Cosine Basis Poisson Regression

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1 Poisson

1.1 Model

$$
\begin{aligned}
y &\sim \mathrm{Poi}\left(\exp\left(W\beta + \varphi_J\theta_J\right)\right) \\
\theta_j|\sigma,\tau,\psi &\sim \mathcal{N}\left(0, \sigma^2\tau^2\exp\left[-j\left|\psi\right|\right]\right) \\
\tau^2 &\sim \mathrm{IG}\left(\frac{r_{0,\tau}}{2}, \frac{s_{0,\tau}}{2}\right) \\
\sigma^2 &\sim \mathrm{IG}\left(\frac{r_{0,\sigma}}{2}, \frac{s_{0,\sigma}}{2}\right) \\
\beta &\sim \mathcal{N}\left(\mu_\beta^0, \sigma^2\Sigma_\beta^0\right) \\
\psi &\sim \mathrm{DE}\left(0,\omega_0\right)
\end{aligned}
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1.2 Likelihood

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\begin{aligned}
\ln p\left(y,\Theta\right) = & \, y^\top\left(W\beta+\varphi_J\theta_J\right) - 1_n^\top\exp\left(W\beta+\varphi_J\theta_J\right) - 1_n^\top\ln\left(y!\right) \\
& - \frac{1}{2}\sum_{j=1}^J\left[\ln\left(2\pi\right)+\ln\sigma^2+\ln\tau^2 - j\left|\psi\right| + \frac{\exp\left(j\left|\psi\right|\right)\theta_j^2}{\sigma^2\tau^2}\right] \\
& + \frac{r_{0,\tau}}{2}\ln\frac{s_{0,\tau}}{2} - \ln\Gamma\left(\frac{r_{0,\tau}}{2}\right) - \left(\frac{r_{0,\tau}}{2}+1\right)\ln\tau^2 - \frac{s_{0,\tau}}{2}\frac{1}{\tau^2} \\
& + \frac{r_{0,\sigma}}{2}\ln\frac{s_{0,\sigma}}{2} - \ln\Gamma\left(\frac{r_{0,\sigma}}{2}\right) - \left(\frac{r_{0,\sigma}}{2}+1\right)\ln\sigma^2 - \frac{s_{0,\sigma}}{2}\frac{1}{\sigma^2} \\
& - \frac{1}{2}\left\{\left(p+1\right)\left(\ln\left(2\pi\right)+\ln\sigma^2\right)+\ln\left|\Sigma_\beta^0\right|\right\} - \frac{1}{2\sigma^2}\left(\beta-\mu_\beta^0\right)^\top{\Sigma_\beta^0}^{-1}\left(\beta-\mu_\beta^0\right) \\
& - \ln\frac{\omega_0}{2} - \omega_0\left|\psi\right|
\end{aligned}
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1.3 Update

1.3.1 θ_J

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\begin{aligned}
\mathrm{E}_{-\theta_J}\left(\ln p\left(\theta_J|\text{rest}\right)\right) &\propto \mathrm{E}_{-\theta_J}\left(y^\top\varphi_J\theta_J - 1_n^\top\exp\left(W\beta+\varphi_J\theta_J\right) - \frac{1}{2}\sum_{j=1}^J\frac{\theta_j^2e^{j|\psi|}}{\sigma^2\tau^2}\right) \\
&\propto y^\top\varphi_J\theta_J - 1_n^\top\left(\mathrm{E}\left(\exp\left(W\beta\right)\right)\odot\exp\left(\varphi_J\theta_J\right)\right) - \frac{1}{2}\frac{r_{0,\tau}}{s_{0,\tau}}\frac{r_{0,\sigma}}{s_{0,\sigma}}\theta_J^\top\,\mathrm{Dg}\left(Q_{1:J}\right)\theta_J
\end{aligned}
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