The triangletools package

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Introduction

This package aims to help you construct special points in a triangle directly in a short and easy way. Using this package, you can construct most important points listed in Clark Kimberling's Encyclopedia of Triangle Centers (ETC). Currently, all points numbered from X_1 and X_{10} , as well as the excenter, are supported; however with other utilities in this package (see Section 3.3) and a bit of knowledge in geometry and expl3 programming, you can construct even more.

Loading the package

This package can be loaded as usual.

```
1 \usepackage{triangletools}
```

It will load tikz and expl3 automatically.

User interface

The user interface of this package, including that of the utilities, is provided as pgf keys under the tree /tikz/triangletools.

Note that, in the following sections, a *coordinate* means a *named* TikZ coordinate. That is, in the following example,

```
1 \begin{tikzpicture}
  \downarrow (0,0) -- (3,0) coordinate (a);
3 \end{tikzpicture}
```

a is a named coordinate, while 0,0 or 3,0 are not named coordinates. The current implementation of this package only allows named coordinates in the user interface. It is like the angles TikZ library.

3.1 Accessing the keys

/tikz/trt /tikz/trt= $\{\langle keys\rangle\}$

It executes keys with the key path set to /tikz/triangletools, which is the main key tree of this package.

This key is used to access all other keys in the user interface.

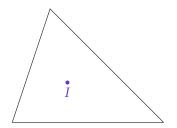
3.2 Triangle centers

incenter \trt_sp_incenter:nnnn

```
/tikz/triangletools/incenter=(\langle coor 1 \rangle)(\langle coor 2 \rangle)(\langle coor 3 \rangle)
\trt_sp_incenter:nnnn {\langle coor 1 \rangle} {\langle coor 2 \rangle} {\langle coor 3 \rangle} {\langle name \rangle}
```

Find the incenter X_1 of the triangle joining TikZ coordinates $\langle coor 1 \rangle$, $\langle coor 2 \rangle$ and $\langle coor 3 \rangle$. The incenter is saved to TikZ coordinate $\langle name \rangle$.

If you use the key (why do you use the function anyway), (name) is set to trt output by default. You can change that using output name, see Section 3.4.



excenter
\trt_sp_excenter:nnnn

/tikz/triangletools/excenter= $(\langle coor 1 \rangle) (\langle coor 2 \rangle) (\langle coor 3 \rangle)$ \trt_sp_excenter:nnnn $\{\langle coor 1 \rangle\} \{\langle coor 2 \rangle\} \{\langle coor 3 \rangle\} \{\langle name \rangle\}$

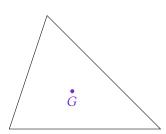
Find the excenter of the triangle. The returned point will be on the internal angular bisector at $\langle coor 1 \rangle$. Note that the order matters: excenter=(a)(b)(c) is *different* from excenter=(b)(a)(c).



L

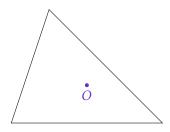
centroid
\trt_sp_centroid:nnnn

/tikz/triangletools/centroid=($\langle coor\ 1 \rangle$)($\langle coor\ 2 \rangle$)($\langle coor\ 3 \rangle$) \trt_sp_centroid:nnnn $\{\langle coor\ 1 \rangle\}\{\langle coor\ 2 \rangle\}\{\langle coor\ 3 \rangle\}\{\langle name \rangle\}$ Find the centroid X_2 of the triangle.



circumcenter
\trt_sp_circumcenter:nnnn

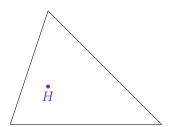
/tikz/triangletools/circumcenter=($\langle coor\ 1 \rangle$)($\langle coor\ 2 \rangle$)($\langle coor\ 3 \rangle$) \trt_sp_circumcenter:nnnn { $\langle coor\ 1 \rangle$ }{ $\langle coor\ 2 \rangle$ }{ $\langle coor\ 3 \rangle$ }{ $\langle name \rangle$ } Find the circumcenter X_3 of the triangle.



orthocenter
\trt_sp_orthocenter:nnnn

 $\label{eq:coor_lambda} $$ \dot \cdot = (\langle coor\ 1 \rangle) (\langle coor\ 2 \rangle) (\langle coor\ 3 \rangle) \ \\ \dot \cdot = \sup_{0 \le i \le n} \{\langle coor\ 1 \rangle\} \{\langle coor\ 2 \rangle\} \{\langle coor\ 3 \rangle\} \{\langle name \rangle\} \} $$$

Find the orthocenter X_4 of the triangle.



triangle_center

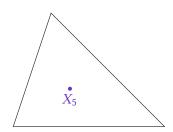
/tikz/triangletools/triangle center= $(\langle coor 1 \rangle)(\langle coor 2 \rangle)(\langle coor 3 \rangle)(\langle index \rangle)$

Find the point $X_{\langle index \rangle}$ of the triangle. Currently $\langle index \rangle$ can be any integer between and including 1 and 10.

\trt_sp_ninepointcenter:nnnn

/tikz/triangletools/triangle center= $(\langle coor\ 1 \rangle)(\langle coor\ 2 \rangle)(\langle coor\ 3 \rangle)(5)$ \trt_sp_ninepointcenter:nnnn $\{\langle coor\ 1 \rangle\}\{\langle coor\ 2 \rangle\}\{\langle coor\ 3 \rangle\}\{\langle name \rangle\}$

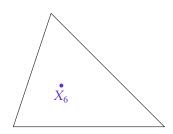
Find the nine-point center X_5 of the triangle.



\trt_sp_symmedian:nnnn

/tikz/triangletools/triangle center= $(\langle coor 1 \rangle)(\langle coor 2 \rangle)(\langle coor 3 \rangle)$ (6) \trt_sp_symmedian:nnnn $\{\langle coor 1 \rangle\}\{\langle coor 2 \rangle\}\{\langle coor 3 \rangle\}\{\langle name \rangle\}$

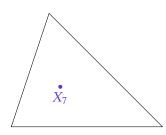
Find the symmedian point X_6 (*aka.* the Lemoine point or Grebe point) of the triangle.



\trt_sp_gergonne:nnnn

/tikz/triangletools/triangle center= $(\langle coor\ 1 \rangle)(\langle coor\ 2 \rangle)(\langle coor\ 3 \rangle)(7)$ \trt_sp_gergonne:nnnn $\{\langle coor\ 1 \rangle\}\{\langle coor\ 2 \rangle\}\{\langle coor\ 3 \rangle\}\{\langle name \rangle\}$

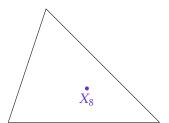
Find the Gergonne point X_7 of the triangle.



\trt_sp_nagel:nnnn

/tikz/triangletools/triangle center= $(\langle coor 1 \rangle)(\langle coor 2 \rangle)(\langle coor 3 \rangle)(8)$ \trt_sp_nagel:nnnn $\{\langle coor 1 \rangle\}\{\langle coor 2 \rangle\}\{\langle coor 3 \rangle\}\{\langle name \rangle\}$

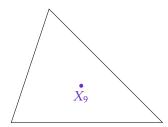
Find the Nagel point X_8 of the triangle.



\trt_sp_mittenpunkt:nnnn

/tikz/triangletools/triangle center= $(\langle coor 1 \rangle)(\langle coor 2 \rangle)(\langle coor 3 \rangle)(9)$ \trt_sp_mittenpunkt:nnnn $\{\langle coor 1 \rangle\}\{\langle coor 2 \rangle\}\{\langle coor 3 \rangle\}\{\langle name \rangle\}$

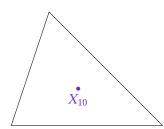
Find the *mittenpunkt* X_9 of the triangle.



\trt_sp_spieker:nnnn

/tikz/triangletools/triangle center= $(\langle coor\ 1 \rangle)(\langle coor\ 2 \rangle)(\langle coor\ 3 \rangle)(10)$ \trt_sp_spieker:nnnn $\{\langle coor\ 1 \rangle\}\{\langle coor\ 2 \rangle\}\{\langle coor\ 3 \rangle\}\{\langle name \rangle\}$

Find the Spieker center X_{10} of the triangle.



3.3 Other utilities

3.3.1 Line tools

The line tools utility can helps you play with some (very basic) operations related to lines.

intersection

```
/tikz/triangletools/intersection=(\langle coor 1 \rangle)(\langle coor 2 \rangle) - (\langle coor 3 \rangle)(\langle coor 4 \rangle)
```

There are two lines, the first joins $\langle coor 1 \rangle$ and $\langle coor 2 \rangle$, and the other joins $\langle coor 3 \rangle$ and $\langle coor 4 \rangle$. This finds the intersection of these lines.

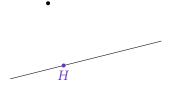
If the two lines are parallel, trt output is set to (0,0), and the package will report a warning.

```
1 \begin {tikzpicture}
2 \draw (0,0) coordinate (a) -- (0,1) coordinate (b);
3 \draw (1,0) coordinate (c) -- (.5,1) coordinate (d);
4 \coordinate (e) at (1,1);
5 \fill[maincolor,trt={intersection=(a)(b)--(c)(d)}] (trt output)
6 circle (1.5pt) node[above] {$I$};
7 \fill[maincolor,trt={intersection=(a)(b)--(c)(e)}] (trt output)
8 circle (1.5pt) node[left] {$J$};
9 \end{tikzpicture}
```

foot_of_perpendicular

/tikz/triangletools/foot of perpendicular= $(\langle coor 1 \rangle) - (\langle coor 2 \rangle) (\langle coor 3 \rangle)$

Find the foot of perpendicular of point $\langle coor 1 \rangle$ to the line joining $\langle coor 2 \rangle$ and $\langle coor 3 \rangle$.



```
1 \begin {tikzpicture}
2 \draw (0,0) coordinate (b) -- (4,1) coordinate (c);
3 \fill (1,2) circle (1.5pt) coordinate (a);
4 \fill[maincolor,trt={foot of perpendicular=(a)--(b)(c)}] (trt output)
5 circle (1.5pt) node[below] {$H$};
6 \end{tikzpicture}
```

You can do much more using these expl3 functions.

\trt_lt_get_line_equation:nnNNN

```
\trt_lt_get_line_equation:nnNNN{\langle coor 1 \rangle}{\langle coor 2 \rangle}{\langle a \rangle \langle b \rangle \langle c \rangle}
```

Find the equation of the line joining $\langle coor 1 \rangle$ and $\langle coor 2 \rangle$, in the form of ax + by = c. The l3fp *local* variables $\langle a \rangle$, $\langle b \rangle$ and $\langle c \rangle$ will be set accordingly.

Note that for any pair of points $\langle coor 1 \rangle$ and $\langle coor 2 \rangle$, there are infinitely many solutions for $\langle a \rangle$, $\langle b \rangle$ and $\langle c \rangle$. This function will produce one of such solution. While the solution is likely to be the simplest of all possible ones, this is not guaranteed.

```
1x + -3y = 1
```

```
1 \begin {tikzpicture}
    \coordinate (a) at (1,0);
    \coordinate (b) at (4,1);
    \ExplSyntax0n
    \fp_new:N \l_foo_tmpa_fp
    \fp_new:N \l_foo_tmpb_fp
    \fp_new:N \l_foo_tmpc_fp
    \trt_lt_get_line_equation:nnNNN {a} {b}
      \l_foo_tmpa_fp \l_foo_tmpb_fp \l_foo_tmpc_fp
    \fp_new:N \c_foo_cm_fp
    \fp_set:Nn \c_foo_cm_fp {28.45275590551181}
    \def \resultequation {
      fp_eval:n {round (\l_foo_tmpa_fp / \c_foo_cm_fp)}x +
14
       \fp_eval:n {round (\l_foo_tmpb_fp / \c_foo_cm_fp)}y
      =\fp_eval:n {round (\l_foo_tmpc_fp / (\c_foo_cm_fp * \c_foo_cm_fp))}$
15
    }
16
    \ExplSyntax0ff
    \draw (a) -- (b) node[midway,sloped,below] {\resultequation};
19 \end{tikzpicture}
```

\trt_lt_get_intersection_line:NNNNNNN

```
\trt_lt_get_intersection_line:NNNNNNNN(a1)\langle b1\rangle\langle c1\rangle\langle a2\rangle\langle b2\rangle\langle c2\rangle\langle x\rangle\langle y\rangle
```

This function finds the intersection of lines $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$, afterwards store the dimensions of the intersection in variables $\langle x \rangle$ and $\langle y \rangle$.

All arguments are floating points variables, $\langle x \rangle$ and $\langle y \rangle$ needs to be local variables.

\trt_lt_get_intersection_coordinate:nnnnNN

```
\label{eq:coordinate:nnnnNN} $$ {\langle coor 1 \rangle} {\langle coor 2 \rangle} {\langle coor 3 \rangle} {\langle coor 4 \rangle} {\langle x \rangle} $$
```

This function is a wrapper of \trt_lt_get_intersection_line:NNNNNNN. It finds the intersection of the line joining $\langle coor\ 1 \rangle$, $\langle coor\ 2 \rangle$ and the line joining $\langle coor\ 3 \rangle$, $\langle coor\ 4 \rangle$. The dimensions of the returned point is stored in $\langle x \rangle$ and $\langle y \rangle$, which are local l3fp variables.

A warning will be raised if the lines are parallel, in that case $\langle x \rangle$ and $\langle y \rangle$ are set to zero.

This is the base of intersection.

```
1 \begin {tikzpicture}
2 \draw (0,0) coordinate (a) -- (0,1) coordinate (b);
3 \draw (1,0) coordinate (c) -- (.5,1) coordinate (d);
4 \ExplSyntaxOn
5 \fp_new:N \l_foo_tmpa_fp
6 \fp_new:N \l_foo_tmpb_fp
7 \trt_lt_get_intersection_coordinate:nnnnNN {a} {b} {c} {d}
8 \l_foo_tmpa_fp \l_foo_tmpb_fp
9 \coordinate (i) at (\fp_to_dim:N \l_foo_tmpa_fp,
10 \fp_to_dim:N \l_foo_tmpb_fp);
11 \ExplSyntaxOff
12 \fill[maincolor] (i) circle (1.5pt);
13 \end{tikzpicture}
```

\trt_lt_get_perpendicular_equation:nNNNNNN

 $\label{eq:coor} $$ \operatorname{trt_lt_get_perpendicular_equation:nNNNNN}(\langle coor\ 1\rangle) \langle a1\rangle \langle b1\rangle \langle c1\rangle \langle a2\rangle \langle b2\rangle \langle c2\rangle $$$

This function finds the line of equation $a_2x + b_2y = c_2$ that passes coordinate $\langle coor 1 \rangle$ and is perpendicular to $a_1x + b_1y = c_1$.

```
| \begin {tikzpicture}
    draw (-1,0) coordinate (b) node[left] {$(-1,0)$} --
          (3,1) coordinate (c) node[right] {$(3,1)$};
    \fill (0.5,4) circle (1.5pt) coordinate (a) node[above] {$(0.5,4)$};
    \ExplSyntax0n
    \fp_new:N \l_foo_tmpa_fp
    \fp_new:N \l_foo_tmpb_fp
    \fp_new:N \l_foo_tmpc_fp
    \fp_new:N \l_foo_tmpd_fp
    \fp_new:N \l_foo_tmpe_fp
    \fp_new:N \l_foo_tmpf_fp
11
    \trt_lt_get_line_equation:nnNNN {b} {c}
      \l_foo_tmpa_fp \l_foo_tmpb_fp \l_foo_tmpc_fp
    \trt_lt_get_perpendicular_equation:nNNNNNN {a}
      \l_foo_tmpa_fp \l_foo_tmpb_fp \l_foo_tmpc_fp
      \verb|\l_foo_tmpd_fp \ | l_foo_tmpe_fp \ | l_foo_tmpf_fp |
    fp_new:N \c_foo_cm_fp
    fp_set:Nn \c_foo_cm_fp {28.45275590551181}
    \def \resultequation {
      fp_eval:n {round (\l_foo_tmpd_fp / \c_foo_cm_fp)}x +
21
       \fp_eval:n {round (\l_foo_tmpe_fp / \c_foo_cm_fp)}y
      =fp_eval:n {round (\l_foo_tmpf_fp / (\c_foo_cm_fp * \c_foo_cm_fp))}
    }
23
    \ExplSyntax0ff
    \path (1,0) node[below=3mm,align=center]
      {Equation of perpendicular line:\\\resultequation};
27 \end{tikzpicture}
```

(0.5, 4)

(-1,0) (3,1)

Equation of perpendicular line: 4x + 1y = 6

\trt_lt_get_perpendicular_coordinate:nnnNN

 $\trt_lt_get_perpendicular_coordinate:nnnNN{\langle coor 1 \rangle}{\langle coor 2 \rangle}{\langle coor 3 \rangle}{\langle x \rangle}{\langle y \rangle}$

Find the dimensions of the foot of perpendicular from $\langle coor 1 \rangle$ to the line joining $\langle coor 2 \rangle$ and $\langle coor 3 \rangle$. Afterwards store the dimensions found in $\langle x \rangle$ and $\langle y \rangle$.

This is the base of foot of perpendicular.

3.3.2 The barycentric coordinate system

initialize_barycentric

/tikz/triangletools/initialize barycentric=($\langle coor\ 1 \rangle$)($\langle coor\ 2 \rangle$)($\langle coor\ 3 \rangle$)

Use the three coordinates as "anchors" of the barycentric coordinate system.

```
bc3 (bc3 cs:\langle l1 \rangle,\langle l2 \rangle,\langle l3 \rangle)
```

Using the barycentric coordinate system. Note that the system needs to be initialized in advance using initialize barycentric, and an error message will be reported if you do otherwise.

The sum of $\langle l1 \rangle$, $\langle l2 \rangle$ and $\langle l3 \rangle$ is not necessarily 1 – the package will take care of that internally.

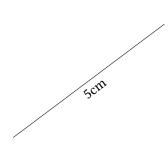
```
P_b
P_c
P_a
```

3.3.3 Distance-finding utility

\trt_distance:nnN

```
\trt_distance:nnN {\langle coor 1 \rangle} {\langle coor 2 \rangle} {\langle fp \ var \rangle}
```

Find distance between $\langle coor 1 \rangle$ and $\langle coor 2 \rangle$, and store that value to $\langle fp \ var \rangle$.



```
1 \begin {tikzpicture}
2 \path (0,0) coordinate (a) (4,3) coordinate (b);
3 \ExplSyntaxOn
4 \fp_new:N \l_foo_tmpa_fp
5 \trt_distance:nnN {a} {b} \l_foo_tmpa_fp
6 \fp_new:N \c_foo_cm_fp
7 \fp_set:Nn \c_foo_cm_fp {28.45275590551181}
8 \draw (a) -- (b) node[midway,sloped,below]
9 { \fp_eval:n {round(\l_foo_tmpa_fp / \c_foo_cm_fp)} cm };
10 \ExplSyntaxOff
11 \end{tikzpicture}
```

 $\verb|\trt_distance_triangle:nnnNNN||$

 $\label{eq:coor_2} $$ \operatorname{distance_triangle:nnnNNN} {\langle coor\ 1 \rangle} {\langle coor\ 2 \rangle} {\langle coor\ 3 \rangle} {\langle a \rangle \langle b \rangle \langle c \rangle} $$$

\trt_distance:nnN is needed to find the side lengths in a triangle (these side lengths are very helpful in many areas, for instance in this package to find special points based on the barycentric system). However, using it three times in a row is not quite elegant; this function is defined to automate that process.

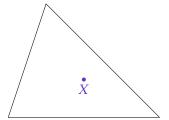
 $\langle a \rangle$ is set to the distance between $\langle coor 2 \rangle$ and $\langle coor 3 \rangle$, similar things happen for $\langle b \rangle$ and $\langle c \rangle$.

3.4 Customization

output_name

/tikz/triangletools/output name= $\langle name \rangle$

This key can be used to change the name of the returned coordinates. The initial value of this key is trt output.



```
1 \begin {tikzpicture}[trt={output name=hello world}]
   \draw (1,3) coordinate (a) --
         (0,0) coordinate (b) --
          (4,0) coordinate (c) -- cycle;
   \fill[maincolor,trt={circumcenter=(a)(b)(c)}] (hello world)
     circle[radius=1.5pt] node[below] {$X$};
7 \end{tikzpicture}
```

Implementation

```
1 (@@=trt)
```

4.1 The main package file

```
2 (*triangletools)
3 \RequirePackage{tikz}
4 \RequirePackage{xparse}
5 \ProvidesExplPackage {triangletools} {2020/04/30} {0.1}
   {TikZ support for triangular geometry}
```

\trt@tmp@ii dinate.

\trt@tmp@i We will use these dimensions many times to extract the dimensions of a TikZ coor-

```
7 \newdimen\trt@tmp@i
8 \newdimen\trt@tmp@ii
```

(End definition for \trt@tmp@i and \trt@tmp@ii. These functions are documented on page ??.)

Let's load the necessary subpackage files.

```
9 \input {trtmessages.code.tex}
10 \input {trtlinetools.code.tex}
in \input {trtbarycentric.code.tex}
12 \input {trtdistance.code.tex}
13 \input {trtspecialpoints.code.tex}
14 \input {trtfrontend.code.tex}
15 (/triangletools)
```

4.2 Errors and warnings

```
16 (*messages)
17 \ProvidesExplFile {trtmessages.code.tex} {2020/04/30} {0.1}
    {The ~ triangletools ~ package: ~ Messages}
```

We also need to declare some helpful messages that we will use later on.

In trtlinetools.code.tex, when we find the intersection of two lines, a warning will be shown if the lines are parallel. The warning is based on intersection-not-found.

```
19 \msg_new:nnnn {triangletools} {intersection-not-found}
    {
20
       Intersection \sim not \sim found.
21
22
    }
23
       You ~ told ~ me ~ to ~ find ~ the ~ intersection ~ of ~ the ~ line ~
24
       joining \sim #1 \sim and \sim #2 \sim and \sim the \sim line \sim joining \sim #3 \sim and \sim #4, \sim
       however ~ these ~ lines ~ are ~ parallel ~ so ~ I ~ can't ~ find ~ any ~
       intersection. ~ The ~ return ~ point ~ is ~ set ~ to ~ the ~ origin ~
27
       (0, \sim 0).
28
    }
```

When the barycentric coordinate system, implemented in trtbarycentric.code. tex, is used, it should already be initialized, i.e. we should already know what are the three "anchor" coordinates of the system. If the coordinate system is not yet initialized, this error will be shown.

We do let the user to find triangle center X_i for any i. However this package obviously can't implement all points in ETC (in fact, I will implement only some most important points). An error will be raised if the user tries to use an unimplemented point.

4.3 The backend layer

4.3.1 The line tools utility

```
49 (*linetools)
50 \ProvidesExplFile {trtlinetools.code.tex} {2020/04/30} {0.1}
51 {The ~ triangletools ~ package: ~ Utilities ~ for ~ lines}
```

In trtlinetools.code.tex, we will implement the necessary functions to handle lines in a mathematical way.

Firstly, let's declare some internal variables that we will use later.

(End definition for $\l_{-trt_lt_linei_a_fp}$ and others.)

```
63 \fp_new:N \l__trt_lt_tmpa_fp
64 \fp_new:N \l__trt_lt_tmpb_fp

(End definition for \l__trt_lt_tmp_fp, \l__trt_lt_tmpa_fp, and \l__trt_lt_tmpb_fp.)
```

\trt_lt_get_line_equation:nnNNN

Find the equation of the line passing #1 and #2, and store the values of a, b, c found to #3, #4 and #5, which are floating point variables, respectively.

```
65 \cs_new:Npn \trt_lt_get_line_equation:nnNNN #1 #2 #3 #4 #5
66 {
67    \pgfextractx {\trt@tmp@i} {\pgfpointanchor{#1}{center}}
68    \pgfextracty {\trt@tmp@ii} {\pgfpointanchor{#1}{center}}
69    \fp_set:Nn \l__trt_i_pointi_x_fp {\trt@tmp@i}
70    \fp_set:Nn \l__trt_i_pointi_y_fp {\trt@tmp@ii}
71    \pgfextractx {\trt@tmp@ii} {\pgfpointanchor{#2}{center}}
72    \pgfextracty {\trt@tmp@ii} {\pgfpointanchor{#2}{center}}
73    \fp_set:Nn \l__trt_i_pointii_x_fp {\trt@tmp@i}
74    \fp_set:Nn \l__trt_i_pointii_y_fp {\trt@tmp@i}
75    \fp_set:Nn \l__trt_i_pointii_y_fp {\trt@tmp@ii}
76    \fp_set:Nn \l__trt_i_pointii_y_fp {\trt@tmp@ii}
```

There is a simple hack here. We have $ax_1 + by_1 = c = ax_2 + by_2$, which is equivalent to $a(x_1 - x_2) = b(y_2 - y_1)$. Therefore $a = y_2 - y_1$ and $b = x_1 - x_2$ can be used.

```
\fp_set:Nn #3
75
76
77
           \l__trt_i_pointii_y_fp - \l__trt_i_pointi_y_fp
78
         }
79
      \fp_set:Nn #4
80
           \\l__trt_i_pointi_x_fp - \\l__trt_i_pointii_x_fp
81
      \fp_set:Nn #5
83
84
           #3 * \l__trt_i_pointi_x_fp + #4 * \l__trt_i_pointi_y_fp
86
87
    }
```

(End definition for \trt_lt_get_line_equation: nnNNN. This function is documented on page 5.)

 $\verb|\trt_lt_get_intersection_line:NNNNNNN||$

Find the intersection of two lines with given equation, after that store the intersection coordinate to floating point variables #7 and #8.

```
88 \cs_new:Npn \trt_lt_get_intersection_line:NNNNNNNN #1 #2 #3 #4 #5 #6 #7 #8
89 {
90 \fp_set:Nn \l__trt_lt_tmp_fp { #1 * #5 - #4 * #2 }
```

If $\l_{\text{trt_lt_tmp_fp}}$ is zero, the two lines are parallel. In that case, we will issue a warning, and set the intersection coordinate to (0,0). Otherwise, continue computing as usual.

```
fp_compare:nNnTF {\l_t_tmp_fp} = {0}
92
          \msg_warning:nnnnnn {triangletools} {intersection-not-found}
93
             {(#1)} {(#2)} {(#3)} {(#4)}
          \fp_set:Nn #7 {0}
          \fp_set:Nn #8 {0}
96
97
        }
98
          \fp_set:Nn #7 { ( #5 * #3 - #2 * #6 ) / \l__trt_lt_tmp_fp }
          \fp_set:Nn #8 { ( #1 * #6 - #4 * #3 ) / \l__trt_lt_tmp_fp }
100
        }
101
```

(End definition for \trt_lt_get_intersection_line: NNNNNNNN. This function is documented on page 6.)

finds the intersection of two lines between #1, #2 and #3, #4, and store the coordinates to #5 and #6. We still use floating point variables here, as they might be useful in the future.

```
103 \cs_new:Npn \trt_lt_get_intersection_coordinate:nnnnNN #1 #2 #3 #4 #5 #6
       \trt_lt_get_line_equation:nnNNN {#1} {#2}
105
         \\__trt_\lt_\linei_a_fp \\\__trt_\lt_\linei_b_fp \\\__trt_\lt_\linei_c_fp
106
       \trt_lt_get_line_equation:nnNNN {#3} {#4}
         \\__trt_\lt_\lineii_a_fp \\\_trt_\lt_\lineii_b_fp \\\\_trt_\lt_\lineii_c_fp
108
       \trt_lt_get_intersection_line:NNNNNNNN
109
         \\__trt_\lt_\linei_a_fp \\\__trt_\lt_\linei_b_fp \\\\_trt_\lt_\linei_c_fp
110
         \l__trt_lt_lineii_a_fp \l__trt_lt_lineii_b_fp \l__trt_lt_lineii_c_fp
111
     }
```

(End definition for \trt_lt_get_intersection_coordinate:nnnnNN. This function is documented on page

__trt_lt_return_intersection:nnnnn

Now, let's TikZify the above function! Note that I use overlay because I don't want to affect the bounding box. The user can use the returned coordinate to change the bounding box in whatever way he wants to.

```
114 \cs_new:Npn \__trt_lt_return_intersection:nnnnn #1 #2 #3 #4 #5
       \trt_lt_get_intersection_coordinate:nnnnNN {#1} {#2} {#3} {#4}
116
         \\\__trt_\t_tmpa_fp \\\\_trt_\t_tmpb_fp
       \coordinate[overlay] (#5) at
118
         (\fp_to_dim:N \l__trt_lt_tmpa_fp, \fp_to_dim:N \l__trt_lt_tmpb_fp);
119
120
    }
```

(End definition for __trt_lt_return_intersection:nnnnn.)

Next, let's make some implementation regarding perpendicularity.

\trt_lt_get_perpendicular_equation:nNNNNNN

This function finds the equation of the line passing point and being perpendicular to a line having a given equation. The task is not quite complicated: note that lines ax + by = c and ay - bx = d are perpendicular.

```
121 \cs_new:Npn \trt_lt_get_perpendicular_equation:nNNNNNN #1 #2 #3 #4 #5 #6 #7
      \fp_set:Nn #5 { -#3 }
      \fp_set:Nn #6 { #2 }
124
      \pgfextractx {\trt@tmp@i} {\pgfpointanchor{#1}{center}}
      \pgfextracty {\trt@tmp@ii} {\pgfpointanchor{#1}{center}}
126
      \fp_set:Nn \l__trt_lt_tmpa_fp {\trt@tmp@i}
      \fp_set:Nn \l__trt_lt_tmpb_fp {\trt@tmp@ii}
128
      \fp_set:Nn #7 { #5 * \l__trt_lt_tmpa_fp + #6 * \l__trt_lt_tmpb_fp }
130
```

(End definition for \trt_lt_get_perpendicular_equation: nNNNNNN. This function is documented on page

\trt_lt_get_perpendicular_coordinate:nnnNN

The base implemented, let's find the foot of perpendicular from a point to a segment.

```
isi \cs_new:Npn \trt_lt_get_perpendicular_coordinate:nnnNN #1 #2 #3 #4 #5
      \trt_lt_get_line_equation:nnNNN {#2} {#3}
        \l__trt_lt_linei_a_fp \l__trt_lt_linei_b_fp \l__trt_lt_linei_c_fp
134
135
      \trt_lt_get_perpendicular_equation:nNNNNNN {#1}
136
        \l__trt_lt_linei_a_fp \l__trt_lt_linei_b_fp \l__trt_lt_linei_c_fp
        \l__trt_lt_lineii_a_fp \l__trt_lt_lineii_b_fp \l__trt_lt_lineii_c_fp
138
```

```
139
140  \trt_lt_get_intersection_line:NNNNNNNN
141  \\\__trt_lt_linei_a_fp \\\__trt_lt_linei_b_fp \\\_trt_lt_lineii_c_fp
142  \\\_trt_lt_lineii_a_fp \\\_trt_lt_lineii_b_fp \\\_trt_lt_lineii_c_fp
143  #4 #5
144  }
```

(End definition for \trt_lt_get_perpendicular_coordinate:nnnNN. This function is documented on page 7.)

rt_lt_return_perpendicular_coordinate:nnnn

This is just a wrapper of \trt_lt_get_perpendicular_coordinate:nnnNN.

```
145 \cs_new:Npn \__trt_lt_return_perpendicular_coordinate:nnnn #1 #2 #3 #4
146 {
147 \trt_lt_get_perpendicular_coordinate:nnnNN {#1} {#2} {#3}
148 \l__trt_lt_tmpa_fp \l__trt_lt_tmpb_fp
149 \coordinate[overlay] (#4) at
150 \(\fp_to_dim:N \l__trt_lt_tmpa_fp, \fp_to_dim:N \l__trt_lt_tmpb_fp);
151 \}
152 \(\frac{\linetools}\)
```

 $(\textit{End definition for \setminus_-trt_lt_return_perpendicular_coordinate:nnnn.})$

4.3.2 The barycentric coordinate system utility

```
153 (*barycentric)
154 \ProvidesExplFile {trtbarycentric.code.tex} {2020/04/30} {0.1}
155 {
156    The ~ triangletools ~ package: ~ Utilities ~ for ~ the ~ barycentric ~
157    coordinate ~ system.
158 }
```

In trtbarycentric.code.tex, we will implement the three-point barycentric coordinate system, which is essential in constructing many special points in a triangle.

(End definition for $\l_{-trt_bc_anchor_ix_fp}$ and others.)

\l__trt_bc_lambda_ii_fp
\l__trt_bc_lambda_iii_fp
\l__trt_bc_lambda_iii_fp

We use these variables to store the user input coordinate. Note that our system is a three-point one, hence exactly three number is required.

Why lambda λ ? Well, I don't know. Wikipedia uses that, so I do the same.

```
165 \fp_new:N \l__trt_bc_lambda_i_fp
166 \fp_new:N \l__trt_bc_lambda_ii_fp
167 \fp_new:N \l__trt_bc_lambda_iii_fp
(End definition for \l__trt_bc_lambda_i_fp, \l__trt_bc_lambda_ii_fp, and \l__trt_bc_lambda_iii_-
```

\l__trt_bc_initialized_bool

We need to guard against using the system before initializing. This boolean variable does that job: if it is set to false (default), do nothing.

```
168 \bool_new:N \l__trt_bc_initialized_bool
(End definition for \l__trt_bc_initialized_bool.)
```

```
\l__trt_bc_tmp_fp A temporary variable.
```

```
169 \fp_new:N \l__trt_bc_tmp_fp
(End definition for \l__trt_bc_tmp_fp.)
```

__trt_bc_initialize:nnn

Initialize the barycentric coordinate system. This is the only place where \l__trt_-bc_initialized_bool can be set to true, so this function must be executed before everything else in this file.

```
\cs_new:Npn \__trt_bc_initialize:nnn #1 #2 #3
171
      \bool_set_true:N \l__trt_bc_initialized_bool
      \pgfextractx {\trt@tmp@i} {\pgfpointanchor{#1}{center}}
173
174
       \pgfextracty {\trt@tmp@ii} {\pgfpointanchor{#1}{center}}
       \fp_set:Nn \l__trt_bc_anchor_ix_fp {\trt@tmp@i}
       \fp_set:Nn \l__trt_bc_anchor_iy_fp {\trt@tmp@ii}
       \pgfextractx {\trt@tmp@i} {\pgfpointanchor{#2}{center}}
       \pgfextracty {\trt@tmp@ii} {\pgfpointanchor{#2}{center}}
      \fp_set:Nn \l__trt_bc_anchor_iix_fp {\trt@tmp@i}
      \fp_set:Nn \l__trt_bc_anchor_iiy_fp {\trt@tmp@ii}
180
      \pgfextractx {\trt@tmp@i} {\pgfpointanchor{#3}{center}}
      \pgfextracty {\trt@tmp@ii} {\pgfpointanchor{#3}{center}}
182
      \fp_set:Nn \l__trt_bc_anchor_iiix_fp {\trt@tmp@i}
183
      \fp_set:Nn \l__trt_bc_anchor_iiiy_fp {\trt@tmp@ii}
184
```

(End definition for __trt_bc_initialize:nnn.)

bc3 The bc3 coordinate system implementation. We will guard against using it when __trt_bc_initialize:nnn is not yet executed – in that case, uninitialized error will be raised.

We will receive arguments of bc3 as #1,#2,#3, so a simple parser is needed. All interesting things will be done with that parser.

(End definition for bc3. This function is documented on page 7.)

__trt_bc_parse:w

This is the parser we use for bc3.

The conversion from λ_i to the Cartesian format is pretty simple, we have $x = \lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3$ and the formula for y is similar. However, first we have to change the value of λ_i so that $\lambda_1 + \lambda_2 + \lambda_3 = 1$.

```
196 \cs_new:Npn \__trt_bc_parse:w #1,#2,#3 \q_stop
197 {
198    \fp_set:Nn \l__trt_bc_tmp_fp { (#1) + (#2) + (#3) }
199    \fp_set:Nn \l__trt_bc_lambda_i_fp { (#1) / (\l__trt_bc_tmp_fp) }
200    \fp_set:Nn \l__trt_bc_lambda_ii_fp { (#2) / (\l__trt_bc_tmp_fp) }
201    \fp_set:Nn \l__trt_bc_lambda_iii_fp { (#3) / (\l__trt_bc_tmp_fp) }
202    \fp_set:Nn \l__trt_tmp_a_fp
203     {
204    \l__trt_bc_anchor_ix_fp * \l__trt_bc_lambda_i_fp +
```

Floating point variables are not TEX dimensions, hence \fp_to_dim:N is used.

```
\pgf@x = \fp_to_dim:N \l__trt_tmp_a_fp
\pgf@y = \fp_to_dim:N \l__trt_tmp_b_fp

\delta \
\frac{barycentric}
```

(End definition for __trt_bc_parse:w.)

4.3.3 Distance-finding utility

```
218 (*distance)
219 \ProvidesExplFile {trtdistance.code.tex} {2020/04/30} {0.1}
220 {The ~ triangletools ~ package: ~ Utilities ~ for ~ 2d ~ distance}
```

This file implements functions to find the distance between (2d) TikZ coordinates.

\l__trt_d_pointi_x_fp
\l__trt_d_pointi_y_fp
\l__trt_d_pointii_x_fp
\l__trt_d_pointii_y_fp

These variables are used to store the coordinates of the points between which we are finding the distance.

```
221 \fp_new:N \l__trt_d_pointi_x_fp
222 \fp_new:N \l__trt_d_pointii_x_fp
223 \fp_new:N \l__trt_d_pointii_y_fp
224 \fp_new:N \l__trt_d_pointii_y_fp
```

(End definition for \l_trt_d_pointi_x_fp and others.)

\trt_distance:nnN Find the distance between TikZ coordinates #1 and #2.

```
225 \cs_new:Npn \trt_distance:nnN #1 #2 #3
226
       \pgfextractx {\trt@tmp@i} {\pgfpointanchor{#1}{center}}
       \pgfextracty {\trt@tmp@ii} {\pgfpointanchor{#1}{center}}
228
       \fp_set:Nn \l__trt_d_pointi_x_fp {\trt@tmp@i}
       \fp_set:Nn \l__trt_d_pointi_y_fp {\trt@tmp@ii}
230
       \pgfextractx {\trt@tmp@i} {\pgfpointanchor{#2}{center}}
       \pgfextracty {\trt@tmp@ii} {\pgfpointanchor{#2}{center}}
       \fp_set:Nn \l__trt_d_pointii_x_fp {\trt@tmp@i}
       \fp_set:Nn \l__trt_d_pointii_y_fp {\trt@tmp@ii}
234
       \fp_set:Nn #3
           sqrt((
             (\\l_trt_d_pointi_x_fp - \\l_trt_d_pointii_x_fp) *
238
             (\\l_trt_d_pointi_x_fp - \\l_trt_d_pointii_x_fp)
239
             (\\l_trt_d_pointi_y_fp - \\l_trt_d_pointii_y_fp) *
241
             (\l__trt_d_pointi_y_fp - \l__trt_d_pointii_y_fp)
242
243
           ))
     }
245
```

(End definition for \trt_distance:nnN. This function is documented on page 8.)

\trt_distance_triangle:nnnNNN

We mainly need the above function to find the side length in a triangle. Let's create a function that do so automatically.

```
246 \cs_new:Npn \trt_distance_triangle:nnnNNN #1 #2 #3 #4 #5 #6
247 {
248  \trt_distance:nnN {#2} {#3} #4
249  \trt_distance:nnN {#3} {#1} #5
250  \trt_distance:nnN {#1} {#2} #6
251  }
252 \( \frac{distance}{} \)
```

(End definition for \trt_distance_triangle:nnnNNN. This function is documented on page 8.)

4.4 Construction of triangle centers

```
253 (*specialpoints)
254 \ProvidesExplFile {trtspecialpoints.code.tex} {2020/04/30} {0.1}
255 {The ~ triangletools ~ package: ~ Triangle ~ center ~ construction}
```

This file will use the utility implemented in the above sections to find some most important triangle centers described in the ETC.

```
These variables may also be helpful for triangle centers for which a simple formula
 \l__trt_sp_coordinatei_x_fp
                             doesn't exist, e.g. the circumcenter.
 \l__trt_sp_coordinatei_y_fp
\l__trt_sp_coordinateii_x_fp
                             259 \fp_new:N \l__trt_sp_coordinatei_x_fp
\l__trt_sp_coordinateii_y_fp
                             260 \fp_new:N \l__trt_sp_coordinatei_y_fp
\l__trt_sp_coordinateiii_x_fp
                             261 \fp_new:N \l__trt_sp_coordinateii_x_fp
\l__trt_sp_coordinateiii_y_fp 262 \fp_new:N \l__trt_sp_coordinateii_y_fp
       \l__trt_sp_linei_a_fp 263 \fp_new:N \l__trt_sp_coordinateiii_x_fp
       \l__trt_sp_linei_b_fp 264 \fp_new:N \l__trt_sp_coordinateiii_y_fp
       266 \fp_new:N \l__trt_sp_linei_b_fp
      \l__trt_sp_lineii_a_fp
                             267 \fp_new:N \l__trt_sp_linei_c_fp
      \l__trt_sp_lineii_b_fp
                             268 \fp_new:N \l__trt_sp_lineii_a_fp
      \l__trt_sp_lineii_c_fp
                             269 \fp_new:N \l__trt_sp_lineii_b_fp
                             ^{270}\ \fp_new:N\ \l_-trt_sp_lineii_c_fp
```

 $(\textit{End definition for $\l_-trt_sp_coordinatei_x_fp$ and others.})$

```
\l__trt_sp_tmpa_fp
\l__trt_sp_tmpb_fp
\l__trt_sp_tmpc_fp
```

Some additional temporary variables.

```
271 \fp_new:N \l__trt_sp_tmpa_fp
272 \fp_new:N \l__trt_sp_tmpb_fp
273 \fp_new:N \l__trt_sp_tmpc_fp
```

(End definition for $\l_-trt_-sp_-tmpa_-fp$, $\l_-trt_-sp_-tmpb_-fp$, and $\l_-trt_-sp_-tmpc_-fp$.)

4.4.1 X_1 - The incenter

Each center will have a function taking four arguments. The first three arguments are the TikZ coordinates of the triangle vertices; the last argument is the name of the return TikZ coordinate.

To prevent conflict between these sister functions when they are used together, I put each of them inside a TEX group.

\trt_sp_incenter:nnnn Return the incenter. It is based on the barycentric coordinate of the incenter, (a, b, c).

```
274 \cs_new:Npn \trt_sp_incenter:nnnn #1 #2 #3 #4
276
       \group_begin:
         \__trt_bc_initialize:nnn {#1} {#2} {#3}
         \trt_distance_triangle:nnnNNN {#1} {#2} {#3}
278
           \\\__trt_sp_a_fp \\\__trt_sp_b_fp \\\__trt_sp_c_fp
279
         \path[overlay] (bc3 ~ cs \c_colon_str
           \fp_eval:n {\l__trt_sp_a_fp},
           \fp_eval:n {\l__trt_sp_b_fp},
282
           \fp_eval:n {\l__trt_sp_c_fp}) coordinate (#4);
283
       \group_end:
285
```

(End definition for \trt_sp_incenter:nnnn. This function is documented on page 2.)

\trt_sp_excenter:nnnn

Return the excenter of the triangle, with respect to vertex #1. This center is just a derivation of the incenter; also it is not unique, so it is not assigned a number. Barycentric coordinate of the excenter is (-a, b, c), where a is the length of the side joining #2 and #3.

Note that this is the only function in this series in which argument order is important.

```
\cs_new:Npn \trt_sp_excenter:nnnn #1 #2 #3 #4
      \group_begin:
288
         \__trt_bc_initialize:nnn {#1} {#2} {#3}
289
         \trt_distance_triangle:nnnNNN {#1} {#2} {#3}
           \\\__trt_sp_a_fp \\\\_trt_sp_b_fp \\\\\_trt_sp_c_fp
291
         \path[overlay] (bc3 ~ cs \c_colon_str
292
           \fp_eval:n {- \l__trt_sp_a_fp},
           fp_eval:n {\l_trt_sp_b_fp},
           \fp_eval:n {\l__trt_sp_c_fp}) coordinate (#4);
295
       \group_end:
296
297
    }
```

(End definition for \trt_sp_excenter:nnnn. This function is documented on page 2.)

4.4.2 X_2 - The centroid

\trt_sp_centroid:nnnn This is perhaps the simplest of all. Barycentric coordinate: (1, 1, 1).

(End definition for \trt_sp_centroid:nnnn. This function is documented on page 3.)

4.4.3 X_3 – The circumcenter

\trt_sp_circumcenter:nnnn

This is opposite to X_2 : perhaps this is the most complex of all. The barycentric coordinate formula is not simple enough for me, so I construct this point purely manually: find the intersection of the perpendicular bisectors.

```
305 \cs_new:Npn \trt_sp_circumcenter:nnnn #1 #2 #3 #4
306 {
307 \group_begin:
```

Firstly, let's store the coordinate of the vertices.

```
\pgfextractx {\trt@tmp@i} {\pgfpointanchor{#1}{center}}
308
        \pgfextracty {\trt@tmp@ii} {\pgfpointanchor{#1}{center}}
309
        \fp_set:Nn \l__trt_sp_coordinatei_x_fp {\trt@tmp@i}
        \fp_set:Nn \l__trt_sp_coordinatei_y_fp {\trt@tmp@ii}
        \pgfextractx {\trt@tmp@i} {\pgfpointanchor{#2}{center}}
        \pgfextracty {\trt@tmp@ii} {\pgfpointanchor{#2}{center}}
        \fp_set:Nn \l__trt_sp_coordinateii_x_fp {\trt@tmp@i}
314
        \fp_set:Nn \l__trt_sp_coordinateii_y_fp {\trt@tmp@ii}
        \pgfextractx {\trt@tmp@i} {\pgfpointanchor{#3}{center}}
316
        \pgfextracty {\trt@tmp@ii} {\pgfpointanchor{#3}{center}}
317
        \fp_set:Nn \l__trt_sp_coordinateiii_x_fp {\trt@tmp@i}
318
319
        \fp_set:Nn \l__trt_sp_coordinateiii_y_fp {\trt@tmp@ii}
```

Now, let's change point #2 to the midpoint between #1 and #2, and do the same for #3.

```
\fp_set:Nn \l__trt_sp_coordinateii_x_fp
          {
322
            (\l__trt_sp_coordinatei_x_fp + \l__trt_sp_coordinateii_x_fp) / 2
          }
323
        \fp_set:Nn \l__trt_sp_coordinateii_y_fp
324
            (\l__trt_sp_coordinatei_y_fp + \l__trt_sp_coordinateii_y_fp) / 2
          }
327
        \coordinate[overlay] (trt@tmp@ii) at (
328
          \fp_to_dim:N \l__trt_sp_coordinateii_x_fp,
          \fp_to_dim:N \l__trt_sp_coordinateii_y_fp);
        {
            (\l__trt_sp_coordinatei_x_fp + \l__trt_sp_coordinateiii_x_fp) / 2
334
        \fp_set:Nn \l__trt_sp_coordinateiii_y_fp
336
          {
            (\l__trt_sp_coordinatei_y_fp + \l__trt_sp_coordinateiii_y_fp) / 2
          }
        \coordinate[overlay] (trt@tmp@iii) at (
339
          \fp_to_dim:N \l__trt_sp_coordinateiii_x_fp,
          \fp_to_dim:N \l__trt_sp_coordinateiii_y_fp);
```

All we have to do now is to find the equations of the bisectors and their intersection.

```
\trt_lt_get_line_equation:nnNNN {#1} {#2}
342
           \l__trt_sp_tmpa_fp \l__trt_sp_tmpb_fp \l__trt_sp_tmpc_fp
343
         \trt_lt_get_perpendicular_equation:nNNNNNN {trt@tmp@ii}
344
           \\l__trt_sp_tmpa_fp \\l__trt_sp_tmpb_fp \\l__trt_sp_tmpc_fp
           \l__trt_sp_linei_a_fp \l__trt_sp_linei_b_fp \l__trt_sp_linei_c_fp
346
         \label{line_equation:nnNNN} $$ \{\#1\} $$ \{\#3\} $$
347
           \verb|\l_trt_sp_tmpa_fp \l_trt_sp_tmpb_fp \l_trt_sp_tmpc_fp|
         \trt_lt_get_perpendicular_equation:nNNNNNN {trt@tmp@iii}
349
           \l__trt_sp_tmpa_fp \l__trt_sp_tmpb_fp \l__trt_sp_tmpc_fp
350
351
           \l__trt_sp_lineii_a_fp \l__trt_sp_lineii_b_fp \l__trt_sp_lineii_c_fp
         \trt_lt_get_intersection_line:NNNNNNNN
352
           \l__trt_sp_linei_a_fp \l__trt_sp_linei_b_fp \l__trt_sp_linei_c_fp
353
           \l__trt_sp_lineii_a_fp \l__trt_sp_lineii_b_fp \l__trt_sp_lineii_c_fp
354
           \l__trt_sp_tmpa_fp \l__trt_sp_tmpb_fp
355
         \coordinate[overlay] (#4) at (
           \fp_to_dim:N \l__trt_sp_tmpa_fp, \fp_to_dim:N \l__trt_sp_tmpb_fp);
357
       \group_end:
358
```

Quite surprisingly, the function is still very fast after all this. On my machine it never exceeds 10ms in execution time.

(End definition for \trt_sp_circumcenter:nnnn. This function is documented on page 3.)

4.4.4 X_4 – The orthocenter

\trt_sp_orthocenter:nnnn

Return the orthocenter of the triangle. This point is also constructed manually instead of using a proved formula. However, the utilities help making the construction look very simple.

```
360 \cs_new:Npn \trt_sp_orthocenter:nnnn #1 #2 #3 #4
361 {
362  \group_begin:
363  \__trt_lt_return_perpendicular_coordinate:nnnn {#1} {#2} {#3} {trt@tmp@i}
364  \__trt_lt_return_perpendicular_coordinate:nnnn {#2} {#1} {#3} {trt@tmp@ii}
365  \__trt_lt_return_intersection:nnnnn
366  {#1} {trt@tmp@i} {#2} {trt@tmp@ii} {#4}
367  \group_end:
368 }
```

(End definition for \trt_sp_orthocenter:nnnn. This function is documented on page 3.)

4.4.5 X_5 – The nine-point center

\trt_sp_ninepointcenter:nnnn

Return the center of the nine-point circle.

```
369 \cs_new:Npn \trt_sp_ninepointcenter:nnnn #1 #2 #3 #4
370 {
371 \group_begin:
```

 X_5 is is the midpoint of X_3 and X_4 . Therefore, for simplicity, X_3 and X_4 are constructed first. This causes some run-time overhead, however the overall execution time is still below 15ms, which is, in my opinion, still good.

Note that we already used trt@tmp@i and trt@tmp@ii coordinates in the construction of X_3 and X_4 , so to prevent conflict, trt@tmp@ii and trt@tmp@iv are used.

```
\trt_sp_circumcenter:nnnn {#1} {#2} {#3} {trt@tmp@iii}
        \trt_sp_orthocenter:nnnn {#1} {#2} {#3} {trt@tmp@iv}
        \pgfextractx {\trt@tmp@i} {\pgfpointanchor{trt@tmp@iii}{center}}
        \pgfextracty {\trt@tmp@ii} {\pgfpointanchor{trt@tmp@iii}{center}}
        \fp_set:Nn \l__trt_sp_tmpa_fp {\trt@tmp@i}
377
        \fp_set:Nn \l__trt_sp_tmpb_fp {\trt@tmp@ii}
        \pgfextractx {\trt@tmp@i} {\pgfpointanchor{trt@tmp@iv}{center}}
378
        \pgfextracty {\trt@tmp@ii} {\pgfpointanchor{trt@tmp@iv}{center}}
        \fp_set:Nn \l__trt_sp_tmpa_fp { (\trt@tmp@i + \l__trt_sp_tmpa_fp) / 2 }
380
        381
        \coordinate[overlay] (#4) at (\fp_to_dim:N \l__trt_sp_tmpa_fp,
383
         \fp_to_dim:N \l__trt_sp_tmpb_fp);
      \group_end:
384
385
    }
```

(End definition for \trt_sp_ninepointcenter:nnnn. This function is documented on page 3.)

4.4.6 X_6 – The symmedian point

\trt_sp_symmedian:nnnn

Return the symmedian point (aka. the Lemoine point or Grebe point). The barycentric coordinate of the point is (a^2 , b^2 , c^2).

(End definition for \trt_sp_symmedian:nnnn. This function is documented on page 4.)

4.4.7 X_7 – The Gergonne point

\trt_sp_gergonne:nnnn Return the Gergonne point of the triangle. The barycentric coordinate of the point is $(\frac{1}{b+c-a}, \frac{1}{c+a-b}, \frac{1}{a+b-c})$.

```
398 \cs_new:Npn \trt_sp_gergonne:nnnn #1 #2 #3 #4
       \group_begin:
400
         \__trt_bc_initialize:nnn {#1} {#2} {#3}
401
         \trt_distance_triangle:nnnNNN {#1} {#2} {#3}
402
           \\\__trt_sp_a_fp \\\__trt_sp_b_fp \\\__trt_sp_c_fp
         \path[overlay] (bc3 ~ cs \c_colon_str
404
           fp_eval:n { 1/(\l_trt_sp_b_fp + \l_trt_sp_c_fp - \l_trt_sp_a_fp) },
405
           \fp_eval:n { 1/(\l__trt_sp_c_fp + \l__trt_sp_a_fp - \l__trt_sp_b_fp) },
           fp_eval:n { 1/(\l_trt_sp_a_fp + \l_trt_sp_b_fp - \l_trt_sp_c_fp) }
         ) coordinate (#4);
408
       \verb|\group_end:|
409
410
    }
```

(End definition for \trt_sp_gergonne:nnnn. This function is documented on page 4.)

4.4.8 X_8 – The Nagel point

\trt_sp_nagel:nnnn Return the Nagel point. The barycentric coordinate of the point is (b+c-a, c+a-b, a+b-c).

```
411 \cs_new:Npn \trt_sp_nagel:nnnn #1 #2 #3 #4
412
413
                                                                     \group_begin:
                                                                                       \__trt_bc_initialize:nnn {#1} {#2} {#3}
414
                                                                                       \trt_distance_triangle:nnnNNN {#1} {#2} {#3}
415
                                                                                                             \l__trt_sp_a_fp \l__trt_sp_b_fp \l__trt_sp_c_fp
                                                                                       \path[overlay] (bc3 ~ cs \c_colon_str
417
                                                                                                            \label{eq:condition} $$ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} ds \ ds = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} ds \ ds = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty
418
419
                                                                                                            \fp_eval:n { \l__trt_sp_c_fp + \l__trt_sp_a_fp - \l__trt_sp_b_fp },
                                                                                                            \fp_eval:n { \l__trt_sp_a_fp + \l__trt_sp_b_fp - \l__trt_sp_c_fp }
                                                                                       ) coordinate (#4);
421
                                                                     \group_end:
422
423
                                               }
```

(End definition for \trt_sp_nagel:nnnn. This function is documented on page 4.)

4.4.9 X_9 – The mittenpunkt

\trt_sp_mittenpunkt:nnnn Return the *mittenpunkt* of the triangle – its barycentric coordinate is $(a \times (b + c - a), b \times (c + a - b), c \times (a + b - c))$.

```
\l_{-trt_sp_a_fp} * (
433
                    \l_{trt\_sp\_b\_fp} + \l_{trt\_sp\_c\_fp} - \l_{trt\_sp\_a\_fp}
434
               },
436
             \fp_eval:n
437
438
               {
                  \l_{\text{trt\_sp\_b\_fp}} * (
439
                    \l_{trt_sp_c_fp} + \l_{trt_sp_a_fp} - \l_{trt_sp_b_fp}
441
               },
             \fp_eval:n
444
               {
                  \l_{-trt_sp_c_fp} * (
445
                    \l_{trt_sp_a_fp} + \l_{trt_sp_b_fp} - \l_{trt_sp_c_fp}
446
                  )
447
               }
448
          ) coordinate (#4);
449
450
        \group_end:
451
```

(End definition for \trt_sp_mittenpunkt:nnnn. This function is documented on page 4.)

4.4.10 X_{10} – The Spieker point

\trt_sp_spieker:nnnn Return the Spieker point. The barycentric coordinate of the point is (b+c, c+a, a+b).

```
452 \cs_new:Npn \trt_sp_spieker:nnnn #1 #2 #3 #4
453
    {
454
       \group_begin:
         \__trt_bc_initialize:nnn {#1} {#2} {#3}
455
         \trt_distance_triangle:nnnNNN {#1} {#2} {#3}
456
           \\\__trt_sp_a_fp \\\__trt_sp_b_fp \\\__trt_sp_c_fp
         \path[overlay] (bc3 ~ cs \c_colon_str
458
           fp_eval:n {  \l_trt_sp_b_fp + \l_trt_sp_c_fp },
459
           fp_eval:n {  \l_trt_sp_c_fp + \l_trt_sp_a_fp },
460
           \fp_eval:n { \l__trt_sp_a_fp + \l__trt_sp_b_fp }
         ) coordinate (#4);
462
       \group_end:
463
    }
464
465 (/specialpoints)
```

(End definition for \trt_sp_spieker:nnnn. This function is documented on page 5.)

4.5 The frontend layer

```
466 (*frontend)
467 \ProvidesExplFile {trtfrontend.code.tex} {2020/04/30} {0.1}
468 {The ~ triangletools ~ package: ~ The ~ front-end ~ layer}
```

The user interface of the package, which consists solely of pgf keys, will be implemented in this file.

\l__trt_fr_output_name_tl Store the name of the output coordinate. Default to trt_output.

```
469 \tl_new:N \l__trt_fr_output_name_tl
470 \tl_set:Nn \l__trt_fr_output_name_tl {trt ~ output}

(End definition for \l_trt_fr_output_name_tl.)
```

 $\label{local_local_local_local_local} $$ l_-trt_fr_center_number_int $$$

We only provide specific key for X_1 , X_2 , X_3 and X_4 . All other points can be referenced using a single generic key. We need to store the index of that point so that we can choose the right function for the point.

```
471 \int_new:N \l__trt_fr_center_number_int
```

```
Now it's time for the keys. They will be stored under /tikz/triangletools and can
                          be accessed at trt=\{\langle keys \rangle\}.
                          472 \tikzset {
                               triangletools/.is ~ family,
                               trt/.code={\pgfkeys{/tikz/triangletools/.cd,#1}},
                               triangletools/.cd,
                          (End definition for /tikz/trt. This function is documented on page 2.)
                         Change the output name of all returned coordinates.
           output name
                               output ~ name/.code={
                                 \tl_set:Nn \l__trt_fr_output_name_tl {#1}
                          477
                          478
                               },
                          (End definition for output name. This function is documented on page 8.)
                         The front-end of the line tools utility.
          intersection
 foot_of_perpendicular
                               intersection/.code \sim args=\{(\#1)(\#2)--(\#3)(\#4)\}\{
                                 \__trt_lt_return_intersection:nnnnn {#1} {#2} {#3} {#4}
                          481
                                   {\tl_use:N \l__trt_fr_output_name_tl}
                               },
                          482
                               foot \sim of \sim perpendicular/.code \sim args={(#1)--(#2)(#3)}{
                          483
                                 \__trt_lt_return_perpendicular_coordinate:nnnn {#1} {#2} {#3}
                                   {\tl_use:N \l__trt_fr_output_name_tl}
                          485
                          486
                               },
                          (End definition for intersection and foot of perpendicular. These functions are documented on page
initialize_barycentric The front-end of the barycentric coordinate system.
                               initialize ~ barycentric/.code ~ args={(#1)(#2)(#3)}{
                                 \__trt_bc_initialize:nnn {#1} {#2} {#3}
                          489
                               },
                          (End definition for initialize barycentric. This function is documented on page 7.)
                         The front-end of the triangle centers X_1 to X_4 and the excenter.
               excenter
                               incenter/.code \sim args=\{(#1)(#2)(#3)\}\{
               centroid
                                 \trt_sp_incenter:nnnn {#1} {#2} {#3}
                          491
          circumcenter
                         492
                                   {\tl_use:N \l__trt_fr_output_name_tl}
            orthocenter
                               excenter/.code ~ args={(#1)(#2)(#3)}{
                          494
                                 \trt_sp_excenter:nnnn {#1} {#2} {#3}
                          495
                                   {\tl_use:N \l__trt_fr_output_name_tl}
                          496
                               centroid/.code ~ args={(#1)(#2)(#3)}{
                          498
                                 \trt_sp_centroid:nnnn {#1} {#2} {#3}
                          499
                                   {\tl_use:N \l__trt_fr_output_name_tl}
                          501
                               circumcenter/.code ~ args={(#1)(#2)(#3)}{
                          502
                                 \trt_sp_circumcenter:nnnn {#1} {#2} {#3}
                          503
                          504
                                   {\tl_use:N \l__trt_fr_output_name_tl}
                          505
                               orthocenter/.code ~ args={(#1)(#2)(#3)}{
                          506
                                 \trt_sp_orthocenter:nnnn {#1} {#2} {#3}
                          507
                                   {\tl_use:N \l__trt_fr_output_name_tl}
```

509

},

triangle_center This key is used to access all centers. I don't give any centers from X_5 a key – this key is necessary to construct them.

```
triangle \sim center/.code \sim args={(#1)(#2)(#3)(#4)}{
       \int_case:nnF {#4}
512
         {
           {1} {
514
             \trt_sp_incenter:nnnn {#1} {#2} {#3}
                {\tl_use:N \l__trt_fr_output_name_tl}
           }
516
           {2} {
             \trt_sp_centroid:nnnn {#1} {#2} {#3}
519
                {\tl_use:N \l__trt_fr_output_name_tl}
           }
520
           {3} {
521
             \trt_sp_circumcenter:nnnn {#1} {#2} {#3}
                {\tl_use:N \l__trt_fr_output_name_tl}
           }
524
           {4} {
             \trt_sp_orthocenter:nnnn {#1} {#2} {#3}
                {\tl_use:N \l__trt_fr_output_name_tl}
528
529
           {5} {
             \trt_sp_ninepointcenter:nnnn {#1} {#2} {#3}
                {\tl_use:N \l__trt_fr_output_name_tl}
           }
           {6} {
             \trt_sp_symmedian:nnnn {#1} {#2} {#3}
                {\tl_use:N \l__trt_fr_output_name_tl}
           }
536
           {7} {
             \trt_sp_gergonne:nnnn {#1} {#2} {#3}
538
539
                {\tl_use:N \l__trt_fr_output_name_tl}
           }
540
           {8} {
             \trt_sp_nagel:nnnn {#1} {#2} {#3}
542
                {\tl_use:N \l__trt_fr_output_name_tl}
543
544
           }
545
           {9} {
             \trt_sp_mittenpunkt:nnnn {#1} {#2} {#3}
                {\tl_use:N \l__trt_fr_output_name_tl}
547
           }
           {10} {
             \trt_sp_spieker:nnnn {#1} {#2} {#3}
550
                {\tl_use:N \l__trt_fr_output_name_tl}
551
           }
         }
554
           \msg_error:nnn {triangletools} {center-not-found} {#4}
         }
556
557
558 }
559 (/frontend)
```

(End definition for triangle center. This function is documented on page 3.)

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