Could an economy get stuck in a rational pessimism bubble? The case of Japan

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Abstract

Developed economies have experienced slower growth since the 2008 financial crisis, creating fears of "secular stagnation." Rational expectations models have forward-looking bubble solutions, which could cause this; here we investigate the case of Japan. We show that a New Keynesian model with a weak equilibrium growth path driven by pessimism sunspot belief shocks matches economic behaviour. Another possibility is a conventional model where productivity growth has simply slowed down for unknown reasons. Nevertheless, a welfare-optimising approach implies fiscal policy should commit to eliminating the potential sunspot while being prepared to revert to normal policy if inflation rises.

JEL classification: E5; E6; E32

Keywords: Secular stagnation; Pessimism sunspot; Indirect Inference; DSGE model

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1 Introduction

It is a well-known feature of rational expectations models of the future that they imply bubble solutions where expectations of future explosive reactions to current sunspots underpin these current sunspot effects. Thus people can expect a current shock to prices which is underpinned by the expectation it will lead to future price movements; a hyperinflation can occur with such a shock, which is self-validating. Prices rise today, with people expecting future rises which stimulate the demand rise today that creates the price shock — Minford and Peel (2015, chapter 2) set out these mechanics and explain how these bubbles can be eliminated from the model solution by a monetary commitment to stable prices by the central bank.

For output it is usual to assume that it tends to a natural rate equilibrium gradually over time and that it is not affected by expectations of future events; thus typically output does not depend on the future via a forward root in the model, as prices do. This implies that there will be no bubble element in the output solution. However, in recent decades we have observed puzzling stagnation episodes in output in several economies, especially since the financial crisis, leading to a wide range of views suggesting that the modern economy is vulnerable to weak self-perpetuating demand — termed 'secular stagnation'. In particular, Summers (2014) has urged in a series of policy-oriented papers that there should be expansionary fiscal policy to ward off the threat of secular stagnation. He suggests that weak demand can become converted into low long term growth by hysteresis. He postulates a world savings glut as the cause of this demand weakness. Eichengreen (2015) and Backhouse and Boianovsky (2016) review various possible causes of secular stagnation, a theory with a long history much associated with Alvin Hansen (e.g. Hansen, 1954). However, the main problem with applying any of these theories to the current situation is that today's stagnation only appeared after the financial crisis, whereas before growth seemed strong and assured. This link to the crisis is more suggestive of a sudden sunspot cause, triggered by a crisis contraction that destroyed business confidence. Meanwhile, in another series of papers Farmer (e.g. Farmer 2020), together with a wide group of authors reviewed below who focus particularly on asset bubbles, has argued persistently for the presence in the economy of real sunspot equilibria, such as those that can occur in rational expectations models — in this case as real bubbles fed by a forward root in output determination. If permitted by models through such a forward root, such equilibria are usually ruled out by transversality and other terminal conditions, such as general business knowledge of true long run productive potential or a government commitment to force this as the equlibrium via fiscal policy; plainly if no such conditions exist then these sunspot equilibria can occur and achieve the effects postulated by Summers for hysteresis — for which in itself there is not really a clear theoretical cause.

Episodes of persistently weak growth have occurred in many western economies after the financial crisis — including the US and the UK. However, in one economy, Japan, this type of stagnationary behaviour has occurred systematically since the 'bursting of the financial bubble' in 1989, a long period now stretching over more than thirty years. It therefore seems a prime candidate for a sunspot-created secular stagnation. In this paper we ask: did the bursting of a monetary bubble in Japan thirty years ago lead to the start of a real deflationary bubble in output? We investigate this question through a DSGE model of Japan estimated over a sample from 1989 to 2022. In this model, the Bank of Japan (BOJ) eliminates the possibility of a monetary bubble via its well-known commitment to price stability in the form of a low inflation target, which has the side-effect of preventing a higher inflation equilibrium; but the Japanese business sector does not appear to believe in a high output potential equilibrium while the government of Japan (GOJ) refuses to commit via fiscal policy to eliminatiing persistently weak growth, so that there is nothing to prevent a stagnationary output bubble fed by sunspots. At the heart of our model is an output production function in which the current natural rate depends on expectations of future output via their effects on capital and labour supply.

The rest of the paper is organised as follows. Section 2 reviews the literature, Section 3 introduces a DSGE with sunspots, Section 4 discusses the estimation procedure, Section 5 shows the empirical results, Section 6 reviews some policy discussions, and finally, Section 7 concludes.

2 Related literature

Our paper broadly relates to the strand of literature on liquidity traps following Summers (2014). Eggertsson, Mehrotra and Robbins (2019) formalise and quantify the secular stagnation hypothesis by constructing a series of analytic and quantitative overlapping generation models where the full-employment real interest rate is permanently negative and under certain conditions, this leads to secular stagnation, characterised by a chronically binding Zero Lower Bound (ZLB), subpar growth and inflation below target. The persistent fall in interest rates can be driven by slow-moving secular forces, such as a slowdown in population growth, an increase in life expectancy, rising income inequality, a fall in the relative price of investment goods, a slowdown in productivity growth or a deleveraging cycle. These forces alter the relative supply of savings and investment, and subsequently have effects on interest rates. Since this episode can be permanent, policymakers should not wait for a ZLB episode to end itself. They find that an aggressive fiscal policy can eliminate the secular stagnation equilibrium more effectively than a monetary policy of raising the inflation target. This works when the fiscal policy reduces the oversupply of savings and raises the natural rate of interest. Other works have also explored the relationship between demographic changes and secular growth. Gagnon, Johannsen and Lopez-Salido (2016) calibrate an overlapping-generation model with a rich demographic structure to the changes in US population, family composition, life expectancy and labour market activity. They find that these factors' declines were pronounced recently because of demographic declines due to the post-war baby boom and suggest that US real GDP and real interest rates will remain low in the coming decades. Away from the overlapping-generation models, Jones (2021) develops and estimates, using Bayesian estimation, a New Keynesian model with demographic changes, real and monetary shocks and an occasionally binding ZLB on nominal rates for the US. He finds that demographic shocks can generate slow-moving trends in real and nominal interest rates, employment and productivity in the US. This decline in interest rates brings the policy rate rate closer to the ZLB. These papers however do not consider the role of bubbles in the stagnation.

Second, there is work that relates to the branch of the literature that allows for the existence of bubbles and investigates their impact on policies and the economy. This literature recognises the fact that economic bubbles are possible important sources of macroeconomic instability. Beyond the more traditional models of rational bubbles (Samuelson, 1958; Tirole, 1985), the recent literature, such as Farhi and Tirole (2012), Martin and Ventura (2012), Miao and Wang (2012, 2014, 2018), Aoki and Nikolov (2015), Bengui and Phan (2018), and Ikeda and Phan (2019), have extended the analysis to study the role of bubbles in real models with financial frictions. In these models the main idea is that bubbles can help to relax financial constraints and improve investment efficiency, firms' borrowing capacity, investment and output.

To investigate the role of monetary policy and its potential impact on bubbles, nominal rigidities have been embedded into overlapping generations models with rational bubbles (Biswas et al., 2020; Gali, 2014, 2020; Asriyan et al., 2021). This type of model enables analysis of whether central banks should raise interest rates if they are faced with a potential bubble, referred to as a "leaning against the bubble" policy. Gali (2014) studies the interaction between rational bubbles and monetary policy in a two-period overlapping generations model with nominal rigidities. This model produces multiple equilibria and in some equilibria the "leaning against the bubble" policy is counterproductive. The reason is that the bubble component of an asset price has no payoffs to discount and the only equilibrium requirement on its size is that it grows at the rate of interest and thus a monetary tightening would increase the bubble size or fluctuations even though the objective may have been to dampen it. As a result, it concludes that if the average size of the bubble is sufficiently large, the central bank should lower interest rates to stabilise the bubble. This result is controversal and challenges the "leaning against the bubble" idea in the literature. However, there are shortcomings in this model because it only allows the bubble to have redistribution effects, and excludes any possibility of bubble-driven fluctuations, since employment and output are constant in equilibrium and the two-period overlapping generations assumption means that it is difficult to consistently match the bubble behaviour with the data. To analyse the impact of bubble fluctuations on economic activity and reconcile them with the data, Gali (2020) extends the model to incorporate an overlapping generation of finite-lived consumers into a New Keynesian model with stochastic transition to retirement. This model allows for the existence of rational expectations equilibiria with asset price bubbles depending on the retirement incidence. In particular, if the probability of retirement is sufficiently high, beside a bubbles balanced growth path, there exist multiple bubbly balanced growth paths which result in bubble-driven fluctuations. This framework allows for a much more important role for monetary policy. It finds that a "leaning against the bubble" interest rate policy can insulate output and inflation from the effects of the bubble and if aggressive enough, it can rule out bubbles themselves. However, the risk is that if policy does not succeed in eliminating bubble fluctuations, it might increase volatility and the persistence of these fluctuations. The paper also finds that large bubbles are good because expected lifetime utility increases along a deterministic balanced growth path, but their fluctuations are generally not. On the other hand, Allen et al. (2018) challenged the findings of Gali (2014). If they modified the model by Gali (2014) so that asset prices are uniquely determined, then higher rates unambiguously dampen the bubbles, but the problem remains that there is no benefit from leaning against bubbles because the friction that allows bubbles creates dynamic inefficiency in this model, and bubbles ameliorate this inefficiency. Therefore, Allen et al. (2018) propose to modify Gali's model in such a way that the economy is dynamically efficient and they introduce credit and information frictions that interfere with lending. In this environment bubbles arise when agents borrow, and against the interests of their lenders who cannot monitor them, buy risky assets and bid up their price. Asset bubbles reduce welfare by crowding out productive lending and associated activities. In this model, monetary intervention that leans against the wind can dampen a possible bubble to prevent its impact once it bursts, but it would result in a contraction across the economy too. Therefore, the benefits from intervention by monetary policy can outweigh its costs if and only if the default costs are sufficiently high because despite the adverse impact on the economy, higher rates would reduce the amount agents can borrow against bubble assets and reduce the possible default amount if the bubble burst, and make society as a whole better off.

Other studies also find an importance for monetary policy in stabilising economic bubble conditions. Asriyan et al. (2021) investigate the role of monetary policy in a bubbly world using an overlapping generations model with bubbles and money. The model includes financial frictions where entrepreneurs raise funds by issuing debts, which is backed by future output. It also includes two types of unbacked assets, one is referred to as a bubble asset issued by entrepreneurs but only backed by expectations of their future value and another is money issued by a central bank. Savers want to purchase and hold both assets issued by the entrepreneurs and also money, if inflation is low enough. Financial frictions restrict the supply of debts, but given the demand for assets, the entrepreneurs would issue unbacked assets. As long as the expected return to these unbacked assets is sufficiently high, the world is a bubbly one and savers will be willing to hold bubble assets in equilibrium. This framework creates an interaction between unbacked assets, bubbles and money. It finds that central banks' interventions are effective and welfare-enhancing because they expand the net supply of assets available to the private sector and decrease inefficient investment. Biswas et al. (2018) also construct a rational bubbles model combined with financial frictions and downward wage rigidities at the ZLB. The model assumes a stochastic bubble so that in each period the price of the bubbly asset can collapse to the fundamental value with an exogenous probability. The collapse of expansionary bubbles can trigger a collapse of output and cause persistent involuntary unemployment, because net worth falls, leading to contractions in credit and investment. Thus, the demand for labour from firms also contracts and given downward rigid wages, there will be rationing in the labour market, resulting in involuntary unemployment. In turn, this unemployment can lead to an endogenous and protracted recession by eroding the intertemporal allocation of resources. This then further leads to a contraction in capital investment, since entrepreneurs' ability to borrow and invest depends on their net worth. Therefore, the future capital stock will decline, causing further downward pressure on labour demand and wages, thus reducing future capital accumulation. The vicious cycle repeats and only stops when the capital stock has fallen enough below the bubbleless steady-state level. This model thus implies that a "leaning against the bubble" type of macroprudential policy intervention is warranted for excessively large bubbles. It is this model that comes closest to our work in this paper, where entrepreneurs' confidence in future returns drives investment and so current output.

Although the literature on rational bubbles is extensive, many studies, including the above, investigate the issue using theoretical models, with only a few addressing the matter using estimation. Miao et al. (2015) provide the first structural analysis of bubbles using a Bayesian DSGE framework. They construct and estimate a real business cycle model with infinitely lived agents, habit formation, investment adjustment cost and variable capacity utilisation. Firms are subject to idiosyncratic investment efficiency shocks and face endogenous credit constraints. A sentiment shock reflecting households' beliefs about the relative size of the old and the new bubble, drives the fluctuations in the bubble and transmits them through the economy via the credit constraint. The bubble emerges through this feedback loop between the credit constraint and self-fulfilling beliefs. They find that the sentiment shock explains most fluctuations in the stock market and other real variables. It generates the comovement between stock prices and the real economy and can explain the internet bubbles and Great Recession. Ikeda (2013) introduces monetary policy and wage rigidities into this model of Miao et al. (2015). He addresses moderate inflation in asset price booms in a DSGE framework. Based on Bayesian estimation on US data, he finds that nominal wage rigidities and an asset price bubble work together to cause an inefficiently excessive boom, but inflation remains moderate because they relax the financial constraints and apply downward pressure on inflation. Strict inflation targeting fails to effectively address inefficiencies caused by bubbles and also exacerbates them in the short run. He shows that the optimal monetary policy is a monetary tightening to restrain the boom at the cost of greater inflation volatility. The use of Bayesian estimation gives weight to priors that may be rejected by the data and so lead to bias in estimation.

Here we use indirect inference both to estimate and test our model rigorously against the data behaviour; the aim is to compare the reduced form data behaviour generated by the unknown true model with the simulated reduced form behaviour generated by our candidate models. In estimation we search for the paramaters of our candidate model that can most closely match the data behaviour from the true model. Le et al. (2016) and Meenagh et al. (2019) show that in the small samples we encounter here indirect inference exhibits low bias in estimation and high power in testing.

3 A DSGE model with real sunspots

Our model (for more details, see Appendix 1) is a small open DSGE model of Japan with real and nominal rigidities, financial frictions and a central bank that pursues monetary policy either by setting interest rates on short term government bonds or, if that is nullified by a zero lower bound, by asset purchases (QE). It also rules out price and other nominal bubbles, including in asset markets, via terminal conditions. This distances our model from the literature reviewed above. Our focus is on a real output bubble arising through a sunspot of pessimism about future output. This produces self-validating falls in capital investment, labour supply (including via lower births), creating a real bubble solution for output demand and also for output supply potential from a standard production function.

The idea we explore is that Japanese households and firms suffer from random bouts of pessimism about the future fundamental prospects for the economy; these bouts we assume started to happen after the crisis of the end-1980s when the 'financial bubble' was burst by brutally deflationary monetary policy, plunging the economy into deep recession and leaving the banks with weak balance sheets. We model these bouts as sunspots, which affect the expected long term performance of output. These enter the model as the terminal expected value of output. This terminal belief in turn feeds into expected values for earlier consumption, investment and labour supply which are determined by expected output. Expected output in turn reflects these variables; and then goes on to influence expected consumption etc. for still earlier periods. In this way the original bout of pessimism about the long term future cascades back down to current behaviour of the economy.

We solve our DSGE model in repeated bootstrap simulations based on errors retrieved from the model and the data since 1990, the era since the bursting of the nominal 1980s bubble — which we hypothesize caused the situation in which business confidence became prone to bubbles. We then test it by indirect inference against a VECM representation of the Japanese macro data for output, inflation and credit interest rates. Finally, we estimate it by indirect inference — by a search procedure that finds the parameters yielding the closest match of simulated VECM coefficients to those in the data.

The main challenge in this empirical process is finding the bubble simulation solution paths. We do this by postulating a bubble path whose terminal rate of expansion is then imposed on the model to find the self-validating solution consistent with it: consumption and investment demand respond to these output expectations, and in turn they generate output demand that determines output under sticky prices; the model then solves backwards from this future state. The model solves for the optimising capital, labour and innovation strategies of private agents, conditional on this evolving solution path.

In the following subsections, we first write a note on sunspot solutions, we then use a simple example to illustrate the sunspot model of output, and then extend this to the full DSGE model for Japan.

3.1 A note on sunspot solutions in rational expectations models

Rational expectations of future events introduce a forward root into the model solution which is a saddle-path in the case of the usual model where all the forward and backward roots lie within the unit circle. In this solution, the saddle path is the unique stable path, and there is also an infinite number of explosive paths, any one of which can be selected by chance, unless there is some mechanism preventing such a selection.

For example, if current inflation depends on future inflation through the demand for money, the forward root this introduces permits explosive hyperinflationary or hyperdeflationary solutions, that can be selected by agents at random; such a random selection is known as a sunspot, whereby agents spontaneously come to believe an inflation or deflation will happen in a future period, triggering inflation or deflation in the current period, and implying that in later future periods inflation or deflation will increase along the chosen explosive path. Such spontaneous beliefs could be the result of pure sentiment taking hold — potentially triggered by some current event or a rumour of a future event. What the models imply is that once such a sentiment takes hold, it is validated by future events occurring as expected; hence it is a rationally expected solution.

The mathematics of these solutions is relatively simple. We follow the exposition in Minford and Peel (2015, chapter 2). Suppose we take a simple New Classical model of prices (p_t), money (m_t) and output (y_t):

$$m_t = p_t + y_t - \alpha(E_{t-1}p_{t+1} - E_{t-1}p_t)\ (\alpha > 0) \tag{1}$$

$$p_t = E_{t-1}p_t + \delta(y_t - y^*) \tag{2}$$

$$m_t = \overline{m} + \varepsilon_t \tag{3}$$

The general solution for expected prices is:

$$E_{t-1}p_{t+i} = \overline{m} - y^* + A_1(\frac{1+\alpha}{\alpha})^i\ (i \geqslant 0) \tag{4}$$

where $A_1 = [E_{t-1}p_t - (\overline{m} - y^*)]$ from the initial condition. This solution can be conveniently found by solving for expected future prices in terms of lagged prices (the 'backward solution' whereby the forward root, being applied backwards, is inverted and becomes unstable.) The unique stable path is found by setting A_1 to zero. The model's behaviour can be illustrated by a phase diagram where the arrowed line is the saddlepath and the arrows off it indicate the explosive paths.

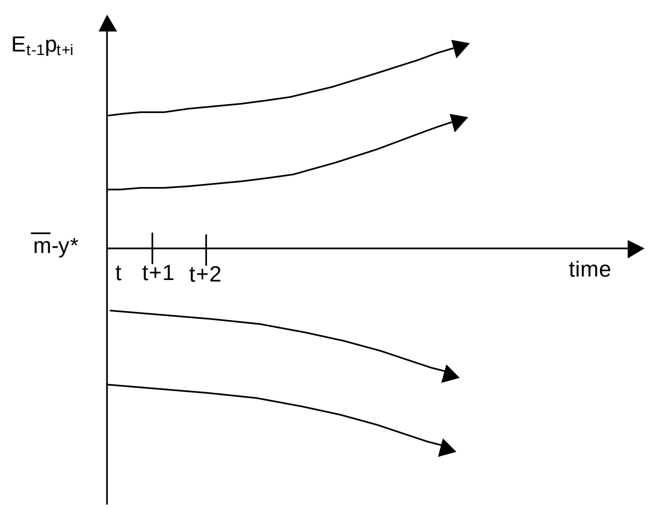

Figure 1: The solution paths for the price level expected at $t-1$, as in equation 4

In models like this with saddlepath stability and an infinity of unstable paths for prices, a natural way to rule out all paths except the saddlepath is for the central bank to commit itself to stopping them via a terminal condition in which future prices away from the stable path would be overidden by money supply movements.

In this paper we are dealing not with unstable price paths but rather unstable output paths. In the model above these do not arise as expected output equals potential output which is an exogenous constant. However, it is possible for output to depend on expected future events through model behaviour. Thus, in a New Keynesian model output depends on demand which in turn depends on expected future events; also potential output depends on capital investment which in turn depends on expected future output, creating a forward root giving rise to a saddle-path solution for output, with an infinity of explosive paths, similar to the above one for prices. These explosive paths could be similarly eliminated by a government commitment, e.g. through fiscal policy, to prevent them.

3.2 A simple illustrative sunspot model of output

To illustrate the workings of a simple model of this type, suppose that natural rate output, y_t^*, depends on capital invested and that this in turn depends on expectations of future y_t^* via the marginal product of capital; thus current y_t^* depends on expected future y_{t+1}^* via a forward root, σ. We can write the equation for natural rate output as:

$$y_t^* = \sigma E_t y_{t+1}^*$$

For the rest of the model, we will assume here the simple New Classical structure as in Section 3.1:

$$m_t = p_t + y_t \tag{5}$$

$$p_t = E_{t-1} p_t + \delta(y_t - y_t^*) \tag{6}$$

$$m_t = \overline{m} + \varepsilon_t \tag{7}$$

To solve this model, we can first solve for y_t^*. This yields a first order difference equation for the future as a function of the previous expected value as:

$$E_t y_{t+i+1}^* = \sigma^{-1} E_t y_{t+i}^*$$

whose general solution is:

$$E_t y_{t+i}^* = A_1 \sigma^{-i} \qquad (i \geq 1)$$

where A_1 is given by the initial value, $E_t y_t^* = A_1$. But this value can be chosen arbitrarily, as nothing in the model ties it down.

We can then find the expected solution for output as $E_t y_{t+i} = \sigma^{-i} E_t y_t^*$ where $E_t y_t^*$ can be chosen at will, and can therefore be treated as a sunspot. It follows that expected prices are $E_t p_{t+i} = E_t m_{t+i} - \sigma^{-i} E_t y_t^*$. This implies that prices can also have a sunspot solution, absent any monetary response. This can be eliminated by setting expected money supply to eliminate the effect of the sunspot on prices, forcing them to a target path, p_{t+i}^*, as follows:$E_t m_{t+i} = p_{t+i}^* + \sigma^{-i} E_t y_t^*$. Once these expected paths are established, actual price and output deviate from them according to the demand shock to the money supply, obtaining: $p_{t+i}^{ue} = \frac{\delta}{1+\delta} \varepsilon_{t+i}$ and $y_{t+i}^{ue} = \frac{1}{1+\delta} \varepsilon_{t+i}$, where the ue superscript denotes 'unexpected.'

It is useful to examine the solution of this very simple model, under plausible assumptions for the sunspot path of the natural rate. Notice that according to this model, output, ignoring iid errors, is equal to expected natural rate output. Due to pessimism about the expected future natural rate output, $E_t y_{t+i}^*$, future output is depressed and continues to diverge from the true potential output path.

In the period after the post-war 'miracle' (when growth was 10%) growth fell to 4.5%; then in the bubble-bursting period from 1990 it fell to about 1%. Since 2000 it has averaged about 0.6%, rising slightly after the financial crisis when fiscal policy became generally stimulative. Most developed countries manage to grow about 2%, which we take to be a reasonable estimate also of the true potential (natural rate) growth in Japan. In the Figure 2 one can see the way GDP has fallen steadily further below this 2% trend path from 1992.[1]

[1] Some argue that growth on a per capita basis should be the policy target; growth per capita, with a declining population, has not slowed so markedly. However, this assumes that population growth does not respond to growth of GDP, which is unlikely. The willingness to have children is likely to respond to the outlook for output and jobs — Kearney and Levine (2020), Black et al. (2013), Autor et al. (2019). The worse the outlook, the more parents must invest in education and other support for their children, to create lifetime opportunities for them. In this model we focus on GDP growth as the key policy outcome.

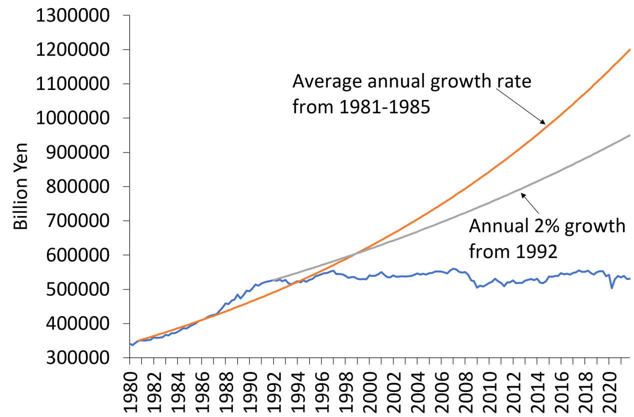

Figure 2: Real GDP 1980-2020 (blue line)

We now show the type of sunspot path this simple model implies given Japanese data. If we take the true potential output path to follow the 2% growth trend, then the sunspot-affected path is actual output; and the sunspot can be derived as the gap betwen this and the 2% growth trend. Using this logic, we get the sunspot series in Figure 3 reflecting this gap between output and the 2% trend.

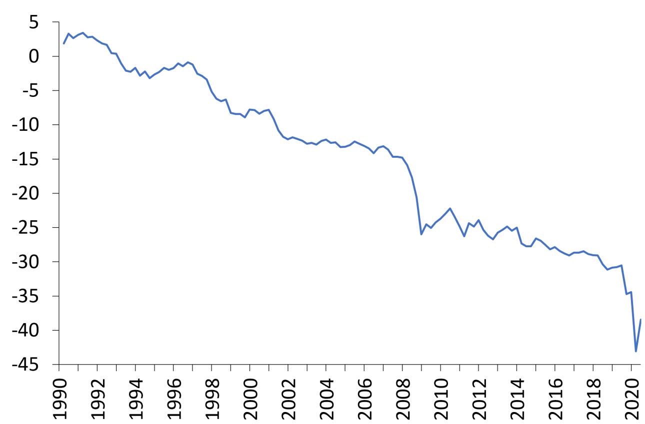

Figure 3: Sunspot Series (output deviations from natural rate)

When we further decompose this into the sequence of sunspot innovations, that is the necessary revision each period compared with the value implied by the previous sunspot, we obtain Figure 4.

It can be seen that up to 2007 the innovations fluctuate moderately around a negative mean, driving expected output steadily downwards. However the financial crisis triggers a large negative shock. This was followed by a long period of further pessimism, culminating in another large negative shock in 2020, the period of Covid, with the post-Covid bounceback, followed by the resumption of further moderate pessimism. With a full set of shocks providing the effect of non-sunspot shocks on output, we would expect these two shocks to be eliminated from the sunspot.

We have shown the type of path this simple model implies given Japanese data. Over the sample period from 1992 the economy has evolved with a succession of negative sunspots, launching the economy on a slowly exploding downward path. We show the contribution of each dated sunspot to the later GDP evolution. It can be seen that the original sunspot around the bursting of the monetary bubble accounts for much of the later evolution. However additional negative and occasionally positive shocks occurred later.

In the subsequent subsections we develop a full model with the investment path set by future expectations

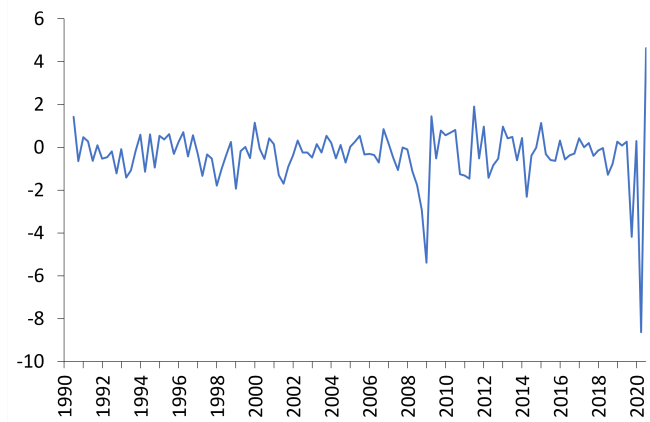

Figure 4: Sunspot Innovations

of the marginal product of capital; and with labour supply responding to expectations of future wages. We generate the expected path of y_t^* according to the natural rate model equation set out above. We then embed this in a New Keynesian model of Japan which comes closest to explaining Japanese data behaviour. The natural rate enters the model via the terminal conditions when output has converged on it. Prior to that date output is demand-determined under the usual New Keynesian assumptions. Hence the effect of the natural rate sunspot solution is to change expectations of output from the terminal date backwards towards the initial period. Unlike in the simple New Classical example developed above, expected output is not equal to the expected natural rate, rather it is impacted by it through the demand effects of expected output.

3.3 Solving the Japanese model with a pessimism bubble

Now we apply this analysis to an estimated DSGE model of Japan. The full model for Japan is based on that of Le et al. (2016a), but with the addition of open economy elements to deal with trade and capital flows — a full account is given in Appendix 1. The Le et al. (2016a) model is developed from the model of Smets and Wouters (2007): it adopts a hybrid price-setting structure, with a fraction of goods markets assumed to be flexprice while the rest set prices for longer durations; similarly with labour markets. Beyond these frictions in labour and goods markets, the model also incorporates financial frictions as proposed by Bernanke et al. (1999) and at the zero lower interest rate bound allows for cheap money collateral to make monetary policy effective via largescale asset purchases, 'Quantitative Easing'. Trade equations enter via an Armington (1969) nested CES utility function of differentiated home and foreign consumption goods, as in Meenagh et al. (2010). On the capital account of the balance of payments uncovered interest parity is assumed between home and foreign bonds.

In this model the central equation driving investment in capital, in log-linearised form, is[2]:

$$E_t r_{t+1}^k - (r_t - E_t \pi_{t+1}) = \chi (pk_t + k_t - n_t) - \vartheta m_t^0 + \epsilon_t^{prem}$$

In this expression expectations of future output demand determine networth (n) so the expected bubble feeds into n. This in turn depresses investment and capital, reducing current potential output through the production function, and current output via reduced demand. Next we look for the bubble solution for output and its terminal value that permits the model to generate the data path.

3.4 Calculating the sunspot shock in the full model

We take the 2% growth path from 1990 to be illustrative of the unknown true natural rate growth path, as explained above. To obtain our estimate of the actual bubble we argue as follows. Our bubble hypothesis is

[2]Equation 22 in Appendix 1, where r^k is the external financing rate, r is nominal interest rate, π is inflation, pk is price of capital, k is capital stock, n is networth, m^0 is monetary base, and ϵ_t^{prem} is the risk premium shock.

that the economy is driven by the model and a combination of the normal shocks and the sunspot shock to its lower predicted path, which according to the model should get close to matching the economy's average (HP-filtered) actual behaviour, y_t^*. Thus according to this model, $y_t^* = \widehat{y}_t^{normal\ shocks} + \xi_t$, and thus we create the sunspot shock series as $\xi_t = y_t^* - \widehat{y}_t^{normal\ shocks}$. The resultant sunspot shock series is plotted in Figure 5, and its innovations in Figure 6.

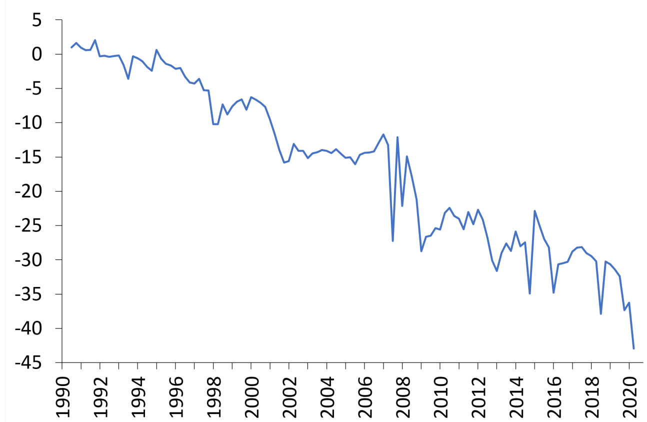

Figure 5: Sunspot Series Generated from Full Model

Figure 6: Sunspot Innovations Generated from Full Model

The sunspot innovations tend to be negative, pushing the economy further and further away from the supposed true potential path. Thus, if our bubble hypothesis is correct, Japanese business is beset with steadily increasing pessimism with no basis in the truth. The policy implications of this situation are stark and challenging; we will revert to them in the final section.

3.5 How financial market sunspots can arise and impact the economy

The analysis above illustrates how sunspots arise from the presence of a forward root in the model, i.e. where a variable depends on its future. The sunspot solution arising from this can be found by solving the model backwards, so making the future depend on the past, inverting this forward root; the solution implies that the future variable value explodes from any arbitrarily chosen initial expectation.

In our literature review, we noted a wide variety of papers in which financial market bubble/sunspots are assumed and cause economy-wide effects. It is useful to give an example of how this could arise in our

Japan model, even though we do not use this financial market mechanism.

We can take a sector of the equity market, such as tech stocks; let the market value of a share be s_t. Their value will be equal to discounted (by discount factor d) future profits, $E_t\pi_{t+1}$, thus $e_t = dE_t\pi_{t+1}$. Now assume that the larger the firm's asset value and production size, the lower are costs and the higher are profits, so that $\pi_t = \theta e_t (\theta \leq 1)$. It follows that $e_t = d\theta E_t e_{t+1}$. The sunspot solution this creates is then $E_t e_{t+i} = (d\theta)^{-i} E_t e_t$.

So far this is simply a 'micro sunspot' for a single sector. However as the sector will enjoy explosively rising value, it will attract investment and become an increasing share of the country's equity portfolio. This in turn will increase the overall net worth of the entrepreneurial sector, raising the return on credit in our central model equation determining investment finaced by credit, as noted above:

$$E_t r^k_{t+1} - (r_t - E_t\pi_{t+1}) = \chi\left(pk_t + k_t - n_t\right) - \vartheta m^0_t + \epsilon^{prem}_t$$

In this way the financial market sunspot can feed into the economy, fuelling derived sunspot behaviour in the economy. It can be seen from this example that there could be other ways that sunspots could arise at the micro level, and from there feed back into bank credit and investment. All that is needed is some assumed micro process that creates a forward root for asset values and hence for the supply of credit.

However, as noted above, we do not use this type of mechanism here. Instead we assume that general confidence in the future potential of the economy is the channel by which a sunspot takes hold. It is this that distinguishes our work here from the large literature on financial-origin sunspots.

4 Estimation and testing by Indirect Inference

We use the approach of Indirect Inference proposed by Le et al. (2011) to assess the model's ability to match the data. Le et al. (2016b) provide a full description of the procedure. To generate a description of the data against which the theory's performance is indirectly evaluated, the approach uses an auxiliary model that is fully independent of the theoretical one. The estimated parameters of the auxiliary model, or functions of these, might be used to summarise such a description; we name them as the 'data descriptors'. While they are viewed as 'reality,' the theoretical model under consideration is simulated to determine its suggested values. In estimation the parameters of the structural model are chosen so that when the model is simulated, the auxiliary model estimates are similar to those obtained from the real data. The structural model parameters that minimise the distance between a given function of the two sets of estimated coefficients of the auxiliary model are the best.

When evaluating the model's data fit, the structural model is simulated, and the auxiliary model is fitted to each set of simulated data, yielding a sample distribution of the auxiliary model's coefficients. A Wald statistic is computed to see if functions of the auxiliary model's parameters calculated on actual data fall within a confidence interval given by the sampling distribution.

The auxiliary model should be a process that describes how the data evolves under any applicable model. It is well known that the reduced form of a macro model with non-stationary data is a VARMA, in which non-stationary forcing factors are used as conditioning variables to accomplish cointegration (i.e. ensuring that the stochastic trends in the endogenous vector are picked up so that the errors in the VAR are stationary). This in turn can be approximated as a VECM. As an auxiliary model, we utilise a VECM with a temporal trend and the productivity residual inserted as an exogenous non-stationary process, which we re-express as a VAR(1) for the three macro variables of interest (interest rate, output, and inflation). The two exogenous elements have the effect of achieving cointegration. The VAR coefficients on the lagged dependent variables and the VAR error variances are treated as the data descriptors, and the Wald statistic is computed from them. Thus, we are essentially determining whether the observed dynamics and volatility of the selected variables can be explained by the simulated joint distribution of these variables at a given confidence level. The Wald statistic is given by:

$$(\Phi - \overline{\Phi})' \sum\nolimits^{-1}_{(\Phi\Phi)} (\Phi - \overline{\Phi}) \tag{8}$$

where Φ is the vector of VAR estimates of the chosen descriptors yielded in each simulation, with $\overline{\Phi}$ and $\sum_{(\Phi\Phi)}$ representing the corresponding sample means and variance-covariance matrix of these calculated across simulations, respectively.

The joint distribution of the Φ is obtained by bootstrapping the innovations implied by the data and the theoretical model; it is thus a small sample distribution estimate. For small samples, this distribution is usually more accurate than the asymptotic distribution.

This testing procedure is applied to a set of (structural) parameters put forward as the true ones (H_0, the null hypothesis). The test then asks: could these coefficients within this model structure be the true (numerical) model generating the data? We extend our procedure by a further search algorithm, in which we seek other coefficient sets that minimise the Wald test statistic — in doing this we are carrying out indirect estimation.

Thus we calculate the minimum-value Wald statistic using a powerful algorithm based on Simulated Annealing (SA), in which the search takes place over a large range around the initial values, with the search being optimised by random jumps around the space. The benefit of this extended method is that when we ultimately compare model compatibility with data, we use the best possible version of the model.

5 Empirical results

We have found parameters for this model that can match Japanese data (in the form of a VECM) well with a p-value of 0.0994 showing that we do not reject the null hypothesis that the model can generate the data. The key forward root we discover is 0.9929 which gives rise to a moderately exploding solution. What we see is that, similarly to our simple model, the Japanese economy tends to contract systematically towards a very slowly growing 'natural rate' growth path. Besides other normal shocks driving it, there is a sequence of sunspot shocks that push it along this path. These shocks we calculate in a similar way to that in the simple model but with a difference reflecting the greater number of demand channels. There we assumed that gloomy expectations of future potential output drove investment demand: here we assume that these gloomy expectations drive private actions across the whole model. The rational expectations (RE) loop in the model forces the actual output path we observe into equality with the expected output generated by the terminal expected value of potential output, itself determined by the sunspot shock. Thus the RE solution of the model embeds both the effects of normal shocks and those of the sunspot shocks. The economy's demand and supply behaviour responds to this expected path in a way that must replicate the data-based VECM descriptive model to pass the indirect inference test.

The model therefore works as follows. The bubble output expectations feed into entrepreneurial net worth, and depress investment which reduces output through lower demand. Employment falls also via demand. Real wages fall, clearing the labour market. Labour supply also falls, with the fall in income, consumption and real wages. Output potential falls due to lower capital and labour supply. The output gap versus potential is little changed so inflation moves little. Inflation fluctuations come from demand shocks interacting with this output potential.

Table 1 reports the estimated parameters and Wald statistic are reported.

Table 2 shows the auxiliary model estimates on the actual data alongside the mean and 95% confidence bounds from the simulations. Most parameters are within the bounds, but we find that the model under-predicts the output response to lagged output. Similarly for the interest rate response to lagged interest rate. Conversely, the inflation response to lagged inflation is over predicted, by a small margin. The data for interest rate variance is just outside the model's 95% confidence bounds. However, the model fits overall, as shown by the p-value above.

The example simulations (shown in blue) in Figure 7 reveal the prevalence of output, output gap and inflation fluctuations, together with regular switches into the zero lower bound, alongside a slow growth trend. They tend to match the nature of the fluctuations in the actual data (shown in red), so accounting for the good p-value.

	Sunspot model coefficients
Steady-state elasticity of capital adjustment	8.0073
Elasticity of consumption	2.5387
External habit formation	0.4636
Probability of not changing wages	0.4402
Inverse of Frisch elasticity of labour supply (σ_l)	3.3006
Probability of not changing prices	0.8066
Wage indexation	0.1905
Price indexation	0.4457
Elasticity of capital utilisation	0.9435
Share of fixed costs in production (+1)	1.7772
Taylor Rule response to inflation	1.0612
Interest rate smoothing	0.9735
Taylor Rule response to output	0.0060
Taylor Rule response to change in output	0.0126
Share of capital in production	0.3338
Proportion of sticky wages	0.2384
Proportion of sticky prices	0.4806
Elasticity of the premium with respect to leverage	0.0103
Monetary response in crisis time	0.0978
Monetary response in normal time	0.0406
Elasticity of premium with respect to money	0.0494
Bubble forward root	0.9929
Fiscal response	0.6470
Wald	21.1524
Transformed Wald (t-stat)	1.1492
P-Value	0.0994

Table 1: Coefficient Estimates

	Actual	Mean	2.5th Percentile	97.5th Percentile	In/Out
Y_Y	0.8031	0.2770	−0.3191	0.5659	OUT
Y_PI	0.0554	0.0975	−0.4318	0.7039	IN
Y_R	−0.1540	0.0417	−6.5507	4.8900	IN
PI_Y	0.0810	0.0198	−0.0837	0.1066	IN
PI_PI	−0.3040	0.0129	−0.2768	0.3085	OUT
PI_R	0.6767	0.3366	−0.9463	2.6129	IN
R_Y	0.0021	−0.0098	−0.0264	0.0022	IN
R_PI	0.0051	0.0063	−0.0257	0.0385	IN
R_R	0.9320	0.6866	0.1467	0.9308	OUT
Var(Y)	1.5653	2.8773	1.0596	7.8494	IN
Var(PI)	0.2339	0.3869	0.2328	0.5765	IN
Var(R)	0.0009	0.0081	0.0012	0.0177	OUT

Table 2: Auxiliary Model Parameter Bounds

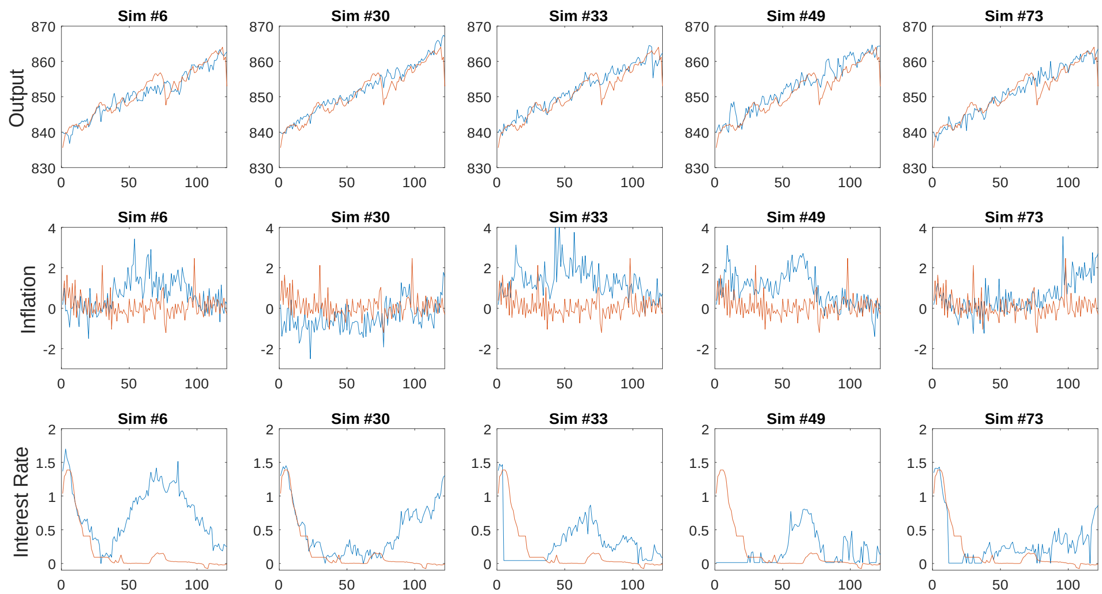

Figure 7: A Selection of Simulations

5.1 The behaviour of the model

The IRF for a negative sunspot shock in Figure 8 shows a persistent and sizeable fall in output, employment and real wages accompanying the slowly exploding sunspot in expected equilibrium output, y^*.

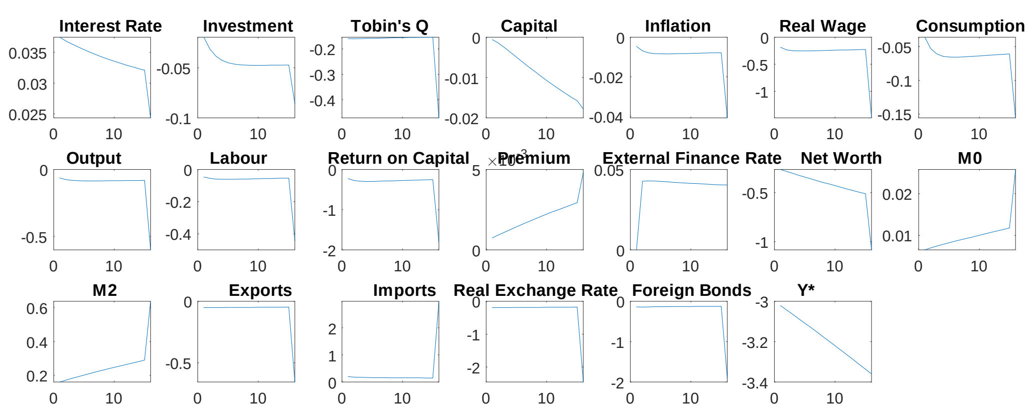

Figure 8: IRFs for a Sunspot Shock

The sunspot enters expected equilibrium output at the terminal date and then induces a fall in consumption, employment and capital investment which in turn depress demand and output. The Taylor rule responds to a rise in $y - y^*$, with output falling less than y^*, by raising interest rates, so also causing real exchange rate appreciation, forcing down net exports which add to the decline in demand and output.

We now turn to the effect of a positive productivity shock (Figure 9). The productivity shock in non-stationary, so has permanent effects. This paradoxically reduces output. However, it does so through a mechanism already well-known in the New Keynesian literature (an early finding is by Gali, 1999): at given demand, fixing output at pre-set prices, a rise in productivity lowers labour demand, hence employment. This in turn depresses consumption and inflation. However, after initially rising due to the drop in inflation, real interest rates are gradually reduced by the Taylor Rule, pushing up consumption and investment, also net exports via real depreciation; output demand therefore gradually recovers, but both capital and employment remain depressed, largely offsetting the productivity effect on y^*.

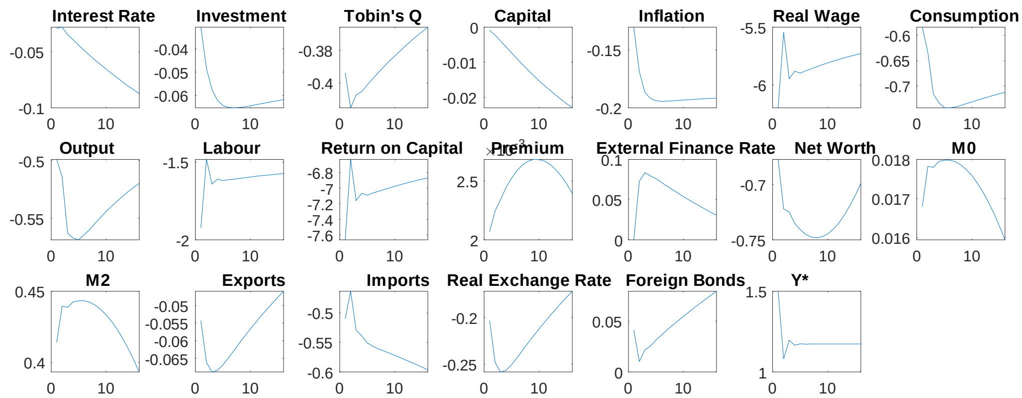

Figure 9: IRFs for a Permanent Productivity Shock

Figure 10 shows the IRF for a positive shock to government spending — a classic demand shock. This has a strong immediate effect on output, but since it raises inflation, the Taylor rule response raises real interest rates which crowd out consumption and investment and (via real appreciation) net export demand. Furthermore the fiscal feedback response offsets some of the initial spending rise. Hence the output expansion dies away fairly rapidly. Similarly, with the other demand shocks to consumption and investment, the output rises are shortlived.

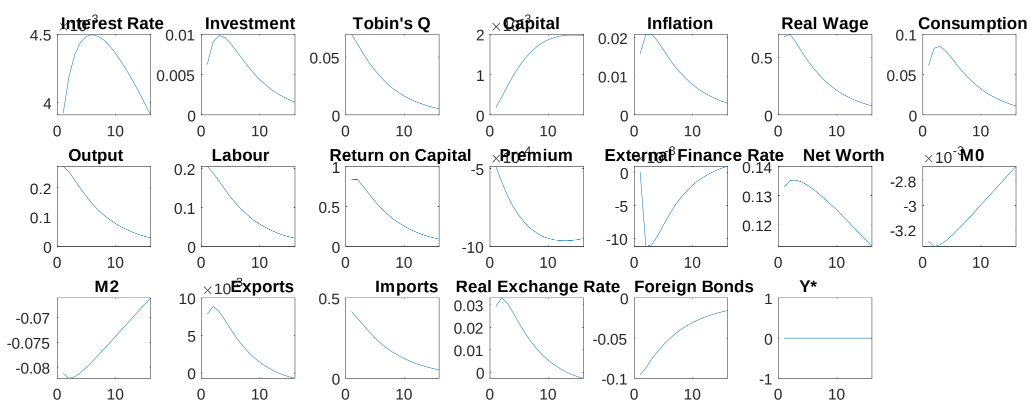

Figure 10: IRFs for a Government Spending Shock

A contractionary monetary shock (Figure 11) has an IRF that similarly reduces demand sharply on impact but that is quickly reversed via the Taylor rule and fiscal feedback responses. Output, employment, home demand and net exports all recover rapidly; the rise in real interest rates and so also the real exchange rate are steadily reversed.

Cost shocks similarly have a sharp short run stimulationary effect on inflation, which via the resulting

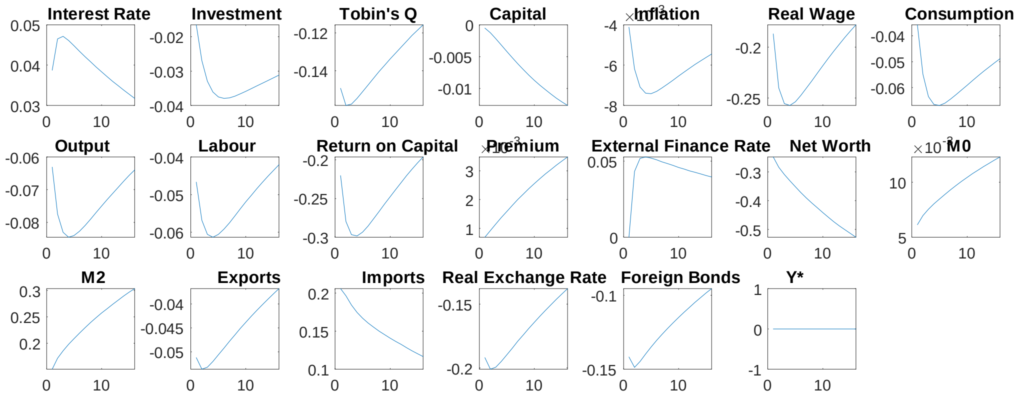

Figure 11: IRFs for a Monetary Policy Shock

fall in the real interest rate also stimulates net export demand and output, before the Taylor rule intervenes to raise real rates and reverse the expansion.

In general, throughout these IRFs we see the combination of the Taylor rule and the fiscal feedback response impart a strongly stabilising impact to the economy.

Overall, it can be seen from the simulations and the IRFs that the model responds to normal shocks in the usual way, since these do not trigger sunspot movements. We can also see that the sunspot shock triggers modestly exploding paths which generate their own inflation/output gap effects, as demand effects fail exactly to match effects on potential output.

5.2 Which shocks cause most variation in the model?

We perform a variance decomposition exercise to see which shocks are the most important. The short-run and long-run analysis are shown in Tables 3 and 4 respectively, where the short-run is 5 periods ahead, and the long-run is 20 periods ahead.

Shock	Interest Rate	Investment	Tobin's Q	Capital	Inflation	Real Wage	Consumption	Output	Labour
Government Spending	0.0017	0.0001	0.0054	0.0000	0.0035	0.0073	0.0088	0.0359	0.0085
Consumer Preference	0.0004	0.0000	0.0001	0.0000	0.0006	0.0065	0.0897	0.0122	0.0026
Investment	0.0011	0.9851	0.0032	0.9994	0.0015	0.0089	0.0084	0.0454	0.0122
Monetary Policy	0.1814	0.0006	0.0430	0.0000	0.0004	0.0011	0.0052	0.0043	0.0010
Productivity	0.1158	0.0019	0.2962	0.0001	0.3107	0.6851	0.7079	0.2051	0.8109
Price Mark-up	0.0218	0.0000	0.0041	0.0000	0.6334	0.0002	0.0006	0.0018	0.0006
Wage Mark-up	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0000	0.0000	0.0000
Labour Supply	0.0020	0.0000	0.0088	0.0000	0.0226	0.1568	0.0002	0.0019	0.0004
Premium	0.0000	0.0007	0.0624	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000
Net Worth	0.0006	0.0013	0.1228	0.0000	0.0014	0.0024	0.0022	0.0118	0.0027
M0	0.0000	0.0010	0.0630	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000
Exports	0.0162	0.0000	0.0064	0.0000	0.0154	0.0933	0.1068	0.4939	0.1173
Imports	0.0052	0.0000	0.0024	0.0000	0.0051	0.0293	0.0323	0.1535	0.0364
Sunspot	0.6539	0.0092	0.3822	0.0004	0.0054	0.0089	0.0379	0.0339	0.0076

Table 3: Short Variance Decomposition

One striking feature of this shock decomposition is the dominant effects on output of the net trade shocks in the short run. In the long run the sunspot shock dominates, with productivity contributing much of the rest. Both shocks are nonstationary which accounts for their relative long run importance. Demand shocks have very limited importance for output, being heaviliy suppressed by fiscal and monetary feedback.

The price mark-up is the overwhelming source of inflation variation in the short run, followed by productivity. Otherwise inflation is largely unaffected by shocks, remaining essentially stable around costs. Productivity also destabilises wages and employment as we see from the IRF, contributing most of their

Shock	Interest Rate	Investment	Tobin's Q	Capital	Inflation	Real Wage	Consumption	Output	Labour
Government Spending	0.0013	0.0002	0.0012	0.0000	0.0017	0.0023	0.0006	0.0095	0.0030
Consumer Preference	0.0001	0.0000	0.0000	0.0000	0.0002	0.0013	0.0040	0.0021	0.0006
Investment	0.0037	0.7205	0.0024	0.9835	0.0049	0.0154	0.0040	0.0216	0.0230
Monetary Policy	0.1189	0.0023	0.0214	0.0004	0.0006	0.0014	0.0010	0.0059	0.0017
Productivity	0.4125	0.0086	0.1743	0.0013	0.4482	0.5643	0.1419	0.1808	0.7484
Price Mark-up	0.0135	0.0004	0.0023	0.0000	0.2086	0.0007	0.0002	0.0033	0.0011
Wage Mark-up	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Labour Supply	0.0075	0.0005	0.0023	0.0000	0.0139	0.0537	0.0004	0.0091	0.0032
Premium	0.0001	0.0017	0.0230	0.0003	0.0000	0.0004	0.0001	0.0021	0.0006
Net Worth	0.0018	0.0050	0.0781	0.0009	0.0020	0.0020	0.0005	0.0086	0.0025
M0	0.0001	0.0041	0.0425	0.0007	0.0001	0.0006	0.0001	0.0023	0.0008
Exports	0.0074	0.0005	0.0013	0.0000	0.0064	0.0237	0.0062	0.1067	0.0343
Imports	0.0045	0.0005	0.0006	0.0000	0.0029	0.0112	0.0029	0.0512	0.0165
Sunspot	0.4284	0.2557	0.6505	0.0128	0.3106	0.3229	0.8382	0.5967	0.1641

Table 4: Long Variance Decomposition

variation. Interest rates and the real exchange rate are mainly disturbed by monetary policy, productivity and the price mark-up, the last two via their effects on costs and so inflation.

We can now consider the effects of the sunspot shock on the longer term trends in the model. We show in Figure 12 the timelines in response to this shock as it evolves, for the main variables: output is depressed by 54% at the terminal date; in other words growth over the sample is depressed by 1.8% p.a.. This in turn depresses investment by 13%, Tobin's Q by 80%, and employment by 39%.

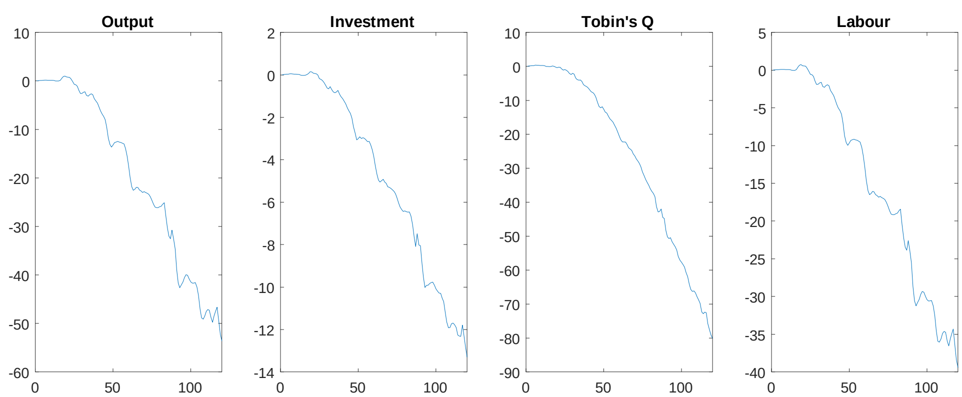

Figure 12: Longrun Effects of the Sunspot Shock

When we accumulate the effects of the sunspot shocks on expected terminal output, we find the picture above of a steadily worsening expected future growth rate. This is how the sunspot undermines Japanese growth. Investment in Japan is linked to these expectations via the marginal product of capital equation with the cost of capital. Employment is depressed by much the same percentage as output, and since consumption falls with output, lowering the demand for leisure, labour supply is reduced at the same time, forcing down real wages strongly.

6 Policy discussion

At the centre of this model fitted to Japanese data is its sunspot solution in which the forward root drives the model downwards in an explosive way towards a very slow growth path. The model generates poor growth outcomes because of this 'pessimism bubble'. It is a key function of government to eliminate such bubbles. It has long been recognised that central banks need to enunciate terminal conditions that eliminate nominal bubbles. Real bubbles like the one here are however rare because usually eliminated by private sector recognition of the true trend in output potential — what Keynes termed 'animal spirits' — so little attention has been given to any government role in eliminating real bubbles. Yet we have shown here that

Japan's weak growth performance can be accounted for by a real bubble not suppressed by private sector beliefs. It follows that in this situation the government needs to step in to suppress it. It cannot do so via monetary means, instructing the central bank, as this only affects nominal outcomes; of course we have observed that the BOJ's policy of targeting higher inflation has, as the model would imply, been ineffective in raising growth. It follows that fiscal policy needs to create the necessary terminal condition, which could take the form of a commitment to raise demand up to the true output potential level. We now show how the model behaves when this commitment is included, eliminating the sunspot shocks. This complements the existing fiscal feedback rule in which fiscal policy responds to the output gap; this helps to create output stability but on its own cannot eliminate the sunspot that creates long term pessimism and undermines long run growth.

In Table 5 we show the variances of output and inflation with and without the sunspot ('without' implying that the government suppresses it via a terminal condition on ystar). It can be seen that suppressing the sunspot reduces the variance of output markedly and makes little difference to the variance of inflation, so that welfare overall improves substantially.

	$var(Y)$	$var(\pi)$	$var(R)$	Welfare Cost
Sunspot	3.2989	0.8154	0.1572	4.2715
No Sunspot	2.6270	0.8532	0.1262	3.6064

Table 5: Effects on Volatility of Sunspot

This suggests that fiscal policy should be committed to suppressing any sunspot. So far the GOJ has expressed no interest in such a policy. This suggests it does not believe there is a sunspot. In the next section we review a model without a sunspot that they implicitly must believe.

6.1 Can we account for Japan's weak growth without the pessimism sunspot?

Japan shifted in the 1990s to a sharply slower growth rate than before. In this paper we have suggested that this came from a sunspot pessimism shock engendered by the economy's collapse when the 'financial bubble' was burst by a brutal monetary squeeze in 1989. What we have shown is that this model cannot be rejected by the behaviour of Japanese data since 1990. However, of course this does not also imply that there is no alternative model of Japan, with no sunspot, that could also match Japanese data behaviour.

In fact we can also match this behaviour with a 'normal model' in which agents correctly expected the trend in output that actually occurred. Under this model Japan would have experienced an exogenous slowdown in productivity growth which in turn produced slowing of capital and labour supply growth. We have also fitted this model to the data and its estimated parameters turn out to be the same as for the sunspot model; the model also matches the data behaviour about as well, with a p-value of 11%, as compared with 10% for the sunspot model — we have put the results in an Appendix 3.

What this reveals is that, though we cannot reject the sunspot model, an equally probable model of Japan's low growth recent history is one in which productivity growth simply slowed down. As this slowdown rolled out over time, expectations of the low productivity continuing into the future rolled out with it, given that productivity is nonstationary, with its innovations following a low order negative AR process.

As noted in the previous section, the GOJ must believe in this no-sunspot model since it has not taken action to eliminate a sunspot. The operative question for policymakers, given that both models are roughly equally probable, is what risks are taken in getting the model wrong. In Table 5 above, we showed that if the sunspot model was the true one, there would be a welfare gain of 32% from suppressing the sunspot. Now we must ask what the loss, if any, would be of doing so if the no-sunspot model is true. In this case fiscal policy would force output to lie above the true ystar value and equal the supposed higher ystar path. It might well be thought that such a policy would be highly inflationary and would be resisted by the BOJ via sharp interest rate increases; since this would be a conflict that cannot be won by either government or Bank, we model fiscal policy as strongly pushing downwards the output difference from its targeted ystar path. Table 6 below shows our simulated welfare results.

For this simulation the no-sunspot model has a ystar trend given by an HP filter of actual output and the fiscal suppression policy has strong fiscal feedback on the log difference of output from this HP ystar augmented to match the 2% growth trend. This feedback rule replaces the one assumed in the sunspot model; it is therefore substantially more aggressive, as besides stabilising output fluctuations, it is forcing output up to the assumed no-sunspot trend. However, our simulation results for this aggressive fiscal policy in the no-sunspot model also reveal a gain in welfare, with the variance of output falling sharply. Furthermore interest rate variance hardly changes either; and the inflation trend is barely affected; it turns out that this aggressive fiscal policy raises average inflation across all simulations by 0.24% p.a., which is pretty small. This might seem highly surprising. But when one considers how little inflation reacted to the huge monetary stimuli applied by the BOJ during our sample period it is less surprising that inflation reacts little to the very large fiscal stimuli implied by our simulated policy, or that in consequence interest rate policy also changes little.

No Sunspot model — HP=y^*	$var(Y)$	$var(\pi)$	$var(R)$	Welfare Cost
No Sunspot base line	2.6270	0.8532	0.1262	3.6064
Fiscal policy	0.1837	0.8160	0.1074	1.1071

Table 6: Effects on Volatility of Sunspot Suppression in No-Sunspot Model

The rather surprising policy conclusion we reach from this analysis is that the GOJ should act to suppress sunspot pessimism on the assumption that the sunspot model is correct; this is because even in the equally probable case that it is incorrect, such an aggressive fiscal policy would still give welfare gains. We may note further that in the event the sunspot model is incorrect, the government will have evidence from the higher inflation rate- even though only slightly higher- that it is indeed incorrect, and it would then abandon its aggressive policy. Equally it may conclude from the evidence of only slightly higher inflation that the sunspot model is correct; in this case it can trumpet its strategy of killing any sunspot, and — this generally understood — can revert to a normal fiscal policy, confident that any sunspot can no longer arise. We would therefore argue that the GOJ would be best advised to embark on such an aggressive fiscal policy.

If we examine the policy rule we estimated the GOJ was following, it reveals that this was a long way from such an aggressive policy. The GOJ has instead weakly stabilised output around a slowly rising potential output trend. It might be argued that this relative restraint was forced on it by fear of insolvency. Yet throughout our sample since the end of the 1990s long term Japanese bond yields have been close to zero, creating little threat to solvency. Debt issues have been held domestically, with Japanese residents unwilling to shift their holdings to foreign stocks. In this paper we assume the model estimated on this sample holds for our policy experiments.

7 Conclusions

In recent decades following the financial crisis the major economies of the OECD have slowed down and lost momentum, leading some economists to argue that 'secular stagnation' has emerged and should be fought by stimulative policies, including fiscal stimulus given that monetary stimulus has been undermined by interest rates hitting the zero lower bound. One possible reason for this loss of momentum since the financial crisis could be a rise in pessimistic beliefs about future potential output growth triggered by a loss of business confidence from the crisis — such a rise could trigger a sunspot equilibrium in a rational expectations New Keynesian model. We examine in this paper whether this theory could account for the slow growth behaviour of the Japanese economy since the crisis of 1989 produced there by the 'bursting of the financial bubble' that had arisen due to the loose money policies of the 1980s. We show in this paper that a New Keynesian model with a weak equilibrium growth path driven by pessimism sunspot belief shocks does match the behaviour of the Japanese economy as represented by a VAR. We also found that an equally probable model of Japan post-1989 remains a conventional one where productivity growth simply slowed down for unknown reasons. Nevertheless, we find that under a welfare-optimising approach Japanese fiscal policy should have committed to eliminating the possible sunspot but stood ready to revert to a normal policy in the face of rising inflation.

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8 Appendix 1: The DSGE model

8.1 Households

The economy is populated by a continuum of households indexed by $j \in [0,1]$. Each household j consumes a composite consumption $C_{t+s}(j)$, made up of final goods produced domestically $C_t^d(j)$ and imported goods $C_t^f(j)$, supplies labour service $N_{t+s}(j)$, and chooses to hold domestic $(B_t(j))$ and foreign $\left(B_t^f(j)\right)$ bonds to maximise the following present discounted utility function:

$$E_t \sum_{s=0}^{\infty} \beta^s \left[\frac{1}{1-\sigma_c} [C_{t+s}(j) - hC_{t+s-1}(j)]^{1-\sigma_c} \right] \exp\left(\frac{\sigma_c - 1}{1+\sigma_l} N_{t+s}(j)^{1+\sigma_l} \right)$$

where β is the discount factor, h is the degree of the external consumption habit formation, σ_c is the degree of relative risk aversion and σ_l denotes the inverse of the Frisch elasticity labour supply.

The budget constraint in real term is

$$C_t(j) + \frac{B_t(j)}{\varepsilon_t^b R_t P_t} + \frac{S_t B_t^f(j)}{\varepsilon_t^b R_t^f P_t} + T_t \leq \frac{W_t^h(j) N_t(j)}{P_t} + \frac{B_{t-1}(j)}{P_t} + \frac{S_t B_{t-1}^f(j)}{P_t} + \frac{\Pi_t}{P_t}$$

where R_t and R_t^f are nominal interest rate on deposit and foreign bonds respectively. P_t denotes the domestic price level and S_t is the nominal exchange rate measured as the amount of domestic currecy needed to purchase one unit of foreign currency, so that a rise in S_t means a depreciation in domestic currency. W_t^h is the nominal wage. Π_t is the dividends and T_t is a lump sum tax/transfer. ε_t^b is an financial assets prefence shock and it follows an AR(1) process $\ln \varepsilon_t^b = \rho_b \ln \varepsilon_{t-1}^b + \eta_t^b$.

The optimal conditions on $C_t(j)$ and $L_t(j)$ gives us the standard consumption Euler equation and labour supply, respectively. They replicate those of Smets and Wouters (2007). The optimal conditions in $B_t(j)$ and $B_t^f(j)$ together gives a uncovered interest parity condition

$$\frac{1}{r_t} = \frac{Q_t}{Q_{t+1} r_t^f} \tag{9}$$

where r_t is the real domestic interest rate on domestic bonds, r_t^f is the real foreign interest rate, Q_t is the real exchange rate$\left(Q_t = \frac{S_t P_t^f}{P_t}\right)$, P_t^f is the general foreign price level.

Households choose to consume domestically produced and imported goods to maximise their consumption composite $C_t = [\omega \left(C_t^d\right)^{\frac{\vartheta-1}{\vartheta}} - (1-\omega)\,\varepsilon_t^m \left(C_t^f\right)^{\frac{\vartheta-1}{\vartheta}}]^{\frac{\vartheta}{\vartheta-1}}$, subject to the budget constraint:

$$C_t = p_t^d C_t^d + Q_t C_t^f$$

where ω is the share of domestically produced goods in aggregate consumption, $p_t^d \equiv \frac{P_t^d}{P_t}$ is the relative price of domestic goods, and ϑ is an elasticity of substitution across consumption goods. ε_t^m is the imports demand shock that follows an AR(1) process, $\ln \varepsilon_t^m = \rho_b \ln \varepsilon_{t-1}^m + \eta_t^m$, $\eta_t^m \sim N(0,\ \sigma_m)$. The optimisation problem gives the demand for imports

$$IM_t = C_t^f = \left(\frac{(1-\omega)\varepsilon_t^m}{Q_t} \right)^{\vartheta} C_t \tag{10}$$

and the demand for domestically produced goods

$$C_t^d = \left(\frac{\omega}{p_t^d} \right)^{\vartheta} C_t \tag{11}$$

Symmetrically, the foreign demand for domestically produced goods is given as follows:

$$EX_t = \left(\frac{(1-\omega^F)\varepsilon_t^{ex}}{Q_t^f} \right)^{\vartheta^F} C_t^* \tag{12}$$

where ω^F, $\vartheta^F$and ε_t^{m*} is foreign equivalent to ω, ϑ and ε_t^m. C_t^* is the aggregate consumption in foreign country.

The balance of payments is defined as

$$\frac{S_t B_t^f}{R_t^f} - S_t B_{t-1}^f = P_t^d EX_t - S_t IM_t \tag{13}$$

that links the net foreign assets position to the trade balance.

8.2 Labour Market

Following Le, Meenagh and Minford (2016), the labour in production function combines in a fixed proportion labour inputs N_{1t} bought from imperfectly competitive market and labour inputs N_{2t} from perfectly competitive market. The aggregate labour supply N_t is:

$$N_t = N_{1t} + N_{2t}$$

where the share of unionised labour is ω_{NK}^w, so that $N_{1t} = \omega_{NK}^w N_t$ and $N_{2t} = (1 - \omega_{NK}^w) N_t$. These labour inputs enter each household's utility in the same way. Therefore, the aggregate hybrid wage is:

$$W_t = \omega_{NK}^w W_{1t} + (1 - \omega_{NK}^w) W_{2t} \tag{14}$$

W_{2t} where W_{2t} is the wage in a perfectly competitive market and equals to the marginal disutility of work; W_{1t} is the New Keynesian wage following Smets and Wouters (2007). These labour inputs are passed on to labour packer who offers the weighted wage for each unit of aggregate labour to intermediate goods firms. We also assume that household's utility includes these two types of labour in the same way. The wages are determined as below.

8.2.1 New Keynesian wage-setting

As in Smets and Wouters (2007), we assume that households supply a part of their labour services to labour unions, which then differentiate labour services and have the market power to set wages a la Calvo (1983). These differentiated labour services are gathered into labour input and sold on to the labour packer. Labour packer maximises the profit

$$\max_{N_t(j), N_t} W_t N_{1t} - \int_0^1 W_t(l) N_t(l)\, dl$$

where $N_t = \left(\int_0^1 N_t^{\frac{1}{1+\lambda_{w,t}}}(j)\, dj\right)^{1+\lambda_{w,t}}$. This then gives the demand for labour as $N_t(l) = \left(\frac{W_t(l)}{W_t}\right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} N_t$ and the wage cost in this imperfect labour market as $W_t = \left(\int_0^1 W_t^{\frac{1}{\lambda_{w,t}}}(j)\, dj\right)^{\lambda_{w,t}}$. $\lambda_{w,t}$ follows the exogenous ARMA process.

The labour unions purchase labour services from the households at the marginal rate of substitution between leisure and consumption, but then while exercising their market power in setting wages, they are subject to nominal rigidities, i.e. they can readjust wages with a probability $(1 - \zeta_w)$ in each period. For those that cannot adjust wages, $W_t(l)$ will increase at the weighted average of steady state inflation, π_*, and of last period's inflation, π_{t-1}. For those that can adjust, the problem is to choose a wage $\widetilde{W}_t(l)$ that maximises the wage income in all states of nature where the union is stuck with that wage in the future

$$\max_{\widetilde{W}_t(l)} E_t \sum_{s=0}^{\infty} \zeta_w^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+1}} \left[W_{t+s}(l) - W_{t+1}^h\right] N_{t+s}(l)$$

where $N_{t+s}(l) = \left(\frac{W_{t+s}(l)}{W_{t+s}}\right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} N_{t+s}$ and $W_{1t+s}(l) = \widetilde{W}_t(l) \left(\prod_{l=1}^{s} \pi_{t+l-1}^{\iota_w} \pi_*^{1-\iota_w}\right)$ for $s = 1, ...\infty$. Ξ_t is the lagrangean multiplier in the household's optimisation problem.

The imperfect labour market's aggregate wage is

$$W_{1t} = \left[(1-\zeta_w) \widetilde{W}_{1t}^{\frac{1}{\lambda_{w,t}}} + \zeta_w \left(\pi_{t+l-1}^{\iota_w} \pi_*^{1-\iota_w} W_{1t-1} \right)^{\frac{1}{\lambda_{w,t}}} \right]^{\lambda_{w,t}} \tag{15}$$

8.2.2 Perfectly competitive market

Households supply the other part of labour services to the competitive labour market, which would be gathered by the labour packer and resold to intermediate goods producers. The perfect market's aggregate wage is given as the marginal rate of substitution between leisure and consumption.

8.3 Goods Market

8.3.1 Final Goods Producers

Final goods producers assemble intermediate goods produced by firms. We follow Le, Meenagh and Minford (2016) in assuming that final goods Y_t is made up of a fixed proportion of intermediate goods sold in imperfectly competitive markets Y_{1t} and those sold in perfectly competitive market Y_{2t}. Hence, final output is given by:

$$Y_t = Y_{1t} + Y_{2t}$$

where ω_{NK}^P is the share of goods from imperfectly competitive markets, so that $Y_{1t} = \omega_{NK}^P Y_t$ and $Y_{1t} = \left(1-\omega_{NK}^P\right) Y_t$. The hydrid average weighted price equation is

$$P_t = \omega_{NK}^P P_{1t} + \left(1-\omega_{NK}^P\right) P_{2t} \tag{16}$$

where P_{1t} follows the New Keynesian price setting as in Smets and Wouters (2007) and P_{2t} is set at marginal costs. The final goods producers combine these two types of intermediate goods as a bundle and sell them at the above weighted average price.

8.3.2 New Keynesian Price setting

Intermediate goods producers/entrpreneurs choose ${K_t}^s(j)$ and $N_t(j)$ to minimise their cost of production

$$\min_{{K_t}^s(j), N_t(j)} R_t^k K_t(j) + W_t N_t(j),$$

subject to the production function

$$Y_t(j) = A_t {K_t}^s(j)^\alpha [\gamma^t N_t(j)]^{1-\alpha} - \Phi \tag{17}$$

where ${K_t}^s(j) = U_t(j) K_{t-1}(j)$ and $N_t(j)$ are the capital and labour input used in production respectively. α is capital share and Φ is fixed cost of producing products. A_t is total factor productivity and follows a nonstationary process. The cost minimisation problem gives an equal capital-labour ratio across firms

$$K_t^s = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t^k} N_t \tag{18}$$

This also gives the same marginal cost for all firms

$$MC_t = \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)} W_t^{1-\alpha} R_t^{ka} (A_t)^{-1} \tag{19}$$

Intermediate goods producers purchase capital and then choose the optimal level of capital utilization for production. The maximizing problem about the optimal degree of capital utilization is shown as:

$$\max_{U_t(j)} \frac{R_t^k U_t(j) K_{t-1}(j)}{P_t} - R_t^k \Upsilon \left[\exp\left(\frac{U_t(j)-1}{\Upsilon} \right) - 1 \right] K_{t-1}(j)$$

where $\Upsilon(U_t(j))$ is the adjustment cost with $\Upsilon(1) = 1$, $\Upsilon'(1) = R_t^k$ and $\frac{\Upsilon'(1)}{\Upsilon''(1)} = \psi$.

Intermediate goods producers sells goods to a part of the goods to final good producers in imperfect goods market and the rest to those in the perfect goods market. In the imperfect goods market, final good producers, would gather the intermediate goods, differentiate them, set prices and sell on. However, they are subject to Calvo pricing rule. Under this rule, the optimal price would be set from the following optimisation problem

$$\max_{\widetilde{P}_t(i)} E_t \sum_{s=0}^{\infty} \zeta_p^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+1}} \left[\widetilde{P}_t(i) \left(\prod_{l=1}^{s} \pi_{t+s-1}^{\iota_p} \pi_*^{1-\iota_p} \right) - MC_{t+s} \right] Y_{t+s}(i)$$

where $Y_{t+s}(i) = Y_{t+s} \left(\frac{P_{t+s}^i}{P_{t+s}} \right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}}$.

The aggregate price for the imperfect competitive goods market is

$$P_t^{\frac{1}{\lambda_{pt}}} = \left(1 - \zeta_p^s\right) \widetilde{P}_t^{\frac{1}{\lambda_{pt}}} + \zeta_p^s \left(\pi_{t-1}^{\iota_p} \pi_*^{1-\iota_p} P_{t-1} \right)^{\frac{1}{\lambda_{pt}}} \tag{20}$$

8.3.3 Perfectly competitive goods market

Intermediate goods producers also sell their products through the perfectly competitive goods market to the final goods producers. These final goods producers resell the goods to households at the price that is equal to the marginal cost of producing the goods.

8.4 Capital Producer

Capital producers are competitive. At the end of each period t, they buy undepreciated capital goods from intermediate good producer at price P_t^K and invest I_t to produce new capital. These capital goods would be sold to intermediate producer, as one of input, to produce products in period $t+1$. The capital producers maximise their profits

$$\sum_{s=0}^{\infty} \beta^s \left[P_t^K K_t - I_t - P_t^K (1-\delta) K_{t-1} \right]$$

subject to the capital evoluation

$$K_t = (1-\delta) K_{t-1} + \varepsilon_t^i \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) \right] I_t$$

where $\left(S'(\bullet) > 0,\ S''(\bullet) > 0, S(1) = S'(1) = 0 \right)$ and ε_t^i is the investment specific shock that affects the efficiency in transforming investment into new capital and follows an AR(1) process.

8.5 Financial Frictions and Quantitative easing

The intermediate goods producers are engaged in loan contracts to finance their capital purchase, choosing the level of capital utilisation and producing intermediate goods. We will look at these activites in turn.

8.5.1 Capital Purchase and Financial Frictions

The model introduces the financial friction a la Bernanke et al (1999). To facilitate this financial friction mechanism, it distinguishes between the intermediate goods producers and the capital goods producers. At period t, intermediate goods producers buy capital K_t from capital producers at price P_t^K and use it in next period production. In the end of period $t+1$, intermediate good producers receive $(1-\delta)P_{t+1}^K$ from reselling undepreciated capital back to capital producers and the marginal product of capital MPK_{t+1} from operating capital goods. The expected rate of return of capital is given by:

$$E_t \left[R_{t+1}^k \right] = E_t \left[\frac{MPK_{t+1} + (1-\delta) P_{t+1}^K}{P_t^K} \right] \tag{21}$$

It states that the expected rate of return on holding a unit of capital from t to $t+1$ consists of the marginal product of capital and the capital gain and it is also equal to the externally finance cost.

In order to purchase capital K_t, intermediate goods producers borrow from financial intermediaries (lenders). There is an asymmetric information problem between borrowers and lenders. The return on capital is sensitive to idiosyncratic risk which is known to intermediate producers but not to lenders, that could cause intermediate goods producers to default their loans and a costly process for lenders to recover funds in this situation. The assumption explains the reason of why external finance is more expensive than internal finance, and this gap between external and internal finance rates is derived from an optimal contract between borrowers and lenders. This gap between external and internal finance ($prem_t$) depends inversely on the share of the intermediate goods producers' capital investment ($pk_t + k_t$) that is financed by their own net worth (n_t). However, to facililate a role of money and effective quantitative easing under the zero lower bound of nominal interest rate, the model follows Le et al. (2016), where financial intermediaries also require intermediate good producers to provide money collateral upfront, which is easier to recover in case of default. The idea of money in the model works as follows. The central bank issues M0 in exchange for short-term bonds held by households. Once households have received this M0, they place it in banks as deposits and obtain the risk-free interest rate. Firms wish to acquire as much M0 as possible from banks to use as collateral for their borrowing. The M0 will appear on firms' balance sheet as the most liquid collateral pledged to banks in the case of bankcruptcy. The central bank can influence the credit market and its credit premium by varying the supply of M0. The loglinearised equation for the credit premium is as below

$$prem_t = E_t r_{t+1}^k - (r_t - E_t \pi_{t+1}) = \chi\left(pk_t + k_t - n_t\right) - \vartheta m_t^0 + \epsilon_t^{prem} \tag{22}$$

where $\vartheta > 0$ denotes the credit easing effect of M0 on the loans. The evolution of networth, in turn, is determined as

$$n_t = \theta n_{t-1} + \frac{K}{N}\left(r_t^k - E_{t-1} r_t^k\right) + E_{t-1} r_t^k + \epsilon_t^n \tag{23}$$

where θ is the survival rate of firms and $\frac{K}{N}$ is the steady-state ratio of capital to networth. Networth depends on past net worth of surviving firms plus their total return on capital minus the expected return (which is paid out in borrowing costs to the bank) on the externally financed part of their capital stock.

The detailed derivation for the credit premium is presented as follows. Suppose the quantity of capital produced by capital producer is exactly same as quantity needed by intermediate firms. In the end of period t, the entrepreneur who manages intermediate firms purchases K_{t+1} amount of capital at a unit price of P_t^K . The entrepreneur has net worth NW_t, which is not sufficient to finance the expenditure on capital goods. So that he must sign in a loan contract with banks. The loan rate is Z_{t+1}. Banks require the c proportion of net worth as collateral and a φ proportion of collateral is used up in liquidating collateral. The amount of borrowing is given by

$$B_{t+1} = P_t^K K_{t+1} - (1-c)\,\mathrm{NW}_t$$

The capital is homogeneous and the gross return on capital of entrepreneur is ωR_{t+1}^k, where R_{t+1}^k is the ex post aggregate return on capital and ω is idiosyncratic disturbance to firm's return. ω is an identically independent distribution (i.i.d.) random variable and it follows the cumulative distribution function (c.d.f.) $F(\omega)$. $\omega \in (0,\ \infty)$ and $E\left(\omega\right) = 1$. $f(\omega)$ is the pdf of ω . At time $t+1$, entrepreneurs choose either pay off the loan or default. If the entrepreneur stays in business, he repays $Z_{t+1}B_{t+1}$ to the banks and receives $\omega R_{t+1}^k P_t^K K_{t+1} + c\mathrm{NW}_t$. If the entrepreneur defaults, the situation is just the opposite. Thus, the threshold level of $\overline{\omega}$ is defined by:

$$\overline{\omega} R_{t+1}^k P_t^K K_{t+1} + c\mathrm{NW}_t = Z_{t+1} B_{t+1}$$

Denote $L_t = \frac{P_t^K{}_t K_{t+1}}{\mathrm{NW}_t}$ as leverage, the loan rate Z_{t+1} can be written as:

$$Z_{t+1} = \frac{\overline{\omega} R_{t+1}^k\ L_t + c}{L_t - (1-c)}$$

The higher threshold value of $\overline{\omega}_{t+1}$, the higher leverage and higher loan rates. When $\omega > \overline{\omega}$, the entrepreneur repays the promised repayment to the banks and receives the net revenue $\omega R_{t+1}^k P_t^K K_{t+1} +$

$c\mathrm{NW}_t - Z_{t+1}B_{t+1}$. When $\omega < \overline{\omega}$, return on capital is smaller than the opportunity cost which borrows from bank, the entrepreneur chooses to default. In this case, the entrepreneur gets nothing, banks receive the collateral and $(1-\mu)$ of the gross return on capital where μ is monitor cost.

The optimal debt contract maximizes entrepreneurial welfare subject to banks' feasibility constraint. We first discuss the lender's expected return. Banks are competitive, which ensures that, banks make zero profits in equilibrium. Banks' expected return of lending equals to its opportunity cost of those funds. The opportunity cost is $R_{t+1}B_{t+1}$, where R_{t+1} is riskless nominal interest rate. Thus, banks' zero profit condition is given by:

$$[1-F(\overline{\omega})]\, Z_{t+1}B_{t+1} + (1-\mu)\int_0^{\overline{\omega}} \omega R^k_{t+1} P^K_t K_{t+1} \partial F(\omega) + F(\overline{\omega})(1-\varphi)\, c\mathrm{NW}_t = R_{t+1}B_{t+1}$$

where $F(\overline{\omega})$ is the probability of default. On LHS, the first term is the repayment to the bank when the entrepreneur operates the firm well; the second term is the expected capital return when entrepreneur chooses to default; the third term is the collateral after liquidation under bankruptcy. Assume that $\Gamma(\overline{\omega})$ is the share of entrepreneurial expected capital return accrued to banks, $\Gamma(\overline{\omega}) = \overline{\omega}[1-F(\overline{\omega})] + G(\overline{\omega})$; and $G(\overline{\omega}) = \int_0^{\overline{\omega}} \omega \partial F(\omega)$, $\Gamma'(\overline{\omega}) = 1-F(\overline{\omega})$, $\Gamma''(\overline{\omega}) = -f(\overline{\omega})$, the banks' zero profit condition is rewritten as:

$$[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})]\, R^k_{t+1} P^K_t K_{t+1} + (1-\varphi F(\overline{\omega}))\, c\mathrm{NW}_t = R_{t+1}\left(P^K_t K_{t+1} - (1-c)\,\mathrm{NW}_t\right),$$

which results in the banks' leverage offer curve as:

$$L_t \left(= \frac{P^K_t K_{t+1}}{NW_t}\right) = \frac{R_{t+1} - c\,[R_{t+1} - 1 + \varphi F(\overline{\omega})]}{R_{t+1} - (\Gamma(\overline{\omega}) - \mu G(\overline{\omega}))R^k_{t+1}} = \frac{R_{t+1} - c\,[R_{t+1} - 1 + \varphi F(\overline{\omega})]}{R_{t+1} - \Theta(\overline{\omega})\, R^k_{t+1}} \tag{24}$$

where it is an increasing and convex curve with respect to $\overline{\omega}$.

The entrepreneur can only make a profit if he does not breach the contract, i.e. drawing $\omega > \overline{\omega}$. The expected entrepreneurial earning from getting a loan is:

$$\omega R^k_{t+1} P^K_t K_{t+1} dF(\omega) - [1-F(\overline{\omega})]\, Z_{t+1}B_{t+1} + [1-F(\overline{\omega})]\, c\mathrm{NW}_t$$

The first term is the expected return on capital, the second term is the expected repayment to banks and the third term is the collateral required in the contract. It can be rewritten as

$$\left[\int_{\overline{\omega}}^{\infty} \omega dF(\omega) - [1-F(\overline{\omega})]\,\overline{\omega}\right] R^k_{t+1} P^K_t K_{t+1} = (1-\Gamma(\overline{\omega}))\, R^k_{t+1} P^K_t K_{t+1}$$

The terms of collateral have been eliminated, which mean that the firm's expected return is unaffected by the amount of collateral. Using the definition of leverage above, we can write the firm's expected return as:

$$(1-\Gamma(\overline{\omega}))\, R^k_{t+1} L_t$$

Hence the formal contracting problem for the entrepreneur is shown as:

$$\max_{L_t,\overline{\omega}} (1-\Gamma(\overline{\omega}))\, R^k_{t+1} L_t$$

s.t.

$$L_t = \frac{R_{t+1} - c\,[R_{t+1} - 1 + \varphi F(\overline{\omega})]}{R_{t+1} - \Theta(\overline{\omega})\, R^k_{t+1}}$$

The FOC is:

$$[R_{t+1} - c\,(R_{t+1} - 1 + \varphi F(\overline{\omega}))]\left[R_{t+1} - \Omega' R^k_{t+1}\right] = \left[\frac{-c\varphi F'(\overline{\omega})\,(1-\Gamma(\overline{\omega}))}{\Gamma'(\overline{\omega})}\right]\left[R_{t+1} - \Theta(\overline{\omega})\, R^k_{t+1}\right]$$

where $\Omega^{'} = \frac{\Theta^{'}(\overline{\omega})}{\Gamma^{'}(\overline{\omega})} + \left[1 - \frac{\Theta^{'}(\overline{\omega})}{\Gamma^{'}(\overline{\omega})}\right]\Theta(\overline{\omega}) \approx 1$. Therefore, the entrepreneur's optimal choice is

$$L_t\left[R_{t+1} - \Omega^{'} R^k_{t+1}\right] = \frac{-c\varphi F^{'}(\overline{\omega})(1 - \Gamma(\overline{\omega}))}{\Gamma'(\overline{\omega})} \tag{25}$$

There are now two equations () and () in $(\overline{\omega}, L)$ space. We investigate the comparative static properties of changes around the equilibrium by taking the total differentiation of these two equations in $\partial L, \partial\overline{\omega}, \partial\varphi$ and ∂R^k. We evaluate the derivatives at an equilibirum where $\varphi = 0$. The total differentiations are expressed respectively as

$$\left[R - \Omega^{'} R^k\right]\partial L_t - L\Omega^{'}\partial R^k = \frac{-cF^{'}(\overline{\omega})(1 - \Gamma(\overline{\omega}))}{\Gamma'(\overline{\omega})}\partial\varphi \tag{26}$$

$$\partial L = L\left[\frac{\Theta(\overline{\omega})}{R - \Theta(\overline{\omega}) R^k}\right]\partial R^k_{t+1} + \left[\frac{-cF(\overline{\omega})}{R - \Theta(\overline{\omega}) R^k}\right]\partial\varphi + L\left[\frac{\Theta^{'}(\overline{\omega}) R^k}{R - \Theta(\overline{\omega}) R^k}\right]\partial\overline{\omega} \tag{27}$$

Since in the rest of the DSGE model, L is determined by capital and networth, and φ is determined by the provision of M0 as an alternative illiquid collateral, we consider them as exogenous to the financial sector analysis here, and then just need to solve for R^k and $\overline{\omega}$. Our interest lies in finding out the effect of φ on the equilibirum value of R^k and $\overline{\omega}$. These two elements are internal to the bank contract decision and unobservable in the public domain but in turn from these we can solve for the observable cost of the bank credit, Z, from the bankcruptcy threshold as $Z = \frac{R^k\overline{\omega}L + c}{L - 1 + c}$. We note from eq() that

$$\frac{\partial R^k}{\partial\varphi} = \frac{cF^{'}(\overline{\omega})(1 - \Gamma(\overline{\omega}))}{L\Omega'\Gamma'(\overline{\omega})} > 0$$

and from eq() that

$$\frac{\partial\overline{\omega}}{\partial\varphi} = \frac{cF(\overline{\omega})}{L\Theta'(\overline{\omega}) R^k}\left[1 - \frac{F^{'}(\overline{\omega})\Theta(\overline{\omega})(1 - \Gamma(\overline{\omega}))}{F(\overline{\omega})\Omega'\Gamma'(\overline{\omega})}\right] > 0.$$

The latter expression is positive, proven numerically in Le et al (2016). These two conditions means that

$$\frac{\partial Z}{\partial\varphi} = \frac{c}{L - 1 + c}\left[\frac{F(\overline{\omega})}{\Omega'(\overline{\omega})}\left(1 - \frac{F^{'}(\overline{\omega})\Theta(\overline{\omega})(1 - \Gamma(\overline{\omega}))}{F(\overline{\omega})\Omega'\Gamma'(\overline{\omega})}\right) + \left(\frac{\overline{\omega}F^{'}(\overline{\omega})(1 - \Gamma(\overline{\omega}))}{\Theta'(\overline{\omega})(1 - \Gamma(\overline{\omega}))}\right)\right] > 0$$

Since φ is reduced by M0 injections, we can say that a rise in M0 will reduce the required return on capital and also the credit premium.

8.6 Monetary Policy

Following Le et al. (2016) ,the model assumes that the central bank would conduct monetary policy via the short-term interest rate according to the Taylor rule formulation normally. However, under the zero lower bound of normal interest rate, the central bank would resort to using Quantitative easing policy to regulate the economy. The model facilitates the switch between these two states endogenously.

When the nominal interest rate is above zero percent, the central bank set interest rate according to the following Taylor rule

$$r_t = \rho r_{t-1} + (1 - \rho)(r_p\pi_t + r_y(y_t - y^*_t)) + r_{\Delta y}\Delta(y_t - y^*_t) + \varepsilon^r_t r > 0 \tag{28}$$

where ρ measures the degree of interest rate smoothing. r_p, r_y and $r_{\Delta y}$ represents Taylor's rule responses to inflation, output and change in output, respectively. The monetary policy shock follows an AR(1) process $\varepsilon^r_t = \rho_r\varepsilon^r_{t-1} + \eta^r_t$. The supply of M0 is set as

$$m^0_t - m^0_{t-1} = \vartheta_{m2}\left(m^2_t - m^2_{t-1}\right) + \epsilon^{m0}_t \tag{29}$$

to accommodate the broad money supply M2, which is determined by the firms balance sheet quantities

$$m_t^2 = \left(1 - \frac{M0}{M2} + \frac{N}{M2}\right) k_t + \frac{M0}{M2} m_t^0 - \frac{N}{M2} n_t, \tag{30}$$

where $\epsilon_t^{m0} = \rho_{m0}\epsilon_{t-1}^{m0} + \eta_t^{m0}$, $\frac{M0}{M2}$ and $\frac{N}{M2}$ are the steady state ratios of M0 and networth to M2, respectively. The money supply is defined as $M2 = credit + bank\ deposit$,where credit is equal to capital expenditure and collateral in excess of networth, resulting in $M2 = M0 + (K - N + collateral)$. Equation 30) is therefore derived given that collateral is a fixed proportion of money.

When the nominal interest rate reaches 0%, the interest rate Taylor rule ceased to operate, the central bank has to use unconventional monetary policies such as quantitative easing. In this model, we assume that the central bank issue M0, which are transfered to households who then deposit with banks who lend M0 to intermediate producers who want to hold it as much as for the collateral purposes to reduce the cost of borrowing. In turn, the central bank uses the quantitative easing to response to the aggregate demand which depends on the credit premium. The monetary policy under the zero lower bound ($r_t = 0$) is characterised by

$$m_t^0 - m_{t-1}^0 = \vartheta_{prem}\left(prem_t - prem^*\right) + \epsilon_t^{m0} \tag{31}$$

where ϑ_{prem} is the elasticity of M0 to the credit premium.

8.7 Closing the Model

As set in Smet and Wouters(2007), government spending relate to the steady state output path $\varepsilon_g = \frac{G_t}{Y\gamma^t}$, it follows the process that

$$\varepsilon_t^g = \left(1 - \rho_g\right)\varepsilon^g + \rho_g \varepsilon_{t-1}^g + \rho_{ga}\varepsilon_t^a - \rho_{ga}\varepsilon_{t-1}^a + \eta_t^g,\ \eta_t^g \sim (0,\ \sigma_g)$$

where $0 < \rho_g < 1$ and the government spending is affected by the productivity process. The government spending comes from collecting lump sum taxes T_t and issues B_t. The budget constraint is shown as:

$$P_t G_t + B_{t-1} = T_t + \frac{B_t}{R_t}$$

By integrating the behaviours among households, firms, entrepreneurs, central bank and government, the good market clearing condition in log-linearized form is:

$$y_t = \frac{C}{Y}c_t + \frac{I}{Y}i_t + \frac{K}{Y}MPK^*\frac{1-\psi}{\psi}mpk_t + \frac{C^e}{Y}c_t^e + \frac{EX}{Y}ex_t - \frac{IM}{Y}im_t + \varepsilon_t^g \tag{32}$$

where c_t^e is consumption of the bankcrupt firms, but in log it is equal to the networth variable.

9 Appendix 2: Terminal conditions

At the terminal date, two main conditions must be satisfied:

- UIP, such that q, the real exchange rate, is constant, equilibrating the current account, so that $ex = q.im$ at T. $lnex = w.lnWorldGDP + \sigma_x \ln q. \ln im = m. \ln y^* - \sigma_m \ln q$. Hence $lnex = w.lnWorldGDP + (\sigma_x + \sigma_m - 1)\ln q^* - m. \ln y^*$

  so that $\ln q^* = [+m. \ln y^* - w.lnWorldGDP]/(\sigma_x + \sigma_m - 1)$

- consumption must be given by market-clearing:$y^* = c^* + I^* + G^*$; hence $\ln c^* = \frac{c}{y}[\ln y^* - \frac{i}{y}\ln(I) - \frac{g}{y}\ln G]$

Hence Y^* determines q^* and C^*. These then feed into the exchange rate, exports, imports at $T-1$ etc; and into C at $T-1$ etc via Euler equation.

To obtain I^* we note that at T $(r_F + \delta)K^* = (1-\alpha)y^*$ so that $I^* = \Delta K^* = [(1-\alpha)/(r_F + \delta)]\Delta y^*$

To find labour market outcomes note that $\ln Ls = \frac{1}{\sigma_l}(\ln w^* - \ln c); \ln L_D = \ln \alpha + \ln y^* - \ln w^*$; hence $\ln w^* = (\frac{1}{1+1/\sigma_l})[\ln \alpha + \ln y^* + 1/\sigma_l \ln c^*]$

Note that via Cobb-Douglas: $y^* = [K^{*1-\alpha} L^{*\alpha}]A^*$ and $y = wL + rK$.

The Taylor Rule is assumed to be satisfied with inflation at its target and the real interest rate equal to the world level.

10 Appendix 3: Empirical results from model without sunspot

	Sunspot model coefficients
Steady-state elasticity of capital adjustment	8.0073
Elasticity of consumption	2.5387
External habit formation	0.4636
Probability of not changing wages	0.4402
Inverse of Frisch elasticity of labour supply (σ_l)	3.3006
Probability of not changing prices	0.8066
Wage indexation	0.1905
Price indexation	0.4457
Elasticity of capital utilisation	0.9435
Share of fixed costs in production (+1)	1.7772
Taylor Rule response to inflation	1.0612
Interest rate smoothing	0.9735
Taylor Rule response to output	0.0060
Taylor Rule response to change in output	0.0126
Share of capital in production	0.3338
Proportion of sticky wages	0.2384
Proportion of sticky prices	0.4806
Elasticity of the premium with respect to leverage	0.0103
Monetary response in crisis time	0.0978
Monetary response in normal time	0.0406
Elasticity of premium with respect to money	0.0494
Bubble forward root	0.9929
Fiscal response	0.6470
Wald	19.5040
Transformed Wald (t-stat)	0.9898
P-Value	0.1134

Table 7: Coefficient Estimates for Model with No Sunspot

	Actual	Mean	2.5th Percentile	97.5th Percentile	In/Out
Y_Y	0.8031	0.3062	−0.3038	0.5697	OUT
Y_PI	0.0554	0.0819	−0.3580	0.5623	IN
Y_R	−0.1540	2.0262	−15.1630	24.7305	IN
PI_Y	0.0810	0.0325	−0.0697	0.1299	IN
PI_PI	−0.3040	0.0385	−0.2288	0.3424	OUT
PI_R	0.6767	0.2716	−4.0278	5.0695	IN
R_Y	0.0021	−0.0038	−0.0122	0.0032	IN
R_PI	0.0051	−0.0026	−0.0181	0.0133	IN
R_R	0.9320	0.8115	0.0533	0.9932	IN
Var(Y)	1.5653	2.1700	1.0108	7.7991	IN
Var(PI)	0.2339	0.3870	0.2397	0.5710	OUT
Var(R)	0.0009	0.0025	0.0000	0.0108	IN

Table 8: Auxiliary Model Parameter Bounds for Model with No Sunspot