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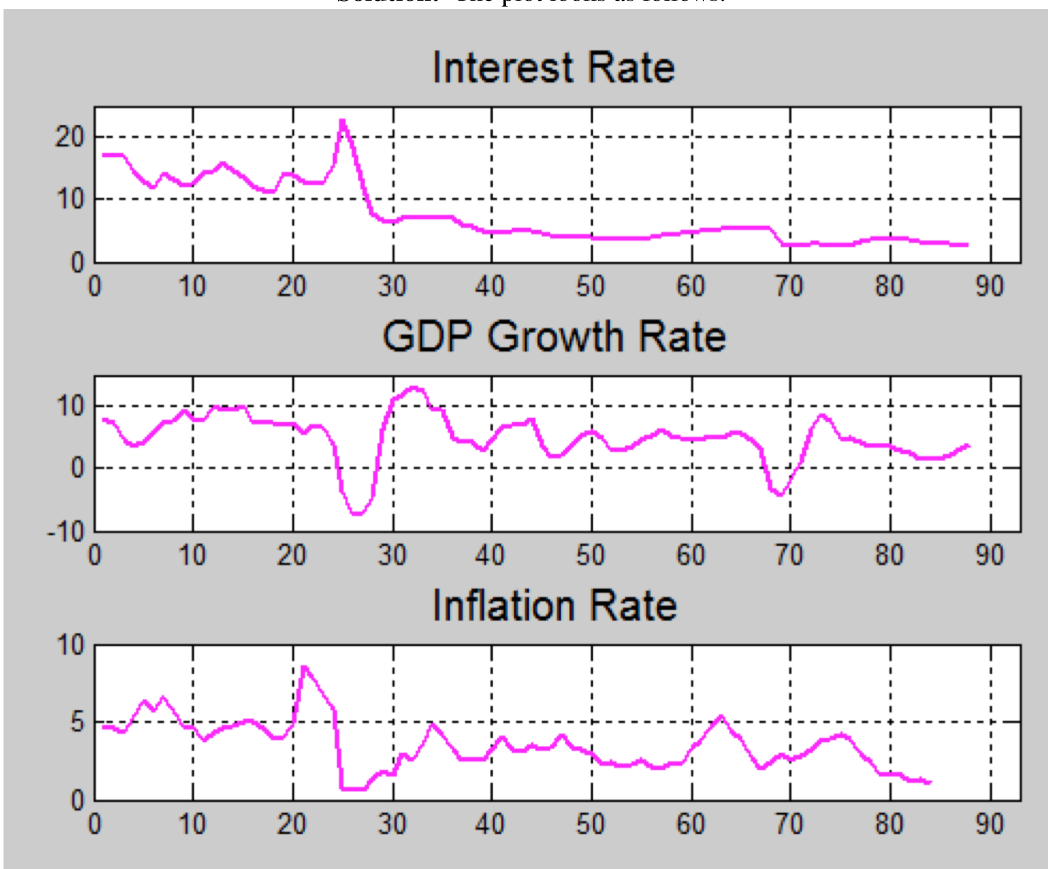
Economic Time Series

Fall Semester, 2016

Download CD.txt, GDP.txt, CPI.txt from the course webpage. CD.txt contains quarterly CD (91 days) rates from 1991.Q1 to 2013.Q4; GDP.txt contains GDP growth rates (year-on-year) from 1971.Q1 to 2013.Q4; and CPI.txt contains consumer price index from 1965.Q1 to 2013.Q4.

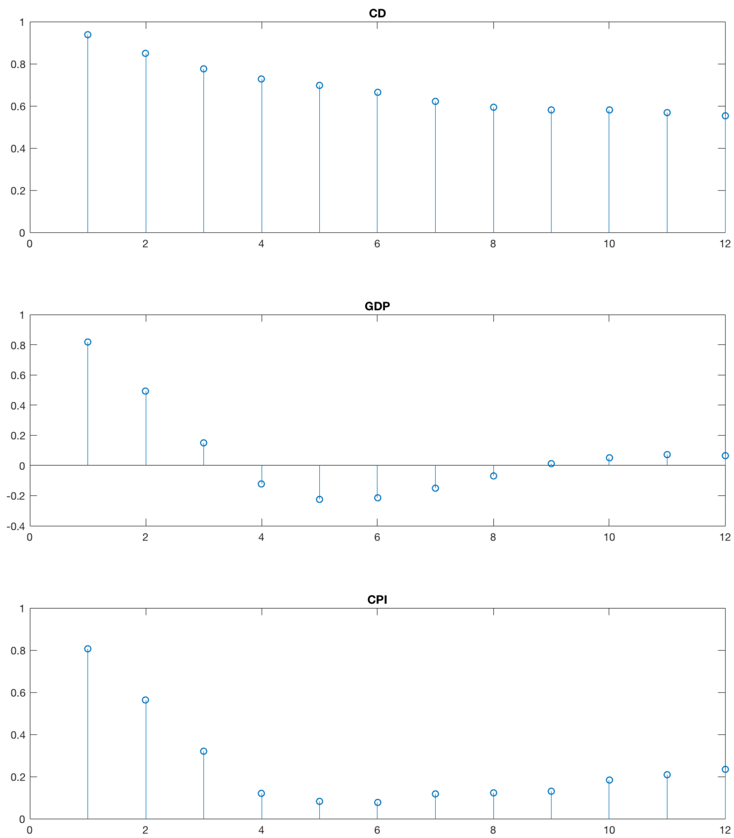
1. Plot interest rates (CD rates), growth rates (GDP growth rates), and inflation rates from 1991.Q1 to 2013.Q4 in a figure with 3 by 1 format. Inflation rates are obtained as $100 \times (\log \mathrm{CPI}_t - \log \mathrm{CPI}_{t-4})$.

Solution: The plot looks as follows.



2. Compute the autocorrelation functions of the three variables and report the results as the following figure.

Solution: The plot is as follows.



3. Test the hypothesis of white noise for each of the three variables, $H_0 : \rho(1) = \cdots = \rho(\tau_{\max}) = 0$, where $\tau_{\max} = 6$. Report the p-values.

Solution: P-values are all zero. Therefore, we can conclude that the interest rate, GDP growth rate, and inflation rate are not white noise processes.

4. Suppose that each of the three variables is an ARMA(2,1) process,

$$X_t = \mu + a_1 X_{t-1} + a_2 X_{t-2} + e_t + b_1 e_{t-1} \tag{1}$$

$$e_t \sim \mathcal{N}\left(0, \sigma^2\right) \tag{2}$$

Estimate $(a_1, a_2, b_1, \sigma^2)$ for each of the three variables using `chap2_Estimation.m` in the lecture note.

Solution:

- Inflation rate

μ	a_1	a_2	b_1	σ^2
0.5402	−0.0831	0.9895	0.3406	1.7917

- GDP growth rate

μ	a_1	a_2	b_1	σ^2
1.2697	−0.6726	1.4113	−0.1978	3.2555

- Inflation rate

μ	a_1	a_2	b_1	σ^2
1.0489	0.4700	0.2485	0.9734	0.6697

5. Now, consider a VAR system consisting of inflation rates, growth rates, and interest rates.

   (a) Test if one variable Granger-causes another variable. Note that there are 6 permutations in total.

   (b) Plot impulse response functions and forecast error variance decompositions up to 16 horizons. In doing so, identify structural parameters using short-run restrictions by assuming that the causal chain holds in the order of inflation rates, growth rates, and inflation rates. Interpret your results.

   (c) Repeat (b) with long-run restrictions. How do your results change? Explain main differences.

Solution:

(a) Testing if there is a Granger causality between the three variables, the results are as follows.

- H_0 : GDP growth rate does not Granger-cause the interest rate.

p-value
0.0006

- H_0 : The inflation rate does not Granger-cause the interest rate.

p-value
0.9514

- H_0 : The interest rate does not Granger-cause the GDP growth rate.

p-value
0.0.1712

- H_0 : The inflation rate does not Granger-cause the GDP growth rate.

p-value
0.2523

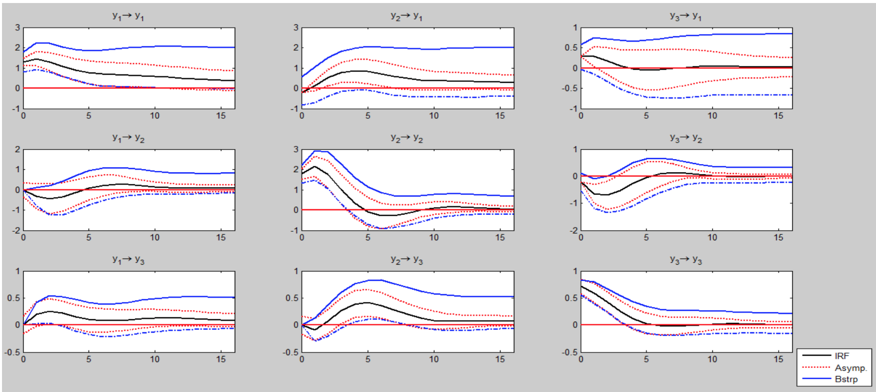
- H_0 : The interest rate does not Granger-cause the inflation rate.

p-value
0.0490

- H_0 : The GDP growth rate does not Granger-cause the inflation rate.

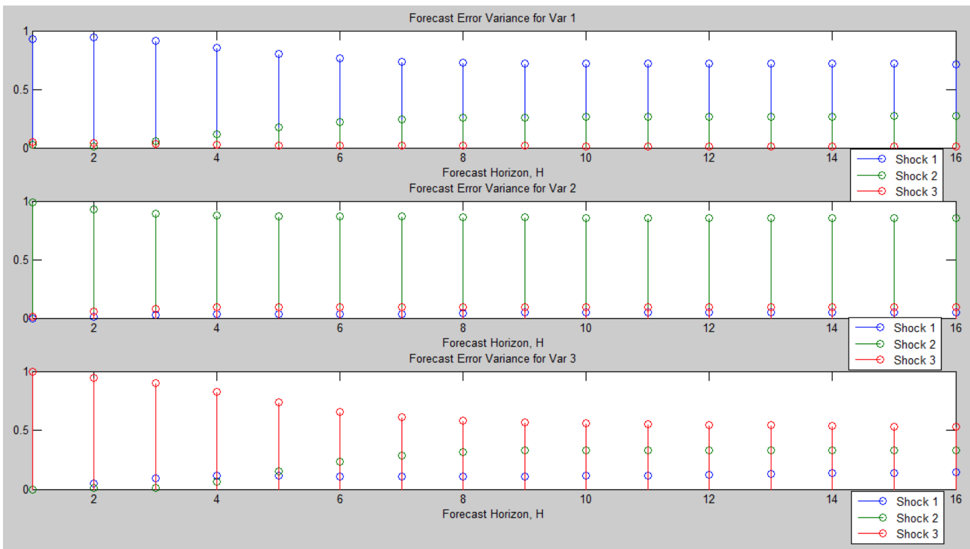
p-value
0.0538

(b) The plot is as follows.



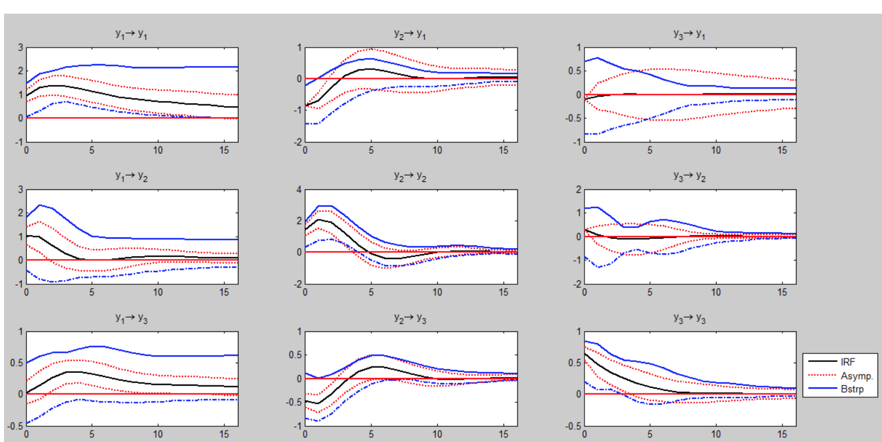
Accordingly to common knowledge, it is known that the interest rate and the growth rate have negative correlation and so do the interest rate and the inflation rate. On the other hand, the inflation rate and the growth rate are known to be correlated positively. However, when we look at the above plots, the $(2, 3)^{\text{th}}$ plot demonstrates that the inflation rate and the growth rate are moving in the opposite direction. Likewise, $(3, 1)^{\text{th}}$ plot tells us that the inflation rate increases—rather than decrease—as the error of the interest rate increases. These results that are in conflict with our common sense showed up because we have omitted some relevant variables from the regression.

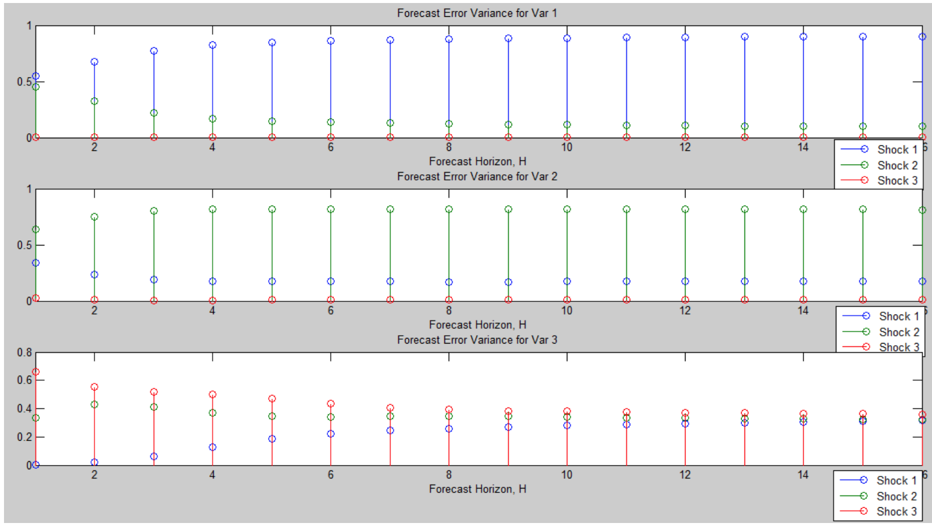
Moving on to the forecast error variance decompositions, below are the plots.



Carefully examining the above plots, we are able to figure out that the contribution of the interest rate to the forecast error variance diminishes over time and at some point becomes stable. The plot with respect to the growth rate shows an increasing trend in the first half and stablizing later. The contribution of the inflation rate is hardly visible in comparison.

(c) The below are the IRF and the variance decompositions under the long-run restrictions.





The IRF plots have changed except for the $(3,1)^{\text{th}}$ one and the diagonal plots. For the $(1,2)^{\text{th}}$ plot, it starts negative unlike the previous one although the overall shape hasn't changed much. The $(1,3)^{\text{th}}$ plot gives us the information that the interest rate is hardly ever affected by the inflation shock.

The main difference of the first variable from the one under short-run restrictions is that the interest rate's contribution to the forecast error variance no longer decreases but rather increases and the growth rate's now decreases and does not increase. The second variable under the long-run restrictions has some influence on the interest rate. The third variable with respect to the growth rate shows decreasing influence and at some point later the three variables have equal contribution.

6. Show that a vector autoregression is a special case of a seemingly unrelated regression.

Solution: For simplicity, consider VAR(1).

$$y_{1t} = \mu_1 + a_{11}y_{1t-1} + a_{12}y_{2t-1} + u_{1t} \tag{3}$$

$$y_{2t} = \mu_2 + a_{21}y_{1t-1} + a_{22}y_{2t-1} + u_{2t} \tag{4}$$

where $\mathrm{Cov}(u_{1t}, u_{2t}) = \sigma_{12}$ for $t = s$, 0 otherwise. The model consists of 2 regression equations with different dependet variables but same independent variables. We could estimate the model using OLS estimator separately for each equation. This falls under the category of a seemingly unrelated regression where variables are contemporaneously correlated—which exactly means $\mathrm{Cov}(u_{1t}, u_{2t}) \neq 0$

for $t = s$—but uncorrelated over time—$\mathrm{Cov}(u_{1t}, u_{2t}) = 0$ otherwise.

7. Exercise #3 on page 59 of the lecture note.

Solution: