Cosine basis

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1   Model specifications

$$y_i = w_i^\top \beta + f(x_i) + \epsilon_i, \qquad \epsilon_i \sim \mathcal{N}\left(0, \sigma^2\right)$$
$$\theta_j | \sigma, \tau, \gamma \sim \mathcal{N}\left(0, \sigma^2 \tau^2 \exp\left[-j\gamma\right]\right)$$
$$\tau^2 \sim \mathrm{IG}\left(\frac{r_{0,\tau}}{2}, \frac{s_{0,\tau}}{2}\right)$$
$$\sigma^2 \sim \mathrm{IG}\left(\frac{r_{0,\sigma}}{2}, \frac{s_{0,\sigma}}{2}\right)$$
$$\beta \sim \mathcal{N}\left(\mu_\beta^0, \Sigma_\beta^0\right)$$
$$\gamma \sim \mathrm{Exp}\left(\omega_0\right)$$
$$|\psi| = \gamma, \quad \psi \sim \mathrm{DE}\left(0, \omega_0\right)$$
$$\varphi_j(x) = \sqrt{2}\cos\left(\pi j x\right)$$

Joint density:

$$p(y, \Theta) = \mathcal{N}\left(y | W\beta + f_J, \sigma^2 I_n\right) \left\{ \prod_{j=1}^{J} \mathcal{N}\left(\theta_j | 0, \sigma^2 \tau^2 \exp\left[-j |\psi|\right]\right) \right\} \mathrm{IG}\left(\tau^2 \Big| \frac{r_{0,\tau}}{2}, \frac{s_{0,\tau}}{2}\right) \mathrm{IG}\left(\sigma^2 \Big| \frac{r_{0,\sigma}}{2}, \frac{s_{0,\sigma}}{2}\right) \mathcal{N}\left(\beta | \mu_\beta^0, \Sigma_\beta^0\right)$$
$$\mathrm{DE}\left(\psi | 0, \omega_0\right)$$

We will use the joint density to derive the LB and updating algorithm. The variational distributions are

$$q_1(\beta) = \mathcal{N}\left(\mu_\beta^q, \Sigma_\beta^q\right)$$
$$q_2(\theta_J) = \mathcal{N}\left(\mu_\theta^q, \Sigma_\theta^q\right)$$
$$q_3(\sigma^2) = \mathrm{IG}\left(\frac{r_{q,\sigma}}{2}, \frac{s_{q,\sigma}}{2}\right)$$
$$q_4(\tau^2) = \mathrm{IG}\left(\frac{r_{q,\tau}}{2}, \frac{s_{q,\tau}}{2}\right)$$
$$q_5(\psi) = \mathrm{NCVMP}.$$

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2 Lower bound

2.1 LB: $\mathsf{E}\left[\ln p\left(y|\Theta\right)\right]$

$$\mathsf{E}\left[\ln p\left(y|\Theta\right)\right] = -\frac{n}{2}\ln\left(2\pi\sigma^2\right) - \frac{1}{2}\mathsf{E}\left[\left(y - W\beta - \varphi_J^\top\theta\right)^\top\left(y - W\beta - \varphi_J^\top\theta\right)\right]$$

$$= -\frac{n}{2}\ln\left(2\pi\sigma^2\right) - \frac{1}{2}\left(y - W\mu_\beta^q - \varphi_J^\top\mu_\theta^q\right)^\top\left(y - W\mu_\beta^q - \varphi_J^\top\mu_\theta^q\right) - \frac{1}{2}\left(\mathrm{Tr}\left(W^\top W\Sigma_\beta^q\right) + \mathrm{Tr}\left(\varphi_J\varphi_J^\top\Sigma_\theta^q\right)\right)$$

2.2 LB: $\mathsf{E}\left[\ln p\left(\theta_j|\sigma,\tau,\psi\right)\right]$

$$\sum_{j=1}^{J}\mathsf{E}\left[\ln p\left(\theta_j|\sigma,\tau,\psi\right)\right] = \sum_{j=1}^{J}\mathsf{E}\left[-\frac{1}{2}\ln\left(2\pi\right) - \ln\sigma - \ln\tau + \frac{j}{2}\left|\psi\right| - \frac{\theta_j^2 e^{j|\psi|}}{2\sigma^2\tau^2}\right]$$