Cosine Basis Logistic

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1 Logistic

1.1 Model

$$
\begin{aligned}
y &\sim \mathrm{Ber}\left(\mathrm{logit}^{-1}\left(\varphi_J \theta_J\right)\right) \\
\theta_j | \tau, \psi &\sim \mathcal{N}\left(0, \tau^2 \exp\left[-j\left|\psi\right|\right]\right) \\
\tau^2 &\sim \mathrm{IG}\left(\frac{r_{0,\tau}}{2}, \frac{s_{0,\tau}}{2}\right) \\
\psi &\sim \mathrm{DE}\left(0, \omega_0\right)
\end{aligned}
$$

1.2 Likelihood

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\begin{aligned}
\ln p\left(y, \Theta\right) = & \, y^\top \varphi_J \theta_J - 1_n^\top \ln\left(1_n + \exp\left(\varphi_j \theta_J\right)\right) \\
& - \frac{1}{2} \sum_{j=1}^{J} \ln\left(2\pi\tau^2\right) - j\left|\psi\right| - \frac{\theta_j^2 e^{j|\psi|}}{\tau^2} \\
& + \frac{r_{0,\tau}}{2} \ln\left(\frac{s_{0,\tau}}{2}\right) - \ln\Gamma\left(\frac{r_{0,\tau}}{2}\right) + \left(\frac{r_{0,\tau}}{2} + 1\right) \ln\frac{1}{\tau^2} - \frac{s_{0,\tau}}{2}\frac{1}{\tau^2} \\
& - \ln\frac{\omega_0}{2} - \omega_0\left|\psi\right|
\end{aligned}
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1.3 Getting around the intractability

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\begin{aligned}
-\ln\left(1 + e^x\right) &= \max_{\xi \in \mathbb{R}} \left\{\lambda\left(\xi\right) x^2 - \frac{1}{2}x + \Psi\left(\xi\right)\right\}, \quad \forall x \in \mathbb{R} \\
\lambda\left(\xi\right) &= -\tanh\left(\xi/2\right) / \left(4\xi\right) \\
\Psi\left(\xi\right) &= \xi/2 - \ln\left(1 + e^{\xi}\right) + \xi \tanh\left(\xi/2\right)/4
\end{aligned}
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then

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\begin{aligned}
-1_n^\top \ln\left(1_n + \exp\left(\varphi_J\theta_J\right)\right) &\geq 1_n^\top \left\{\lambda\left(\xi\right) \odot \left(\varphi_J \theta_J\right)^2 - \frac{1}{2}\varphi_J\theta_J + \Psi\left(\xi\right)\right\} \\
&= \theta_J^\top \varphi_J^\top \mathrm{Dg}\left\{\lambda\left(\xi\right)\right\} \varphi_J \theta_J - \frac{1}{2} 1_n^\top \varphi_J \theta_J + 1_n^\top \Psi\left(\xi\right)
\end{aligned}
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\begin{aligned}
\ln \underline{p}\left(y, \Theta ; \xi\right) = {} & y^{\top} \varphi_J \theta_J - \frac{1}{2} 1_n^{\top} \varphi_J \theta_J + \theta_J^{\top} \varphi_J^{\top} \operatorname{Dg}\left\{\lambda\left(\xi\right)\right\} \varphi_J \theta_J + 1_n^{\top} \Psi\left(\xi\right) \\
& - \frac{1}{2} \sum_{j=1}^{J} \ln\left(2\pi\tau^2\right) - j\left|\psi\right| - \frac{\theta_j^2 e^{j|\psi|}}{\tau^2} \\
& + \frac{r_{0,\tau}}{2} \ln\left(\frac{s_{0,\tau}}{2}\right) - \ln\Gamma\left(\frac{r_{0,\tau}}{2}\right) + \left(\frac{r_{0,\tau}}{2} + 1\right) \ln\frac{1}{\tau^2} - \frac{s_{0,\tau}}{2}\frac{1}{\tau^2} \\
& - \ln\frac{\omega_0}{2} - \omega_0\left|\psi\right|
\end{aligned}
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