The triangletools package

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Contents

Index

Loading	the package
User int	erface
3.1 Acc	essing the keys
3.2 Cir	cles associated with triangle centers
	angle centers
3.4 Oth	ner utilities
3.4.1	Line tools
3.4.2	The barycentric coordinate system
3.4.3	Distance-finding utility
3.5 Cus	stomization
Implem	entation
4.1 The	e main package file
	ors and warnings
	e backend layer
4.3.1	The line tools utility
4.3.2	The barycentric coordinate system utility
4.3.3	Distance-finding utility
4.4 Cor	nstruction of triangle centers
4.4.1	X_1 – The incenter
4.4.2	X_2 – The centroid
4.4.3	X_3 – The circumcenter
4.4.4	X_4 – The orthocenter
4.4.5	X_5 – The nine-point center
4.4.6	X_6 – The symmedian point
4.4.7	X_7 – The Gergonne point
	X_8 – The Nagel point
4.4.8	V The witten be what
4.4.8 4.4.9	X_9 – The mittenpunkt

25

1 Introduction

This package aims to help you construct special points in a triangle directly in a short and easy way. Using this package, you can construct most important points listed in Clark Kimberling's Encyclopedia of Triangle Centers (ETC). Currently, all points numbered from X_1 and X_{10} , as well as the excenter, are supported; however with other utilities in this package (see Section 3.4) and a bit of knowledge in geometry and expl3 programming, you can construct even more.

2 Loading the package

This package can be loaded as usual.

```
\usepackage{triangletools}
```

It will load TikZ and expl3 automatically.

3 User interface

The user interface of this package, including that of the utilities, is provided as pgf keys under the tree /tikz/triangletools.

Note that, in the following sections, a *coordinate* means a *named* TikZ coordinate. That is, in the following example,

```
1 \begin{tikzpicture}
2 \draw (0,0) -- (3,0) coordinate (a);
3 \end{tikzpicture}
```

a is a named coordinate, while 0.0 or 3.0 are *not* named coordinates. The current implementation of this package only allows named coordinates in the user interface. It is like the angles TikZ library.

3.1 Accessing the keys

/tikz/trt /tikz/trt= $\{\langle keys \rangle\}$

It executes *keys* with the key path set to /tikz/triangletools, which is the main key tree of this package.

This key is used to access all other keys in the user interface.

3.2 Circles associated with triangle centers

\trtradius

Some points, for example the incenter and the circumcenter, are associated with some special circles. If the requested point is associated with a circle, this macro stores the radius of that circle, in points (pt).

This macro is assigned *globally* every time a point is requested. Therefore, it stores the radius related to the last point that has a circle. So beware that while it always gives you some values once you have drawn such points, that value might not be what you want. It is recommended to use this macro *immediately after* the execution of triangle center keys.

In Section 3.3, if a point has a \trtradius associated to it, the circle will be drawn in the code example. Currently X_1 , the excenter, X_3 , X_5 and X_{10} can change the value of \trtradius.

If the macro is used before any center with a circle is constructed, an error message will be issued.

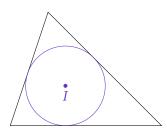
3.3 Triangle centers

incenter
\trt_sp_incenter:nnnn

```
/tikz/triangletools/incenter=(\langle coor\ 1 \rangle)(\langle coor\ 2 \rangle)(\langle coor\ 3 \rangle) \trt_sp_incenter:nnnn {\langle coor\ 1 \rangle}{\langle coor\ 2 \rangle}{\langle coor\ 3 \rangle}{\langle name \rangle}
```

Find the incenter X_1 of the triangle joining TikZ coordinates $\langle coor 1 \rangle$, $\langle coor 2 \rangle$ and $\langle coor 3 \rangle$. The incenter is saved to TikZ coordinate $\langle name \rangle$.

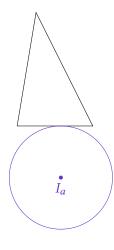
If you use the key (why do you use the function anyway), $\langle name \rangle$ is set to trt output by default. You can change that using output name, see Section 3.5.



excenter
\trt_sp_excenter:nnnn

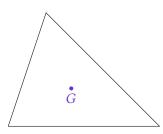
```
/tikz/triangletools/excenter=(\langle coor\ 1 \rangle)(\langle coor\ 2 \rangle)(\langle coor\ 3 \rangle) \trt_sp_excenter:nnnn {\langle coor\ 1 \rangle}{\langle coor\ 2 \rangle}{\langle coor\ 3 \rangle}{\langle name \rangle}
```

Find the excenter of the triangle. The returned point will be on the internal angular bisector at $\langle coor 1 \rangle$. Note that the order matters: excenter=(a)(b)(c) is *different* from excenter=(b)(a)(c).



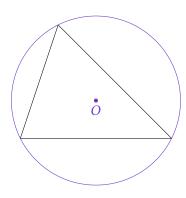
centroid
\trt_sp_centroid:nnnn

/tikz/triangletools/centroid= $(\langle coor\ 1 \rangle)(\langle coor\ 2 \rangle)(\langle coor\ 3 \rangle)$ \trt_sp_centroid:nnnn $\{\langle coor\ 1 \rangle\}\{\langle coor\ 2 \rangle\}\{\langle coor\ 3 \rangle\}\{\langle name \rangle\}$ Find the centroid X_2 of the triangle.



circumcenter
\trt_sp_circumcenter:nnnn

/tikz/triangletools/circumcenter=($\langle coor\ 1 \rangle$)($\langle coor\ 2 \rangle$)($\langle coor\ 3 \rangle$) \trt_sp_circumcenter:nnnn { $\langle coor\ 1 \rangle$ }{ $\langle coor\ 2 \rangle$ }{ $\langle coor\ 3 \rangle$ }{ $\langle name \rangle$ } Find the circumcenter X_3 of the triangle.



```
begin {tikzpicture}

draw (1,3) coordinate (a) --

(0,0) coordinate (b) --

(4,0) coordinate (c) -- cycle;

fill[maincolor,trt={circumcenter=(a)(b)(c)}] (trt output)

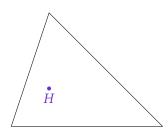
circle[radius=1.5pt] node[below] {$0$};

draw[maincolor] (trt output) circle (\trtradius);

end{tikzpicture}
```

orthocenter
\trt_sp_orthocenter:nnnn

/tikz/triangletools/orthocenter= $(\langle coor\ 1 \rangle)(\langle coor\ 2 \rangle)(\langle coor\ 3 \rangle)$ \trt_sp_orthocenter:nnnn $\{\langle coor\ 1 \rangle\}\{\langle coor\ 2 \rangle\}\{\langle coor\ 3 \rangle\}\{\langle name \rangle\}$ Find the orthocenter X_4 of the triangle.



triangle_center

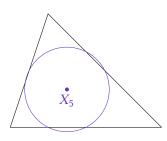
/tikz/triangletools/triangle center= $(\langle coor\ 1 \rangle)(\langle coor\ 2 \rangle)(\langle coor\ 3 \rangle)(\langle index \rangle)$

Find the point $X_{\langle index \rangle}$ of the triangle. Currently $\langle index \rangle$ can be any integer between and including 1 and 10.

\trt_sp_ninepointcenter:nnnn

/tikz/triangletools/triangle center= $(\langle coor\ 1 \rangle)(\langle coor\ 2 \rangle)(\langle coor\ 3 \rangle)$ (5) \trt_sp_ninepointcenter:nnnn $\{\langle coor\ 1 \rangle\}\{\langle coor\ 2 \rangle\}\{\langle coor\ 3 \rangle\}\{\langle name \rangle\}$

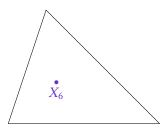
Find the nine-point center X_5 of the triangle.



\trt_sp_symmedian:nnnn

/tikz/triangletools/triangle center= $(\langle coor 1 \rangle)(\langle coor 2 \rangle)(\langle coor 3 \rangle)$ (6) \trt_sp_symmedian:nnnn $\{\langle coor 1 \rangle\}\{\langle coor 2 \rangle\}\{\langle coor 3 \rangle\}\{\langle name \rangle\}$

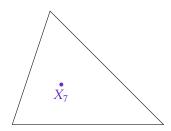
Find the symmedian point X_6 (aka. the Lemoine point or Grebe point) of the triangle.



\trt_sp_gergonne:nnnn

/tikz/triangletools/triangle center= $(\langle coor\ 1 \rangle)(\langle coor\ 2 \rangle)(\langle coor\ 3 \rangle)(7)$ \trt_sp_gergonne:nnnn $\{\langle coor\ 1 \rangle\}\{\langle coor\ 2 \rangle\}\{\langle coor\ 3 \rangle\}\{\langle name \rangle\}$

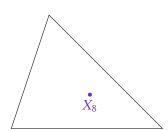
Find the Gergonne point X_7 of the triangle.



\trt_sp_nagel:nnnn

/tikz/triangletools/triangle center= $(\langle coor\ 1\rangle)(\langle coor\ 2\rangle)(\langle coor\ 3\rangle)(8)$ \trt_sp_nagel:nnnn $\{\langle coor\ 1\rangle\}\{\langle coor\ 2\rangle\}\{\langle coor\ 3\rangle\}\{\langle name\rangle\}$

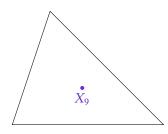
Find the Nagel point X_8 of the triangle.



 $\verb|\trt_sp_mittenpunkt:nnnn||$

/tikz/triangletools/triangle center= $(\langle coor\ 1 \rangle)(\langle coor\ 2 \rangle)(\langle coor\ 3 \rangle)(9)$ \trt_sp_mittenpunkt:nnnn $\{\langle coor\ 1 \rangle\}\{\langle coor\ 2 \rangle\}\{\langle coor\ 3 \rangle\}\{\langle name \rangle\}$

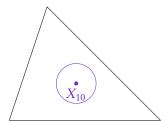
Find the *mittenpunkt* X_9 of the triangle.



\trt_sp_spieker:nnnn

/tikz/triangletools/triangle center= $(\langle coor\ 1 \rangle)(\langle coor\ 2 \rangle)(\langle coor\ 3 \rangle)(10)$ \trt_sp_spieker:nnnn $\{\langle coor\ 1 \rangle\}\{\langle coor\ 2 \rangle\}\{\langle coor\ 3 \rangle\}\{\langle name \rangle\}$

Find the Spieker center X_{10} of the triangle.



3.4 Other utilities

3.4.1 Line tools

The line tools utility can helps you play with some (very basic) operations related to lines.

intersection

```
/\text{tikz/triangletools/intersection} = (\langle coor 1 \rangle) (\langle coor 2 \rangle) - - (\langle coor 3 \rangle) (\langle coor 4 \rangle)
```

There are two lines, the first joins $\langle coor 1 \rangle$ and $\langle coor 2 \rangle$, and the other joins $\langle coor 3 \rangle$ and $\langle coor 4 \rangle$. This finds the intersection of these lines.

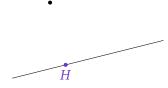
If the two lines are parallel, trt output is set to (0,0), and the package will report a warning.

```
1 \begin {tikzpicture}
2 \draw (0,0) coordinate (a) -- (0,1) coordinate (b);
3 \draw (1,0) coordinate (c) -- (.5,1) coordinate (d);
4 \coordinate (e) at (1,1);
5 \fill[maincolor,trt={intersection=(a)(b)--(c)(d)}] (trt output)
6 circle (1.5pt) node[above] {$I$};
7 \fill[maincolor,trt={intersection=(a)(b)--(c)(e)}] (trt output)
8 circle (1.5pt) node[left] {$J$};
9 \end{tikzpicture}
```

foot_of_perpendicular

/tikz/triangletools/foot of perpendicular= $(\langle coor 1 \rangle) - (\langle coor 2 \rangle) (\langle coor 3 \rangle)$

Find the foot of perpendicular of point $\langle coor 1 \rangle$ to the line joining $\langle coor 2 \rangle$ and $\langle coor 3 \rangle$.



```
1 \begin {tikzpicture}
2 \draw (0,0) coordinate (b) -- (4,1) coordinate (c);
3 \fill (1,2) circle (1.5pt) coordinate (a);
4 \fill[maincolor,trt={foot of perpendicular=(a)--(b)(c)}] (trt output)
5 circle (1.5pt) node[below] {$H$};
6 \end{tikzpicture}
```

You can do much more using these expl3 functions.

\trt_lt_get_line_equation:nnNNN

```
\label{eq:coor_2} $$ \operatorname{line\_equation:nnNNN} {\langle coor\ 1 \rangle} {\langle coor\ 2 \rangle} {\langle a \rangle \langle b \rangle \langle c \rangle} $$
```

Find the equation of the line joining $\langle coor 1 \rangle$ and $\langle coor 2 \rangle$, in the form of ax + by = c. The l3fp *local* variables $\langle a \rangle$, $\langle b \rangle$ and $\langle c \rangle$ will be set accordingly.

Note that for any pair of points $\langle coor 1 \rangle$ and $\langle coor 2 \rangle$, there are infinitely many solutions for $\langle a \rangle$, $\langle b \rangle$ and $\langle c \rangle$. This function will produce one of such solution. While the solution is likely to be the simplest of all possible ones, this is not guaranteed.

```
\coordinate (b) at (4,1);
                       \ExplSyntax0n
                       \fp_new:N \l_foo_tmpa_fp
                       \fp_new:N \l_foo_tmpb_fp
                       \fp_new:N \l_foo_tmpc_fp
                       \trt_lt_get_line_equation:nnNNN {a} {b}
1x + -3y = 1
                         \l_foo_tmpa_fp \l_foo_tmpb_fp \l_foo_tmpc_fp
                       \fp_new:N \c_foo_cm_fp
                       \fp_set:Nn \c_foo_cm_fp {28.45275590551181}
                       \def \resultequation {
                          fp_eval:n {round (\l_foo_tmpa_fp / \c_foo_cm_fp)}x +
                          fp_eval:n {round (\l_foo_tmpb_fp / \c_foo_cm_fp)}y
                          =\fp_eval:n {round (\l_foo_tmpc_fp / (\c_foo_cm_fp * \c_foo_cm_fp))}$
                   16
                       }
```

1 \begin {tikzpicture}

\ExplSyntax0ff

19 \end{tikzpicture}

17

\coordinate (a) at (1,0);

\trt_lt_get_intersection_line:NNNNNNN

 $\trt_lt_get_intersection_line:NNNNNNN(a1)\langle b1\rangle\langle c1\rangle\langle a2\rangle\langle b2\rangle\langle c2\rangle\langle x\rangle\langle y\rangle$

\draw (a) -- (b) node[midway,sloped,below] {\resultequation};

This function finds the intersection of lines $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$, afterwards store the dimensions of the intersection in variables $\langle x \rangle$ and $\langle y \rangle$.

All arguments are floating points variables, $\langle x \rangle$ and $\langle y \rangle$ needs to be local variables.

\trt_lt_get_intersection_coordinate:nnnnNN

```
\label{local_coordinate:nnnnNN} $$ {\langle coor 1 \rangle} {\langle coor 2 \rangle} {\langle coor 3 \rangle} {\langle coor 4 \rangle} {\langle x \rangle} $$
```

This function is a wrapper of \trt_lt_get_intersection_line:NNNNNNNN. It finds the intersection of the line joining $\langle coor\ 1 \rangle$, $\langle coor\ 2 \rangle$ and the line joining $\langle coor\ 3 \rangle$, $\langle coor\ 4 \rangle$. The dimensions of the returned point is stored in $\langle x \rangle$ and $\langle y \rangle$, which are local l3fp variables.

A warning will be raised if the lines are parallel, in that case $\langle x \rangle$ and $\langle y \rangle$ are set to zero.

This is the base of intersection.

```
1 \begin {tikzpicture}
2 \draw (0,0) coordinate (a) -- (0,1) coordinate (b);
3 \draw (1,0) coordinate (c) -- (.5,1) coordinate (d);
4 \ExplSyntaxOn
5 \fp_new:N \l_foo_tmpa_fp
6 \fp_new:N \l_foo_tmpb_fp
7 \trt_lt_get_intersection_coordinate:nnnnNN {a} {b} {c} {d}
8 \l_foo_tmpa_fp \l_foo_tmpb_fp
9 \coordinate (i) at (\fp_to_dim:N \l_foo_tmpa_fp,
10 \fp_to_dim:N \l_foo_tmpb_fp);
11 \ExplSyntaxOff
12 \fill[maincolor] (i) circle (1.5pt);
13 \end{tikzpicture}
```

\trt_lt_get_perpendicular_equation:nNNNNNN

 $\verb|\trt_lt_get_perpendicular_equation:nNNNNNN{$\langle coor\ 1\rangle$} \\ \langle a1\rangle\langle b1\rangle\langle c1\rangle\langle a2\rangle\langle b2\rangle\langle c2\rangle \\$

This function finds the line of equation $a_2x + b_2y = c_2$ that passes coordinate $\langle coor 1 \rangle$ and is perpendicular to $a_1x + b_1y = c_1$.

```
(0.5, 4)
(3, 1)
```

Equation of perpendicular line: 4x + 1y = 6

(-1,0)

```
1 \begin {tikzpicture}
    \draw (-1,0) coordinate (b) node[left] {$(-1,0)$} --
          (3,1) coordinate (c) node[right] {$(3,1)$};
    \fill (0.5,4) circle (1.5pt) coordinate (a) node[above] {$(0.5,4)$};
    \ExplSyntax0n
    \fp_new:N \l_foo_tmpa_fp
    \fp_new:N \l_foo_tmpb_fp
    \fp_new:N \l_foo_tmpc_fp
    \label{local_state} $$ \fp_new:N \l_foo_tmpd_fp $$
    \fp_new:N \l_foo_tmpe_fp
    \fp_new:N \l_foo_tmpf_fp
    \trt_lt_get_line_equation:nnNNN {b} {c}
      \l_foo_tmpa_fp \l_foo_tmpb_fp \l_foo_tmpc_fp
    \trt_lt_get_perpendicular_equation:nNNNNNN {a}
      \l_foo_tmpa_fp \l_foo_tmpb_fp \l_foo_tmpc_fp
      \l_foo_tmpd_fp \l_foo_tmpe_fp \l_foo_tmpf_fp
    \fp_new:N \c_foo_cm_fp
    fp_set:Nn \c_foo_cm_fp \{28.45275590551181\}
    \def \resultequation {
      fp_eval:n {round (\l_foo_tmpd_fp / \c_foo_cm_fp)}x +
       fp_eval:n {round (\l_foo_tmpe_fp / \c_foo_cm_fp)}y
      =fp_eval:n {round (\l_foo_tmpf_fp / (\c_foo_cm_fp * \c_foo_cm_fp))}
    }
23
    \ExplSyntax0ff
24
    \path (1,0) node[below=3mm,align=center]
      {Equation of perpendicular line:\\\resultequation};
27 \end{tikzpicture}
```

\trt_lt_get_perpendicular_coordinate:nnnNN

 $\trt_lt_get_perpendicular_coordinate:nnnNN{\langle coor 1\rangle}{\langle coor 2\rangle}{\langle coor 3\rangle}{\langle x\rangle}{\langle y\rangle}$

Find the dimensions of the foot of perpendicular from $\langle coor 1 \rangle$ to the line joining $\langle coor 2 \rangle$ and $\langle coor 3 \rangle$. Afterwards store the dimensions found in $\langle x \rangle$ and $\langle y \rangle$.

This is the base of foot of perpendicular.

3.4.2 The barycentric coordinate system

 $\verb"initialize_barycentric"$

/tikz/triangletools/initialize barycentric= $(\langle coor 1 \rangle)(\langle coor 2 \rangle)(\langle coor 3 \rangle)$

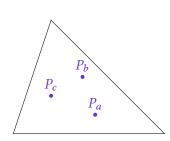
Use the three coordinates as "anchors" of the barycentric coordinate system.

bc3

```
(bc3 cs:\langle l1 \rangle, \langle l2 \rangle, \langle l3 \rangle)
```

Using the barycentric coordinate system. Note that the system needs to be initialized in advance using initialize barycentric, and an error message will be reported if you do otherwise.

The sum of $\langle l1 \rangle$, $\langle l2 \rangle$ and $\langle l3 \rangle$ is not necessarily 1 – the package will take care of that internally.

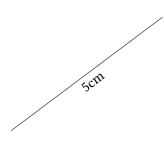


3.4.3 Distance-finding utility

\trt_distance:nnN

```
\trt_distance:nnN {\langle coor 1 \rangle} {\langle coor 2 \rangle} {\langle fp \ var \rangle}
```

Find distance between $\langle coor 1 \rangle$ and $\langle coor 2 \rangle$, and store that value to $\langle fp \ var \rangle$.



```
1 \begin {tikzpicture}
    \path (0,0) coordinate (a) (4,3) coordinate (b);
    \ExplSyntax0n
    \fp_new:N \l_foo_tmpa_fp
    \trt_distance:nnN {a} {b} \l_foo_tmpa_fp
    \fp_new:N \c_foo_cm_fp
    fp_set:Nn \c_foo_cm_fp {28.45275590551181}
    \draw (a) -- (b) node[midway,sloped,below]
      { fp_eval:n {round(\l_foo_tmpa_fp / \c_foo_cm_fp)} cm };
    \ExplSyntax0ff
11 \end{tikzpicture}
```

\trt_distance_triangle:nnnNNN

 $\trt_distance_triangle:nnnNNN{\langle coor 1 \rangle}{\langle coor 2 \rangle}{\langle coor 3 \rangle}{\langle a \rangle \langle b \rangle \langle c \rangle}$

\trt_distance:nnN is needed to find the side lengths in a triangle (these side lengths are very helpful in many areas, for instance in this package to find special points based on the barycentric system). However, using it three times in a row is not quite elegant; this function is defined to automate that process.

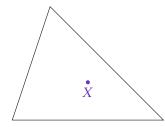
 $\langle a \rangle$ is set to the distance between $\langle coor 2 \rangle$ and $\langle coor 3 \rangle$, similar things happen for $\langle b \rangle$ and $\langle c \rangle$.

3.5 Customization

output_name

/tikz/triangletools/output name= $\langle name \rangle$

This key can be used to change the name of the returned coordinates. The initial value of this key is trt output.



```
\begin {tikzpicture}[trt={output name=hello world}]
  \draw (1,3) coordinate (a) --
        (0,0) coordinate (b) --
        (4,0) coordinate (c) -- cycle;
  \fill[maincolor,trt={circumcenter=(a)(b)(c)}] (hello world)
    circle[radius=1.5pt] node[below] {$X$};
\end{tikzpicture}
```

Implementation

1 (@@=trt)

4.1 The main package file

```
2 (*triangletools)
3 \RequirePackage{tikz}
4 \RequirePackage{expl3}
5 \ProvidesExplPackage {triangletools} {2020/04/30} {0.1}
   {TikZ support for triangular geometry}
```

\trt@tmp@ii

\trt@tmp@i We will use these dimensions many times to extract the dimensions of a TikZ coordinate.

- 7 \newdimen\trt@tmp@i
- 8 \newdimen\trt@tmp@ii

Let's load the necessary subpackage files.

```
9 \input {trtmessages.code.tex}
10 \input {trtlinetools.code.tex}
11 \input {trtbarycentric.code.tex}
12 \input {trtdistance.code.tex}
13 \input {trtspecialpoints.code.tex}
14 \input {trtfrontend.code.tex}
15 \langle /triangletools\rangle
```

4.2 Errors and warnings

```
16 (*messages)
17 \ProvidesExplFile {trtmessages.code.tex} {2020/04/30} {0.1}
18 {The ~ triangletools ~ package: ~ Messages}
```

We also need to declare some helpful messages that we will use later on.

In trtlinetools.code.tex, when we find the intersection of two lines, a warning will be shown if the lines are parallel. The warning is based on intersection-not-found.

When the barycentric coordinate system, implemented in trtbarycentric.code. tex, is used, it should already be initialized, *i.e.* we should already know what are the three "anchor" coordinates of the system. If the coordinate system is not yet initialized, this error will be shown.

We do let the user to find triangle center X_i for any i. However this package obviously can't implement all points in ETC (in fact, I will implement only some most important points). An error will be raised if the user tries to use an unimplemented point.

```
39 \msg_new:nnnn {triangletools} {center-not-found}
40     {
41          Triangle ~ center ~ not ~ found.
42     }
43     {
44          I ~ can't ~ find ~ the ~ requested ~ triangle ~ center, ~ because ~
45          point ~ X(#1) ~ is ~ not ~ yet ~ implemented ~ in ~ the ~ triangletools ~
46          package. ~ Try ~ to ~ construct ~ it ~ yourself.
47     }
```

We need to guard against using \trtradius before the macro stores something.

```
48 \msg_new:nnnn {triangletools} {no-radius-found}
49
50
      No ~ circles ~ can ~ be ~ constructed.
    }
51
52
    {
      I ~ can't ~ construct ~ the ~ requested ~ circle, ~ because ~ you ~ have ~
53
54
      not ~ request ~ me ~ to ~ construct ~ any ~ triangle ~ centers ~ that ~
      are ~ associated ~ to ~ a ~ circle. ~ I ~ will ~ set ~
      \protect\trtradius\space to ~ zero ~ now.
56
   }
57
58 (/messages)
```

4.3 The backend layer

4.3.1 The line tools utility

```
59 (*linetools)
60 \ProvidesExplFile {trtlinetools.code.tex} {2020/04/30} {0.1}
61 {The ~ triangletools ~ package: ~ Utilities ~ for ~ lines}
```

In trtlinetools.code.tex, we will implement the necessary functions to handle lines in a mathematical way.

Firstly, let's declare some internal variables that we will use later.

(End definition for \l_trt_lt_linei_a_fp and others.)

71 \fp_new:N \l__trt_lt_lineii_c_fp

```
\label{localization} $$ \sum_{t=1}^{72} fp_new:N \leq t_t fp_new:N \leqslant t
```

(End definition for \l_trt_lt_tmp_fp, \l_trt_lt_tmpa_fp, and \l_trt_lt_tmpb_fp.)

\trt_lt_get_line_equation:nnNNN

Find the equation of the line passing #1 and #2, and store the values of a, b, c found to #3, #4 and #5, which are floating point variables, respectively.

```
75 \cs_new:Npn \trt_lt_get_line_equation:nnNNN #1 #2 #3 #4 #5
76 {
77    \pgfextractx {\trt@tmp@i} {\pgfpointanchor{#1}{center}}
78    \pgfextracty {\trt@tmp@ii} {\pgfpointanchor{#1}{center}}
79    \fp_set:Nn \l__trt_i_pointi_x_fp {\trt@tmp@ii}
80    \fp_set:Nn \l__trt_i_pointi_y_fp {\trt@tmp@ii}
81    \pgfextractx {\trt@tmp@i} {\pgfpointanchor{#2}{center}}
82    \pgfextracty {\trt@tmp@ii} {\pgfpointanchor{#2}{center}}
```

```
\fp_set:Nn \l__trt_i_pointii_x_fp {\trt@tmp@i}

fp_set:Nn \l__trt_i_pointii_y_fp {\trt@tmp@ii}
```

There is a simple hack here. We have $ax_1 + by_1 = c = ax_2 + by_2$, which is equivalent to $a(x_1 - x_2) = b(y_2 - y_1)$. Therefore $a = y_2 - y_1$ and $b = x_1 - x_2$ can be used.

```
\fp_set:Nn #3
86
         {
           \l__trt_i_pointii_y_fp - \l__trt_i_pointi_y_fp
87
         }
88
       \fp_set:Nn #4
         {
90
           \\l__trt_i_pointi_x_fp - \\l__trt_i_pointii_x_fp
91
92
       \fp_set:Nn #5
93
94
         {
           #3 * \l__trt_i_pointi_x_fp + #4 * \l__trt_i_pointi_y_fp
95
96
97
    }
```

(End definition for \trt_lt_get_line_equation:nnNNN. This function is documented on page 6.)

\trt_lt_get_intersection_line:NNNNNNNN

Find the intersection of two lines with given equation, after that store the intersection coordinate to floating point variables #7 and #8.

```
98 \cs_new:Npn \trt_lt_get_intersection_line:NNNNNNNN #1 #2 #3 #4 #5 #6 #7 #8
99 {
100 \fp_set:Nn \l__trt_lt_tmp_fp { #1 * #5 - #4 * #2 }
```

If l_tmp_fp is zero, the two lines are parallel. In that case, we will issue a warning, and set the intersection coordinate to (0,0). Otherwise, continue computing as usual.

```
fp_compare:nNnTF {\l_t_tmp_fp} = {0}
101
102
           \msg_warning:nnnnnn {triangletools} {intersection-not-found}
103
             {(#1)} {(#2)} {(#3)} {(#4)}
104
105
           fp_set:Nn #7 {0}
           \fp_set:Nn #8 {0}
106
         }
107
108
           \fp_set:Nn #7 { ( #5 * #3 - #2 * #6 ) / \l__trt_lt_tmp_fp }
           \fp_set:Nn #8 { ( #1 * #6 - #4 * #3 ) / \l__trt_lt_tmp_fp }
110
    }
```

(End definition for \trt_lt_get_intersection_line: NNNNNNNN. This function is documented on page 7.)

\trt_lt_get_intersection_coordinate:nnnnNN

Let's generalize \trt_lt_get_intersection_line:NNNNNNN. The following function finds the intersection of two lines between #1, #2 and #3, #4, and store the coordinates to #5 and #6. We still use floating point variables here, as they might be useful in the future.

```
113 \cs_new:Npn \trt_lt_get_intersection_coordinate:nnnnNN #1 #2 #3 #4 #5 #6
114 {
115 \trt_lt_get_line_equation:nnNNN {#1} {#2}
116 \l__trt_lt_linei_a_fp \l__trt_lt_linei_b_fp \l__trt_lt_linei_c_fp
117 \trt_lt_get_line_equation:nnNNN {#3} {#4}
118 \l__trt_lt_lineii_a_fp \l__trt_lt_lineii_b_fp \l__trt_lt_lineii_c_fp
119 \trt_lt_get_intersection_line:NNNNNNNN
120 \l__trt_lt_linei_a_fp \l__trt_lt_linei_b_fp \l__trt_lt_linei_c_fp
121 \l__trt_lt_lineii_a_fp \l__trt_lt_lineii_b_fp \l__trt_lt_lineii_c_fp
122 #5 #6
123 }
```

(End definition for \trt_lt_get_intersection_coordinate:nnnnNN. This function is documented on page 7.)

__trt_lt_return_intersection:nnnnn

Now, let's TikZify the above function! Note that I use overlay because I don't want to affect the bounding box. The user can use the returned coordinate to change the bounding box in whatever way he wants to.

```
124 \cs_new:Npn \__trt_lt_return_intersection:nnnnn #1 #2 #3 #4 #5
125 {
126  \trt_lt_get_intersection_coordinate:nnnnNN {#1} {#2} {#3} {#4}
127  \l__trt_lt_tmpa_fp \l__trt_lt_tmpb_fp
128  \coordinate[overlay] (#5) at
129  (\fp_to_dim:N \l__trt_lt_tmpa_fp, \fp_to_dim:N \l__trt_lt_tmpb_fp);
130 }
```

(End definition for __trt_lt_return_intersection:nnnnn.)

Next, let's make some implementation regarding perpendicularity.

\trt_lt_get_perpendicular_equation:nNNNNNN

This function finds the equation of the line passing point and being perpendicular to a line having a given equation. The task is not quite complicated: note that lines ax + by = c and ay - bx = d are perpendicular.

```
131 \cs_new:Npn \trt_lt_get_perpendicular_equation:nNNNNNN #1 #2 #3 #4 #5 #6 #7
132 {
133    \fp_set:Nn #5 { -#3 }
134    \fp_set:Nn #6 { #2 }
135    \pgfextractx {\trt@tmp@i} {\pgfpointanchor{#1}{center}}
136    \pgfextracty {\trt@tmp@ii} {\pgfpointanchor{#1}{center}}
137    \fp_set:Nn \l__trt_lt_tmpa_fp {\trt@tmp@i}
138    \fp_set:Nn \l__trt_lt_tmpb_fp {\trt@tmp@ii}
139    \fp_set:Nn #7 { #5 * \l__trt_lt_tmpa_fp + #6 * \l__trt_lt_tmpb_fp }
140 }
```

(End definition for \trt_lt_get_perpendicular_equation:nNNNNNN. This function is documented on page 7.)

\trt_lt_get_perpendicular_coordinate:nnnNN

The base implemented, let's find the foot of perpendicular from a point to a segment.

```
\cs_new:Npn \trt_lt_get_perpendicular_coordinate:nnnNN #1 #2 #3 #4 #5
142
       \trt_lt_get_line_equation:nnNNN {#2} {#3}
143
         \l__trt_lt_linei_a_fp \l__trt_lt_linei_b_fp \l__trt_lt_linei_c_fp
144
145
       \trt_lt_get_perpendicular_equation:nNNNNNN {#1}
146
         \l__trt_lt_linei_a_fp \l__trt_lt_linei_b_fp \l__trt_lt_linei_c_fp
147
         \l__trt_lt_lineii_a_fp \l__trt_lt_lineii_b_fp \l__trt_lt_lineii_c_fp
149
       \trt_lt_get_intersection_line:NNNNNNNN
150
         \\l_trt_\lt_\linei_a_fp \\l_trt_\lt_\linei_b_fp \\l_trt_\lt_\linei_c_fp
151
         \\__trt_\lt_\lineii_a_fp \\\_trt_\lt_\lineii_b_fp \\\\_trt_\lt_\lineii_c_fp
         #4 #5
    }
154
```

(End definition for \trt_lt_get_perpendicular_coordinate:nnnNN. This function is documented on page 8.)

rt_lt_return_perpendicular_coordinate:nnnn

This is just a wrapper of \trt_lt_get_perpendicular_coordinate:nnnNN.

```
155 \cs_new:Npn \__trt_lt_return_perpendicular_coordinate:nnnn #1 #2 #3 #4
156 {
157 \trt_lt_get_perpendicular_coordinate:nnnNN {#1} {#2} {#3}
158 \l__trt_lt_tmpa_fp \l__trt_lt_tmpb_fp
159 \coordinate[overlay] (#4) at
```

```
(\fp_to_dim:N \l__trt_lt_tmpa_fp, \fp_to_dim:N \l__trt_lt_tmpb_fp);
(Ind definition for \__trt_lt_return_perpendicular_coordinate:nnnn.)
```

4.3.2 The barycentric coordinate system utility

In trtbarycentric.code.tex, we will implement the three-point barycentric coordinate system, which is essential in constructing many special points in a triangle.

(End definition for \l__trt_bc_anchor_ix_fp and others.)

\l__trt_bc_lambda_ii_fp
\l__trt_bc_lambda_iii_fp
\l__trt_bc_lambda_iii_fp

We use these variables to store the user input coordinate. Note that our system is a three-point one, hence exactly three number is required.

Why lambda λ ? Well, I don't know. Wikipedia uses that, so I do the same.

```
175 \fp_new:N \l__trt_bc_lambda_i_fp
176 \fp_new:N \l__trt_bc_lambda_ii_fp
177 \fp_new:N \l__trt_bc_lambda_iii_fp

(End definition for \l__trt_bc_lambda_i_fp, \l__trt_bc_lambda_ii_fp, and \l__trt_bc_lambda_iii_fp.)
```

\l__trt_bc_initialized_bool

We need to guard against using the system before initializing. This boolean variable does that job: if it is set to false (default), do nothing.

```
\lambda \bool_new:\N\\l__trt_bc_initialized_bool\
\(End definition for \l__trt_bc_initialized_bool.\)
\\l__trt_bc_tmp_fp A temporary variable.
\(\frac{179}{fp_new:\N\\l__trt_bc_tmp_fp}\)
```

(End definition for $\l_{-trt_bc_tmp_fp}$.)

__trt_bc_initialize:nnn

Initialize the barycentric coordinate system. This is the only place where $\l_{-trt-bc_initialized_bool}$ can be set to true, so this function must be executed before everything else in this file.

```
180 \cs_new:Npn \__trt_bc_initialize:nnn #1 #2 #3
181 {
182    \bool_set_true:N \l__trt_bc_initialized_bool
183    \pgfextractx {\trt@tmp@i} {\pgfpointanchor{#1}{center}}
184    \pgfextracty {\trt@tmp@ii} {\pgfpointanchor{#1}{center}}
185    \fp_set:Nn \l__trt_bc_anchor_ix_fp {\trt@tmp@i}
```

```
\fp_set:Nn \l__trt_bc_anchor_iy_fp {\trt@tmp@ii}
186
       \pgfextractx {\trt@tmp@i} {\pgfpointanchor{#2}{center}}
187
       \pgfextracty {\trt@tmp@ii} {\pgfpointanchor{#2}{center}}
       \fp_set:Nn \l__trt_bc_anchor_iix_fp {\trt@tmp@i}
      \fp_set:Nn \l__trt_bc_anchor_iiy_fp {\trt@tmp@ii}
190
      \pgfextractx {\trt@tmp@i} {\pgfpointanchor{#3}{center}}
191
      \pgfextracty {\trt@tmp@ii} {\pgfpointanchor{#3}{center}}
192
       \fp_set:Nn \l__trt_bc_anchor_iiix_fp {\trt@tmp@i}
      \fp_set:Nn \l__trt_bc_anchor_iiiy_fp {\trt@tmp@ii}
194
195
    }
```

bc3 The bc3 coordinate system implementation. We will guard against using it when

(End definition for __trt_bc_initialize:nnn.)

__trt_bc_initialize:nnn is not yet executed – in that case, uninitialized error will be raised.

We will receive arguments of bc3 as #1,#2,#3, so a simple parser is needed. All interesting things will be done with that parser.

(End definition for bc3. This function is documented on page 8.)

__trt_bc_parse:w Thi

This is the parser we use for bc3.

The conversion from λ_i to the Cartesian format is pretty simple, we have $x = \lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3$ and the formula for y is similar. However, first we have to change the value of λ_i so that $\lambda_1 + \lambda_2 + \lambda_3 = 1$.

```
206 \cs_new:Npn \__trt_bc_parse:w #1,#2,#3 \q_stop
207
       fp_set:Nn \\l_trt_bc_tmp_fp { (#1) + (#2) + (#3) }
208
       \fp_set:Nn \l__trt_bc_lambda_i_fp { (#1) / (\l__trt_bc_tmp_fp) }
209
       \fp_set:Nn \l__trt_bc_lambda_ii_fp { (#2) / (\l__trt_bc_tmp_fp) }
210
       \fp_set:Nn \l__trt_bc_lambda_iii_fp { (#3) / (\l__trt_bc_tmp_fp) }
       \fp_set:Nn \l__trt_tmp_a_fp
           \label{local_trt_bc_anchor_ix_fp} $$ l__trt_bc_lambda_i_fp + $$
214
           \\l__trt_bc_anchor_iix_fp * \\l__trt_bc_lambda_ii_fp +
           \l__trt_bc_anchor_iiix_fp * \l__trt_bc_lambda_iii_fp
216
         }
       fp_set:Nn \\l_trt_tmp_b_fp
218
           \\l__trt_bc_anchor_iy_fp * \\l__trt_bc_lambda_i_fp +
           \\l__trt_bc_anchor_iiy_fp * \\l__trt_bc_lambda_ii_fp +
           \\l__trt_bc_anchor_iiiy_fp * \\l__trt_bc_lambda_iii_fp
         }
```

Floating point variables are not TeX dimensions, hence \fp_to_dim:N is used.

```
\pgf@x = \fp_to_dim:N \l__trt_tmp_a_fp
\pgf@y = \fp_to_dim:N \l__trt_tmp_b_fp
\}
\text{226} \
\text{barycentric}
\text{(End definition for \__trt_bc_parse:w.)}
\text{\text{}}
\text{\text{\text{}}}
\text{\text{\text{}}}
\text{\text{\text{}}}
\text{\text{\text{}}}
\text{\text{\text{\text{}}}}
\text{\text{\text{\text{\text{}}}}
\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\tet
```

4.3.3 Distance-finding utility

```
228 (*distance)
229 \ProvidesExplFile {trtdistance.code.tex} {2020/04/30} {0.1}
230 {The ~ triangletools ~ package: ~ Utilities ~ for ~ 2d ~ distance}
```

This file implements functions to find the distance between (2d) TikZ coordinates.

\l__trt_d_pointi_x_fp
\l__trt_d_pointi_y_fp
\l__trt_d_pointii_x_fp
\l__trt_d_pointii_y_fp

These variables are used to store the coordinates of the points between which we are finding the distance.

```
231 \fp_new:N \l__trt_d_pointi_x_fp
232 \fp_new:N \l__trt_d_pointii_x_fp
233 \fp_new:N \l__trt_d_pointii_y_fp
234 \fp_new:N \l__trt_d_pointii_y_fp
```

(End definition for \l__trt_d_pointi_x_fp and others.)

\trt_distance:nnN Find the distance between TikZ coordinates #1 and #2.

```
235 \cs_new:Npn \trt_distance:nnN #1 #2 #3
236
                          \pgfextractx {\trt@tmp@i} {\pgfpointanchor{#1}{center}}
                          \pgfextracty {\trt@tmp@ii} {\pgfpointanchor{#1}{center}}
238
239
                         \fp_set:Nn \l__trt_d_pointi_x_fp {\trt@tmp@i}
                         \fp_set:Nn \l__trt_d_pointi_y_fp {\trt@tmp@ii}
240
241
                          \pgfextractx {\trt@tmp@i} {\pgfpointanchor{#2}{center}}
                          \pgfextracty {\trt@tmp@ii} {\pgfpointanchor{#2}{center}}
242
                          \fp_set:Nn \l__trt_d_pointii_x_fp {\trt@tmp@i}
243
                          \fp_set:Nn \l__trt_d_pointii_y_fp {\trt@tmp@ii}
                          \fp_set:Nn #3
245
                                 {
246
247
                                         sart((
                                                (\\l_trt_d_pointi_x_fp - \\l_trt_d_pointii_x_fp) *
249
                                                (\\l_trt_d_pointi_x_fp - \\l_trt_d_pointii_x_fp)
250
                                                 (\label{local_pointi_y_fp} - \label{local_pointi} - \label{local_p
                                        ))
254
                                 }
255
                  }
```

(End definition for \trt_distance: nnN. This function is documented on page 9.)

\trt_distance_triangle:nnnNNN

We mainly need the above function to find the side length in a triangle. Let's create a function that do so automatically.

```
256 \cs_new:Npn \trt_distance_triangle:nnnNNN #1 #2 #3 #4 #5 #6
257 {
258  \trt_distance:nnN {#2} {#3} #4
259  \trt_distance:nnN {#3} {#1} #5
260  \trt_distance:nnN {#1} {#2} #6
261  }
262 \( \frac{distance}{distance} \)
```

(End definition for \trt_distance_triangle:nnnNNN. This function is documented on page 9.)

4.4 Construction of triangle centers

```
263 (*specialpoints)
264 \ProvidesExplFile {trtspecialpoints.code.tex} {2020/04/30} {0.1}
265 {The ~ triangletools ~ package: ~ Triangle ~ center ~ construction}
```

This file will use the utility implemented in the above sections to find some most important triangle centers described in the ETC.

```
\l__trt_sp_a_fp We will need the side length of the triangle for some centers.
               \l__trt_sp_b_fp
                                 266 \fp_new:N \l__trt_sp_a_fp
               \l__trt_sp_c_fp
                                 267 \fp_new:N \l__trt_sp_b_fp
                                 268 \fp_new:N \l__trt_sp_c_fp
                                 (End definition for \lower l_- trt_sp_a_fp, \lower l_- trt_sp_b_fp, and \lower l_- trt_sp_c_fp.)
  \l__trt_sp_coordinatei_x_fp
                                 These variables may also be helpful for triangle centers for which a simple formula
                                 doesn't exist, e.g. the circumcenter.
  \l__trt_sp_coordinatei_y_fp
 \l__trt_sp_coordinateii_x_fp
                                 269 \fp_new:N \l__trt_sp_coordinatei_x_fp
 \l__trt_sp_coordinateii_y_fp
                                 270 \fp_new:N \l__trt_sp_coordinatei_y_fp
\l__trt_sp_coordinateiii_x_fp
                                 271 \fp_new:N \l__trt_sp_coordinateii_x_fp
\l__trt_sp_coordinateiii_y_fp
                                 272 \fp_new:N \l__trt_sp_coordinateii_y_fp
        \l__trt_sp_linei_a_fp
                                273 \fp_new:N \l__trt_sp_coordinateiii_x_fp
        \l__trt_sp_linei_b_fp 274 \fp_new:N \l__trt_sp_coordinateiii_y_fp
                                275 \fp_new:N \l__trt_sp_linei_a_fp
        \l__trt_sp_linei_c_fp
                                 276 \fp_new:N \l__trt_sp_linei_b_fp
       \l__trt_sp_lineii_a_fp
                                 277 \fp_new:N \l__trt_sp_linei_c_fp
       \l__trt_sp_lineii_b_fp
                                 278 \fp_new:N \l__trt_sp_lineii_a_fp
       \l__trt_sp_lineii_c_fp
                                 279 \fp_new:N \l__trt_sp_lineii_b_fp
                                 280 \fp_new:N \l__trt_sp_lineii_c_fp
                                 (End definition for \l_trt_sp_coordinatei_x_fp and others.)
                                 Some additional temporary variables.
           \l__trt_sp_tmpa_fp
           \l__trt_sp_tmpb_fp
                                 281 \fp_new:N \l__trt_sp_tmpa_fp
           \l__trt_sp_tmpc_fp
                                 282 \fp_new:N \l__trt_sp_tmpb_fp
                                 283 \fp_new:N \l__trt_sp_tmpc_fp
                                 (End definition for \l_{-trt\_sp\_tmpa\_fp}, \l_{-trt\_sp\_tmpb\_fp}, and \l_{-trt\_sp\_tmpc\_fp}.)
```

4.4.1 X_1 – The incenter

Each center will have a function taking four arguments. The first three arguments are the TikZ coordinates of the triangle vertices; the last argument is the name of the return TikZ coordinate.

To prevent conflict between these sister functions when they are used together, I put each of them inside a T_EX group.

\trt_sp_incenter:nnnn Return the incenter. It is based on the barycentric coordinate of the incenter, (a, b, c).

```
284 \cs_new:Npn \trt_sp_incenter:nnnn #1 #2 #3 #4
285
       \group_begin:
287
         \__trt_bc_initialize:nnn {#1} {#2} {#3}
         \trt_distance_triangle:nnnNNN {#1} {#2} {#3}
288
           \l_{trt_sp_a_fp \l_trt_sp_b_fp \l_trt_sp_c_fp}
289
         \path[overlay] (bc3 ~ cs \c_colon_str
           \fp_eval:n {\l__trt_sp_a_fp},
291
           \fp_eval:n {\l__trt_sp_b_fp},
292
           fp_eval:n {\l_trt_sp_c_fp}) coordinate (#4);
       \group_end:
295
    }
```

(End definition for \trt_sp_incenter:nnnn. This function is documented on page 3.)

\trt_sp_excenter:nnnn

Return the excenter of the triangle, with respect to vertex #1. This center is just a derivation of the incenter; also it is not unique, so it is not assigned a number. Barycentric coordinate of the excenter is (-a, b, c), where a is the length of the side joining #2 and #3.

Note that this is the only function in this series in which argument order is impor-

```
296 \cs_new:Npn \trt_sp_excenter:nnnn #1 #2 #3 #4
297
       \group_begin:
         \__trt_bc_initialize:nnn {#1} {#2} {#3}
299
         \trt_distance_triangle:nnnNNN {#1} {#2} {#3}
300
           \l_{trt_sp_a_fp} \l_{trt_sp_b_fp} \l_{trt_sp_c_fp}
         \path[overlay] (bc3 ~ cs \c_colon_str
           \fp_eval:n {- \l__trt_sp_a_fp},
303
           fp_eval:n {\l_trt_sp_b_fp},
304
           fp_eval:n {\l_trt_sp_c_fp}) coordinate (#4);
       \group_end:
306
    }
307
```

(End definition for \trt_sp_excenter:nnnn. This function is documented on page 3.)

4.4.2 X_2 - The centroid

\trt_sp_centroid:nnnn This is perhaps the simplest of all. Barycentric coordinate: (1, 1, 1).

```
308 \cs_new:Npn \trt_sp_centroid:nnnn #1 #2 #3 #4
309 {
310    \group_begin:
311    \__trt_bc_initialize:nnn {#1} {#2} {#3}
312    \path[overlay] (bc3 ~ cs \c_colon_str 1, 1, 1) coordinate (#4);
313    \group_end:
314 }
```

(End definition for \trt_sp_centroid:nnnn. This function is documented on page 3.)

4.4.3 X_3 – The circumcenter

\trt_sp_circumcenter:nnnn

This is opposite to X_2 : perhaps this is the most complex of all. The barycentric coordinate formula is not simple enough for me, so I construct this point purely manually: find the intersection of the perpendicular bisectors.

```
315 \cs_new:Npn \trt_sp_circumcenter:nnnn #1 #2 #3 #4
316 {
317 \group_begin:
```

Firstly, let's store the coordinate of the vertices.

```
\pgfextractx {\trt@tmp@i} {\pgfpointanchor{#1}{center}}
318
        \pgfextracty {\trt@tmp@ii} {\pgfpointanchor{#1}{center}}
        \fp_set:Nn \l__trt_sp_coordinatei_x_fp {\trt@tmp@i}
320
        \fp_set:Nn \l__trt_sp_coordinatei_y_fp {\trt@tmp@ii}
        \pgfextractx {\trt@tmp@i} {\pgfpointanchor{#2}{center}}
        \pgfextracty {\trt@tmp@ii} {\pgfpointanchor{#2}{center}}
        \fp_set:Nn \l__trt_sp_coordinateii_x_fp {\trt@tmp@i}
        \fp_set:Nn \l__trt_sp_coordinateii_y_fp {\trt@tmp@ii}
326
        \pgfextractx {\trt@tmp@i} {\pgfpointanchor{#3}{center}}
327
        \pgfextracty {\trt@tmp@ii} {\pgfpointanchor{#3}{center}}
        \fp_set:Nn \l__trt_sp_coordinateiii_x_fp {\trt@tmp@i}
328
        \fp_set:Nn \l__trt_sp_coordinateiii_y_fp {\trt@tmp@ii}
```

Now, let's change point #2 to the midpoint between #1 and #2, and do the same for #3.

```
330 \fp_set:Nn \l__trt_sp_coordinateii_x_fp
```

```
{
                                                                                                    (\l__trt_sp_coordinatei_x_fp + \l__trt_sp_coordinateii_x_fp) / 2
                                                                    \fp_set:Nn \l__trt_sp_coordinateii_y_fp
334
                                                                                  {
                                                                                                    (\l__trt_sp_coordinatei_y_fp + \l__trt_sp_coordinateii_y_fp) / 2
336
                                                                   \coordinate[overlay] (trt@tmp@ii) at (
                                                                                  \fp_to_dim:N \l__trt_sp_coordinateii_x_fp,
339
                                                                                  \label{local_sp_coordinate} $$ \int_{-\infty}^{\infty} 
                                                                   \fp_set:Nn \l__trt_sp_coordinateiii_x_fp
342
                                                                                                    (\l__trt_sp_coordinatei_x_fp + \l__trt_sp_coordinateiii_x_fp) / 2
343
                                                                                  }
344
                                                                   \fp_set:Nn \l__trt_sp_coordinateiii_y_fp
345
346
                                                                                                    (\l__trt_sp_coordinatei_y_fp + \l__trt_sp_coordinateiii_y_fp) / 2
347
                                                                                  }
3/18
                                                                    \coordinate[overlay] (trt@tmp@iii) at (
                                                                                   \fp_to_dim:N \l__trt_sp_coordinateiii_x_fp,
351
                                                                                  \fp_to_dim:N \l__trt_sp_coordinateiii_y_fp);
```

All we have to do now is to find the equations of the bisectors and their intersection.

```
\trt_lt_get_line_equation:nnNNN {#1} {#2}
352
353
           \\l__trt_sp_tmpa_fp \\l__trt_sp_tmpb_fp \\l__trt_sp_tmpc_fp
         \trt_lt_get_perpendicular_equation:nNNNNNN {trt@tmp@ii}
           \\l__trt_sp_tmpa_fp \\l__trt_sp_tmpb_fp \\l__trt_sp_tmpc_fp
           \l__trt_sp_linei_a_fp \l__trt_sp_linei_b_fp \l__trt_sp_linei_c_fp
356
         \trt_lt_get_line_equation:nnNNN {#1} {#3}
357
           \verb|\l_trt_sp_tmpa_fp \l_trt_sp_tmpb_fp \l_trt_sp_tmpc_fp|
         \trt_lt_get_perpendicular_equation:nNNNNNN {trt@tmp@iii}
           \l__trt_sp_tmpa_fp \l__trt_sp_tmpb_fp \l__trt_sp_tmpc_fp
360
           \l__trt_sp_lineii_a_fp \l__trt_sp_lineii_b_fp \l__trt_sp_lineii_c_fp
361
         \trt_lt_get_intersection_line:NNNNNNNN
           \l_trt_sp_linei_a_fp \l_trt_sp_linei_b_fp \l_trt_sp_linei_c_fp
363
           \l__trt_sp_lineii_a_fp \l__trt_sp_lineii_b_fp \l__trt_sp_lineii_c_fp
364
           \\\__trt_sp_tmpa_fp \\\\_trt_sp_tmpb_fp
         \coordinate[overlay] (#4) at (
           \fp_to_dim:N \l__trt_sp_tmpa_fp, \fp_to_dim:N \l__trt_sp_tmpb_fp);
367
       \verb|\group_end:|
368
    }
369
```

Quite surprisingly, the function is still very fast after all this. On my machine it never exceeds 10ms in execution time.

(End definition for \trt_sp_circumcenter:nnnn. This function is documented on page 3.)

4.4.4 X_4 – The orthocenter

 $\verb|\trt_sp_orthocenter:nnnn|$

Return the orthocenter of the triangle. This point is also constructed manually instead of using a proved formula. However, the utilities help making the construction look very simple.

(End definition for \trt_sp_orthocenter:nnnn. This function is documented on page 4.)

4.4.5 X_5 – The nine-point center

\trt_sp_ninepointcenter:nnnn

Return the center of the nine-point circle.

```
379 \cs_new:Npn \trt_sp_ninepointcenter:nnnn #1 #2 #3 #4
380 {
381 \group_begin:
```

 X_5 is is the midpoint of X_3 and X_4 . Therefore, for simplicity, X_3 and X_4 are constructed first. This causes some run-time overhead, however the overall execution time is still below 15ms, which is, in my opinion, still good.

Note that we already used trt@tmp@i and trt@tmp@ii coordinates in the construction of X_3 and X_4 , so to prevent conflict, trt@tmp@ii and trt@tmp@iv are used.

```
\trt_sp_circumcenter:nnnn {#1} {#2} {#3} {trt@tmp@iii}
        \trt_sp_orthocenter:nnnn {#1} {#2} {#3} {trt@tmp@iv}
383
        \pgfextractx {\trt@tmp@i} {\pgfpointanchor{trt@tmp@iii}{center}}
        \pgfextracty {\trt@tmp@ii} {\pgfpointanchor{trt@tmp@iii}{center}}
        \fp_set:Nn \l__trt_sp_tmpa_fp {\trt@tmp@i}
386
        \fp_set:Nn \l__trt_sp_tmpb_fp {\trt@tmp@ii}
387
        \pgfextractx {\trt@tmp@i} {\pgfpointanchor{trt@tmp@iv}{center}}
388
        \pgfextracty {\trt@tmp@ii} {\pgfpointanchor{trt@tmp@iv}{center}}
        \fp_set:Nn \l__trt_sp_tmpa_fp { (\trt@tmp@i + \l__trt_sp_tmpa_fp) / 2 }
390
        \fp_set:Nn \l__trt_sp_tmpb_fp { (\trt@tmp@ii + \l__trt_sp_tmpb_fp) / 2 }
391
        \coordinate[overlay] (#4) at (\fp_to_dim:N \l__trt_sp_tmpa_fp,
           \fp_to_dim:N \l__trt_sp_tmpb_fp);
       \group_end:
394
    }
395
```

(End definition for \trt_sp_ninepointcenter:nnnn. This function is documented on page 4.)

4.4.6 X_6 – The symmedian point

\trt_sp_symmedian:nnnn

Return the symmedian point (aka. the Lemoine point or Grebe point). The barycentric coordinate of the point is (a^2 , b^2 , c^2).

```
\cs_new:Npn \trt_sp_symmedian:nnnn #1 #2 #3 #4
397
     {
       \verb|\group_begin||
398
         \__trt_bc_initialize:nnn {#1} {#2} {#3}
         \trt_distance_triangle:nnnNNN {#1} {#2} {#3}
400
           \l_{trt\_sp\_a\_fp} \ell_{trt\_sp\_b\_fp} \ell_{trt\_sp\_c\_fp}
401
         \path[overlay] (bc3 ~ cs \c_colon_str
402
           fp_eval:n {\l_trt_sp_a_fp * \l_trt_sp_a_fp},
404
           fp_eval:n {\l_trt_sp_b_fp * \l_trt_sp_b_fp},
           fp_eval:n {\l_trt_sp_c_fp * \l_trt_sp_c_fp}) coordinate (#4);
405
406
       \group_end:
     }
```

(End definition for \trt_sp_symmedian:nnnn. This function is documented on page 4.)

4.4.7 X_7 – The Gergonne point

\trt_sp_gergonne:nnnn

Return the Gergonne point of the triangle. The barycentric coordinate of the point is $(\frac{1}{b+c-a}, \frac{1}{c+a-b}, \frac{1}{a+b-c})$.

```
408 \cs_new:Npn \trt_sp_gergonne:nnnn #1 #2 #3 #4
409 {
410   \group_begin:
411   \__trt_bc_initialize:nnn {#1} {#2} {#3}
412   \trt_distance_triangle:nnnNNN {#1} {#2} {#3}
413   \l__trt_sp_a_fp \l__trt_sp_b_fp \l__trt_sp_c_fp
414   \path[overlay] (bc3 ~ cs \c_colon_str
```

(End definition for \trt_sp_gergonne:nnnn. This function is documented on page 4.)

4.4.8 X_8 – The Nagel point

\trt_sp_nagel:nnnn Return the Nagel point. The barycentric coordinate of the point is (b+c-a, c+a-b, a+b-c).

```
421 \cs_new:Npn \trt_sp_nagel:nnnn #1 #2 #3 #4
422
 423
                                            \group_begin:
                                                         \__trt_bc_initialize:nnn {#1} {#2} {#3}
                                                         \trt_distance_triangle:nnnNNN {#1} {#2} {#3}
 425
                                                                      \label{lambda} $$ \l_-trt_sp_a_fp \l_-trt_sp_b_fp \l_-trt_sp_c_fp $$
 426
                                                         \path[overlay] (bc3 ~ cs \c_colon_str
                                                                      \label{eq:condition} $$ \int_{-\infty}^{\infty} \{ \_{-\infty}^{\infty} + \_{-\infty}^{\infty} - \_{-\infty}^{\infty} \}, $$
 428
                                                                      \label{eq:condition} $$ \int_{-\infty}^{p_e}  + \lim_{sp_e}  + 
 429
 430
                                                                      \label{eq:continuous_power_sp} $$ \int_{-\infty} { \_trt_sp_a_fp + \_trt_sp_b_fp - \_trt_sp_c_fp } $$
                                                         ) coordinate (#4);
 431
                                             \group_end:
 432
                               }
 433
```

(End definition for \trt_sp_nagel:nnnn. This function is documented on page 5.)

4.4.9 X_9 – The mittenpunkt

\trt_sp_mittenpunkt:nnnn Return the *mittenpunkt* of the triangle – its barycentric coordinate is $(a \times (b + c - a), b \times (c + a - b), c \times (a + b - c))$.

```
434 \cs_new:Npn \trt_sp_mittenpunkt:nnnn #1 #2 #3 #4
435
        \group_begin:
436
          \__trt_bc_initialize:nnn {#1} {#2} {#3}
          \trt_distance_triangle:nnnNNN {#1} {#2} {#3}
438
            \label{lambda} $$ \l_{trt_sp_a_fp \ l_trt_sp_c_fp \ } l_trt_sp_c_fp $$
440
          \path[overlay] (bc3 ~ cs \c_colon_str
            \fp_eval:n
               {
442
                 \l_{-trt_sp_a_fp} * (
443
                   \l_{trt_sp_b_fp} + \l_{trt_sp_c_fp} - \l_{trt_sp_a_fp}
                 )
              },
446
            \fp_eval:n
447
448
               {
                 \l_{-trt_sp_b_fp} * (
                   \l_{trt_sp_c_fp} + \l_{trt_sp_a_fp} - \l_{trt_sp_b_fp}
450
                 )
451
               },
            \fp_eval:n
               {
454
                 \l_{-trt_sp_c_fp} * (
455
456
                    \l_{trt\_sp\_a\_fp} + \l_{trt\_sp\_b\_fp} - \l_{trt\_sp\_c\_fp}
457
               }
458
          ) coordinate (#4);
        \group_end:
461
     }
```

4.4.10 X_{10} – The Spieker point

\trt_sp_spieker:nnnn Return the Spieker point. The barycentric coordinate of the point is (b+c, c+a, a+b).

```
462 \cs_new:Npn \trt_sp_spieker:nnnn #1 #2 #3 #4
464
      \group_begin:
        \__trt_bc_initialize:nnn {#1} {#2} {#3}
465
        \trt_distance_triangle:nnnNNN {#1} {#2} {#3}
          \l_{trt_sp_a_fp} \ell_{trt_sp_b_fp} \ell_{trt_sp_c_fp}
        \path[overlay] (bc3 ~ cs \c_colon_str
468
          469
          \fp_eval:n { \l__trt_sp_c_fp + \l__trt_sp_a_fp },
470
          \fp_eval:n { \l__trt_sp_a_fp + \l__trt_sp_b_fp }
        ) coordinate (#4);
472
      \verb|\group_end:|
473
   }
474
475 (/specialpoints)
```

(End definition for \trt_sp_spieker:nnnn. This function is documented on page 5.)

The frontend layer

```
476 (*frontend)
477 \ProvidesExplFile {trtfrontend.code.tex} {2020/04/30} {0.1}
     {The ~ triangletools ~ package: ~ The ~ front-end ~ layer}
```

The user interface of the package, which consists solely of pgf keys, will be implemented in this file.

\l__trt_fr_output_name_tl

Store the name of the output coordinate. Default to trt_output.

```
479 \tl_new:N \l__trt_fr_output_name_tl
480 \tl_set:Nn \l__trt_fr_output_name_tl {trt ~ output}
(End definition for \ l__trt_fr_output_name_tl.)
```

\l__trt_fr_center_number_int

We only provide specific key for X_1 , X_2 , X_3 and X_4 . All other points can be referenced using a single generic key. We need to store the index of that point so that we can choose the right function for the point.

```
481 \int_new:N \l__trt_fr_center_number_int
(End definition for \ l__trt_fr_center_number_int.)
```

\trtradius This macro will store the radius if a circle is associated. Of course firstly we need a \l_trt_fr_radius_fp floating point variable specified for that purpose.

```
482 \fp_new:N \l__trt_fr_radius_fp
483 \cs_gset_nopar:Npn \trtradius
      \msg_error:nn {triangletools} {no-radius-found} Opt
485
```

 $(\textit{End definition for \verb|\trt-adius|} \ \textit{and \verb|\l_-trt_fr_radius_fp}. \ \textit{This function is documented on page 2.})$

/tikz/trt Now it's time for the keys. They will be stored under /tikz/triangletools and can be accessed at trt= $\{\langle keys \rangle\}$.

```
487 \tikzset {
488 triangletools/.is ~ family,
    trt/.code={\pgfkeys{/tikz/triangletools/.cd,#1}},
    triangletools/.cd,
```

```
Change the output name of all returned coordinates.
output_name
```

```
output ~ name/.code={
      \tl_set:Nn \l__trt_fr_output_name_tl {#1}
492
493
     }.
```

(End definition for output name. This function is documented on page 9.)

intersection foot of perpendicular

The front-end of the line tools utility.

```
intersection/.code \sim args=\{(#1)(#2)--(#3)(#4)\}\{
495
      {\tl_use:N \l__trt_fr_output_name_tl}
496
    },
497
    foot \sim of \sim perpendicular/.code \sim args={(#1)--(#2)(#3)}{
498
     \__trt_lt_return_perpendicular_coordinate:nnnn {#1} {#2} {#3}
500
       {\tl_use:N \l__trt_fr_output_name_tl}
    },
501
```

(End definition for intersection and foot of perpendicular. These functions are documented on page *6.*)

initialize_barycentric The front-end of the barycentric coordinate system.

```
initialize ~ barycentric/.code ~ args={(#1)(#2)(#3)}{
      \__trt_bc_initialize:nnn {#1} {#2} {#3}
503
504
    },
```

(End definition for initialize barycentric. This function is documented on page 8.)

incenter

The front-end of the triangle centers X_1 to X_4 and the excenter.

```
excenter
    centroid
circumcenter
 orthocenter
```

```
incenter/.code \sim args=\{(#1)(#2)(#3)\}\{
      \trt_sp_incenter:nnnn {#1} {#2} {#3} {trt@tmp@center}
       \polinimes pgfkeysalso{foot ~ of ~ perpendicular=(trt@tmp@center)--(#1)(#2)}
      \trt_distance:nnN {\tl_use:N \l__trt_fr_output_name_tl} {trt@tmp@center}
509
         \l__trt_fr_radius_fp
       \cs_gset_nopar:Npx \trtradius { \fp_to_dim:N \l__trt_fr_radius_fp }
510
       \coordinate (\tl_use:N \l__trt_fr_output_name_tl) at (trt@tmp@center);
    },
    excenter/.code ~ args={(#1)(#2)(#3)}{
```

\coordinate (\tl_use:N \l__trt_fr_output_name_tl) at (trt@tmp@center);

```
514
      \trt_sp_excenter:nnnn {#1} {#2} {#3} {trt@tmp@center}
       \pgfkeysalso{foot ~ of ~ perpendicular=(trt@tmp@center)--(#2)(#3)}
       \trt_distance:nnN {\tl_use:N \l__trt_fr_output_name_tl} {trt@tmp@center}
516
         \l__trt_fr_radius_fp
517
       \cs_gset_nopar:Npx \trtradius { \fp_to_dim:N \l__trt_fr_radius_fp }
518
```

```
520
     centroid/.code ~ args={(#1)(#2)(#3)}{
      \trt_sp_centroid:nnnn {#1} {#2} {#3}
        {\tl_use:N \l__trt_fr_output_name_tl}
```

```
},
524
     circumcenter/.code ~ args={(#1)(#2)(#3)}{
      \trt_sp_circumcenter:nnnn {#1} {#2} {#3}
        {\tl_use:N \l__trt_fr_output_name_tl}
```

```
\trt_distance:nnN {\tl_use:N \l__trt_fr_output_name_tl} {#1}
528
         \l__trt_fr_radius_fp
529
530
       \cs_gset_nopar:Npx \trtradius { \fp_to_dim:N \l__trt_fr_radius_fp }
     },
     orthocenter/.code ~ args={(#1)(#2)(#3)}{
```

\trt_sp_orthocenter:nnnn {#1} {#2} {#3} {\tl_use:N \l__trt_fr_output_name_tl} 535 },

triangle_center This key is used to access all centers. I don't give any centers from X_5 a key – this key is necessary to construct them.

```
triangle \sim center/.code \sim args={(#1)(#2)(#3)(#4)}{
536
       \int_case:nnF {#4}
537
538
         {
           {1} {
             \pgfkeysalso{incenter=(#1)(#2)(#3)}
           }
541
542
           {2} {
             \pgfkeysalso{centroid=(#1)(#2)(#3)}
544
           {3} {
545
             \pgfkeysalso{circumcenter=(#1)(#2)(#3)}
           {4} {
548
             \pgfkeysalso{orthocenter=(#1)(#2)(#3)}
549
           }
           {5} {
             \trt_sp_ninepointcenter:nnnn {#1} {#2} {#3}
                {\tl_use:N \l__trt_fr_output_name_tl}
             \group_begin:
                \__trt_bc_initialize:nnn {#1} {#2} {#3}
               \coordinate (trt@tmp@mid) at (bc3 \sim cs \c_colon_str 1,1,0);
             \group_end:
             \trt_distance:nnN {\tl_use:N \l__trt_fr_output_name_tl} {trt@tmp@mid}
558
               \l__trt_fr_radius_fp
             \cs_gset_nopar:Npx \trtradius { \fp_to_dim:N \l__trt_fr_radius_fp }
560
           }
           {6} {
             \trt_sp_symmedian:nnnn {#1} {#2} {#3}
563
                {\tl_use:N \l__trt_fr_output_name_tl}
564
565
           }
           {7} {
             \trt_sp_gergonne:nnnn {#1} {#2} {#3}
567
                {\tl_use:N \l__trt_fr_output_name_tl}
568
           }
           {8}
             \trt_sp_nagel:nnnn {#1} {#2} {#3}
571
                {\tl_use:N \l__trt_fr_output_name_tl}
573
           }
574
           {9} {
             \trt_sp_mittenpunkt:nnnn {#1} {#2} {#3}
                {\tl_use:N \l__trt_fr_output_name_tl}
576
           }
           {10} {
             \trt_sp_spieker:nnnn {#1} {#2} {#3} {trt@tmp@center}
580
             \group_begin:
               \__trt_bc_initialize:nnn {#1} {#2} {#3}
               \coordinate (trt@tmp@midi) at (bc3 \sim cs \c_colon_str 1,1,0);
582
               \coordinate (trt@tmp@midii) at (bc3 \sim cs \c_colon_str 0,1,1);
583
             \group_end:
             \pgfkeysalso{
                foot~of~perpendicular=(trt@tmp@center)--(trt@tmp@midi)(trt@tmp@midii)
586
             }
587
             \trt_distance:nnN {\tl_use:N \l__trt_fr_output_name_tl} {trt@tmp@center}
588
               \l__trt_fr_radius_fp
             \cs_gset_nopar:Npx \trtradius { \fp_to_dim:N \l__trt_fr_radius_fp }
590
             \coordinate (\tl_use:N \l__trt_fr_output_name_tl) at (trt@tmp@center);
591
           }
593
         }
         {
594
```

```
595 \msg_error:nnn {triangletools} {center-not-found} {#4}
596     }
597   }
598 }
599 (/frontend)
```

(End definition for triangle center. This function is documented on page 4.)

Index

The italic numbers denote the pages where the corresponding entry is described, numbers underlined point to the definition, all others indicate the places where it is used.

Symbols	245, 320, 321, 324, 325, 328, 329,
/tikz/trt 2, 487	330, 334, 341, 345, 386, 387, 390, 391
_	\fp_to_dim:N 15, 129,
В	160, 224, 225, 339, 340, 350, 351,
bc3	367, 392, 393, 510, 518, 530, 560, 590
bool commands:	
\bool_if:NTF 198	G
\bool_new:N 178	group commands:
\bool_set_true:N 182	\group_begin:
	286, 298, 310, 317, 372,
C	381, 398, 410, 423, 436, 464, 554, 580
centroid $\dots 3, \underline{505}$	\group_end: 294, 306, 313, 368, 377,
circumcenter	394, 406, 419, 432, 460, 473, 557, 584
\coordinate 128, 159, 338, 349,	
366, 392, 511, 519, 556, 582, 583, 591	I
cs commands:	incenter $\dots 3, \underline{505}$
\cs_gset_nopar:Npn 483	initialize_barycentric $8, \underline{502}$
\cs_gset_nopar:Npx	\input 9, 10, 11, 12, 13, 14
510, 518, 530, 560, 590	int commands:
\cs_new:Npn	$\int \int \int dx $
75, 98, 113, 124, 131, 141, 155,	\int_new:N 481
180, 206, 235, 256, 284, 296, 308,	intersection $6, \underline{494}$
315, 370, 379, 396, 408, 421, 434, 462	
	M
E	msg commands:
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	\msg_error:nn 203, 485
excenter	\msg_error:nn
excenter $3, 505$	\msg_error:nn
excenter	\msg_error:nn
excenter	\msg_error:nn
excenter	$\begin{tabular}{lllllllllllllllllllllllllllllllllll$
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	\msg_error:nn
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	\msg_error:nn
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	\msg_error:nn
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$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	\msg_error:nn
	$\begin{tabular}{lllllllllllllllllllllllllllllllllll$
F foot_of_perpendicular 6, 494 fp commands: \fp_compare:nNnTF 101 \fp_eval:n	\msg_error:nn
F foot_of_perpendicular 6, 494 fp commands: \fp_compare:nNnTF 101 \fp_eval:n	$\begin{tabular}{lllllllllllllllllllllllllllllllllll$
F foot_of_perpendicular 6, 494 fp commands: \fp_compare:nNnTF 101 \fp_eval:n	\msg_error:nn
F foot_of_perpendicular 6, 494 fp commands: \fp_compare:nNnTF 101 \fp_eval:n	$\begin{tabular}{lllllllllllllllllllllllllllllllllll$
F foot_of_perpendicular 6, 494 fp commands: \fp_compare:nNnTF 101 \fp_eval:n	\msg_error:nn
F foot_of_perpendicular 6, 494 fp commands: \fp_compare:nNnTF 101 \fp_eval:n	\msg_error:nn
F foot_of_perpendicular 6, 494 fp commands: \fp_compare:nNnTF 101 \fp_eval:n	\msg_error:nn
F foot_of_perpendicular 6, 494 fp commands: \fp_compare:nNnTF 101 \fp_eval:n	\msg_error:nn
F foot_of_perpendicular 6, 494 fp commands: \fp_compare:nNnTF 101 \fp_eval:n	\msg_error:nn

192, 237, 238, 241, 242, 318, 319,	<pre>\trt_sp_ninepointcenter:nnnn</pre>
322, 323, 326, 327, 384, 385, 388, 389	
\protect 56	<pre>\trt_sp_orthocenter:nnnn</pre>
\ProvidesExplFile 17, 60, 164, 229, 264, 477	
\ProvidesExplPackage 5	$\text{trt_sp_spieker:nnnn} \dots 5, \underline{462}, 579$
	\trt_sp_symmedian:nnnn 4, <u>396</u> , 563
Q	trt internal commands:
quark commands:	\ltrt_bc_anchor_iiix_fp
\q_stop 200, 206	<u>169</u> , 193, 216
.	\ltrt_bc_anchor_iiiy_fp
R	
\RequirePackage	\ltrt_bc_anchor_iix_fp <u>169</u> , 189, 215
S	\ltrt_bc_anchor_iiy_fp <u>169</u> , 190, 221
\space	\ltrt_bc_anchor_ix_fp <u>169</u> , 185, 214
str commands:	\ltrt_bc_anchor_iy_fp <u>169</u> , 186, 220
\c_colon_str 290, 302, 312,	\trt_bc_initialize:nnn
402, 414, 427, 440, 468, 556, 582, 583	<i>15</i> , <u>180</u> , 287, 299, 311,
102, 111, 127, 110, 100, 330, 302, 303	399, 411, 424, 437, 465, 503, 555, 581
T	<pre>\ltrt_bc_initialized_bool</pre>
T _E X and L ^o T _E X 2_{ε} commands:	
\pgf@x 224	\ltrt_bc_lambda_i_fp
\pgf@y 225	<u>175,</u> 209, 214, 220
\trt@tmp@i 7, 77, 79, 81, 83,	\ltrt_bc_lambda_ii_fp
135, 137, 183, 185, 187, 189, 191,	<u>175</u> , 210, 215, 221
193, 237, 239, 241, 243, 318, 320,	\ltrt_bc_lambda_iii_fp
322, 324, 326, 328, 384, 386, 388, 390	
\trt@tmp@ii 7, 78, 80, 82, 84,	\trt_bc_parse:w 200, <u>206</u>
136, 138, 184, 186, 188, 190, 192,	\ltrt_bc_tmp_fp
194, 238, 240, 242, 244, 319, 321,	<u>179</u> , 208, 209, 210, 211
323, 325, 327, 329, 385, 387, 389, 391	\ltrt_d_pointi_x_fp
\tikzdeclarecoordinatesystem 196	
\tikzset 487	\ltrt_d_pointi_y_fp
tl commands:	231, 240, 251, 252
\tl_new:N 479	\ltrt_d_pointii_x_fp
\tl_set:Nn 480, 492	\\trt_d_pointii_y_fp
\tl_use:N 496, 500, 508,	
511, 516, 519, 523, 527, 528, 534,	\\trt_fr_center_number_int 481
553, 558, 564, 568, 572, 576, 588, 591 triangle_center	\ltrt_fr_output_name_tl
trt commands:	479, 492, 496, 500, 508,
\trt_distance:nnN 9, 9, 235,	511, 516, 519, 523, 527, 528, 534,
258, 259, 260, 508, 516, 528, 558, 588	553, 558, 564, 568, 572, 576, 588, 591
\trt_distance_triangle:nnnNNN 9,	\ltrt_fr_radius_fp 482, 509, 510,
256, 288, 300, 400, 412, 425, 438, 466	517, 518, 529, 530, 559, 560, 589, 590
\trt_lt_get_intersection	\ltrt_i_pointi_x_fp 79,91,95
coordinate:nnnnNN 7, 113, 126	\ltrt_i_pointi_y_fp 80,87,95
\trt_lt_get_intersection	\ltrt_i_pointii_x_fp 83,91
line:NNNNNNN	\ltrt_i_pointii_y_fp 84,87
	\ltrt_lt_linei_a_fp
\trt_lt_get_line_equation:nnNNN	<u>66</u> , 116, 120, 144, 147, 151
6, <u>75</u> , 115, 117, 143, 352, 357	\ltrt_lt_linei_b_fp
<pre>\trt_lt_get_perpendicular</pre>	<u>66</u> , 116, 120, 144, 147, 151
coordinate:nnnNN 8, 13, <u>141</u> , 157	<pre>\ltrt_lt_linei_c_fp</pre>
<pre>\trt_lt_get_perpendicular</pre>	<u>66</u> , 116, 120, 144, 147, 151
equation:nNNNNNN	\ltrt_lt_lineii_a_fp
7, <u>131</u> , 146, 354, 359	
$\trt_sp_centroid:nnnn 3, 308, 522$	\ltrt_lt_lineii_b_fp
\trt_sp_circumcenter:nnnn	
3, <u>315</u> , 382, 526	\ltrt_lt_lineii_c_fp
\trt_sp_excenter:nnnn 3, <u>296</u> , 514	
\trt_sp_gergonne:nnn 4, <u>408</u> , 567	\ltrt_lt_pointi_x_fp62
\trt_sp_incenter:nnn 3, <u>284</u> , 506	\\trt_\t_pointi_y_fp62
\trt_sp_mittenpunkt:nnnn . 5, <u>434</u> , 575	\\trt_\t_pointii_x_fp62
$\trt_sp_nagel:nnnn \dots 5, 421, 571$	\ltrt_lt_pointii_y_fp <u>62</u>

\trt_lt_return_intersection:nnnnn	
<u>124,</u> 375, 495	<pre>\ltrt_sp_coordinateii_x_fp</pre>
\trt_lt_return_perpendicular	<u>269,</u> 324, 330, 332, 339
coordinate:nnnn <u>155</u> , 373, 374, 499	\ltrt_sp_coordinateii_y_fp
\ltrt_lt_tmp_fp	
<i>12</i> , <u>72</u> , 100, 101, 109, 110	\ltrt_sp_coordinateiii_x_fp
\ltrt_lt_tmpa_fp	
<u>72</u> , 127, 129, 137, 139, 158, 160	\ltrt_sp_coordinateiii_y_fp
\ltrt_lt_tmpb_fp	
72, 127, 129, 138, 139, 158, 160	\ltrt_sp_linei_a_fp <u>269</u> , 356, 363
\ltrt_sp_a_fp <u>266</u> ,	\ltrt_sp_linei_b_fp <u>269</u> , 356, 363
289, 291, 301, 303, 401, 403, 413,	\ltrt_sp_linei_c_fp <u>269</u> , 356, 363
415, 416, 417, 426, 428, 429, 430,	\ltrt_sp_lineii_a_fp . <u>269</u> , 361, 364
439, 443, 444, 450, 456, 467, 470, 471	\ltrt_sp_lineii_b_fp . <u>269</u> , 361, 364
\ltrt_sp_b_fp <u>266</u> ,	\ltrt_sp_lineii_c_fp . <u>269</u> , 361, 364
289, 292, 301, 304, 401, 404, 413,	\ltrt_sp_tmpa_fp <u>281</u> , 353,
415, 416, 417, 426, 428, 429, 430,	355, 358, 360, 365, 367, 386, 390, 392
439, 444, 449, 450, 456, 467, 469, 471	\ltrt_sp_tmpb_fp <u>281</u> , 353,
\ltrt_sp_c_fp <u>266</u> ,	355, 358, 360, 365, 367, 387, 391, 393
289, 293, 301, 305, 401, 405, 413,	\ltrt_sp_tmpc_fp
415, 416, 417, 426, 428, 429, 430,	<u>281</u> , 353, 355, 358, 360
439, 444, 450, 455, 456, 467, 469, 470	\ltrt_tmp_a_fp 212, 224
\ltrt_sp_coordinatei_x_fp	\ltrt_tmp_b_fp 218, 225
<u>269</u> , 320, 332, 343	\trtradius 2,
\ltrt_sp_coordinatei_y_fp	10, 56, 482, 510, 518, 530, 560, 590