

Challenges in visualizing results from more modern finite element formulations: high-order, DG, $H(\text{div})$, $H(\text{curl})$...

22 juin 2023, Journée Visu

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65% staff with PhD or Master

High Level customer expertise

20+ years of expertise

Kitware USA, 1998
Kitware Europe, 2010



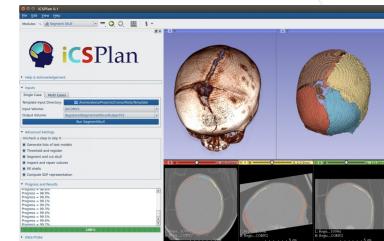
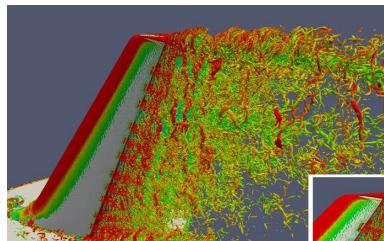
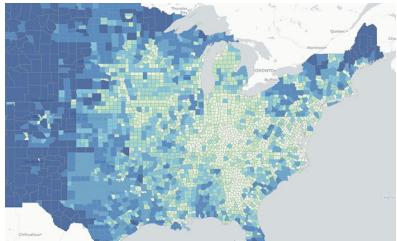
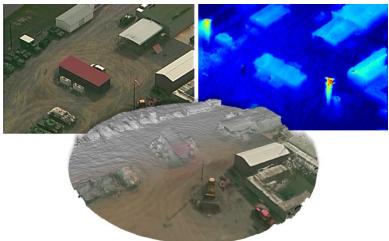
Revenue 2020

\$39M consolidated

Kitware / Services



Areas of expertise / Built on open source



Computer
Vision



Data and
Analytics

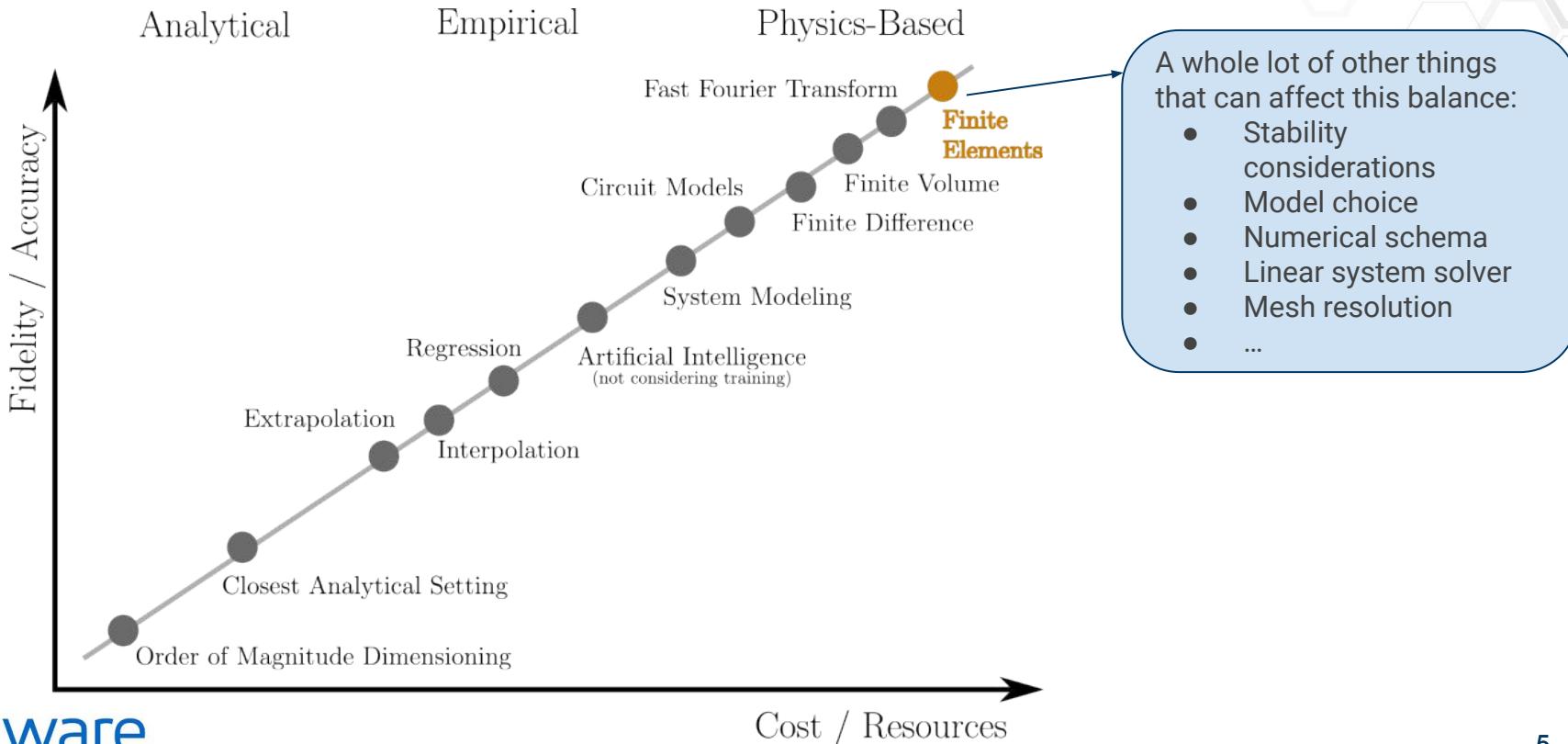


Medical
Computing



Software
Solutions

Numerical Methods: a cost/accuracy balancing act

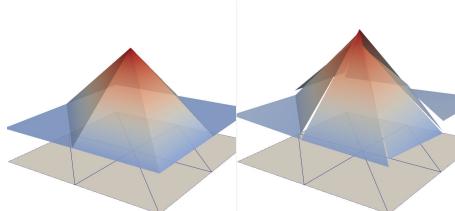
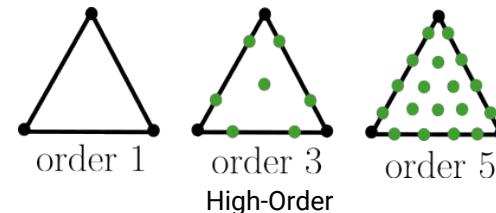


What do we mean by "modern" finite elements?

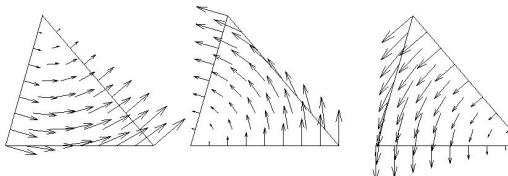
*Anything that is **not** classical finite elements.*

Classical finite elements methods usually have the following properties:

- Piecewise linear per element solutions
- At least C^0 Continuity between elements
- Nodal degrees of freedom
- Scalar basis functions



Discontinuous



Conforming

Jacobsson, Per. "Nedelec elements for computational electromagnetics." (2007).

Reasons for using more complex formulations

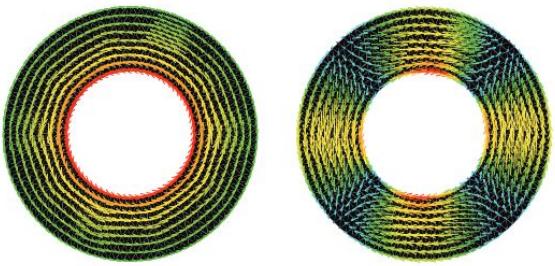
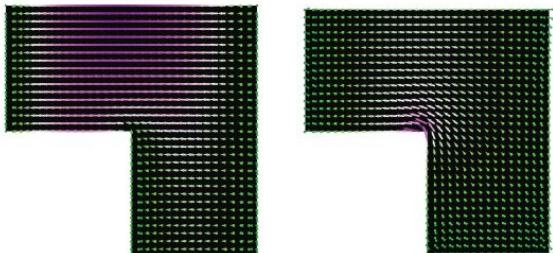


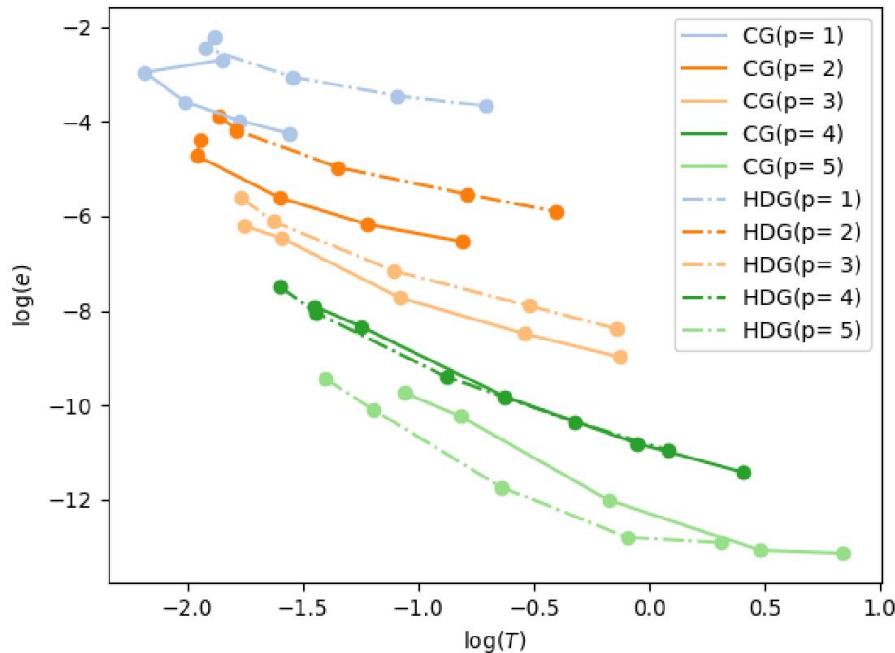
FIGURE 2.4. Approximation of the vector Laplacian on an annulus. The true solution shown here on the right is an (accurate) approximation by a mixed method. It is orthogonal to the harmonic fields and satisfies the differential equation only modulo harmonic fields. The standard Galerkin solution using continuous piecewise linear vector fields, shown on the left, is totally different.



Arnold, Douglas, Richard
Falk, and Ragnar Winther.
"Finite element exterior
calculus: from Hodge theory
to numerical stability."
*Bulletin of the American
mathematical society* 47.2
(2010): 281-354.



FIGURE 2.3. Approximation of the vector Laplacian by the standard finite element method (left) and a mixed finite element method (right). The former method totally misses the singular behavior of the solution near the reentrant corner.

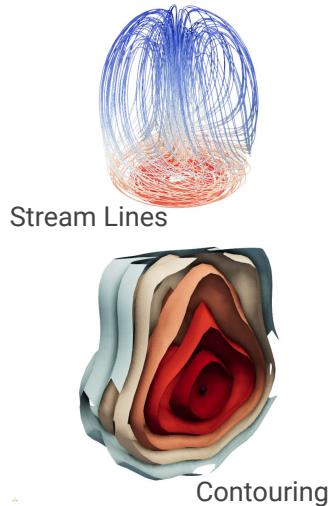
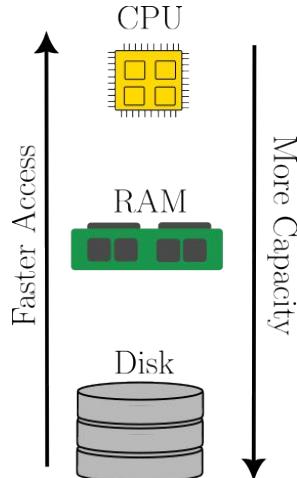
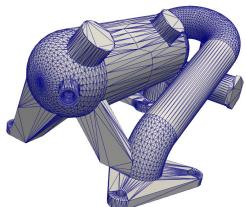


Logarithmic evolution of the error as a function of computational time for an analytical parabolic test case: a comparison between Continuous Galerkin (CG) and Hybrid Discontinuous Galerkin (HDG) approaches at high orders

The big steps from simulation to visualization



$$\begin{cases} \int_{\Omega} F(u, \phi) d\Omega = \int_{\partial\Omega} b(\phi) d\partial\Omega \\ \int_{\partial\Omega} B(u, \phi) d\partial\Omega = \int_{\partial\Omega} f \cdot \phi d\partial\Omega \end{cases}$$



The OpenFoam Motorbike, rendered with the NVIDIA OptiX path tracer in ParaView.

Formalizing the Challenges

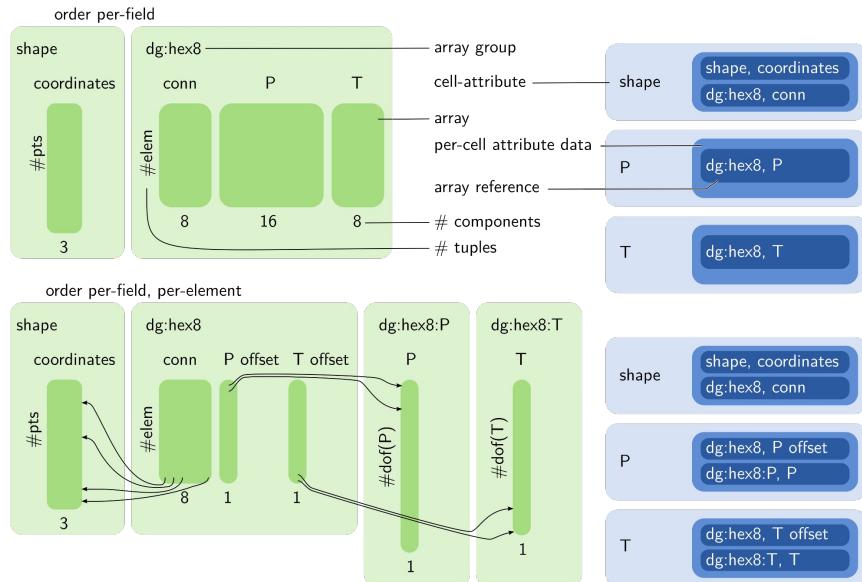


I/O: formatting and interfacing

What format can solutions from modern finite element codes use to describe themselves?

Characteristics of solutions from more modern formulations breaking the mould:

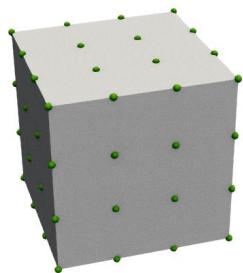
- A zoology of element types
 - Modal vs. nodal degrees of freedom
 - Non-standard interpolation primitives
- Discontinuous point data



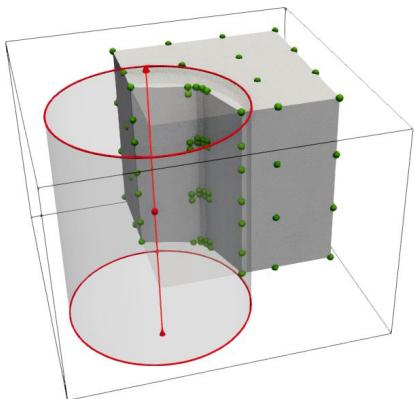
Design for discontinuous galerkin data structure taken from:
[https://discourse.vtk.org/t/discontinuous-galerkin-elements-and-other-novel-cell-type s-function-spaces/9209](https://discourse.vtk.org/t/discontinuous-galerkin-elements-and-other-novel-cell-type-s-function-spaces/9209)

Post-processing: reducing and transforming data

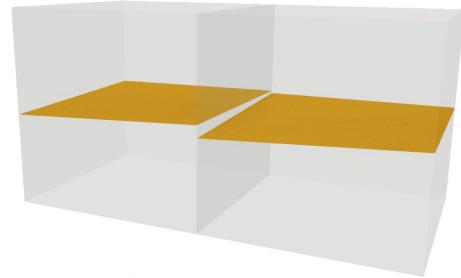
Do basic processing operations have the same implementations for more complex data?



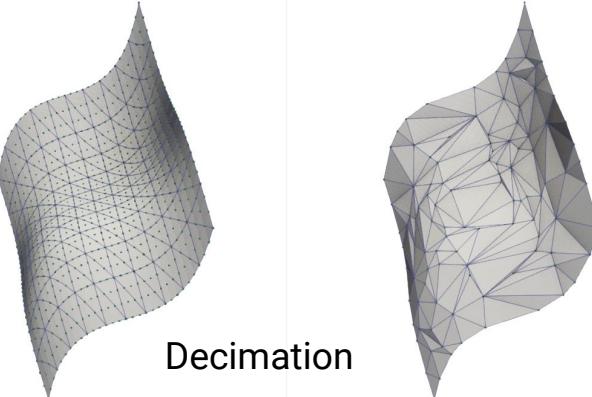
Clipping



Contouring



Decimation



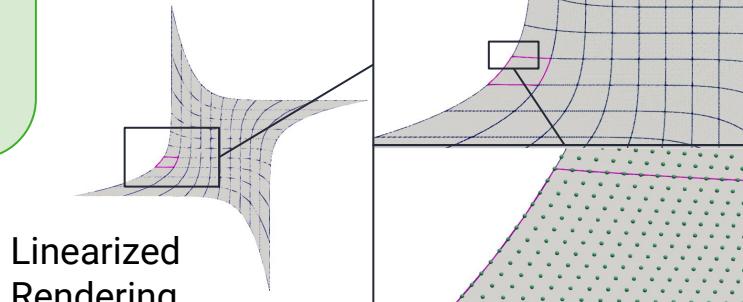
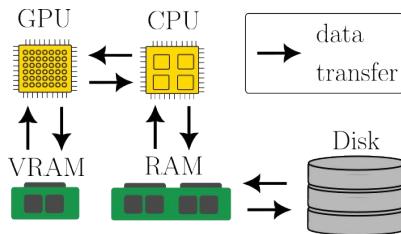
Rendering: from data to image



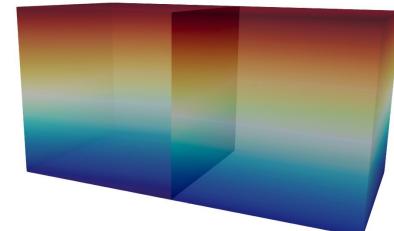
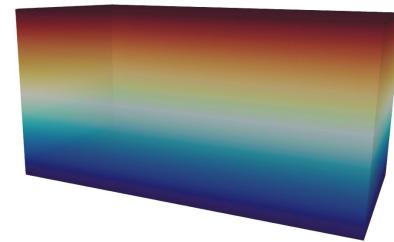
How do we implement graphics primitives based on user formulations? How should graphics primitives behave in discontinuous settings?

2 main challenges:

- Translating special interpolation primitives to the GPU
- Rendering discontinuous data



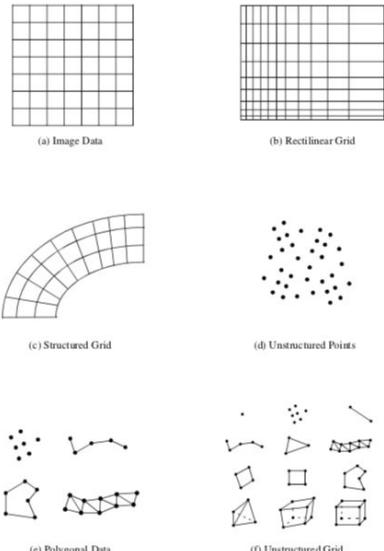
Discontinuous
Rendering
Artifacts



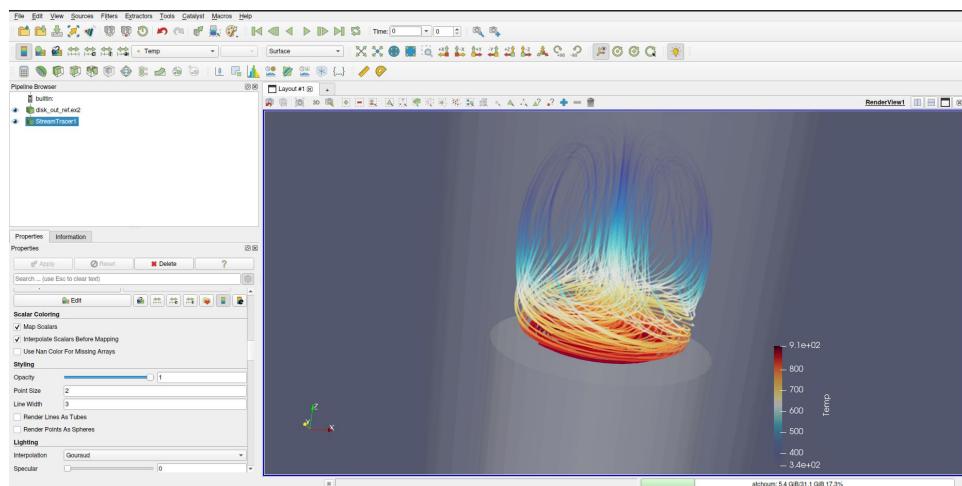
Infrastructure: refactoring tools for new applications

How do we migrate existing visualization tools towards supporting these new data models?

These more modern finite element formulations necessitate fundamental design changes in existing APIs, formats and computational kernels.



VTK Dataset model taken from:
<https://examples.vtk.org/site/VTKBook/05Chapter5/>

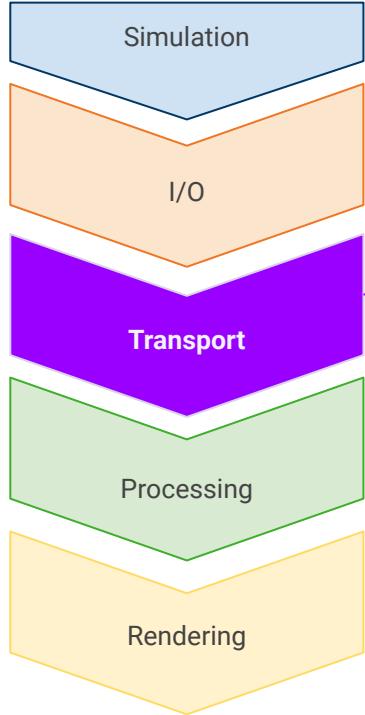


ParaView Screenshot

Promising Solutions



Intermediate solution framework



Extra step to project the high-order / exotic data onto a linear mesh dedicated to visualization

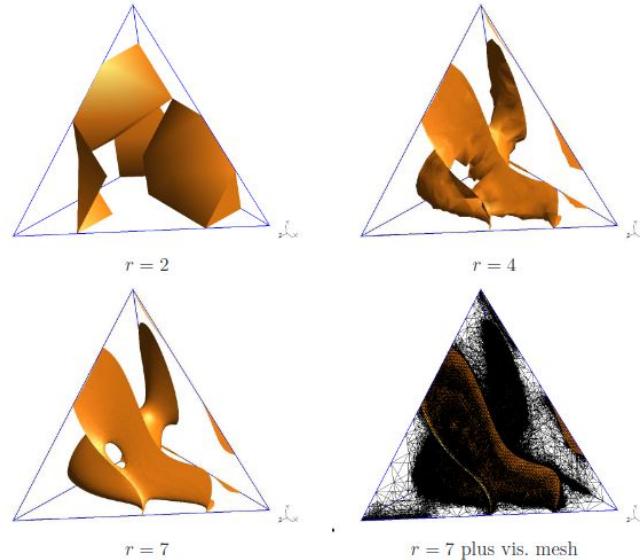


Figure 5. Visualization of one 4th order function on one single tetrahedron. Figures show one iso-surface using different error thresholds. The last Figure (bottom-right) show the visualization mesh together with the iso-surface.

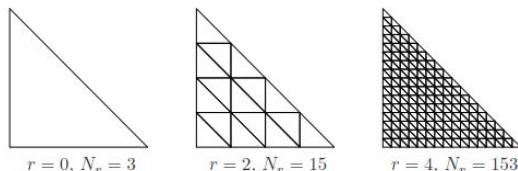
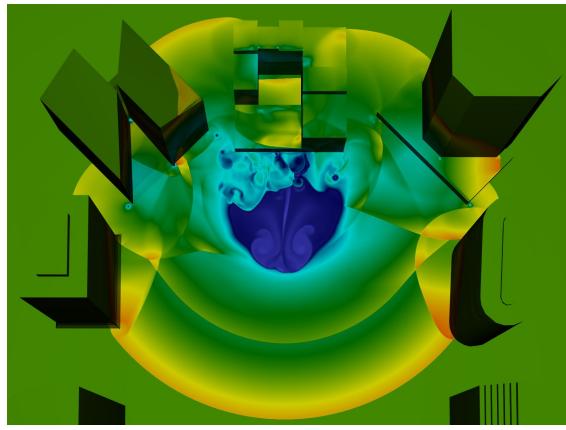


Figure 3. Some AMR refinement templates for triangles at different resolution levels.

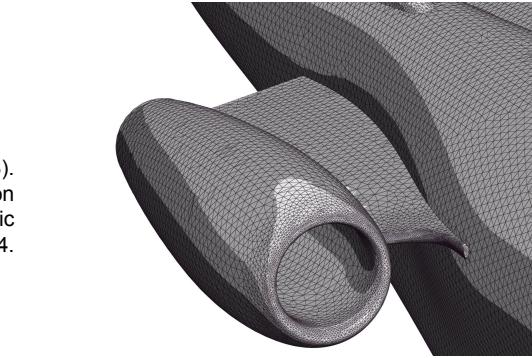
J.-F. Remacle, N. Chevaugeon, E. Marchandise, and C. Geuzaine. Efficient visualization of high-order finite elements. International Journal for Numerical Methods in Engineering, 69(4):750–771, 2007.

Long-term solution framework

Pixel exact approaches
using appropriate
computational
primitives on the GPU.

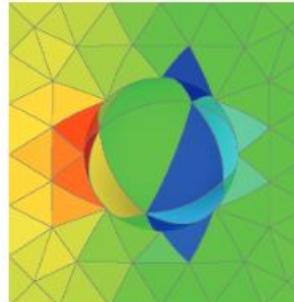


Loseille, Adrien & Feuillet, Rémi. (2018).
Vizir: High-order mesh and solution
visualization using OpenGL 4.0 graphic
pipeline. 10.2514/6.2018-1174.

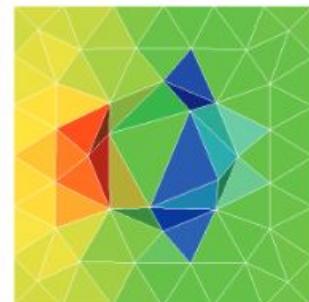


VizIR: https://pyamg.saclay.inria.fr/images/vizir/falconP3_zoom.png

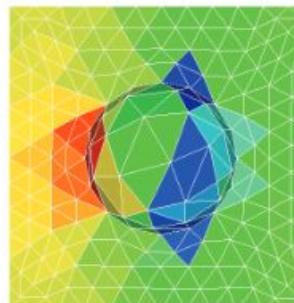
B. Nelson, E. Liu, R. M. Kirby, and R. Haines. Elvis: A system for the accurate and interactive visualization of high-order finite element solutions. *IEEE transactions on visualization and computer graphics*, 18(12):2325–2334, 2012.



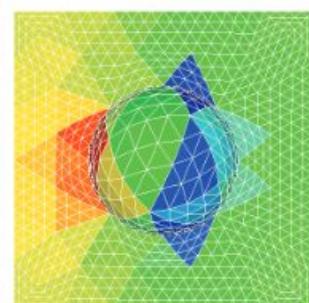
(a) ElVis



(b) Visual3 0 Refinements



(c) Visual3 1 Refinement



(d) Visual3 2 Refinements

Fig. 7. A top-down view of the hemisphere from Case 3. The mesh plotting tools of Visual3 and ElVis are enabled.

Conclusion and Outlook

Conclusions:

- Modern finite element formulations are becoming more mainstream (with good reason)
- Challenges in visualizing these kind of datasets can be divided into 4 categories:
 - Absence of existing standard for **I / O** formatting and design
 - Classical **post-processing** transformations do not behave correctly for more complex data
 - **Rendering** primitives need to be re-designed for these types of datasets
 - Porting current **infrastructure** is not trivial
- 2 generic approaches to these problems stand out: linear projections and native support

Outlook:

- A clear and iterative roadmap needs to be elaborated at the community level
- Best to source dedicated project financing for affecting coherent change



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- Kitware one-to-one hands-on sessions
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Thank you for your attention!

Questions?

Or/and contact us at kitware@kitware.fr or julien.fausty@kitware.com