Let the model be

$$B = (c, A_1, \dots, A_p); \quad (m \times (mp+1)) \tag{1}$$

$$x_t = \begin{bmatrix} 1 \\ y_{t-1} \\ \vdots \\ y_{t-p} \end{bmatrix}; \quad ((mp+1) \times 1) \tag{2}$$

$$\tag{3}$$

and define

$$X = \begin{bmatrix} x_1' \\ x_2' \\ \vdots \\ x_T' \end{bmatrix}; \quad (T \times (mp+1)) \tag{4}$$

$$y = \begin{bmatrix} (y_{11} \dots y_{1T})' \\ \vdots \\ (y_{m1} \dots y_{mT})' \end{bmatrix}; \quad (Tm \times 1) \tag{5}$$

and $\alpha = \text{vec}(B)$, that is, $m(mp+1) \times 1$ vector. VAR(p) can be written as

$$y = (I_m \otimes X)\alpha + \epsilon, \tag{6}$$

where $\epsilon \sim \mathcal{N}(0, \Omega^{-1} \otimes I_T)$.

- $L(\Omega, \alpha \mid y, X) \propto |\Omega|^{-T/2} \exp\left[-\frac{1}{2}(y - (I_m \otimes X)\alpha)'(\Omega^{-1} \otimes I_T)(y - (I_m \otimes X)\alpha)\right]$
- $\pi(\alpha) \propto \exp\left[-\frac{1}{2}(\alpha - \mu_\alpha)'\Sigma_\alpha^{-1}(\alpha - \mu_\alpha)\right]$
- $\pi(\Omega) \propto |\Omega|^{-(b-m-1)/2} \exp\left[-\frac{1}{2}\,\text{tr}(D^{-1}\Omega^{-1})\right]$

0.1 Estimating α

$$\pi(\alpha \mid \text{rest}) \propto \exp\left[-\frac{1}{2}\left(\alpha'\left((I_m \otimes X)'\left(\Omega^{-1} \otimes I_T\right)(I_m \otimes X) + \Sigma_\alpha^{-1}\right)\alpha - 2\alpha'\left(\Sigma_\alpha^{-1}\mu_\alpha + (I_m \otimes X)'\left(\Omega^{-1} \otimes I_m\right)y\right)\right)\right] \tag{7}$$

Thus, $\alpha \mid \text{rest} \sim \mathcal{N}\left(\mu_\alpha^T, \Sigma_\alpha^T\right)$ where

$$\Sigma_\alpha^T = \left(\left(\Omega^{-1} \otimes X'X\right) + \Sigma_\alpha^{-1}\right)^{-1} \tag{8}$$

$$\mu_\alpha^T = \Sigma_\alpha^p \left(\Sigma_\alpha^{-1}\mu_\alpha + \left(\Omega^{-1} \otimes X'\right) y\right) \tag{9}$$

0.2 Estimating Ω

If we define

$$Y = \begin{bmatrix} y_{11} & y_{21} & \cdots & y_{m1} \\ y_{12} & y_{22} & \cdots & y_{m2} \\ \cdots & \cdots & \ddots & \vdots \\ y_{1T} & y_{2T} & \cdots & y_{mT} \end{bmatrix}; \quad (T \times m) \tag{10}$$

In fact, $y = \operatorname{vec} Y$. By the identity

$$\operatorname{tr}(A_1 A_2 A_3) = \operatorname{vec}(A_1)'(A_3 \otimes I) \operatorname{vec}(A_2) \tag{11}$$

$$= \operatorname{vec}(A_2)'(A_1 \otimes I) \operatorname{vec}(A_3) \tag{12}$$

$$= \operatorname{vec}(A_3)'(A_2 \otimes I) \operatorname{vec}(A_1) \tag{13}$$

we can get the following relation.

$$(y - (I_m \otimes X)\,\alpha)' \left(\Omega^{-1} \otimes I_T\right) (y - (I_m \otimes X)\,\alpha) = \operatorname{tr}\left(\left(Y' - BX'\right)\left(Y - XB'\right)\Omega^{-1}\right) \tag{14}$$

$$\pi(\Omega \mid \text{rest}) \propto |\Omega|^{-(T+b-m-1)/2} \exp\left[-\frac{1}{2}\left(\operatorname{tr}\left(D^{-1}\Omega^{-1}\right) + \operatorname{tr}\left((Y' - BX')(Y - XB')\Omega^{-1}\right)\right)\right] \tag{15}$$

$$= |\Omega|^{-(T+b-m-1)/2} \exp\left[-\frac{1}{2}\operatorname{tr}\left(\left(D^{-1} + \left(Y' - BX'\right)\left(Y - XB'\right)\right)\Omega^{-1}\right)\right] \tag{16}$$

Therefore, $\Omega \mid \text{rest} \sim \mathcal{W}_m\left(T + b, R_T\right)$ where

$$R_T = \left(D^{-1} + \left(Y' - BX'\right)\left(Y - XB'\right)\right)^{-1} \tag{17}$$