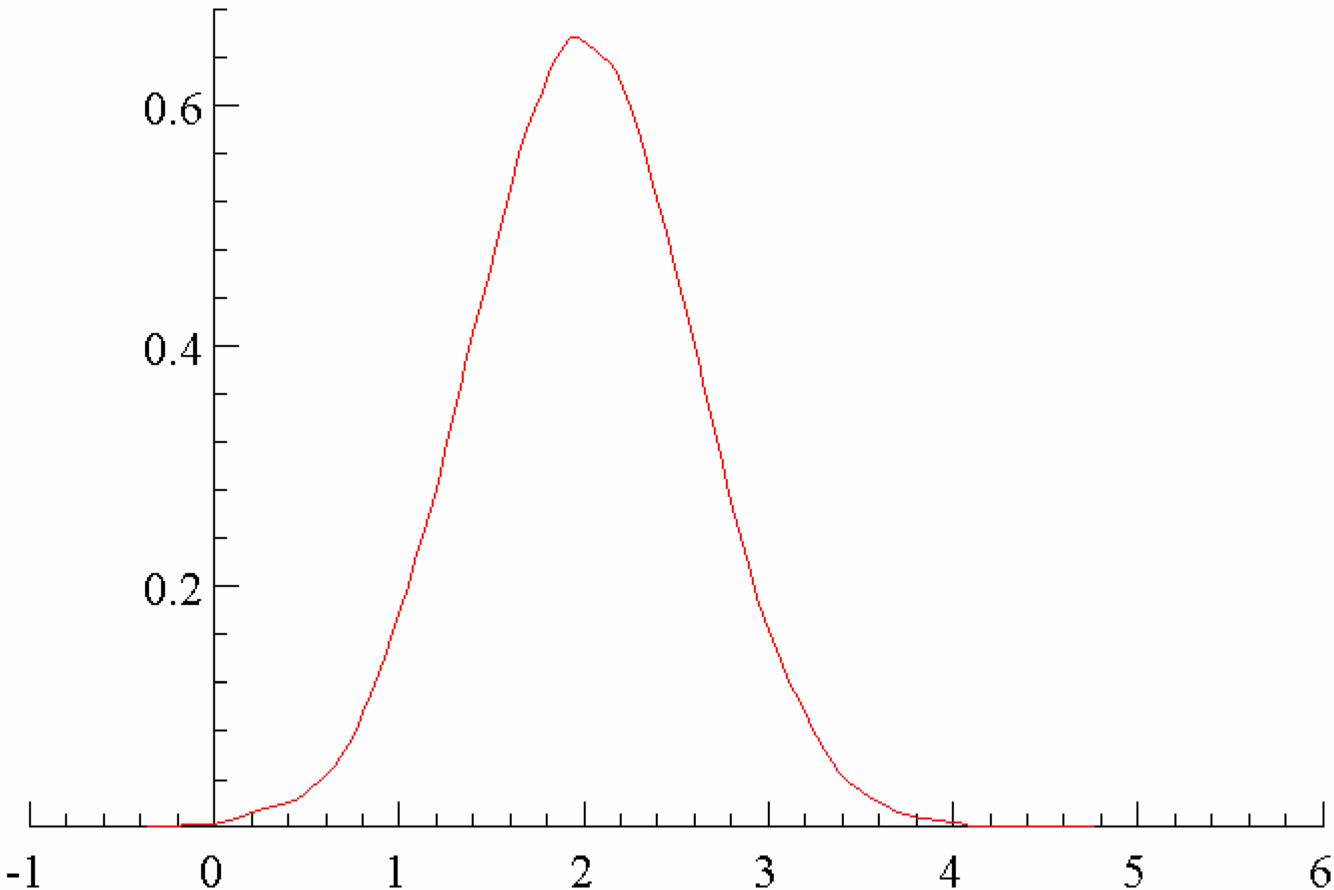
Markov chain Monte Carlo

$$y_t = x_t\beta + \varepsilon_t$$

$$\max.\ f(y|\beta) \rightarrow \hat{\beta}$$

likelihood



$$y_t = x_t\beta + \varepsilon_t$$

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$$\beta \sim N(\mu_\beta, \sigma_\beta^2)$$

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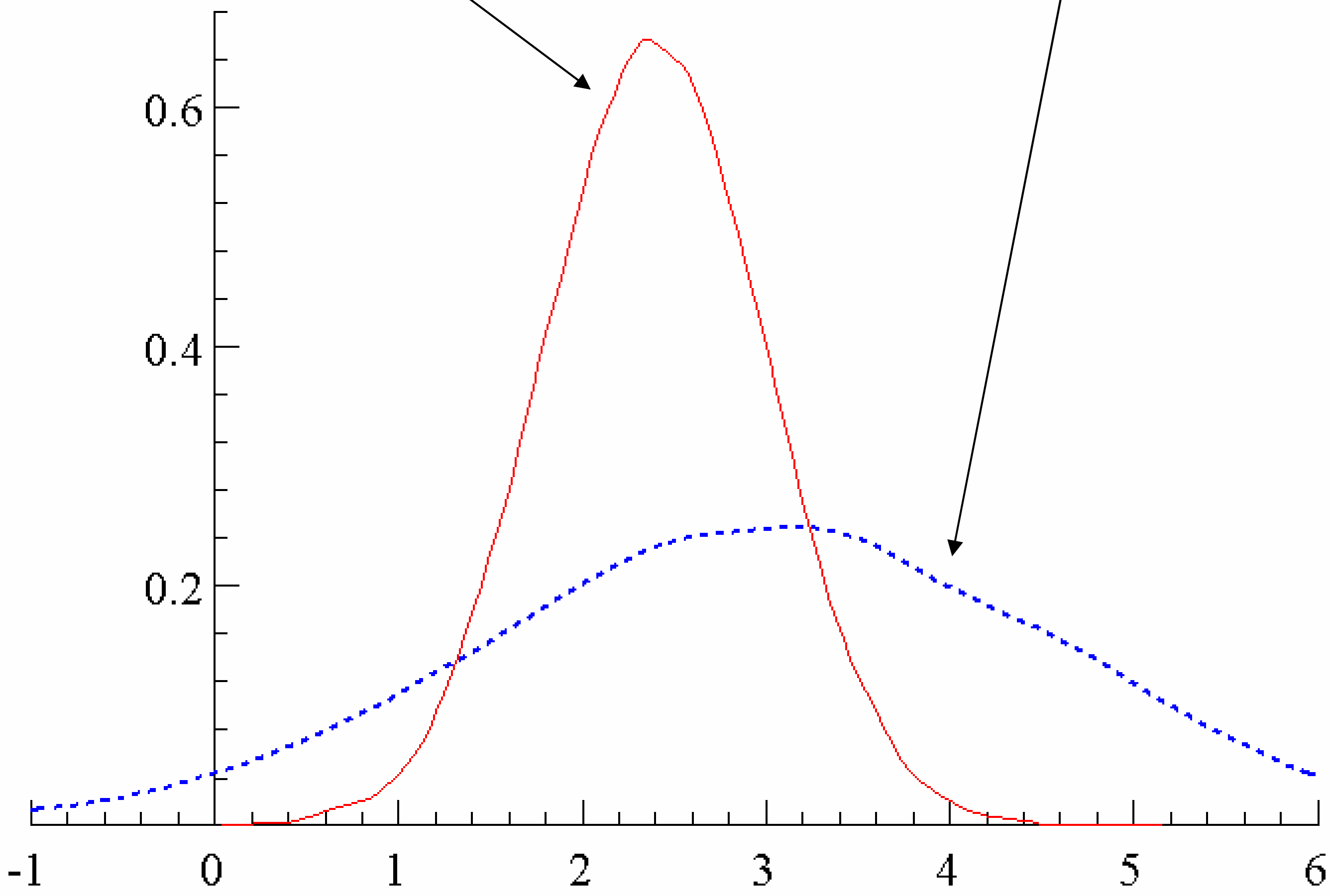
$$\beta \sim N(\mu_\beta, \sigma_\beta^2)$$

Bayesian inference

$$\underbrace{\pi(\beta|y)}_{\text{posterior}} \propto \underbrace{f(y|\beta)}_{\text{likelihood}} \times \underbrace{\pi(\beta)}_{\text{prior}}$$

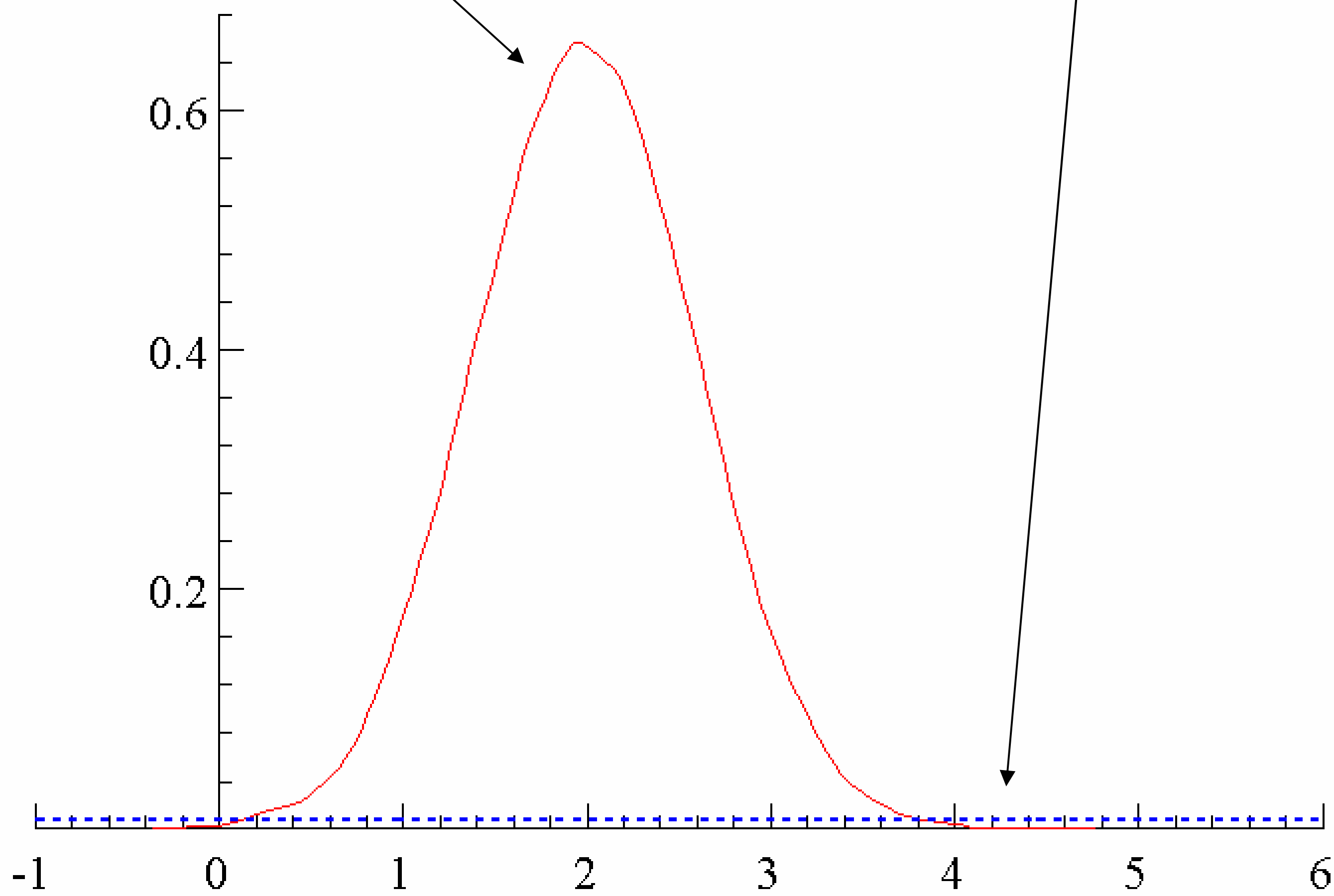
$$\pi(\beta|y) \;\propto\; f(y|\beta) \times \pi(\beta)$$

posterior likelihood prior



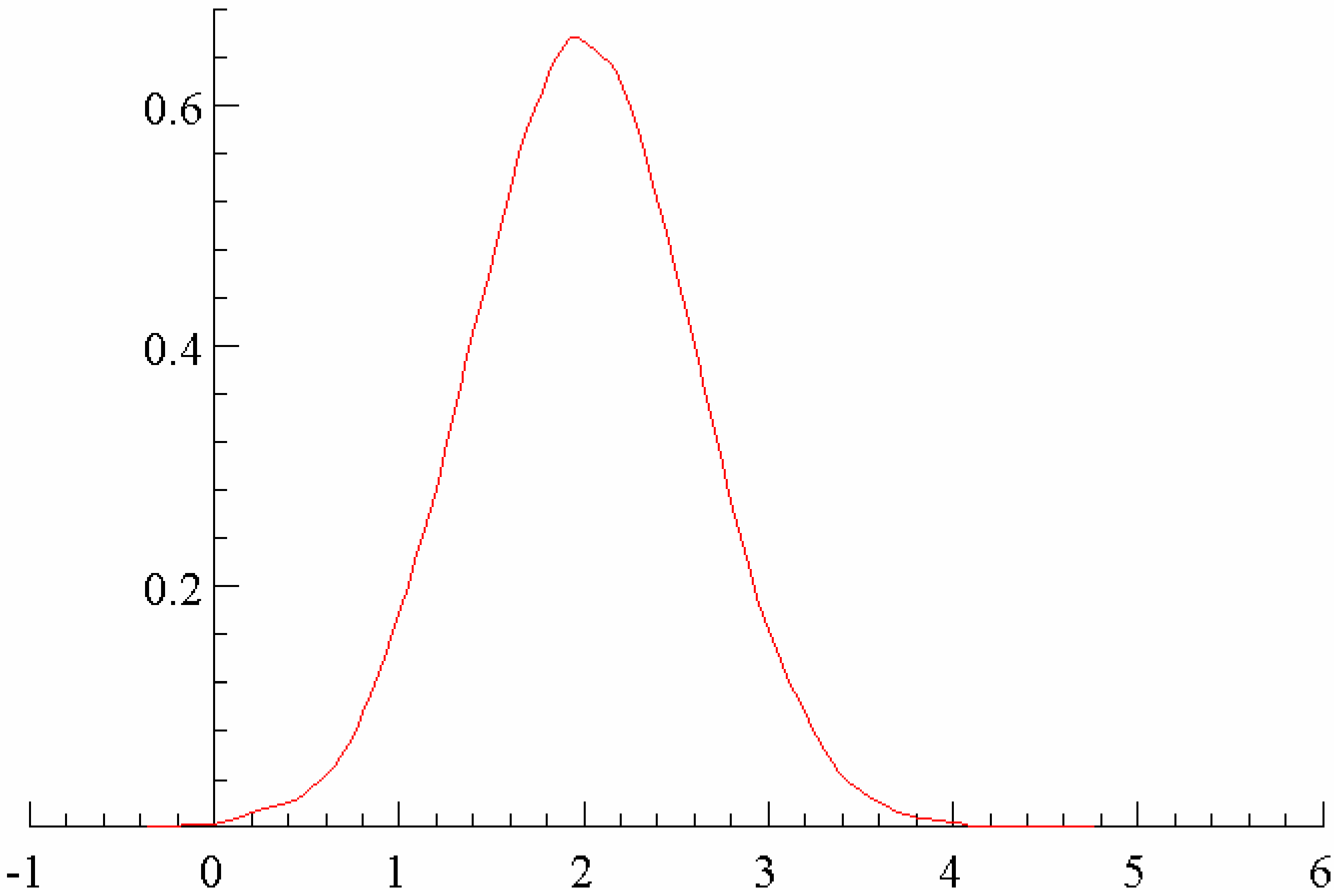
$$\pi(\beta|y) \;\propto\; f(y|\beta) \times \pi(\beta)$$

posterior likelihood prior



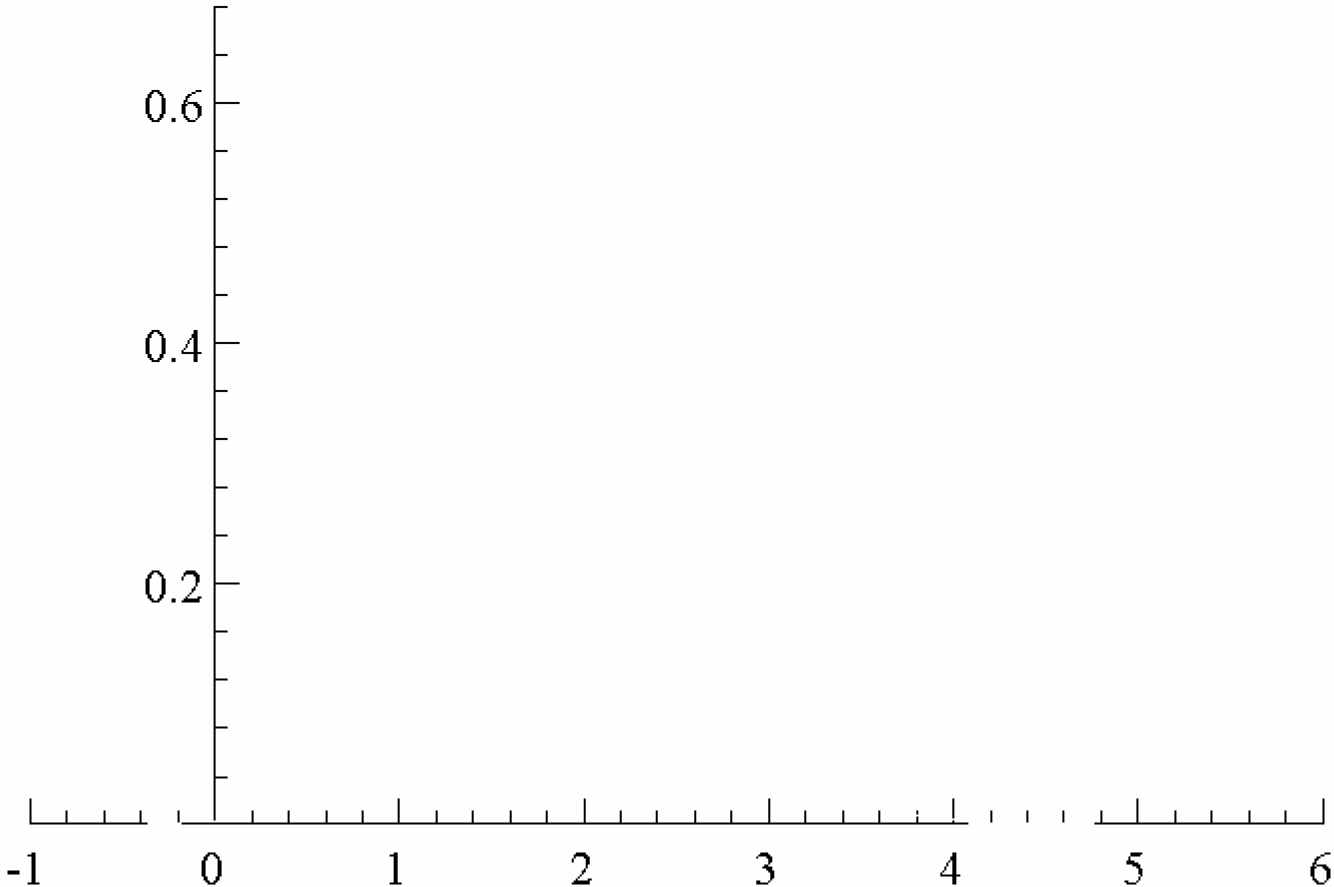
$$\text{max. } f(y|\beta) \rightarrow \hat{\beta}$$

likelihood



$$\beta^{(i)} \sim \pi(\beta|y)$$

sample from posterior



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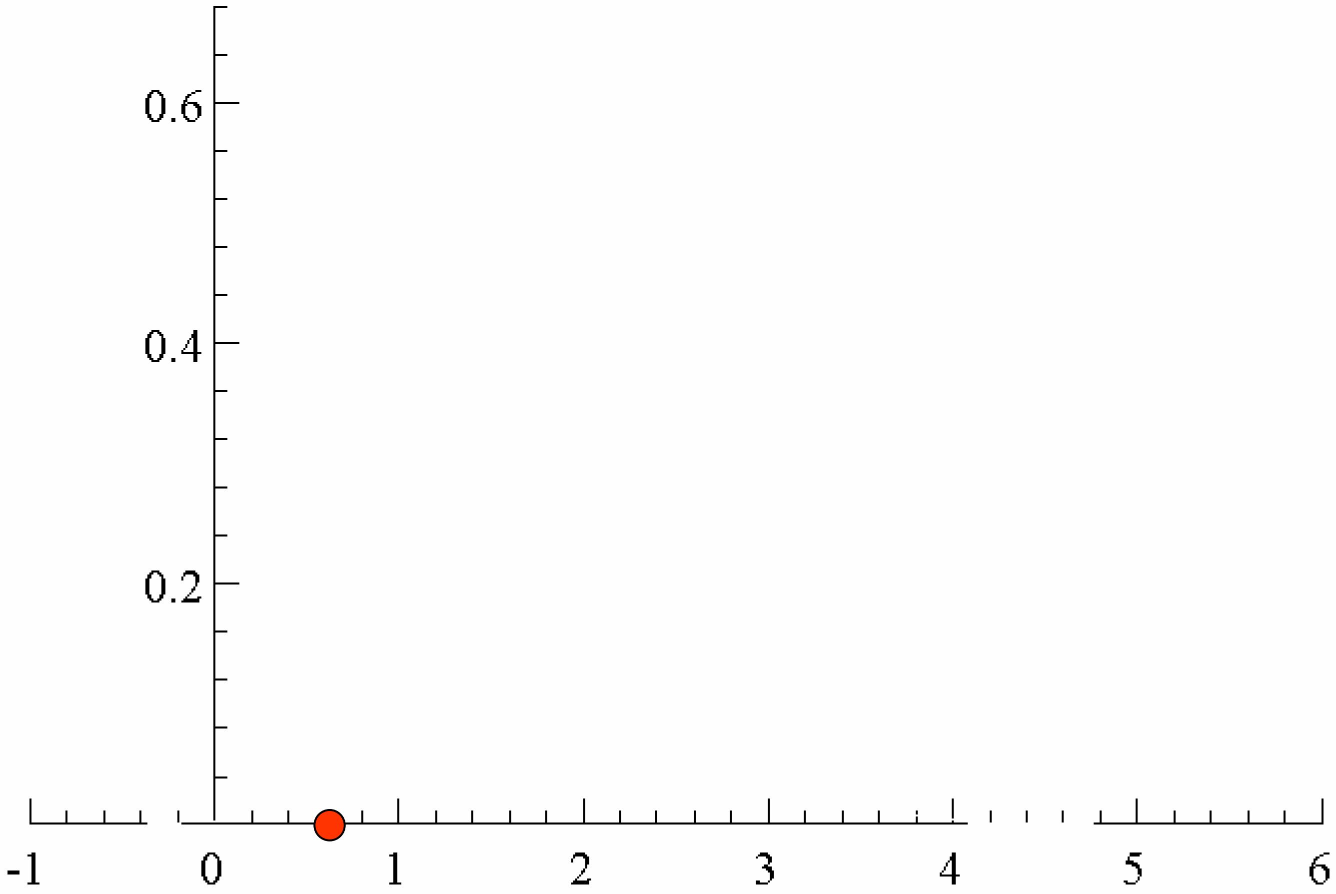

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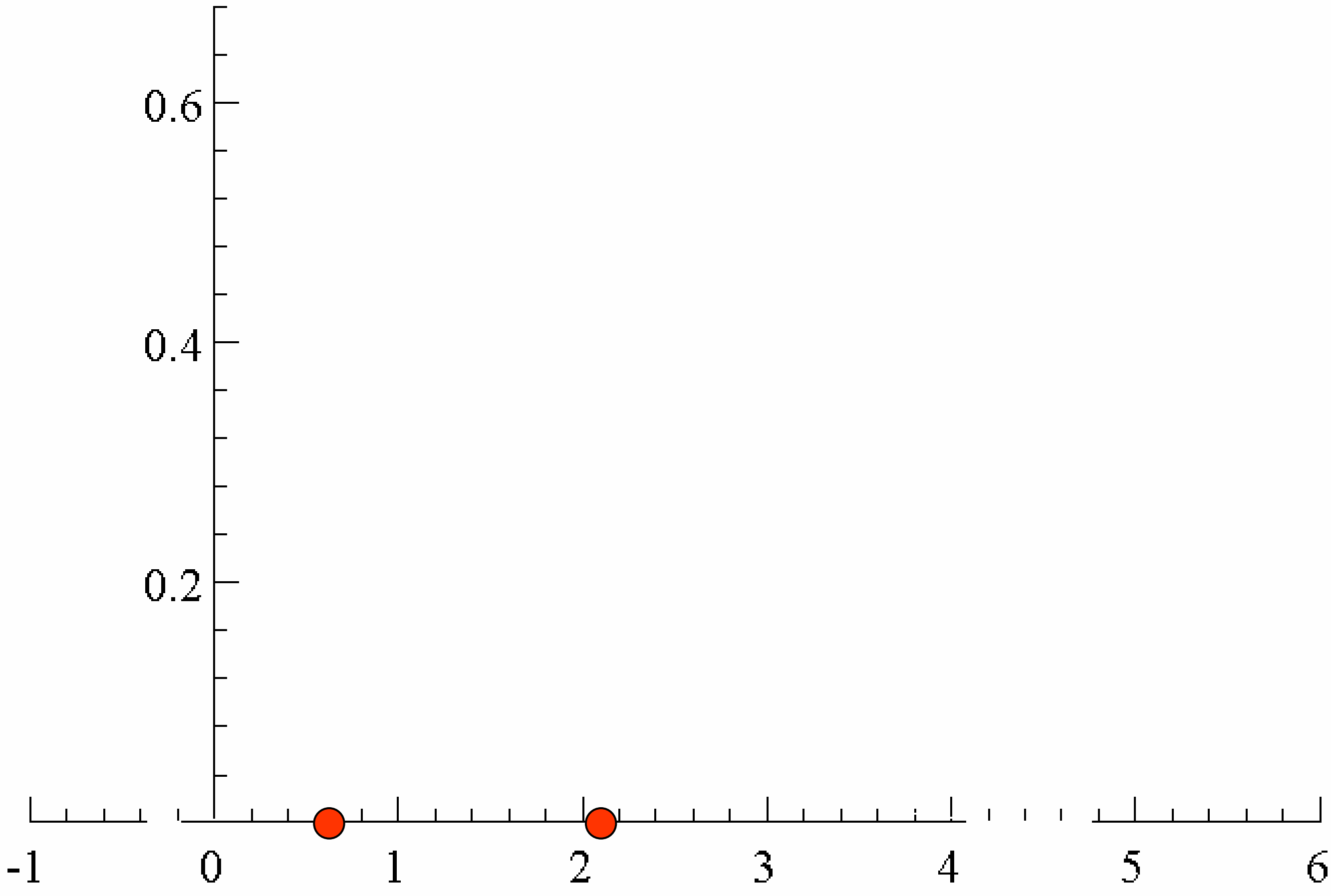

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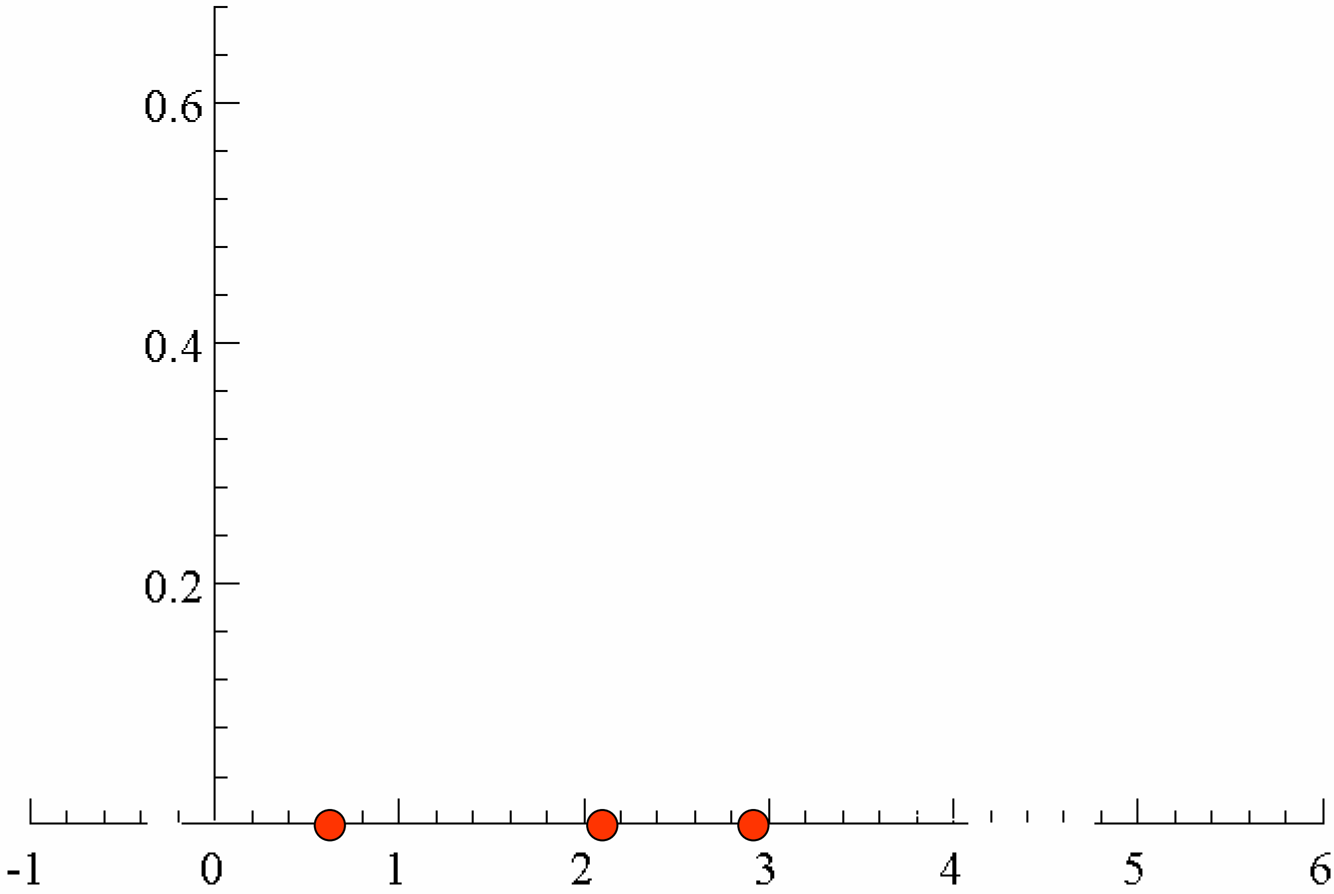

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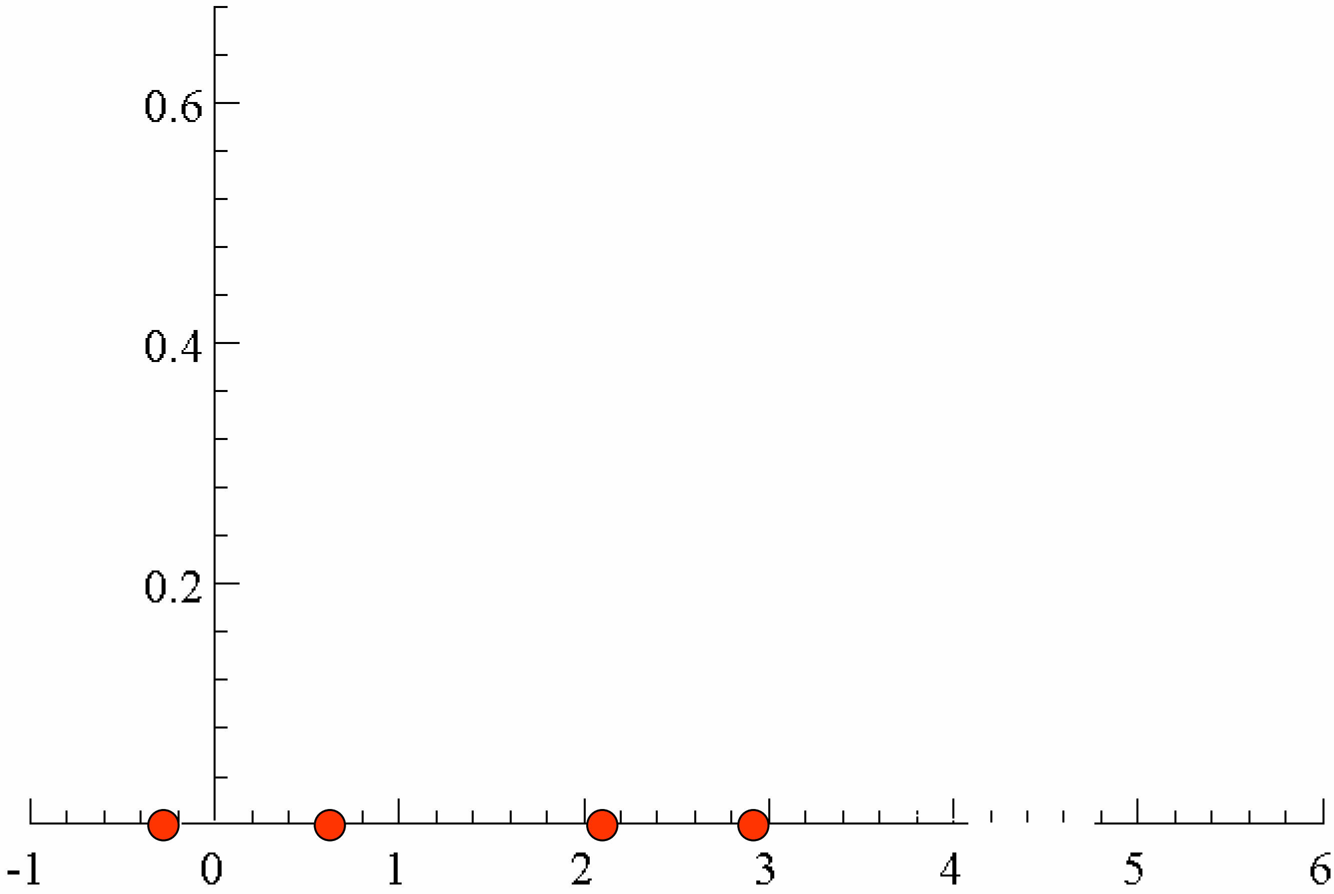
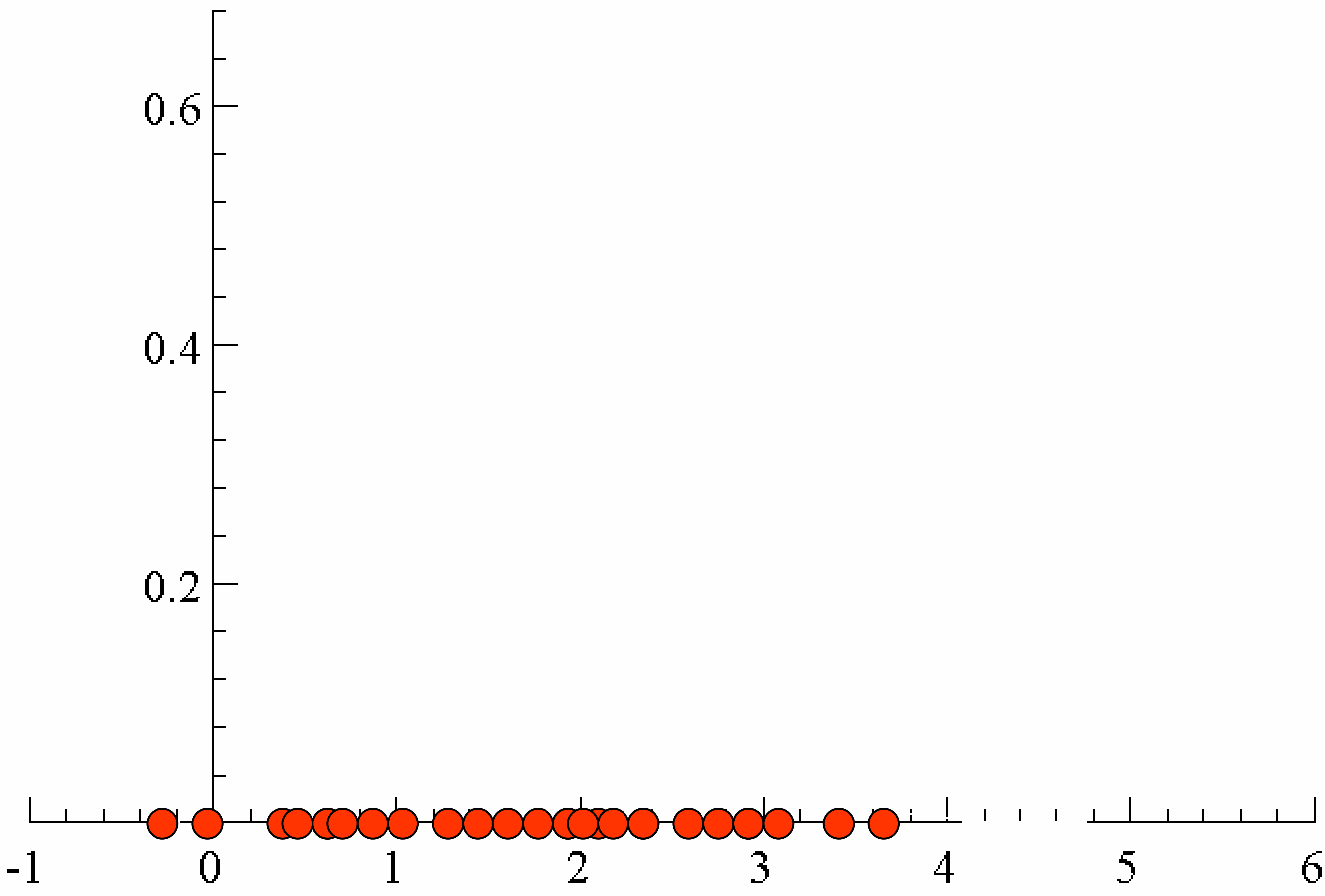
sample from posterior



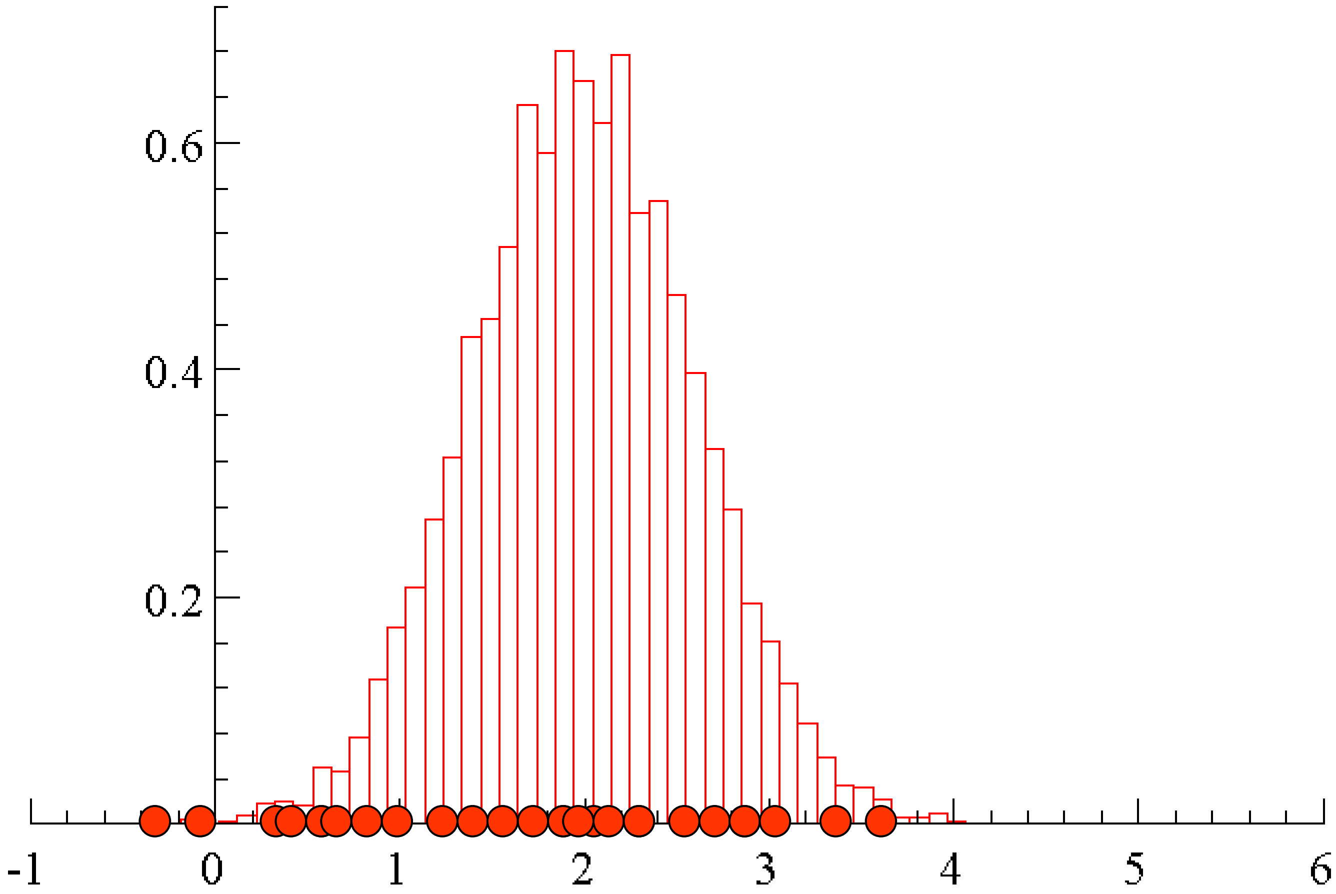
$$\beta^{(i)} \sim \pi(\beta|y)$$

sample from posterior



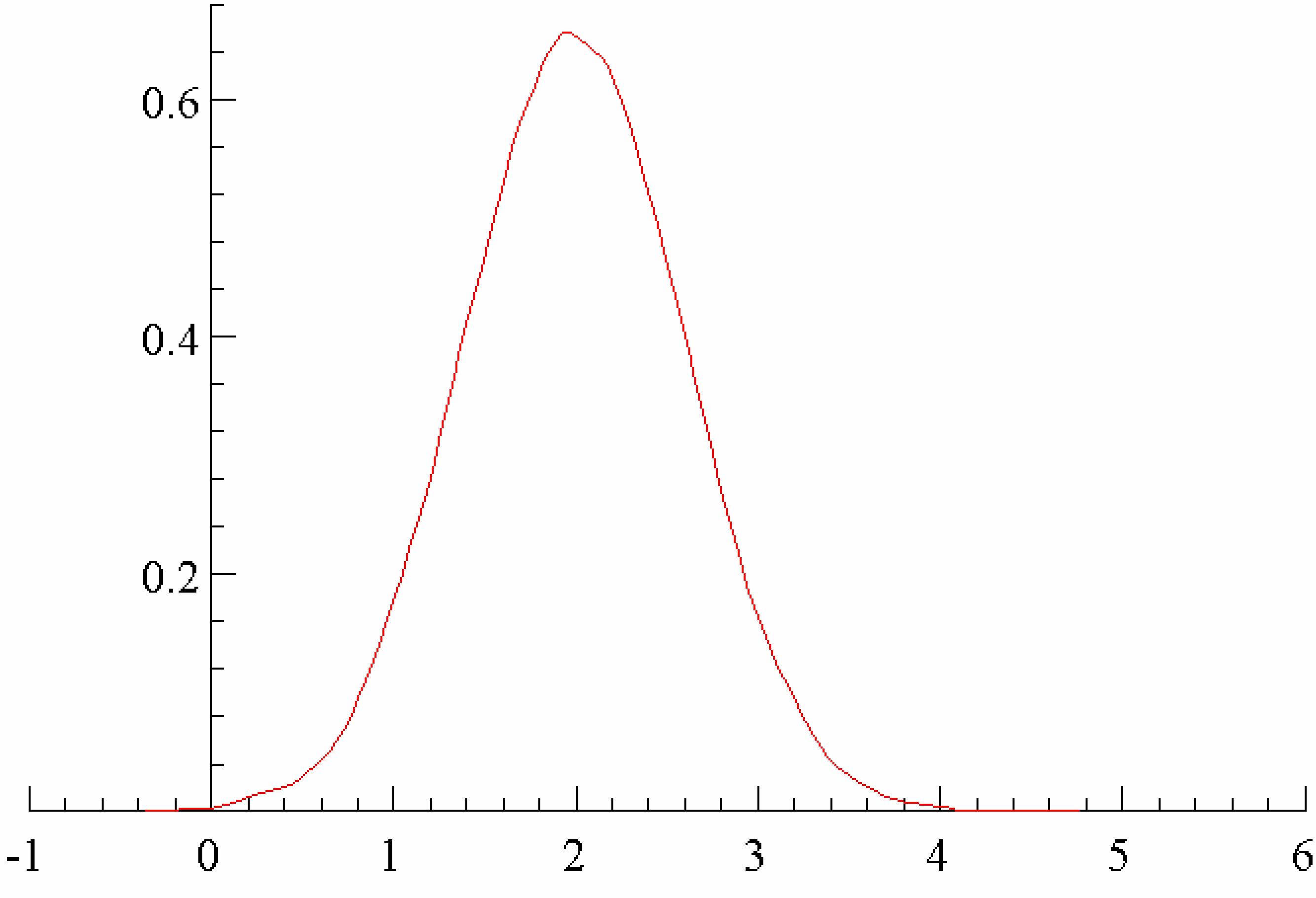
$$\beta^{(i)} \sim \pi(\beta|y)$$

sample from posterior



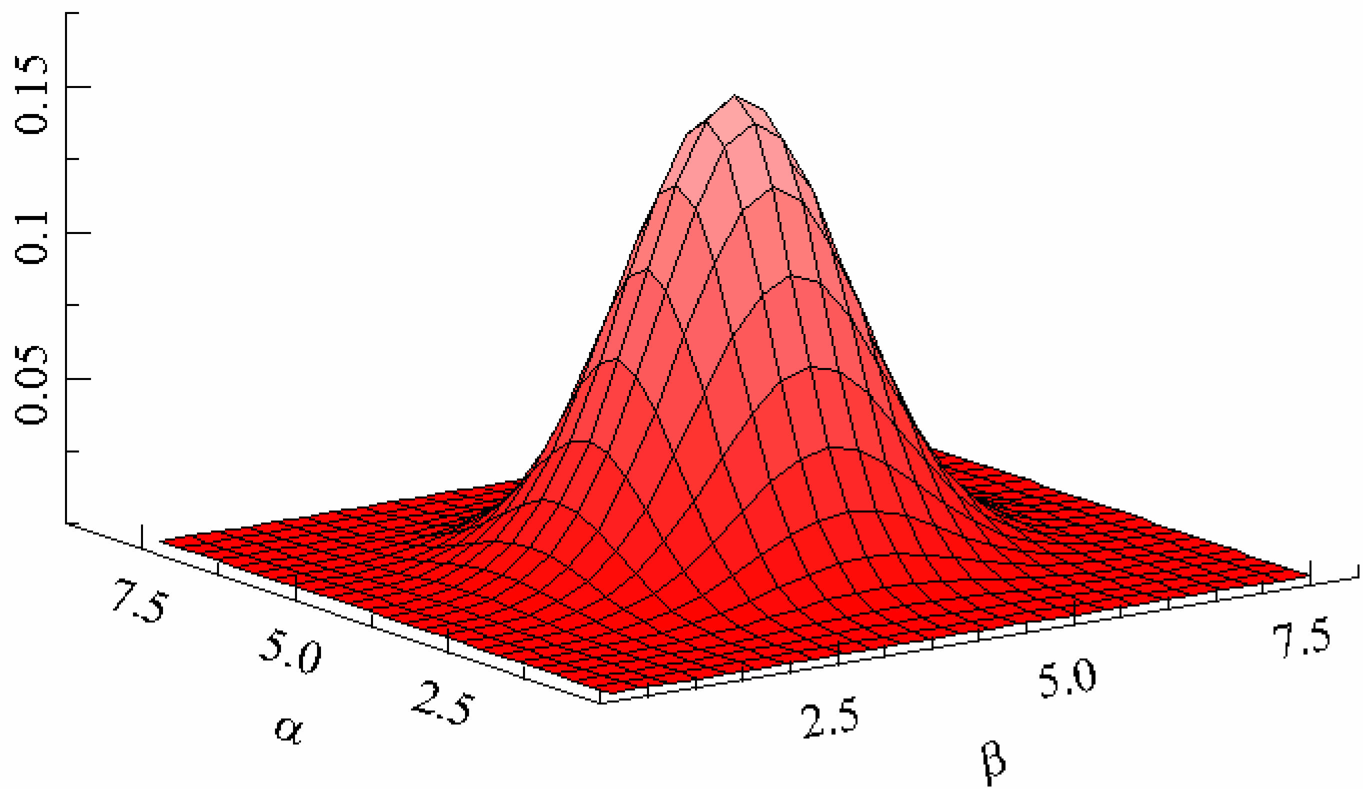
$$\beta^{(i)} \sim \pi(\beta|y)$$

sample from posterior



$$(\alpha, \beta)^{(i)} \sim \pi(\alpha, \beta|y)$$

$$(\alpha,\ \beta)^{(i)} \sim \pi(\alpha,\ \beta|y)$$



$$(\alpha,\, \beta)^{(i)} \sim \pi(\alpha,\, \beta | y)$$

MCMC algorithm

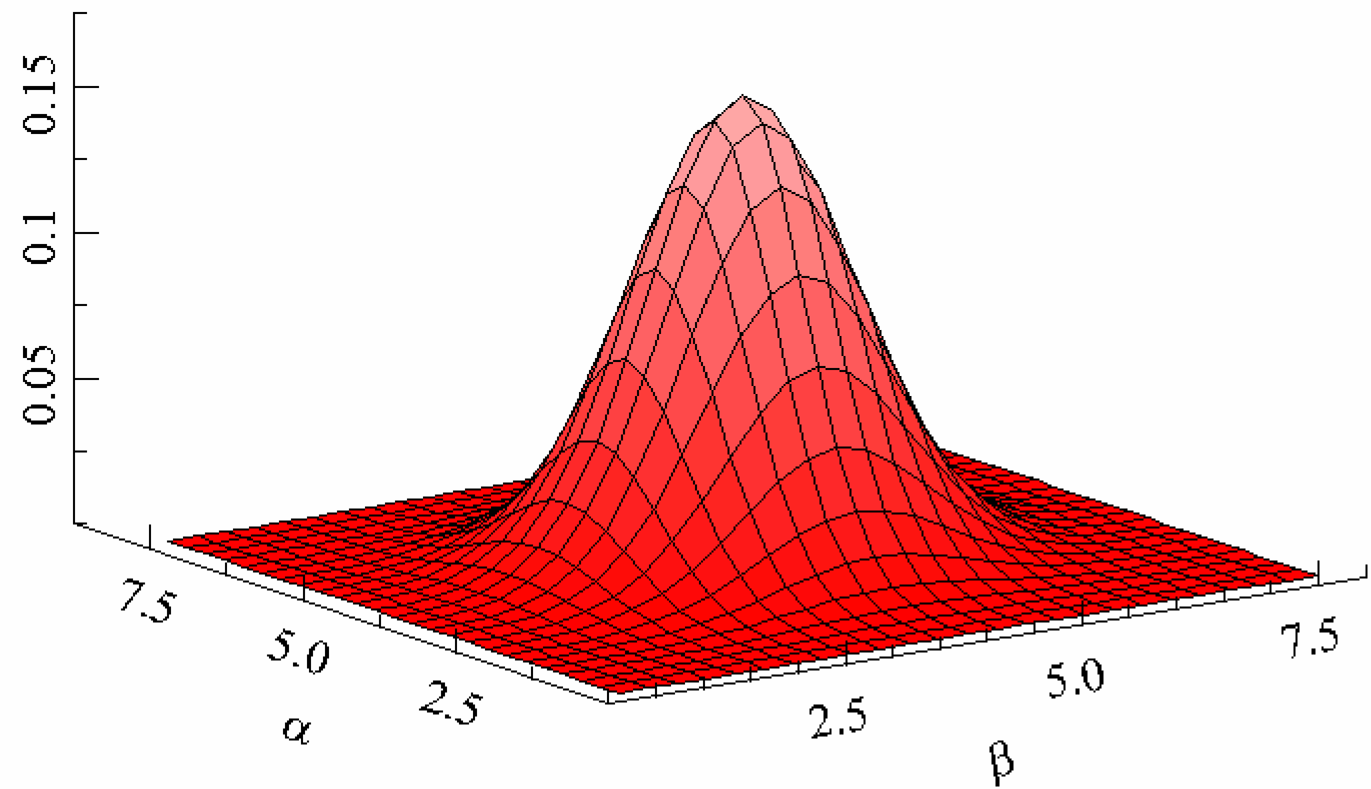
1. Initialize $\alpha^{(0)},\ \beta^{(0)}$

2. Sample $\alpha^{(1)} \sim \pi(\alpha | \beta^{(0)}, y)$

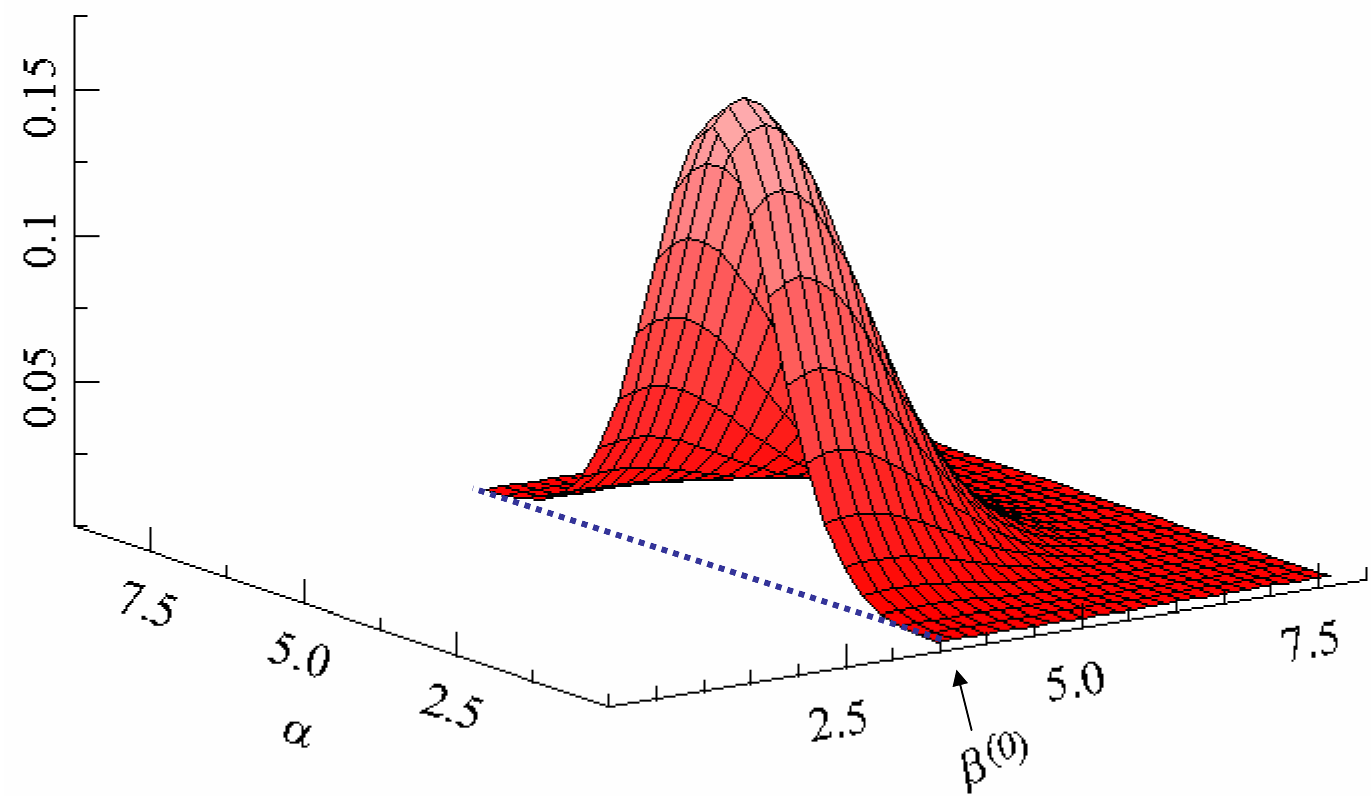
3. Sample $\beta^{(1)} \sim \pi(\beta | \alpha^{(1)}, y)$

4. Sample $\alpha^{(2)} \sim \pi(\alpha | \beta^{(1)}, y)$ ...

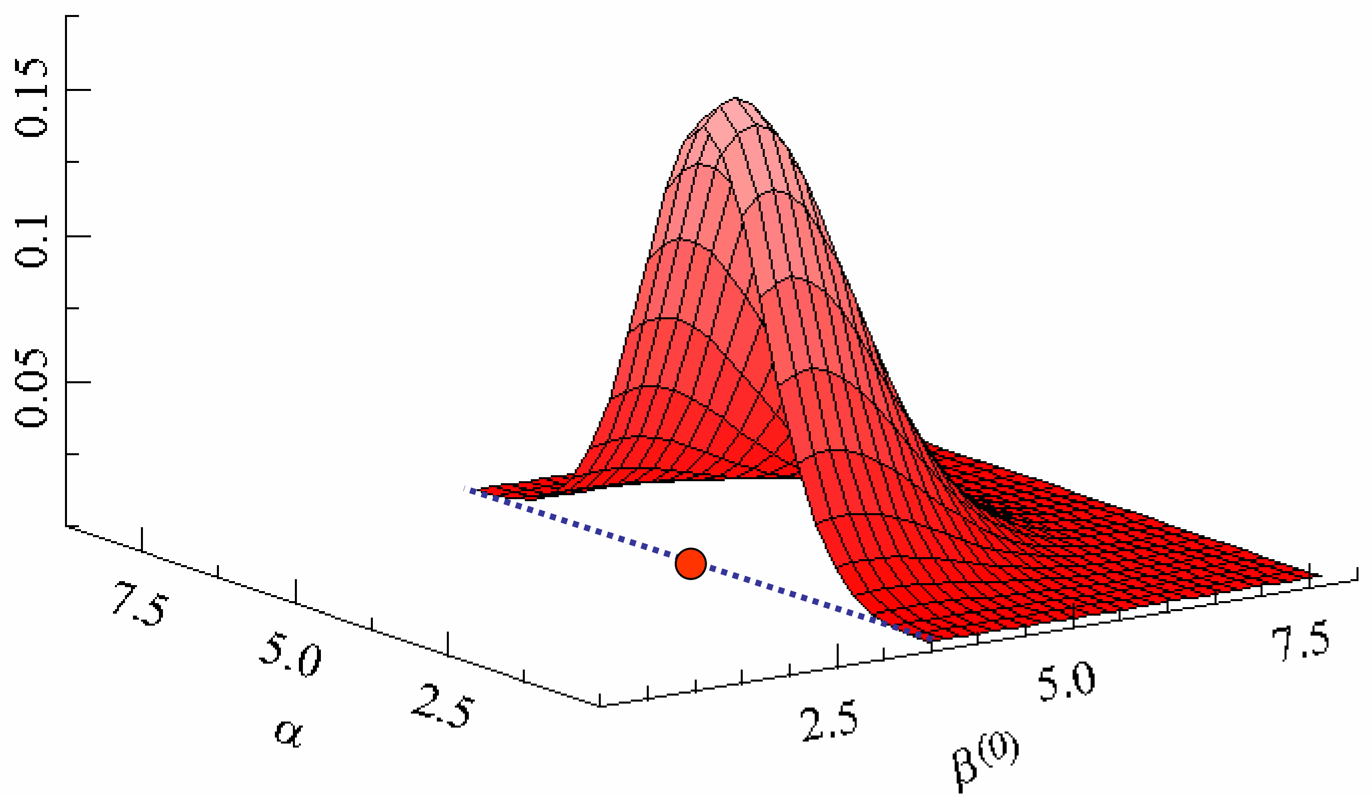
$$(\alpha,\ \beta)^{(i)} \sim \pi(\alpha,\ \beta|y)$$



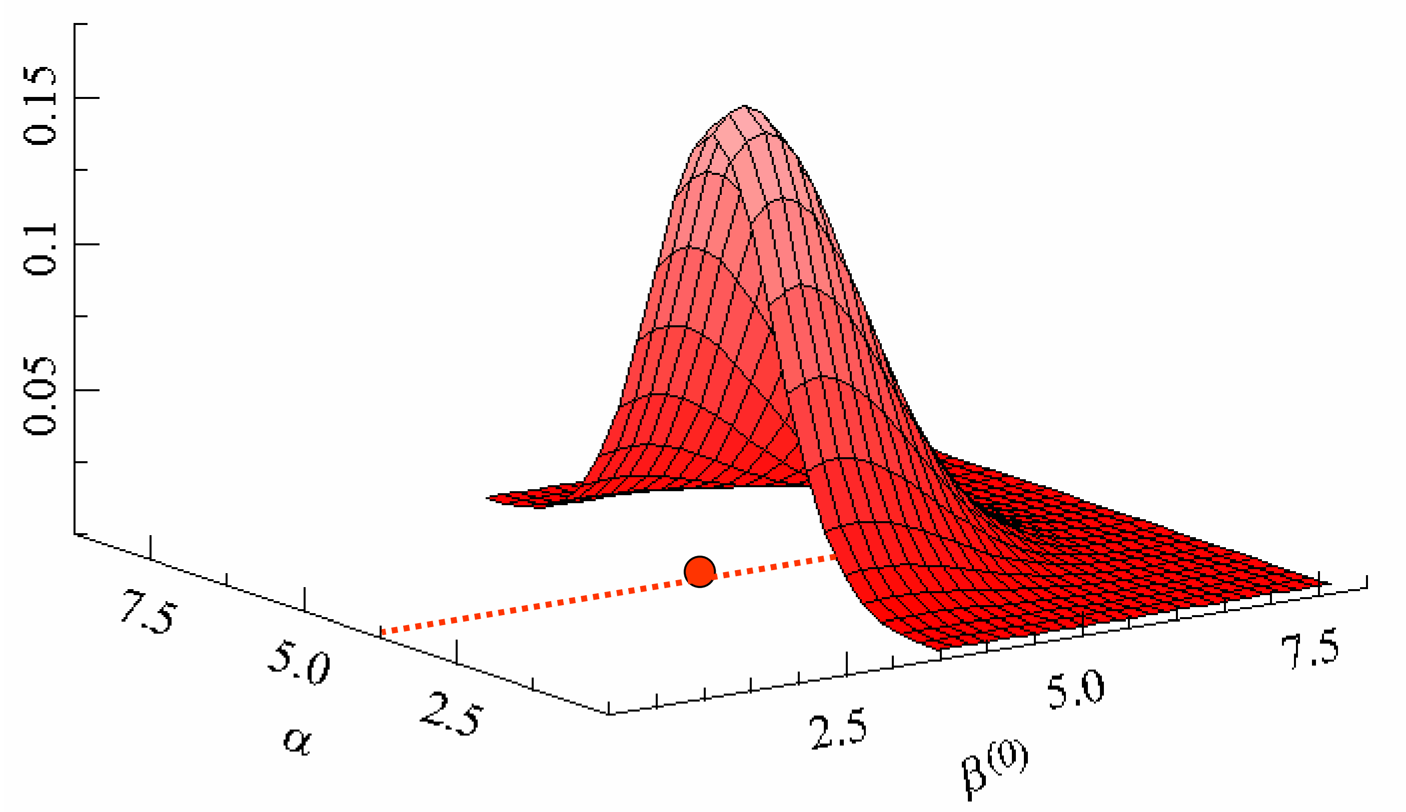
$$(\alpha,\ \beta)^{(i)} \sim \pi(\alpha,\ \beta|y)$$



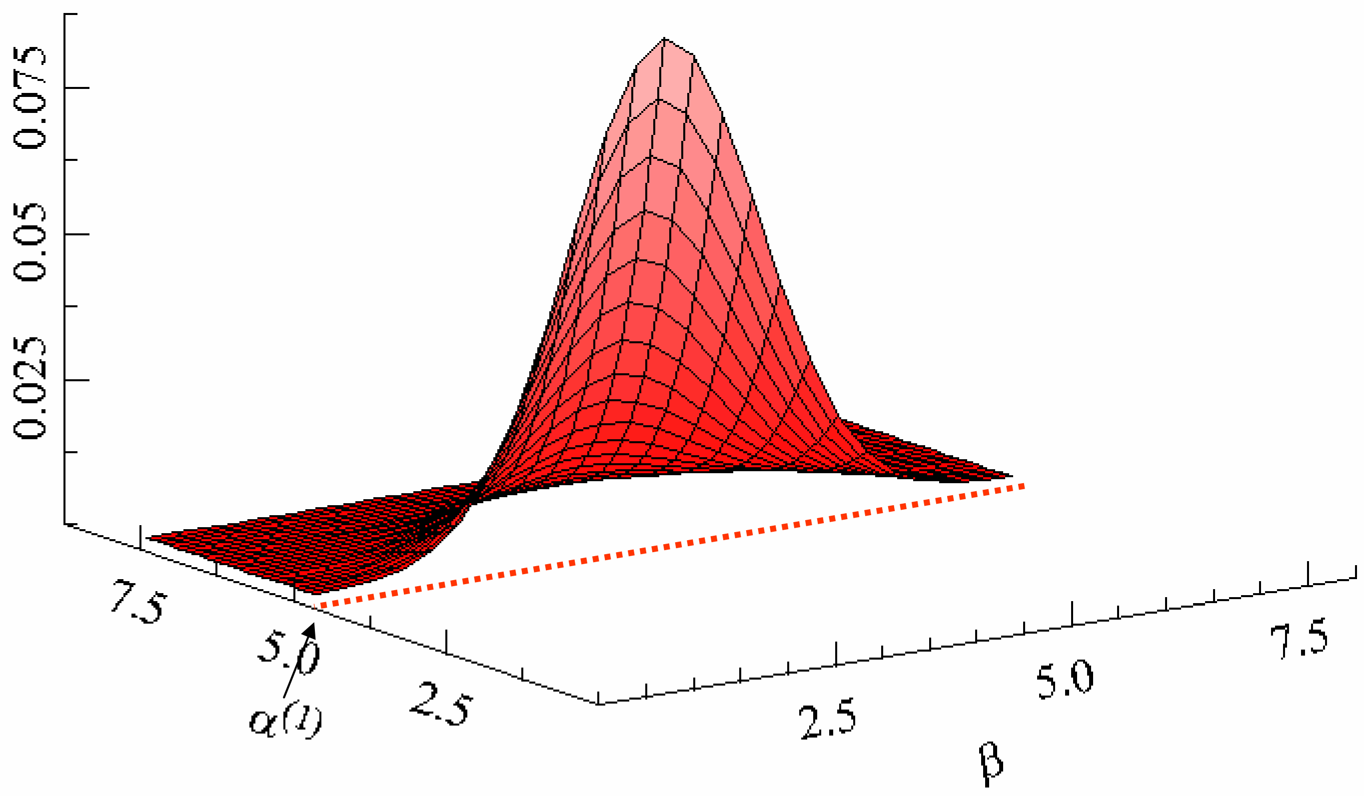
$$(\alpha,\ \beta)^{(i)} \sim \pi(\alpha,\ \beta|y)$$



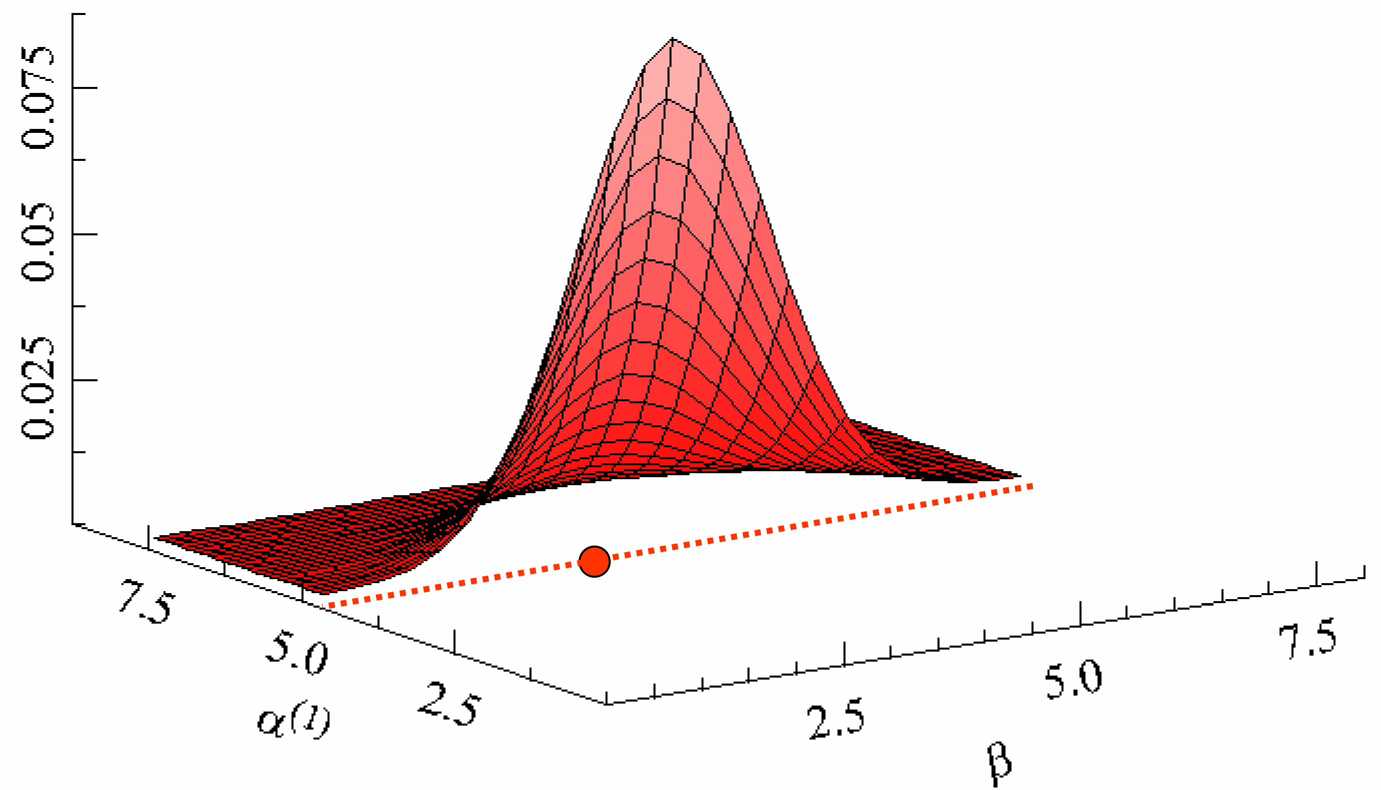
$$(\alpha,\ \beta)^{(i)} \sim \pi(\alpha,\ \beta|y)$$



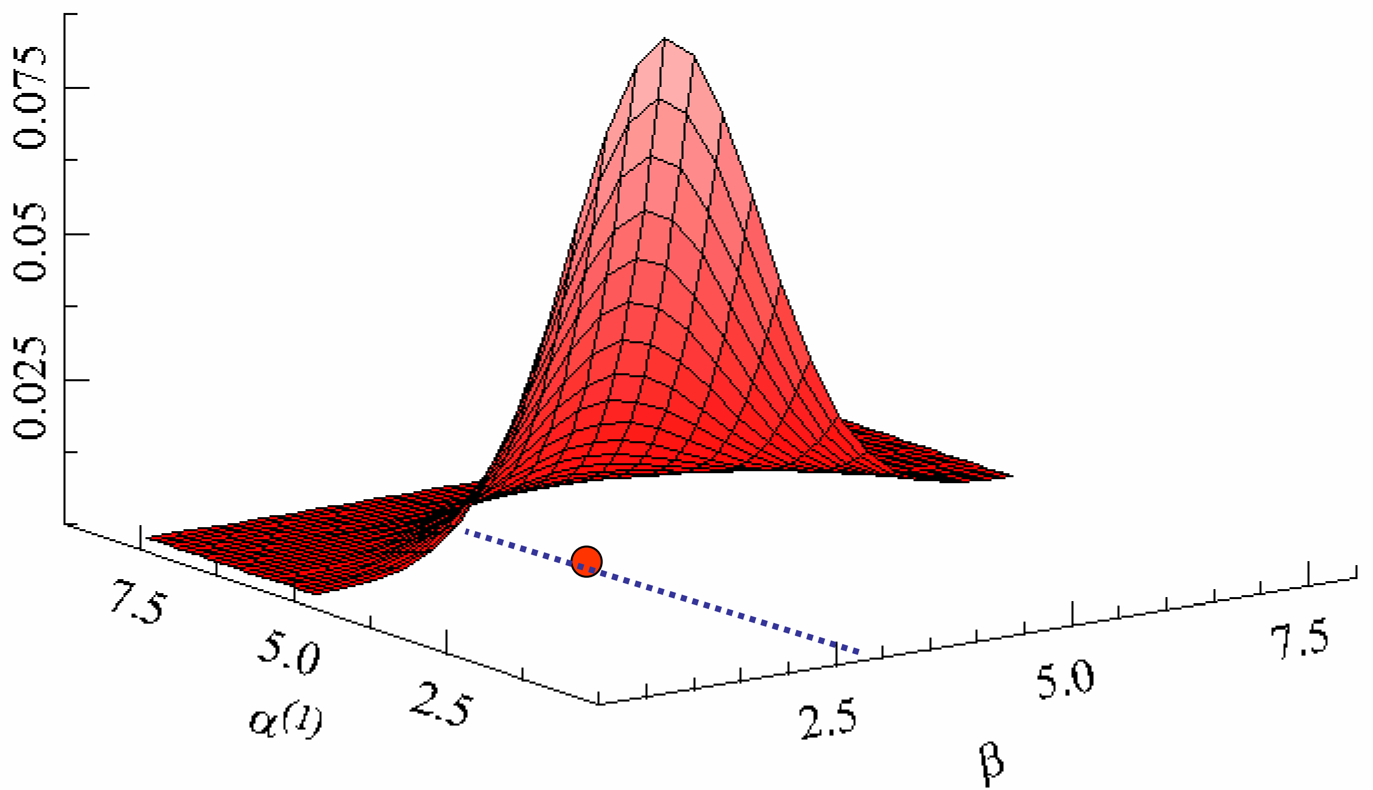
$$(\alpha,\ \beta)^{(i)} \sim \pi(\alpha,\ \beta|y)$$



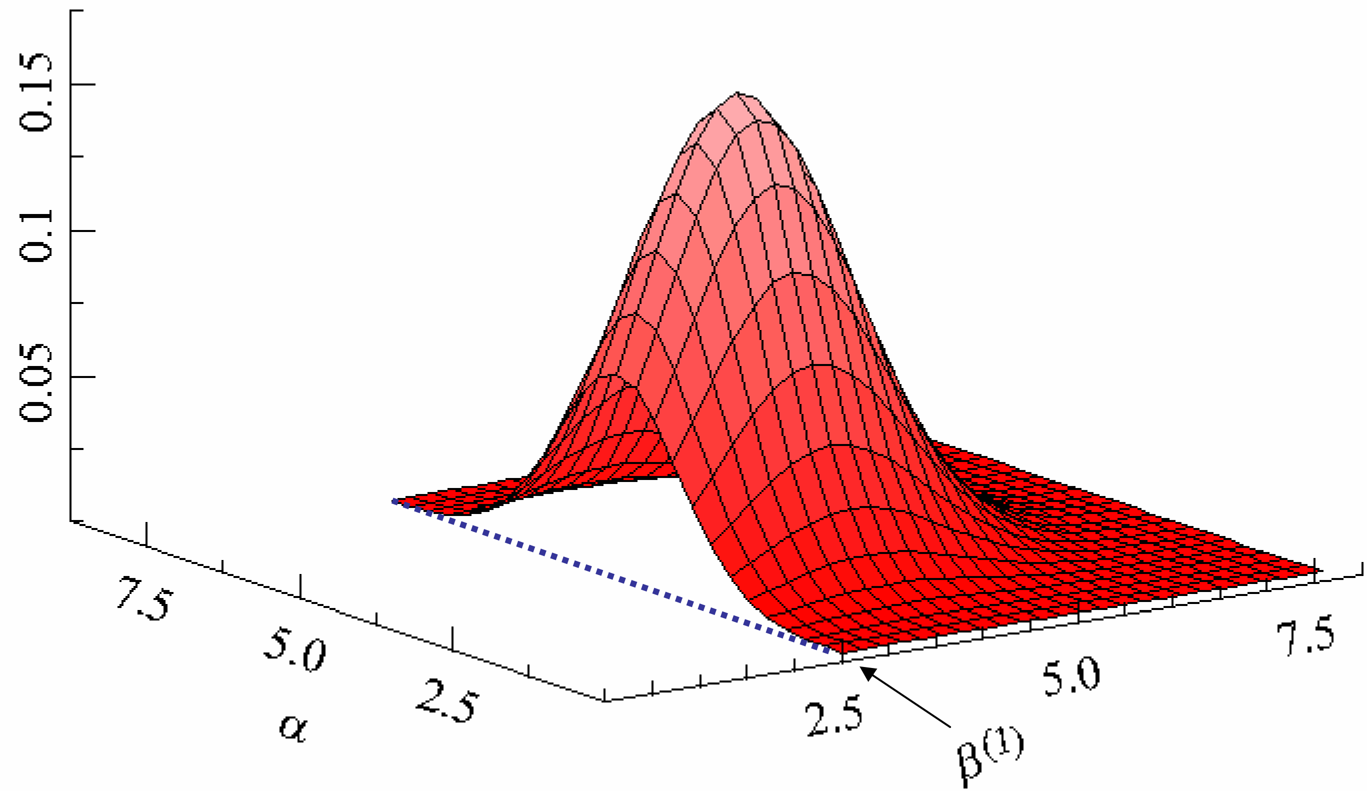
$$(\alpha,\ \beta)^{(i)} \sim \pi(\alpha,\ \beta|y)$$



$$(\alpha,\ \beta)^{(i)} \sim \pi(\alpha,\ \beta|y)$$



$$(\alpha,\ \beta)^{(i)} \sim \pi(\alpha,\ \beta | y)$$



1. Initialize $\alpha^{(0)},\ \beta^{(0)}$

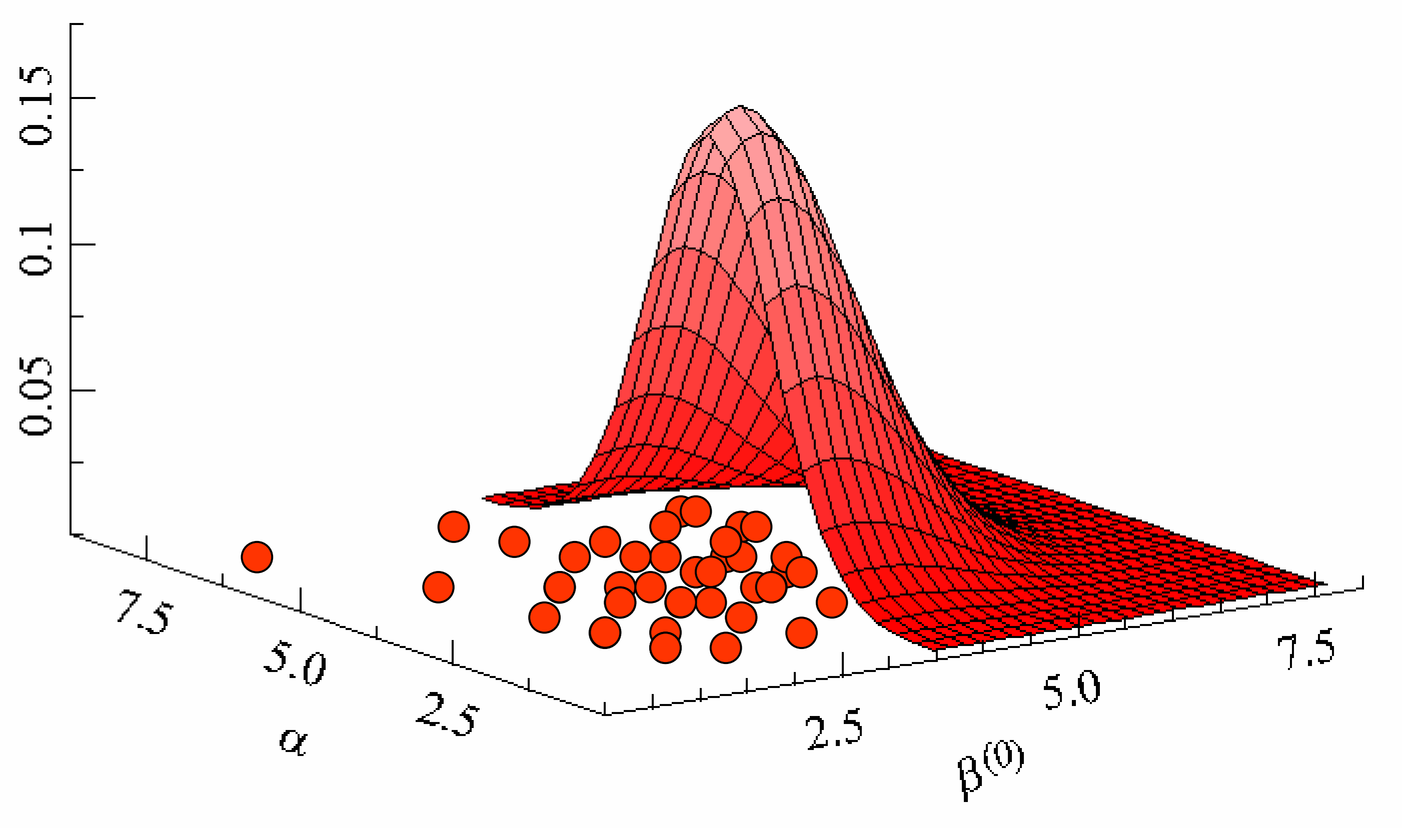
2. Sample $\alpha^{(1)} \sim \pi(\alpha|\beta^{(0)}, y)$

3. Sample $\beta^{(1)} \sim \pi(\beta|\alpha^{(1)}, y)$

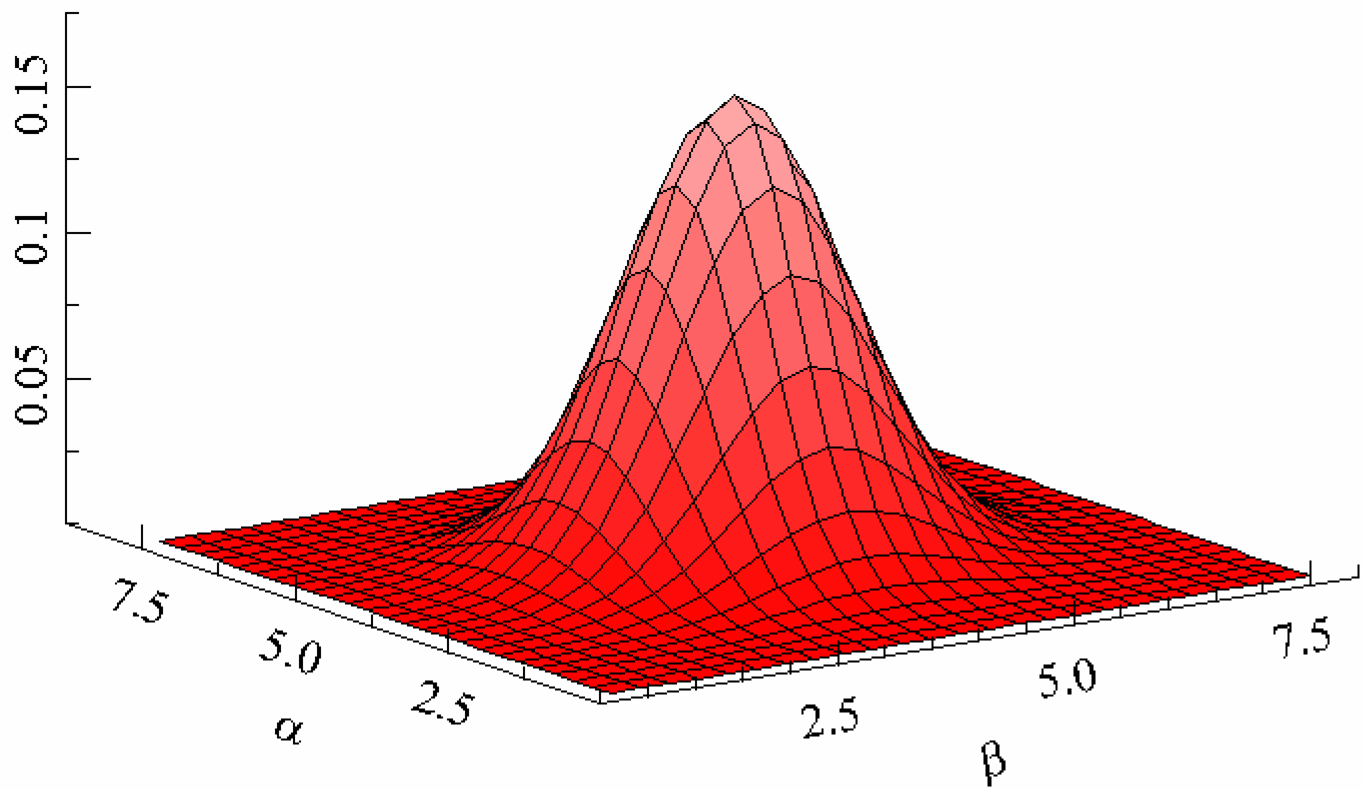
4. Sample $\alpha^{(2)} \sim \pi(\alpha|\beta^{(1)}, y)$ ...

$\rightarrow\ (\alpha,\ \beta)^{(i)} \sim \pi(\alpha,\ \beta|y)$

$$(\alpha,\ \beta)^{(i)} \sim \pi(\alpha,\ \beta|y)$$



$$(\alpha,\ \beta)^{(i)} \sim \pi(\alpha,\ \beta|y)$$



$$(\alpha, \beta, \gamma)^{(i)} \sim \pi(\alpha, \beta, \gamma | y)$$

1. Initialize α, β, γ
2. Sample $\alpha \sim \pi(\alpha|\beta, \gamma, y)$
3. Sample $\beta \sim \pi(\beta|\alpha, \gamma, y)$
4. Sample $\gamma \sim \pi(\gamma|\alpha, \beta, y)$
5. Go to 2.

1. Initialize $\alpha^{(0)}, \ \beta^{(0)}$

2. Sample $\alpha^{(1)} \sim \pi(\alpha|\beta^{(0)}, y)$

3. Sample $\beta^{(1)} \sim \pi(\beta|\alpha^{(1)}, y)$

4. Sample $\alpha^{(2)} \sim \pi(\alpha|\beta^{(1)}, y) \ \ldots$

$\longrightarrow \ (\alpha, \beta)^{(i)} \sim \pi(\alpha, \beta|y)$

Convergence

Markov chain

$$x_{t+1} = Px_t$$

Markov chain

$$x_{t+1} = Px_t$$

$$P = \mathsf{Pr}(x_{t+1}|x_t)$$

Markov chain

$$x = Px$$

stationary distribution