IE531: Algorithms for Data Analytics
Spring, 2018
Homework 3: SVD and Related Topics
Due Date: March 10, 2020

Instructions

1. You can modify any of the code on Compass to solve these problems, if you want. It might help you with honing your programming skills.
2. You will submit a PDF-version of your answers on Compass on-or-before midnight of the due date.

Instructions

1. (*15 points*) Exercise 3.7 (Text).
2. (*15 points*) Exercise 3.13 (Text)
3. (*15 points*) Exercise 3.14 (Text)
4. (*15 points*) Exercise 3.16 (Text)
5. (*40 points*) Let $\mathbf{A}$ be an $n \times d$ matrix (of real numbers) that can be partitioned as

$$\underbrace{\mathbf{A}}_{n \times d} = \begin{pmatrix} \underbrace{\mathbf{A}_1}_{n_1 \times d_1} & \underbrace{\mathbf{A}_2}_{n_1 \times d_2} \\ \underbrace{\mathbf{A}_3}_{n_2 \times d_1} & \underbrace{\mathbf{A}_4}_{n_2 \times d_2} \end{pmatrix},$$

where $n = n_1 + n_2$ and $d = d_1 + d_2$ (obviously).

(a) (*30 points*) Show that

$$rank(\mathbf{A}) \leq rank(\mathbf{A}_1) + rank(\mathbf{A}_2) + rank(\mathbf{A}_3) + rank(\mathbf{A}_4)$$

(b) (*10 points*) Suppose for each $\{\mathbf{A}_i\}_{i=1}^4$ there is a corresponding set of matrices $\{\mathbf{B}_i\}_{i=1}^4$ such that $\forall i, \|\mathbf{A}_i - \mathbf{B}_i\|_F \leq \epsilon$ show that

$$\left\| \begin{pmatrix} \mathbf{B}_1 & \mathbf{B}_2 \\ \mathbf{B}_3 & \mathbf{B}_4 \end{pmatrix} \right\| \leq 4\epsilon$$