IE531 Homework 3

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Exercise 3.7

Given A has orthonormal rows:

Therefore, we have

Therefore:

Therefore

Columns of A are orthonormal.

Exercise 3.13

Frobenius norm for

2-norm for

Frobenius norm for

2 norm for

Exercise 3.14

Therefore, the left singular vector of is the right singular vector of and vice versa.

Since A is symmetric, and thus ,

According to singular value decompositions’ uniqueness (due to distinct singular values), therefore, .

Exercise 3.16

The first singular vector is the normalized column vector of

Therefore, we define

Q5.

1. A non-square matrix has rank

We can disassemble the four blocks into 4 matrices with one of the four blocks in it and other elements equal to 0.

All these 4 matrices have ranks no bigger than

Therefore, the equality only holds when such as the case of an identity matrix.

1. Due to the best rank-k approximations of singular vectors, we have , which is the projection of the rows of A onto the subspace spanned by the first k singular vectors of A.

We can treat as projection of on arbitrary subspace with dimension no bigger, with projection length of A no larger than .Since for any , , we can say that is the largest distance possible between and , we have the following equation holds for arbitrary A and split of A.

Therefore, the minimum value of is =.( can be in subspace spanned by singular vectors number less than K, which will be the same result, will merely change .

Therefore, we would have the following result:

And the equality holds when are best rank r approximation of (r is the rank of ).