Linear Stability Analysis of Plane Channel Flow

Part of the course Computational Fluid Mechanics II

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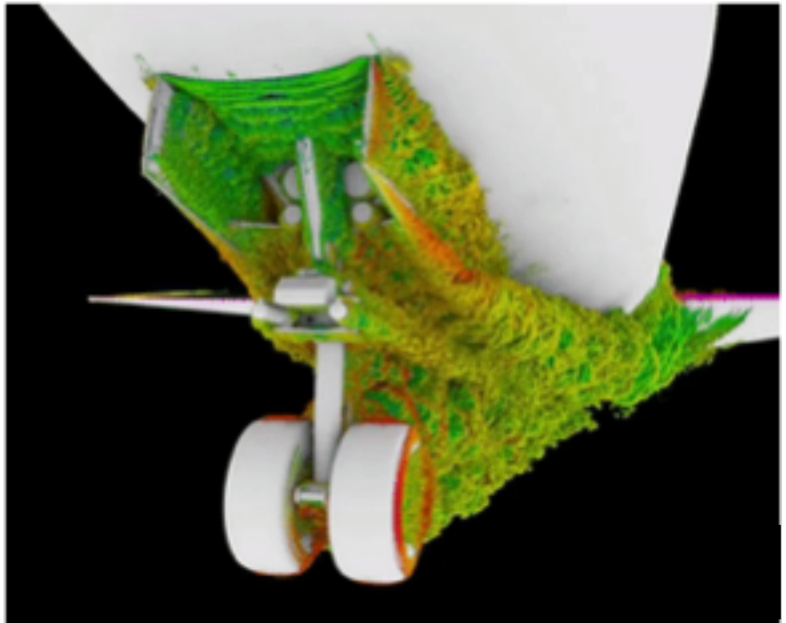
Institute for Hydrodynamics – Prof. Dr. Markus Uhlmann



Outline

1. Motivation
2. History & Theoretical Background
3. Assignment
4. Implementation
5. Difficulties
6. Results & Outlook

Motivation

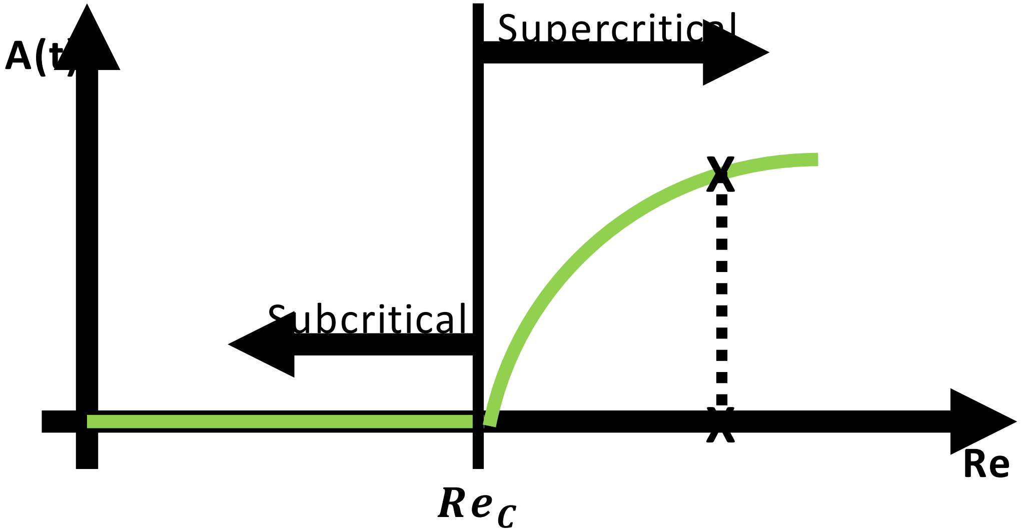
- OS Equations govern the stability of shear and related flows
- Climate Modelling:
  - Mid-latitude cyclone → high and low pressure regions
- Shear Flows in electrohydrodynamic (EHD)
  - Clutch development or high voltage generators
- Flow over an airfoil covered in de-icer

Motivation

"Linear growth mechanisms are the only responsible for disturbance growth in shear flows, where nonlinear terms redistribute energy and give zero net contribution when integrated over the whole control domain" – T. Ellingsen, E.Palm

History & Theoretical Background

- Reynolds (1883)



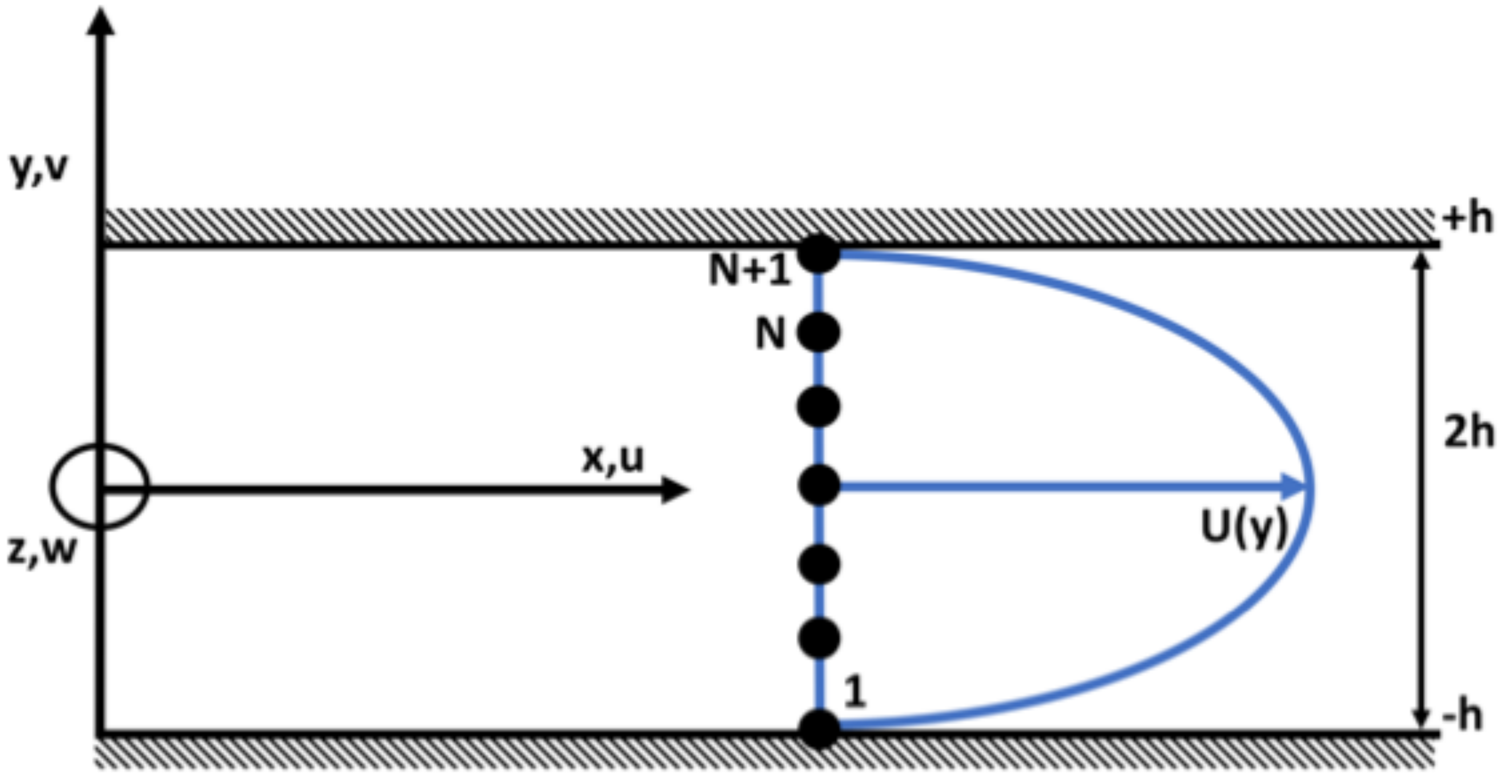
- G.I. Taylor vs. Prandtl-Tollmien-Schlichting → Schubauer & Skramstad experiment HTS waves

Assignment

Laminar Flow Solution $(\frac{dp}{dx} = const)$:

$$U(y) = 1 - y^2$$

$$U(i) = U_0 * (1 - (y(i))^2)$$



Boundary Conditions:

- No-slip
  → f(-1) = f(1) = 0
- No-Penetration
  → f'(-1) = f'(1) = 0

Squire's theorem

To obtain the minimum critical Reynolds number it is sufficient to consider only two-dimensional disturbances!

Assignment

$$\nabla \cdot \boldsymbol{u} = 0$$

$$\partial_t \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} + \nabla p = \frac{1}{Re} \nabla^2 \boldsymbol{u}$$

To – Do:

- Introduce mode ansatz
- Discretize
- Matrix-Vector Form $\bar{\bar{\boldsymbol{A}}}\boldsymbol{f} = c\bar{\bar{\boldsymbol{B}}}\boldsymbol{f}$

Orr-Sommerfeld Equation

$$k^4 f - 2k^2 \frac{d^2 f}{dy^2} + \frac{d^4 f}{dy^4} = i\nu k \left((U(y) - c) \left(\frac{d^2 f}{dy^2} - k^2 f \right) - \frac{d^2 U}{dy^2} f \right)$$

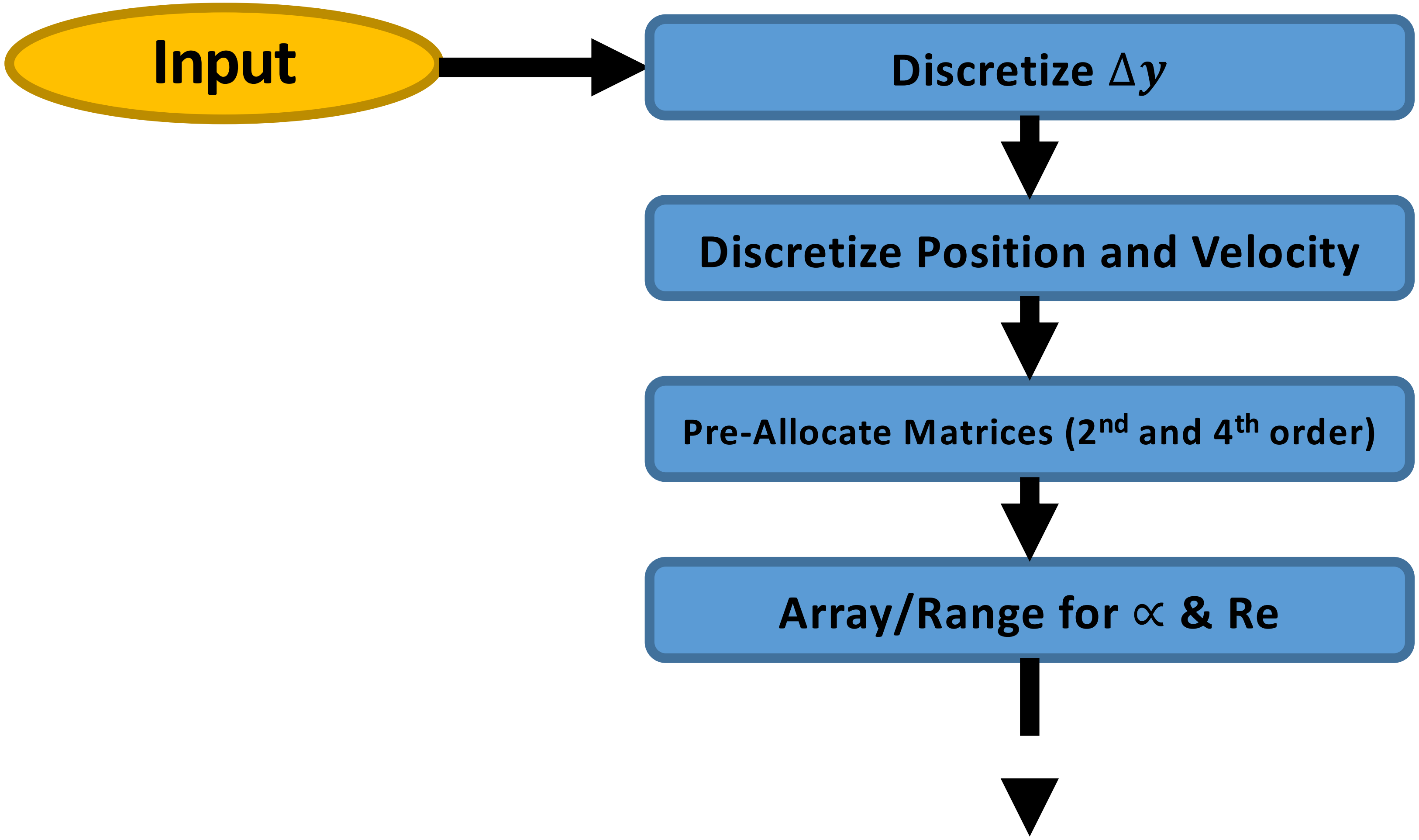
$$i\nu k \left(-2 \frac{d^2 f}{dy^2} + k^2 f \right) + \frac{i\nu}{k} \frac{d^4 f}{dy^4} + U(y) \left(\frac{d^2 f}{dy^2} - k^2 f \right) - \frac{d^2 U}{dy^2} f = c \left(\frac{d^2 f}{dy^2} - k^2 f \right)$$

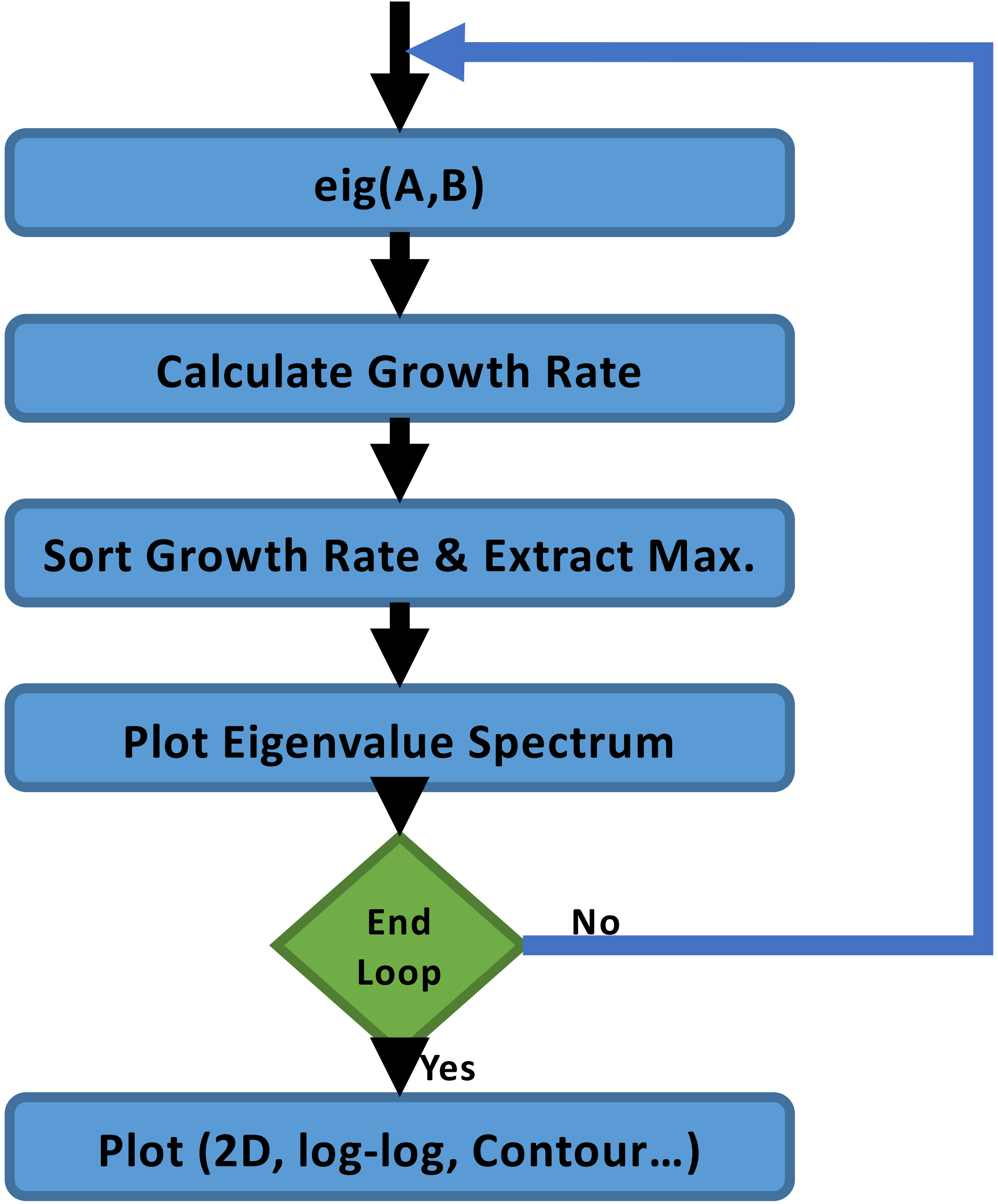
Finite-Difference Schemes

$$\left.\frac{\partial^2 f}{\partial y^2}\right|_{y_i} = \frac{f_{i-1} - 2f_i - f_{i+1}}{\Delta y^2} - \frac{\Delta y^2}{12}\left.\frac{\partial^4 f}{\partial y^4}\right|_{f_i} + \ldots$$

$$\left.\frac{\partial^4 f}{\partial y^4}\right|_{y_i} = \frac{f_{i-2} - 4f_{i-1} + 6f_i - 4f_{i+1} + f_{i+2}}{\Delta y^4} - \frac{\Delta y^2}{6}\left.\frac{\partial^4 f}{\partial y^6}\right|_{y_i} + \ldots$$

Implementation (Flow Chart)




eig(A,B)
Calculate Growth Rate
Sort Growth Rate & Extract Max.
Plot Eigenvalue Spectrum
End Loop
No
Yes
Plot (2D, log-log, Contour…)

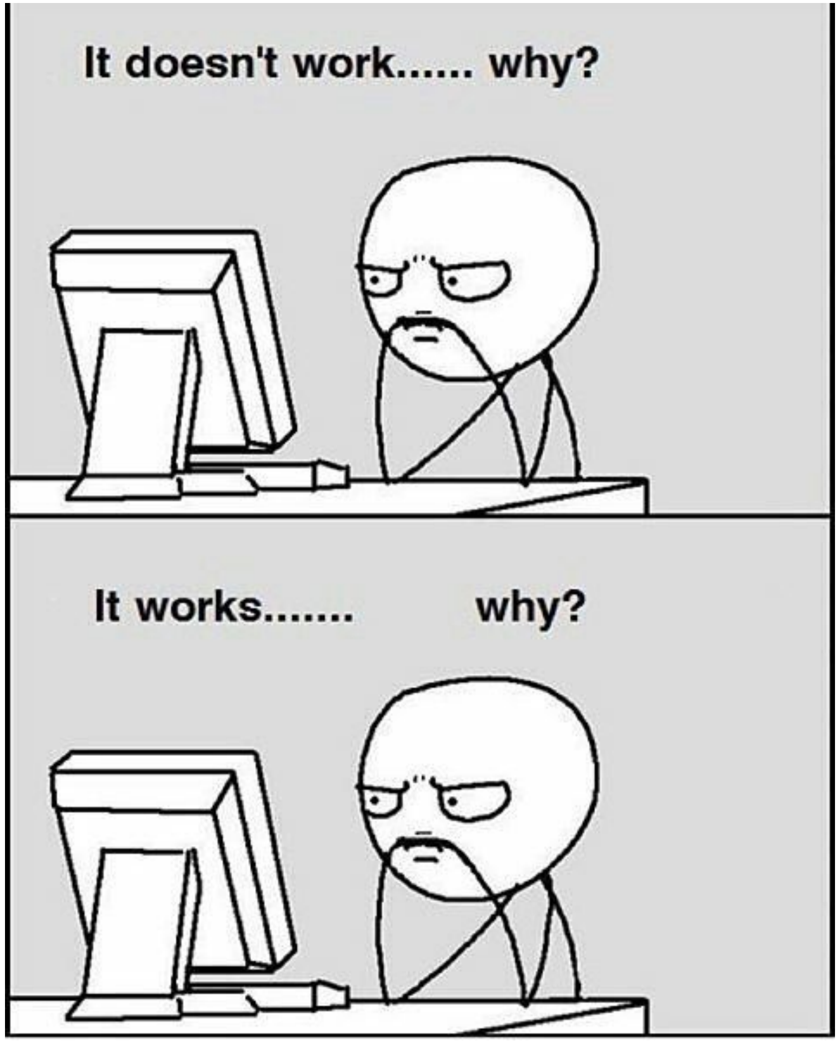
Implementation

$$iνk\left(-2\frac{d^2 f}{dy^2} + k^2 f\right) + \frac{iν}{k}\frac{d^4 f}{dy^4} + U(y)\left(\frac{d^2 f}{dy^2} - k^2 f\right) - \frac{d^2 U}{dy^2} f = c\left(\frac{d^2 f}{dy^2} - k^2 f\right)$$

$$\bar{\bar{\boldsymbol{A}}}\boldsymbol{f} = c\bar{\bar{\boldsymbol{B}}}\boldsymbol{f}$$

Difficulties

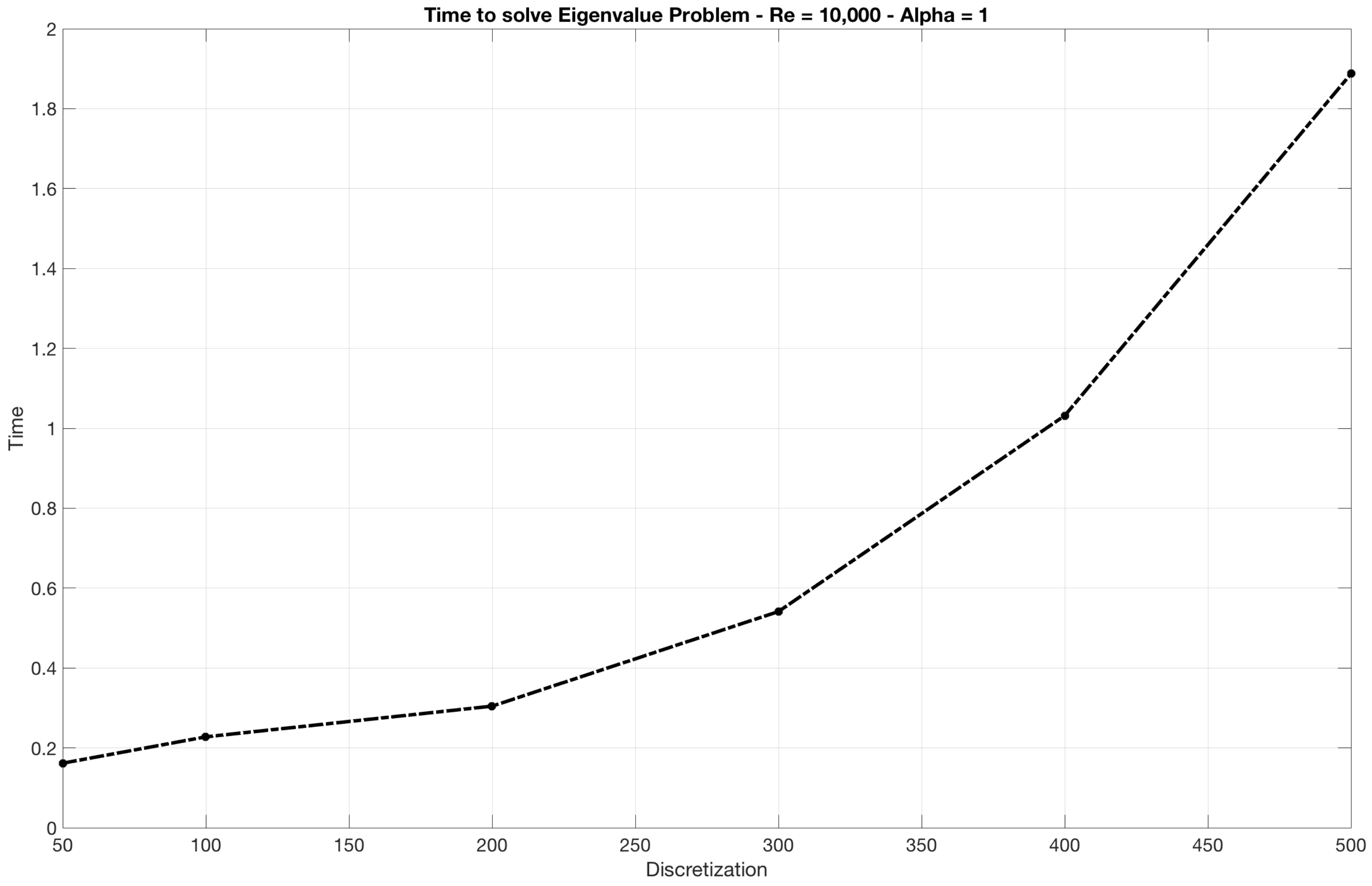
- Debugging tricky
- Convergence Study & Plots



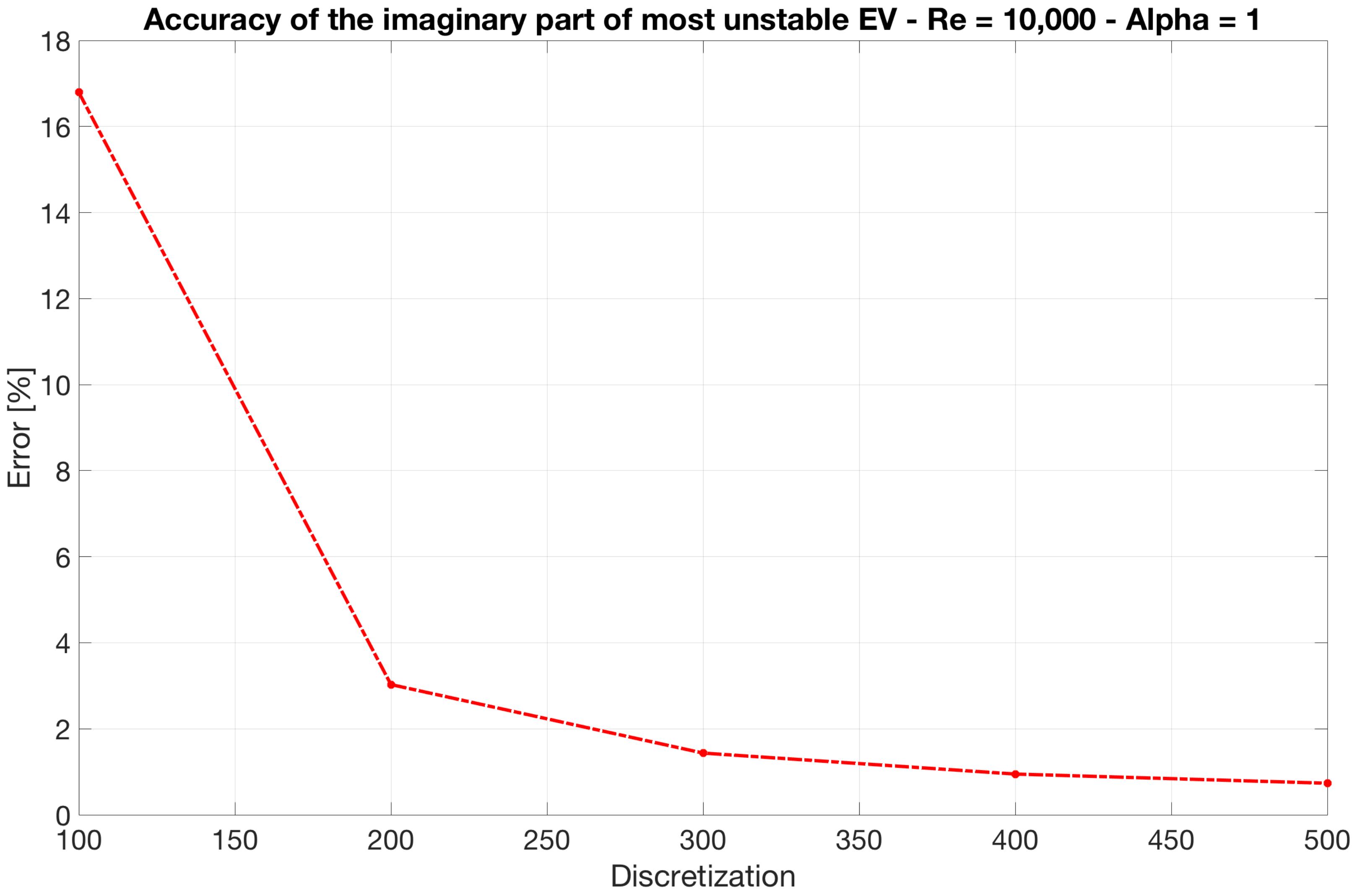
Results

- Time to solve Eigenvalue problem
- Accuracy of imaginary Part
- Convergence Study
- Eigenvalue Spectrum
- Stability

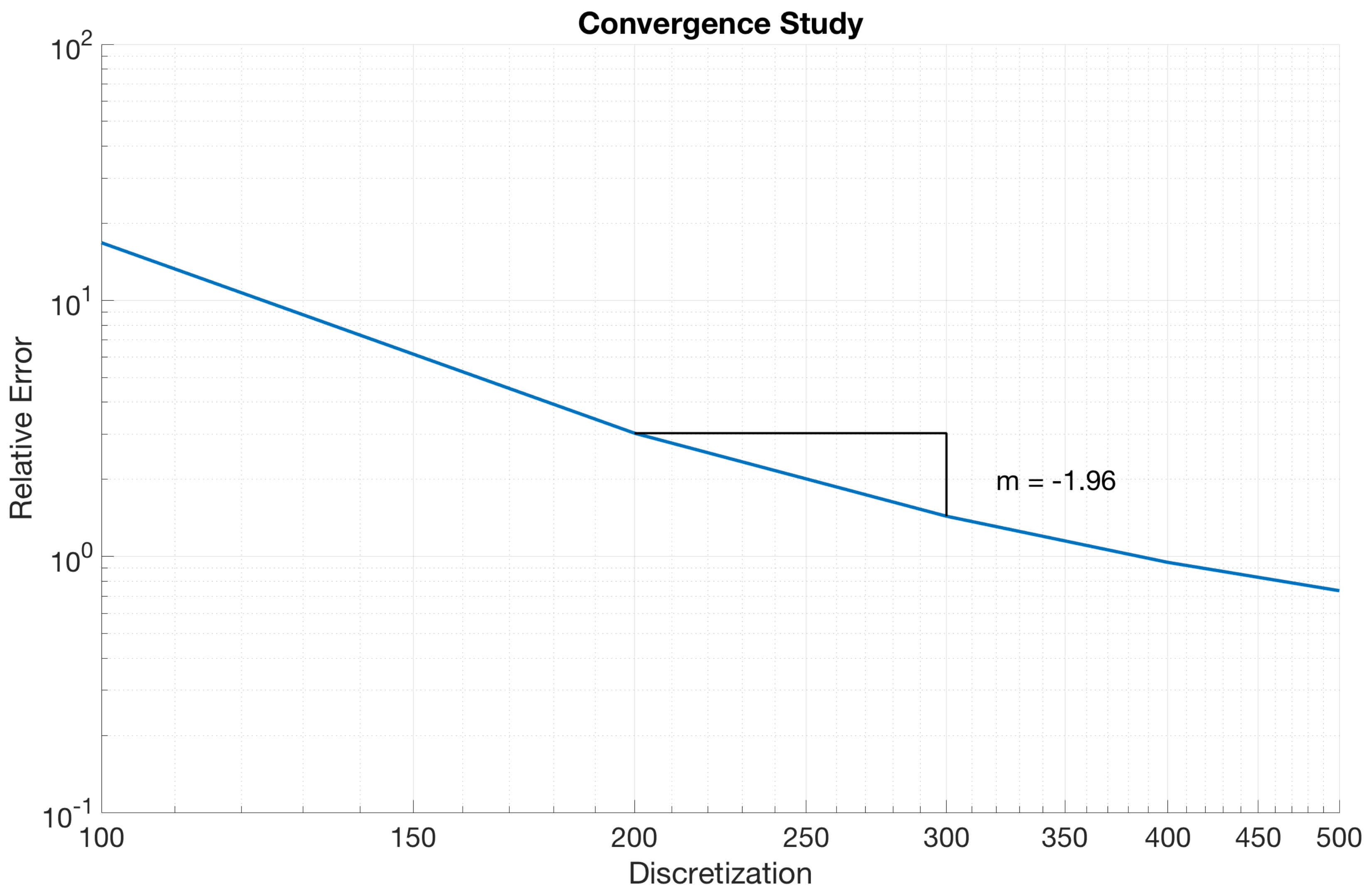
Time to solve Eigenvalue Spectrum



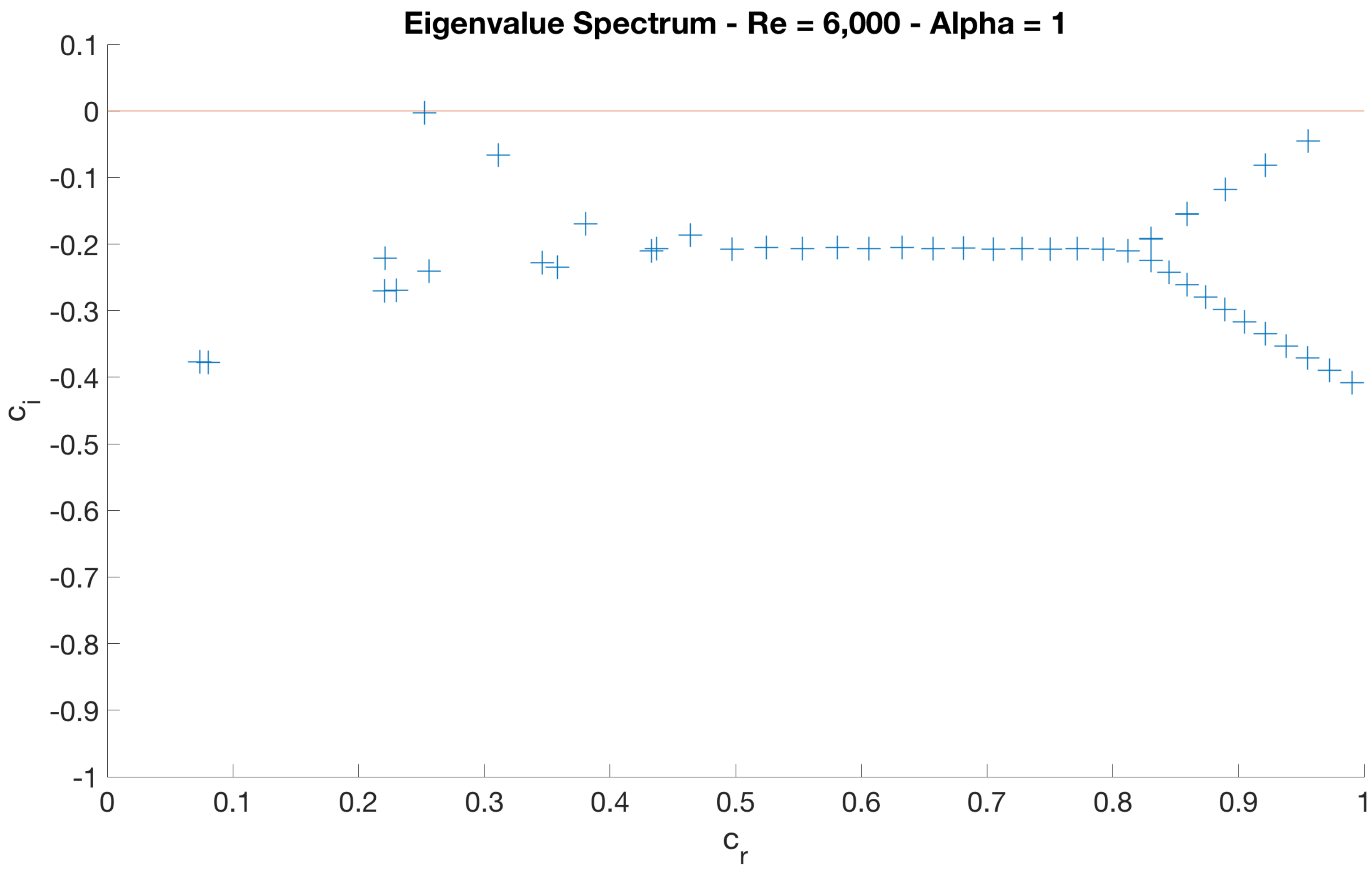
Accuracy of imaginary part



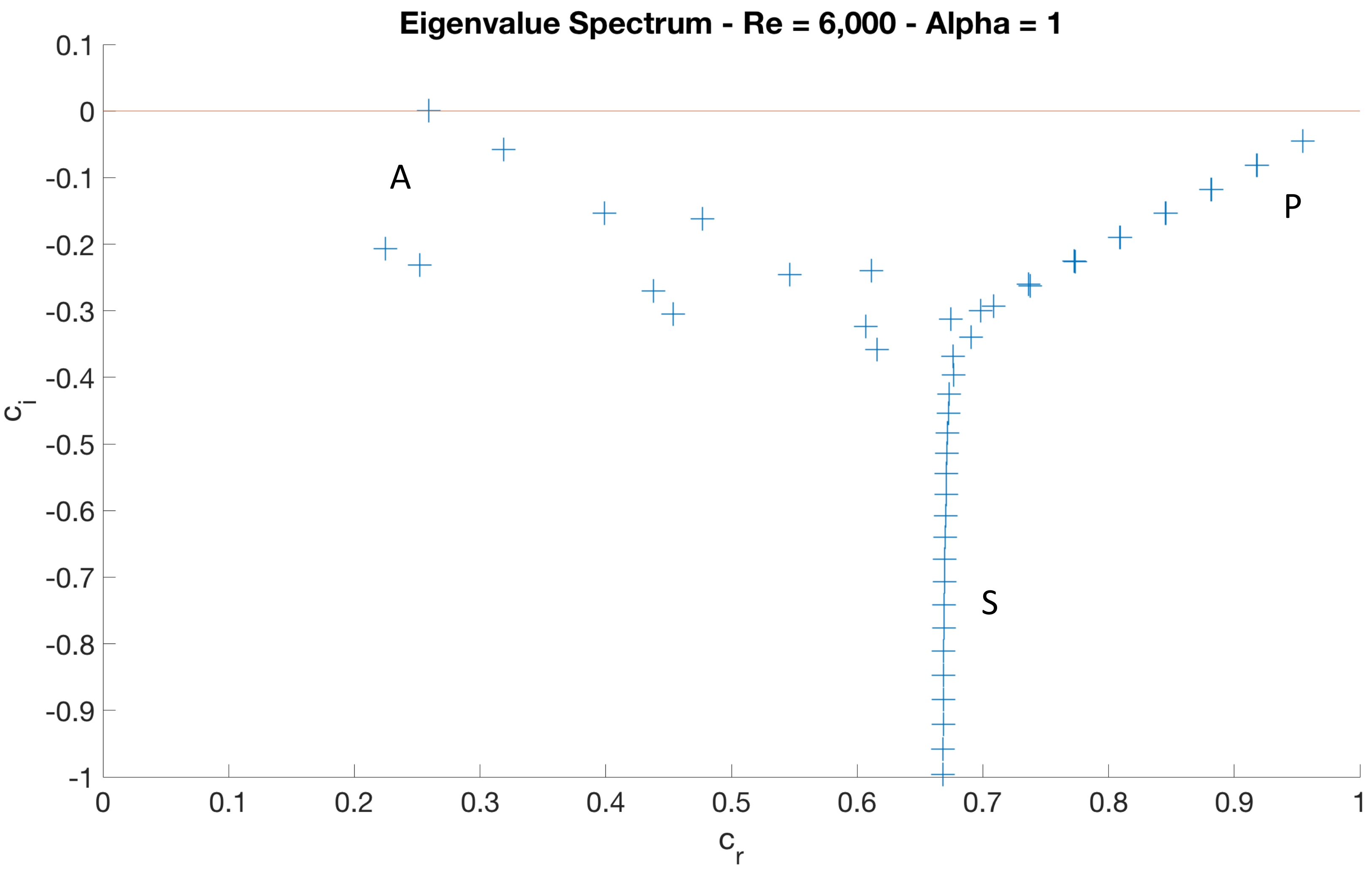
Convergence Study



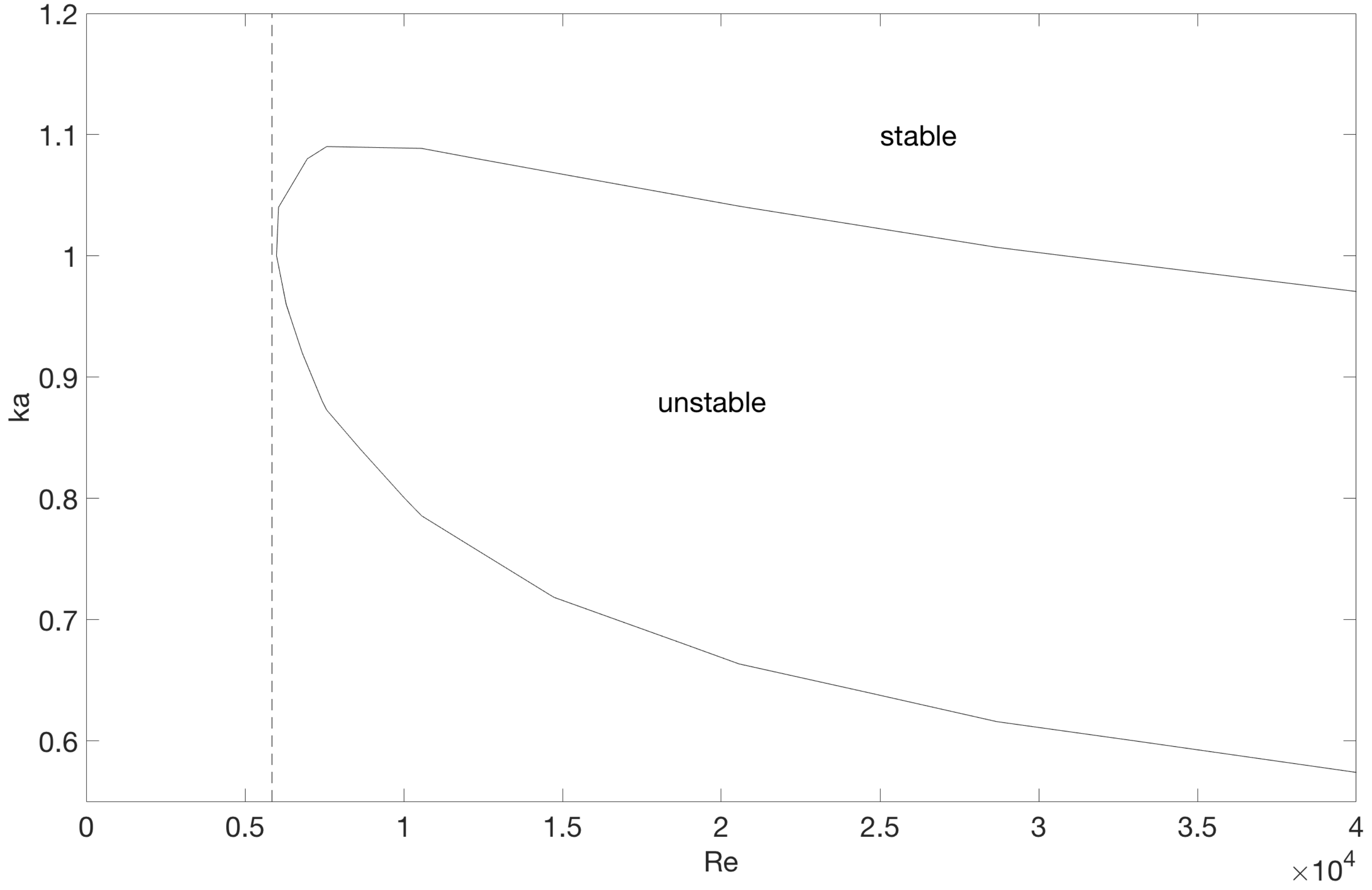
Eigenvalue Spectrum N = 50



Eigenvalue Spectrum N = 200



Stability Contour Plot (Re_c = 5850)



Outlook

- Include spanwise wave number β
- 2D → 3D
- Optimal initial conditions
- Non-Uniform Grid & Optimal stretching factor (Asai & Nakasuji)
- Transient Growth

References

- NASA Video: https://www.youtube.com/watch?v=-D5N_OnZ_Tg

Source Code

- Code and Documentation available at GitHub: https://github.com/jousefm/

Questions?