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1. 다음과 같은 다중선형회귀모형

$$Y = \beta_0 + \beta_1 X_1 + \cdots + \beta_p X_p + \epsilon$$

을 고려하자. 여기서, Y는 독립변수를, $X_1, \ldots, X_p$는 설명변수들을, 그리고 $\beta_0, \beta_1, \ldots, \beta_p$는 회귀계수들을 의미하며 ϵ은 평균이 0, 분산이 σ^2인 오차항을 의미한다.

   (a) 이 선형회귀모형을 적합한 후 얻어진 예측값을 $\widehat{Y}$으로 나타낼 때, $\widehat{Y}$의 기댓값 및 분산을 구하시오.

   (b) 이 회귀모형의 잔차(residual)를 $e = Y - \widehat{Y}$으로 나타낼 때, e의 기댓값 및 분산을 구하시오.

   (c) 잔차제곱합(residual sum of squares)의 기댓값을 구하시오.

Solution:

2. Suppose that the conditional distribution of X given that $P = p$ has a binomial distribution wit parameters 5 and p, $X \mid P = p \sim \text{Bin}\left(5, p\right)$ and the marginal distribution of P is a uniform distribution on $(0, 1)$, $P \sim \text{Unif}\left(0, 1\right)$. We would like to calculate the correlation coefficient between X and P.

   (a) Compute variance of X.

   (b) Compute covariance of X and P.

   (c) Compute $\text{Cor}\left(X, P\right)$.

Solution:

3. Let $X_1, X_2, \ldots, X_n \quad (n > 2)$ be a random sample from the following density

$$f\left(x; \theta\right) = \begin{cases} \theta x^{\theta - 1}, & 0 < x < 1,\ 0 < \theta < \infty \\ 0, & \text{elsewhere} \end{cases}$$

   (a) Find the maximum likelihood estimator (MLE) $\widehat{\theta}$ of θ.

   (b) Compare variacne of $\widehat{\theta}$ with the Cramér-Rao lower bound.

Solution:

4. Let $X_1, \ldots, X_n$ be a random sample from the following probability density function(pdf),

$$f(x;\theta) = \theta/x^2, \quad 0 < \theta \leq x < \infty.$$

   1. Find a sufficient statistic for θ.
   2. Find the maximum likelihood estimator (MLE) of θ.
   3. Find the method of moments estimator (MME) of θ.

Solution:

5. Let $X_1, \ldots, X_n$ be a random sample from the following probability density function (pdf),

$$f(x;\theta) = \theta e^{-\theta x}, \quad 0 < x < \infty,$$

where $\theta = \theta_0$ or $\theta = \theta_1$. We assume that known fixed numbers $\theta_1 > \theta_0$.

   (a) Explain the *Neyman-Pearson lemma* briefly.
   (b) Explain the most powerful (MP) test briefly.
   (c) Obtain the MP test for testing $H_0 : \theta = \theta_0$ versus $H_1 : \theta = \theta_1$ by the *Neyman-Pearson lemma*.

Solution: