(a) Example dataset

S	A	B	Y	rank(Y)	ART_A	ART_B	ART_{AB}
s_1	a_1	b_1	0.4	3	7	3	3
s_1	a_1	b_2	3.0	7	6	6	6
s_1	a_2	b_1	0.8	4	3	4	8
s_1	a_2	b_2	1.6	5	1	5	1
s_2	a_1	b_1	0.3	1.5	5	2	2
s_2	a_1	b_2	3.1	8	8	7	7
s_2	a_2	b_1	0.3	1.5	2	1	5
s_2	a_2	b_2	2.3	6	4	8	4

(b) ART calculation

$$ART_A(y_{sij}) = rank(y_{sij} - \overline{y}_{ij} + \overline{y}_{a_i} - \overline{y})$$ estimated main effect a_i

$$ART_B(y_{sij}) = rank(y_{sij} - \overline{y}_{ij} + \overline{y}_{b_j} - \overline{y})$$ estimated main effect b_j

$$ART_{AB}(y_{sij}) = rank(y_{sij} - \overline{y}_{ij} + \overline{y}_{a_i b_j} - \overline{y} - (\overline{y}_{a_i} - \overline{y}) - (\overline{y}_{b_j} - \overline{y}))$$

residual

estimated interaction effect $a_i b_j$

(c) Mean terms

$$\overline{y} = \sum_{sij} y_{sij}$$ grand mean

$$\overline{y}_{a_i} = \sum_{sj} y_{sij}$$ mean for $A = a_i$

$$\overline{y}_{b_j} = \sum_{si} y_{sij}$$ mean for $B = b_j$

$$\overline{y}_{a_i b_j} = \overline{y}_{ij} = \sum_{s} y_{sij}$$ cell mean

mean for $A = a_i$ & $B = b_j$