Online Variational Bayes Inference for High-Dimensional Correlated Data Review

Daeyoung Lim[*]
Department of Statistics
Korea University

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1 Model specification

The mdoel that was proposed in this paper naturally assumes image data that consist of voxels where a voxel represents a single sample, or a data point, on a regularly spaced, three-dimensional grid. This data point can consist of a single piece of data such as an opacity or multiple pieces of data, such as a color in addition to opacity. The model is as follows:

$$\mathbf{Y}_{it} = \boldsymbol{\eta}_i \boldsymbol{\mu}_{it} + \mathbf{X}_{it} \boldsymbol{\beta}_i + \boldsymbol{\epsilon}_{it} \tag{1}$$

where

$$\mathbf{Y}_{it} : K \times 1 \tag{2}$$
$$\mathbf{X}_{it} : K \times g \tag{3}$$
$$\boldsymbol{\mu}_{it} : m \times 1 \, (m < K) \tag{4}$$
$$\boldsymbol{\eta}_i : K \times m \tag{5}$$
$$\boldsymbol{\beta}_i : g \times 1. \tag{6}$$

1.1 Priors

The priors are as follows:

$$\boldsymbol{\mu}_{it} \sim \mathcal{N}\left(\boldsymbol{\mu}_{i,t-1}, \theta_i^{-1}\mathbf{I}_m\right), \ \boldsymbol{\mu}_{i0} \sim \mathcal{N}\left(\boldsymbol{\mu}_0, v\mathbf{I}_m\right) \tag{7}$$
$$\boldsymbol{\epsilon}_{it} \sim \mathcal{N}\left(\mathbf{0}, \sigma^{-2}\mathbf{I}_K\right) \tag{8}$$
$$\boldsymbol{\eta}_{ij} \sim \mathcal{N}\left(\mathbf{0}, \tau^{-1}\left(\mathbf{I}_K - \rho\mathbf{C}\right)^{-1}\boldsymbol{\Omega}\right)^{*} \tag{9}$$
$$(\boldsymbol{\beta}_i, \theta_i) \sim G, \ G \sim \mathrm{DP}\left(\alpha, \mathcal{N}\left(\boldsymbol{\beta}_{0r}, \boldsymbol{\Sigma}_{0r}\right)\mathrm{Ga}\left(a_\theta, b_\theta\right)\right) \tag{10}$$
$$\sigma^2 \sim \mathrm{Ga}\left(a_\sigma, b_\sigma\right) \tag{11}$$
$$\tau \sim \mathrm{Ga}\left(a_\tau, b_\tau\right) \tag{12}$$
$$\rho_\ell = \frac{\ell}{M}, \ \ell = 0, 1, \ldots, M-1, M-\epsilon \tag{13}$$
$$\phi_\ell = \mathrm{P}\left(\rho = \rho_\ell\right) = \frac{1}{M+1} \tag{14}$$

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[*]Prof. Taeryon Choi

in which elements in ***** need clarification as below.

$$\mathbf{\Omega} = \mathrm{Diag}\left(\frac{1}{d_{1+}}, \ldots, \frac{1}{d_{K+}}\right), \ d_{r+} = \sum_s d_{rs} \tag{15}$$

$$\mathbf{C} : K \times K, \ c_{rs} = \frac{d_{rs}}{d_{r+}}, \quad \mathbf{D} = (d_{rs}) : \text{proximity matrix} \tag{16}$$

$$d_{rs} = \begin{cases} 0 & \text{if } r = s \\ \|\varphi_r - \varphi_s\|^{-\phi}, \ \phi > 0 & \text{otherwise} \end{cases}. \tag{17}$$

2 Variational Bayes

Instead of directly dealing with the Dirichlet process itself, we will use the stick-breaking representation and reformulate it as follows. If we let $\Theta_i = \left(\boldsymbol{\beta}_i', \theta_i'\right)'$,

$$\Theta_i \sim G, \quad G = \sum_{r=1}^{\infty} \pi_r \delta_{\Theta_r^*}, \quad \pi_r = v_r \prod_{\ell < r} (1 - v_\ell), \quad \Theta_r^* = \left(\boldsymbol{\beta}_r^{*\prime}, \theta_r^{*\prime}\right)' \tag{18}$$

$$v_r \sim \mathrm{Be}\left(1, \alpha\right), \quad \alpha \sim \mathrm{Ga}\left(a_\alpha, b_\alpha\right), \boldsymbol{\beta}_r^* \sim \mathcal{N}\left(\boldsymbol{\beta}_{0r}, \boldsymbol{\Sigma}_{0r}\right), \ \theta_r^* \sim \mathrm{Ga}\left(a_\theta, b_\theta\right). \tag{19}$$

Then the parameters of interest are $\mathbf{W} = \left(V, \boldsymbol{\Theta}^*, \boldsymbol{z}, \boldsymbol{\eta}, \boldsymbol{\mu}, \rho, \tau, \sigma^2, \alpha\right)$. Then by mean-field assumption,

$$q\left(\mathbf{W}\right) = q\left(\sigma^2\right) q\left(\alpha\right) q\left(\rho\right) q\left(\tau\right) \left(\prod_{r=1}^{R} q\left(\theta_r^*\right) q\left(\boldsymbol{\beta}_r^*\right) q\left(v_r\right)\right) \left(\prod_{i=1}^{n} q\left(z_i\right)\right)$$
$$\times \left(\prod_{i=1}^{n} \prod_{i=1}^{T} q\left(\mu_{it}\right)\right) \left(\prod_{i=1}^{n} q\left(\boldsymbol{\eta}_i\right)\right)$$

where R is the truncation point of the Dirichlet process.

2.1 Online Variational Bayes

Online variational Bayes only considers one data point at a time which makes it much more practical for large datasets. Sato presented a discount term for the parameters that depend on the ith subject or observation. The only modification occurs here where the variational objective function embraces the discount term and also when updating the variational parameters, we do so by updating the